

Complementarity Endures: No Firewall for an Infalling Observer

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Abstract

We argue that the complementarity picture, as interpreted as a reference frame change represented in quantum gravitational Hilbert space, does not suffer from the “firewall paradox” recently discussed by Almheiri, Marolf, Polchinski, and Sully. A quantum state described by a distant observer evolves unitarily, with the evolution law well approximated by semi-classical field equations in the region away from the (stretched) horizon. And yet, a classical infalling observer does not see a violation of the equivalence principle, and thus a firewall, at the horizon. The resolution of the paradox lies in careful considerations on how a (semi-)classical world arises in unitary quantum mechanics describing the whole universe/multiverse.

1 Introduction: Complementarity and Firewall

In the past decades, it has become increasingly clear that the concept of spacetime must receive substantial revisions when it is treated in a fully quantum mechanical manner. Consider describing a process in which an object falls into a black hole, which eventually evaporates, from the viewpoint of a distant observer. Unitarity of quantum mechanics, together with the semi-classical analysis of the process, suggests that the information content of the object will first be stored on the (stretched) horizon, and then emitted back to distant space in the form of quantum correlations among Hawking radiation quanta. On the other hand, the equivalence principle implies that the object should not find anything special at the horizon, when the process is described by an observer falling with the object. These two pictures are obviously incompatible if we formulate quantum mechanics in global spacetime; in particular, the full quantum information content of the falling object will be duplicated into internal spacetime and the horizon/Hawking radiation, violating the no-cloning theorem of quantum mechanics [1].

Black hole complementarity [2, 3] asserts that there is no contradiction between the two pictures, since the statements by the two observers cannot be operationally compared due to the nontrivial structure of the geometry. Specifically, no physical observer can collect the information from both Hawking radiation and the fallen object, avoiding a violation of the no-cloning theorem [4, 5]. This requires a deviation from the naive expectation that global spacetime provides a universal platform on which unitary quantum mechanics is formulated preserving (approximate) locality. Extensions of the complementarity picture to other spacetimes are discussed in Refs. [6, 7, 8, 9, 10]. In particular, it has profound implications on how the eternally inflating universe/multiverse must be described without leading to many apparent problems/paradoxes [9, 11].

Recently, the complementarity picture has been challenged by Almheiri, Marolf, Polchinski, and Sully (AMPS) [12]. These authors consider a black hole that forms from collapse of some pure state and discuss what an observer falling into this black hole after the Page time (the time when the black hole emits a half of its initial Bekenstein-Hawking entropy) will see. AMPS argue that the observer measures high energy quanta at the horizon, a phenomenon referred to as a “firewall,” contradicting the equivalence principle. The argument (which we will review in more detail in the next section) goes as follows: because of the entanglement between early radiation and other degrees of freedom, implied by the purity of a state, an infalling observer may select a state that is *incompatible* with the equivalence principle by making measurements on early radiation, which can be causally accessed by the observer. This puzzling conclusion is called the firewall paradox. In a subsequent paper [13] Susskind has argued that the location and formation time scale of the firewall are different from what AMPS imagine, but it still requires a violation of the equivalence principle in a low-curvature region. More recently, Bousso has refuted these arguments by considering that each causal diamond has its “own theory” [14]. This, however, still leaves the

question of why the infalling observer does not select a state that violates the equivalence principle by measuring early radiation.

In this paper, we argue that the firewall paradox can be resolved by considering carefully how a semi-classical world appears in a fully quantum mechanical description of the entire universe/multiverse. In Section 2, we review the argument by AMPS and discuss how the paradox can be avoided by treating quantum measurements as self-consistent dynamical processes occurring in unitary quantum mechanics. In Section 3, we consider the issue in the context of an explicit realization of complementarity described in Ref. [10], which applies to arbitrary dynamical spacetimes including cosmological ones. In this framework, complementarity is understood as a reference frame change represented in quantum gravitational Hilbert space, which corresponds to only limited regions in spacetime associated with a fixed reference frame. This implies that complementarity, as interpreted in Ref. [10], is a relation between full quantum states as viewed from different reference frames, and not a relation of different views in a fixed semi-classical background. We argue that the framework does not suffer from the firewall paradox. In Section 4 we conclude: the complementarity picture endures in the fully quantum mechanical context.

2 Resolution of the Paradox—Quantum Measurement and the Emergence of a Classical World

Following AMPS, let us consider a black hole that has formed from collapse of some pure state and has already emitted more than a half of its initial Bekenstein-Hawking entropy in the form of Hawking radiation (a black hole that is older than the Page time $t_{\text{Page}} \approx O(M^3)$, where M is the mass of the original black hole). Because of the purity of the state, the system can be written as

$$|\Psi\rangle = \sum_i c_i |\psi_i\rangle \otimes |i\rangle, \quad (1)$$

where $|\psi_i\rangle \in \mathcal{H}_{\text{rad}}$ represent states associated with Hawking radiation emitted early on and thus macroscopically away from the black hole, while $|i\rangle \in \mathcal{H}_{\text{BH}}$ the degrees of freedom associated with the region near the horizon. Because the black hole is old, the dimensions of the Hilbert space factors \mathcal{H}_{rad} and \mathcal{H}_{BH} satisfy $\dim \mathcal{H}_{\text{rad}} \gg \dim \mathcal{H}_{\text{BH}}$ [15]. Then, as argued by AMPS, states $|\psi_i\rangle$ for different i are expected to be nearly orthogonal, so that one can construct a projection operator P_i that acts only on \mathcal{H}_{rad} (*not* on \mathcal{H}_{BH}) but selects a particular term in Eq. (1) corresponding to a specific state $|i\rangle$ in \mathcal{H}_{BH} when operated on $|\Psi\rangle$:

$$P_i |\Psi\rangle \propto |\psi_i\rangle \otimes |i\rangle. \quad (2)$$

(Note that there is no summation on i here.) The point is that one can construct such an operator for an arbitrary state $|i\rangle$ in \mathcal{H}_{BH} .

Now, consider a distant observer who is located far away from the black hole horizon and an infalling observer who enters into the black hole after the Page time is passed. These two observers can both access early Hawking radiation, i.e. elements in \mathcal{H}_{rad} , causally. From the viewpoint of the distant observer, the remaining degrees of freedom in \mathcal{H}_{BH} are those located near/on the stretched horizon that later evolve into Hawking radiation quanta emitted into distant space. For the infalling observer, on the other hand, the degrees of freedom in \mathcal{H}_{BH} are those associated with the spacetime region near the horizon which he/she is traveling through.¹

The essence of the argument by AMPS is that since the infalling observer can access the early radiation, he/she can select a particular term in Eq. (1) by making a measurement on those degrees of freedom (due to the ability of constructing an operator P_i corresponding to an arbitrary $|i\rangle$). In particular, AMPS imagine that such a measurement would select a term in which $|i\rangle$ in \mathcal{H}_{BH} is an eigenstate of the number operator $b^\dagger b$ of a Hawking radiation mode as viewed from a distant observer, which we denote generically by $|\tilde{i}\rangle$ ($\in \mathcal{H}_{\text{BH}}$):

$$b^\dagger b |\tilde{i}\rangle \propto |\tilde{i}\rangle. \quad (3)$$

If this were true, then the infalling observer must find physics represented by $|\tilde{i}\rangle$ near the horizon, and since an eigenstate of $b^\dagger b$ cannot be a vacuum for the infalling modes a_ω , related to b by

$$b = \int_0^\infty d\omega (B(\omega)a_\omega + C(\omega)a_\omega^\dagger) \quad (4)$$

with some functions $B(\omega)$ and $C(\omega)$, the infalling observer must experience nontrivial physics at the horizon (i.e. $a_\omega |\tilde{i}\rangle \neq 0$ for infalling modes with the frequencies much larger than the inverse horizon size). This obviously contradicts the equivalence principle.

What can be wrong with this argument? The point is that any measurement that leads to a classical world is *a dynamical process* dictated by unitary evolution of a state, and *not* something we can impose from outside by acting with some projection operator on the state. (A detailed explanation of this point in the context of dynamical spacetimes/cosmology, see Ref. [10].) In particular, *the existence of the projection operator P_i for an arbitrary i does not imply that a measurement—in a sense that it leads to a classical world—can occur to pick up the corresponding state $|i\rangle$* . This point can be understood relatively easily if we consider a state corresponding

¹Following AMPS (and some other literature), here we take the “Heisenberg picture” in which both the distant and infalling observers access the same Hilbert space factor \mathcal{H}_{BH} ; these observers interpret it differently because the operator sets they use to extract physical effects on other systems, e.g. a freely falling object, are different (which must be the case because of relative acceleration between the two observers). On the other hand, in Refs. [9, 10], the “Schrödinger picture” has been employed; in this picture, the operator sets associated with the two observers are the same, but the Hilbert space factors are different: \mathcal{H}_{BH} and \mathcal{H}'_{BH} . These two pictures are equivalent—in the Heisenberg picture, changing the viewpoint corresponds to mapping the operator set associated with one observer to that associated with the other, while in the Schrödinger picture, it corresponds to mapping elements of one Hilbert space factor to those in the other. For more discussions on this and related points, see Section 3.

to a superposition of two different macroscopic configurations, for example that of upward and downward chairs (relative to some other object, e.g. the ground, which we omit):

$$|\Psi_{\text{chair}}\rangle = |\uparrow\rangle + |\downarrow\rangle. \quad (5)$$

An observer interacting with this system evolves following the unitary, deterministic Schrödinger equation; in particular, the combined chair and observer state becomes

$$|\Psi_{\text{chair+observer}}\rangle = |\uparrow\rangle \otimes |\uparrow_{\text{obs}}\rangle + |\downarrow\rangle \otimes |\downarrow_{\text{obs}}\rangle. \quad (6)$$

This does *not* lead to a classical world in which the chair is in a superposition state but to two different worlds in which the chair is upward and downward, respectively.² Namely, the measurement is performed in the particular basis $\{|\uparrow\rangle, |\downarrow\rangle\}$, which is *determined by the dynamics*—the existence of an operator projecting onto a superposition chair state does not mean that a measurement can be performed in that basis. For a sufficiently macroscopic object, the appropriate basis for measurements is almost always the one in which the object has well-defined configurations in classical phase space (within some errors, which must exist because of the uncertainty principle). This is because the Hamiltonian has the form that is local in spacetime [10].

In the specific context of the firewall argument, a measurement of the early radiation by an infalling observer will select a state in \mathcal{H}_{rad} that represents a well-defined classical configuration of Hawking radiation quanta $|\psi_I\rangle$ (which will be entangled with other classical objects, e.g. a measuring device monitoring the quanta). The point is that *there is no reason that the state $|\psi_I\rangle$ selected in this way is a state that is maximally entangled with $|\tilde{i}\rangle$, i.e. $|\psi_{\tilde{i}}\rangle$* . Indeed, since the states that represent well-defined classical worlds are very special among all the possible states³ and since the dimension of the Hilbert space factor spanned by $b^\dagger b$ is very small ($\dim \mathcal{H}_{\text{BH}} \ll \dim \mathcal{H}_{\text{rad}}$), we expect that $|\psi_I\rangle$ does not coincide with $|\psi_{\tilde{i}}\rangle$ for any \tilde{i} ; see the Appendix for a sample calculation demonstrating this. Namely, the basis in \mathcal{H}_{rad} selected by entanglement with the eigenstates of $b^\dagger b$ is different from the one selected by entanglement with the infalling observer, i.e. by measurements. This implies that the state projected by $P_{\tilde{i}}$, considered by AMPS, does *not* correspond to a classical world for the infalling observer, but rather to a superposition of macroscopically different worlds. And since the equivalence principle is a statement about a (semi-)classical world, there is no contradiction between the complementarity picture and the equivalence principle as envisioned by

²Note that a classical world can be defined as the world in which the information/observation is stable/reproducible [16, 17]. In the example here, the information that the chair is upward or downward is reproduced—or “amplified”—in many physical systems, e.g. in the chair itself, the brain state of the observer, in the conversation he/she has about the chair, etc. This, however, cannot be the case for the information that the chair is in a superposition state because of the property of the dynamics.

³In the example of a chair, those are the states having $\alpha \approx 0$ or $\pi/2$ when we write a general chair state as $|\Psi_{\text{chair}}\rangle = \cos \alpha |\uparrow\rangle + e^{i\varphi} \sin \alpha |\downarrow\rangle$.

AMPS. In other words, in a classical world for the infalling observer, in which the equivalence principle holds, the state in \mathcal{H}_{rad} is always in a superposition of $|\psi_{\tilde{i}}\rangle$'s for different \tilde{i} 's, so that the corresponding state in \mathcal{H}_{BH} is *not* an eigenstate of $b^\dagger b$ —in particular, there is no contradiction if it is a simultaneous eigenstate of a_ω 's with the eigenvalue zero.

In fact, in order to make the above argument, we need not consider a physical observer who is actually falling into the black hole. The Hawking radiation quanta in the early radiation has a natural basis $|\Psi_I\rangle$ in which the information is amplified, i.e. the basis corresponding to (emergent) classical worlds; and if we focus on one of these terms, then the conclusion by AMPS is avoided. A state projected by $P_{\tilde{i}}$ corresponds to a superposition of different macroscopic worlds, which is not the one described by classical general relativity. Conversely, a state corresponding to a well-defined classical world is one in which the state in \mathcal{H}_{rad} is a superposition of different $|\psi_{\tilde{i}}\rangle$'s, so that the corresponding state in \mathcal{H}_{BH} is not an eigenstate of $b^\dagger b$.

The picture described here can also be made explicit in the context of the framework in Ref. [10]. In this framework, complementarity is realized for general dynamical spacetimes as a reference frame change represented in quantum gravitational Hilbert space \mathcal{H}_{QG} , whose elements correspond only to limited spacetime regions as viewed from a fixed (local Lorentz) reference frame. We now turn to discussions on this picture.

3 Complementarity as a Reference Frame Change

The complementarity picture refers to the fact that assigning physical degrees of freedom in all the global spacetime regions as naively suggested by quantum field theory is overcounting. These degrees of freedom must be divided by gauge redundancies associated with a consistent quantum theory of gravity, which are expected to be much larger than those in general relativity. The reduction of degrees of freedom appearing in a black hole geometry discussed so far is expected to be one manifestation of this more general phenomenon.

In the treatment in the previous section, the reduction has been realized in such a way that the horizon degrees of freedom as viewed from the distant observer and the degrees of freedom located near the horizon as viewed from the infalling observer are represented by the same Hilbert space factor \mathcal{H}_{BH} . These two observers interpret states in \mathcal{H}_{BH} differently because the operator sets used to extract physical implications, e.g. on objects carried by them, differ for the two observers because of relative acceleration between them. Equivalently, we may formulate the same phenomenon such that the two observers use the same set of operators \mathcal{O}_X but the Hilbert space factors they access are different: \mathcal{H}_{BH} and \mathcal{H}'_{BH} , whose elements represent only *limited*, and *different* spacetime regions (see also footnote 1). This is the picture adopted in Refs. [9, 10], and corresponds to describing all the phenomena from the viewpoint of a fixed, freely falling—or local Lorentz—reference frame

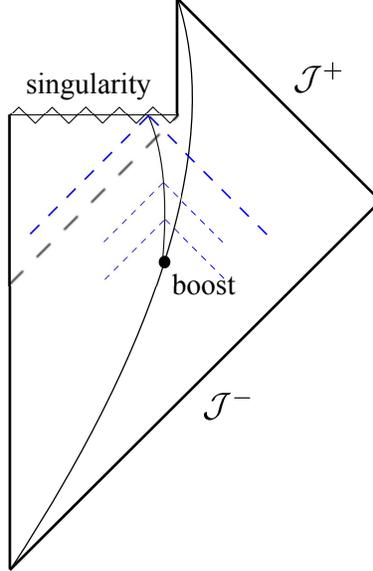


Figure 1: A Penrose diagram showing the trajectories of the reference point p for two reference frames—distant (right curve) and infalling (left curve)—in a black hole background. The two frames can be related by a boost acted at some time, represented by the dot on which the two curves merge. The dashed hats attached to the left curve illustrate (null) hypersurfaces on which states, as viewed from the infalling frame, are defined. Here, we represented the setup in a fixed classical geometry for presentation purposes, but a full quantum state is in general a superposition of terms representing well-defined classical geometries.

(if one chooses the fixed operator set \mathcal{O}_X to be that of the inertial frame).

In this picture, complementarity between physics described by two observers—or rather, in two reference frames—is nothing but a unitary transformation represented in the full Hilbert space \mathcal{H}_{QG} that contains both \mathcal{H}_{BH} and \mathcal{H}'_{BH} . In particular, building on earlier progresses in quantum gravity, such as holography [18, 19, 20] and complementarity [2, 3], Ref. [10] has proposed a particular structure for the Hilbert space for quantum gravity \mathcal{H}_{QG} . According to this proposal, \mathcal{H}_{QG} contains a component \mathcal{H} that represents general spacetimes as well as $\mathcal{H}_{\text{sing}}$ that represents spacetime singularities: $\mathcal{H}_{\text{QG}} = \mathcal{H} \oplus \mathcal{H}_{\text{sing}}$. (The $\mathcal{H}_{\text{sing}}$ component is needed to preserve unitarity of the evolution of a generic state.) In particular, elements in \mathcal{H} correspond to states defined on the past light cone of a fixed reference point (the “spatial” origin of the reference frame) in and on the apparent horizon. This interpretation of states in spacetime arises through responses of these states to the action of operators \mathcal{O}_X , and the use of the past light cone, although not mandatory, allows us to formulate the theory (the algebra among \mathcal{O}_X ’s) independently of the background spacetime. (More details of this construction will be discussed elsewhere [22].)

The framework of Ref. [10] is sufficiently general and explicit that we can describe the picture advocated in the previous section in a more concrete form. To be specific, let us consider a

process in which a physical (test) object falls into an old black hole after the Page time. Suppose we describe this process using a reference frame whose spatial origin p stays outside the horizon throughout the entire history of the black hole, until it completely evaporates. (Such a trajectory of p is depicted in Fig. 1.) Then for a generic initial state $|\Psi_0\rangle$, the state describing the system evolves as

$$|\Psi_0\rangle \xrightarrow{\text{object falls}} |\Psi\rangle = \sum_i |\psi_i\rangle \otimes |i\rangle \xrightarrow{\text{BH evaporates}} |\Psi_\infty\rangle, \quad (7)$$

where $|\psi_i\rangle \in \mathcal{H}_{\text{rad}}$, $|i\rangle \in \mathcal{H}_{\text{BH}}$, and $|\Psi_\infty\rangle$ is a final state after the black hole evaporates.⁴ In this description, once the test object falls into the black hole, it is absorbed into the (stretched) horizon, which will eventually give the information about the object back in late Hawking radiation.

Now, imagine that the falling object is entangled with a particular term in $|\Psi\rangle$ in Eq. (7), with the corresponding $|\psi_i\rangle$ representing a well-defined configuration of radiation quanta in classical phase space. Given the structure of the Hamiltonian, i.e. locality, this is a perfectly legitimate case to consider. The relevant part of the evolution is then

$$|\Psi_0\rangle \xrightarrow{\text{object falls}} |\Psi\rangle = |\psi_i\rangle \otimes |i\rangle \xrightarrow{\text{BH evaporates}} |\Psi_\infty\rangle, \quad (8)$$

where we have kept only the term of the form $|\psi_i\rangle \otimes |i\rangle$ in $|\Psi\rangle$, without a summation on i . The picture discussed in the previous section then says that $|i\rangle$ in $|\Psi\rangle$ does *not* evolve into an eigenstate of the number operator $b^\dagger b$ for a late Hawking radiation mode. From the viewpoint of the distant observer, this simply says that the final state $|\Psi_\infty\rangle$ in Eq. (8) is a superposition of different worlds in which the configuration of late Hawking radiation differ.

What happens if we describe the classical world/history represented in Eq. (8) from a reference frame whose spatial origin p falls into the black hole together with the test object? The resulting description can be obtained by first boosting the entire state (or equivalently the reference frame) at the initial moment, $|\Psi_0\rangle \rightarrow |\Psi'_0\rangle = U_{\text{boost}} |\Psi_0\rangle$, and then following the evolution of that state to the future (see Fig. 1 for a schematic depiction of this procedure):

$$|\Psi'_0\rangle \xrightarrow{\text{object falls}} |\Psi'\rangle = |\psi'_i\rangle \otimes |i'\rangle \xrightarrow{p \text{ hits singularity}} |\Psi'_\infty\rangle, \quad (9)$$

where $|\psi'_i\rangle \in \mathcal{H}_{\text{rad}}$ (i.e. $|\psi_i\rangle$ as viewed from the new reference frame), $|i'\rangle \in \mathcal{H}'_{\text{BH}}$, and $|\Psi'_\infty\rangle \in \mathcal{H}_{\text{sing}}$. Since $|i\rangle$ in Eq. (8) is not an eigenstate of $b^\dagger b$, there is no contradiction if $|i'\rangle$ is a simultaneous zero-eigenvalue eigenstate of the annihilation operators a_ω 's in the infalling frame: $a_\omega |i'\rangle = 0$, as suggested by the equivalence principle. The complementarity picture endures.

⁴Evolution of a state is possible despite the fact that time translation is a gauge transformation, since the state has a boundary (the apparent horizon) so that it may not be an eigenstate of the Hamiltonian. This is analogous to the situation in flat space, where the contribution from surface terms [23] allows a state not to vanish under the action of the Hamiltonian: $H|\psi\rangle \neq 0$. (This is the reason why the S -matrix can ever be discussed in string theory in the flat background.) For more discussions on this point, see Ref. [11].

The setup considered by AMPS corresponds to projecting $|\Psi\rangle$ in Eq. (7) onto a state that is an eigenstate $|\tilde{i}\rangle$ of $b^\dagger b$:

$$|\Psi\rangle \rightarrow P_{\tilde{i}} |\Psi\rangle = |\psi_{\tilde{i}}\rangle \otimes |\tilde{i}\rangle. \quad (10)$$

What is this state? To answer this question, note that the argument in the Appendix implies that $|\psi_{\tilde{i}}\rangle$ is a superposition of terms representing different classical worlds:

$$|\psi_{\tilde{i}}\rangle = \sum_a d_a |\psi_a\rangle, \quad (11)$$

where $|\psi_a\rangle \in \mathcal{H}_{\text{rad}}$ represent states having well-defined classical configurations of emitted Hawking quanta, which will be maximally entangled with other classical objects. This state, therefore, is not the one described by general relativity, which is a theory for a classical world, i.e. a basis state in which the information is naturally amplified by the local Hamiltonian.⁵

Finally, we point out an interesting aspect of the black hole evolution represented in Eq. (8). To discuss it, let us regard emission of Hawking radiation quanta by the black hole as a stochastic process in which the black hole performs a random walk in momentum space due to backreaction of the Hawking emission. In a timescale of order the Page time, the black hole emits order $N_H \sim M/T_H \sim M^2$ Hawking quanta, where $T_H \sim 1/M$ is the Hawking temperature. This implies that each emission occurs with the average time interval of order $t_{\text{Page}}/N_H \sim M$, with each emission providing $\Delta v \sim \Delta p/M \sim T_H/M \sim 1/M^2$ kick in velocity space, leading to $|v| \sim \Delta v \sqrt{t_{\text{Page}}/M} \sim 1/M$ at $t \sim t_{\text{Page}}$, whose direction stays almost constant in each kick. The location of the black hole, therefore, has large uncertainty of order $|v| t_{\text{Page}} \sim M^2 \gg r_S$ after the Page time, giving huge variations of the geometries.⁶ More detailed discussion of the implications of this large uncertainty will be given in a forthcoming paper.

4 Conclusions

In this paper we have argued that the complementarity picture endures despite the “firewall challenge” recently posed by AMPS. The point is that states corresponding to well-defined classical worlds are very special ones in quantum mechanics, which are determined by the dynamics as a result of amplification of information, i.e. by the evolution of the system with an environment

⁵The argument here implies that one cannot treat a carefully-crafted quantum device that can collect the information in the Hawking radiation quanta to learn that they are in a $|\psi_{\tilde{i}}\rangle$ state and send it to the infalling observer, within the context of a classical description of physics. Since the information is encoded in Hawking quanta in a highly scrambled form, such a device would have to be very large collecting many quanta spread in space without losing their coherence, and would be in a superposition of different classical configurations as indicated by Eq. (11). Given that $\dim \mathcal{H}_{\text{rad}} \gg \dim \mathcal{H}_{\text{BH}}$, this requires an exponentially fine-tuned (and intrinsically quantum mechanical) initial condition for the device state, and physics occurring to the infalling observer entangled with such a device cannot be well described by general relativity, which is a theory of a classical world.

⁶This large uncertainty of the black hole location was noted earlier in Ref. [24].

(such as a measuring device, observer, etc.). A vast majority of the states in general Hilbert space, including the ones considered by AMPS, are superpositions of different classical worlds. And since general relativity is a theory describing a classical world, results obtained in it need not persist in those general superposition states.

Another issue we have discussed is that complementarity, as interpreted in Ref. [10], is a relation between full quantum states, which can involve superpositions of classical spacetimes, as viewed from different reference frames—it is not a relation of different views in a fixed semi-classical background, as might have been imagined in some earlier work. Further implications of this fact for physics of black holes will be discussed in a forthcoming paper.

Note added:

While completing this paper, we received a paper by Harlow [21] which also discusses the firewall paradox.

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A Misalignment between the Classical State Basis and the Basis Selected by $b^\dagger b$

In this appendix we argue that if the black hole is sufficiently old, there is an exponential suppression of the probability that $|\psi_{\tilde{i}}\rangle$, corresponding to *some* $b^\dagger b$ eigenstate $|\tilde{i}\rangle$, represents a well-defined classical world.

Let $N_R \equiv \dim \mathcal{H}_{\text{rad}}$ and $N_{\text{BH}} \equiv \dim \mathcal{H}_{\text{BH}}$, and consider the expansion of a pure state in Hilbert space $\mathcal{H}_{\text{rad}} \otimes \mathcal{H}_{\text{BH}}$ in the form

$$|\Psi\rangle = \sum_{\tilde{i}=1}^{N_{\text{BH}}} c_{\tilde{i}} |\psi_{\tilde{i}}\rangle \otimes |\tilde{i}\rangle, \quad (12)$$

where $|\tilde{i}\rangle$ represent the $b^\dagger b$ eigenstates. We assume that \mathcal{H}_{rad} has an orthonormal “classical state basis” $\{e_n \mid n = 1, \dots, N_R\}$, where e_n represent states that have well-defined configurations in classical phase space (within some errors). In practice, this basis is determined by the interaction between the early Hawking radiation and (classical) environment.

We now estimate the probability that there exists an \tilde{i} such that the corresponding $|\psi_{\tilde{i}}\rangle$ is “close” to one of the vectors $|e_n\rangle$. To proceed, it is necessary to introduce the (arbitrary) notion of a state being “almost classical.” To this end, we introduce a small positive real number ϵ and say that $|\psi_{\tilde{i}}\rangle$ is close to $|e_n\rangle$ if $|\langle e_n|\psi_{\tilde{i}}\rangle| > 1 - \epsilon$.

Since \mathcal{H}_{rad} is an N_{R} -dimensional complex vector space, we can identify it with a $2N_{\text{R}}$ -dimensional real vector space. Choose a fixed $|\psi_{\tilde{j}}\rangle$ and $|e_m\rangle$, and rotate the coordinates such that $|e_m\rangle$ is the $2N_{\text{R}}$ -dimensional vector $(1, 0, \dots, 0)$. The vector $|\psi_{\tilde{j}}\rangle$ can then be identified as a normalized vector in the $2N_{\text{R}}$ -dimensional space i.e. a point on the $(2N_{\text{R}} - 1)$ -dimensional unit sphere, $S^{2N_{\text{R}}-1}$. We wish to compute the probability, $P(\tilde{j}, m)$, that the condition $|\langle e_m|\psi_{\tilde{j}}\rangle| > 1 - \epsilon$ is satisfied. Assuming that $|\psi_{\tilde{j}}\rangle$ is equally likely to lie on any point on the sphere, $P(\tilde{j}, m)$ is the twice the area of the spherical cap

$$C = \{(x_1, \dots, x_{2N_{\text{R}}}) \in S^{2N_{\text{R}}-1} \mid x_1 > 1 - \epsilon\}, \quad (13)$$

divided by the area of $S^{2N_{\text{R}}-1}$ for normalization. (The factor of 2 arises because we must consider both the $x_1 > 1 - \epsilon$ and the $x_1 < -(1 - \epsilon)$ caps.) Since ϵ is small, C is approximately a solid ball of radius $\sqrt{2\epsilon}$. Carrying out this calculation and using the fact that $N_{\text{R}} \gg 1$, one finds that $P(\tilde{j}, m) \sim (2\epsilon)^{N_{\text{R}}} / \sqrt{N_{\text{R}}}$.

Finally, the probability that any of the $|\psi_{\tilde{i}}\rangle$'s is close to any of the $|e_n\rangle$'s is $P = N_{\text{R}} N_{\text{BH}} P(\tilde{j}, m)$. (The factor of N_{R} arises because there are N_{R} classical basis vectors and the factor of N_{BH} because there are N_{BH} distinct $|\psi_{\tilde{i}}\rangle$.) We thus conclude that

$$P \sim \sqrt{N_{\text{R}}} N_{\text{BH}} (2\epsilon)^{N_{\text{R}}}. \quad (14)$$

The probability that any of the $|\psi_{\tilde{i}}\rangle$'s represents a classical world is exponentially suppressed in N_{R} for a sufficiently old black hole.

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