From vacuum nonsingular black hole to variable cosmological constant

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In the spherically symmetric case the dominant energy condition, together with the requirement of regularity at the center, asymptotic flatness and finiteness of the ADM mass, defines the family of asymptotically flat globally regular solutions to the Einstein minimally coupled equations which includes the class of metrics asymptotically de Sitter as $r \to 0$ and asymptotically Schwarzschild as $r \to \infty$. A source term connects smoothly de Sitter vacuum in the origin with the Minkowski vacuum at infinity and corresponds to anisotropic vacuum defined macroscopically by the algebraic structure of its stress-energy tensor invariant under boosts in the radial direction. In the range of masses $m \geq m_{crit}$, de Sitter-Schwarzschild geometry describes a vacuum nonsingular black hole, and for $m < m_{crit}$ it describes G-lump which is a vacuum self-gravitating particle-like structure without horizons. Quantum energy spectrum of G-lump is shifted down by the binding energy, and zero-point vacuum mode is fixed at the value corresponding to the Hawking temperature from the de Sitter horizon. Space-time symmetry changes smoothly from the de Sitter group near the center to the Lorentz group at infinity and the standard formula for the ADM mass relates it to de Sitter vacuum replacing a central singularity at the scale of symmetry restoration. This class of metrics is easily extended to the case of nonzero cosmological constant at infinity. The source term connects then smoothly two de Sitter vacua which makes possible to relate a spherically symmetric anisotropic vacuum with an $r-$dependent cosmological term $\Lambda_{\mu\nu}$.

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I. INTRODUCTION

Confrontation of models with observations in cosmology as well as the inflationary paradigm, compellingly require treating a cosmological constant as a variable dynamical quantity. Big value of the cosmological constant at the very early stage of the Universe evolution is needed to explain the reason for expansion [1] as well as puzzles of the standard hot big bang model [2,3]. The key cosmological parameter to decide if cosmological constant is zero or not today, is the product of the Hubble parameter and the age of the Universe Ht. In the standard cosmology without cosmological constant this product never exceeds the unity, but it is possible in the presence of a nonzero cosmological constant [3]. Therefore, if the Hubble parameter and the age of the Universe are found in observations to satisfy the bound $Ht > 1$, it requires a term in the expansion rate equation that acts as a cosmological constant [4]. With taking into account uncertainties in models the best fit to achieve consensus between observational constraints is [4–6]

$$H = (70-80) km\ s^{-1} Mpc^{-1}, \quad t = [(13-16) \pm 3] Gy,$$

$$\Omega_{matter} = 0.3 - 0.4, \quad \Omega_\Lambda = 0.6 - 0.7,$$

where $\Omega = \rho_{today}/\rho_{crit}$, and the critical density ρ_{crit} corresponds to $\Omega = 1$.

A cosmological term was introduced by Einstein in 1917 into his equations describing gravity as space-time geometry (G-field) generated by matter

$$G_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{1}$$

to make them consistent with Mach's principle which was one of his primary motivations [7]. Einstein's formulation of Mach's principle was that some matter has the property of inertia only because there exists also some other matter in the Universe ([8], Ch.2). When Einstein found that Minkowski geometry is the regular solution to (1) perfectly describing inertial motion in the absence of any matter, he modified his equations by adding the cosmological term $\Lambda g_{\mu\nu}$ in the hope that modified equations

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \tag{2}$$

will have reasonable regular solutions only when matter is present - if matter is the source of inertia, then in case of its absence there should not be any inertia [9]. The primary task of Λ was thus to eliminate inertia in case when matter is absent by eliminating regular G-field solutions in case when $T_{\mu\nu} = 0$.

Soon after introducing $\Lambda g_{\mu\nu}$, de Sitter found quite reasonable solution with $\Lambda g_{\mu\nu}$ and without $T_{\mu\nu}$ [10],

$$ds^2 = \left(1 - \frac{\Lambda}{3} r_2\right) dt^2 - \frac{dr^2}{\left(1 - \frac{\Lambda}{3} r_2\right)} - r^2 d\Omega^2 \tag{3}$$

whose nowadays triumphs are well known [2,3].

In de Sitter geometry Λ must be constant by virtue of the Bianchi identities $G^{\mu\nu}_{;\nu} = 0$. It plays the role of a universal repulsion whose physical sense remained obscure during several decades when de Sitter metric has been mainly used in quantum field theory as a simple testing ground for developing the quantum field technics in curved space-time.

Almost fifty years later, in 1965, two papers appeared in the same issue of the Soviet Physics JETP, which shed some light on the physical nature of the de Sitter geometry. The first was the paper by Sakharov [11], in which he suggested that gravitational effects dominate the equation of state of a cold baryon-lepton superdense matter at densities $\rho \sim 10^{74} g\ cm^{-3}$ (GUT density is of order $\rho \sim 10^{77} g\ cm^{-3}$ for GUT scale $\sim 10^{15}$GeV) and that one of possible equations of state in such a regime could be

$$p = -\rho \tag{4}$$

formally corresponding to the equation of state for $\Lambda g_{\mu\nu}$ shifted to the right hand side of the Einstein equation (2) as some stress-energy tensor. The physical sense of this operation has been clarified in the second 1965 paper by Gliner [12] who interpreted $\Lambda g_{\mu\nu}$ as corresponding to a stress-energy tensor of a superdense vacuum

$$T^{\mu\nu}_{vac} = (8\pi G)^{-1} \Lambda g_{\mu\nu} \tag{5}$$

In the Petrov classification scheme [13] stress-energy tensors are classified on the basis of their algebraic structure. When the elementary divisors of the matrix $T_{\mu\nu} - \lambda g_{\mu\nu}$ are real, the eigenvectors of $T_{\mu\nu}$ are non-isotropic and form a comoving reference frame with the timelike vector representing a velocity. A comoving reference frame is defined uniquely if and only if none of the spacelike eigenvectors $\lambda_a (a = 1, 2, 3)$ coincides with a timelike eigenvalue λ_0. A stress-energy tensor (5) with all eigenvalues equal, has an infinite set of comoving reference frames and hence no preferred one. An observer moving through de Sitter vacuum (5) cannot in principle measure his velocity with respect to it, since an observer's comoving frame is also comoving for (5) [12]. Gliner suggested that at superhigh densities a continual medium is formed with attraction between its elements, which is phenomenologically described by stress-energy tensor (5) with the negative pressure and the equation of state (4). The other very important hypothesis suggested by Gliner in this paper was that such a state could be achieved in a gravitational collapse [12].

In 1967 De Witt found that quantum effects in one-loop approximation lead to the vacuum stress-energy tensor of a form [14]

$$< 0|T_{\mu\nu}|0 >= \rho_{vac} g_{\mu\nu} \tag{6}$$

In 1968 Zel'dovich proposed to relate $\Lambda g_{\mu\nu}$ to gravitational interaction of virtual particles in vacuum [15].

In 80-s several attempts have been made to eliminate a black hole singularity by replacing it at the Planck scale curvature with de Sitter metric using direct matching of Schwarzschild metric outside to de Sitter metric inside a short spacelike transitional layer of the Planckian depth [16–20]. The matched solutions typically have a jump at the junction surface which comes from singularity of a tangential pressure there. The situation with de Sitter-Schwarzschild transition has been analyzed by Poisson and Israel who suggested to introduce a transitional layer of "noninflationary material" of uncertain depth in which geometry can be self-regulatory and describable semiclassically down to a few Planckian radii by the Einstein equations with a source term representing vacuum polarization effects [21].

Generic properties of "noninflationary material" have been considered in Ref. [22] for the case of a smooth transition from de Sitter vacuum at the origin to Minkowski vacuum at infinity, and the exact analytical solution has been found describing a vacuum nonsingular black hole in a simple semiclassical model for vacuum polarization in the gravitational field. In the course of Hawking evaporation such a black hole evolves towards a self-gravitating particle-like vacuum structure without horizons [23], kind of gravitational vacuum soliton called G-lump [24].

Model-independent analysis of the Einstein spherically symmetric minimally coupled equations has shown [24] which kind of geometry they can describe in principle (no matter which matter source is responsible for a stress-energy tensor) if certain general requirements are satisfied:

a) regularity of metric and density at the center;

b) asymptotic flatness at infinity and finiteness of the ADM mass;

c) dominant energy condition for $T_{\mu\nu}$.

The requirements (a)-(c) define the family of asymptotically flat solutions with the regular center which includes the class of metrics asymptotically de Sitter as $r \to 0$ and asymptotically Schwarzschild as $r \to \infty$. A source term belongs to the class of stress-energy tensors invariant under boosts in the radial direction and connects de Sitter vacuum in the origin with the Minkowskli vacuum at infinity. Space-time symmetry changes smoothly from de Sitter group at the center to the Lorentz group at infinity through the radial boosts in between, and the standard formula for the ADM mass relates it (generically, since a matter source can be any from considered class) to both de Sitter vacuum replacing a singularity and breaking of space-time symmetry.

This class of metrics can be extended to the case of non-zero cosmological term at infinity [25] corresponding to extension of the Einstein cosmological term $\Lambda g_{\mu\nu}$ to an $r-$dependent second rank symmetric tensor $\Lambda_{\mu\nu}$ [26] connecting in a smooth way two de Sitter vacua with different values of a cosmological constant.

This talk is organized as follows. In Section II we outline de Sitter-Schwarzschild geometry, and in Section III its extension to the case of non-zero cosmological constant at infinity. In Section IV we present variable cosmological term $\Lambda_{\mu\nu}$ and quantum energy spectrum of G-lump. In Section V we outline the results concerning connection between the ADM mass and cosmological term.

Section VI contains discussion.

II. DE SITTER-SCHWARZSCHILD GEOMETRY

A static spherically symmetric line element can be written in the standard form (see, e.g., [27], p.239)

$$ds^2 = e^{\mu(r)}dt^2 - e^{\nu(r)}dr^2 - r^2 d\Omega^2 \tag{7}$$

where $d\Omega^2$ is the metric of a unit 2-sphere.

The Einstein equations (1) reduce to ([27], p.244)

$$8\pi G T_t^t = 8\pi G\rho(r) = e^{-\nu}\left(\frac{\nu'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \tag{8}$$

$$8\pi G T_r^r = -8\pi G p_r(r) = -e^{-\nu}\left(\frac{\mu'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2} \tag{9}$$

$$8\pi G T_\theta^\theta = 8\pi G T_\phi^\phi = -8\pi G p_\perp(r) =$$

$$-e^{-\nu}\left(\frac{\mu''}{2} + \frac{\mu'^2}{4} + \frac{(\mu' - \nu')}{2r} - \frac{\mu'\nu'}{4}\right) \tag{10}$$

Here $\rho(r) = T_t^t$ is the energy density (we adopted $c = 1$ for simplicity), $p_r(r) = -T_r^r$ is the radial pressure, and $p_\perp(r) = -T_\theta^\theta = -T_\phi^\phi$ is the tangential pressure for anisotropic perfect fluid ([27], p.243). A prime denotes differentiation with respect to r.

Integration of Eq.(8) gives [29]

$$e^{-\nu(r)} = 1 - \frac{2GM(r)}{r}; \quad M(r) = 4\pi\int_0^r \rho(x)x^2 dx \tag{11}$$

which has for large r the Schwarzschild asymptotic $e^{-\nu} = 1 - 2Gm/r$, where the mass parameter m is given by

$$m = 4\pi\int_0^\infty \rho(r)r^2 dr \tag{12}$$

Analysis of this system in the case when requirements (a)-(c) are satisfied leads to the following results [24]:

The dominant energy condition $T^{00} \geq |T^{ab}|$ for each $a, b = 1, 2, 3$, which holds if and only if [28]

$$\rho \geq 0; \quad -\rho \leq p_k \leq \rho; \quad k = 1, 2, 3 \tag{13}$$

implies that the local energy density is non-negative and each principal pressure never exceeds the energy density. In the limit $r \to \infty$ the condition of finiteness of the mass (12) requires density profile $\rho(r)$ to vanish at infinity quicker than r^{-3}, and the dominant energy condition (13) requires both radial and tangential pressures to vanish as $r \to \infty$. Then $\mu' = 0$ and μ =const at infinity, and the standard boundary condition $\mu \to 0$ as $r \to \infty$ leads to asymptotic flatness needed to identify (12) as the ADM mass [29]. As a result we get the Schwarzschild asymptotic at infinity

$$T_{\mu\nu} = 0; \; ds^2 = \left(1 - \frac{2Gm}{r}\right) - \frac{dr^2}{\left(1 - \frac{2Gm}{r}\right)} - r^2 d\Omega^2 \tag{14}$$

From Eq.(8)-(10) we derive the equation (see also [30])

$$p_\perp = p_r + \frac{r}{2}p_r' + (\rho + p_r)\frac{GM(r) + 4\pi G r^3 p_r}{2(r - 2GM(r))} \tag{15}$$

which is generalization of the Tolman-Oppenheimer-Volkoff equation ([29], p.127) to the case of different principal pressures, and the equation [31]

$$T_t^t - T_r^r = p_r + \rho = \frac{1}{8\pi G}\frac{e^{-\nu}}{r}(\nu' + \mu') \tag{16}$$

From Eq.(11) it follows that for any regular value of $e^{\nu(r)}$ $M(r) = 0$ at $r = 0$ and thus $\nu(r) \to 0$ as $r \to 0$ [31]. The dominant energy condition allows us to fix asymptotic behavior of a mass function and of a metric at approaching the regular center. Requirement of regularity of density $\rho(r = 0) < \infty$, leads, by Eq. (13), to regularity of pressures. Requirement of regularity of the metric, $e^{\nu(r)} < \infty$, leads then, by (16), to $\nu' + \mu' = 0$ and $\nu + \mu = \mu(0)$ at $r = 0$ with $\mu(0)$ playing the role of the family parameter.

The weak energy condition, $T_{\mu\nu}\xi^\mu\xi^\nu \geq 0$ for any timelike vector ξ^μ, which is satisfied if and only if $\rho \geq 0; \rho + p_k \geq 0, k = 1, 2, 3$ and which is contained in the dominant energy condition [28], defines, by Eq.(16), the sign of the sum $\mu' + \nu'$. In the case when $e^{\nu(r)} > 0$ everywhere, it demands $\mu' + \nu' \geq 0$ everywhere. In case when $e^{\nu(r)}$ changes sign, the function $T_t^t - T_r^r$ is zero, by Eq.(16), at the horizons where $e^{-\nu} = 0$. In the regions inside the horizons, the radial coordinate r is timelike and T_t^t represents a tension, $p_r = -T_t^t$, along the axes of the spacelike 3-cylinders of constant time r=const [21], then $T_t^t - T_r^r = -(p_r + \rho)$, and the weak energy condition still demands $\nu' + \mu' \geq 0$ there. As a result the function $\mu + \nu$ is a function growing from $\mu = \mu(0)$ at $r = 0$ to $\mu = 0$ at $r \to \infty$, which gives $\mu(0) \leq 0$.

The well known example of solution from this family is boson stars [32] (for review [33]).

The range of family parameter $\mu(0)$ dictated by the weak energy condition, includes the value $\mu(0) = 0$, which corresponds to $\nu + \mu = 0$ at the center. In this case the function $\phi(r) = \nu(r) + \mu(r)$ is zero at $r = 0$ and at $r \to \infty$, its derivative is non-negative, it follows that $\phi(r) = 0$, i.e., $\nu(r) = -\mu(r)$ everywhere. The weak energy condition defines also equation of state and thus asymptotic behavior as $r \to 0$. The function $\phi(r) = \mu(r) + \nu(r)$, which is equal zero everywhere for $0 \leq r < \infty$, cannot have extremum at $r = 0$, therefore $\mu'' + \nu'' = 0$ at $r = 0$ (this is easily proved by contradiction using the Maclaurin rule for even derivatives in the extremum). It leads, by using L'Hopital rule in Eq.(16), to $p_r + \rho = 0$ at $r = 0$. In the limit $r \to 0$ Eq.(15) becomes $p_\perp = -\rho - \frac{r}{2}\rho'$. The energy dominant condition (13) requires $\rho' \leq 0$, while regularity of ρ requires $p_k + \rho < \infty$ and thus $|\rho'| < \infty$. Then the equation of state near the center becomes $p = -\rho$, which gives de Sitter asymptotic (3) as $r \to 0$ [24].

Summarizing, we conclude that if we require asymptotic flatness, regularity of a density and metric at the center and finiteness of the ADM mass, then the dominant energy condition defines the family of asymptotically flat solutions with the regular center which includes the class of metrics

$$e^{\mu(r)} = e^{-\nu(r)} = g(r) = 1 - 2GM(r)/r \qquad (19)$$

with $M(r)$ given by Eq.(11), whose behavior in the origin - asymptotically de Sitter as $r \to 0$, is dictated by the weak energy condition.

For this class a source term has the algebraic structure

$$T_t^t = T_r^r : \quad T_\theta^\theta = T_\phi^\phi \qquad (20)$$

and the equation of state is

$$p_r = -\rho : \quad p_\perp = -\rho - (r/2)\rho' \qquad (21)$$

It connects de Sitter vacuum $T_{\mu\nu} = \rho_0 g_{\mu\nu}$ in the origin with the Minkowski vacuum $T_{\mu\nu} = 0$ at infinity, and generates de Sitter-Schwarzschild geometry [23] asymptotically de Sitter as $r \to 0$ and asymptotically Schwarzschild as $r \to \infty$.

Note, that if we postulate regularity also for pressures, then the weak energy condition is enough to distinguish the class of metrics (19) [24].

The weak energy condition $p_\perp + \rho \geq 0$ gives $\rho' \leq 0$, so that it demands monotonic decreasing of a density profile. By Eq.(10) it leads to the important fact that, except the point $r = 0$ where $g(r)$ has the maximum, in any other extremum $g'' > 0$, so that the function $g(r)$ has in the region $0 < r < \infty$ only minimum and the metric (19) can have not more than two horizons [24].

To find explicit form of $M(r)$ we have to choose some density profile leading to the needed behavior of $M(r)$ as $r \to 0$, $M(r) \simeq (4\pi/3)\rho_0 r^3$. The simplest choice [22]

$$\rho(r) = \rho_0 e^{-r^3/r_0^2 r_g}; \quad r_0^2 = 3/\Lambda; \quad r_g = 2Gm \qquad (22)$$

can be interpreted [23] as due to vacuum polarization in the spherically symmetric gravitational field as described semiclassically by the Schwinger formula $w \sim e^{-F_{crit}/F}$ (see, e.g., [34]) with tidal forces $F \sim r_g/r^3$ and $F_{crit} \sim 1/r_0^2$, in agreement with the basic idea suggested by Poisson and Israel that in Schwarzschild-de Sitter transition space-time geometry can be self-regulatory as a result of vacuum polarization effects [21].

The key point is the existence of two horizons, a black hole event horizon r_+ and an internal horizon r_-. A critical value of a mass parameter exists, m_{crit}, at which the horizons come together and which puts a lower limit on a black hole mass [23]. For the model (22)

$$m_{crit} \simeq 0.3 m_{Pl} \sqrt{\rho_{Pl}/\rho_0} \qquad (23)$$

De Sitter-Schwarzschild configurations are shown in Fig.1.

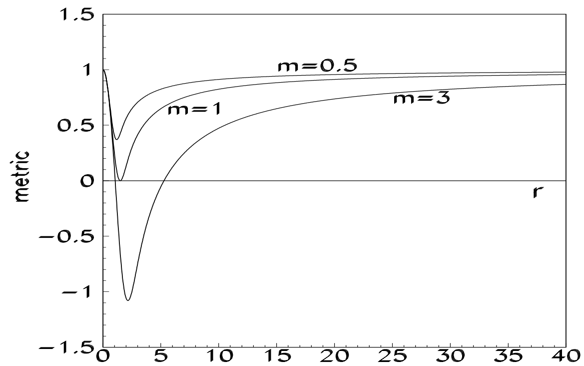

FIG. 1. The metric $g(r)$ for de Sitter-Schwarzschild configurations plotted for the case of the density profile (22). The mass parameter m is normalized to m_{crit}.

For $m \geq m_{crit}$ de Sitter-Schwarzschild geometry describes the vacuum nonsingular black hole (ΛBH) [22], and global structure of space-time, shown in Fig.2 [23], contains an infinite sequence of black and white holes whose future and past singularities are replaced with regular cores $\mathcal{RC}$ asymptotically de Sitter as $r \to 0$.

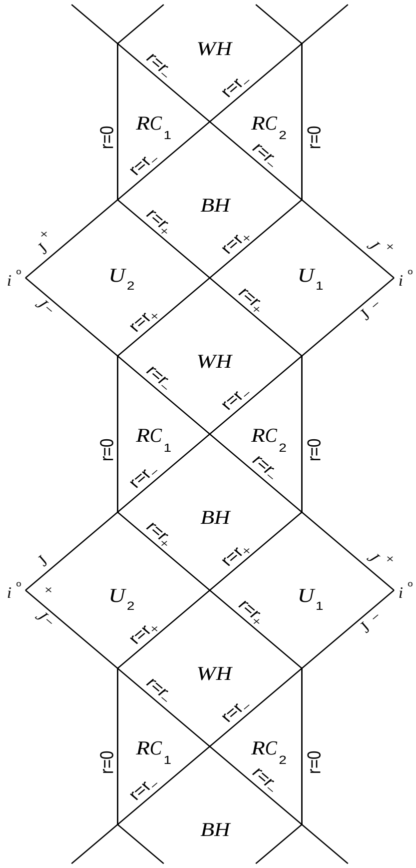

FIG. 2. Penrose-Carter diagram for Λ black hole.

A ΛBH emits Hawking radiation from both black hole and cosmological horizons with the Gibbons-Hawking temperature $T = \hbar\kappa(2\pi kc)^{-1}$ [35] where κ is the surface gravity and k is the Boltzmann constant. For a ΛBH the temperature from horizons is given by [23]

$$T_h = \frac{\hbar G}{2\pi ckr_0}\left(\frac{M(r_h)}{r_h^2} - \frac{M'(r_h)}{r_h}\right) \tag{24}$$

In the limit $r_g/r_0 \gg 1$, the temperature tends to the Schwarzschild value $T_{Schw} = \hbar c^3/8\pi Gkm$ on the black hole horizon and to the de Sitter value $T_{DeS} = -\hbar c/2\pi kr_0$ on the internal horizon.While a ΛBH loses its mass, horizons come together, and configuration evolves towards a self-gravitating particle-like structure without horizons [23]. Temperature-mass diagram is shown in Fig.3. Its form is generic for de Sitter-Schwarzschild geometry and does not depend on particular form of a density profile. The temperature T_+ on BH horizon r_+ is positive by general laws of BH thermodynamics [29]. It drops to zero at $m = m_{crit}$, while the Schwarzschild asymptotic requires $T_+ \to 0$ as $m \to \infty$. As a result the temperature-mass diagram should have a maximum between m_{crit} and $m \to \infty$ [23]. In a maximum a specific heat is broken and changes sign testifying to a second-order phase transition in the course of Hawking evaporation and suggesting symmetry restoration to the de Sitter group in the origin [36].

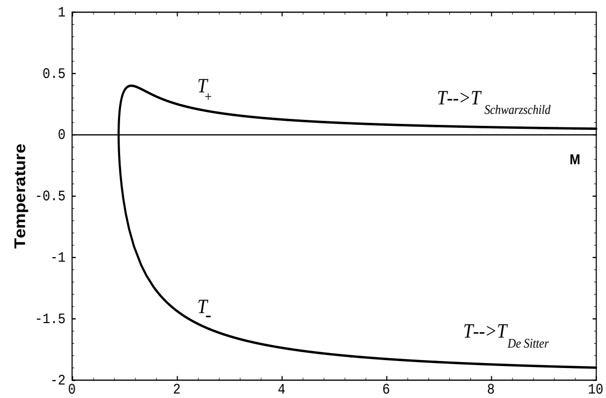

FIG. 3. Temperature-mass diagram for Λ black hole.

For masses $m < m_{crit}$ de Sitter-Schwarzschild geometry describes a self-gravitating particle-like vacuum structure without horizons, globally regular and globally neutral. It resembles Coleman's lumps - non-singular, non-dissipative solutions of finite energy, holding themselves together by their own self-interaction [37]. The lump idea goes back to the Einstein proposal to describe an elementary particle by regular solution of nonlinear field equations as "bunched field" located in the confined region where field tension and energy are particularly high [38]. De Sitter-Schwarzschild lump is regular solution to the Einstein equations, perfectly localized (see Fig.4) in a region where field tension and energy are particularly high (this is the region of the former singularity), so we can call it G-lump [24].

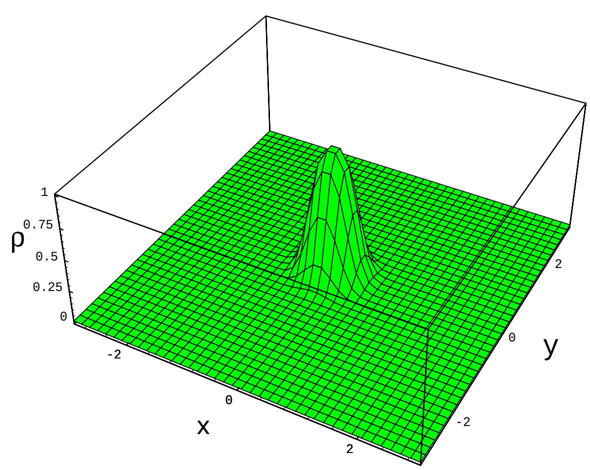

FIG. 4. G-lump in the case $r_g = 0.1r_0$ ($m \simeq 0.06m_{crit}$).

It holds itself together by gravity due to balance between gravitational attraction outside and gravitational repulsion inside of zero-gravity surface $r = r_c$ beyond which the strong energy condition of singularities theorems [28], $(T_{\mu\nu} - Tg_{\mu\nu}/2)\xi^\mu\xi^\nu) \geq 0$, is violated [23]. The surface of zero gravity is defined by $2\rho + r\rho' = 0$. It is depicted in Fig.5 together with horizons and with the surface $r = r_s$ of zero scalar curvature $R(r_s) = 0$ which represents the characteristic curvature size in the de Sitter-Schwarzschild geometry. In the case of the density profile (22) the characteristic size r_s is given by

$$r_s = \left(\frac{4}{3}r_0^2 r_g\right)^{1/3} = \left(\frac{m}{\pi\rho_0}\right)^{1/3} \tag{25}$$

and confines about 3/4 of the mass m.

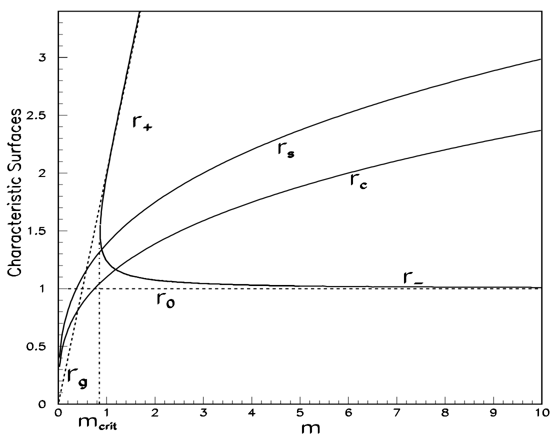

FIG. 5. Horizons of ΛBH, surface of zero scalar curvature $r = r_s$ and surface of zero gravity $r = r_c$.

The question of stability of de Sitter-Schwarzschild configurations is currently under investigation. De Sitter-Schwarzschild black hole configuration obtained by direct matching of the Schwarzschild metric outside to de Sitter metric inside of a spacelike three-cylindrical short transitional layer [20] is a stable configuration in a sense that the three-cylinder does not tend to shrink down under perturbations [39]. De Sitter-Schwarzschild configurations considered above represent general case of a smooth transition with a distributed density profile. The heuristic argument in favor of their stability comes from comparison of the ADM mass with the proper mass [29] $\mu = 4\pi\int_0^\infty \rho(r)\left(1 - \frac{2GM(r)}{r}\right)^{-1/2} r^2 dr$. In the spherically symmetric case the ADM mass represents the total energy, $m = \mu$+binding energy [29]. In de Sitter-Schwarzschild geometry μ is bigger than m. This suggests that the configuration might be stable since energy is needed to break it up [26]. Analysis of stability of a ΛBH as an isolated system by Poincare's method, with the total energy m as a thermodynamical variable and the inverse temperature as the conjugate variable [40], shows immediately its stability with respect to spherically symmetric perturbations. The analysis by Chandrasekhar method [41] is straightforward for a ΛBH stability to external perturbations, in close similarity with the Schwarzschild and Reissner-Nordström cases. The potential barriers in one-dimensional wave equations governing perturbations, external to the event horizon, are real and positive, and stability follows from this fact [41]. Preliminary results suggest stability also for the case of G-lump. In the context of catastrophe-theory analysis, de Sitter-Schwarzschild configuration resembles high-entropy neutral type in the Maeda classification, in which a non-Abelian structure may be approximated as a sphere of uniform vacuum density ρ_{vac} whose radius is the Compton wavelength of a massive non-Abelian field, and self-gravitating particle approaches the particle solution in the Minkowski space [42].

III. TWO-LAMBDA GEOMETRY

The class of metrics (19)-(20 is easily extended to the case of nonzero background cosmological constant λ, by introducing

$$T_t^t(r) = \rho(r) + (8\pi G)^{-1}\lambda \tag{26}$$

Then the metric function $g(r)$ in Eq.(19) is given by [25]

$$g(r) = 1 - \frac{2GM(r)}{r} - \frac{\lambda r^2}{3} \tag{27}$$

For $r \ll (3r_g/\Lambda)^{1/3}$, the metric (27) behaves like de Sitter metric with cosmological constant $\Lambda + \lambda$, while for $r \gg (3r_g/\Lambda)^{1/3}$ it aproaches the Kottler-Trefftz metric [43]

$$ds^2 = \left(1 - \frac{r_g}{r} - \frac{\lambda r^2}{3}\right)dt^2 - \left(1 - \frac{r_g}{r} - \frac{\lambda r^2}{3}\right)^{-1} dr^2$$

$$-r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2), \tag{28}$$

which is frequently referred to in the literature as the Schwarzschild-de Sitter geometry describing cosmological black hole. The metric (27) represents thus its nonsingular modification.

The two-lambda space-time has in general three horizons: a cosmological horizon r_{++}, a black hole horizon r_+ and an internal horizon r_- which can be formally identified as the Cauchy horizon (see also [21]) as formed by zero generators inextendible to the past. The metric function (27) is plotted in Fig.6.

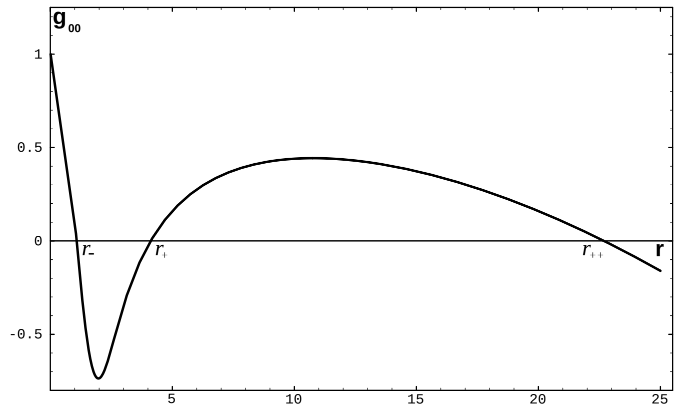

FIG. 6. Two-lambda spherically symmetric solution for the case $q = \sqrt{\Lambda/\lambda} = 10$ and $m = 4\sqrt{3/\Lambda}(c^2/G)$.

In the range of horizons $r_h \ll (\Lambda r_g/3)^{1/3}$ the internal horizon is given approximately by [25]

$$r_- \simeq \sqrt{\frac{3}{\Lambda+\lambda}}\left[1+\frac{1}{4r_g}\sqrt{\frac{3}{\Lambda+\lambda}}\left(\frac{\Lambda}{\Lambda+\lambda}\right)^2\right.$$

$$\left.\left[1+\frac{5}{4r_g}\sqrt{\frac{3}{\Lambda+\lambda}}\left(\frac{\Lambda}{\Lambda+\lambda}\right)^2\right]\right] \tag{29}$$

for $r_g \gg \sqrt{3/\Lambda+\lambda}(\Lambda/\Lambda+\lambda)^2$.

In the range of $r_h \gg (\Lambda r_g/3)^{1/3}$, the cosmological horizon is located approximately at

$$r_{++} \simeq \sqrt{\frac{3}{\lambda}} - \frac{r_g}{2} \tag{30}$$

for $r_g \ll \sqrt{3/\lambda}(\Lambda/\lambda)$.

In the interface a horizon can be written in the form $r_h = r_g + \varepsilon,\ \varepsilon \ll r_g$ which gives the black hole horizon

$$r_+ \simeq r_g\left[1+\frac{\lambda r_g^2}{3} - \exp\left(-\frac{\Lambda r_g^2}{3}\right)\right] \tag{31}$$

for r_g within the range $\sqrt{3/\Lambda} \ll r_g \ll \sqrt{3/\lambda}$.

Horizon-mass diagram is plotted in Fig.7 for the case of the density profile given by (22).

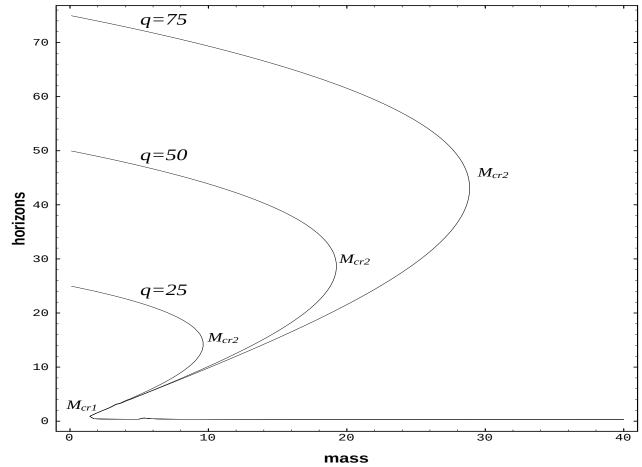

FIG. 7. Horizon-mass diagram for the metric (27). The parameter M is the mass m normalized to $(3/G^2\Lambda)^{1/2}$.

There are two critical values of the mass m, restricting the mass of a nonsingular cosmological black hole from below and from above. Within the range of masses $m_{cr1} < m < m_{cr2}$, the metric (27) has three horizons and describes a nonsingular cosmological black hole. Its global structure is shown in Fig.8.

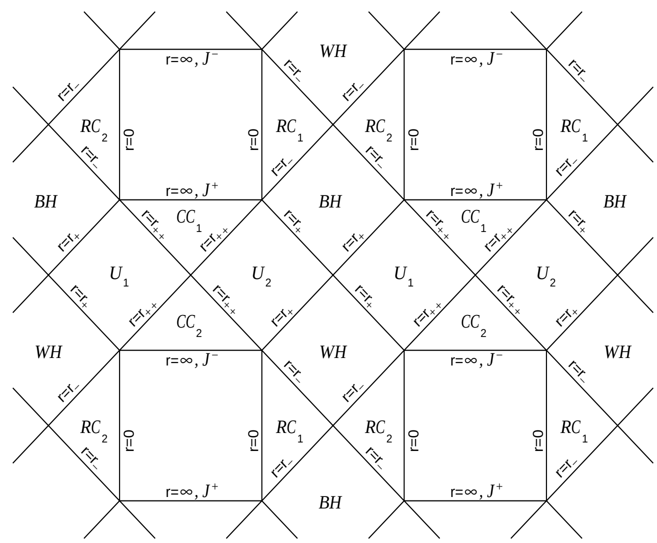

FIG. 8. Penrose-Carter diagram for two-lambda black hole.

This diagram is similar to the case of Reissner-Nordström-de Sitter geometry [45]. The essential difference is that the timelike surface $r = 0$ is regular in our case. The global structure of nonsingular cosmological black hole contains an infinite sequence of asymptotically de Sitter (small background λ) universes $\mathcal{U}_1$, $\mathcal{U}_2$, black and white holes $\mathcal{BH}$, $\mathcal{WH}$ whose singularities are replaced with future and past regular cores $\mathcal{RC}_1$, $\mathcal{RC}_2$ (with $\Lambda+\lambda$ at $r \to 0$), and "cosmological cores"

$\mathcal{CC}$ (regions between cosmological horizons and spacelike infinities). Rectangular regions confined by the surfaces $r=0$ and $r=\infty$ do not belong to the diagram.

Specification of these regions [44] is given by the invariant quantity [34,46] $\Delta = g^{\mu\nu} r_{,\mu} r_{,\nu}$. Dependently on the sign of Δ, space-time is divided into R and T regions (see [34,46]): In the R regions the normal vector to the surfaces $r=$const, $N_\mu = r_{,\mu}$ is spacelike, and an observer on those surfaces can send radial signals directed to both inside and outside of them. In the T regions the normal vector N_μ is timelike, surfaces $r=$const are spacelike, and both signals propagate on the same side of this surface, and any observer can cross the surface $r=$const only once and only in the same direction. The R and T regions are separated by horizons, where $\Delta = 0$. For the two-lambda space-times, the regions $\mathcal{RC}$ and $\mathcal{U}$ are R regions, while the regions $\mathcal{BH}$, $\mathcal{WH}$ and $\mathcal{CC}$ are T regions. For the metric in the Kruskal form $\Delta = (1/2) g_{00}^{-1} r_{,u} r_{,v}$; in the T regions $\Delta > 0$, the vector $r_{,u}$ cannot be zero, and the conditions $r_{,u} > 0$ and $r_{,u} < 0$ are invariant [46]. For $r_{,u} < 0$, T region is T_- region of contraction, so that $\mathcal{BH}$ are T_- regions, while $\mathcal{WH}$ are expanding T_+ regions, since $r_{,u} > 0$ there.

Five types of globally regular spherically symmetric configurations described by two-lambda geometry, are plotted in Fig.9 for the case $q \equiv \sqrt{\Lambda/\lambda} = 10$.

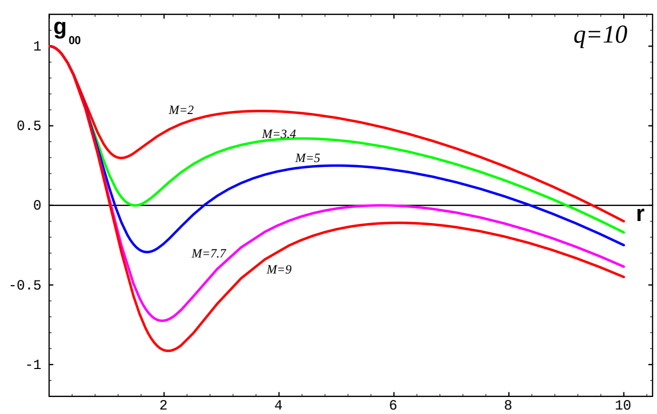

FIG. 9. Two-lambda configurations for the case $q = 10$. The parameter M is a mass m normalized to $(3/G^2\Lambda)^{1/2}$.

The critical value of mass at which the internal horizon r_- coincides with the black hole horizon r_+, defines the first extreme black hole state. The value m_{cr1} puts the lower limit for a black hole mass. It practically does not depend on the parameter $q = \sqrt{\Lambda/\lambda}$ and is given by (23). This extreme black hole is shown in Fig.9 (the curve $M = 3.4$). The Schwarzschild-de Sitter family of singular black holes contains masses between zero and the size of the cosmological horizon (see, e.g., [47]). Replacing a black hole singularity with a cosmological constant Λ results in appearance of the lower limit on a mass of cosmological black hole which is almost the same as in the case of nonsingular Λ black hole at the Minkowski space background [23]. It represents the new type spherically symmetric configuration - the extreme neutral nonsingular cosmological black hole whose internal horizon coincides with a black hole horizon.

An upper limit m_{cr2}, at which the black hole horizon r_+ coincides with the cosmological horizon r_{++}, corresponds to the nonsingular modification of the Nariai solution [48], with the additional internal horizon which is absent in the Nariai case. The value of m_{cr2} depends essentially on the parameter $q = \sqrt{\Lambda/\lambda}$ (see Fig.7).

Beyond the limiting masses m_{cr1} and m_{cr2}, there exist two different types of globally regular spherically symmetric configurations:

(i) A spherically symmetric self-gravitating particle-like structure at the de Sitter background in the range of masses $m < m_{cr1}$. This G-lump differs from the case of Minkowski space background Fig.1 by existence of the cosmological horizon.

(ii) The case $m > m_{cr2}$ differs essentially from the Schwarzschild-de Sitter case by existence of an internal horizon. Configuration of this type which we called "de Sitter bag" [25], corresponds to cosmology with the same global structure as for de Sitter geometry, but with cosmological constant smoothly evolving from Λ in the past to λ in the future.

IV. VARIABLE COSMOLOGICAL TERM

Stress-energy tensors (20) for the considered class of metrics belong to the Petrov type [(II)(II)]. The first symbol in the brackets denotes the eigenvalue related to the timelike eigenvector representing a velocity. Parentheses combine equal eigenvalues. A stress-energy tensor of this type has an infinite set of comoving reference frames, since it is invariant under boosts in the radial direction, and can be thus identified as describing a spherically symmetric anisotropic vacuum (an observer moving through such a medium cannot in principle measure the radial component of his velocity with respect to it), i.e., vacuum with variable energy density and pressures, macroscopically defined by the algebraic structure of its stress-energy tensor $T^{vac}_{\mu\nu}$ [22]. In the case of nonzero background λ it connects smoothly two de Sitter vacua with different values of cosmological constant. This makes it possible to interpret $T^{vac}_{\mu\nu}$ as corresponding to the extension of the algebraic structure of the cosmological term from $\Lambda g_{\mu\nu}$ (with Λ=const) to an r-dependent cosmological term $\Lambda_{\mu\nu} = 8\pi G T^{vac}_{\mu\nu}$, evolving from $\Lambda_{\mu\nu} = \Lambda g_{\mu\nu}$ as $r \to 0$ to $\Lambda_{\mu\nu} = \lambda g_{\mu\nu}$ as $r \to \infty$, and satisfying the equation of state (21) with $8\pi G \rho^\Lambda = \Lambda^t_t$, $8\pi G p^\Lambda_r = -\Lambda^r_r$ and $8\pi G p^\Lambda_\perp = -\Lambda^\theta_\theta = -\Lambda^\phi_\phi$ [26].

In quantum field theory cosmological constant Λ is related to zero-point vacuum energy. A zero-point energy of G-lump which clearly represents an elementary spherically symmetric excitation of a vacuum defined macroscopically by (20), can be evaluated in simple quantum minisuperspace model [24]. Since de Sitter vacuum is trapped within a G-lump, we can model it by a spherical bubble whose density decreases with a distance. In the Finkelstein coordinates, de Sitter-Schwarzschild geometry is described by the metric

$$ds^2 = d\tau^2 - \frac{2GM(r(R,\tau))}{r(R,\tau)} - r^2(R,\tau)d\Omega^2 \tag{32}$$

The equation of motion $\dot{r}^2 + 2r\ddot{r} - 8\pi G\rho(r)r^2 = f(R)$ [49], where dot denotes differentiation with respect to τ and $f(R)$ is constant of integration, has the first integral

$$\dot{r}^2 - \frac{2GM(r)}{r} = f(R) \tag{33}$$

which resembles the equation of a particle in the potential $V(r) = -\frac{GM(r)}{r}$, with the constant of integration $f(R)$ playing the role of the total energy $f = 2E$.

A spherical bubble can be described by the minisuperspace model with a single degree of freedom [50]. The momentum operator is introduced by $\hat{p} = -i{l_{Pl}}^2 d/dr$, and the equation (33) transforms into the Wheeler-DeWitt equation in the minisuperspace [50] which reduces to the Schrödinger equation

$$\frac{\hbar^2}{2m_{Pl}}\frac{d^2\psi}{dr^2} - (V(r) - E)\psi = 0 \tag{34}$$

with the potential (in the Planckian units)

$$V(r) = -\frac{GM(r)}{r} \tag{35}$$

depicted in Fig.10.

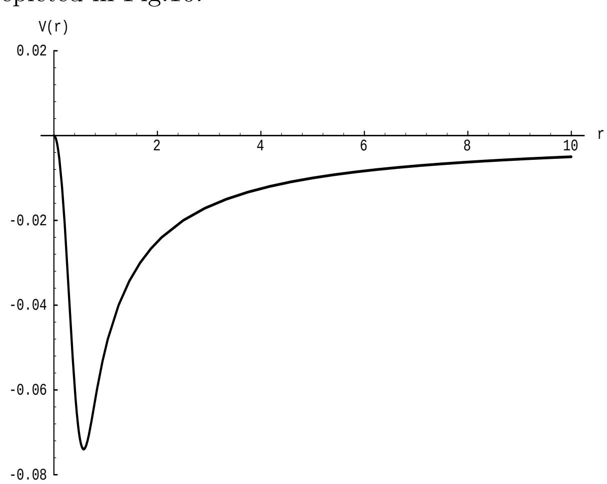

FIG. 10. The plot of the potential (35) for G-lump with $r_g = 0.1r_0 (m \simeq 0.07m_{crit})$.

Near the minimum $r = r_m$ the potential takes the form $V(r) = V(r_m) + 4\pi G p_\perp(r_m)(r - r_m)^2$. Introducing the variable $x = r - r_m$ we reduce Eq.(34) to the equation for a harmonic oscillator

$$\frac{d^2\psi}{dx^2} - \frac{m_{Pl}^2\omega^2 x^2}{\hbar^2}\psi + \frac{2m_{Pl}\tilde{E}}{\hbar^2}\psi = 0 \tag{36}$$

where $\tilde{E} = E - V(r_m)$, $\omega^2 = \Lambda c^2 \tilde{p}_\perp(r_m)$, and $\tilde{p}_\perp$ is the dimensionless pressure normalized to vacuum density ρ_0 at $r = 0$; for the density profile (22) $\tilde{p}_\perp(r_m) \simeq 0.2$. The energy spectrum

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) - \frac{GM(r_m)}{r_m}E_{Pl} \tag{37}$$

is shifted down by the minimum of the potential $V(r_m)$ which represents the binding energy. The energy of zero-point vacuum mode [24]

$$\tilde{E}_0 = \frac{\sqrt{3\tilde{p}_\perp}}{2}\frac{\hbar c}{r_0} \tag{38}$$

never exceeds the binding energy $V(r_m)$. It remarkably agrees with the Hawking temperature from the de Sitter horizon $kT_H = \frac{1}{2\pi}\frac{\hbar c}{r_0}$ [35], representing the energy of virtual particles which could become real in the presence of the horizon. In the case of G-lump which is structure without horizons, kind of gravitational vacuum exciton, they are confined by the binding energy $V(r_m)$.

V. COSMOLOGICAL TERM AS A SOURCE OF MASS

The mass of both G-lump and ΛBH is directly connected to cosmological term $\Lambda_{\mu\nu}$ by the ADM formula (12) which in this case reads

$$m = (2G)^{-1}\int_0^\infty \Lambda_t^t(r)r^2 dr \tag{39}$$

and relates mass to the de Sitter vacuum at the origin (which is thus evidently trapped within an object) [24].

The Minkowski geometry allows existence of inertial mass as the Lorentz invariant $m^2 = p_\mu p^\mu$ of a test body. High symmetry of this geometry allows both existence of inertial frames and of quantity m as the measure of inertia, but geometry tells nothing about this quantity.

In the Schwarzschild geometry the parameter m is responsible for geometry, it is identified as a gravitational mass of a source by asymptotic behavior of the metric at infinity. By the equivalence principle, gravitational mass is equal to inertial mass. The inertial mass is represented thus by a purely geometrical quantity, the Schwarzschild radius r_g which is geometrical fact of the Schwarzschild geometry [51]. However it still does not tell about origin of a mass.

The geometrical fact of de Sitter-Schwarzschild geometry is that a mass m (identified by Schwarzschild asymptotic at infinity) is related to cosmological term, since Schwarzschild singularity is replaced with a de Sitter vacuum. The operation of introducing mass by the ADM formula (39) is impossible in the de Sitter geometry, since symmetry of the source term $T_{\mu\nu} = \rho_0 g_{\mu\nu} = (8\pi G)^{-1}\Lambda g_{\mu\nu}$ is too high and ρ_0=const everywhere. In the case of de Sitter-Schwarzschild geometry symmetry of a source term is reduced from the full Lorentz group to the Lorentz boosts in the radial direction only. Together with asymptotic flatness this allows introducing a distinguished point as the center of an object whose ADM mass is defined by the standard formula (12). The reduced symmetry of a source and the asymptotic flatness of geometry are responsible for mass of an object given by (39).

Let us note that this observation does not depend on identification of a vacuum tensor of the algebraic structure (20) as corresponding to variable cosmological term $\Lambda_{\mu\nu}$. Any stress-energy tensor for this class of metrics (no matter interpreted as $\Lambda_{\mu\nu}$ or not) is invariant under full Lorentz group in the origin and at infinity but only under radial boosts in between. And for any source from this class the standard formula (12) for the ADM mass relates it to both de Sitter vacuum trapped in the origin and breaking of space-time symmetry.

This picture seems to be in remarkable conformity with the basic idea of the Higgs mechanism for generation of mass via spontaneous breaking of symmetry of a scalar field vacuum from a false vacuum (where $T_{\mu\nu} = V(0)g_{\mu\nu}$, and $p = -\rho$), to a true vacuum $T_{\mu\nu} = 0$. In both cases de Sitter vacuum is involved and vacuum symmetry is broken. Even graphically the gravitational potential $g(r)$ resembles a Higgs potential (see Fig.11).

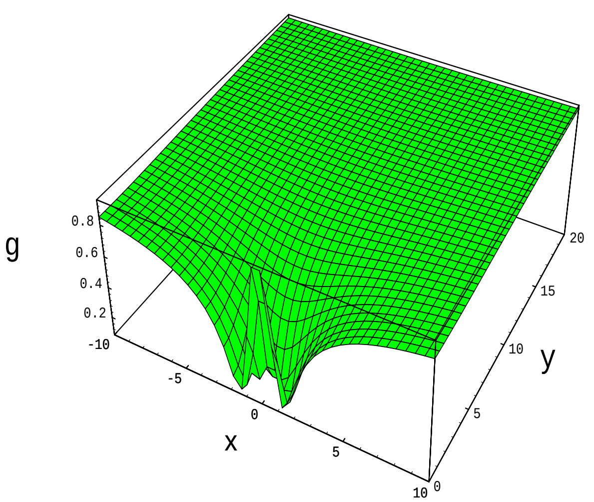

FIG. 11. The gravitational potential $g(r)$ for the case of G-lump with the mass a little bit less than m_{crit}.

The difference is that in case of a mass coming from $\Lambda_{\mu\nu}$ by (39), the gravitational potential $g(r)$ is generic, and de Sitter vacuum supplies a particle with a mass via smooth breaking of space-time symmetry from the de Sitter group in its center to the Lorentz group at its infinity.

This leads to the natural assumption [52] that whatever would be particular mechanism for mass generation, a fundamental particle (a particle which does not display substructure, like a lepton or quark) may have an internal vacuum core (at the scale where it gets mass) related to its mass and a geometrical size defined by gravity. Such a core with de Sitter vacuum at the origin and Minkowski vacuum at infinity can be approximated by de Sitter-Schwarzschild geometry. Characteristic size in this geometry is given by (25). It depends on vacuum density at $r = 0$ and presents modification of the Schwarzschild radius r_g to the case when singularity is replaced by de Sitter vacuum. While application of the Schwarzschild radius to elementary particle size is highly speculative since obtained estimates are many orders of magnitude less than l_{Pl}, the characteristic size r_s gives reasonable numbers (e.g., $r_s \sim 10^{-18}$ cm for the electron getting its mass from the vacuum at the electroweak scale) close to estimates obtained in experiments (see Fig.12 [52] where they are compared with electromagnetic (EM) and electroweak (EW) experimental limits).

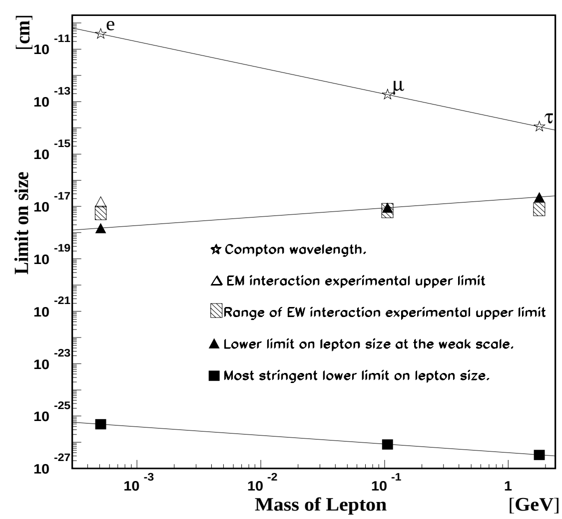

FIG. 12. Characteristic sizes for leptons [52].

VI. DISCUSSION

The main point outlined here is the existence of the class of globally regular solutions to the minimally coupled GR equations (8)-(10), with a source term of the algebraic structure (20) interpreted as spherically symmetric anisotropic vacuum with variable density and pressures $T_{\mu\nu}^{vac}$ associated with a time-dependent and spatially inhomogeneous cosmological term $\Lambda_{\mu\nu} = 8\pi G T_{\mu\nu}^{vac}$, whose asymptotic in the origin, dictated by the weak energy condition, is the Einstein cosmological term $\Lambda g_{\mu\nu}$.

The key difference of $\Lambda_{\mu\nu}$ from the quintessence which is introduced as a time-varying spatially inhomogeneous component of matter content with negative pressure is in the algebraic structure of stress tensors. Quintessence is defined by the equation of state $p = -\alpha\rho$ with $\alpha < 1$ [54]. This corresponds to such a stress-energy tensor $T_{\mu\nu}$ for which a comoving reference frame is defined uniquely. The quintessence represents thus a non-vacuum negative-pressure isotropic alternative to a cosmological constant Λ while the cosmological tensor $\Lambda_{\mu\nu}$ represents the extension of the algebraic structure of the Einstein cosmological term $\Lambda g_{\mu\nu}$ which makes it variable and anisotropic.

De Sitter-Schwarzschild geometry (19) describes generic properties of any configuration satisfying (20) and requirements (a)-(c), obligatory for any particular model in the same sense as de Sitter geometry (3) is obligatory for any matter source satisfying (4).

In the inflationary cosmology which is based on generic properties of de Sitter vacuum $\Lambda g_{\mu\nu}$ independently on where Λ comes from [1], several mechanisms are investigated relating $\Lambda g_{\mu\nu}$ to matter sources (for review see [2,53]). Most frequently considered is a scalar field

$$S = \int d^4x\sqrt{-g}\left[R + (\partial\phi)^2 - 2V(\phi)\right] \qquad (40)$$

where R is the scalar curvature, $(\partial\phi)^2 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, with various forms for a scalar field potential $V(\phi)$.

The question whether a regular black hole can be obtained as a false vacuum configuration described by (40), has been addressed in the paper [55], where "the no-go theorem" has been proved: Asymptotically flat regular black hole solutions are absent in the theory (40) with any non-negative potential $V(\phi)$. This result has been extended to the case of any $V(\phi)$ and any asymptotic and then generalized to the case of a theory with the action $S = \int d^4x\sqrt{-g}\left[R + F[(\partial\phi)^2, \phi]\right]$, where F is an arbitrary function [56], to the multi-scalar theories of sigma-model type, and to scalar-tensor and curvature-nonlinear gravity theories [57]. It has been shown that the only possible regular solutions are either de Sitter-like with a single cosmological horizon or those without horizons, including asymptotically flat ones. The latter do not exist for $V(\phi) \geq 0$, so that the set of causal false vacuum structures is the same as known for $\phi = const$ case, namely Minkowski (or anti-de Sitter), Schwarzschild, de Sitter, and Schwarzschild-de Sitter [56,57], and thus does not include de Sitter-Schwarzschild configurations.

In the case of *complex* massive scalar field the regular structures can be obtained in the minimally coupled theory with positive $V(\phi)$ [58]. These are boson stars ([33] and references therein), but in this case algebraic structure of the stress-energy tensor [33] does not satisfy Eq.(20), and asymptotic at $r = 0$ is not de Sitter.

The considered connection between r-dependent cosmological term $\Lambda_{\mu\nu}$ and the ADM mass seems to satisfy Einstein's version of Mach's principle - no matter, no inertia - if we explicitly separate two aspects of the problem of inertia: existence of inertial frames and existence of inertial mass. In empty space, $T_{\mu\nu}^{vac} = 0$, inertial frames exist due to high symmetry of Minkowski geometry, but to prove it we need a measure of inertia, a test particle with the inertial mass, i.e. a region in space where Minkowski vacuum is a little bit disturbed. For the considered class of metrics with the regular center $T_{\mu\nu}^{vac} \neq 0$ and the inertial mass is generically related to both reduced symmetry of a source term (20) (no matter interpreted as $\Lambda_{\mu\nu}$ or not) and de Sitter vacuum trapped in the origin. In other words, full symmetry of Minkowski space-time is responsible for existence of inertial frames, while its breaking to Lorentz boosts in the radial direction only is responsible for inertial mass.

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