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Black Hole Entropy and the Problem of Universality

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Abstract

A key test of any quantum theory of gravity is its ability to reproduce the known thermodynamic properties of black holes. A statistical mechanical description of the Bekenstein-Hawking entropy once seemed remote, but today we suffer an embarrassment of riches: many different approaches to quantum gravity yield the same entropy, despite counting very different states. This "universality" suggests that some underlying feature of the classical theory may control the quantum density of states. I discuss the possibility that this feature is an approximate two-dimensional conformal symmetry near the horizon.

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1. Introduction

In the continuing search for a quantum theory of gravity, black hole thermodynamics may be the nearest thing we have to an "experimental" result. While Hawking radiation has not yet been directly observed, the derivations of the Hawking temperature [1] and the Bekenstein-Hawking entropy [2] are robust enough that any purported quantum theory that failed to reproduce these results would be viewed with deep skepticism.

The Bekenstein-Hawking entropy depends on both Newton's constant G and Planck's constant $\hbar$, and an understanding of the microscopic states responsible for this entropy might provide fundamental insights into quantum gravity. A decade ago, the question of what these microstates were had a simple answer: "We don't know." Several interesting ideas had been proposed—entanglement entropy, for example [3], or the entropy of quantum fields near the horizon [4]—but none seemed adequate.

Today, by contrast, we suffer from an embarrassment of riches. The Bekenstein-Hawking entropy of a black hole can apparently be explained by

- weakly coupled string and D-brane states [5, 6];
- horizonless "fuzzball" geometries [7];
- states in a dual conformal field theory "at infinity" [8];
- spin network states at the horizon [9];
- spin networks *inside* the horizon [10];
- "heavy" degrees of freedom in induced gravity [11];
- elements of a causal set in the domain of dependence of a horizon [12];
- inherently global characteristics of a black hole spacetime [13];
- no microscopic states: the entropy can be obtained from quantum field theory in a fixed background, with no information about quantum gravity needed [1].

None of these explanations is complete—string theory, for example, typically gives exact answers only for supersymmetric black holes and their duals, while loop quantum gravity requires a choice of a poorly understood adjustable parameter. But within their domains of applicability, all seem to work. This is a puzzle: even if one rejects the existing approaches to quantum gravity, one must still explain why Hawking's original calculation, which required no detailed assumptions about quantum gravity, should agree with the enumeration of states in *any* quantum theory.

To put this puzzle in context, it is useful to compare "ordinary" thermodynamics. The quantum theory of an ideal gas allows us to specify and count states, and the resulting entropy agrees, up to typically small corrections, with the classical prediction. But this is to be expected: the correspondence principle relates the quantum states to the classical phase space, forcing an approximate agreement between the two theories. A classical black hole, on the other hand, has no hair—there *is* no classical phase space to explain the thermodynamics. The states responsible for black hole entropy must be fundamentally quantum mechanical, and there is no obvious reason for them to have any preconceived behavior.

The ideal gas analogy suggests, however, that perhaps some *different* classical property of general relativity might control the density of quantum states. It is thus natural to search for some classical aspect of general relativity—perhaps a symmetry—that might govern the state-counting of quantum gravity.

2. Conformal symmetry and the Cardy formula

There is, to the best of my knowledge, only one known case in which a classical symmetry determines a universal form for the density of states of a quantum theory. A two-dimensional conformal field theory is characterized by two symmetry generators $L[\xi]$ and $\bar{L}[\bar{\xi}]$, which generate holomorphic and antiholomorphic diffeomorphisms. The Poisson bracket algebra of these generators is given by the unique central extension of the group of two-dimensional diffeomorphisms, the Virasoro algebra:

$$\begin{aligned} \{L[\xi], L[\eta]\} &= L[\eta\xi' - \xi\eta'] + \frac{c}{48\pi}\int dz\,(\eta'\xi'' - \xi'\eta'') \\ \{\bar{L}[\bar{\xi}], \bar{L}[\bar{\eta}]\} &= \bar{L}[\bar{\eta}\bar{\xi}' - \bar{\xi}\bar{\eta}'] + \frac{\bar{c}}{48\pi}\int d\bar{z}\,(\bar{\eta}'\bar{\xi}'' - \bar{\xi}'\bar{\eta}'') \\ \{L[\xi], \bar{L}[\bar{\eta}]\} &= 0\ , \end{aligned} \tag{2.1}$$

where the central charges c and $\bar{c}$ (the "conformal anomalies") depend on the particular theory. The zero-mode generators $L_0 = L[\xi_0]$ and $\bar{L}_0 = \bar{L}[\bar{\xi}_0]$ are conserved charges, roughly analogous to energies.

In 1986, Cardy discovered a remarkable characteristic of such theories [14, 15]. Given *any* two-dimensional conformal field theory for which the lowest eigenvalues Δ_0 of L_0 and $\bar{\Delta}_0$ of $\bar{L}_0$ are nonnegative, the asymptotic density of states at large eigenvalues Δ and $\bar{\Delta}$ takes the form

$$\ln\rho(\Delta,\bar{\Delta}) \sim 2\pi\sqrt{\frac{(c-24\Delta_0)\Delta}{6}} + 2\pi\sqrt{\frac{(\bar{c}-24\bar{\Delta}_0)\bar{\Delta}}{6}}\ . \tag{2.2}$$

Higher order corrections to this relation are also uniquely determined [16–18]. The entropy is thus fixed by symmetry, independent of any details of the states being counted.

Black holes are not, of course, two-dimensional, and neither are they conformally invariant. But there is a sense in which the near-horizon region of a black hole is *almost* two-dimensional and *almost* conformally invariant. For example, a quantum fields near the horizon can be approximately described by two-dimensional conformal field theory [19–21]; indeed, such a description has recently been shown to determine the Hawking temperature, by a procedure closely related to the computation of the conformal anomaly [22, 23]. Further, the surface gravity, and therefore the Hawking temperature, of a black hole may be expressed in a conformally invariant manner [24], and a generic black hole metric admits an approximate conformal Killing vector near the horizon [25, 26]. So it is at least possible that results from two-dimensional conformal field theory may be relevant.

3. The BTZ black hole

The first demonstration that conformal field theory techniques could explain black hole thermodynamics came from the study of the BTZ black hole [27–30], a three-dimensional, asymptotically anti-de Sitter black hole with cosmological constant $\Lambda = -1/\ell^2$ and metric

$$\begin{gathered} ds^2 = -(N^\perp)^2dt^2 + f^{-2}dr^2 + r^2\left(d\phi + N^\phi dt\right)^2 \\ \text{with}\quad N^\perp = f = \left(-8GM + \frac{r^2}{\ell^2} + \frac{16G^2J^2}{4r^2}\right)^{1/2}, \quad N^\phi = -\frac{4GJ}{r^2}\ . \end{gathered} \tag{3.1}$$

Although the spacetime described by (3.1) is one of constant negative curvature, it is a genuine black hole, with a Carter-Penrose diagram virtually identical to that of the usual four-dimensional

Schwarzschild-AdS black hole. The BTZ black hole has an event horizon at r_+ and an inner Cauchy horizon at r_-, with

$$M = \frac{r_+^2 + r_-^2}{8G\ell^2} \ , \qquad J = \frac{r_+ r_-}{4G\ell} \ . \tag{3.2}$$

Most important for our purposes, the BTZ black hole exhibits conventional thermodynamic behavior, with an entropy

$$S = \frac{2\pi r_+}{4\hbar G} \tag{3.3}$$

equal to a quarter of its horizon size.

The existence of such thermodynamic properties is, at first sight, a mystery: general relativity in three spacetime dimensions has no local degrees of freedom [31], and there seems to be no room for microscopic states to account for the entropy. A partial resolution was discovered independently by Strominger [32] and Birmingham, Sachs, and Sen [33]. The asymptotic boundary of three-dimensional anti-de Sitter space is a flat two-dimensional cylinder, so it is not too surprising that the diffeomorphisms that preserve the asymptotic behavior of the BTZ metric form a Virasoro algebra. It is, perhaps, surprising that this algebra has a classical central charge, but Brown and Henneaux showed in 1986 that it does [34], with

$$c = \bar{c} = \frac{3\ell}{2G} \quad \text{and} \quad \Delta = \frac{1}{16G\ell}(r_+ + r_-)^2 \ , \quad \bar{\Delta} = \frac{1}{16G\ell}(r_+ - r_-)^2 \ . \tag{3.4}$$

Substituting these values into the Cardy formula (2.2), and assuming that $\Delta_0 = \bar{\Delta}_0 = 0$, one easily obtains the entropy (3.3).

The entropy thus seems to be related to "boundary conditions" at infinity. We can gain further insight by considering the Chern-Simons formulation of three-dimensional gravity [35, 36]. While a Chern-Simons theory is gauge-invariant on a *compact* manifold, the presence of a boundary breaks this symmetry. The result is a Goldstone-like mechanism, in which "would-be pure gauge" degrees of freedom become physical at the boundary [37], leading to a dynamical Wess-Zumino-Witten model, in this case at infinity [38, 39]. A similar emergence of Goldstone-like dynamical degrees of freedom can be demonstrated, with a bit more difficulty, in the metric formalism [40, 41]. Whether these degrees of freedom can actually explain the entropy (3.3) remains an open question—for a review, see [30]—but they are certainly good candidates.

4. How to ask the right question

While these results for the BTZ black hole are certainly suggestive, they are also clearly not good enough. The Chern-Simons formulation of gravity is possible only in three dimensions, and it is only in three dimensions that the boundary will admit a nice two-dimensional description. While many string theoretical black holes have a near-horizon structure that looks like that of a BTZ black hole [42], allowing the use of the results of the preceding section, most black holes do not have such nice characteristics.

Moreover, it would be useful to find black hole microstates at the horizon rather than at infinity. Although a "holographic" description at infinity may be possible, such a description would make it rather difficult to disentangle degrees of freedom in a spacetime with more than one black hole. There are, for example, multi-black hole solutions in three dimensions with the same asymptotic symmetries as the BTZ black hole [43]; a complete microscopic description of black hole entropy should surely allow us to distinguish these.

We might thus take away the following lessons from the BTZ black hole:

- we should look for "broken gauge invariance" to provide new degrees of freedom;
- we should at least *hope* for an effective two-dimensional picture, which would allow us to use the Cardy formula;
- but we should look near the horizon for our new Goldstone-like modes.

To proceed further, we must first address a general but somewhat delicate issue: how does one ask a question about a black hole in a quantum theory of gravity? This question is seldom asked, because the "usual" answer seems obvious: we fix a black hole background, and then ask about quantum fields, gravitational perturbations, and the like in that background. In a full quantum theory of gravity, however, we cannot do this: there is no fixed background, and the uncertainty relations prevent us from simply imposing a black hole metric.

A question about a black hole in quantum gravity is a question about a conditional probability: "If [some defining characteristic] is present, what is the probability of [some black hole property]?" To answer, we must first choose a defining characteristic (*not* the full classical metric!), and then figure out how to require its presence. For the BTZ black hole computation of the preceding section, for example, the defining characteristic was the asymptotic behavior of the metric. We may instead impose boundary conditions at past null infinity $\mathcal{I}^-$ [44]. To search for horizon degrees of freedom, we might impose conditions on the near-horizon behavior of the metric at constant time (see, for example, [45–48]), or on the behavior of the metric near a null "isolated horizon" (see, for example, [49–52]). In each case, the hope is that such conditions break gauge (or diffeomorphism) invariance in a manner that leads to new degrees of freedom.

I know of two ways to impose such conditions. One, first introduced in [37], is to treat the horizon as a sort of "boundary" at which suitable boundary conditions are prescribed. In a path integral approach, for instance, we can split spacetime into two regions along a hypersurface $\mathcal{H}$ and perform separate path integrals over fields on each side, with fields at $\mathcal{H}$ restricted by our horizon conditions. Such split path integral has been studied in detail in three dimensions [53], where it leads to the same WZW models that were discussed in the preceding section. The horizon is not, of course, a genuine boundary, but it is a hypersurface upon which we impose "boundary conditions," and this turns out to be good enough.

For the black hole, this "horizon as boundary" approach has been shown to correctly reproduce the Bekenstein-Hawking entropy by way of the Cardy formula [45–48]. On the other hand, the diffeomorphisms whose algebra yields that central charge—essentially those that leave the lapse function invariant—are generated by vector fields that blow up at the horizon, and the significance of this divergence is not yet clear [54–60]. It has also been difficult to apply these methods directly to the two-dimensional black hole.

A second, newer, approach is to literally impose the desired "defining characteristics" of the black hole as added constraints in the canonical theory. We can then use the standard tools of constrained Hamiltonian dynamics [61–63] to study the resulting model. This approach was first proposed in [52], and has had some successes, but is still in its early stages. In the following section, I will briefly illustrate it for the two-dimensional dilaton black hole.

5. Dilaton gravity with horizon constraints

Two-dimensional dilaton gravity is described by the action

$$I = \frac{1}{16\pi G}\int d^2x\sqrt{g}\,(\varphi R + V[\varphi]) \tag{5.1}$$

with a metric g_{ab} and a scalar dilaton φ with an arbitrary potential $V[\varphi]$ [64, 65]. This action appears in many contexts; in particular, it can be obtained by dimensionally reducing higher-dimensional general relativity, in which case φ has an interpretation as the "transverse area." Although the restriction to two dimensions is a strong one, the action (5.1) arguably describes the generic behavior of a black hole near enough to the horizon.

Let us now look at "Euclidean" dilaton gravity, and consider the method of "radial quantization" used in string theory. That is, we take the metric to be positive definite,

$$ds^2 = N^2 f^2 dr^2 + f^2 \left(dt + \alpha dr\right)^2 \ , \tag{5.2}$$

and evolve radially outward from the origin. Using standard canonical methods, it is straightforward to check that the momenta conjugate to f and φ are

$$\pi_f = \frac{1}{Nf}\left(\varphi' - \alpha\dot{\varphi}\right) \ , \qquad \pi_\varphi = \frac{1}{Nf}\left(f' - (\alpha f)^{\cdot}\,\right) \ , \tag{5.3}$$

and that the generators of diffeomorphisms are

$$\mathcal{H}_\perp = f\pi_f\pi_\varphi + f\left(\frac{\dot{\varphi}}{f}\right)^{\cdot} - f^2 V \ , \qquad \mathcal{H}_\parallel = \pi_\varphi\dot{\varphi} - f\dot{\pi}_f \ . \tag{5.4}$$

It is also not hard to show that these symmetries form a Virasoro algebra (with $c = \bar{c} = 0$), with

$$L = \frac{1}{2}\left(\mathcal{H}_\parallel + i\mathcal{H}_\perp\right) \ , \qquad \bar{L} = \frac{1}{2}\left(\mathcal{H}_\parallel - i\mathcal{H}_\perp\right) \ . \tag{5.5}$$

Let us now impose "horizon constraints" on our initial surface. Teitelboim [66] has shown that the condition for the origin to be a horizon in Euclidean gravity is that $\varphi' = \dot{\varphi} = 0$, or, equivalently, $s = \bar{s} = 0$, where

$$\vartheta = \left(\dot{\varphi} - if\pi_f\right)/\varphi \tag{5.6}$$

is the Euclidean version of the expansion and $s = \varphi\vartheta$. This is not quite suitable for our purposes, though, since it gives a constraint at a single point rather than an initial surface. To define radial evolution, we must instead start at a "stretched horizon," a small circle around the horizon. The appropriate constraint—determined, for example, by the requirement that the Hamiltonian be well-defined [50]—can be written in terms of s and the surface gravity κ as [52, 67]

$$K = s - a\varphi_+(\kappa - \kappa_+) = 0, \qquad \bar{K} = \bar{s} - a\varphi_+(\bar{\kappa} - \bar{\kappa}_+) = 0 \tag{5.7}$$

where

$$\kappa = \frac{\dot{f}}{f} - i\pi_\varphi, \tag{5.8}$$

φ_+ is the value of the dilaton at the horizon, and a and κ_+ are constants.

The constraints (5.7) are not preserved by the generators L and $\bar{L}$ of the conformal symmetry—that is, Poisson brackets such as $\{L[\xi], K\}$ do not vanish. We can fix this, though, by using a trick first introduced by Bergmann and Komar to handle second class constraints [63]: we add "zero," in the form of multiples of K and $\bar{K}$, to the symmetry generators to make their brackets with the constraints vanish. A fairly straightforward computation shows that as one approaches the horizon, the modified generators again obey a Virasoro algebra, but now with

$$c = \bar{c} = 6\pi a\varphi_+ \ , \qquad \Delta = \bar{\Delta} = \frac{\pi a\varphi_+}{4}\left(\frac{\kappa_+\beta}{2\pi}\right)^2 \ , \tag{5.9}$$

where we assume that our diffeomorphisms are periodic in t with period β. Smoothness of the metric (5.2) at the origin requires a periodicity $\beta = 2\pi/\kappa_+$, yielding the usual Hawking temperature. The Cardy formula (2.2) then gives an entropy

$$S = \pi a \frac{\varphi_+}{4G} \tag{5.10}$$

It remains for us to determine the constant a. For a dimensionally reduced spherically symmetric black hole,

$$f^2 = 1 - \left(\frac{r_+}{r}\right)^{D-3}, \quad \varphi = \varphi_+ \left(\frac{r}{r_+}\right)^{D-2} , \tag{5.11}$$

and an easy computation gives $a = 2$. More generally, this value appears to be determined by demanding that the stretched horizon be affinely parametrized, but this issue is not quite settled.

Given a value $a = 2$, the entropy (5.10) is precisely 2π times the standard Bekenstein-Hawking entropy. A similar factor of 2π was found in [68]. I believe it arises because in radial quantization we are computing the entropy over "all times," on an initial surface consisting of a circle of circumference 2π.

Up to this factor of 2π, the central charge (5.9) agrees with that of the "horizon as boundary" approach [46], and is closely related to the BTZ central charge (3.4). At first sight, the conformal charges Δ and $\bar{\Delta}$ in (5.9) look rather different from the corresponding BTZ quantities in (3.4). But this difference is easily explained: the asymptotic diffeomorphisms that generate Δ and $\bar{\Delta}$ for the BTZ black hole depend on $t \pm \ell\phi$, and have periodicities [69]

$$\beta_\pm = \left(\frac{r_+ \pm r_-}{r_+}\right)\beta \ . \tag{5.12}$$

Substituting these expressions into (5.9), we find differing "left" and "right" conformal charges of the same form as (3.4).

While these results are intriguing, we still have a long way to go to establish (or reject) the proposal that black hole thermodynamics can be understood in terms of horizon symmetries. Perhaps the most important open question is whether the "Goldstone-like" modes can couple to matter to produce Hawking radiation. In three dimensions, this has been shown to occur [70]: a classical matter source coupled to the boundary degrees of freedom of the BTZ black hole causes transitions among boundary states, and detailed balance arguments can then be used to derive the Hawking radiation spectrum. For more general black holes, the anomaly-based computations of Hawking radiation [22,23] are suggestive (see also [71,72]), but the full story will require more work.

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