

Satellite-based quantum steering under the influence of spacetime curvature of the Earth

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Spacetime curvature of the Earth deforms wavepackets of photons sent from the Earth to satellites, thus influencing the quantum state of light. We show that Gaussian steering of photon pairs, which are initially prepared in a two-mode squeezed state, is affected by the curved spacetime background of the Earth. We demonstrate that quantum steerability of the state increases for a specific range of height h and then gradually approaches a finite value with further increasing height of the satellite's orbit. Comparing with the peak frequency parameter, the Gaussian steering changes more for different squeezing parameters, while the gravitational frequency effect leads to quantum steering asymmetry between the photon pairs. In addition, we find that the influence of spacetime curvature on the steering in the Kerr spacetime is very different from the non-rotating case because special relativistic effects are involved.

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INTRODUCTION

Einstein-Podolsky-Rosen (EPR) steering, which describes how the state of one subsystem in an entangled pair is manipulated by local measurements performed on the other part, was proposed by Schrödinger in 1935 [1, 2]. The EPR steering differs from both entanglement and the Bell nonlocality, because it has inherent asymmetric features. Therefore, the EPR steering has potential applications in the one-side device-independent quantum key distribution [3]. Recently, quantum steering and its asymmetry have been theoretically studied [4–12] and experimentally demonstrated [13–24] in different quantum systems. However, little is known about behaviors of quantum steering in relativistic settings. Most recently, Navascues and Perez-Garcia studied quantum steering between space-like separated parties in the frame of algebraic quantum field theory [25]. In addition, more attention has been given to the dynamics of quantum steering under the influence of the dynamical Casimir effect [26], the Hawking radiation [27], and relativistic motions [28].

Since a realistic quantum system cannot be prepared and transmitted in a curved spacetime without any gravitational and relativistic effects, the study of quantum steerability in a relativistic framework is necessary. Such studies are of practical and fundamental importance to understand the influence of gravitational effects on the steerability-type quantum resource when the parties involved are located at large distances in the curved space time [29–33]. It has been shown that the curved background spacetime of the Earth affects the running of quantum clocks [34], is employed as witnesses of general relativistic proper time in laser interferometric [35], and influences the implementation of quantum metrology [36, 37] in satellite-based setups. Furthermore, Kish and Ralph found that there would be inevitable losses of quantum resources in the estimation of the Schwarzschild radius [38]. We studied

how the curved background spacetime of the Earth influences the satellite-based quantum clock synchronization [39]. Most recently, an experimental test of photonic entanglement in an accelerated setting was realized [40], where a genuine quantum state of entangled photon pairs was exposed to different accelerations.

In this work, we present a quantitative investigation of Gaussian quantum steerability for correlated photon pairs which are initially prepared in a two-mode squeezed state in the curved background spacetime of the Earth. We assume that one of entangled photons is sent to Alice (at the Earth station) and the other propagates to Bob (at the satellite). During this propagation, the photons' wave-packet will be deformed by the curved background spacetime of the Earth, and these deformations effects on the quantum state of the photons can be modeled as a lossy quantum channel [39, 41]. Since the initial state is Gaussian and the transformations involved are linear and unitary, we can restrict our state to the Gaussian scenarios and employ the covariance matrix formalism. We calculate the Gaussian quantum steering from Alice to Bob, which quantifies to what extent Bob's mode can be steered by Alice's measurements. We also discuss Gaussian quantum steering from Bob to Alice to verify the asymmetric property of steerability in the curved spacetime.

This work is organized as follows. In section II, we introduce the quantum field theory of a massless uncharged bosonic field which propagates from the Earth to a satellite. In section III, we briefly introduce the definition and measure of the bipartite Gaussian quantum steering. In section IV, we show a scheme to test quantum steering between the Earth and satellites and study the behaviors of quantum steering in the curved spacetime. The last section is devoted to a brief summary. Throughout the whole paper we employ the natural units $G = c = \hbar = 1$.

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LIGHT WAVE-PACKETS PROPAGATING IN THE CURVED SPACE-TIME

In this section we will describe the propagation of photons from the Earth to satellites under the influence of the Earth's gravity [29]. The Earth's spacetime can be approximately described by the Kerr metric [42]. For the sake of simplicity, our work will be constrained to the equatorial plane $\theta = \frac{\pi}{2}$. The reduced metric in Boyer-Lindquist coordinates (t, r, ϕ) reads [42]

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{\Delta} dr^2 + \left(r^2 + a^2 + \frac{2Ma^2}{r}\right) d\phi^2 - \frac{4Ma}{r} dt d\phi, \quad (1)$$

$$\Delta = 1 - \frac{2M}{r} + \frac{a^2}{r^2}, \quad (2)$$

where $M, r, J, a = \frac{J}{M}$ are the mass, radius, angular momentum and Kerr parameter of the Earth, respectively.

A photon is sent from Alice on Earth's surface to Bob at time τ_A , Bob will receive this photon at $\tau_B = \Delta\tau + \sqrt{f(r_B)/f(r_A)}\tau_A$ in his own reference frame, where $f(r_A) = 1 - \frac{r_S}{r_A}$ and $f(r_B) = 1 - \frac{r_S}{r_B}$. Here $r_S = 2M$ is the Schwarzschild radius of the Earth and $\Delta\tau$ is the propagation time of the light from the Earth to the satellite by taking curved effects of the Earth into account. In general, a photon can be modeled by a wave packet of excitations of a massless bosonic field with a distribution $F_{\Omega_{K,0}}^{(K)}$ of mode frequency Ω_K and peaked at $\Omega_{K,0}$ [44, 45], where $K = A, B$ denote the modes in Alice's or Bob's reference frames, respectively. The annihilation operator of a photon for an observer far from Alice or Bob takes the form

$$a_{\Omega_{K,0}}(t_K) = \int_0^{+\infty} d\Omega_K e^{-i\Omega_K t_K} F_{\Omega_{K,0}}^{(K)}(\Omega_K) a_{\Omega_K}. \quad (3)$$

Alice's and Bob's operators in Eq. (3) can be used to describe the same optical mode in different altitudes. By considering the curved spacetime of the Earth, the wave packet received is modified. The relation between a_{Ω_A} and a_{Ω_B} was discussed in [29, 30, 39], and can be used to calculate the relation between the frequency distributions $F_{\Omega_{K,0}}^{(K)}$ of the photons before and after the propagation [29, 30, 39]

$$F_{\Omega_{B,0}}^{(B)}(\Omega_B) = \sqrt[4]{\frac{f(r_B)}{f(r_A)}} F_{\Omega_{A,0}}^{(A)} \left(\sqrt{\frac{f(r_B)}{f(r_A)}} \Omega_B \right). \quad (4)$$

From Eq. (4), we can see that the effect induced by the curved spacetime of the Earth cannot be simply corrected by a linear shift of frequencies. Therefore, it may be challenging to compensate the transformation induced by the curvature in realistic implementations.

Indeed, such a nonlinear gravitational effect is found to influence the fidelity of the quantum channel between Alice and Bob [29, 30, 39]. It is always possible to decompose the mode

\bar{a}' received by Bob in terms of the mode a' prepared by Alice and an orthogonal mode a'_\perp (i.e. $[a', a'_\perp] = 0$) [46]

$$\bar{a}' = \Theta a' + \sqrt{1 - \Theta^2} a'_\perp, \quad (5)$$

where Θ is the wave packet overlap between the distributions $F_{\Omega_{B,0}}^{(B)}(\Omega_B)$ and $F_{\Omega_{A,0}}^{(A)}(\Omega_B)$,

$$\Theta := \int_0^{+\infty} d\Omega_B F_{\Omega_{B,0}}^{(B)*}(\Omega_B) F_{\Omega_{A,0}}^{(A)}(\Omega_B), \quad (6)$$

and we have $\Theta = 1$ for a perfect channel. From this expression we can see that the spacetime curvature of the Earth would affect the the fidelity $\mathcal{F} = |\Theta|^2$ as well as the quantum resource of EPR steering.

We assume that Alice employs a real normalized Gaussian wave packet

$$F_{\Omega_0}(\Omega) = \frac{1}{\sqrt[4]{2\pi\sigma^2}} e^{-\frac{(\Omega - \Omega_0)^2}{4\sigma^2}}, \quad (7)$$

with wave packet width σ . In this case the overlap Θ is given by (6) where we have extended the domain of integration to all the real axis. We note that the integral should be performed over strictly positive frequencies. However, since $\Omega_0 \gg \sigma$, it is possible to include negative frequencies without affecting the value of Θ . Using Eqs. (3) and (7) one finds that [29, 30, 39]

$$\Theta = \sqrt{\frac{2}{1 + (1 + \delta)^2}} \frac{1}{1 + \delta} e^{-\frac{\delta^2 \Omega_{B,0}^2}{4(1 + (1 + \delta)^2)\sigma^2}}, \quad (8)$$

where the new parameter δ quantifying the shifting is defined by

$$\delta = \sqrt[4]{\frac{f(r_A)}{f(r_B)}} - 1 = \sqrt{\frac{\Omega_B}{\Omega_A}} - 1. \quad (9)$$

The expression for $\frac{\Omega_B}{\Omega_A}$ in the equatorial plane of the Kerr spacetime has been shown in [33]

$$\frac{\Omega_B}{\Omega_A} = \frac{1 + \epsilon \frac{a}{r_B} \sqrt{\frac{M}{r_B}}}{C \sqrt{1 - 3\frac{M}{r_B} + 2\epsilon \frac{a}{r_B} \sqrt{\frac{M}{r_B}}}}, \quad (10)$$

where $C = [1 - \frac{2M}{r_A}(1 + 2a\omega) + (r_A^2 + a^2 - \frac{2Ma^2}{r_A})\omega^2]^{-\frac{1}{2}}$ is the normalization constant, ω is the Earth's equatorial angular velocity and $\epsilon = \pm 1$ stand for the direct of orbits (i.e., when $\epsilon = +1$ for the satellite co-rotates with the Earth). In the Schwarzschild limit $a, \omega \rightarrow 0$, Eq. (10) coincides to the result found in [29], which is

$$\frac{\Omega_B}{\Omega_A} = \sqrt{\frac{1 - \frac{2M}{r_A}}{1 - \frac{3M}{r_B}}}. \quad (11)$$

Noticing that $(r_A\omega)^2 > a\omega$, therefore we can retain second order terms in $r_A\omega$. Expanding Eq. (10) we obtain the

following perturbative expression for δ . This perturbative result does not depend on whether the Earth and the satellite are co-rotating or not

$$\begin{aligned} \delta &= \delta_{Sch} + \delta_{rot} + \delta_h \\ &= \frac{1}{8} \frac{r_S}{r_A} \left(\frac{1 - 2\frac{h}{r_A}}{1 + \frac{h}{r_A}} \right) - \frac{(r_A \omega)^2}{4} - \frac{(r_A \omega)^2}{4} \left(\frac{3}{4} \frac{r_S}{r_A} - \frac{4Ma}{\omega r_A^3} \right), \end{aligned}$$

where $h = r_B - r_A$ is the height between Alice and Bob, δ_{Sch} is the first order Schwarzschild term, δ_{rot} is the lowest order rotation term and δ_h denotes all higher order correction terms. If the parameter $\delta = 0$ (the satellite moves at the height $h \simeq \frac{r_A}{2}$), we have $\Theta = 1$. That is to say, the received photons at this height will not experience any frequency shift, and the effects of gravity of the Earth and the effects of special relativity completely compensates each other.

GAUSSIAN QUANTUM STEERING

In this section we briefly review the measurement of quantum steering for a general two-mode Gaussian state ρ_{AB} . The character of a bipartite Gaussian state ρ_{AB} can be described by its covariance matrix (CM)

$$\sigma_{AB} = \begin{pmatrix} A & C \\ C^\top & B \end{pmatrix}, \quad (12)$$

with elements $\sigma_{ij} = \text{Tr}[\{\hat{R}_i, \hat{R}_j\}_+ \rho_{AB}]$. Here the submatrices A and B are the CMs corresponding to the reduced states of A 's and B 's subsystems, respectively. The *bona fide* condition should be satisfied for a physical CM, which is

$$\sigma_{AB} + i(\Omega_A \oplus \Omega_B) \geq 0. \quad (13)$$

Let us continue by giving the definition of steerability. For a bipartite state, it is steerable from A to B *iff* it is *not* possible for every pair of local observables $R_A \in \mathcal{M}_A$ on A and R_B (arbitrary) on B , with respective outcomes r_A and r_B , to express the joint probability as [50] $P(r_A, r_B | R_A, R_B, \rho_{AB}) = \sum_\lambda \wp_\lambda \wp(r_A | R_A, \lambda) P(r_B | R_B, \rho_\lambda)$. That is to say, there exists at least one measurement pair between R_A and R_B that can violate this expression when \wp_λ is fixed across all measurements. Here \wp_λ and $\wp(r_A | R_A, \lambda)$ are arbitrary probability distributions and $P(r_B | R_B, \rho_\lambda)$ is a probability distribution restricted to the extra condition of being evaluated on a quantum state ρ_λ . It has been proven that a necessary and sufficient condition for Gaussian $A \rightarrow B$ steerability is *iff* the condition

$$\sigma_{AB} + i(0_A \oplus \Omega_B) \geq 0, \quad (14)$$

is violated [50]. To quantify how much a bipartite Gaussian state with CM σ_{AB} is steerable (by Gaussian measurements on Alice's side), the following quantity has been performed [47]

$$\mathcal{G}^{A \rightarrow B}(\sigma_{AB}) := \max \left\{ 0, - \sum_{j: \bar{\nu}_j^B < 1} \ln(\bar{\nu}_j^B) \right\}, \quad (15)$$

where $\bar{\nu}_j^B$ are the symplectic eigenvalues of the Schur complement of A in the covariance matrix σ_{AB} . By defining the Schur complement $\det \sigma_{AB} = \det A \det M_\sigma^B$ and employing the Rényi-2 entropy, Eq. (15) can be written as

$$\begin{aligned} \mathcal{G}^{A \rightarrow B}(\sigma_{AB}) &= \max \left\{ 0, \frac{1}{2} \ln \frac{\det A}{\det \sigma_{AB}} \right\} \\ &= \max \left\{ 0, \mathcal{S}(A) - \mathcal{S}(\sigma_{AB}) \right\}, \quad (16) \end{aligned}$$

where the Rényi-2 entropy \mathcal{S} reads $\mathcal{S}(\sigma) = \frac{1}{2} \ln(\det \sigma)$ [51] for a Gaussian state with CM σ . However, unlike quantum entanglement, quantum steering is asymmetric [47]. To obtain the measurement of Gaussian steering $B \rightarrow A$, one can swap the roles of A and B and get an expression like Eq. (16).

THE INFLUENCE OF GRAVITATIONAL EFFECTS ON QUANTUM STEERABILITY AND ENTANGLEMENT

In this section we propose a scheme to test large distance quantum steering between the Earth and satellites and discuss how quantum steerability is affected by the curved spacetime of the Earth. Firstly, we consider a pair of entangled photons which are initially prepared in a two-mode squeezed state with modes b_1 and b_2 at the ground station. Then we send one photon with mode b_1 to Alice. The other photon in mode b_2 propagates from the Earth to the satellite and is received by Bob. Due to the curved background spacetime of the Earth, the wave packet of photon with mode b_2 is deformed. Finally, one can test how the quantum state of Alice's photon is manipulated by local Gaussian measurements performed by Bob at the satellite and verifies the quantum steerability from b_2 to b_1 , and vice versa.

Considering that Alice receives the mode b_1 and Bob receives the mode b_2 at different satellite orbits, we should take the curved spacetime of the Earth into account. As discussed in [29, 30, 39], the influence of the Earth's gravitational effect can be modeled by a beam splitter with orthogonal modes $b_{1\perp}$ and $b_{2\perp}$. The covariance matrix of the initial state is given by

$$\sum_0^{b_1 b_2 b_{1\perp} b_{2\perp}} = \begin{pmatrix} \tilde{\sigma}(s) & 0 \\ 0 & I_4 \end{pmatrix}, \quad (17)$$

where I_4 denotes the 4×4 identity matrix and $\tilde{\sigma}(s)$ is the covariance matrix of the two-mode squeezed state

$$\tilde{\sigma}(s) = \begin{pmatrix} \cosh(2s)I_2 & \sinh(2s)\sigma_z \\ \sinh(2s)\sigma_z & \cosh(2s)I_2 \end{pmatrix}, \quad (18)$$

where σ_z is Pauli matrix and s is the squeezing parameter. The effect induced by the curved spacetime of the Earth on Bob's mode b_2 can be model as a lossy channel, which is described by the transformation [29, 30, 39]

$$\bar{b}_2 = \Theta_2 b_2 + \sqrt{1 - \Theta_2^2} b_{2\perp}, \quad (19)$$

while the mode b_1 received by Alice is unaffected because Alice stays at the ground station. This process can be represented

as a mixing (beam splitting) of modes $b_1(b_2)$ and $b_{1\perp}(b_{2\perp})$. Therefore, for the entire state, the symplectic transformation can be encoded into the Bogoliubov transformation

$$S = \begin{pmatrix} I_2 & 0 & 0 & 0 \\ 0 & \Theta_2 I_2 & 0 & \sqrt{1 - \Theta_2^2} I_2 \\ 0 & 0 & -I_2 & 0 \\ 0 & \sqrt{1 - \Theta_2^2} I_2 & 0 & -\Theta_2 I_2 \end{pmatrix}.$$

The final state $\Sigma^{b_1 b_2 b_{1\perp} b_{2\perp}}$ after the transformation is $\Sigma^{b_1 b_2 b_{1\perp} b_{2\perp}} = S \Sigma_0^{b_1 b_2 b_{1\perp} b_{2\perp}} S^T$. Then we trace over the orthogonal modes $b_{1\perp}, b_{2\perp}$ and obtain the covariance matrix $\Sigma^{b_1 b_2}$ for the modes b_1 and b_2 after the propagation

$$\Sigma^{b_1 b_2} = \begin{pmatrix} (1 + 2 \sinh^2 s) I_2 & \sinh(2s) \Theta_2 \sigma_z \\ \sinh(2s) \Theta_2 \sigma_z & (1 + 2 \sinh^2 s) \Theta_2^2 I_2 \end{pmatrix}. \quad (20)$$

The form of the two-mode squeezed state under the influence of the effects of gravity of the Earth is given by Eq. (20). Then employing the measurement of Gaussian steering, we obtain an specific mathematic expression of the mode $b_1 \rightarrow b_2$ Gaussian steering under the curved spacetime of the Earth

$$\mathcal{G}^{b_1 \rightarrow b_2} = \max \left\{ 0, \ln \frac{1 + 2 \sinh^2(2s)}{1 + 2(1 - \Theta_2^2) \sinh^2 s} \right\}. \quad (21)$$

We notice that the wave packet overlap Θ in the above equation is determined by the parameters δ , σ and $\Omega_{B,0}$. Since the Schwarzschild radius of the Earth is $r_s = 9$ mm, we have $\delta \sim -\frac{1}{2} \left(\frac{r_s}{r_B} - \frac{r_s}{r_A} \right) \sim 10^{-10}$. Here we consider a typical PDC source with a wavelength of 598 nm (corresponding to the peak frequency $\Omega_{B,0} = 500$ THz) and Gaussian bandwidth $\sigma = 1$ MHz [48, 49]. Under these constraints, $\delta \ll \left(\frac{\Omega_{B,0}}{\sigma} \right)^2 \ll 1$ is satisfied. Therefore, the wave packet overlap Θ can be expand by the parameter δ . Then we obtain $\Theta \sim 1 - \frac{\delta^2 \Omega_{B,0}^2}{8\sigma^2}$ by keeping the second order terms. The Eq. (21) has following form in the second order of perturbation for the parameter δ

$$\mathcal{G}^{b_1 \rightarrow b_2} \simeq \max \left\{ 0, \mathcal{G}_0 - \frac{\delta^2 \Omega_{B,0}^2}{2\sigma^2} \sinh^2(s) \right\}, \quad (22)$$

where higher order contributions are neglected. To ensure the validity of perturbative expansion, we should estimate the values of the last term in Eq. (22). Considering $\frac{\delta^2 \Omega_{B,0}^2}{2\sigma^2} \sim 1.25 \times 10^{-7}$, we find that even if the value of the squeezing parameter is $s \ll 7.6$ (corresponding to $\sinh^2(s) \ll 10^6$), the perturbative expansion is valid. Therefore, we can safely prelimit the value of the squeezing parameter as $s < 3$ hereafter. In the case of flat spacetime, this expression reduces to $\mathcal{G}_0 := \ln[1 + 2 \sinh^2(2s)]$. As showed in Eq. (22), the Gaussian steering $b_1 \rightarrow b_2$ not only depends on the squeezing parameter, the peak frequency, and the Gaussian bandwidth, but also the height of the orbiting satellite. This means that the curved spacetime of the Earth will influence the $b_1 \rightarrow b_2$ steerability because the parameter δ contains the height h of the satellite. It is clear that δ approaches to a constant value when the height $h \rightarrow \infty$ and the squeezing parameter s is

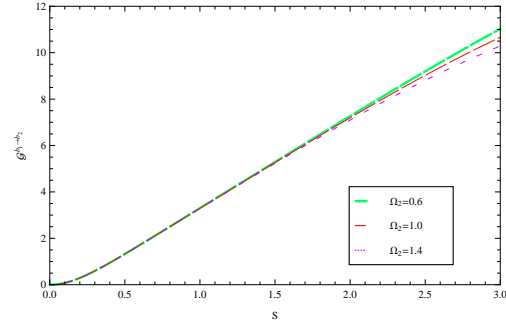


FIG. 1: (Color online) The Gaussian steering $\mathcal{G}^{b_1 \rightarrow b_2}$ of two-mode squeezed state as a function of the squeezing parameter s for different peak frequencies, $\Omega_2 = 0.6$ (green dashed line), $\Omega_2 = 1$ (red dashed line) and $\Omega_2 = 1.4$ (violet dotted line), respectively. The orbit height of the satellite and the Gaussian bandwidth are fixed as $h = 20000$ km and $\sigma = 1$.

a fixed value. Therefore, quantum steering $\mathcal{G}^{b_1 \rightarrow b_2}$ also becomes a constant.

For convenience, we will work with dimensionless quantities by rescaling the peak frequency and the Gaussian bandwidth

$$\Omega \rightarrow \tilde{\Omega} \equiv \frac{\Omega}{\Omega_{B,0}}, \sigma \rightarrow \tilde{\sigma} \equiv \frac{\sigma}{\sigma_0}, \quad (23)$$

where $\Omega_{B,0} = 500$ THz and $\sigma_0 = 1$ MHz. For simplicity, we abbreviate the dimensionless parameter $\tilde{\Omega}$ as Ω_2 and abbreviate $\tilde{\sigma}$ as σ , respectively.

In Fig. (1) we plot the Gaussian steering $\mathcal{G}^{b_1 \rightarrow b_2}$ as a function of the squeezing parameter s for the fixed orbit height $h = 20000$ km and Gaussian bandwidth $\sigma = 1$. We can see that quantum steering monotonically increases with the increase of the squeezing parameter s . It is also shown that, comparing with the peak frequency parameter, the Gaussian steering $\mathcal{G}^{b_1 \rightarrow b_2}$ changes more for different squeezing parameters, which indicates that the initial quantum resource plays a more important role in the quantum steering.

The Gaussian steering $\mathcal{G}^{b_1 \rightarrow b_2}$ in terms of the orbit height h and the Gaussian bandwidth σ for the fixed values $s = 1$ and $\Omega_2 = 1$ has been shown in Fig. (2). We can see that the quantum steerability $\mathcal{G}^{b_1 \rightarrow b_2}$ decreases with increasing the Gaussian bandwidth σ . In addition, comparing with the squeezing parameter, the Gaussian steering is not easy to change with changing orbit height parameter and Gaussian bandwidth. This allows us to choose appropriate physical parameters and perform more reliable quantum steering tasks between the Earth to a satellite.

One of the most distinguishable properties of quantum steering is its asymmetry, which has been recently experimentally demonstrated in flat spacetime [16, 17]. To understanding this properties in the curved spacetime, we also calculate the steerability $\mathcal{G}^{b_2 \rightarrow b_1}$, which is

$$\mathcal{G}^{b_2 \rightarrow b_1} = \max \left\{ 0, \ln \frac{1 + 2 \sinh^2(2s) \Theta_2^2}{1 + 2(1 - \Theta_2^2) \sinh^2 s} \right\}, \quad (24)$$

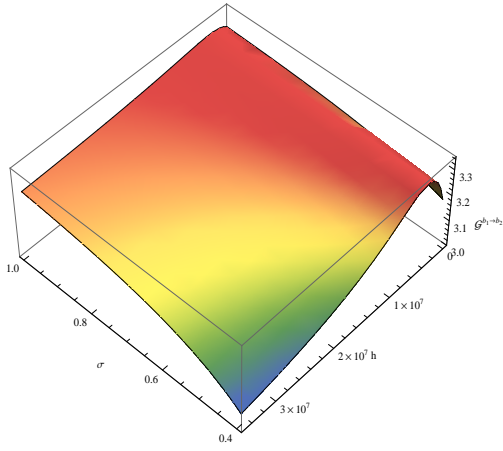


FIG. 2: (Color online) The Gaussian steering $\mathcal{G}_{b_1 \rightarrow b_2}$ in terms of the orbit height h and the Gaussian bandwidth σ , for the fixed values $s = 1$ and $\Omega_2 = 1$.

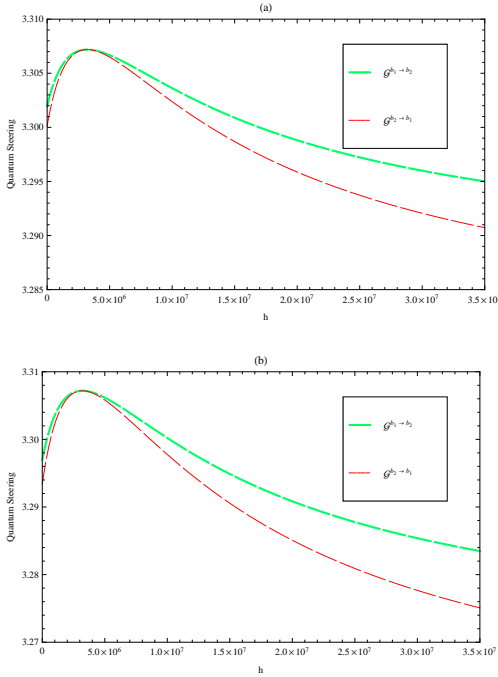


FIG. 3: (Color online). The Gaussian steering $\mathcal{G}^{b_1 \rightarrow b_2}$ (green lines) and $\mathcal{G}^{b_2 \rightarrow b_1}$ (orange lines) as functions of the height between Alice and Bob under the influence of the Earth's gravity. Here the Gaussian bandwidth of the initial state is fixed as $\sigma = 1$, the dimensionless peak frequencies of the mode b_2 are fixed as (a) $\Omega_2 = 0.6$, (b) $\Omega_2 = 1$, and the squeezing parameter is $s = 1$.

Similarly, this equation can be rewritten in its perturbative expansion form as

$$\mathcal{G}^{b_2 \rightarrow b_1} \simeq \max \left\{ 0, \mathcal{G}_0 - \frac{\delta^2 \Omega_{B,0}^2}{2\sigma^2} \left(\sinh^2(s) + \frac{\sinh^2(2s)}{\cosh(4s)} \right) \right\} \quad (25)$$

This equation gives us a quantitative way to evaluate the con-

tributions of the curved background spacetime of the Earth to the steering for the $b_2 \rightarrow b_1$ scenario, when the satellites are far away to the Earth. It is clearly shown that $\mathcal{G}^{b_2 \rightarrow b_1}$ is equal to \mathcal{G}_0 when the $\delta \rightarrow 0$, which means that the effect induced by the curved background spacetime of the Earth vanishes in this limit.

The typical distance between the ground station and the geostationary satellite is about 3.6×10^4 km, which yields the height $r_B = 4.237 \times 10^4$ km for the satellite. Since the height of current GPS (Global Position System) satellites is $r_B \approx 2.7 \times 10^4$ km. For this distance the influence of relativistic disturbance of the spacetime curvature on quantum steerability cannot be ignored for the quantum information tasks at current level technology [31, 32, 52]. Hence, in this work the plotting range of the satellite height will be constrained to geostationary satellites height.

In Fig. (3), we plot the quantum steerability $\mathcal{G}^{b_1 \rightarrow b_2}$, as well as $\mathcal{G}^{b_2 \rightarrow b_1}$ of the final state as a function of the height h . The plot range is limited to geostationary Earth orbits $r_B(GEO) = r_A + 35784$ km. Here, the range of peak frequency parameter Ω_2 is fixed from 0.6 to 1 to satisfy $\delta \ll \left(\frac{\delta \Omega_2}{\sigma}\right)^2 \ll 1$. It is shown that both the $b_1 \rightarrow b_2$ and $b_2 \rightarrow b_1$ steering increase for a specific range of height parameter h and then gradually approach to a finite value with increasing h . This is because the total frequency shift in Eq. (10) both includes the Schwarzschild term and the rotation term. The parameter δ in the Kerr spacetime is $\delta = \frac{1}{8} \frac{r_S}{r_A} \left(\frac{1-2\frac{r_A}{r_A}}{1+\frac{h}{r_A}} \right)$ which is different from the Schwarzschild case $\delta'_{Sch} = -\frac{r_S}{4r_A} \frac{h}{(r_A+h)}$ [29, 30] since special relativistic effects are involved [33]. When the satellite moves at the height $h = \frac{r_A}{2}$, the Schwarzschild term δ_{Sch} vanishes and photons received on satellites will generate a very small frequency shift dominated by special relativistic effects, therefore the lowest order rotation term δ_{rot} needs to be considered. In addition, we can see that whatever $\mathcal{G}^{b_1 \rightarrow b_2}$ or $\mathcal{G}^{b_2 \rightarrow b_1}$ both reduce with increasing h after reaches the peak. That's why we say the gravitational frequency shift effect is a lossy channel. This losing degree of quantum steering depends on the dimensionless peak frequency of mode b_2 , which means that this lossy channel not only depends on curvature of the Earth.

In fact, the peak value of quantum steering indicates the fact that the photon's frequency received by satellites experiences a transformation from blue-shift to red-shift, which causes the Gaussian steering between the photon pairs to increase first and then to reduce with increasing height [33]. In the Schwarzschild limit $a, \omega_A \rightarrow 0$, the frequency shift simplifies to $\frac{\Omega_B}{\Omega_A} = \sqrt{\frac{1-\frac{2M}{r_A}}{1-\frac{3M}{r_B}}}$, from which we can see that the received photon's frequency on satellites do not experience any frequency shift at $h = \frac{r_A}{2}$. On the other hand, the frequency of photon received at orbits with height $h < \frac{r_A}{2}$ will experience blue-shift, while the frequencies of photons received at height $h > \frac{r_A}{2}$ experience red-shift. For this reason, the photons experience different frequency shifts when the satellite

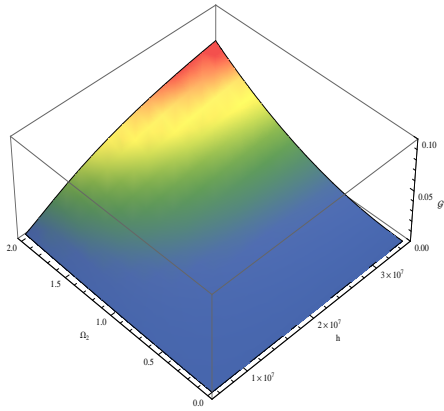


FIG. 4: (Color online) The Gaussian steering asymmetry \mathcal{G}^Δ as a function of the height h of the satellite and the peak frequency Ω_2 of mode b_2 . The Gaussian bandwidth and the squeezing parameter are fixed as $\sigma = 1$ and $s = 1$, respectively.

locates at different heights in the Kerr spacetime. Therefore, the Gaussian steering increases at the beginning, reaches the peak value (corresponds to the satellite at the heights $h \approx \frac{r_A}{2}$, i.e. the parameter $\delta = 0$), and then decreases with increasing height.

To check the asymmetric degree of steerability under the Earth's spacetime curvature, we calculate the Gaussian steering asymmetry

$$\mathcal{G}^\Delta = |\mathcal{G}^{b_1 \rightarrow b_2} - \mathcal{G}^{b_2 \rightarrow b_1}|, \quad (26)$$

and plot it as a function of the peak frequency Ω_2 and the height h of the satellite in Fig. (4). This allows us to have a better understanding of how the peak frequency Ω_2 and the Earth's gravitation affect steering asymmetry. It is shown that the \mathcal{G}^Δ is close to zero, i.e., the steerability is almost symmetric when the height parameter $h \rightarrow 0$ and the peak frequency $\Omega_2 \rightarrow 0$ because $\mathcal{G}^{b_1 \rightarrow b_2} \approx \mathcal{G}^{b_2 \rightarrow b_1}$ in these two cases. In addition, the steering asymmetry monotonically increases with increasing orbit height h of the satellite. The physical support behind this is that gravitational field would reduce quantum resource [53], and the effect of gravitational field on different directions of steering is different [27]. Furthermore, it is not difficult to infer that if the gravitation is strong enough or Bob is close to the horizon of a black hole, the gravitational frequency should lead to completely asymmetry: Alice can steer Bob but Bob cannot steer Alice at all.

CONCLUSIONS

In conclusion, we have studied Gaussian steering for a two-mode Gaussian state when one of the modes propagates from the ground to satellites. We found that the frequency shift induced by the curved spacetime of the Earth reduces the quantum correlation of the steerability between the photon pairs

when one of the entangled photons is sent to the Earth station and the other photon is sent to the satellite. In addition, the influence of spacetime curvature on the steering in the Kerr spacetime is very different from the non-rotation case because special relativistic effects are involved. We also found that Gaussian steering is easier to change with the initial squeezing parameter than the gravitational effect and other parameters. Although the gravitational effect of the Earth is small, it will lead to the Gaussian steering asymmetry between the photon pairs. This is because the influence of gravitational field on the steering of the downlink setup is stronger than the effect of gravitational field on the steering of the uplink setup, which results in the increase of quantum steering asymmetry. Therefore, we can conclude that the effects induced by the curved spacetime of the Earth will generate quantum steering asymmetry. Finally, the peak value is found to be a critical point which indicates the Gaussian steering experiences the blue-shift transforms into the red-shift. According to the equivalence principle, the effects of acceleration are equivalence with the effects of gravity, our results could be in principle apply to dynamics of quantum steering under the influence of acceleration. Since realistic quantum systems will always exhibit gravitational and relativistic features, our results should be significant both for giving more advices to realize quantum information protocols such as quantum key distribution from Earth to satellites and for a general understanding of quantum steering in relativistic quantum systems.

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