Trajectory Design Tools for Libration and Cis-Lunar Environments

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ABSTRACT

Innovative trajectory design tools are required to support challenging multi-body regimes with complex dynamics, uncertain perturbations, and the integration of propulsion influences. Two distinctive tools, Adaptive Trajectory Design and the General Mission Analysis Tool have been developed and certified to provide the astrodynamics community with the ability to design multi-body trajectories. In this paper we discuss the multi-body design process and the capabilities of both tools. Demonstrable applications to confirmed missions, the Lunar IceCube Cubesat lunar mission and the Wide-Field Infrared Survey Telescope (WFIRST) Sun-Earth L_2 mission, are presented.

1. INTRODUCTION

The Sun-Earth libration and Cis-lunar environments are challenging regimes for trajectory designers, with complex multi-body dynamics, perturbation modeling, and integration of propulsion influences. Beginning with libration orbits and research on dynamical systems, several tools with applications to libration orbits and Cis-lunar regions have been developed in cooperation between NASA's Goddard Space Flight Center (GSFC) and Purdue University [1,2,3]. One of these innovative tools, Adaptive Trajectory Design (ATD), is being used in conjunction with NASA developed software, the General Mission Analysis Tool (GMAT), to design multi-body transfer trajectories for the upcoming Lunar IceCube Cubesat mission and the Wide-Field Infrared Survey Telescope (WFIRST) Sun-Earth L_2 mission [4,5]. As a payload deployed by the Exploration Mission-1 (EM-1) on the maiden flight of NASA's Space Launch System, Lunar IceCube will use a lunar-gravity assisted, multi-body transfer trajectory with an innovative RF Ion engine to achieve lunar capture and delivery to the science orbit. WFIRST trajectory design is based on an optimal direct-transfer trajectory to a specific Sun-Earth L_2 quasi-halo orbit.

Trajectory design in support of lunar and libration point missions is becoming more challenging as more complex mission designs are envisioned. To meet these greater challenges, trajectory design software must be developed or enhanced to incorporate improved understanding of the Sun-Earth/Moon dynamical solution space and to encompass new optimal methods. Thus the support community needs to improve the efficiency and expand the capabilities of current trajectory design approaches. For example, invariant manifolds, derived from dynamical systems theory, have been applied to the trajectory design of the Acceleration, Reconnection, Turbulence and Electrodynamics of the Moon's Interaction with the Sun (ARTEMIS) mission and the James Webb Space Telescope (JWST) mission [6]. The dynamical systems approach and related manifold approach offer new insights into the natural dynamics associated with the multi-body problem. Overall, it allows a more rapid and robust methodology to libration orbit and transfer orbit design when used in combination with numerical techniques. Trajectory design approaches should also include improved numerical targeting methods that allow optimization and a dynamical view of the state space allowing the user rapid intuitive feedback.

1.1 Efficient and Flexible Trajectory Design

Improved flexibility in trajectory design tools is essential in accommodating increased complexity in mission requirements. Strategies that offer interactive access to a variety of solutions provide an enhanced perspective of the design space. ATD is intended to provide access to solutions that exist within the

framework of the circular restricted three-body problem (CR3BP) in order to facilitate trajectory design in these regimes in an interactive and automated way [7].
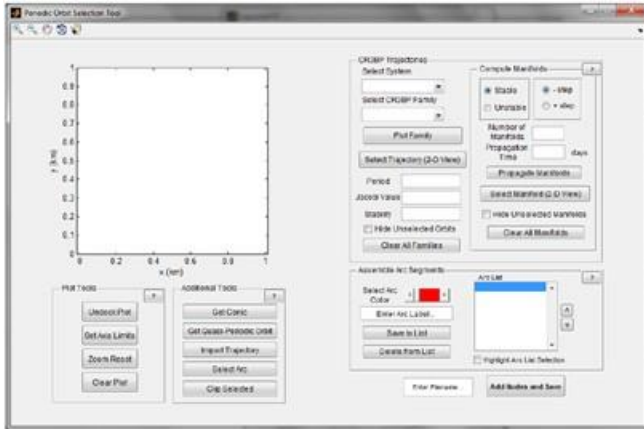

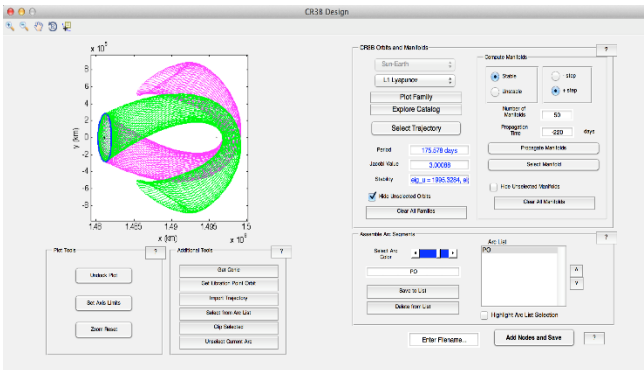**Figure 1**. ATD Graphical Environment

Figure 2. ATD Graphical Interface showing Stable (green) and Unstable (red) Sun – Earth Manifolds

In particular, well-known solutions from the CR3BP, such as libration point orbits and their associated manifolds, are easily constructed using ATD. The ability to blend solutions from a variety of dynamical regimes is provided in several ways using ATD. Conic arcs may be included using the ATD programs, and may be blended with solutions from the CR3BP model. Trajectories that traverse between CR3BP systems may be designed using a "patched CR3BP model" approach. After a trajectory has been assembled in the ATD design programs, it may be transitioned to an ephemeris model to incorporate a realistic dynamical model. Once at that level the preliminary but realistic design can be transferred to operational, high-fidelity software like GMAT. An ATD design environment is shown in Figure 1 and ATD with a Sun-Earth L1 manifold is shown in Figure 2.

In response to an increasing need for a fast and efficient trajectory design process that utilizes well-known multi-body solutions, ATD was initiated to develop an interactive design environment and a composite view of multi-body orbits possessing a variety of characteristics. This interactive design strategy that incorporates a variety of theoretical solutions (e.g., conic arcs, periodic and quasi-periodic libration point orbits, invariant manifolds, primary- and secondary- centered orbits in the CR3BP model, etc.) offers an environment in which exploration of the design space is simple and efficient. Rather than locating single-point solutions, a thorough search of the global solution space is facilitated. User interaction with plots allows for point-and-click arc selection, as well as interactive trajectory 'clipping', in which the desired arc along a longer trajectory may be isolated. A general overview of the ATD strategy can be summarized as follows: Select desired three-body system; Compute and select trajectory arcs of interest; Store any desirable arcs within 'arc list; Arrange all arcs within arc list into appropriate sequence for final design; and Distribute.

2. TOOL AND TRAJECTORY DESIGN PREREQUISITES

It is important that libration trajectories be modeled accurately. The software must integrate spacecraft trajectories very precisely and model all accelerations including both impulsive and finite maneuvers. GMAT provides this capability by incorporating various high-order variable or fixed-step numerical integrators (e.g. Runge-Kutta, Bulirsch-Stoer, etc.). Precise force modeling include an Earth and lunar gravity potential of 360 degree and order, solar radiation pressure, and multiple third-body perturbation effects. Trajectory targeting and optimization is accomplished by varying user-selected parameters to achieve the required goals. A differential corrector (DC) is routinely used as an initial method for targeting to primary or secondary body-related events. These tools can use B-plane and libration coordinate targets as well as intermediate targets such as Cartesian states, energy levels, and even stable and

unstable mode directions. These software tools are excellent for prelaunch analysis and operations. In general they include capabilities for maneuver and launch error analysis, launch window calculations, impulsive and finite maneuver modeling, and ephemeris generation.

Tools that permit the designer to categorize orbits by energy and amplitudes, among other numerous design variables, CR3BP methods, and manifold generation, are essential for the transfer trajectory design process for both the Lunar IceCube and WFIRST missions. Based on the constrained Lunar IceCube EM-1 architecture and deployment, an assessment using ATD and dynamical system research tools has revealed Euclidian regions of Cis-lunar space which permit a transition onto stable/unstable manifolds that encounter the Moon at the prerequisite arrival conditions, resulting in an innovative solution process. Using ATD's powerful Poincaré mapping tools and libration orbit generation via energy or orbit amplitudes, feasible WFIRST science orbits are generated that feed into the selection of optimal transfer manifolds from the low-Earth orbit injection condition. For both missions, these ATD utilities permit the interweaving of manifolds and conics to complete any cis-lunar or libration orbit design. ATD's innovative applications are fully defined and the basic operations and its interface to GSFC's GMAT for high-fidelity modeling have been verified and used for upcoming trajectory design.

2.1 Libration and Lunar Encounter Numerical Trajectory Design

In addition to ATD's dynamical systems approach, the designer also relies on proven operational numerical methods for targeting and to generate data that can be fed back into the dynamical process for further refinement [8]. Obviously, any trajectory design for lunar and libration orbit transfers and stationkeeping can be computed using GMAT without the need for ATD inputs. Tools like GMAT use a direct-shooting approach (forward or backward) or optimization techniques for targeting and meeting mission goals. These numerical methods use partial (first) derivatives to calculate the direction for convergence. The partial derivatives are calculated by numerically propagating to the stopping condition, changing the independent

variable with a small perturbation and re-propagating. The change in the goals divided by the change in the variables is used to compute the partials. The usual sequence of a forward-shooting method is used to vary the initial conditions though predefined perturbations. The initial conditions include the orbital initial conditions; an applied ΔV; and spacecraft design parameters to meet goals that include orbital parameters such as period, position, velocity, amplitude, etc.

A typical libration orbit numerical targeting scenario includes the following steps.

- Target a trajectory energy that yields an escape trajectory towards a libration point with the Moon at the appropriate geometry
- Target the anti-Sun right ascension and declinations at the appropriate launch epoch
- Target the Solar-rotating coordinate system velocity of the Sun- Earth rotating coordinate x-z plane crossing condition to achieve a quasi-libration orbit, L_2 x-axis velocity ~ 0
- Target a second x-z plane crossing velocity which yields a subsequent x-z plane crossing, then target to a one-period revolution at L_2
- In all above conditions, vary the launch injection C3 and parking orbital parameters (ω, Ω, parking orbit coast duration, and inclination)
- Incorporate conditions to achieve the correct orientation of the Lissajous pattern

Basic DC targeting procedures used in developing a baseline lunar gravity assist trajectory for a transfer trajectory to the Sun – Earth L2 are:

- Target the Moon at the appropriate encounter epoch to achieve an anti-Sun outgoing asymptote vector
- Target the lunar B-Plane condition to achieve gravity assist parameters and a perpendicular Sun-Earth rotating coordinate x-z plane crossing
- Target x-z plane crossing velocities which yield a second x-z plane crossing and target to a one-period revolution at L2
- Re-target lunar B-plane conditions to achieve the correct orientation of the Lissajous pattern with respect to the ecliptic plane

In both scenarios, target goals may include time (epoch, durations, and flight time), B-plane conditions (B.T B.R angle, B magnitude, outgoing asymptote vector and energy), libration Sun-Earth line crossing conditions (position, velocity, angle, energy, or a mathematical computation (eigenvectors)), or other parameters at intermediate locations that are often used in the targeting process. Targets may be in a single event string, nested, or branched to allow repeatable targeting. Maneuvers can be inserted where appropriate.

Retargeting conditions via addition of deterministic ΔVs can be used to achieve the correct orientation and Lissajous pattern size with respect to the ecliptic plane. This procedure is duplicated for significant changes in launch date or to include lunar phasing loop strategies.

While these procedures will achieve the required orbit, it is not robust for rapidly changing requirements or it may not provide the intuitive understanding of the general environment necessary for the designer to make educated decisions on design parameters. In order to decrease the difficulty in meeting mission orbit parameters and constraints in a direct targeting approach, the application of a dynamical system approach is investigated and incorporated into the overall trajectory design technique. This procedure can also be used for backward targeting, starting with a predefined libration orbit from ATD and targeting backward in time to the launch / parking orbit conditions. This procedure also involves the use of a DC to calculate maneuvers to attain the mission orbit and parking orbit constraints. Using parametric scans, optimization, DC, and multiple targets, a more robust design can be achieved.

3. THE ADAPTIVE TRAJECTORY DESIGN TOOL

Numerous unique and original designs go undeveloped because the manual trajectory blending process not only takes time, but also limits the designer's options since they must make a choice based on experience or what the software permits. A much wider design space is available for exploration if a fundamental concept over multiple regimes can be evaluated quickly and efficiently from a systems perspective [9,10]. In the last two decades, Goddard and Purdue University have been proactive in exploring new regimes in support of trajectory concepts. Exploiting the inherent dynamical structures that emerge from Dynamical System Theory, ATD represents an innovative next step in trajectory design that will be the future of complex mission design for many programs.

ATD is an original and unique concept for quick and efficient end-to-end trajectory designs using proven piece-wise dynamical methods. ATD provides mission design of cis-lunar and Earth-Moon libration orbits within unstable/stable regions through the unification of individual trajectories from different dynamical regimes. Based on a graphical user interface (GUI) ATD provides access to solutions that exist within the framework of the CR3BP in order to facilitate trajectory design in the Earth-Moon regime in an interactive and automated way. These trajectories can be developed individually via numerical Floquet methods, high-fidelity integration and optimization, or simple conic applications of a fundamentally elliptical orbit. ATD was developed under the FY12 and FY13 NASA GSFC Innovative Research and Development programs. ATD is used by GSFC to support Earth-Moon libration orbit missions and other missions in cis-lunar space including the Transiting Exoplanet Survey Satellite mission, analysis of Earth-Moon habitats, and was also used to aid in the evaluation of the mission design of the Earth-Moon orbits for the Asteroid Redirect Mission

Other mission design approaches using commercial and NASA software tools, such as STK/Astrogator and GMAT, complete each mission design phase in isolation with the beginning/end state information from one regime used to kick-off the design process in the next regime. Such a serial design strategy can be time-consuming and yields a result with the very real possibility that the optimal combination is overlooked. In contrast, ATD allows disconnected arcs to be conceptually devised in different frames (inertial, rotating, libration point) and models (conic, restricted three-body, ephemeris). Then the individual arcs are blended to leverage the advantages of each dynamical environment. The GSFC supported ARTEMIS mission was supported in

this manner since each section/phase of the trajectory, i.e., near Earth, Sun-Earth, and Earth-Moon, was required to be part of a continuous trajectory flow. Current design processes are not automated and, once a continuous solution exists, it is not possible to substantially modify the overall design without a new start and a significant time investment.

ATD provides access to a composite view of multi-body orbits possessing a variety of characteristics within an interactive design setting. The availability of a large assortment of orbit types within one mission design environment offers the user a unique perspective in which various mission design options may be explored, and the effectiveness of different orbits in meeting mission requirements may be evaluated. Once a discontinuous baseline is assembled within the design environment, it is then transitioned into a unified higher-fidelity ephemeris model via interactive ATD differential correction environments. The final trajectory is imported into GMAT where it can be accessed for further high-fidelity analysis.

4.   GENERAL MISSION ANALYSIS TOOL

The GMAT was conceived and developed by an experienced team of NASA Goddard Space Flight Center's aerospace engineers and software designers. Along with private industry, public, and private contributors, GMAT is used for real-world engineering studies, as a tool for education and public engagement, and to fly operational spacecraft. It is an open-source high-fidelity space mission design tool designed to model and optimize spacecraft trajectories in flight regimes ranging from low-Earth orbit to lunar, libration point, and deep space missions. GMAT is a feature-rich system containing high-fidelity space system models, optimization and targeting, built-in scripting and programming infrastructure, and customizable plots, reports and data products to enable flexible analysis and solutions for custom and unique applications. GMAT can be driven from a fully-featured, interactive GUI or from a custom script language.

Analysts model space missions in GMAT by first creating resources such as spacecraft, propagators, estimators, and optimizers. Resources can be configured to meet the needs of specific applications and missions. GMAT contains an extensive set of available resources that can be broken down into physical model resources and analysis model resources. Physical resources include spacecraft, thrusters, tanks, ground stations, formations, impulsive burns, finite burns, planets, comets, asteroids, moons, barycenters, and libration points. Analysis model resources include differential correctors, propagators, optimizers, estimators, 3-D graphics, x-y plots, report files, ephemeris files, user-defined variables, arrays, strings, coordinate systems, custom subroutines, MATLAB functions, and data. Figure 3 below illustrates a recent application using GMAT to solve for a trajectory solution that uses a low-thrust propulsion system for a lunar Cubesat mission.

Figure 3. GMAT Graphical Interface

5.   MODELING FIDELITY

The various models used in trajectory design tools can impact the quality and accuracy of the design and the simulation durations. Fortunately, for most of the design process, a lower-fidelity model can provide an accurate assessment of the overall design challenges and a preliminary set of states and conditions that can be used in high-fidelity tools. An example of this is the use of the CR3BP in the design process. The CR3BP as used in tools such as ATD can provide a user with an efficient preliminary design that can be used for selection criteria and transfer and orbit trades. In addition to the use of ATD, there are additional tools within the ATD database that provide trade information via the use of a reference catalog. This

catalog permits the user to perform trades with long-term simulations without concern for modeling uncertainties. These trades include transfer and maintenance ΔVs, duration, and stability of the orbit.

5.1 Poincaré Maps

A feature of the ATD tool is that it can provide a mapping of the dynamical system using the internal calculations and mathematical equations of a CR3BP [11]. This Poincaré mapping process permits the designer to calculate large areas of possible manifolds by generating a map that shows values of possible target conditions. These include a mapping of the Euclidean space in the areas surrounding all of the manifold entry locations as well as the libration orbit environments that the manifolds are generated from. Shown in Figure 4 is a sample of a Poincaré mapping from ATD that gives information on Sun-Earth transfer manifolds for Lunar IceCube and WFIRST.
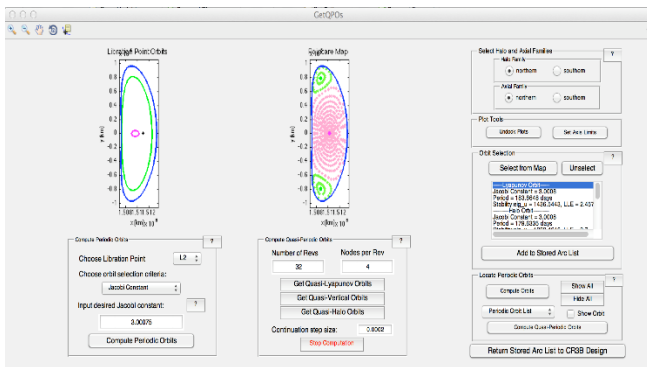
Figure 4. Poincare Mapping Interface based on ATD Calculations
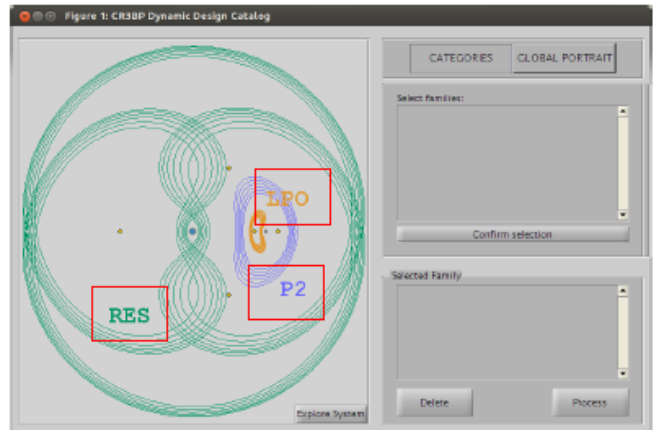
Figure 5. ATD Main Panel of the Graphical Interface for the Reference Catalog.

5.2 Reference Catalogues

In addition to Poincaré Maps, Purdue University and GSFC also integrated a reference catalog into ATD [12, 13]. This catalog provides the user with a simple approach to determine and select libration and other orbits. Based on the ATD CR3BP model capabilities, the catalog contains numerous regenerated orbits, either spatial or Lyapunov, which can be used to generated the transfer invariant manifolds. A sample reference catalog interface and output is presented in Figures 5 and 6.
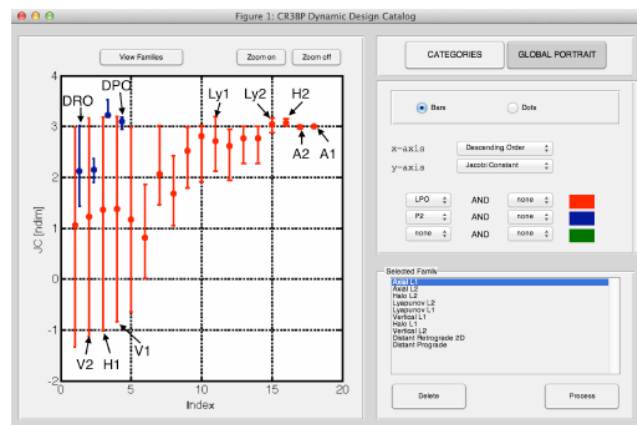
Figure 6. Reference Catalog Interface Comparison of Jacobi Constant Range across Libration Point Orbits and Moon-Centered Families

6. MISSION DESIGN APPLICATIONS

To show ATD and GMAT applications, interfaces, and ease of use, the following examples from Lunar IceCube and WFIRST mission design are demonstrated. The roles of various orbits in facilitating transport in the Earth-Moon system are shown, emphasizing the value in design tools that offer access to a composite view of a variety of orbit types. In addition to considering invariant manifolds associated with libration point orbits, a variety of different orbit types are useful when designing transfers in the Earth-Moon system.

6.1 Lunar Ice Cube Application

Lunar IceCube, a 6U CubeSat shown in Figure 7, has been selected for participation in the Next Space Technologies for Exploration Partnerships, which leverages partnerships between public and private entities to develop the deep space exploration capabilities necessary for the next steps in human spaceflight. The Lunar IceCube mission is led by the Space Science Center at Morehead State University and supported by scientists and engineers from the NASA GSFC, Busek, and Catholic University of America. GSFC is providing the trajectory design and maneuver and navigation support, as well as tracking support.
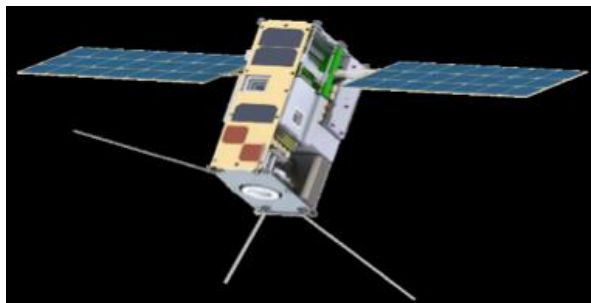
Figure 7. Lunar Ice Cube Spacecraft Design

Lunar IceCube will ride onboard the Orion EM-1 vehicle, currently scheduled for launch in 2018. Secondary payloads are deployed after the Interim Cryogenic Propulsion Stage (ICPS) disposal maneuver. The ICPS places Orion on a lunar free-return trajectory and thus the ICPS is also on a similar high-energy trajectory. Due to uncertainties in the ejection mechanism, Lunar IceCube's exact deployment state is not known in advance. However, with no additional maneuvers, the highly energetic nominal deployment state would result in Lunar IceCube quickly departing the Earth-Moon system. To decrease the spacecraft energy and achieve a transfer that approaches a low-altitude lunar orbit, the Lunar IceCube is equipped with a low-thrust propulsion system. This iodine-fueled engine is a Busek Ion Thruster 3-cm (BIT-3) system, which is currently designed to deliver a maximum 1.2 mN of thrust with an Isp of 2500 s and a fuel mass of approximately 1.5 kg. For the Lunar IceCube mission, the BIT-3 system enables finite-duration low-thrust arcs to be introduced along the transfer trajectory.

6.1.1 Designing the Lunar Ice Cube Trajectory

Although feasible end-to-end transfers may be obtained within a numerical modeling environment, a combined dynamical systems and numerical approach offers significant insight into the available transfer geometries and into the corresponding regions that can be used for the design process. Individual point solutions may be highly sensitive to uncertainties in both the deployment state and epoch, as well as any additional on-orbit perturbations. In fact, for relatively large third-body or lunar-flyby perturbations Lunar IceCube may not possess sufficient propulsive capability to achieve the desired reference trajectory. Alternatively, another transfer geometry may provide an operationally-feasible solution. To facilitate the identification and computation of these solutions, a trajectory design framework in ATD is constructed and demonstrated. First, the complete transfer trajectory is split into three segments: the post-deployment lunar encounter, the Sun-Earth-Moon transfer, and the lunar approach. Concepts from dynamical systems theory are applied to models of varying levels of fidelity, from the CR3BP to ephemeris level, over each segment. Next, mapping techniques are employed to identify connections between available trajectory arcs. Using the resulting analysis, a reasonable initial guess is obtained for corrections in an ephemeris model to obtain a high-fidelity, low-thrust-enabled, end-to-end transfer in GMAT.

Using the numerical sequence for B-plane targeting described above, one can change the flyby distance to reduce the overall system energy and place the CubeSat post-lunar flyby into a region with the correct Jacobi energy level (or Earth-centered 'C3' level) that can map to a dynamical manifold. Once at that energy level and position, there are multiple ways to jump or target onto the manifold, thus providing a natural motion that re-encounters the Moon at a later date.

Sample invariant manifolds used for the Lunar IceCube mission design are shown In Figure 8 and depict the typical output form ATD, which can then be used as a guide to determine target apoapsis locations and energy levels used for the above numerical flyby targeting conditions. These techniques are applied to dynamical models of varying levels of fidelity to explore the construction of a trajectory design framework. Despite an energetic initial deployment state, Lunar IceCube can achieve the desired final science orbit by exploiting solar gravity to modify both its energy and phasing. To supply rapid insight into the potential geometries for the long Sun-Earth phase of the trajectory, the CR3BP is employed. In this autonomous dynamical model, approximate bounds on the motion can be established and transfer geometries can be explained via manifolds of libration point orbits. This analysis is then transitioned to higher-fidelity models including the Bi-circular Four-Body Problem and an ephemeris model that also includes the additional contribution of a low-thrust engine. Boundary conditions such as the initial deployment state and the final science orbit are incorporated into this trajectory design framework to identify regions and geometries corresponding to feasible transfer trajectories for the mission.
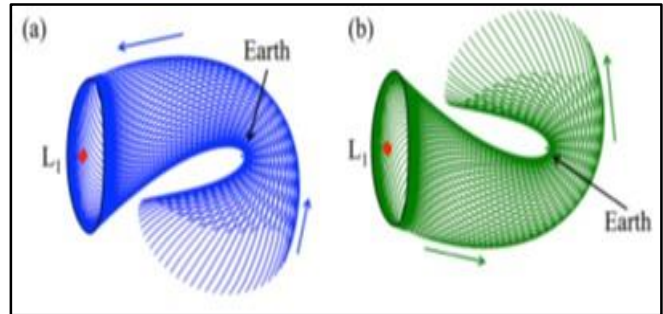
Figure 8. ATD Generated Invariant Manifold Used for Lunar IceCube Transfer Trajectory Design

6.1.2 Manifolds of Periodic Orbits

Motion within the CR3BP is guided by an underlying dynamical structure that includes families of periodic orbits and their associated manifolds. In the Sun-Earth system, well-known periodic orbits in the Earth's vicinity include the planar Lyapunov and three-dimensional halo orbits near the L_1 and L_2 equilibrium points. Both of these families include periodic orbits that possess stable and unstable manifolds, causing nearby trajectories to naturally flow towards or away from the periodic orbit, respectively. Along these manifolds, trajectories can pass through the L_1 and L_2 gateways, departing the Earth's vicinity. For planar motion, the manifold structures associated with the L_1 and L_2 Lyapunov orbits serve as separatrices, identifying the boundary between two types of motion that are qualitatively different. To demonstrate this concept, consider Figure 8 above which displays a sample (a) stable manifold and (b) unstable manifold associated with a Sun-Earth L_1 Lyapunov orbit, as generated in ATD. Using Figure 8(a) as a reference, trajectories on the blue surface lie directly on the stable manifold, which has been integrated backwards in time in a CR3BP model of the Sun-Earth system for approximately 210 days. Accordingly, these trajectories asymptotically approach the reference L_1 Lyapunov orbit. Motion that possesses both position and velocity states that lie within the boundaries of the blue surface pass through the L_1 gateway and depart the Earth's vicinity. When designing CubeSat trajectories that are close to planar, the stable manifolds of the L_1 Lyapunov orbit can supply approximate bounds on motion, i.e., regions

within the stable manifold must be avoided to ensure that a trajectory does not depart the Earth vicinity. Furthermore, this structure may influence motion near the Earth after deployment. On the contrary, motion on the green surface in Figure 8(b) lies on the unstable manifold associated with the L_1 Lyapunov orbit, which is integrated forward in time for 210 days. Trajectories interior to the boundaries of this manifold structure originate from the vicinity of the Sun. However, the unstable manifold may still guide motion that flows towards the Earth. In fact, arcs from both of these manifold structures may be combined to construct nearby trajectories that temporarily depart the Earth vicinity to achieve the necessary energy and phasing parameters to reach the desired lunar science orbit. Although these structures exist in the simplified and autonomous CR3BP, they are approximately retained in the true ephemeris model of the Sun, Earth and Moon, providing rapid and valuable insight into the existence of and the associated boundaries for predominantly natural transfer geometries for the Lunar IceCube mission.

6.1.3 Feasible Transfer Regions

ATD-generated regions in the Earth apoapsis maps in Figure 9, corresponding to transfers that remain in the Earth's vicinity, can be differentiated by their geometries to guide numerically-targeted outgoing lunar flyby conditions which subsequently place the Lunar IceCube on a natural transfer that requires little or no propulsive effort. To demonstrate the identification of feasible transfer regions and their associated geometries, consider an apoapsis map constructed using prograde (counter-clockwise motion about Earth) initial conditions at $C = 3.00088$ for trajectories that complete two revolutions around the Earth, as depicted in Figure 9. The gray-shaded portions of the figure indicate forbidden regions, while red diamonds locate the equilibrium points, the light blue circle at the center indicates the location of the Earth and the purple curve depicts the lunar orbit, approximated as circular. On this apoapsis map, apoapses for each feasible transfer region are colored by the geometry of the subsequent transfer path, determined using the velocity direction at each apoapsis, i.e. prograde or retrograde. Specifically, blue regions in Figure 9 indicate transfers that possess only apoapses that are prograde, such as the transfer

displayed in the bottom left inset. This feasible transfer region lies close to the zero velocity curves of the CR3BP and the transfers resemble the sample end-to-end trajectory in the bottom right corner of Figure 9 constructed as a point solution using an operational modeling environment.
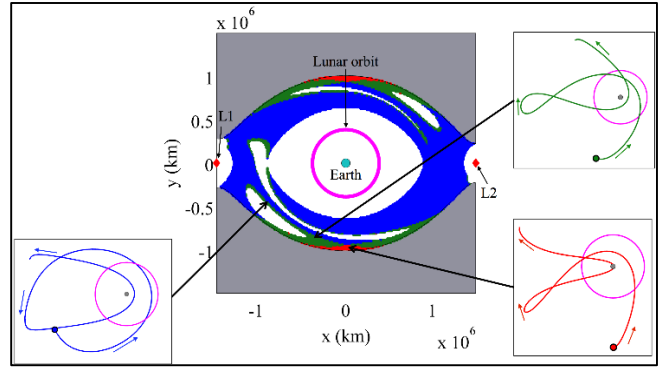

Figure 9. Apoapsis Map in the CR3BP at $C = 3.00088$ for Prograde Initial Conditions. Blue, Red and Green Regions Indicate Initial Apoapses of Feasible Trajectories that Remain within The Earth Vicinity for Two Revolutions, with Each Color Corresponding to A Different Transfer Geometry Illustrated via the Inset Images.

6.1.5 End-to-End Transfer: Connections between Transfer Segments

To validate the proposed ATD trajectory design framework, a sample trajectory is split into the three mission segments and compared to the transfer options identified by the tools within this framework. Consider the previously-developed point solution as seen in the lower-right panel in Figure 9; this solution is constructed using operational-level ephemeris software. This sample trajectory is reproduced in Figure 10. The transfer begins at the current EM-1 deployment state; shortly thereafter, a 3.8 day low-thrust arc is activated until just before lunar periapsis to decrease the orbital energy and to target a lunar B-plane crossing that produces a trajectory which remains within the Earth's vicinity. This multi-day maneuver is represented by a red arc segment in Figure 10. Following the first lunar flyby, the spacecraft initiates a long coast arc (blue) and passes through three apogees over 173 days before beginning a 70 day low-thrust burn, colored red, to capture around the Moon and achieve the desired science orbit.

The end-to-end path requires three arcs, one for each mission segment. Once the arcs are designed, individuals are linked to deliver a continuous path.

6.1.6 High Fidelity Numerical Targeting using ATD data

Based on the above ATD dynamical design properties, these manifolds and the related trajectory data are then provided as a script for execution by the GMAT tool. This scripting is basically a process in which patch points are provided for a numerical targeting process within GMAT. The patch points provide intermediate target locations that are easily achieved by a DC process. Figure 10 presents the ephemeris generated ATD output for a feasible Lunar IceCube design.
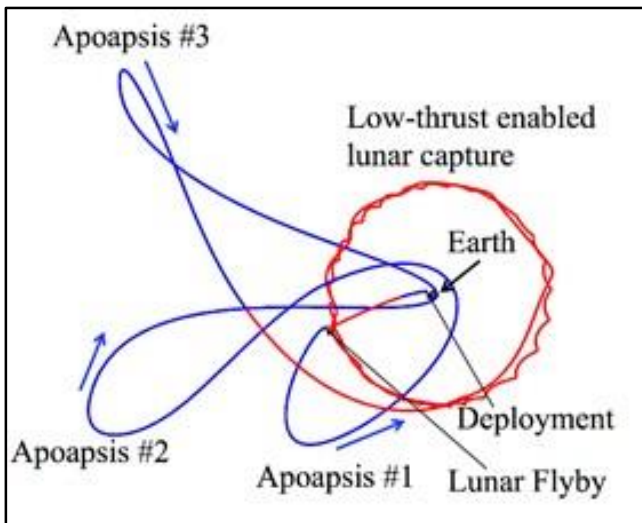
Figure 10. Sample Lunar IceCube Design in Solar Rotating Coordinate Frame Produced using ATD Ephemeris Model, with Blue Segments Indicating Natural Coasting and Red Segments Indicating a Low Thrust Arc.

This ATD / ephemeris information is transferred to a GMAT script which reflects a full high-fidelity modeling of gravity and solar radiation pressure. As the reader can see in Figure 11, the transfer design is nearly identical to that in Figure 10.
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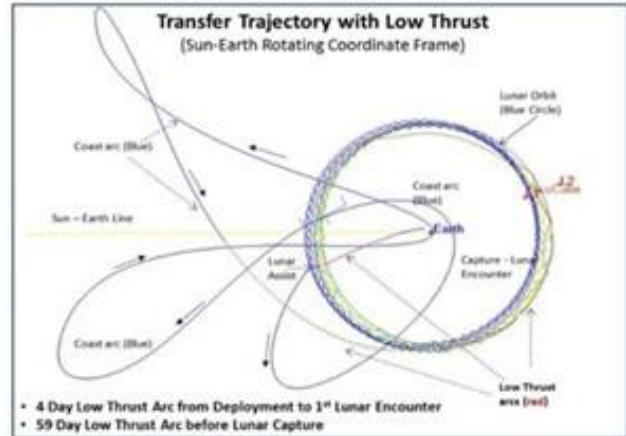
Figure 11. A GMAT generated Lunar IceCube transfer designed in an ATD environment
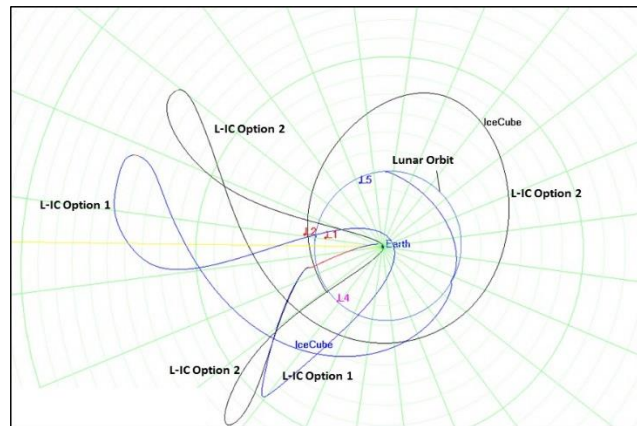
Figure 12. GMAT generated Lunar IceCube transfers designed using ATD Apoapsis Selection Criteria

Thus the Lunar IceCube design process is a result of an initial GMAT numerical lunar B-Plane targeting process that achieves the energy and apoapsis conditions of the invariant manifold that in turn permits a natural flow back to the Moon without the need for any deterministic maneuvers. During operations, navigation uncertainties will require statistical maneuvers to be performed, but instead of targeting back to a 'reference' trajectory, another nearby manifold can be identified that provides a similar lunar encounter which can minimize the overall ΔV and fuel budget. Additionally, for uncertainties in the outbound lunar flyby, one does not necessarily want to target back to the original B-plane target as that may require a ΔV or thrust level that is not achievable with the propulsion system. Using ATD, other nearby manifolds can be identified for

other apoapsis energy levels that can be reached with a different flyby condition. Figure 12 provides an example of two different transfers that were based on two ATD apoapsis energies and two different flyby conditions, but based on the same original deployment state.

6.2 WFIRST Application

WFIRST is a NASA observatory currently in a preliminary design development stage. WFIRST is using an existing re-purposed 2.4 meter telescope along with two main instruments: 1) A wide field instrument (WFI) which is comprised of a wide field camera with a field of view 100 times greater than the Hubble Space Telescope's and an integral field unit, which will help characterize supernovae to trace the evolution of the universe; 2) A coronagraphic instrument (CGI) which will be the first instrument able to characterize the atmospheres of super-Earth planets and Neptune-like planets around nearby Sun-like stars [5]. WFIRST was the highest-ranked large space mission in the 2010 National Academy of Sciences Decadal Survey, New Worlds New Horizons, and addresses all of the following questions identified for astrophysics in the 2014 NASA Science Plan [5]: 1) How does the universe work? 2) How did we get here? 3) Are we alone?

The WFIRST design reference mission is a four-part observing program comprising (1) a high-latitude survey optimized to study dark energy but enabling an enormous variety of other investigations, (2) a galactic bulge survey that will use microlensing observations to complete the planetary census begun by Kepler, (3) coronagraphic observations of nearby planets and proto-planetary systems, and (4) a Guest Observer program that will utilize the power of the WFI and the CGI to address a wide-ranging set of open problems in astrophysics [5]. WFIRST will be making major contributions towards all three of the goals listed for astrophysics in the 2014 NASA Science Plan: 1) Probe the origin and destiny of our universe, including the nature of black holes, dark energy, dark matter and gravity. 2) Explore the origin and evolution of the galaxies, stars and planets that make up our universe. 3) Discover and study planets around other stars, and explore whether they could harbor life. [5]

WFIRST, a NASA-led mission, is a partnership between NASA GSFC and NASA Jet Propulsion Laboratory. The GSFC's Navigation and Mission Design Branch is providing the trajectory design, maneuver and navigation support, as well as tracking support. WFIRST is scheduled for launch on an EELV from Cape Canaveral in 2024. WFIRST's transfer will require a launch towards Sun-Earth Lagrangian 2 (SEL2) and is planned to orbit in a Libration Point Orbit (LPO) for 6 years. With WFIRST still being in its early developmental stages, project and science requirements have not been finalized, leaving the orbit design with only one orbit requirement thus far: no Earth shadows during the 6 year mission orbit at SEL2. This means that the mission orbit is not constrained to be a particular shape, leaving many LPO orbit options available. However, finding the correct LPO that had no Earth shadows and remained conditionally stable (with station-keeping maneuvers) for 6 years would be a challenge without the guidance/assistance of ATD.

6.2.1 Designing a WFIRST Trajectory

With the current design requirement in mind, ATD was used to investigate the orbit options for the LPOs. The only other added constraint placed on the LPO design was that the maximum Sun-Earth L2 Vehicle (SEL2V) angle during the mission orbit must be less than 36 degrees. An SEL2V angle greater than 36 degrees poses communications issues with the ground because the spacecraft could be too far above or below the horizon during the winter and summer seasons. This means that maximum RLP Y amplitude at SEL2 can be no greater than 1,090,927 km (see Figure 13). In order to avoid all Earth shadows during the 6 year mission orbit, a minimum SEL2V angle was determined to be 0.51 degrees (see Figure 14) [14]. The Earth's umbra range does not extend to SEL2 eliminating that shadow constraint, but the Penumbral Earth shadow continues beyond SEL2 and must be accounted for. In order to avoid this shadow, the RLP Y minimum value must be greater than 13,423 km.

With the minimum and maximum RLP Y amplitudes determined, the design process began in ATD. The desired amplitude was set to 1,000,000 km and the LPO options were investigated. Lyapunov Orbits were eliminated due to the fact that they cross

at RLP Y and RLP Z = 0, hence they undergo Earth Penumbral shadows. Axial and Vertical orbits also violate the this requirement. It was found that quasi-halo (non-Planar Lyapunov orbits) and Lissajous LPOs met the current WFIRST requirements. However, a Lissajous LPO would require an avoidance maneuver in the future in order to avoid Earth penumbral shadows.
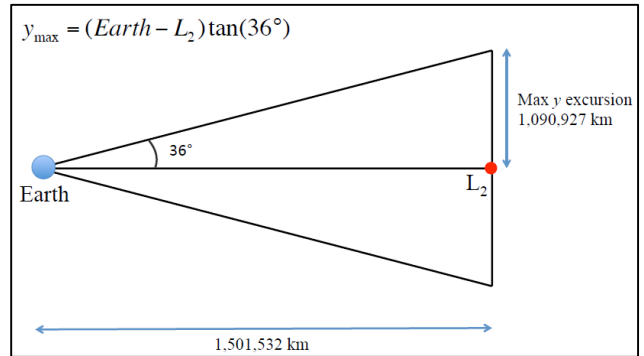

$$y_{max} = (Earth - L_2) \tan(36°)$$

Figure 13: Diagram for the maximum RLP Y value for the given SEL2V angle of 36 degrees
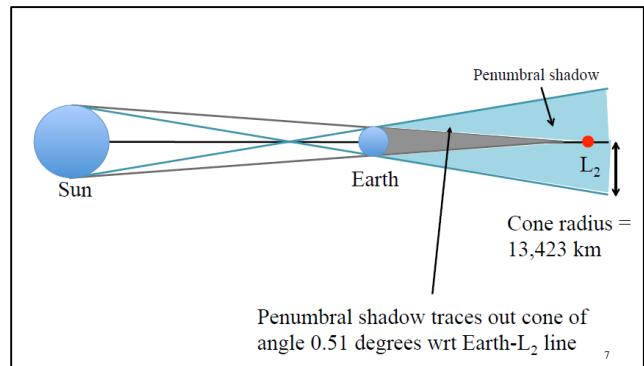
Figure 14: Umbral Earth shadow

6.2.2 WFIRST Manifolds

For simplicity, a SEL2 quasi-halo design was selected. In order to find an end-to-end transfer design, a 185 km, 28.5 degree parking orbit which simulates an EELV launch and coast from Cape Canaveral was used as an initial orbit. Stable manifolds with a duration of 150 days were generated using the desired SEL2 quasi-halo Orbit with a goal of intersecting with or passing close to the parking orbit (see Figure 15 and 16) [14].
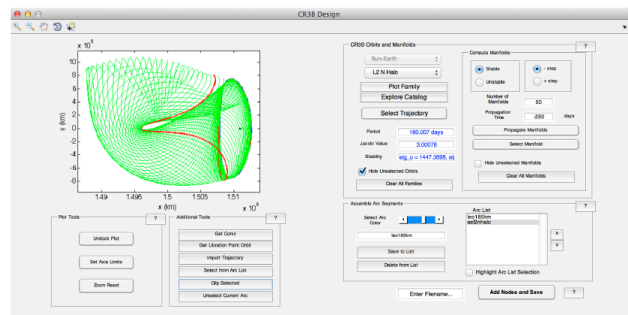
Figure 15: 150 Days of propagated L2 Manifolds (in green) with the selected stable manifold (in red) that approaches the designed 185 km parking orbit.
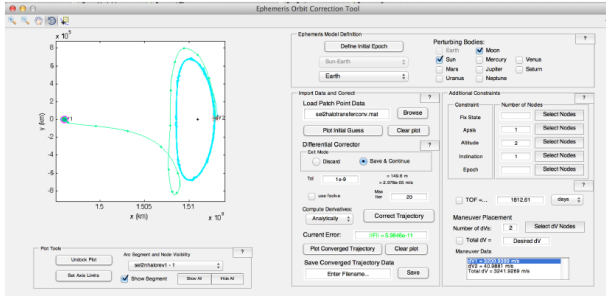

Figure 16: Full ATD WFIRST Trajectory Design with added ΔV's and perturbing bodies

The selected transfer manifold was then clipped so that it began near the parking orbit and ended near the first RLP XZ crossing at SEL2. Additional revolutions at SEL2 were then added to achieve the desired mission lifetime. Individual nodes were added along the desired trajectory so that it could be saved as a MATLAB .mat file. CR3BP corrections were then calculated to find the ΔV necessary to transfer from the parking orbit onto the stable manifold, and from the stable manifold into the L2 Mission Orbit. An initial epoch, and central and perturbing bodies (the Sun and Moon), were then modeled so that a full ephemeris model of the desired trajectory was generated. With the ephemeris model saved, a MATLAB script was written that would load the ephemeris model into GMAT and differential correct to achieve the original trajectory by targeting the evenly distributed nodes. The trajectory in GMAT is shown in Figure 17. Finally, a GMAT ephemeris was saved and loaded into AGI's STK. Using STK's Astrogator and a Differential Corrector, a Mission Sequence was built to target an initial position on the transfer out to SEL2 and an initial SEL2 RLP X and Z crossing that would match the ATD design.
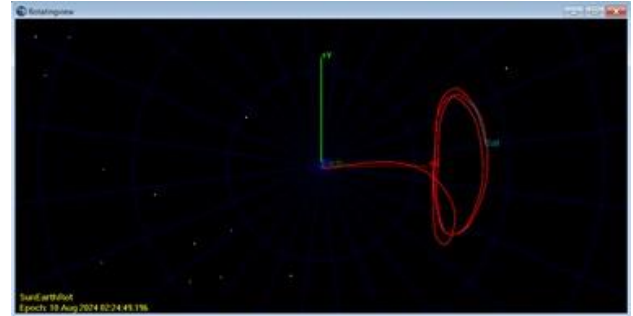

Figure 17: ATD Trajectory Propagated and Differentially Corrected in GMAT

As shown in Figures 18 and 19, the STK-targeted and original ATD designs are nearly identical. By using ATD to design a trajectory for WFIRST, a point-and-shoot (multi-trial) method was eliminate and the mission requirements could be included directly into the design process. The desired LPO orbit requirements and associated stable manifold was generated to minimize the transfer injection ΔV and the SEL2 insertion ΔV.

7. CONCLUDING REMARKS

The combination of the GSFC / Purdue developed ATD tool along with a proven (operational) numerical tool like GMAT provides the mission designer with a unique advantage for trajectories under the influence of Sun-Earth or Earth-Moon multi-body systems. The advantage of an intuitive design process allows an investigation of a full range of possible trajectories along with the possible trajectories that are needed for contingency or in response to sensitivity studies.

With respect to the trajectory design for the Lunar IceCube mission, which is subject to constraints and uncertainties in its deployment state and a limited propulsive capability, a structure is constructed using techniques from ATD, dynamical systems theory, and numerical design tools. Although feasible point solutions can be identified using operational-level modeling software, a dynamical systems approach supplies insight into the sensitivity of these paths and regions of availability for similar transfers. Such analysis is valuable for spacecraft that are unable to implement large corrective maneuvers to remain on a precomputed path. For Lunar IceCube, a flexible design process is constructed that enables rapid

trajectory re-design to mitigate state uncertainties, orbit determination errors, and maneuver execution errors. Once a set of feasible connections has been identified, a corrections scheme may be applied to produce an end-to-end trajectory in operational-level software. For Lunar IceCube, the obvious benefits are the ability to generate an accurate design quickly, and to gain the intuitive trajectory space knowledge that comes with using these tools.
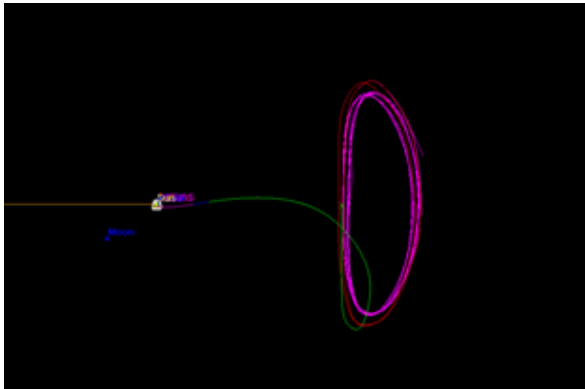

Figure 18: View in the RLP XY Frame of the ATD Ephemeris (in magenta) and STK targeted trajectory (in green and red)
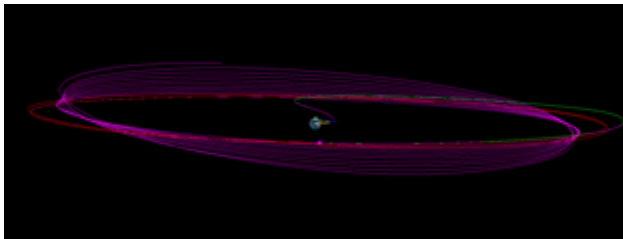

Figure 19: View looking down the SEL2 line of the ATD Ephemeris (in magenta) and STK targeted (in green and red)

The WFIRST mission design was accurately and efficiently generated to meet both the transfer and the science orbit requirements. The design process began with ATD and rapidly expanded beyond the orbit selection and into feasible transfer trajectories (manifolds) that converged on the parking orbit. Once at this level, perturbation modeling and trajectory data can be reassigned to the high-fidelity models of operational tools such as GMAT and STK. As with the Lunar IceCube design, the use of ATD provides the user with an intuitive design process with can easily be incorporated into operational tools.

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