Celest Mech Dyn Astr
DOI 10.1007/s10569-012-9444-9

ORIGINAL ARTICLE

A graph based methodology for mission design

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Received: 9 November 2011 / Revised: 5 June 2012 / Accepted: 30 August 2012

Abstract A space mission design methodology is presented, where initial and final orbits are connected through segments of periodic orbits. After a discretization of the solution space, the problem of mission design is transformed into an equivalent combinatorial optimization problem. Specifically, a graph is constructed that represents periodic orbits connected by the execution of impulsive maneuvers. A low computational complexity algorithm for this transformation is introduced. An efficient combinatorial optimization algorithm that solves the shortest path problem is described. Subject to the initial discretization of the solution space, an optimal sequence of coastal arcs is determined for a low total Delta-V mission. Finally, the proposed methodology is applied to the design of a hypothetical Saturn–Titan system mission.

Keywords Mission design · Periodic orbits · Circular restricted three body problem · Combinatorial optimization · Coastal arcs

1 Introduction

Finding periodic orbits for the three body problem has been the subject of intensive research for many decades (Markellos 1974a; Robin and Markellos 1980; Hénon 1997, 2005; Lam and Whiffen 2005; Dutt and Sharma 2011, among others). Modern computers that offer large main memories, high central processing unit frequency and parallel processing, allow massive computation of orbits. At the same time, algorithmic improvements targeting computational

Electronic supplementary material The online version of this article (doi:10.1007/s10569-012-9444-9) contains supplementary material, which is available to authorized users.

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Published online: 26 September 2012

effectiveness have been introduced (Russell 2005; Tsirogiannis et al. 2009). Space mission designs based on libration point related orbits have led to successful real world missions (Dunham and Farquhar 2003; Hechler and Cobos 2003; Broschart et al. 2009), while a number of such orbits has also been suggested as candidates for future missions (Grebow et al. 2006; Hill et al. 2006; Lindegren et al. 2007; Folta and Lowe 2008). Mainly invariant manifolds have been used for mission designs utilizing techniques that deal with zero cost, or different energy, transfers (Gómez and Masdemont 2000; Koon et al. 2000, 2002; Gómez et al. 2004; Parker and Born 2008; Mingotti et al. 2009; Pergola et al. 2009; Davis et al. 2010, 2011). Alternatively, halo and Lissajous orbits have been utilized (Howell and Hiday-Johnston 1994). Chains of complex periodic orbits in the planar circular restricted three-body problem using the invariant manifolds of unstable orbits, are studied by Parker et al. (2010).

In this paper coastal arcs along periodic orbits are connected through impulsive maneuvers, aiming to a low total Delta-V (DV) mission. Initially, a discretization of the solution space produces a set of periodic orbits that could offer coastal arcs to the mission. These orbits are connected through inexpensive transfers for a network to be formed. A simplified version of primer vector theory (PVT, Lawden 1963) is used in order to estimate single-maneuver connections of members of the base set so as to form a potential network. It is not expected that a single pair of periodic orbits will fulfill the mission needs. A number of periodic orbits need to be connected so as to transfer (spacecraft) from initial orbit to goal orbit. Aiming to a low total DV cost, an optimal sequence of coastal arcs along periodic orbits is determined. This is an optimal sequence only for the initial discretization of the solution space and is computed via a combinatorial optimization algorithm on the network. Thus, the proposed methodology consists of three steps: discretization, graph construction and combinatorial optimization.

The paper is organized as follows: Section 2 presents the discretization of the solution space. In Section 3 the transformation of the solution space to an equivalent graph is introduced. Section 4 describes a low computational complexity algorithm which finds the optimal sequence of coastal arcs. Equivalence and optimality of solution is discussed. Application of the methodology to a hypothetical mission design is demonstrated in Section 5. Finally Section 6 outlines advantages, disadvantages and limitations of the present methodology.

2 Discretization

In this methodology a set of periodic orbits form the base set of coastal arcs. The circular restricted three body problem (CRTBP) possesses an infinity of periodic orbits which are grouped into infinite mono-parametric families. This provides an immense number of possible coastal arcs that can be connected leading to low energy missions. This pluralism provides a variety of orbits that traverse a wide range of locations and have different shapes. A discretization of this huge solution space through sampling of families and orbits is needed.

The necessary discretization is obtained by sampling a wide area of the initial conditions space in order to compute members of many different families. For the case of CRTBP such a methodology has been introduced in Markellos et al. (1974b) and improved by Russell (2005) and Tsirogiannis et al. (2009) for symmetric periodic orbits with respect to Ox axis and/or Oxz plane. At the node points of an orthogonal grid on the initial conditions space, the equations of motion are numerically integrated. Combining the information at the nodes of the grid, exact periodic orbits are computed. This is an effective procedure and has been used for computing millions of periodic orbits. Both planar and three-dimensional symmetric orbits can be computed. By using methods described by Markellos and Halioulias (1977)

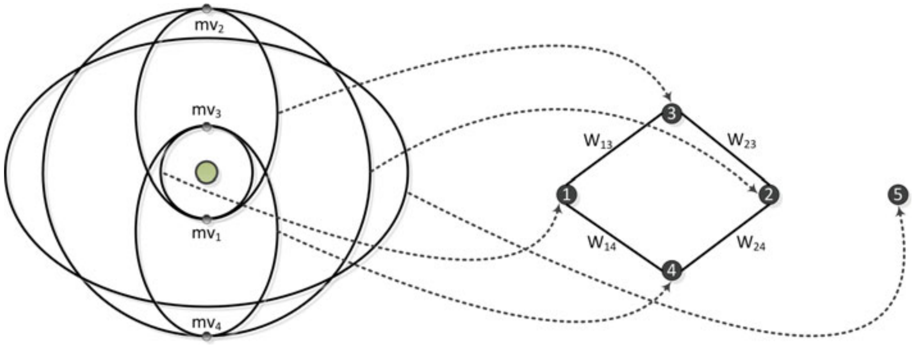

Fig. 1 Graph construction for a simple base set (where $mv_i, i = 1 \dots 5$ indicate the maneuvers' positions)

and Hénon (2005), non-symmetric planar orbits can also be computed. Of course, any other method for computing families of periodic orbits can be used in combination with the above methods (e.g. numerical continuation, see Doedel et al. 2003). Inserting all kinds of orbits (symmetric and non-symmetric, two-dimensional and three-dimensional) into the base set, is advantageous and can probably lead to low DV transfers due to large numbers of possible inexpensive connections between orbits and suitable coastal arcs.

The base set of periodic orbits can be formed according to the mission objectives and constraints. The vast knowledge on families of periodic orbits obtained in the last fifty years, combined with information on the connectivity properties of the resulting graph, should provide guidelines on how to adapt-organize the discretization procedure.

3 Graph construction

In this section the transformation of the base set into a undirected weighted-edge graph, is described.[1] This abstraction of the orbits of the base set, adopted as candidates for inexpensive connections, is suitable for combinatorial optimization which will reveal the optimal sequence of coastal arcs for the mission under design. In this graph the base set of periodic orbits corresponds to set V of the vertices. A weighted-edge w_{ij} is adopted when the vertices i, j (periodic orbits) can be connected to each other inexpensively with an estimated cost DV. Briefly, the transformation into a graph proceeds as follows. Firstly, an unique natural number is associated to each periodic orbit of the base set, up to its cardinality. For each member orbit x, the subset of the base set is found, whose orbits have the property that x can transfer to them with an estimated cost DV less than a threshold. When such a transfer from orbit x to orbit y exists, the edge xy is added to the edge set E. Moreover, the estimated cost w_{xy} of this transfer is added to the edge-weights set W. When this procedure is completed for all base set orbits, the graph G is constructed.

In Fig. 1 a hypothetical base set is plotted. This small set provides a simplified example of the above procedure. Hohmann transfer orbits have been chosen in this example, because of the simplicity of their shapes. For the needs of this illustration, allowed transfers are assumed to have a maximum cost DV_{max} which excludes transfers to and from orbit 5. Under this DV_{max} constraint, orbit 1 transfers to orbit 3 by executing an impulsive maneuver at point

[1] Background material on graph theory and spatial data structures is given in the electronic supplementary material of this paper.

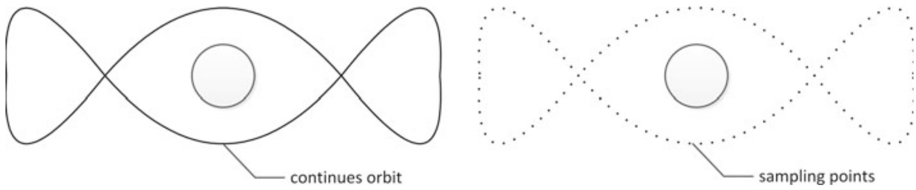

Fig. 2 Sampling an orbit

mv_1 with cost $DV = w_{13}$. An estimation of the transfer cost is needed, not the exact DV. The reverse transfer, from orbit 3 to 1, holds with the same estimated cost because orbits are periodic. This equality of the estimated cost, complying with the assumption of the undirected graph, is based on the constraint that the absolute value of the velocity vector difference of appropriately selected neighboring points of the two orbits is not allowed to exceed DV_{max}. The remaining inexpensive transfers (with $DV < DV_{max}$) are: $2 \leftrightarrows 3$ with cost w_{23}, $1 \leftrightarrows 4$ with cost w_{14} and $2 \leftrightarrows 4$ with cost w_{24}. The constructed graph G is shown in Fig. 1. It is seen that the Hohmann transfers of this example, are represented on graph G in a consistent way: paths $\{1, 3, 2\}$ or $\{1, 4, 2\}$ could be followed in order to transfer from orbit 1 to 2. Here, all allowed transfers correspond to orbits that have common points. This is not a mandatory requirement. In general allowed transfers apply to pairs of orbits that approach each other closely in phase space.

Primer vector theory, besides its elegant mathematical formulation, has a physical meaning which is clearly stated by Marec (1979): "The optimal thrust acceleration ... points towards a neighboring moving point being subjected to the same gravitational field and same thrust acceleration as the moving (spacecraft)." The procedure adopted here for estimating the DVs is based on this concept of PVT. Two problems have to be solved: "neighboring moving points" are to be found (*problem 1*), and those among them must be selected which are "being subjected to the same gravitational field and same thrust acceleration as the moving (spacecraft)" (*problem 2*). These problems are dealt with, in this paper, as follows.

Firstly, the equations of motion are numerically integrated for the base set member orbits using a high accuracy method such as Dormand and Prince (1978). For each member orbit a number of sample points on the orbit path is stored (see Fig. 2). All phase space coordinates (position and velocity components) are stored for the sample points. This sampling is not limited to planar symmetric periodic orbits, but it is applied to three dimensional and non-symmetric orbits, too.

This data set has millions or billions of six-dimensional points. Solving problem 1 for such a huge data set is challenging. Any brute force attempt would require long execution times making it impractical. It is obvious that a suitable data structure is required to solve problem 1. kd-tree (Bentley 1975) solves efficiently the problem: "Given a point $p_i = \{x_i, y_i, z_i\}$ where $p_i \in \mathbb{R}^3$, find these points that are in the neighborhood of p_i, i.e. their euclidian distance is less than r" with expected time complexity $O(\log_2 N + F)$, where N is the total number of points inserted into tree, and F is the number of points in the neighborhood of P_i. Thus, problem 1 can be solved in low computational complexity by using a kd-tree (see supplementary material entitled "Graphs, Shortest Paths and kd-trees" for details). Any other spatial data structure, such as r-tree, could be used. The main advantage of kd-tree is its simplicity. To the author's knowledge kd-tree was firstly used in orbital mechanics by Davis (2009).

It must be pointed out that kd-tree is a main memory data structure, i.e. all data are stored into random access memory (RAM) simultaneously. Even for a top-of-the-line computer, a real world mission will exceed RAM capacity. Modern computational methodologies are needed. kd-tree is suitable for modern computational paradigms such as parallel or distributed processing. A recent publication of Aly et al. (2011) shows that the construction of a distributed kd-tree is algorithmically simple and can handle huge sets of spatial data efficiently.

Problem 2 is a combination of two subproblems: “being subjected to the same gravitational field” and “same thrust acceleration as the moving (spacecraft)”. The “being subjected to the same gravitational field” subproblem is automatically satisfied–solved for neighboring points by the construction phase of data points (samples) since they are numerical output of the same equations of motion (CRTBP). The “same thrust acceleration as the moving (spacecraft)” subproblem is approximately satisfied by setting a maximum DV threshold. The reasoning behind this upper DV threshold is as follows. Neighboring points under the same gravitational forces and being subjected to “same” thrust acceleration, are expected to be those moving points which have their velocities similar too: thus DV smaller than a threshold.

A formal description of the graph construction algorithm (including estimations of DVs) is given in the appendix of the supplementary material of this paper. The adjacency matrix M appearing in the algorithm has some special characteristics: (1) it is a sparse matrix, i.e. the number of connected orbits (non-zero entries) is small, and (2) $w_{i,j} = w_{j,i}$ (graph is undirected) for $i \leftrightarrows j$ transfers. Thus M is symmetric ($M = M^\top$) and only its upper part is required.

Equivalence of transformation

Graph construction entails an exhaustive search of all possible inexpensive transfers between orbits of the base set. For every inexpensively connected pair of orbits (of graph construction algorithm) the minimum estimated DV_{min} is registered as transferring cost and added as edge-weight into the graph. The space flight beginning with the initial orbit and making a number of transfers to other orbits of the base set, is projected (corresponds) to a path or walk on the constructed graph. Moreover, if this tour of the spacecraft has an estimated total cost DV_{total} (for connecting a sequence of coastal arcs), then the projected path on the graph has the same cost. Inversely, a path of the graph, corresponds to a number of maneuvers that transfer the spacecraft form the initial orbit to the final one, because by construction the algorithm 4 (see appendix of supplementary material) includes only allowed transfers i.e. inexpensively connected orbits. Thus, to find the shortest path that connects the two vertices $(v_{initial}, v_{final})$ of the graph is equivalent to finding the optimal sequence of coastal arcs comprising the final mission from orbit labeled with $v_{initial}$, to orbit labeled with v_{final}. Graph construction algorithm, therefore, transforms the mission design problem to an equivalent graph optimization problem. The optimal sequence (shortest path) of graph vertices, bridging initial to final orbit will be adopted as an optimal sequence of coastal arcs for the mission under the following working conditions: (1) the estimation of DV_{min} of each pair is based on a simplified version of PVT which corresponds to a single impulsive maneuver for velocity correction, and (2) the shortest path is implicitly depends on how dense is the discretization of the solution space.

4 Combinatorial optimization

Solving the equivalent shortest path problem for a weighted-edge graph with positive weights using Dijkstra's algorithm (Dijkstra 1959) is a computationally efficient procedure. The principle behind this algorithm is a simple one: expand outward from the starting vertex s, steadily extending the region of the graph in which distances and shortest paths are known. This growth should be orderly, first incorporating the closest vertices and then moving on to those further away. More precisely, when the "known region" is some subset R of vertices that includes s, the next addition to it should be the vertex outside R that is closest to s. Let us call this vertex v. It is identified by the following procedure: Consider u, the vertex just before v in the shortest path from s to v. Since all edge weights are positive, u must be closer to s than v is. This means that $u \in R$, otherwise it would contradict v's status as the closest vertex to s outside R. So, the shortest path from s to v is simply a known shortest path extended by a single edge. Because there will typically be many single-edge extensions of the currently known shortest paths, the shortest of these extended paths identifies v. All single-edge extensions of the currently known shortest paths are tried so as to find the extended shortest path. Its endpoint is claimed to be the next vertex of R. The above description is based on the "known region" approach and was preferred because of its simplicity (Dasgupta 2006). A formal description of the algorithm, that computes not only the cost of shortest path but also the sequence of vertices forming this path, is given in the supplementary material of this paper.

The running time of Dijkstra's algorithm depends on how it is implemented. An obvious implementation of Dijkstra's algorithm has running time $O(|V|^2)$. For, sparse graphs as in the present methodology, a binary heap based implementation gives $O((|V| + |E|) \log |V|)$ running time, while a Fibonacci heap based implementation achieves a running time of $O(|V| \log |V| + |E|)$. Optimality of computed shortest path is guaranteed from Dijkstra's algorithm (see Dasgupta 2006 for a proof).

4.1 Refinement

The result of Dijkstra's algorithm is a minimum cost path that starts at the initial vertex (orbit) of the mission and ends at the goal vertex. This path is a sequence of orbits that are able to offer coastal arcs to the mission. By graph construction every single edge corresponds to a low DV transfer (less than a threshold). The spacecraft transfers from coastal arc segment to coastal arc segment of this sequence and finally transfers to the goal orbit. This preliminary design says nothing about the position of the necessary maneuvers. It simply defines the skeleton on which the coastal arcs lay. A final refinement will be to place the necessary set of maneuvers on the already computed skeleton of the mission. By construction of the graph, valid transfers are those where at least parts of a pair of orbits have very close position and velocity states. Based on this, the simplest approach for connecting each pair of periodic orbits of the optimal sequence (segments of them), is to locate these points where the absolute value of difference of velocity vectors is minimum. In the numerical example of this paper this approach has been used. When a pair of orbits in the optimal sequence consists of orbits that do not intersect (which is the most likely), a local differential corrector should be applied such as the one described in Marchand et al. 2007. For a real world mission, where each m/s of DV is essential, the output of graph based methodology could be considered as a globally good set of coastal arcs (that can inexpensively be connected), each pair could be further optimized locally by PVT (by adding multiple impulses and/or altering the position of the maneuvers).

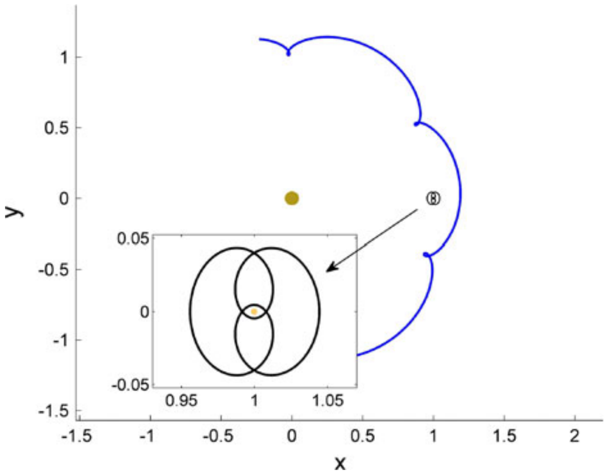

Fig. 3 Initial and final orbit of mission. Units are non-dimensional

5 Application: hypothetical mission design

In this section, a hypothetical mission in the Saturn–Titan system is presented. The objective of this mission is to transfer from an initial non-periodic orbit around both primaries to a periodic orbit that approaches Titan and remains in its region as shown in Fig. 3. Using graph based methodology the optimal sequence of coastal arcs and the exact positions of maneuvers connecting the segment of that sequence are computed.

For this mission the planar CRTBP is used as the base model. CRTBP models the motion of a spacecraft under the influence of two massive bodies. The two bodies of mass m_1 (the primary) and m_2 (the secondary) are assumed to be in circular orbit around their barycenter. The reference frame, centered at the barycenter, rotates at the same rate as the orbital motion of the two massive bodies which appear at rest on the x-axis. The equations describing the coplanar motion of the spacecraft are:

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}, \quad \ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y}$$

where

$$\Omega = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

and $r_1 = \sqrt{(x+\mu)^2 + y^2}$, $r_2 = \sqrt{(x+\mu-1)^2 + y^2}$ are the distances of the spacecraft from the primary and secondary, respectively. The only parameter is $\mu = \frac{m_2}{m_1+m_2}$. This system admits an integral of motion known as the Jacobi integral:

$$C = 2\Omega - \left(\dot{x}^2 + \dot{y}^2\right).$$

C being a constant characterizing any specific orbit. Szebehely (1967) presents the details of derivation of the equations of motion. For this mission in the Saturn–Titan system, the parameter value for μ as well as the constants for transformation to dimensional units have been taken from Koon et al. (2008).

In the discretization phase, only planar symmetric periodic orbits, with respect to Ox axis, are computed due to limitations of the available computer. A grid search as described by Tsirogiannis et al. (2009) is applied, and the characteristics of the base set obtained are as

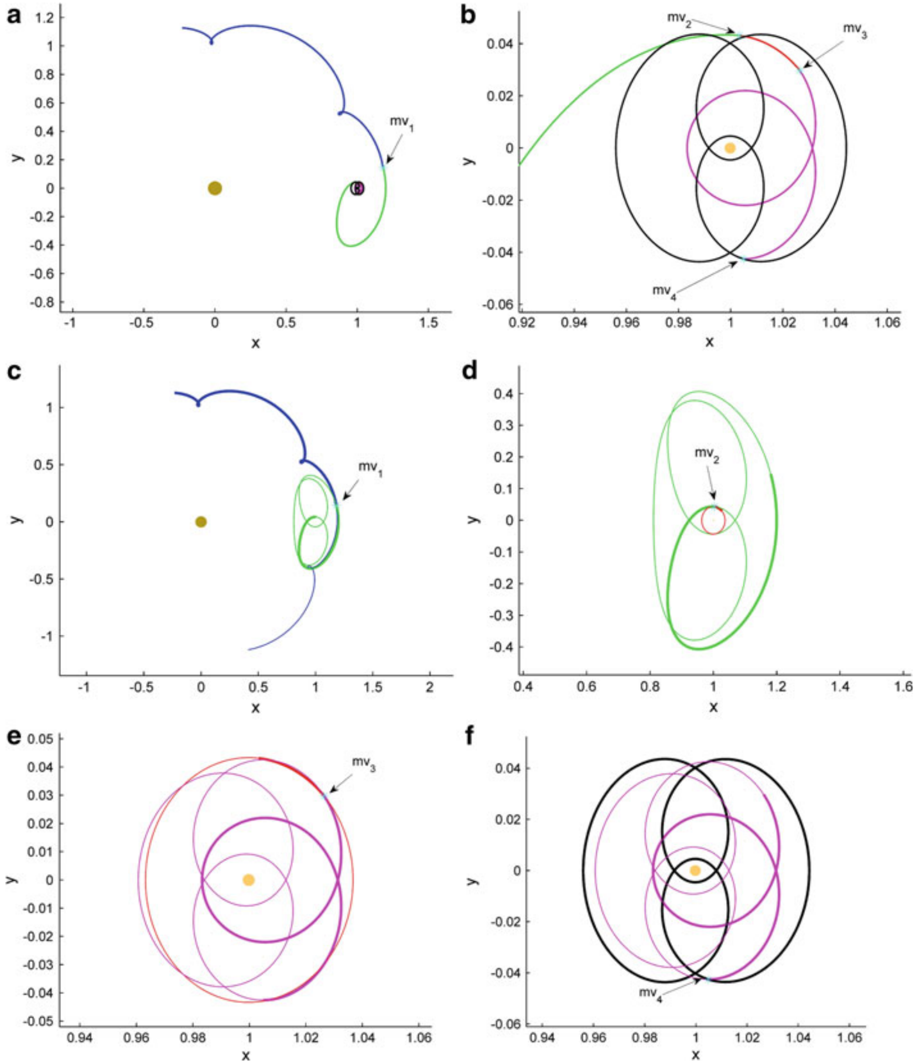

Fig. 4 **a**, **b** The whole mission (only coastal arcs are shown). **c**–f Some details of the mission (*thick lines* are used for coastal arcs along the periodic orbits). Units are non-dimensional

Table 1 Initial conditions of orbits that offer coasting arcs to the mission (non-dimensional)

x_0	y_0	$\dot{x}_0$	$\dot{y}_0$	T
−0.230927367988	1.127594747436	0.248974674735	−0.019360033032	–
0.811217086930	0.0	0.0	0.385632186785	17.072917497030
0.963203154297	0.0	0.0	0.127072498598	2.222278144964
0.960659638792	0.0	0.0	0.108930285864	5.783044487266
0.955951555166	0.0	0.0	0.108571495243	4.848977091564

Table 2 Numerical data for maneuvers $mv_1 - mv_4$ (non-dimensional)

mv_i/DV_i	x	y	$\dot{x}$	$\dot{y}$
mv_1	1.181040241172	0.145961174356	0.052593133380	−0.308763679857
DV_1			0.036814503714	−0.029034073393
mv_2	1.003620671932	0.043070992232	0.183756138917	−0.017286804024
DV_2			−0.084038958792	0.004573187560
mv_3	1.026538287526	0.029632901715	0.071834253549	−0.089233360968
DV_3			−0.019688986364	0.009461036062
mv_4	1.004901763277	−0.042553615510	−0.078758009777	−0.000312670502
DV_4			0.003560879022	0.023728763478

follows. About 10,000 orbits belonging to different families of periodic orbits are included (a subset of them being Lyapunov orbits, while the majority belongs to bifurcations of f and g families, i.e. retrograde and direct satellites of the secondary), with up to 10 intersections of x-axis and period up to 50 days. After sampling the members of the base set approximately 25 millions points are loaded into a kd-tree.

At the combinatorial optimization phase, a set of periodic orbits whose parts form the optimal sequence of coastal arcs, is computed. This sequence is optimal only for the the current discretization of the solution space (10,000 orbits). After a simple refinement for locating the intersection points of orbits, the optimal sequence of coastal arcs shown in Fig. 4a, b, is formed. Maneuvers for connecting these segments are named mv_1 to mv_4. For better illustration of this mission, coastal arcs (thick line) are plotted on their periodic orbits (Fig. 4c–f). Saturn and Titan are plotted with solid color circles preserving their radius ratio (60,268 and 2,576 km, respectively). The total time of the mission is 17.87 days. In Table 1 numerical data for the orbits that participate in mission, are given. Exact numerical data for the maneuvers are given in Table 2.

The estimated costs are $DV_{mv_1} = 0.2620$ km/s, $DV_{mv_2} = 0.4703$ km/s, $DV_{mv_3} = 0.1221$ km/s, $DV_{mv_4} = 0.1341$ km/s with total cost $DV_{total} = 0.9885$ km/s. It must be noted that this total cost is not suitable for a real mission such as Cassini. This is a hypothetical mission based on a crude discretization due to hardware limitations. For a real mission a larger base set of periodic orbits can be used and a proper set of initial-final orbit pair can be selected. Also, for a real mission an extended model can be used such as the Sun–Saturn–Titan model employed by Davis (2011).

6 Concluding remarks

This paper presents a methodology that uses combinatorial optimization for computing an optimal sequence of coastal arcs for a space mission. A base set of periodic orbits, that offers coastal arcs to the mission, is constructed after a discretization of the solution space. Optimality of the sequence of coastal arcs is claimed for this specific discretization. The problem is transformed into an equivalent combinatorial one, which is solved using a low computational complexity algorithm. The methodology is independent of the model problem. Few body problems can be handled in the same way after an appropriate discretization step. Furthermore, different base sets of periodic orbits of different systems, e.g. Sun–Earth, Sun–Saturn and Saturn–Titan, can be combined, after a reduction of data into a common coordinate system. Another advantage is that any costal arc can be transformed to a "parking" orbit, since it is a segment of a periodic orbit. Also, these periodic orbits could be useful for missions that employ constellations of spacecraft. Concerning the flexibility of the designed mission, if requirements change after the launch, alternative paths can be determined in flight exploiting the network of the inexpensive transfers.

At present form, this methodology includes only periodic orbits and optimizes total DV. Thus, it can not be applied for time dependent reference models (unless the dependence is periodic). In future research it will be investigated the use of non-periodic orbits (e.g. invariant manifolds) as members of the base set, and the inclusion of time of flight into optimization criteria. Finally, the problem of a low thrust graph based methodology, remains open. Such a version of the methodology, based on ideas of Russell (2007) and Petropoulos and Russell (2008), is a theme for future research.

Acknowledgments Thanks are due to three anonymous referees for constructive comments which led to significant improvement of the presentation. I also wish to thank V. V. Markellos for useful discussions and critical reading of the manuscript.

References

Aly, M., Munich, M., Perona, P.: Distributed kd-trees for retrieval from very large image collections. In: British Machine Vision Conference, Dundee, Scotland, August (2011)

Bentley, J.L.: Multidimensional binary search trees used for associative searching. Commun. ACM **18**, 509–517 (1975)

Broschart, S., Chung, M., Hatch, S., Ma, J., Sweetser, T., Weinstein-Weiss, S., et al.: Preliminary trajectory design for the Artemis Lunar mission. In: Astrodynamics Specialist Conference, Pittsburgh, PA, August (2009)

Dasgupta, S., Papadimitriou, C., Vazirani, U.: Algorithms. McGraw-Hill, Boston (2006)

Davis, D.C.: Multi-body Trajectory Design Strategies Based on Periapsis Poincaré Maps. PhD Thesis, Purdue University, West Lafayette, Indiana (2011)

Davis, K.E.: Locally Optimal Transfer Trajectories between Libration Point Orbits using Invariant Manifolds. PhD Thesis, University of Colorado, Boulder, CO (2009)

Davis, K.E., Anderson, R.L., Scheeres, D.J., Born, G.H.: The use of invariant manifolds for transfers between unstable periodic orbits of different energies. Celest. Mech. Dyn. Astron. **107**, 471–485 (2010)

Davis, K.E., Anderson, R.L., Scheeres, D.J., Born, G.H.: Optimal transfers between unstable periodic orbits using invariant manifolds. Celest. Mech. Dyn. Astron. **109**, 241–264 (2011)

Dijkstra, E.W.: A Note on two problems in connexion with graphs. Numerische Mathematik **1**, 269–271 (1959)

Doedel, E.J., Paffenroth, R.C., Keller, H.B., Dichmann, D.J., Galn, J., Vanderbauwhede, A.: Continuation of periodic solutions in conservative systems with application to the 3-body problem. Int. J. Bifurcat. Chaos **13**, 1353–1381 (2003)

Dormand, J.R., Prince, P.J.: New Runge–Kutta algorithms for numerical simulation in dynamical astronomy. Celest. Mech. **18**, 223–232 (1978)

Dunham, D.W., Farquhar, R.W.: Libration point missions, 1978–2002. In: Gòmez, G., Lo, M.W., Masdemont, J.J. (eds.) Libration Point Orbits and Applications: Proceedings of the Conference, World Scientific Publishing Company, Aiguablava, Spain (2003)

Dutt, P., Sharma, R.K.: Evolution of periodic orbits in the Sun-Mars system. J. Guid. Control Dyn. **34**, 635–644 (2011)

Folta, D., Lowe, J.: Formation flying of a telescope/occulter system with large separations in an L_2 libration orbit. In: 59th International Astronautical Congress, Glasgow, Scotland, September (2008)

Gómez, G., Masdemont, J.: Some zero cost transfers between libration point orbits. In: AAS/AIAA Spaceflight Mechanics Meeting, Clearwater, FL, January (2000)

Gómez, G., Koon, W.S., Marsden, J.E., Masdemont, J., Ross, S.D.: Connecting orbits and invariant manifolds in the spatial restricted three-body problem. Nonlinearity **17**, 1571–1606 (2004)

Grebow, D., Ozimek, M., Howell, K., Folta, D.: Multi-body orbit architectures for Lunar south pole coverage. In: AIAA/AAS Astrodynamics Specialist Meeting, Tampa, FL, January (2006)
Hechler, M., Cobos, J.: Herschel, Planck and the Gaia orbit design. In: Gómez, G., Lo, M.W., Masdemont, J.J. (eds.) Libration point orbits and applications: Proceedings of the Conference, World Scientific Publishing Company, Aiguablava, Spain (2003)
Hénon, M.: Generating families in the restricted three-body problem. In: Lecture Notes in Physics, Springer, Berlin (1997)
Hénon, M.: Families of asymmetric periodic orbits in Hill's problem of three bodies. Celest. Mech. Dyn. Astron. **93**, 87–100 (2005)
Hill, K., Parker, J.S., Born, G.H., Demandante, N.: A Lunar L_2 navigation, communication, and gravity mission. In: AIAA/AAS Astrodynamics Specialist Conference, Keystone, CO, August (2006)
Howell, K.C., Hiday-Johnston, L.A.: Time-free transfers between libration point orbits in the elliptic restricted problem. Acta Astronaut. **32**, 245–254 (1994)
Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D.: Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics. Chaos. **10**, 427–469 (2000)
Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D.: Constructing a low energy transfer between Jovian Moons. Contemp. Math. **292**, 129–145 (2002)
Koon, W.S., Lo, M.W., Marsden, J.E., Ross, S.D.: Dynamical systems, the three-body problem and space mission design, (2008)
Lawden, D.F.: Optimal Trajectories for Space Navigation. Butterworths and Co Publishers, London (1963)
Lam, T., Whiffen G.J.: Exploration of distant retrograde orbits around Europa. In: AAS/AIAA Spaceflight Mechanics Meeting, Copper Mountain, CO (2005)
Lindegren, L., Babusiaux, C., Bailer-Jones, C., Bastian, U., Brown, A.G.A., Cropper, M. et al.: The Gaia mission: science, organization and present status. Proc. Int. Astron. Union **3**, 217–223 (2007)
Marchand, B.G., Howell, K.C., Wilson, R.S.: An Improved corrections process for constrained trajectory design in the n-body problem. J. Spacecr. Rockets **44**, 884–897 (2007)
Marec, J.P.: Optimal Space Trajectories. Elsevier Scientific Publishing Company, Amsterdam (1979)
Markellos, V.V.: Numerical investigation of the planar restricted three-body problem. II. Regions of stability for retrograde satellites of Jupiter as determined by periodic orbits of the second generation. Celest. Mech. **10**, 87–134 (1974a)
Markellos, V.V., Halioulias, A.A.: Numerical determination of asymmetric periodic solutions. Astrophys. Space Sci. **46**, 183–193 (1977)
Markellos, V.V., Black, W., Moran, P.E.: A grid search for families of periodic orbits in the restricted problem of three bodies. Celest. Mech. **9**, 507–512 (1974b)
Mingotti, G., Topputo, F., Bernelli-Zazzera, F.: Low-energy, low-thrust transfers to the Moon. Celest. Mech. Dyn. Astron. **105**, 61–74 (2009)
Parker, J.S., Born, G.H.: Modeling a low-energy ballistic Lunar transfer using dynamical systems theory. J. Spacecr. Rockets **45**, 1269–1281 (2008)
Parker, J.S., Davis, K.E., Born, G.H.: Chaining periodic three-body orbits in the Earth-Moon system. Acta Astronaut. **67**, 623–638 (2010)
Pergola, P., Geurts, K., Casaregola, C., Andrenucci, M.: Earth-Mars Halo to Halo low thrust manifold transfers. Celest. Mech. Dyn. Astron. **105**, 19–32 (2009)
Petropoulos, A.E., Russell, R.P.: Low-thrust transfers using primer vector theory and a second-order penalty method. In: AIAA/AAS Astrodynamics Specialist Conference and Exhibit, Honolulu, HI, August (2008)
Robin, I.A., Markellos, V.V.: Numerical determination of three-dimensional periodic orbits generated from vertical self-resonant satellite orbits. Celest. Mech. **21**, 395–434 (1980)
Russell, R.P.: Global search for planar and three-dimensional periodic orbits near Europa. In: AAS/AIAA Astrodynamics Specialist Conference, Lake Tahoe, CA, August (2005)
Russell, R.P.: Primer vector theory applied to global low-thrust trade studies. J. Guid. Control Dyn. **32**, 460–472 (2007)
Szebehely, V.: Theory of Orbits. Academic Press, New York (1967)
Tsirogiannis, G.A., Perdios, E.A., Markellos, V.V.: Improved grid search method: an efficient tool for global computation of periodic orbits. Application to Hill's problem. Celest. Mech. Dyn. Astron. **103**, 49–78 (2009)