AN INTEGRATED TOOL FOR LOW THRUST OPTIMAL CONTROL ORBIT TRANSFERS IN INTERPLANETARY TRAJECTORIES

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ABSTRACT

In the last recent years a significant progress has been made in optimal control orbit transfers using low thrust electrical propulsion for interplanetary missions. The system objective is always the same: decrease the transfer duration and increase the useful satellite mass. The optimum control strategy to perform the minimum time to orbit or the minimum fuel consumption requires the use of sophisticated mathematical tools, most of the time dedicated to a specific mission and therefore hardly reusable.

To improve this situation and enable Alcatel Space to perform rather quick trajectory design as requested by mission analysis, we have developed a software tool T-3D dedicated to optimal control orbit transfers which integrates various initial and terminal rendezvous conditions – e.g. fixed arrival time for planet encounter - and engine thrust profiles –e.g. thrust law variation with respect to the distance to the Sun -. This single and quite versatile tool allows to perform analyses like minimum consumption for orbit insertions around a planet from an hyperbolic trajectory, interplanetary orbit transfers, low thrust minimum time multiple revolution orbit transfers, etc…

From a mathematical point of view, the software relies on the minimum principle formulation to find the necessary conditions of optimality. The satellite dynamics is a two body model and relies of an equinoctial formulation of the Gauss equation. This choice has been made for numerical purpose and to solve more quickly the two point boundaries values problem. In order to handle the classical problem of co-state variables initialization, problems simpler than the actual one can be solved straight forward by the tool and the values of the co-state variables are kept as first guess for a more complex problem.

Finally, a synthesis of the test cases is presented to illustrate the capacities of the tool, mixing examples of interplanetary mission, orbit insertion, multiple revolution orbit transfers taken from ESA studies recently studied by Alcatel Space.

1. INTRODUCTION

In the last past years, low-thrust propulsion has evolved from a technological concept to a realistic option for interplanetary missions, considered first for very challenging trajectories to the inner planets, but also for more classical ones, to Mars or the Moon. After Deep-Space 1, the first demonstration mission of the use of low-thrust propulsion for interplanetary missions and Smart 1 to a closer destination, it is the baseline for BepiColombo, the first European mission to Mercury and for Solo, a challenging European mission to observe the Sun from as close as 48 solar radii (0.222 AU).

In that context, Alcatel Space decided to increase its engineering capacity in interplanetary missions to low thrust propulsion by extending a tool developed for the LEOP of geostationary satellites to trajectories in the Solar System. After recalling the mathematical principles used to find low thrust optimal control orbit transfers, the main features of the tool developed to handle interplanetary missions are shortly presented, as well as the man-machine interface which considerably eases the use of the tool.

Finally, some examples of low-thrust interplanetary trajectories inspired from current ESA projects and calculated with the tool are shown and when possible, compared with reference data.

2. CONTROL OPTIMAL MATHEMATICAL FORMULATION

A lot of work has been performed in this field since the 1950's as illustrated in the references [1], [2], [3], [4] and [5]. The performance assessment of a trajectory is one of the important thing to be done during the mission analysis. The objective of the Alcatel Space tool, T_3D, is to perform quickly this analysis.

T_3D tackles the optimal control problem of orbit transfer with continuous thrust propulsion system by solving a two-point boundary problem in minimum time transfer and minimum fuel consumption and fixed time.

Between the different techniques of optimisation the choice has been made on "indirect" methods by using the maximum principle. Such an approach is called "indirect" because, rather trying to solve directly the problem, we try to solve the equations given the necessary conditions of optimality. The advantages are a reduced number of optimisation parameters, which will give a rather quick calculation of the solution. The main drawback is the need of a good initial guess of the optimisation parameters to allow the convergence of the solver.

T_3D satellite dynamic is a two body model. The tool uses the equinoctial formulation of the Gauss equation with parameters:

Equation 1

$$
\begin{aligned}
p &= a\left|1-e^2\right| \\
e_x &= e\times\cos(\omega+\Omega) \\
e_y &= e\times\sin(\omega+\Omega) \\
h_x &= \tan\left(\frac{i}{2}\right)\cos(\Omega) \\
h_y &= \tan\left(\frac{i}{2}\right)\sin(\Omega) \\
l &= \omega+\Omega+\nu
\end{aligned}
$$

The use of the semi-rectum latus p instead of the semi-major axis allows the tool to work either with elliptic or hyperbolic orbits.

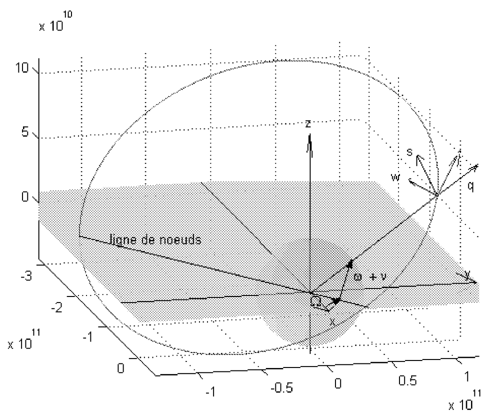

Figure 1: Orbit parameter and local orbital frame

With these parameters and the definition of local orbital frame presented in Figure 1 the dynamics equation is:

$$
\begin{aligned}
\frac{dp}{dt} &= 2\sqrt{\frac{p^3}{\mu}}\frac{1}{Z}S \\
\frac{de_x}{dt} &= \sqrt{\frac{p}{\mu}}\frac{1}{Z}\left(Z\times\sin(l)Q + A\times S - e_y F\times W\right) \\
\frac{de_y}{dt} &= \sqrt{\frac{p}{\mu}}\frac{1}{Z}\left(-Z\times\cos(l)Q + B\times S + e_x F\times W\right) \\
\frac{dh_x}{dt} &= \frac{1}{2}\sqrt{\frac{p}{\mu}}\frac{X}{Z}\cos(l)W \\
\frac{dh_y}{dt} &= \frac{1}{2}\sqrt{\frac{p}{\mu}}\frac{X}{Z}\sin(l)W \\
\frac{dl}{dt} &= \sqrt{\frac{\mu}{p^3}}Z^2 + \sqrt{\frac{p}{\mu}}\frac{1}{Z}F\times W \\
\frac{dm}{dt} &= -\frac{T}{g_0\times Isp}
\end{aligned}
$$

with

$$
\begin{aligned}
Z &= 1+e_x\cos(l)+e_y\sin(l) \\
A &= e_x+(1+Z)\cos(l) \\
B &= e_y+(1+Z)\sin(l) \\
F &= h_x\sin(l)-h_y\cos(l) \\
X &= 1+h_x^{\ 2}+h_y^{\ 2}
\end{aligned}
$$

2.1 General formulation using minimum principle

We introduce J the cost function. The general form of the cost function can be written:

$$
\bar{J} = \phi\left[x(t_f),t_f\right] + \nu^T\psi\left[x(t_f),t_f\right] + \int_{t_0}^{t_f} L[x,u,t] + \lambda^T\left(f(x,u,t)-\dot{x}\right)dt
$$

where:

$\phi\left[x(t_f),t_f\right]$ is the part of the cost function at arrival, for example the final mass,

$\psi\left[x(t_f),t_f\right]$ are the boundary conditions at arrival,

$L[x,u,t]$ is the part of the cost function along the trajectory, for example 1 in case of minimum time problem,

$f(x,u,t)-\dot{x}$ is the dynamic constraint equation along the trajectory.

we define the Hamiltonian by:

$$
H(x,u,\lambda,t) = L(x,u,t) + \lambda^T f(x,u,t)
$$

The necessary condition of optimality is $\partial \bar{J} = 0$, which after the variation calculation gives :

$\dot{x} = f(x,u,t)$ n differential equations

$\dot{\lambda} = -\left(\frac{\partial f}{\partial x}\right)^T \lambda - \left(\frac{\partial L}{\partial x}\right)^T$ n differential equations of the co-statedynamics

the command u(t) being determined by the relation :

$\frac{\partial H}{\partial u} = \left(\frac{\partial f}{\partial u}\right)^T \lambda + \left(\frac{\partial L}{\partial u}\right)^T = 0$ m algebraic relations

and the boundary conditions :

$x_k(t_0)\ given\ at\ t_0\, or\, \lambda_k(t_0) = 0$ n boundary conditions on the state at t_0

$\lambda^T(t_f) = \left(\frac{\partial \phi}{\partial x} + \nu^T \frac{\partial \psi}{\partial x}\right)_{t=t_f}$ n boundary conditions on the co-state at t_f

$$\left[\frac{\partial \phi}{\partial t} + \nu^T \frac{\partial \psi}{\partial t} + \left(\frac{\partial \phi}{\partial x} + \nu^T \frac{\partial \psi}{\partial x}\right) f + L\right]_{t=t_f} = 0$$

1 boundarycondition on the final time in case of minimal time problem

$\psi\left[x(t_f), t_f\right] = 0$ q boundary conditions on the state at t_f.

2.2 Boundary condition solve by T_3D

T_3D solves many types of boundary conditions which cover the main needs of mission analysis.

We can define the departure situation in 2 ways: by giving the 6 orbital parameters at t=0 or by giving the orbit at t=0 and the V_{infiny} departure. The start time t_0 can be free or fixed depending on the launch window for the mission.

They are 11 arrival conditions:

1. transfer on a, e et i
2. transfer on a, e, i, ω et Ω
3. transfer on fixed target on a, e, i, ω, Ω and ν
4. transfer on mobile target on a, e, i, ω, Ω and ν(t_f) modulo 2π
5. transfer on fixed target on a, e, i, ω, Ω and ν modulo 2π
6. transfer on fixed Cartesian space positions x, y and z
7. transfer on mobile Cartesian space positions x(t_f), y(t_f) and z(t_f) (example: planet crossing)
8. transfer on fixed radius r
9. transfer on mobile radius r(t_f)
10. transfer on fixed Cartesian space positions x, y and z and V_{infiny} arrival given
11. transfer on mobile Cartesian space positions x(t_f), y(t_f) and z(t_f) and V_{infiny} arrival given (example planet insertion)

2.3 Thrust law

T_3D uses classical thrust profile like F = constant or F = F_0*(r_0/r)^2 to simulate the influence of the radius distance to the sun on power generation. In fact any thrust profile can be defined by the user in script file.

3. DESCRIPTION OF THE TOOLS

3.1 Overview

The tool T3D is implemented in Matlab®, which allows:

1. To run the tool on various platforms without the need to change the code. T3D can therefore be used in design phase in the same environment as the other engineering tools for interplanetary missions – most of them currently running on workstations – but also on computers more suited for word-processing when writing the study report.

2. To use the graphical interface possibilities of Matlab® to ease the definition of the input parameters. Although the input data can be entered in a file with a given format, it is preferable to let the interface write this file for you when you start the calculations. Moreover, a previous run case can be loaded onto the interface and its parameters changed for further analysis.

3. To generate easily and in a systematic way a series of plots to have an engineering feeling on the solution resulting from the calculations. Apart from the usual time evolution of the parameters of the problem – thrust orientation and level, spacecraft mass, …-, a 3D representation of the trajectory ready to be inserted in a report.

3.2 The Man-Machine Interface (MMI)

A snapshot of the Man-Machine Interface is shownon Figure
2:

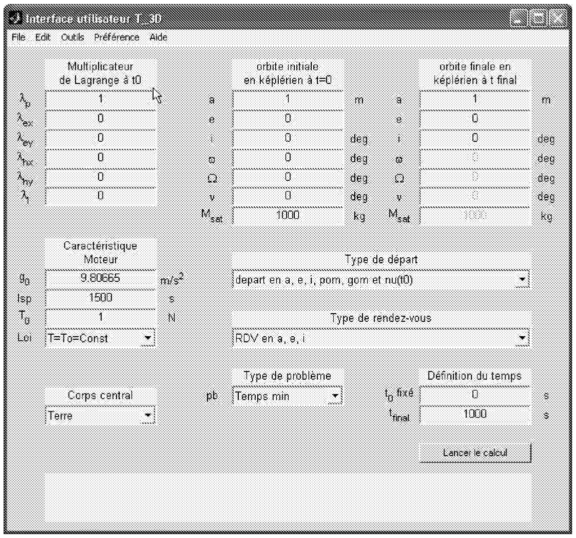

Figure 2: T_3D Man-Machine Interface

4. EXAMPLES OF LOW-THRUST TRAJECTORIES

A few examples are presented in the next paragraphs inspired from studies initiated by the Centre National d'Etudes Spatiales and the European Space Agency. When relevant, a comparison of the trajectory characteristics obtained by T3D with those calculated by the Agencies is provided.

4.1 Earth-Venus Trajectory

Venus is the easiest target for high thrust propulsion since it requires only an infinite departure velocity of about 2.5 km/s. Due to that, apart from being a target on its own, Venus is also often used by a spacecraft on its way to the inner regions of the Solar System to bend its trajectory, even in a context of low-thrust propulsion. Two ESA missions use currently this scheme, BepiColombo and Solo.
A case study inspired from Solo [7] is proposed (only the first leg is simulated) for a spacecraft injected directly on an escape trajectory with an infinite departure velocity of almost 3 km/s on October, 16th 2013. Assuming a launch with Soyuz-Fregat 2B, this corresponds to an initial mass of 1498 kg. The spacecraft is equipped with a thruster of constant Isp of 4200 s and a thrust following an inverse law with a reference level at 1 A.U. of 160 mN (a more complicated could also be considered in T3D). The arrival date is set on February, 7th 2014.

Calculations with T3D based on the maximum mass at Venus allow to determine the optimum injection direction at Earth departure (contrary to missions aiming at inserting a spacecraft around a planet, the departure infinite velocity is not null and its direction is part of the optimization). Once accounting for the direction of the injection velocity, the thrust profile obtained with T3D is similar to the one found by ESA, with around 80 days of thrusting and an arrival infinite velocity w.r.t. Venus of 5.5 km/s. The corresponding trajectory is shown on Figure 3.

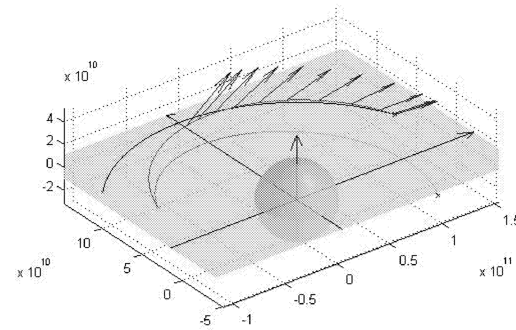

Figure 3: Leg Earth-Venus

4.2 Mars-Earth trajectory

Mars is by far one of the neighbouring planets for which the largest number of missions is envisaged. After a period during which the missions consisted in inserting an orbiter around Mars and/or releasing a lander onto its surface, some missions are envisaged to bring samples of the red planet back to Earth. Such missions are very demanding in terms of propellant mass when considering only chemical propulsion and electric propulsion is an interesting option for the interplanetary phases, in particular for the inbound trajectory. One of them, Mars Sample Return (MSR) is currently under study at ESA and inspired the context of the calculations (considered actual values may be different).

Let us assume that the dry mass of the return vehicle before the Earth atmospheric re-entry is of the order of 970 kg and that the propulsion system is characterised by a fixed specific impulse of 4500 s and a thrust level following an inverse square law with a reference thrust of 300 mN at 1 A.U. The departure from Mars is fixed on October, 1st 2013.

Depending whether the optimisation is performed assuming as criterion a minimum time to Earth or a maximum mass delivered at Earth, the cruise duration can vary between 648 and 750 days and the required mass at Mars be between 1275 and 1142 kg. A representation in the ecliptic plane of the transfer is shown below

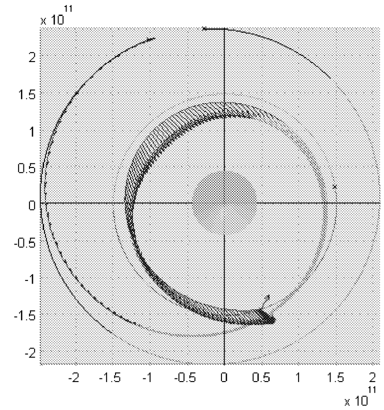

Figure 4: Mars to Earth in minimum time (648 days)

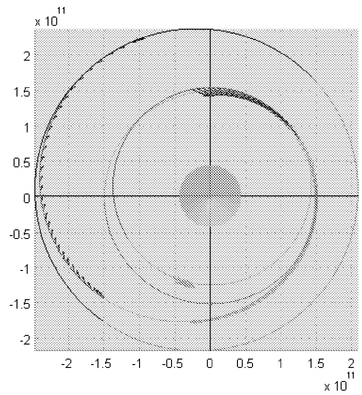

Figure 5: Mars to Earth in maximum mass and fixed 750 days trip

The cruise duration can obviously be reduced by selecting a more adequate Mars departure date. By putting t_0 free in the optimisation process T3D allows to determine the best date departure and achieve a transfer in about 348 days with thrusting all the time, the departure date from Mars shall be either:

- ✓ 97 days before October, 1st 2013 (transfer duration of 348 days)
- ✓ 701 days after October, 1st 2013 (transfer duration of 325.88 days)

4.3 Earth-Jupiter

After a few missions towards Jupiter using its large gravitational field to bend the trajectory and eventually visit other planets or regions of the Solar System, the up-to-now implemented missions aiming at orbiting around Jupiter were implemented using chemical propulsion. Low-thrust propulsion can however provide a significant saving for such missions.

A case study selected from a CNES paper [6] is a benchmark with T_3D.

The satellite mass at departure is fixed to 1500 kg. The thrust level is fixed to 0.33 N and the exhaust velocity to 37278 m/s, which give an Isp of 4500 s

The Earth departure date is December, 12/2008 and the trip time is 1965 days.

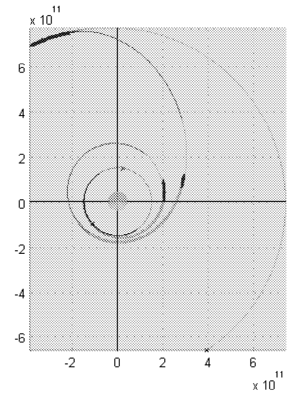

Figure 6 Earth-Jupiter transfer

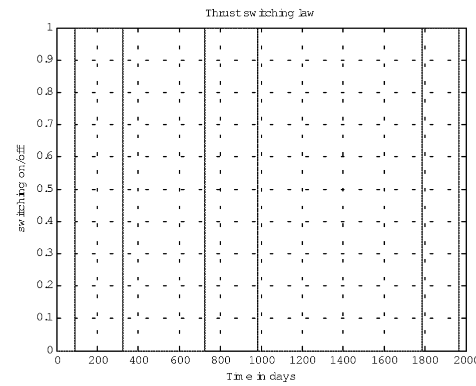

Figure 7 Switch on/off thrust profile

The total propellant mass consumption found by our software is 508.04 kg to be compared with the value of 508.7 kg given in the CNES paper. In conclusion, the 2 calculations are very close with the same thrust strategy.

5. CONCLUSION

A tool to handle the optimisation of low-thrust interplanetary trajectories based on the minimum principle formulation was developed internally at Alcatel Space allowing to design minimum time or maximum mass transfers for a large sets of initial or final conditions (departure with a given or free departure infinite velocity, transfers, rendez-vous…). As required by mission analysis, it is possible to define a variable thrust law to account for the variation of the power with the distance to the Sun: apart from the classical inverse square law, a user-defined law can be used by the tool to fit as much as possible the models provided by the engine manufacturers.

Finally, examples of calculations performed with T_3D were presented for a number of missions currently being studied at Alcatel Space. Note that for ESA inspired missions, the simulation context may differ from the actual one currently considered by the project.

5.1 References

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