Prepared for submission to JCAP

Is there life inside black holes?

Vyacheslav I. Dokuchaev[a]

[a]Institute for Nuclear Research of the Russian Academy of Sciences,
60th October Anniversary prospect 7a, 117312 Moscow, Russia

E-mail: dokuchaev@ms2.inr.ac.ru

Abstract. Inside the rotating or charged black holes there are bound periodic planetary orbits, which not coming out nor terminated at the central singularity. The stable periodic orbits inside black holes exist even for photons. We call these bound orbits by the orbits of the third kind, following to Chandrasekhar classification for particle orbits in the black hole gravitational field. It is shown that an existence domain for the third kind orbits is a rather spacious, and so there is a place for life inside the supermassive black holes in the galactic nuclei. The advanced civilizations of the third kind (according to Kardashev classification) may inhabit the interiors of supermassive black holes, being invisible from the outside and basking as in the light of the central singularity and the orbital photons.

Keywords: GR black holes, massive black holes

Contents

1 Introduction **1**

2 Stable periodic orbits inside BH **1**
2.1 Circular orbits inside nonrotating charged BH 2
2.2 Spherical orbits of neutral particles inside BH 4
2.3 Spherical orbits of photons inside BH 4
2.4 Circular orbits of photons inside BH 6

3 Conclusions **6**

1 Introduction

A voyage inside the Black Hole (BH) event horizon may finished after a finite proper time of the traveler not in the central singularity, but again outside the event horizon. However, it would not be a returning to the native universe, but emerging in the *other* universe due to the complicated internal BH geometry [1–4]. Is it possible live too long inside the BH by avoiding both the downfall to the central singularity and escaping to the other universe? The answer is positive, if a target BH is rotating, charged and massive enough for weakening to the acceptable level of tidal forces and radiation of gravitational waves. After traversing the BH event horizon at radius $r = r_+$, a traveler will appear in the T-region [5], where his radial coordinate r is becoming the temporal one and inevitably diminishing toward the central singularity. The irresistible infall in the T-region will finish soon after traversing the inner Cauchy horizon at $r = r_- < r_+$, which is nonzero for the rotating or charged BH. The internal space-time domain $0 < r < r_-$ between the central singularity and the inner BH horizon is the R-region, where the stationary observers may exist, just as anywhere at the planet Earth. This internal BH domain, hidden by two horizons from the whole external universe, is indeed a suitable place for save inhabitation. The only is needed, is to put your vehicle or your planet to a stable periodic orbit inside BH. Below we discuss some specific properties of stable periodic orbits of planets and photons inside the rotating charged BH, described by the Kerr-Newman metric.

2 Stable periodic orbits inside BH

Subramanyan Chandrasekhar [6] designated two general types of particle orbits in the BH gravitational field: orbits of the *first kind*, which completely confined outside the BH event horizon, and orbits of the *second kind*, which penetrate inside the BH. We propose here to distinguish also the orbits of the *third kind*, which are completely bounded inside the BH, not escaping outside and nor infalling into the central singularity. The bound orbits of the third kind inside the inner BH horizon were found by Jiří Bičák, Zdeněk Stuchlík and Vladimír Balek [7, 8] for charged particles around the rotating charged BHs (see also [9, 10]) and by Eva Hackmann, Claus Lämmerzahl, Valeria Kagramanova and Jutta Kunz [11] for neutral particles around rotating BHs (see also [12]). The bound orbits of the third kind are periodic and stable by neglecting the gravitational and electromagnetic radiation.

Geodesics equations for the neutral test particles and photons and equations of motion for the charged particles in the Kerr-Newman metric were derived by B. Carter [13]. According to these equations the motion of test particle with a mass μ and electric charge ϵ in the background gravitational field of the BH with a mass M, angular momentum $J = Ma$ and electric charge e is completely defined by three integrals of motion: the total particle energy E, the azimuthal component of the angular momentum L and the Carter constant Q, related with a total angular momentum of the particle. The Carter constant is zero, $Q = 0$, or trajectories confined in the BH equatorial plane. In particular, a total angular momentum of the particle is $\sqrt{Q + L^2}$ in the case of nonrotating BH.

An orbital trajectory of test planet is governed in the Boyer-Lindquist coordinates (t, r, θ, φ) by equations of motion [13, 14]:

$$\rho^2 \frac{dr}{d\lambda} = \pm\sqrt{V_r}, \tag{2.1}$$
$$\rho^2 \frac{d\theta}{d\lambda} = \pm\sqrt{V_\theta}, \tag{2.2}$$
$$\rho^2 \frac{d\varphi}{d\lambda} = L \sin^{-2}\theta + a(\Delta^{-1}P - E), \tag{2.3}$$
$$\rho^2 \frac{dt}{d\lambda} = a(L - aE\sin^2\theta) + (r^2 + a^2)\Delta^{-1}P, \tag{2.4}$$

where $\lambda = \tau/\mu$, τ — is a proper time of particle and

$$V_r = P^2 - \Delta[\mu^2 r^2 + (L - aE)^2 + Q], \tag{2.5}$$
$$V_\theta = Q - \cos^2\theta[a^2(\mu^2 - E^2) + L^2\sin^{-2}\theta], \tag{2.6}$$
$$P = E(r^2 + a^2) + \epsilon e r - aL, \tag{2.7}$$
$$\rho^2 = r^2 + a^2\cos^2\theta, \tag{2.8}$$
$$\Delta = r^2 - 2r + a^2 + e^2. \tag{2.9}$$

We will use mainly the normalized dimensionless variables and parameters: $r \Rightarrow r/M$, $a \Rightarrow a/M$, $e \Rightarrow e/M$, $\epsilon \Rightarrow \epsilon/\mu$, $E \Rightarrow E/\mu$, $L \Rightarrow L/(M\mu)$, $Q \Rightarrow Q/(M^2\mu^2)$. The radius of BH event horizon $r = r_+$ and the radius of BH inner horizon $r = r_-$ are both the roots of equation $\Delta = 0$: $r_\pm = 1 \pm \sqrt{1 - a^2 - e^2}$.

The effective potentials V_r and V_θ in (2.5) and (2.6) define the motion of particles in r- and θ-directions [14]. In particular, for a circular orbit at some radius r, equations (2.5) and (2.6) give the conditions:

$$V_r(r) = 0, \quad V_r'(r) \equiv \frac{dV_r}{dr} = 0. \tag{2.10}$$

The circular orbits would be stable if $V_r'' < 0$, i. e. in the minimum of the effective potential. In the case of rotating BH (with $a \neq 0$), the particle on the orbit with $r = const$ may additionally moving in the latitudinal θ-direction. These nonequatorial orbits are called the spherical ones [15]. The purely circular orbits will correspond to the particular case of spherical orbits with parameter $Q = 0$, completely confined in the BH equatorial plane.

2.1 Circular orbits inside nonrotating charged BH

We consider at first the Reissner-Nordström case of nonrotating charged BH. From the joint resolution of equations (2.10) we find two pairs of solutions, respectively, for the energy E

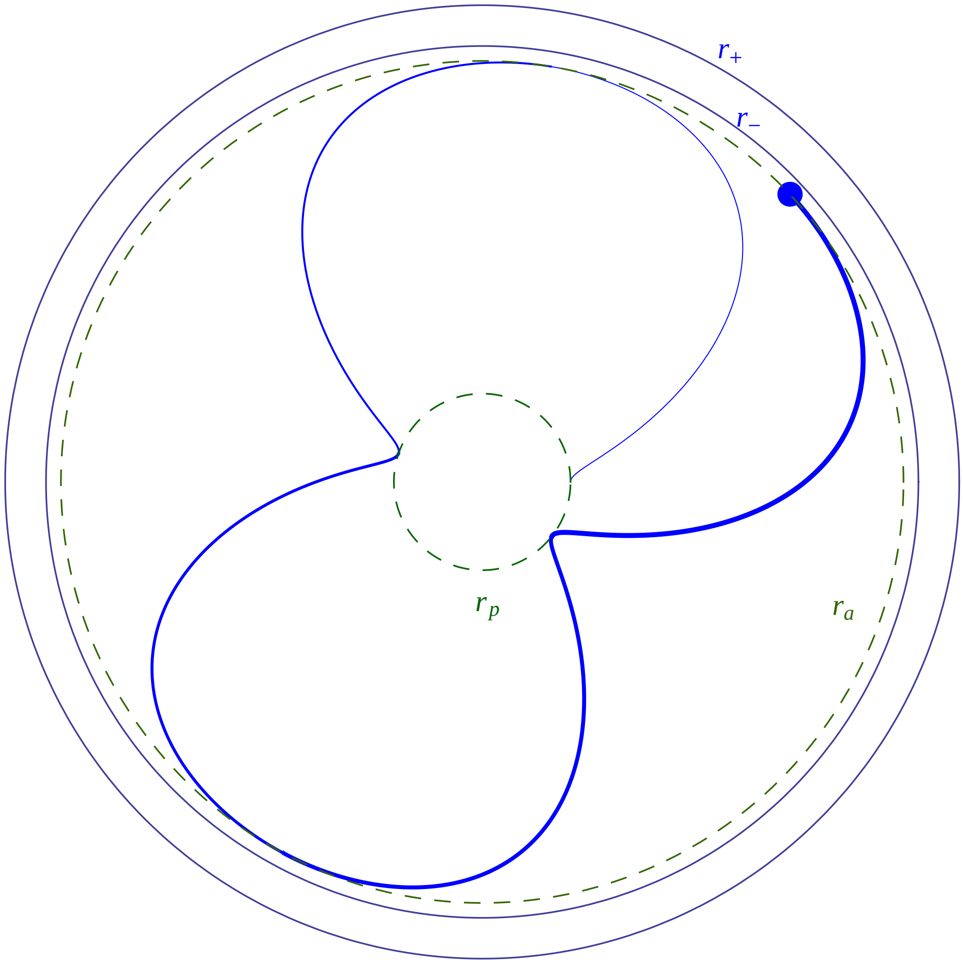

Figure 1. The stable periodic orbit of planet with a mass μ and charge $\epsilon = -1.45\mu$ entirely inside the BH with a mass M and charge $e = 0.999M$. Orbit parameters: $E = 1.5\mu$, $L = 0.2M\mu$, azimuthal and radial periods $(T_\varphi, T_r) = (14.9M, 7.17M)$, the perigee and apogee radii $(r_p, r_a) = (0.19M, 0.92M)$.

and angular momentum L of massive particle with charge ϵ on the circular orbit with radius r:

$$E_{1,2} = \frac{\pm\Delta D_1 - e\epsilon(r^2 - 4r + 3e^2)}{2r(r^2 - 3r + 2e^2)}, \tag{2.11}$$

$$L^2_{1,2} = \frac{r^2}{r^2 - 3r + 2e^2}\left[r - e^2 + \frac{e\epsilon\Delta(e\epsilon \pm D_1)}{2(r^2 - 3r + 2e^2)}\right]. \tag{2.12}$$

where

$$D_1^2 = e^2(\epsilon^2 + 8) + 4r(r - 3). \tag{2.13}$$

The stability condition $V_r''(r, E, L) < 0$ for circular orbits inside the inner horizon, $0 < r < r_-$, is satisfied for the first pair of solution $(E_1, L_1,)$ in equations (2.11) and (2.12), if $\epsilon > \mu$, and for the second pair $(E_2, L_2,)$, if $\epsilon < -\mu$. In figure 1 is an example of the stable periodic orbit of charged particle inside the nonrotating BH, calculated from equations (2.1) and (2.3), by using a general formalism for motion in the central field [16].

2.2 Spherical orbits of neutral particles inside BH

In the Kerr-Newman case the stable circular orbits (and also the stable spherical ones) exist inside the inner horizon not only for charged particles, but also for neutral ones ($\epsilon = 0$) and photons ($\mu = 0$). From equations (2.10) we find the corresponding two pairs of solutions for energy E and azimuthal impact parameter $b = L/E$ of neutral massive particle on the spherical orbit:

$$
\begin{aligned}
E_{1,2}^2 &= \frac{\mp 2D_2 + \beta_1 r^2 + a^2[2(r-e^2)\Delta - r^2(r-1)^2]Q}{r^4[(r^2-3r+2e^2)^2 - 4a^2(r-e^2)]}, \\
b_{1,2} &= \frac{\pm D_2 r - a^2(r-e^2)\{\beta_2 r + [a^2 - r(r-e^2)]Q\}}{a(r-e^2)\{r[(\Delta - a^2)^2 - a^2(r-e^2)] + a^2(1-r)Q\}},
\end{aligned}
\tag{2.14}
$$

where

$$
\begin{aligned}
\beta_1 &= (r^2-3r+2e^2)(r^2-2r+e^2)^2 - a^2(r-e^2)[r(3r-5)+2e^2], \\
\beta_2 &= e^4 - a^2(r-e^2) + 2e^2 r(r-2) - r^2(3r-4), \\
D_2^2 &= [a(r-e^2)\Delta]^2[(r-e^2)r^4 - r^2(r^2-3r+2e^2)Q + a^2Q^2].
\end{aligned}
\tag{2.15}
$$

It can be verified that stable spherical orbits are realized for the first pair of solution (E_1, b_1) with $0 < Q < Q_{\max}$, where $Q_{\max}$ is a root of the marginal stability equation $V_r'' = 0$. All spherical orbits with $Q < 0$ are unstable (see also [15]). The analogous formulae for spherical orbits of the charged particles in the Kerr-Newman case are very cumbersome, and we not reproduce them here.

2.3 Spherical orbits of photons inside BH

The spherical photon orbit correspond to the ultrarelativistic limit for massive particle energy on the spherical orbit, $E \to \infty$. This limit is equivalent to the case $\mu = 0$. Photon orbit depends on two parameters, the azimuthal impact parameter $b = L/E$ and the latitudinal (tangential) impact parameter $q = Q/E^2$. Equations (2.14) in the ultrarelativistic limit are very simplified:

$$
b_1 = \frac{a^2(1+r) + r(r^2-3r+2e^2)]}{a(1-r)}, \qquad b_2 = \frac{a^2+r^2}{a}, \tag{2.16}
$$

$$
q_1 = \frac{r^2[4a^2(r-e^2) - (r^2-3r+2e^2)^2]}{a^2(1-r)^2}, \qquad q_2 = -\frac{r^4}{a^2}. \tag{2.17}
$$

Here the first pair of impact parameters, (b_1, q_1), corresponds to the stable spherical photon orbits, while the second pair, (b_2, q_2), to the unstable ones. The stability condition $V_r'' \leq 0$ for spherical photons with the first pair of impact parameters (b_1, q_1) is written in the form

$$
a^2 + e^2 - r(r^2 - 3r + 3) \leq 0. \tag{2.18}
$$

From here we find the upper limit of impact parameter $q_1 > 0$ for stable spherical photons:

$$
q_1 \leq \left(\frac{1-\delta^{1/3}}{a}\right)^2 [3 - 4e^2 - 2(3-2e^2)\delta^{1/3} + 3\delta^{2/3}], \tag{2.19}
$$

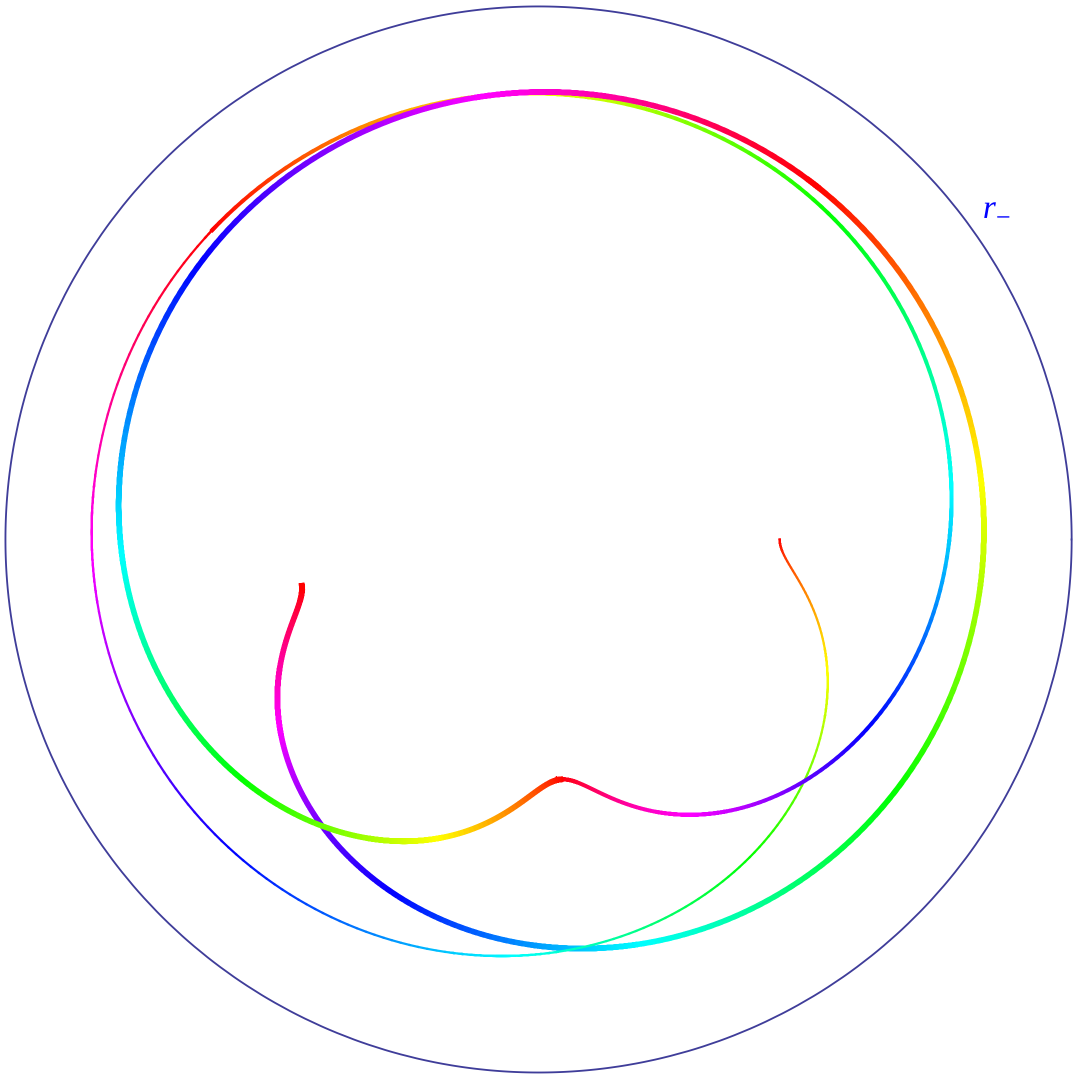

Figure 2. The stable periodic equatorial photon orbit with the impact parameter $b = 1.53$ inside the BH with $a = 0.75$ and $e = 0.6$. Orbit parameters: $(T_r, T_\varphi) = (2.7, 2.1)$, $(r_p, r_a) = (0.33, 0.61)$. The external circle is the inner horizon with $r = r_-$. Note that the equatorial photon, starting at azimuth angle $\varphi = 0$, is orbiting in opposite direction with respect to BH rotation.

where $\delta = 1 - a^2 - e^2$. The maximal allowable value for q_1 is reached for the extreme BH with $a = \sqrt{1 - e^2} \leq 1/2$, $e \leq \sqrt{3}/2$:

$$q_{1,max} = 4 - a^{-2} \leq 3. \tag{2.20}$$

In figure 2 is shown an example of stable periodic equatorial photon orbit inside the slightly charged but near extremely rotating BH. Respectively, see in figures 3 and 4 the examples of stable periodic planet and photon orbits inside a slightly charged but near extremely rotating BH with the canonical specific angular momentum $a_{\rm lim} = 0.09982$ due to untwisting by a thin accretion disk [17].

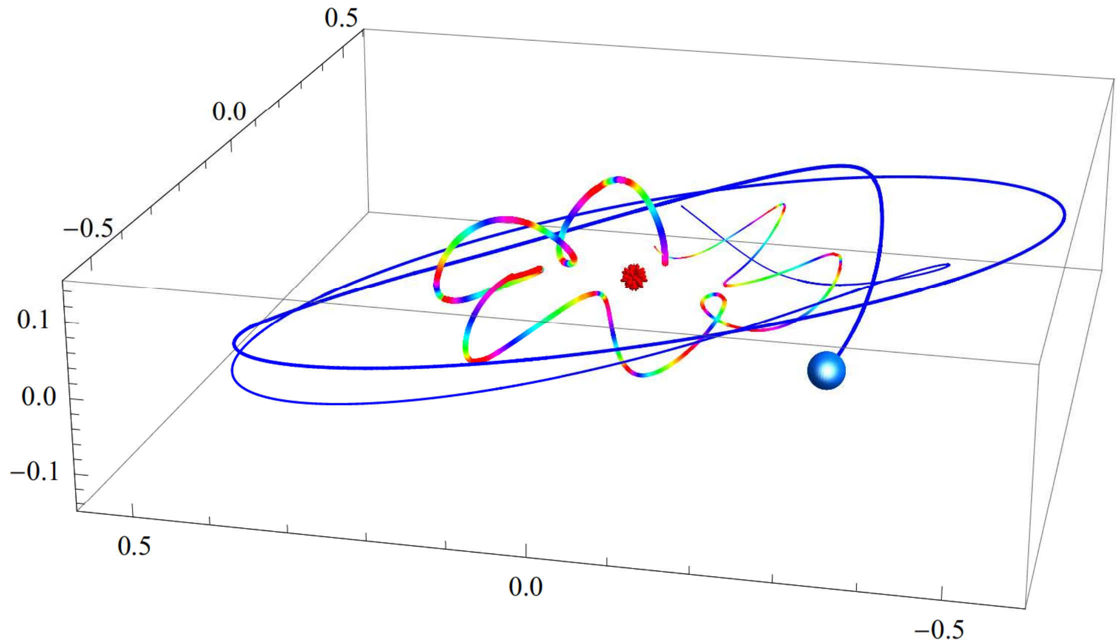

Figure 3. *Outer curve*: the stable periodic orbit of planet with orbital parameters $(E, L, Q) = (0.568, 1.13, 0.13)$, periods $(T_\varphi, T_r, T_\theta) = (1.63, 3.70, 1.17)$, apogee and perigee radii $(r_p, r_a) = (0.32, 0.59)$ and the maximum angle of latitude elevation relative to the equatorial plane $\theta_{\max} = 14.6°$ inside the BH with $a = 0.9982$ and $e = 0.05$. *Inner curve*: the stable periodic nonequatorial photon orbit with orbit parameters: $(b, q) = (1.38, 0.03)$, $(T_\varphi, T_r, T_\theta) = (2.95, 0.49, 0.33)$, $(r_p, r_a) = (0.14, 0.29)$ and $\theta_{\max} = 10.1°$ inside the same BH. The starting parts of orbits are thin, while the ending parts are thick

2.4 Circular orbits of photons inside BH

The spherical orbits become circular in the particular case of $Q = 0$. A corresponding relation for circular photon orbits follows in the relativistic limit from equation (2.14) or from (2.16) and (2.17):

$$4a^2(r - e^2) = (r^2 - 3r + 2e^2)^2. \tag{2.21}$$

with two possible solutions for impact parameter

$$b_{1,2} = \frac{a\beta_2 \pm r^2\sqrt{(r - e^2)\Delta^2}}{(r^2 - 2r + e^2)^2 - a^2(r - e^2)}, \tag{2.22}$$

where Δ and β_2 are, respectively, from (2.9) and (2.15). In (2.22) the first solution q_1 (with a plus sign) corresponds to the stable orbit, whereas q_2 corresponds to the unstable one. See in figure 5 the $3D$ domain of existence for the stable circular photons inside BH. These orbits exist at $e^2 \le r \le (4/3)e^2$, $a \ne 0$, $0 < e \le \sqrt{3}/2$ and $0 < b < 5/2$.

3 Conclusions

Inside the inner Cauchy horizon of rotating charged BH there are the stable periodic orbits of particles (planets) and photons. In the case of nonrotating charged BH the stable periodic orbits exist only for particles with a large enough charge. All stable periodic planet and

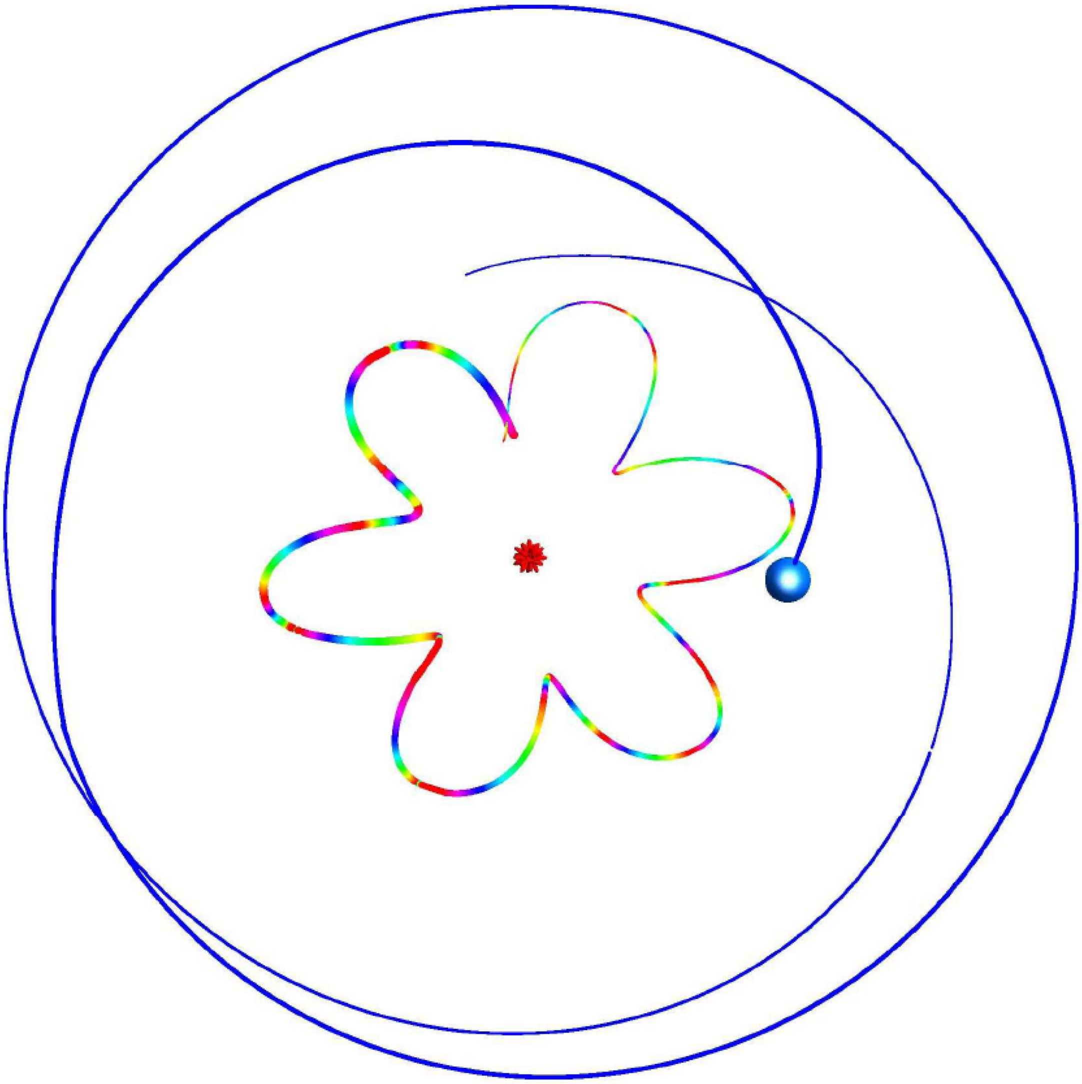

Figure 4. The stable periodic orbits of photon and planet (shown in figure 3) viewed from the coordinate frame north pole. The naked central singularity is glowing in the center. The starting parts of orbits are thin and the finishing parts are thick.

photon orbits inside the rotating and noncharged BH are nonequatorial. The advanced civilizations may live safely inside the supermassive BHs in the galactic nuclei without being visible from outside. The naked central singularity illuminates the orbiting planets and provide the energy supply needed for living. An additional highlighting at night come from the eternally circulating photons. It worth to mention also some troubles (or advantages?) for living inside BH: the possible causality violation [13, 18, 19], and the growing energy density in the very vicinity of Cauchy horizon [20–24].

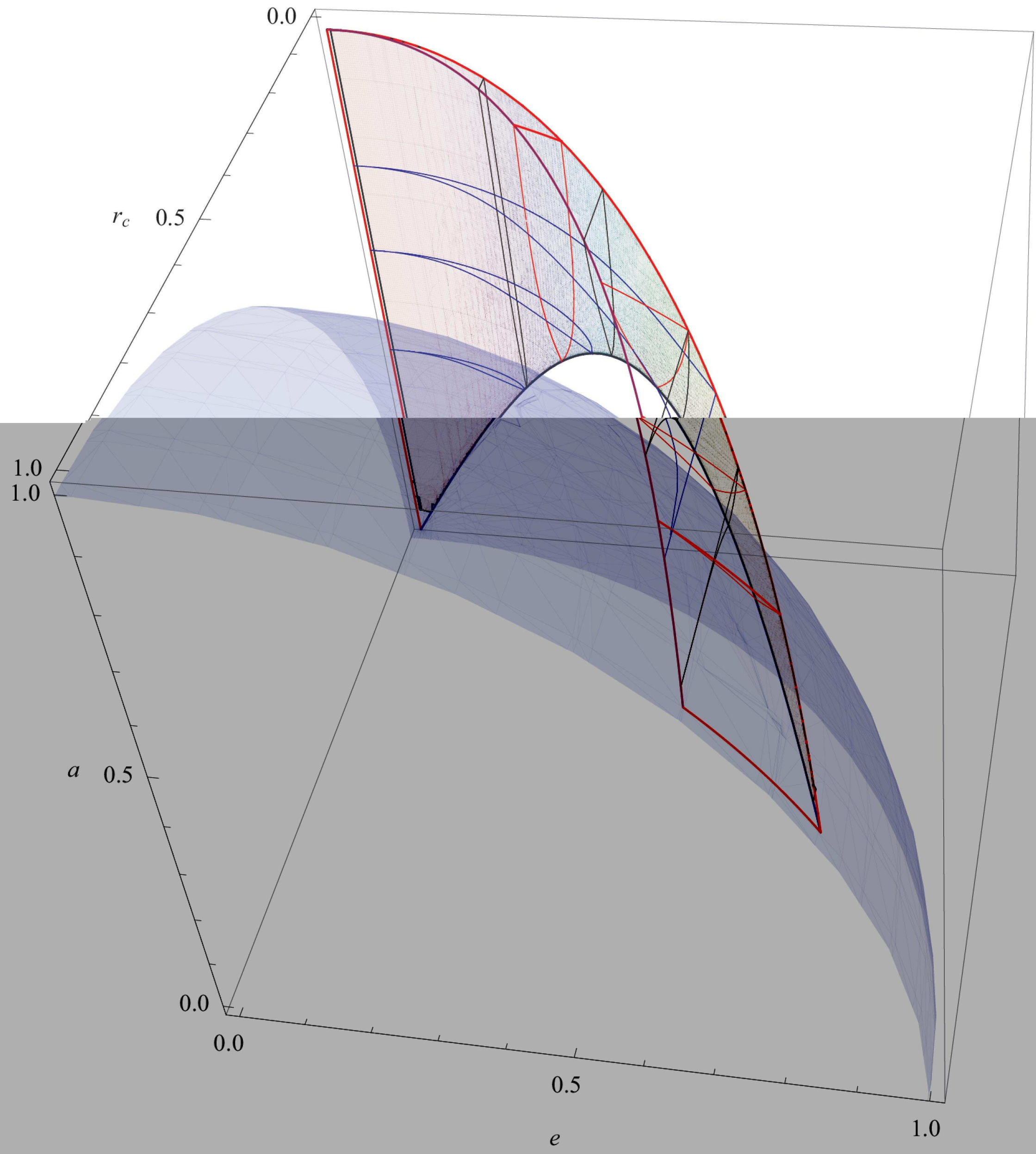

Figure 5. $3D$ domain of existence (marked by the level curves $r = const$, $a = const$, $e = const$) for the stable circular (equatorial) orbits of neutral particles inside the inner BH horizon (a filled part of sphere). The rear filled surface corresponds to the ultrarelativistic limit or circular photon orbits. The stable spherical (nonequatorial) orbits for massive particle exist at radii, limited by the rear surface of circular photons and the inner horizon.

Acknowledgments

Author acknowledges E. O. Babichev, V. A. Berezin and Yu. N. Eroshenko for fruitful discussions. This work was supported in part by the Russian Foundation for Basic Research grant 10-02-00635.

References

[1] C. W. Mizner, K. S. Thorne, J. A. Wheeler *Gravitation*, W. H. Freeman and Company, San Franisco (1973) Chapter 34.

[2] V. P. Frolov, M. A. Markov and V. F. Mukhanov, *Through a black hole into a new universe? Phys. Lett.* **B216** (1989) 272.

[3] V. P. Frolov, M. A. Markov and V. F. Mukhanov, *Black holes as possible sources of closed and semiclosed worlds, Phys. Rev.* **D 41** (1990) 383.

[4] C. Barrabes and V. P. Frolov, *How many new worlds are inside a black hole? Phys. Rev.* **D 53** (1996) 3215, [arXiv:hep-th/9511136].

[5] Ya. B. Zeldovich and I. D. Novikov, *Relativistic astrophysics. Vol.1: Stars and relativity*, Chicago: University of Chicago Press, (1971), Chapter 3.

[6] S. Chandrasekhar, *The mathematical theory of black holes*, Oxford University Press, Oxford (1983), Chapters 5 and 7.

[7] J. Bičák, Z. Stuchlík and V. Balek, *The motion of charged particles in the field of rotating charged black holes and naked singularities, Bull. Astron. Inst. Czechosl.* **40** (1989) 65.

[8] V. Balek, J. Bičák and Z. Stuchlík, *The motion of the charged particles in the field of rotating charged black holes and naked singularities. II - The motion in the equatorial plane. Bull. Astron. Inst. Czechosl.* **40** (1989) 133.

[9] S. Grunau and V. Kagramanova, *Geodesics of electrically and magnetically charged test particles in the Reissner-Nordstro"m space-time: analytical solutions*, [arXiv:1011.5399v1 [gr-qc]].

[10] M. Olivares, J. Saavedra, C. Leiva and J. R. Villanueva, *Motion of charged particles on the Reissner-Nordstro"m (Anti)-de Sitter black holes*, [arXiv:1101.0748v1[gr-qc]].

[11] E. Hackmann, V. Kagramanova, J. Kunz and C. Lam̈merzahl, *Analytical solution of the geodesic equation in Kerr-(anti) de Sitter space-times, Phys. Rev.* **D 81** (2010) 044020.

[12] D. Pugliese, H. Quevedo and R. Ruffini, *Circular motion of neutral test particles in Reissner-Nordstrom̈ spacetime*, [arXiv:1012.5411v1 [astro-ph.HE]].

[13] B. Carter, *Global structure of the Kerr family of gravitational fields, Phys. Rev.* **174** (1968) 1550.

[14] J. M. Bardeen, W. H. Press and S. A. Teukiolsky, *Rotating black holes: locally nonrotating frames, energy extraction and scalar synchrotron radiation. Astrophys. J* **178** (1972) 347.

[15] D. C. Wilkins, *Bound geodesics in the Kerr metric, Phys. Rev.* **D 5** (1972) 814.

[16] L. D. Landau and E. M. Lifshitz, *Mechanics*, Course of theoretical physics, Oxford: Pergamon Press, 2nd ed., (1969), Chapter III.

[17] K. S. Thorne, *Disk accretion onto a black hole. II. Evolution of the hole. Astrophys. J.* **191** (1974) 507.

[18] B. Carter, *Complete analytic extension of the symmetry axis of Kerr's solution of Einstein's equations, Phys. Rev.* **141** (1966) 1242.

[19] B. Carter, *The complete analytic extension of the Reissner-Nordstrom̈ metric in the special case* $e^2 = m^2$*, Phys. Lett.* **21** (1966) 423.

[20] M. Simpson and R. Penrose, *Internal Instability in a Reissner-Nordstrom̈ Black Hole, Intern. J. Theor. Phys.* **7** (1973) 183.

[21] Y. Gursel, V. D. Sandnerg, I. D. Novikov and A. A. Starobinsky, *Evolution of scalar lerturbations near the Cauchy horizon of a charged black hole, Phys. Rev.*, **D 19** (1979) 413.

[22] Y. Gursel, V. D. Sandnerg, I. D. Novikov and A. A. Starobinsky, *The final state of the evolution of the interior of a charged black hole, Phys. Rev.*, **D 20** (1979) 1260.

[23] I. D. Novikov and A. A. Starobinsky, *Quantum-electrodynamic effects inside a charged black hole and the problem of Cauchy horizons, Sov. Phys.- JETP*, **51** (1980) 1.

[24] S. Chandrasekhar and J. B. Hartle, *On crossing the Cauchy horizon of a Reissner-Nordstroem black-hole, Proc. Roy. Soc. Ser. A*, **384** (1982) 301.