Probing Extra Dimensions with Higher Dimensional Gravity-Wave Analogues of Black Holes ?

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The 15th Workshop on Numerical Relativity, Inje University

1/1/2007

Outline

- Introduction and background information
- Metric of higher dimensional black hole analogues
- Properties of higher dimensional gravity-wave analogues of black holes
- Greybody factors , Absorption cross section & Power spectrum
- Summary & Open problem

1. Introduction: motivations for extra-dimensions

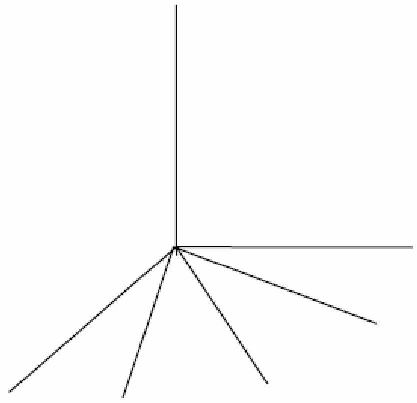
Hierarchy problem:

- EW scale **<<** GUT scale **<<** Planck scale **(~** 21^{3} **GeV, ~** 21^{27} **GeV, ~** $21^{2:}$ **GeV)**
- Why Gravitational Interaction is so weak ?
- **Existing Tev scale gravity models classified by:**

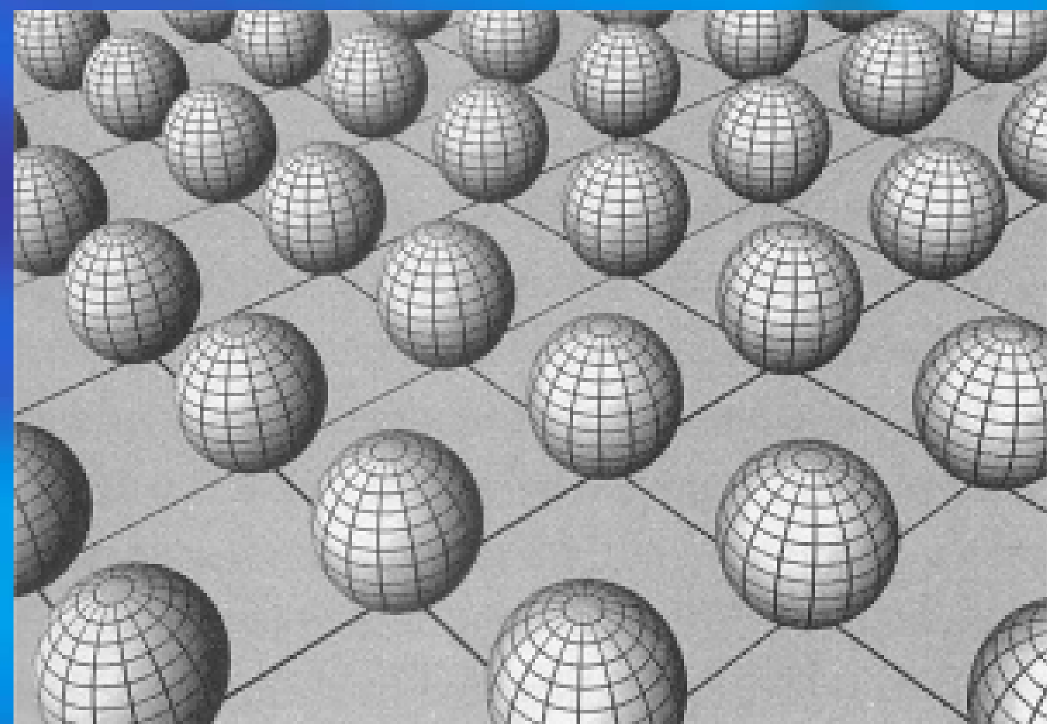
  - Space-time geometry: factorized (flat) (hep-th/9803315) or warped (hep-ph/9905221)

1.2 Framework of Brane-world theory (flat)

- 3+n spacelike dimensions and one for time
- Electroweak and Strong forces confined to our usual 3 spatial dimensions “3-brane”
- Gravity propagates in all dimensions
- Extra n dimensions submillimeter sized, but still “large”



- The traditional Planck scale, N_{Q} is only an effective energy scale derived from the fundamental higher-dimensional one, $N_{¥}$, through the following relation: N_{Q} ¸ $N_{¥}^{3,\circ} S^{\circ}$



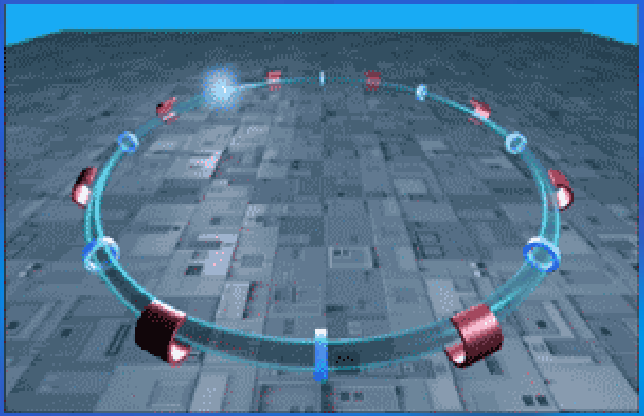
A (4+n)-dimensional World

n	M_g (TeV)	R (cm)
0	10^16	n/a
1	~1	10^13
2	>1.6	<.1
≥3	Interesting	<<1 (undetectable)

1.3 Observing a black hole

- Hawking temperature

$$T_{BH} \approx (n+1)/4\pi R$$



- Black holes decay via Hawking radiation
- Those produced by particle accelerators: mini black holes will decay into roughly 25 particles, mostly hadrons and some leptons with energies of order 50~100GeV
- Large Hadron Collider (LHC): a black hole factory?

2. Acoustic (sonic) black holes

- The concept of acoustic black holes was first proposed by W.G. Unruh (Phys. Rev. Lett. 46, 1351(1981))
- Metric

$$g^{(0)}_{\mu\nu} = -\frac{\rho_0}{c}\begin{pmatrix} c^2 - v_0^2 & \vec{v}_0^T \\ \vec{v}_0 & -I \end{pmatrix}$$

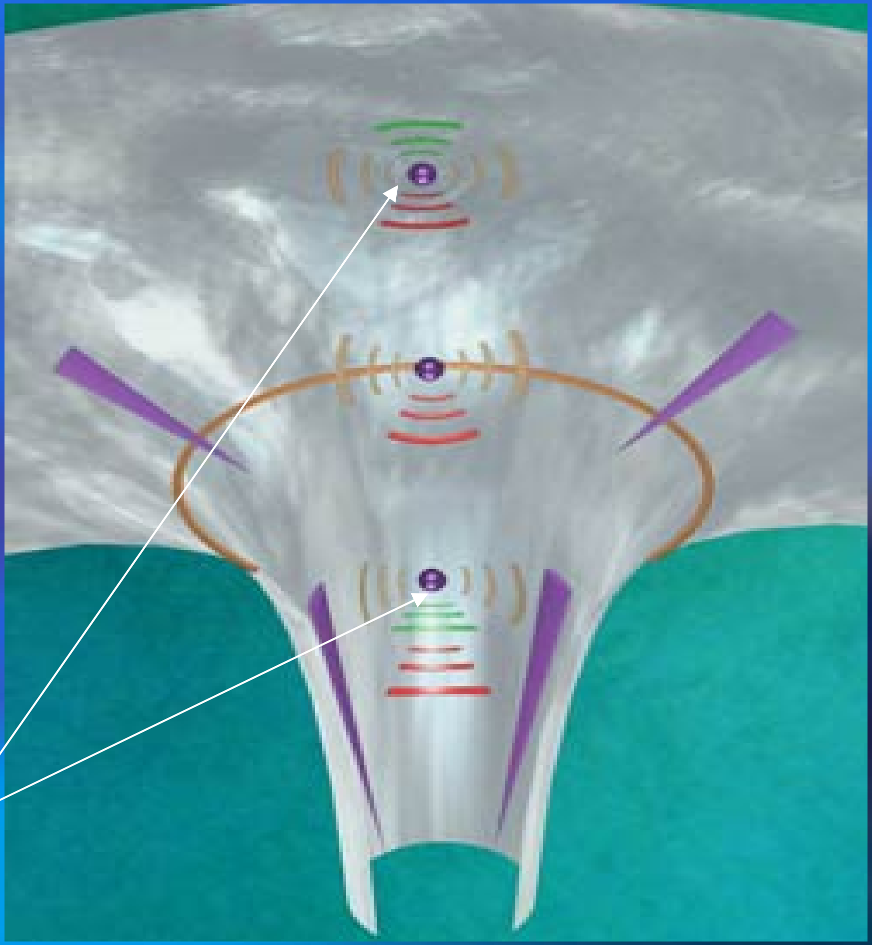
- Recent methods to simulate black holes have been extended to superfluid helium II, atomic Bose-Einstein condensates, one-dimensional (1D) transonic flows, dielectric, gravity waves, waveguide and 1D Fermi-degenerate noninteracting gas etc.

To be more detailed:

- A black hole "analogue"--a fluid flow in which the downstream current acts like a gravitational pull and which is governed by

  black-hole-like equations.

A swimmer in this supersonic region could cry out for help,
but the sound waves would never escape to the stationary region.



Phys. Rev. Focus, Oct.18 2006

3. Questions

- What is the behavior of black hole analogues if extra dimensions do exist ?
- Can we probe extra dimensions by using black hole analogues ?

3.1 The following steps

- We will first extend the metric of 4-dimensional black hole analogues to higher dimensions

- We then find a real system of higher dimensional black hole analogues can be realized by using gravity waves, since only gravity can live in higher dimensional space-time.

3.2 Higher dimensional black hole analogues

- Since the dynamics of effective black hole analogues metric are not described by the laws of gravity (i.e. Einstein equations) in general, to extend the four dimensional effective metric we should start with the equations of fluid dynamics in 4+n flat dimensions,

$$\partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0,$$

$$\rho \frac{d\vec{v}}{dt} \equiv \rho[\partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v}] = \vec{F},$$

where $\nabla = \hat{e}_x \frac{\partial}{\partial x} + \hat{e}_y \frac{\partial}{\partial y} + \ldots + \hat{e}_n \frac{\partial}{\partial n}$ $\quad \vec{F} = -\nabla p - \nabla \rho$

These equations can be linearized in the vicinity of some mean flow solution with $\rho=\rho_0+\varepsilon\rho_1+O(\varepsilon^2),\ \ p=p_0+\varepsilon p_1+O(\varepsilon^2),\ \ \varphi=\varphi_0+\varphi_1+O(\varepsilon^2)$

We can finally get an equation describing the perturbative field φ_1,

$$\boxed{\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\varphi_1)=0}$$

Which is identified with a massless scalar field (Klein-Gordon) equation in the curved space-time background.

Metric

$$h_{\prime\mu} > \left(\frac{{}^{\text{ȿ}}1}{d}\right)^{\frac{3}{o\,,\,3}} \begin{pmatrix} \text{¢}\,d^3\,,\;w_1^3 & \text{¢}\,w_1^j \\ \text{¢}\,w_1^k & O_{jk} \end{pmatrix}$$

where i, j=1,2…n

In canonical coordinates

$$et^3 > \left(\frac{{}^{\text{ȿ}}1}{d}\right)^{\frac{3}{o\,,\,3}} \Big[\text{¢}\,d^3)2\,\text{¢}\,\frac{w_s^3}{d^3}*e^{\text{º}\,3}\,,\;)2\,\text{¢}\,\frac{w_s^3}{d^3}*^{\text{¢}\,2}es^3\,,\;s^3e^{\text{ª}}{}_{o\,,\,3}$$

- If ¸ is position and time independent, the continuity equation s £) ¸w*> 1 indicates ws 0 2 / so , 3 We can rewrite the metric as,

et3 > (¸1 / d)^(3 / o , 3) [¢ d3)2 ¢ s1^(3o , 5) / s^(3o , 5) *eº3 ,)2 ¢ s1^(3o , 5) / s^(3o , 5) *¢ 2es3 , s3eª o , 3]

- Temperature

$$T = -\frac{\hbar}{2\pi k}\frac{\partial v_r}{\partial r}\Big|_{v_r = c},$$

U > 4)o , 3*¤ 21¢ 8L \d>411n >tfd^2n n >s1

3.3 Higher dimensional gravity wave black holes: properties

1).The metric:

The propagation of the perturbation gravity waves yields the following equation [10],

$$\Box\delta\Phi_0 = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\delta\Phi_0) = 0,$$

with the effective metric

$$g_{\mu\nu} = \begin{pmatrix} 1-\frac{v_0^2}{gh_0} & v_0^i \\ v_0^j & -\delta_{ij} \end{pmatrix}, \quad (11)$$

where v_0 is the background flow velocity of the liquid, Φ_0 is the zeroth-order of the perturbation potential, $\vec{g}$ the gravitational acceleration, and h_0 the height of the background fluid. We can easily extend the effective metric (11) to higher dimensions by using the same methods used in section II, which goes as,

$$ds^2 = -g_{4+n}h_0\left(1-\frac{v_r^2}{g_{4+n}h_0}\right)d\tau^2 + \left(1-\frac{v_r^2}{g_{4+n}h_0}\right)^{-1}dr^2 + r^2 d\Omega_{n+2}, \quad (12)$$

note: for “Gravity wave analogues of black holes ” see R. Schutzhold, W.G.Unruh, Phys. Rev.D,66 044019 (2002) where g_{4+n} is the gravitational acceleration in 4+n-dimensions, with

$$g_{4+n} \simeq \frac{GM}{r^{n+2}}$$

For n-dimensional fluids, the continuity s {w > 1 , indicates that $v_r \propto \frac{c_1}{r^{n+2}}$, where c_1 is a normalization constant. we may set

$$c_1 = (GMh_0)^{1/2}$$

Thus, the metric can be rewritten as ,

$$\boxed{ds^2 = -g_{4+n}h_0\left(1-\frac{r_{4+n}^{n+2}}{r^{2n+4}}\right)d\tau^2 + \left(1-\frac{r_{4+n}^{n+2}}{r^{2n+4}}\right)^{-1}dr^2 + r^2 d\Omega_{n+2}}$$

2). Compare the higher dimensional gravity-wave analogues of black holes with the 4-dimensional ones

The line element of 4-dimensional gravity-wave black hole analogues in canonical coordinates reads,

$$\boxed{ds^2 = -g_4 h_0 \left(1-\frac{r_4^2}{r^4}\right) d\tau^2 + \left(1-\frac{r_4^2}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_{n+2}}$$

According to Ann. Phys.172, 304 (1986) P. C. Myers and M. J. Perry, the horizon radius: r_4 , r_{4+n} and the mass M that generates the gravitation acceleration g_4 and g_{4+n} should have the following relations,

$$\boxed{r_4 \sim \frac{1}{M_*}\left(\frac{M}{M_*}\right)\frac{1}{(M_* R)^n}} \qquad \boxed{r_{4+n} \sim \frac{1}{M_*}\left(\frac{M}{M_*}\right)^{1/(n+1)}}$$

3. Properties:

- Temperature: can be colder than its 4-dimensional correspondences

$$\frac{U_{5,o}}{U_5} , \left(\frac{S_{5,o}}{S}\right)^{o} \frac{S_5}{S_{5,o}} , \left(\frac{S_{5,o}}{S}\right)^{3o} \; ' \; 2$$

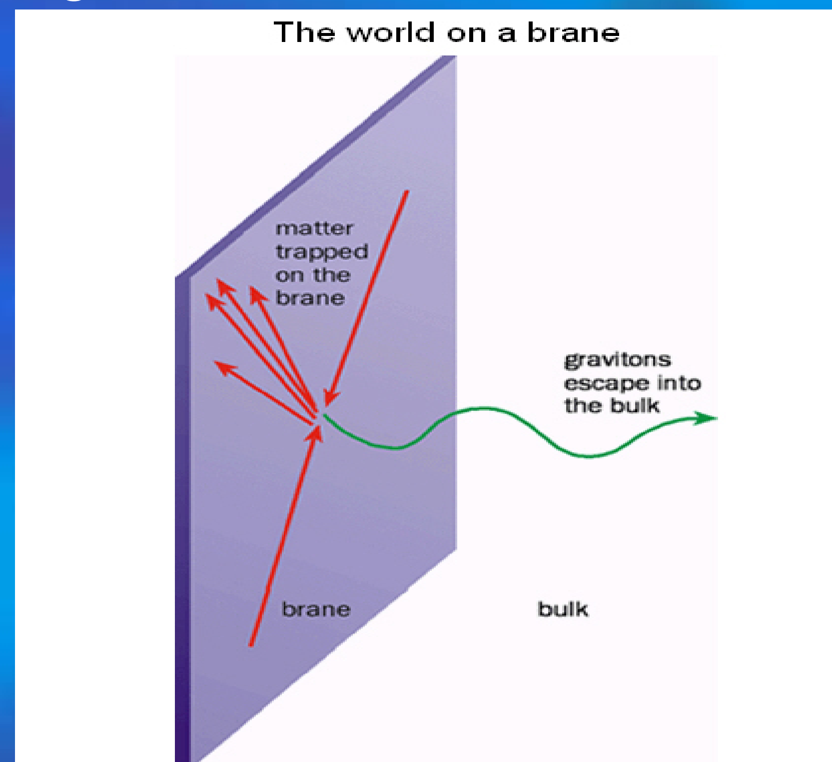
- Horizon radius: can be bigger than its 4-dimensional correspondences

$$S_5 = S_{5,o} = S$$

3.4 Why gravity wave black holes?

- Since the Standard Model particles must to be located to ordinary 4-dimensional spacetime, and extra dimensions can only be probed through the gravitational force. The continuity equation S {W > 1 in higher dimensions should only apply to gravity waves.

  So, a real system of higher dimensional black hole analogues can be realized by using gravity Waves.



4. Greybody factors …

- The power spectrum for boson fields

$$\frac{eF)"*}{eu} > \sum_m 1_{m<o})"*\frac{"}{fyq)">U_I *¢ 2}\frac{e^{o,4}1}{)3\cdot *o,4}$$

where $1_m)"*$ is the greybody factor. To calculate the greybody factor,

$$1_m)"*> \frac{3^{o.|o,2*>3}¢)\frac{o,2}{3}*}{o^{""o,3}}\frac{)3m_, o,2*)m_, o*"}{n"} kB)"*k^3$$

one should first obtain the absorption cross section, $kB)"*k^3$

We will then derive the decay modes which are not spherically symmetric, $l \neq 0$, by rewriting the metric of 4+n-dimensional black hole analogues,

$$\boxed{et^3 > \left(\frac{{}^{s}1}{d}\right)^{\frac{3}{o,3}} \left[¢\, d^3)2 ¢ \frac{s_1^{3o,5}}{s^{3o,5}} *e^{o3} ,)2 ¢ \frac{s_1^{3o,5}}{s^{3o,5}} *^{¢2} es^3 , s^3 e^{a}{}_{o,3} \right]}$$

in the following form,

$$\boxed{ds^2 = -h(r)d\tau^2 + h^{-1}(r)dr^2 + r^2 d\Omega_{n+2}}$$

Where $h(r) = 1 - (\frac{r_0}{r})^{2n+4}$ and we have set $\rho_0 = c = constant$

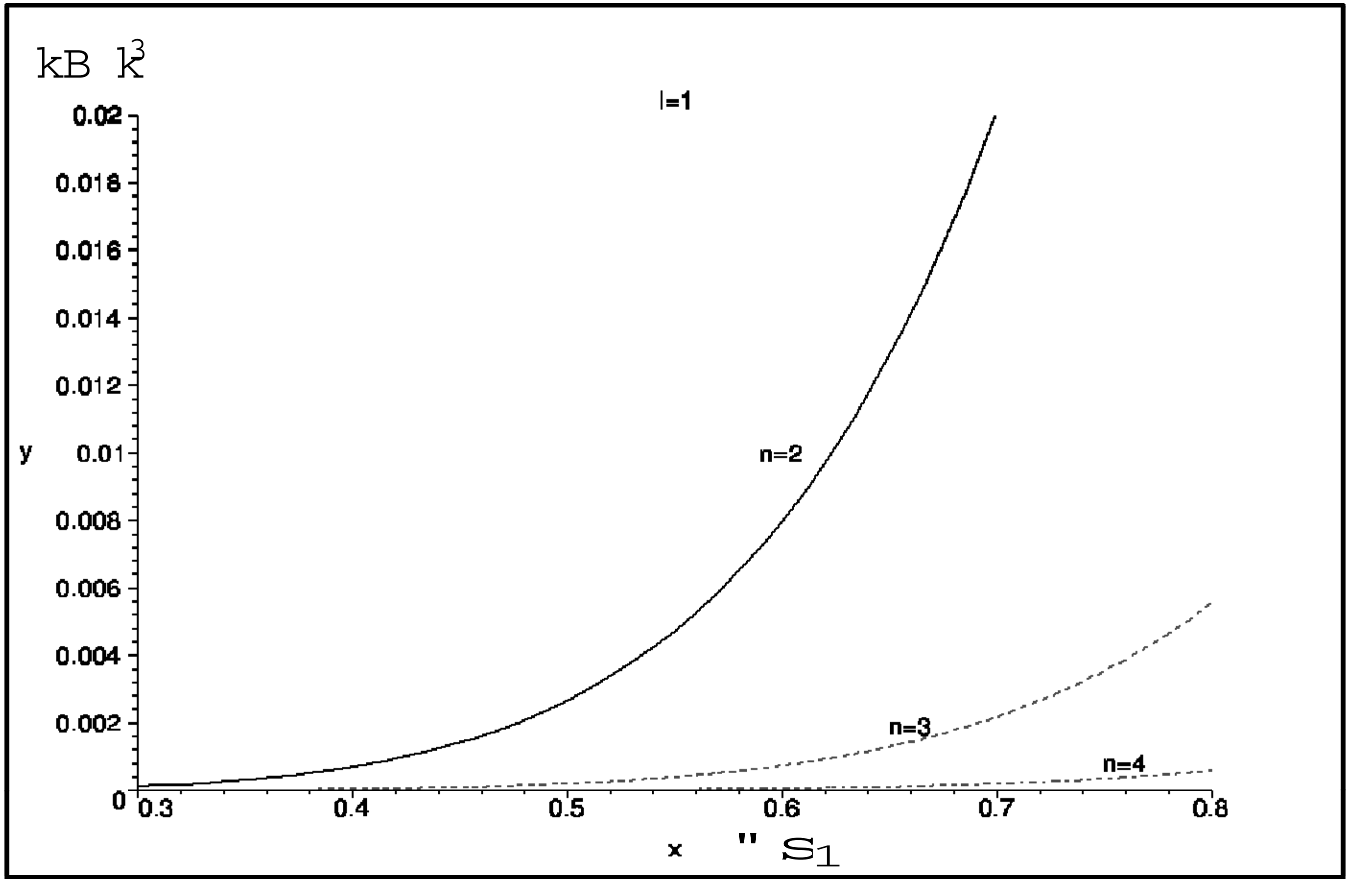
4.1 Absorption cross section for mµ 1 in the bulk

- Methods: Solving the Klein-Gordon equation approximately in three regions: near-horizon regions, far field regions and intermediate regions, and match the solution across the boundaries of the regions, one can then obtain the cross section. The scalar equation is given by,

i)2¢ i*e^3 S / ei^3 , 2¢ 4o, 8 / 3o, 5 i eS / ei , " ^3 s_1^3 /)3o, 5*^3 i)2¢ i* ¢ m)m, o, 2* /)3o, 5*^3)2¢ i* S > 1

kB)" * k^3 (9·)3o, 6¢ 3m* /)m, o, 2 / 3 *)3m¢ 2*^3) " s_1 / 3 *3m, o, 3 ¢) 4/5 , m / 3o, 5 *^3 ¢) o / 5)o, 3* , m / 3o, 5 *^3 / ¢)m, o, 2 / 3 *^3 ¢) 3m¢ 2 / 3o, 5 *^3

4.2 Analytical results



If we fix the angular momentum number and vary only the number of extra dimensions, the absorption probability decreases as n increases, since the expansions of kB k^3 is in powers of " s_1 / 2

Figure1, Analytical results for the absorption probability for a (4+n)-dimensional bulk scalar field

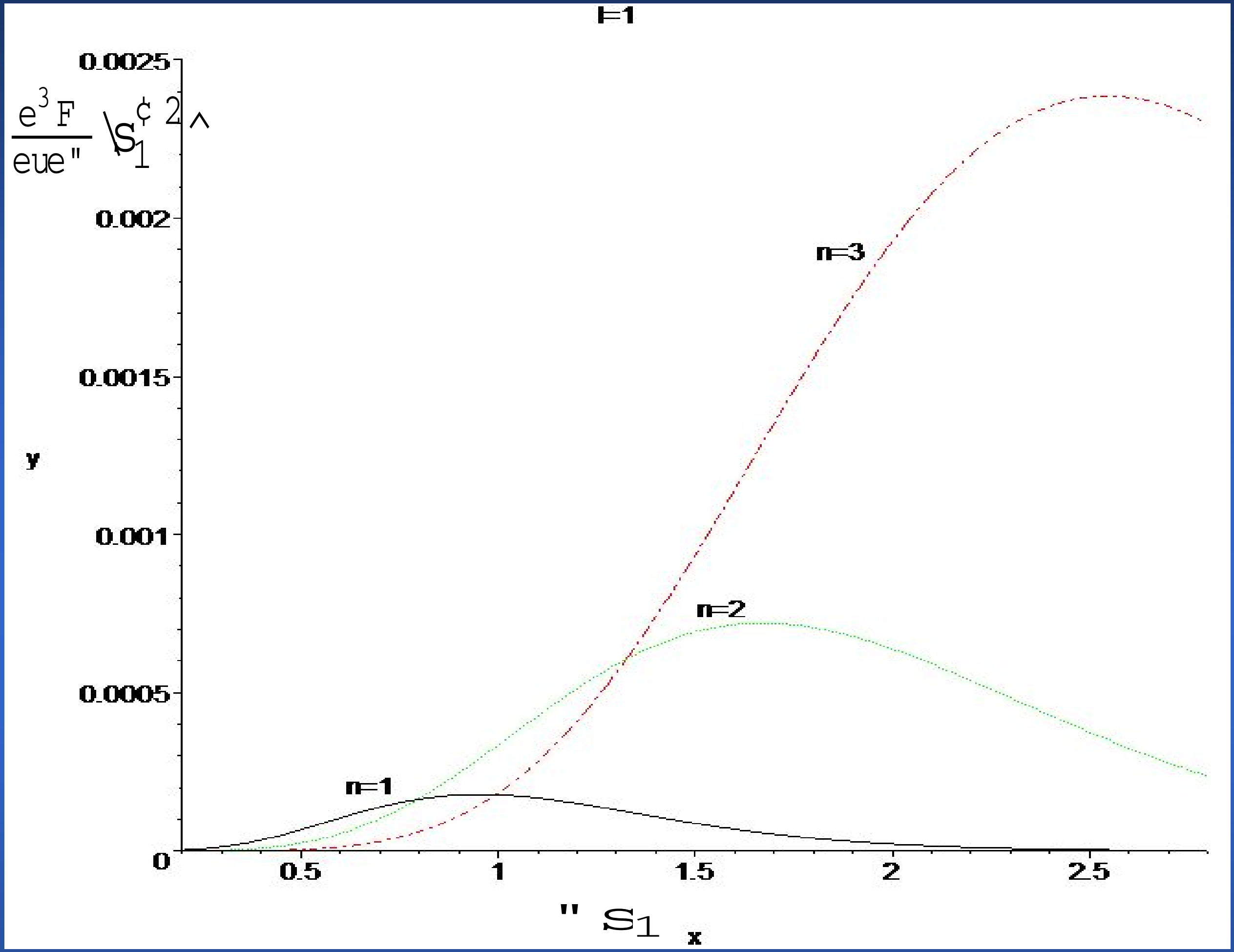


Figure 2. Analytical results for the energy rates for scalars from a (4+n)-dimensional black hole in the bulk for r > 2

The emission rate of scalar fields in the bulk is enhanced as the number of extra dimensions increases. This is caused by the increase of the temperature of black hole analogues, which finally overcome the decreases in the value of the greybody factor and causes the enhancement of the emission rate with n at high energies.

4.3 Brane-localized scalar emission for mμ 1

- Important: By detecting signals via the black hole analogue's evaporation to brane-localized modes, one can in principle determine the exact dimensions of spacetime.
- The radial equation,

$$\text{i)2¢ i*}\frac{\text{e}^3\text{S}}{\text{ei}^3}\text{ , }\left[\text{2¢ }\frac{\text{5o , 8}}{\text{3)o , 3*}}\right]\frac{\text{eS}}{\text{ei}}\text{ , }\left[\frac{\text{"}^3\text{s}^3}{\text{)3o , 5*}^3\text{i)2¢ i*}}\text{ ¢ }\frac{\text{m)m, 2*}}{\text{)3o , 5*}^3\text{)2¢ i*}}\right]\text{S > 1}$$

$$\text{kB)" * k}^3\text{> }\frac{\text{27·}}{\text{)3m, 2*}^3}\left(\text{¢}\frac{\text{" s}_1}{\text{3}}\right)^{\text{£3m, 3}}\frac{\text{¢)}\frac{\text{m, 2}}{\text{3o , 5}}\text{*}^3\text{¢)2, }\frac{\text{m}}{\text{3o , 5}}\text{*}^3}{\text{¢)m, }\frac{2}{3}\text{*}^3\text{¢)2, }\frac{\text{3m, 2}}{\text{3o , 5}}\text{*}^3}$$

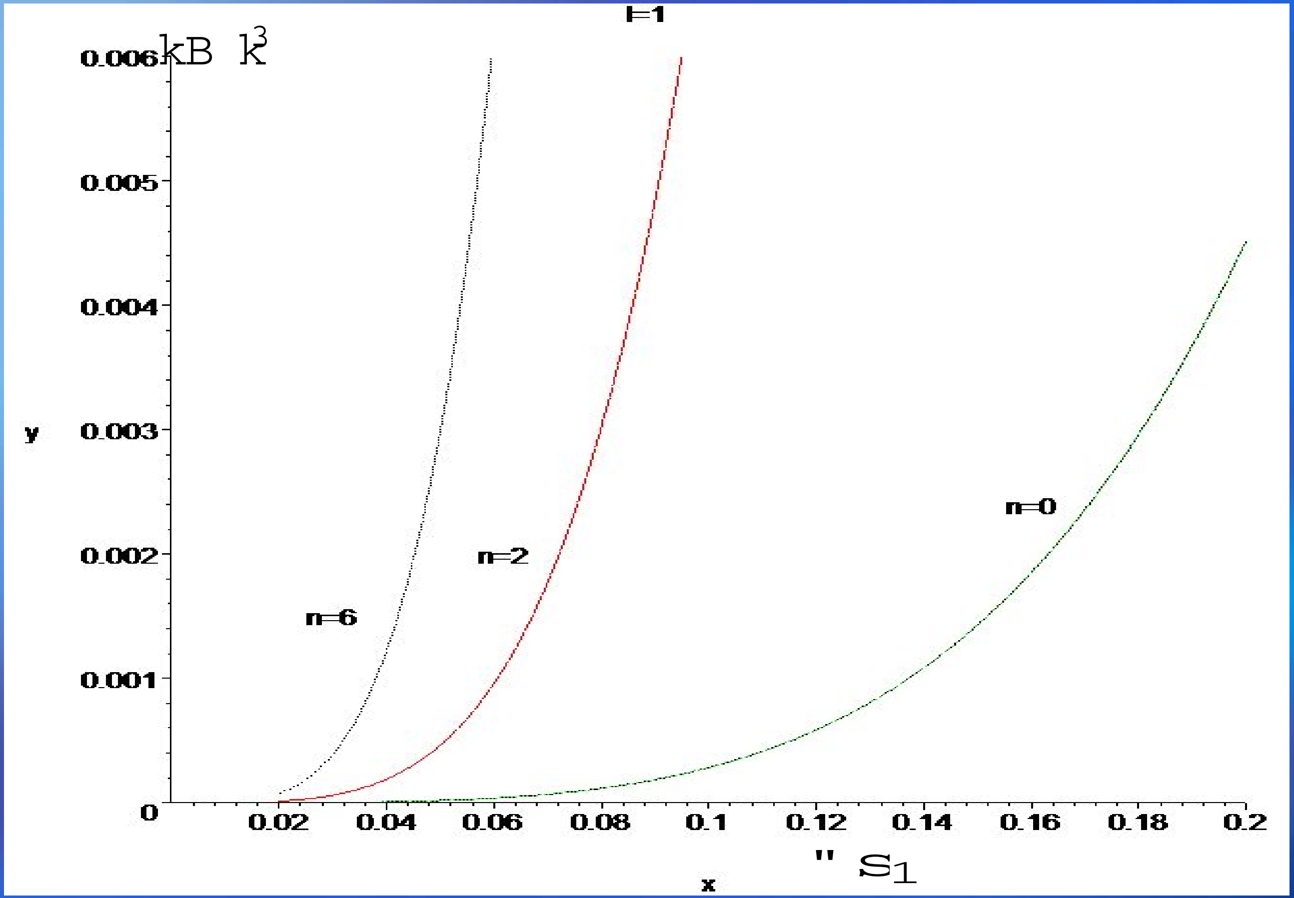


Figure 3 is to compare the results of the absorption probability derived in the case of a brane-localized n>0 scalar field with a purely 4-dimensional (n=0) scalar field. For higher partial waves (l>0), the value of the absorption probability in the case of a brane-localized (n>0) scalar field is larger than the one for a purely 4-dimensional (n=0) field, while for l=0 they are the same.

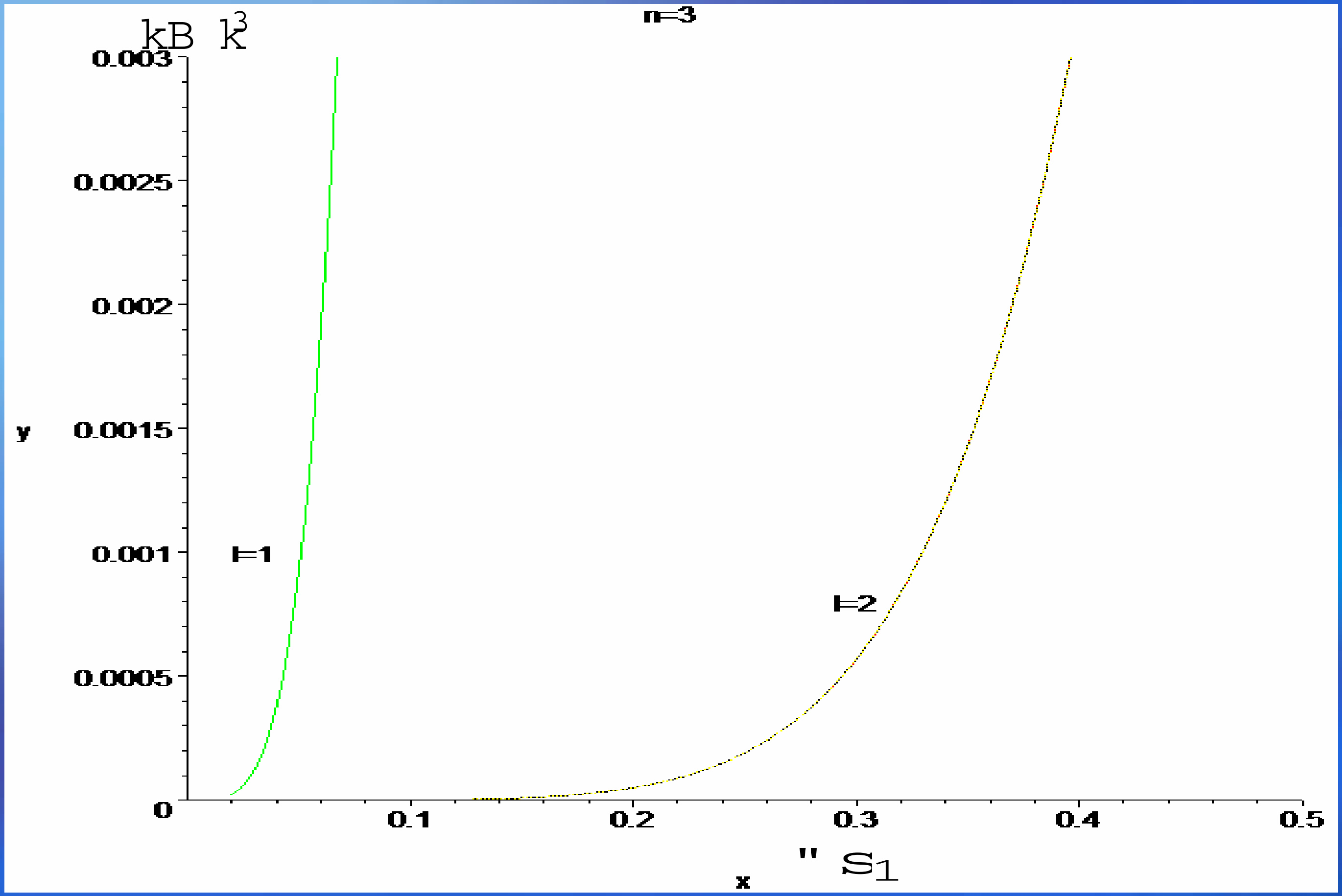


Figure 4 demonstrates that if we keep n fixed and varying r , the absorption probability decreases as r increases.

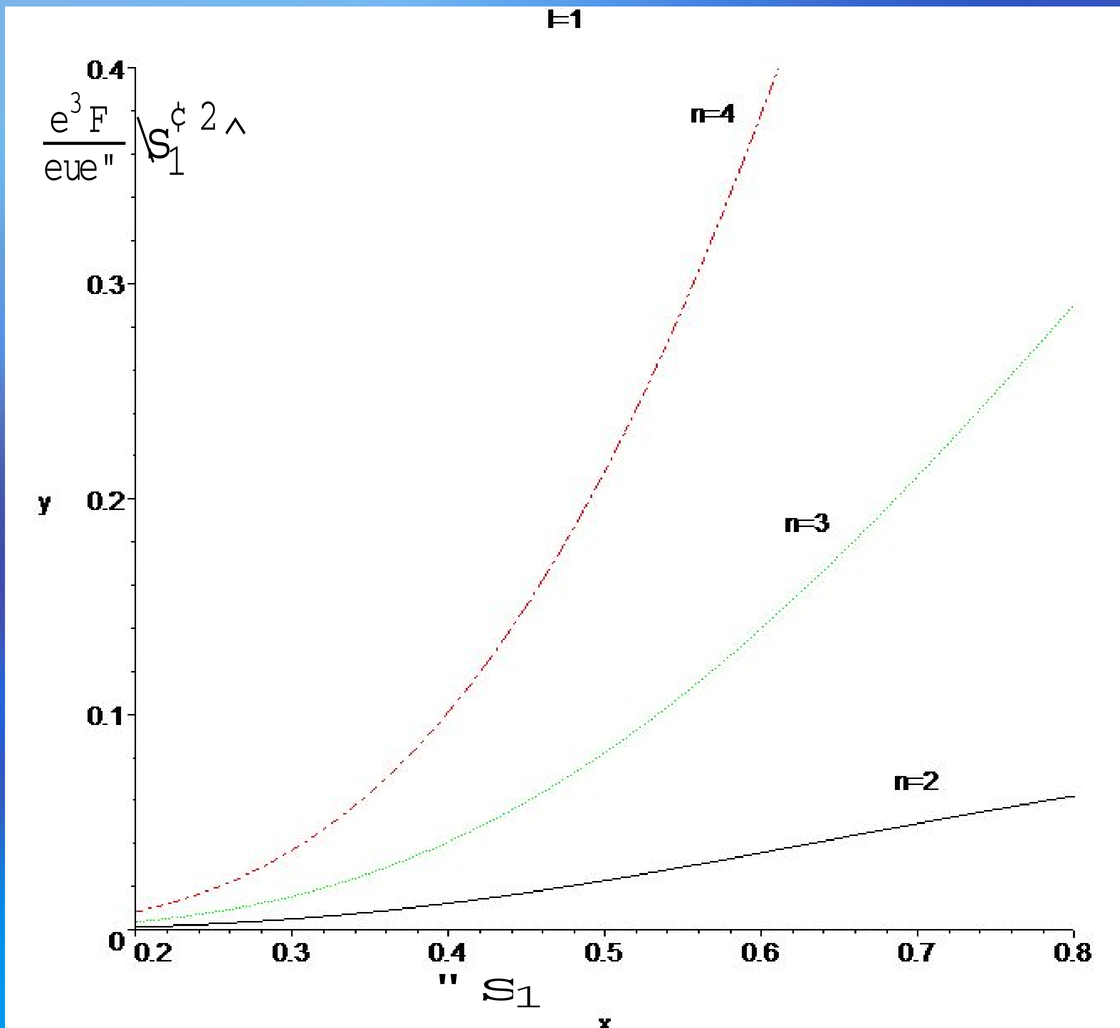


Figure 5 depicts the behavior of the energy emission rates for particles with the angular number r > 2 in the low and intermediate energy regime. The figure shows that the energy and the number of particles, emitted per unit time and energy interval is strongly enhanced, as n increases since the temperature of the black hole is given by the relation

U_I 0)o , 3*>3·s_1

5. Summary & Open Problems

- Summary

-- Metric of higher dimensional black hole analogues

-- Properties of higher dimensional gravity-wave black holes: bigger, cooler…, if the gravitational acceleration is strong enough.

--Greybody factors

- Scalar emission in the 4+n-dimensional bulk for mμ 1
- Brane-localized emission for mμ 1 , important in experiments

-- Experimental verification of higher dimensional black hole analogues might be more difficult than finding mini black holes in LHC or cosmic rays, because the temperature is very low (unless the velocity of gravity-waves in fluids c is large enough)

U > 4)o , 3* ¤ 21¢ 8L \d>411 n >tfd ^\2n n >s1 '

Thank you