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**Electromagnetic Propulsion System for Spacecraft using Geomagnetic Fields and Superconductors**

Student's name: Anang Dadhich

This work and its defense approved by:

Committee chair: Grant Schaffner, Ph.D.

Committee member: George T Black, M.S.

Committee member: Kelly Cohen, Ph.D.



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# **Electromagnetic Propulsion System for Spacecraft using Geomagnetic fields and Superconductors**

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**Anang Dadhich**

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Committee Chair: Grant Schaffner, Ph.D.

# **Electromagnetic Propulsion System for Spacecraft using Geomagnetic fields and Superconductors**

## **Abstract**

This thesis concentrates on developing an innovative method to generate thrust force for spacecraft in localized geomagnetic fields by various electromagnetic systems. The proposed electromagnetic propulsion system is an electromagnet, like normal or superconducting solenoid, having its own magnetic field which interacts with the planet's magnetic field to produce a reaction thrust force. The practicality of the system is checked by performing simulations in order to find the varying radius, velocity, and acceleration changes. The advantages, challenges, various optimization techniques, and viability of such a propulsion system in present day and future are discussed. The propulsion system such developed is comparable to modern MPD Thrusters and electric engines, and has various applications like spacecraft propulsion, orbit transfer and stationkeeping.



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## Nomenclature

N	Total number of turns	$B_c$	Critical Magnetic Field, T
I	Current, Ampere	$T_c$	Critical Temperature, K
A	Area of a single coil of solenoid, $m^2$	T	Applied Temperature, K
L	Length of solenoid, m	$B_c(0)$	Maximum magnetic field for superconductors
$\mu_r$	Relative permeability of substance	$J_c$	Critical Current Density, $A/m^2$
$\mu_0$	Permeability of free space	H	Ampere-Turns
$\theta$	Latitude angle, degrees	a	Area of superconducting surface, $m^2$
R	Distance between spacecraft and the centre of the Earth	b	Reduced induction
$R_e$	Radius of Earth, m	$V_f$	Volume fraction of particles
h	Height of spacecraft from Earth's surface	$\xi$	Coherence length
$M_S$	Magnetic moment of spacecraft	B	Magnetic field
$M_E$	Magnetic moment of Earth	m	mass of electron
$F_r$	Force in radial direction, N	c	speed of light
$F_t$	Force in tangential direction, N	e	charge of electron
F	Resultant force, N	w	angular velocity

# 1 Introduction

## *1.1 Background and Motivation*

The world is advancing and with it the aerospace field is growing rapidly too. With advancements from supersonic to hypersonic aircrafts, and Mars Orbiter to New Horizons spacecraft, the human imagination and thirst for knowledge knows no bounds. The modern day spacecraft are well equipped with latest technologies, gadgets, fastest computer systems, advanced propulsion systems etc. This research is a small contribution to the technology of the aerospace sciences.

While the modern day propulsion systems for spacecraft are efficient and worthy of performing any task delivered to them, there is still a lot of room for improvement, and many shortcomings of such systems needs to be analyzed. While the chemical rockets have high thrust to weight ratio, the size of the system is big and the storage capacity not large enough for long term missions. Similarly, the ion thrusters have high specific impulse but the thrust generated by the system is very low and is thus used for just attitude control in modern technology. Besides that, these systems are high maintenance, heavy, costly, complex, and generate lot of air and sound pollution.

There is a need for a better, efficient, simple, low cost, and environment friendly propulsion system and attitude control system for the modern spacecraft.

A propulsion system is required which can generate thrust forces without any dissipation of masses carried by the spacecraft. The propulsion system without any dissipation of masses will have long range. Also, this type of propulsion system will have constant mass which will make the mathematics involved very simple and logical.

A propulsion system is required which uses less and constant power to run. Doing so will help in increasing the life of the system, and the propulsion system will operate for longer time for the same amount of power used for chemical rockets or ion thrusters.

A propulsion system is required that can use the natural resources, like geomagnetic fields, present around its vicinity to generate power and thrust. This kind of system will thus work on reaction forces and interaction with the localized field, eliminating the need of chemical rockets, thus bringing down its pollution factor to a very low value.

A propulsion system which can also assist in deflecting the radiation ions in space, thus eliminating the need of excess protective layering of the spacecraft. This will reduce the spacecraft's weight drastically, making it swift and easily controllable.

A propulsion system is required which can also act as an attitude control system, providing quick and easy maneuverability.

## ***1.2 Research Goals and Approach***

The aim of this research is to analyze and design conceptually a viable propulsion system for modern day and future spacecraft. The hypothesis of this research is that the electromagnetic propulsion system is feasible enough to provide good values of thrust using the planet's geomagnetic field. The propulsion system used for such a system will thus be of constant mass, environment friendly, and simple. The goal of this research is to show that electromagnetic propulsion is a cost effective means of achieving orbital maneuvering in Lower Earth Orbit (LEO).

The approach for achieving the goal is a two-step process. Firstly, it will be determined that a good level of thrust can be achieved by employing electromagnetic propulsion system by current state of art. This will be done by reviewing the electrodynamic theory of metals and superconductors and applying the required formulas to find out the thrust value generated by the interaction of the Earth's magnetic field and the spacecraft's magnetic field.

Secondly, the practical application of the system in present day technology will be analyzed by demonstrating a simulation containing orbital maneuver in the Lower Earth Orbit (LEO). This will be done by employing basic orbital mechanics and spacecraft dynamics techniques, and making a simulation in MATLAB for graphical results.

## ***1.3 Thesis Outline***

The scope of the thesis aims at understanding the basic laws of electrodynamics and superconductivity, and applying the rules to the propulsion system for finding good values of the Thrust forces and other parameters by simulations.

A lot of books and papers are reviewed for the research, and in literature review the laws of electromagnetism are presented first followed by the superconducting nature and magnetic properties of the material. Type I and Type II superconductors are distinguished, with other techniques to generate magnetic field such as rotating superconductor.

The papers of J.F Engelberger and Valentine Pulatov on Electromagnetic Propulsion System are reviewed. Lastly, the basics of orbital mechanics and simulation techniques are revised in the literature review section.

The methodology of the propulsion system is discussed in section 3. The methods used to find the different values of radius, velocity, acceleration, drag, and positions are described. A number of assumptions are defined for a better approximation of the results.

Section 4 concentrates on showing the results obtained by the simulations in the graphical forms. Here the graphs for Tangential and Radial forces, radius changes, velocity changes, acceleration change etc. can be find.

The thesis ends with discussing the obtained results and recommending ideas how to optimize the system for better values in Section 5 and 6.

## 2 Literature Review

### 2.1 Electromagnetism<sup>[1]</sup>

The science of electricity and magnetism that focuses on charges, forces and interaction of charged particles is known as Electromagnetism. The focus of this section is to understand how a current produces magnetic field which in turn produces a substantial force on electromagnets, conductors, and other electric material. This type of magnetic field which is generated by steady currents, magnets, or electric current changing linearly with time is known as magnetostatic field.

There are three different types of current element;  $I dl$ ,  $\mathbf{J}_s ds$ , and  $\mathbf{J} dv$  which are line, surface, and volume current respectively.  $\mathbf{J}_s$  is also known as surface current density and has the units of Ampere per meter. Similarly, the current element for volume,  $\mathbf{J}$ , is known as current density, and is always perpendicular to the surface  $d\mathbf{s}$ .

It can be easily proven from the theory that

$$I dl = \mathbf{J}_s ds = \mathbf{J} dv$$

The differential magnetic field intensity  $dH$  at a point  $P_2$ , produced by a current element  $I_1 dl_1$  at a point  $P_1$  can be given by the differential form of Biot Savart Law:

$$dH_2 = \frac{I_1 dl_1 \times \hat{a}}{4\pi R_{12}^2}$$

Where,  $R_{12}$  is the distance between point  $P_1$  and  $P_2$ .

It should also be noted that the direction of  $dH_2$  can be found by the right hand rule and is in the direction of the curl of the fingers where the thumb is pointing in the current direction.

The above Biot Savart Law cannot be proved in its differential form as it is impossible to produce an isolated line current element in space, and thus it needs a closed circuit. Thus the total field  $H$  can be found by integrating (2.1) over a closed path of current.

$$H = \oint \frac{I_1 dl_1 \times \hat{a}}{4\pi R^2}$$

For surface, the above equation will become

$$H = \oint \frac{\mathbf{J}_s \times \hat{a} R ds}{4\pi R^2}$$

From the above laws, we can find out that the magnetic field along the axis of a circular current carrying loop, carrying a current  $I$ , is

$$H = \frac{I a^2}{2(a^2 + z^2)^{\frac{3}{2}}} \hat{z}$$

Similarly, the magnetic field intensity can be derived for a solenoid of a length  $l$ , cross section radius  $a$ , and number of windings  $N$ . The cross section area will be  $A=4\pi a^2$  which in turn will give the total surface current as  $NI$ . As the surface current flows throughout the length  $l$ , the surface current density  $J_s$  will become  $NI/l$  ( $\text{Am}^{-1}$ ).

If  $l \gg a$ , the magnetostatic field intensity at the center of the solenoid can thus be reduced to

$$H = \frac{NI}{l} \hat{z} = J_s \hat{z}$$

The field intensity at the end of the solenoid will thus be,

$$H = \frac{NI}{2l} \hat{z} = \frac{J_s}{2} \hat{z}$$

### 2.1.1 Ampere's Law

The Ampere's circuital law is a great way to solve magnetostatic problems in cases of symmetrical current distributions. The formula obtained by this law shows that the enclosed current carrying loop produces a magnetic field through its axis and vice-versa, and is given by

$$I_{\text{en}} = \oint_l H \cdot dl$$

The curl of magnetic field is given as

$$\nabla \times H = J$$

It can also be expressed in determinant form as

$$\text{Curl } H = \mathbf{J} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

The curl outside the conductor will be 0.

$$\nabla \times H = 0$$

### 2.1.2 Stokes' Theorem

With the help of the Stokes' theorem, we can relate the closed loop integral of  $H \cdot dl$  to a surface integral of  $\nabla \times H \cdot ds$ , which is given by

$$\oint_l H \cdot dl = \oint_s (\nabla \times H) \cdot ds$$

Where,  $l$  encloses the large surface area  $s$ .

### 2.1.3 Magnetic Flux Density Vector

The magnetic field intensity  $\mathbf{H}$  in free space is related to magnetic flux density vector  $\mathbf{B}$  in free space by the following equation

$$B = \mu_0 H$$

The unit of  $B$  is webers per square meter ( $\text{Wbm}^{-2}$ ) or in SI units, teslas (T). The term  $\mu_0$  is known as the permeability of free space and has the value of

$$\mu_0 = 4\pi \times 10^{-7} \text{ (Hm}^{-1}\text{)}$$

and holds the units of Henrys per meter.

Also, the magnetic flux  $\Psi$  (unit Weber, Wb), is related to  $B$  by the following equation:

$$\Psi = \int_s B \cdot ds$$

Which means that the magnetic flux  $\Psi$  passes through the surface  $s$ .

Since the magnetic poles in magnetostatics or dynamics are not isolated, therefore we do not have an individual isolated charge where the magnetic flux field lines begin or terminate. It can be proved by integrating the above equation over the closed surface, which gives

$$\Psi = \int_s B \cdot ds = 0$$

Therefore, from the above discussion it is proved that the magnetic field lines are closed and do not terminate on individual magnetic charges. This equation is one of the most defining equations in magnetic history, derived by divergence theorem and is also known as Maxwell's fourth law for static fields.

### 2.1.4 Maxwell's Equations <sup>[2]</sup>

The Maxwell's equations show how different magnetic and electric fields arising from charges and currents change in time and nature. The Maxwell's contribution to the equations made it clear that the Electricity and Magnetism are both connected and can give birth to each another.

The generated fields can travel far from its point of origination and through the space indefinitely with the speed of light!

There are four equations of Maxwell which gives us the laws of Electrostatics, Electrodynamics, Magnetostatics, Magnetodynamics, and Electromagnetism.

#### **2.1.4.1 Maxwell's First Law**

The Maxwell's first equation explains how electric field is generated by individual electric charges and how it is integrated over an area A:

$$\int E \cdot dA = q/\epsilon_0$$

Where dA is the small area vectors perpendicular to the electric field vectors such that its dot product can be taken.

#### **2.1.4.2 Maxwell's Second Law**

The Maxwell's second equation is very similar to the first one, but holds true for magnetostatics. It's the same equation as derived in section 2.1.3, and proves that there are no sources or sinks in magnetics, and thus no magnetic monopoles exist in nature. The number of flux lines that enter a closed surface are equal to the number of flux lines that go out of it. Thus the net flux out of the enclosed surface or volume is always zero.

The second law can be given by:

$$\int_s B \cdot dA = 0$$

The above two equations show the amount of work done to take an electric or magnetic charge over a closed loop of an electric and magnetic field respectively.

#### **2.1.4.3 Maxwell's Third Law**

For electrostatics, the path integral of electric field is zero.

$$\oint E \cdot dl = 0$$

For magnetostatics, the path integral of magnetic field is equal to  $\mu_0$

$$\oint B \cdot dl = \mu_0$$

The third equation also shows the relation between electric and magnetic field and is given by the equation below:

$$\oint E \cdot dl = -\frac{d}{dt} \left( \int B \cdot dA \right)$$

The equation stands true for the varying magnetic fields and shows that the area on the right hand side of the equation covers the closed loop on the left hand side of the equation.

Also, the third equation remains valid for any direction of the path that we choose over the surface.

#### 2.1.4.4 Maxwell's Fourth Law

The fourth equation of Maxwell shows the total magnetic force and varying electric field, i.e, the displacement current, around the circuit in terms of electric current.

$$\oint B \cdot dl = \mu_0 \left( I + \frac{d}{dt} (\epsilon_0 \int E \cdot dA) \right)$$

#### 2.1.5 Vector Magnetic Potential

In magnetostatics,  $\nabla \cdot B = 0$ , on which, when the identity  $\nabla \cdot \nabla \times A = 0$  is applied, gives us the following result:

$$B = \nabla \times A$$

Where A is known as a vector magnetic potential. From this equation, we can show that the curl of vector magnetic potential is equal to magnetic flux density vector.

Now since we have  $\nabla \times H = J$ , and  $B = \mu_0 H$ , we can say that

$$\nabla \times \frac{B}{\mu_0} = J$$



Using another identity,  $\nabla \times (\nabla \times A) = (\nabla \cdot A) - \nabla^2 A$  on the above equation gives us the following equation

$$\nabla(\nabla \cdot A) - \nabla^2 A = \mu_0 J$$

If we let  $A$  decide its divergence, we will have

$$\nabla^2 A = \mu_0 J$$

Thus, the equation above we get in such a manner is called as vector Poisson Equation.

Therefore, the vector magnetic potential we get over a volume is equal to

$$A = \int \frac{\mu_0 J dv}{4\pi R}$$

Similarly, the vector magnetic potential over a line element and a surface element will, respectively, be equal to:

$$A = \int \frac{\mu_0 I dl}{4\pi R}$$

$$A = \int \frac{\mu_0 J ds}{4\pi R}$$

The unit of Vector Magnetic Potential is equal to Weber per meter ( $\text{Wb m}^{-1}$ ).

### 2.1.6 Electromagnetic forces

The forces during Coulomb's time were considered as a direct and an instantaneous interaction between point charges, and the formula was given as

$$F = \frac{Q_1 Q_2}{4\pi R^2}$$

The finding of the electromagnetic force is a 2-step process where first we find the field produced by the charge  $Q_1$  and then we find out the effect of this field on the charge  $Q_2$ , which is the force that we need. Similar method can be used to find the force on charge  $Q_1$  as an effect by the field caused by charge  $Q_2$ , and this kind of solution finding process is known as charge-field-charge solution concept.

It should be kept in mind that this kind of process only works if there are point charges or differential current elements in a field which carry a steady or constant current and this 2-part process becomes much more complicated when complex charge or current distributions are involved. Also since the finite time is not taken into consideration when using this 2-step process, the cases involving accelerating charges or unsteady currents cannot be solved using this method.

The force on a conductor current element  $I dl$ , placed in a magnetic field  $B$ , is equal to

$$dF = I dl \times B$$

The magnetic field  $dH_2$  at point 2, by the effect of  $I_1 dl_1$  at point 1, can be found as

$$dH_2 = \frac{I_1 dl_1}{4\pi R^2}$$

Since  $dB_2$  is equal to  $\mu_0 dH_2$ , the above equation becomes

$$d(dF_2) = I_2 dl_2 \times \frac{I_1 dl_1}{4\pi R^2}$$

Or,

$$dF_2 = I_2 dl_2 \times B_2$$

Or, the force by current carrying conductor loop  $l_1$ ,  $F_2$ , on current carrying loop  $l_2$  is equal to

$$dH_2 = \frac{\mu_0 \cdot I_1 I_2}{4\pi} \oint_{l_2} \left[ \oint_{l_1} \frac{dl_1}{R} \right] \times dl_2$$

Also, for a current-carrying loop conductor in a uniform field  $B$ , the force of translation becomes zero:

$$\mathbf{F} = \oint_l I dl \times B$$

Or,

$$F = -IB \oint_l dl = 0$$

Because,  $\oint_l dl = 0$ .

Being said that, the torque on the current carrying loop may not be zero, even if the force of translation is zero.

### 2.1.7 Torque on Current Loops <sup>[3]</sup>

The torque  $T$  on the current carrying conductor loop ( $Ids$ ) under the influence of the magnetic field  $B$  can be given as

$$dT = Ids \times B$$

The term  $Ids$  is also called as the magnetic moment,  $dm$ , and can be given by

$$dm = Ids$$

Therefore, the torque will become

$$dT = dm \times B$$

If the above equation is integrated, the Torque on a non- differential current loop with vector area  $ds$  is found as,

$$T = I \int_s ds \times B$$

For a uniform magnetic field,  $B$

$$T = m \times B$$

### 2.1.8 Magnetic Material

When a magnetic field is applied to a magnetic material, the atomic magnetic dipoles, which can be combined, of the material gets aligned together and this property of a material is known as magnetic polarization.

The magnetic moment of an atomic magnetic dipole is caused by three effects; the electron spin, the nuclear spin, and the orbiting electron. The electron in an orbit has its own magnetic moment due to its angular momentum, which is same as a magnetic current loop of equal moment. This electron thus generates a current  $I$  which surrounds the area  $ds$ . The total magnetic moment of

the atom is the vector sum of all the moments of all its orbiting electron. The spin magnetic moment of electrons in unfilled shells also contributes to the total magnetic moment of the atom. Also, the spin magnetic moment of the nucleus is taken into account for the same. The net magnetic moment of an atom is thus the vector sum of all these individual magnetic moments, with bound current  $I$ , and can be given as

$$dm = Ids$$

The current  $I$  is bound to the atom through Coulomb forces.

Initially, due to the random orientation of electrons and their dipoles in an atom, the net sum of magnetic moment will be equal to zero. But as soon as a magnetic field,  $B_a$ , is applied, the magnetic moments  $dm_s$  will align instantaneously in the direction of the applied magnetic field due to the effect of applied torque  $dT = dm \times B$ . The net moment of the material,  $M$ , will thus be the vector sum of all the magnetic moments of the atoms which will affect the magnetic field  $B$  inside and outside of the magnetic material.  $M$  is also known as Magnetization vector, is defined as the magnetic dipole moment per unit volume, and for a uniform applied magnetic field can be given as

$$M = ndm = nIds$$

The above magnetization vector  $M$  is uniform and, on the surface of the material, will give rise to a non-zero bound magnetization surface density  $J_{sm}$ . Another current density  $J_m$  will be formed inside the material by a varying magnetization vector  $M$ . The bound magnetization current density  $J_m$  will be zero though because of the current, also called Amperian currents, due to the adjacent current loops formed inside the material.

By derivation, the bound magnetization current density,  $J_m$ , within a magnetic material will be equal to

$$J_m = \nabla \times M$$

By derivation, the bound magnetization surface current density,  $J_m$ , at the surface of the magnetic material will be equal to

$$J_{sm} = M \times n$$

### 2.1.9 Effect of Magnetization on magnetic fields

The above current densities, both bound and bound surface magnetization, will give rise to a magnetic field and will follow the rules of free charge-space equation, which can thus be reformatted as

$$\nabla \times \frac{B}{\mu_0} = J + J_m$$

Where B is the resultant vector magnetic field caused due to currents associated with free charges and with magnetization.

Substituting the value of  $J_m$ , we have

$$\nabla \times \frac{B}{\mu_0} = J + \nabla \times M$$

Rearranging the equation, we have

$$\nabla \times \left( \frac{B}{\mu_0} - M \right) = J$$

If we define,

$$H = \left( \frac{B}{\mu_0} - M \right)$$

Then we have,

$$\nabla \times H = J$$

For free space, since  $M=0$ ;

$$H = \frac{B}{\mu_0}$$

Also,

$$B = \mu_0(M + H)$$

Since, for an isotropic material

$$M = \kappa H$$

Where,  $\kappa$  is the magnetic susceptibility of the material. If we submit M into the magnetic field B equation, we get

$$B = \mu_0 H(1 + \kappa)$$

If we define,

$$\mu_r = 1 + \kappa$$

We get,

$$\mu = \mu_r \mu_0$$

Thus,

$$B = \mu_r \mu_0 H$$

Or,

$$B = \mu H$$

Permeability  $\mu_r$  of iron is 5000.

The resultant magnetic field through a material surface is thus the sum of applied magnetic field  $B_a$  and induced magnetic field  $B_i$ .

$$B = B_a + B_i$$

Where,  $B_i$  is the induced magnetic field due to magnetization only and can be derived by utilizing the conduction properties of the bound surface magnetization current,  $J_{sm}ds$ , and bound volume magnetization current,  $J_m dv$ .

$$B_i = \frac{\mu_0}{4\pi} \oint_s \frac{J_{sm}}{R^2} ds + \frac{\mu_0}{4\pi} \oint_v \frac{J_m}{R^2} dv$$

## 2.2 Superconductivity

[4]Superconductivity has been around the scientific community since the starting of 20<sup>th</sup> century, and has been a topic of an exciting research since 1911. After being stagnant for some years, the discovery of high-temperature superconductivity in 1980s made the field surface again and many considered this as important as the invention of transistors. Superconductors have a great scope in changing the world as we see and it is definitely important for us to understand its basic electromagnetic properties and the advantageous applications that this brilliant piece of technology can bring into our lives.

Superconductors have various advantages; for example, if some current is passed through a superconducting ring, which is kept at a certain low temperature range, the current stays in the ring without any measurable decay. In addition to that, the superconducting ring won't show any electrical resistance to direct currents, and thus there will be no heating and no direct losses will be occurred. Besides that, some strong superconductors can also repel applied magnetic fields, such that the magnetic fields inside the superconductors are always zero, and this is the primary basis for modern day magnetic levitation. The aim of this section is to understand the underlying

dynamics and describing in details some of the electromagnetic properties of the superconductors, along with its advantages.

### 2.2.1 Magnetic properties of materials

<sup>[5]</sup>As discussed earlier, every electron in a magnetic material has its own magnetic moment and which contributes to the total magnetic moment of the material by its vector sum. The electronic magnetic moments align themselves in the direction of the applied magnetic field making the material a strong magnet.

But there are many cases in nature when this is not possible. For instance, in Hydrogen molecules, H<sub>2</sub>, since the atoms are not isolated but bounded by each other through a chemical bond, the angular momentum of the electrons are in opposite direction to each other, which contributes to its weak magnetic properties. We can say that these kind of materials thus lack a permanent magnetic moment, as the bonds are weak due to opposite spins.

The magnetic field applied to such atoms changes the angular momentum of electrons very slightly. Due to this, the individual atoms and electrons having opposite spins don't cancel each other out completely, and the molecule as a whole thus acquires an induced magnetic moment. This kind of behavior is present in all kinds of material known to human kind, and this property of a material to acquire an induced magnetic moment is called *Diamagnetism*. Those materials who only have paired electrons in them are diamagnetic materials, for example, Hydrogen, Bismuth, Copper, Ammonia, Graphite etc, and can be very weakly repelled by applied magnetic fields. In diamagnetic materials, the electrons experience an additional force  $qv \times B$ , when an external field is applied, which alters the central force increasing the orbital speed of the electrons in opposite directions. Thus, the magnetic moments doesn't cancel any more for the electrons and a net dipole moment is acquired by the material which contributes in opposing the applied field.

The other materials, which have unpaired electrons in them, have a net magnetic moment to themselves, even in the absence of an externally applied magnetic field. The applied magnetic field can still alter the electronic angular momentum, providing an induced magnetic moment, but the effect here is more permanent and much stronger than its counterpart, diamagnetism. Such property of a material is known as *Paramagnetism*, and can be found in atomic structures like Oxygen, O<sub>2</sub>. The effect can still be feeble, as the permanent magnetic moments are continually misaligned due to the thermal vibration present inside the materials. The process through which a material's magnetic moment is altered is called magnetization which we discussed earlier. The magnetization of a paramagnetic material depends on the applied field and is inversely proportional to the absolute temperature, and can be explained as Curie's law, named after its discoverer, Pierre Curie:

$$M = C * \frac{B}{T}$$

Where, C is the Curie's constant.

The third and most used type of material in present day physics are ferromagnetic materials, which has a tendency to stay in the orientation after the external magnetic field is removed. The atoms in such materials stay parallel to their neighboring atoms in a lockdown mode, and such a property of a material to retain its magnetization is known as *Ferromagnetism*. This permanent alignment is due to the strong coupling between neighboring atoms, which doesn't exist in anti-ferromagnetic substances, where all the internal magnetic moments have same magnitudes and are in opposite directions. The materials like Iron, Nickel and Cobalt are ferromagnetic at sufficiently low temperatures, and rare Earth materials such as Terbium and Gadolinium are ferromagnetic below room temperatures. Being said that, all transition and rare earth materials become paramagnetic at very high temperatures.

The magnetization properties of a ferromagnetic material strongly depends on its purity, heat treatment, etc. and has other properties like coercivity and remnance, which makes it different from paramagnetic materials. The atoms in ferromagnetic materials also form microscopic groups within which are known as domains, containing upto  $10^{17}$ - $10^{21}$  atoms per unit volume of about  $10^{-12}$  m. When an external field is applied, the domains, as a group, tend to align together in the direction of the magnetic field, and retain a net magnetization in the same direction. This orientation is very strong and mostly permanent, and cannot be easily disrupted by thermal agitation at room temperatures.

The property of a material at which it cannot contribute to the increase in flux is called as saturation. This limitation generally occurs in inductors having ferromagnetic cores under which the current increases lead to a very substantial and smaller increases in the overall flux. Reaching the saturation point in a material should be avoided and can be countered by many engineering techniques such as running the inductor at low current, using larger core with air gaps and lower permeability, and altering the number of turns etc. It can be explained by the Curie's law that at extremely high or super low temperatures the magnetization reaches its saturation or the maximum value with complete alignment of dipoles, at the point where the Curie's law is no longer valid. The temperature at which a ferromagnetic material exceeds a critical temperature and loses its spontaneous magnetization is known as Curie's temperature.

The materials can be classified in terms of how the permeability of free space,  $\mu_0$ , compare to their magnetic permeabilities,  $\mu_m$ .

Paramagnetic substances:  $\mu_m > \mu_0$

Diamagnetic substances:  $\mu_m < \mu_0$

Ferromagnetic substances:  $\mu_m \gg \mu_0$

### 2.2.2 Superconductors

The research in low temperature superconductivity started in 1908 when a Dutch physicist, Kamerlingh Onnes worked on liquefying helium at standard pressure and boiling temperature of 4.2 K. In 1911, Dr. Onnes along with his assistants found a sharp drop in the resistance of Mercury sample when it was subjected to a critical low temperature of 4.5 K. Kamerlingh Onnes



termed this property superconductivity for which he was awarded Nobel Prize in year 1913 in Physics.

The temperature at which the resistivity of a material goes to zero is known as the *Critical Temperature,  $T_c$* , of that material, which is also the characteristic property of individual materials under experiments.

Hans Meissner and Robert Oschensfeld found out that when cooled down below their critical temperatures, the magnetic field is expelled inside the superconductors while experimenting with the magnetic behaviors of superconductors in the year 1933. It was also found that the material loses their superconducting nature when the applied magnetic field is at the temperature dependent value of *Critical Magnetic field,  $B_c$* .

The high temperature superconductivity was first discovered by Dr. Georg Bednorz and Alex Muller in year 1986 when they found evidences of zero resistivity in an oxide of lanthanum, barium, and copper at a shocking temperature of around 30K! This significant discovery caught everybody's attention and it marked as a beginning of a new era in high temperature superconductivity. Many superconductors with the critical temperature upto 150 K have been discovered since in the form of complex oxides, but the reason behind such behaviors of the materials are still a mystery.

### ***2.2.2.1 Type I superconductors or Low temperature Superconductivity***

The superconductors which display their superconducting properties below the critical temperature of 2 K, are known as low temperature superconductors. They are usually basic and simple elements found in nature directly.

The value of critical temperature,  $T_c$ , decreases as the magnetic field B is applied. When this applied magnetic field exceeds the value of critical magnetic field,  $B_c$ , the superconductivity of the material under inspection is lost, and the material from there on behaves like a normal conductor with a finite resistance.

This critical magnetic field varies with temperature and can be given as:

$$B_c(T) = B_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$

The value of  $B_c$  is maximum at 0 K, a temperature which is nearly impossible to maintain in a laboratory setting, and thus  $B_c(0)$  is the maximum magnetic field that is required to destroy superconductivity in any given material.

The Faraday Law of induction states

$$\oint E \cdot ds = -\frac{d\phi}{dt}$$

Since, in a superconductor, the electric field is zero everywhere, the change in magnetic flux over time also becomes zero. Thus, from this we can say that the magnetic field inside a superconductor remains constant everywhere and the magnetic flux in the superconducting material cannot change.

When the temperature is lowered below the critical temperature  $T_c$ , the magnetic field lines are instantaneously expelled from inside the superconductor. This amazing phenomenon of the expulsion of magnetic fields by the superconductors is known as *Meissner effect*. In this effect, the superconductors form induced surface currents, which in turn produces an induced magnetic field which repels the already applied magnetic field and cancels it. These surface currents disappear when the external magnetic field is removed. If the applied field is larger than the critical field, the superconducting state is destroyed and the magnetic field thus enters the material. Also, due to this effect, we can say that the superconductor acts like a perfect diamagnet and can repel a permanent magnet. This property is the basis for magnetic levitation and is used in Maglev trains in Japan.

The surface currents are not formed exactly at the surface, but they penetrate to some extremely small extent. Along this extremely thin layer, the magnetic field decreases exponentially to zero, and is about 100 nm thick.

$$B(x) = B_0 e^{-\frac{x}{\lambda}}$$

The external magnetic field here is assumed to be parallel to the surface of the material.

Here,  $B_0$  is the value of the magnetic field at the surface,  $x$  is the distance from the surface to an interior point, and  $\lambda$  is known as the *penetration depth*.

The penetration depth is temperature dependent and can be given theoretically by the following expression

$$\lambda(T) = \lambda_0 \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]^{-\frac{1}{2}}$$

Where  $\lambda_0$  is the penetration depth at 0 K.

Here we can see that  $\lambda$  approaches infinity as  $T$  becomes  $T_c$ . This simply means that an applied magnetic field penetrates a magnetic sample in superconducting state more and more as  $T$  reaches  $T_c$ .

### ***2.2.2.2 Type II superconductors or High temperature Superconductivity***

The high temperature superconductivity was discovered in late 1980s as we know it, but scientists have guessed about this phenomenon in as early as 1950s. The major difference between a high temperature superconductor and a low temperature superconductor is that its critical values fall within a range. These superconducting materials are characterized by two critical applied magnetic fields named  $B_{c1}$  and  $B_{c2}$ . Below the critical magnetic field  $B_{c1}$ , the material is in superconducting state and behaves purely as a low temperature superconductor and there is no flux penetration. The superconducting state is destroyed above the critical magnetic field  $B_{c2}$ , and the flux penetrates the superconducting material sample completely and the sample becomes normal. Also, the transition for one state to another superconducting state is thermodynamically stable.

The state between both the critical magnetic fields is known as vortex state, and here the material is in mixed state. During this state, the material can have some flux penetration with zero resistance.

Some upper critical magnetic fields for common high temperature superconductors  $Nb_3Al$ ,  $Nb_3Sn$ ,  $NbTi$ , and  $PbMoS$  are 32.4 T, 24.5 T, 15 T, and 60 T.

### ***2.2.2.3 Critical Current Density of Superconductors***

[6]One of the most important properties of superconductors is its critical current density and holds true for all its large scale applications such as electromagnets, power transmission lines, rotating machines, fault-current limiters, SQUIDS etc. The superconducting devices can be made more efficient and smaller in size if anyhow there range of current densities are increased.

When Onnes discovered the superconductivity in Mercury, he found out about a limiting value of current density above which the superconductivity is lost. He called this value critical current density, and its critical value, which was temperature dependent and increases as the temperature reduces, is given as

$$J_c(T) = J_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$

It was noticed that the critical value was less in superconducting coils when compared to small samples of wire. The first superconducting solenoids then made were of tin and lead.

According to Silsbee, the threshold value of the critical current is where the magnetic field due to current is equal to the critical magnetic field. The tangential magnetic field at a distance  $r$  from the line current is equal to

$$H(r) = \frac{I}{2\pi r}$$

For a surface with radius  $a$ ,

$$H(a) = \frac{I}{2\pi a}$$

Thus, the critical current due to Silsbee's Hypothesis will be

$$I_c = 2\pi a H_c$$

And the critical current density will be,

$$J_c = \frac{2H_c}{a}$$

The above formulae are useful for metals and not alloys. Also, critical current is not an intrinsic property of a superconductor but is dependent on the size/ radius of the superconductor. Whereas the critical current density also depends on its size but it decreases as the size of the superconductor is increased.

The superconducting state and its transition from normal conducting state is thermodynamically stable. This shows that the superconducting state has a lower energy than a normal state and can be proven by simple thermodynamics law that energy per unit volume for superconducting state is equal to

$$\Delta G_{ns} = -\frac{1}{2}\mu_0 H_c^2$$

Where  $\Delta G_{ns}$  is the energy required to expel the magnetic field from inside the superconducting material.

The penetration effect discussed earlier was given by London and it showed that the magnetic flux is not totally expelled from the superconducting body but it penetrates inside it exponentially

$$H(r) = H_0 e^{-\frac{r}{\lambda}}$$

The gradient of magnetic field is the current, therefore:

$$J(r) = \frac{\partial H}{\partial r} = \frac{-H_0}{\lambda} \cdot e^{-\frac{r}{\lambda}}$$

The maximum value of this current is at surface ( $r=0$ ), and is given by

$$J(max) = \frac{H_c(0)}{\lambda}$$

From here, it can also be proven that the Silsbee's Hypothesis can be used for critical current related to lower magnetic fields for alloys.

Another thing to keep in mind is that the current density is the sum of the total current over the total area

$$I = \int J \cdot ds$$

Or,

$$J = \frac{Totalcurrent(I)}{Totalarea}$$

The units for linear current density or magnetic field strength is A/m, and for current density is A/m<sup>2</sup>.

The imperfections in the material increase the critical current and not the current density. The current density on the surface layer only increases due to the critical field increasing or penetration depth decreasing.

Therefore, to summarize, the critical current density follows the following rules for lower range,  $J_{c1}$ , and higher range,  $J_{c2}$ , in order to maintain superconductivity <sup>[6]</sup>

$$J_{c1} = \frac{2B_c}{a}$$

$$J_{c2} = \frac{B_c^2 b(1-b)V_f}{4\mu_0 \xi B}$$

Where,  $a$  is the area,  $b$  is the reduced induction,  $V_f$  is the volume fraction of particles,  $\xi$  is the coherence length and  $B$  is the applied field.

#### 2.2.2.4 Rotating Superconductors

[7] A small magnetic field can be generated if a perfect conductor is rotated. If a cylindrical superconducting shell is considered, then the local mean value of the momentum vector,  $p_s$ , integrated over a closed path, can be given by:

$$\oint p_s dl = \oint \left( mV_s + \frac{e}{c} A \right) \cdot dl = Kh$$

Where K and n are integers, and m and e are mass and charge of electron respectively. Thus, from the above equation, if  $n=0$ , then for a superconducting shell rotating with angular velocity  $\omega$ , the magnetic field generated can be given by

$$B = - \left( \frac{2mc}{e} \right) \omega$$

This is also called as London moment which arises from Coriolis forces on a charged particle in a rotating reference frame.

[8] For a thick superconducting ring ( $j=0$  and  $n=0$  inside), the above London's moment can be modified as

$$B = - \left( \frac{2mc}{e} \right) \omega - \left( \frac{m}{e} \right) B_g$$

Where,  $B_g$  is the gravitomagnetic field of the substance.

#### 2.2.2.5 Applications of Superconductivity

Due to its zero resistance and magnetic field expulsion, the high temperature superconductivity has number of modern day applications. One of the major technological advances is the building of lightweight and highly efficient superconducting motors. The property of zero resistance to direct currents can be utilized in building the low-loss power transmission lines for electric power transmission. A very high amount of electrical power is lost due to resistive heating and length of wires when current is passed through normal conducting wires. The DC losses could be easily eliminated resulting in substantial savings in energy costs if the superconducting wires can be made.

The superconductors can totally revolutionize the field of electronics. Due to its brilliant switching qualities, the Josephson function can be used in computer hardware. Also, small sized chips can be made when superconducting films are used to interconnect computer chips, resulting in higher data speeds. More chips can thus be placed on a computer board with very less heat generation.

The Meissner effect explains the concept of magnetic levitation, and on which a prototype train runs in Japan using superconducting magnets. This moving train boasts of having no wheels and high speeds as the train levitates above a normal conducting metal track using eddy surface current repulsions. Toys like *Levitron* also use the same concept and the future of transportation can be envisioned brightly using superconducting magnets.

The high temperature superconductors are also used in particle accelerators such as Large Hadron Collider, where the high magnetic fields are used for accelerating and colliding sub atomic particles for understanding the basic components and history of the universe. Currently all the particle accelerators use liquid based superconducting technology and are advancing rapidly.

The major application of high temperature superconductors yet has happened in the medical field where the superconducting magnets are used in a diagnostic tool called Magnetic Resonance Imaging (MRI). This technology is very safe and secure as compared to the harmful X-ray technology as it produces safe radio-frequency radiation to produce images of body parts. Currently the costs for such a technology is very high. It takes an estimated amount of \$30000 per year to run a helium based MRI. But this cost can be reduced by using nitrogen cooled superconductors and it is estimated that overall \$100,000 can be saved in operating costs of each MRI device.

There are some challenges too for a universal breakthrough applications of superconductors. Firstly, as the superconducting ceramic materials are very brittle in nature, it a difficult challenge to mold it into useful shapes, like wires, ribbons for large-scale applications, and thin films like SQUIDS for small devices. Also because of their low current density, the bulk ceramic compounds are a challenge to build.

### ***2.3 Electromagnetic Propulsion System by J.F. Engelberger <sup>[10]</sup>***

A similar propulsion system is patented by J. F. Engelberger, US, in 1970. The propulsion system proposed is a simple superconducting ring which generates a magnetic field that interacts with the Earth's magnetic field. The interaction such produced can be termed as dipole-dipole interaction and will produce a substantial amount of thrust force to propel the spacecraft in a direction.

The superconducting coil will have its own dipole moment and will act similar as a bar magnet when placed in a field.

The magnetic dipole when kept in a non-uniform magnetic field will exert a translational force which can be expressed in Cartesian coordinates by a number of following equations given as follows

$$F_x = M_x \frac{\partial H_x}{\partial x} + M_y \frac{\partial H_y}{\partial x} + M_z \frac{\partial H_z}{\partial x}$$

$$F_y = M_x \frac{\partial H_x}{\partial y} + M_y \frac{\partial H_y}{\partial y} + M_z \frac{\partial H_z}{\partial y}$$

$$F_z = M_x \frac{\partial H_x}{\partial z} + M_y \frac{\partial H_y}{\partial z} + M_z \frac{\partial H_z}{\partial z}$$

Where, F is the force, M is the dipole moment, and H is the magnetic field intensity of the non-uniform magnetic field.

The above equations can be converted into polar coordinates as the spacecraft is planned to operate in polar orbit. The spacecraft can be assumed to be working at an altitude of r from the center of the earth and  $r_0$  being the earth's radius.  $\theta$  can be the angle between the magnetic north pole and the radius vector r, and B is the angle between the radius vector and the normal.

The above equations, initially in Cartesian coordinates, thus when represented in polar coordinates will look like:

$$F = 5 X \frac{10^{-11} NIA}{\left(\frac{r}{r_0}\right)^4} \left[ -r \left( \cos B \cdot \cos \theta + \frac{1}{2} \sin B \cdot \sin \theta \right) + \theta \left( -\frac{1}{3} \cos B \cdot \sin \theta + \frac{1}{6} \sin B \cdot \sin \theta \right) \right]$$

Since the value of the force F is dependent on the position of the spacecraft through the radius vector, the force values will vary as the spacecraft moves from the North Pole to South Pole via equator. At equator, the axis of the loop will be parallel to the polar axis of the earth as the plane of spacecraft will be in the plane of the equator.

Over North Pole the axis of the spacecraft will be parallel to the polar axis of the spacecraft and a force will be exerted along the radius vector. However at this point, there will be no force acting in the direction of the motion of the spacecraft.

Since angle B is fixed for each value of  $\theta$ , the relation between the angle B and the angle  $\theta$  can be given as

$$\tan B = \frac{1}{2} \tan \theta$$

The propulsion system will generate the maximum propulsive force at the North Pole where both the angles B and  $\theta$  are equal to zero. Therefore the force equation at the poles will thus take the form

$$F = 5 X \frac{10^{-11} NIA}{\left(\frac{r}{r_0}\right)^4}$$



The paper talks about using a single superconducting coil comprising on one single turn and its advantages. The usage of more than one coil can double the force generated but will also double the mass of the system. Since  $a=F/m$ , it will half the value of the acceleration and is thus advised against it.

Also, with a small diameter solenoid, the maximum permissible current in the coil is severely permitted as a higher value of hoop stress will be generated. Also there will be internal forces between adjacent turns of the solenoid. Using a single turn large diameter coil will minimize these effects and a large current in magnitudes of thousands of Amperes can be passed through the magnetic coil.

An example to produce thrust force is provided in the paper. For a radius of 10000 m, current of 4000 A, wire radius of 0.002 m, the force is computed to be 50 N. Assuming the mass of the NbTi superconducting is 64 kg with density being 8400 kgs per cubic meter, the acceleration comes out to be 0.77 m/sec<sup>2</sup>.

#### ***2.4 Electromagnetic propulsion system by Valentine Pulatov<sup>[11]</sup>***

The electromagnetic propulsion system by Valentine Pulatov is similar in concept and design to the propulsion system suggested by Engelberger and this research. Thus, the propulsion system uses the same technology as its counterparts and generates a thrust force by interacting with Earth's magnetic field through dipole-dipole interaction.

In addition to that, Pulatov suggests two more advanced methods through which a spacecraft can be controlled and propelled. The second system doesn't uses the dipole interaction to propel, but a certain kind of electrodynamic cross force which has an arbitrary direction and can be used for spacecraft propulsion in lower earth equatorial orbit by continuous propulsion. This system is called 'Partially controlled system'.

The third system is called 'Total Control' system and through which small arbitrary changes can be made in the direction and the modulus of the thrust electrodynamic force vector. The success of this kind of propulsion system highly depends on the advancements of the solid state physics and superconductivity.

The simple electromagnetic propulsion system follows the same rule as that of Engelberger's system. The force that is generated is the result of dipole-dipole interaction of the Earth's magnetic field and the spacecraft's magnetic field. The magnetic dipole when kept in a non-uniform magnetic field will exert a translational force which can be expressed in Cartesian coordinates by a number of following equations given as follows

$$F_x = M_{1x} \frac{\partial B_x}{\partial x} + M_{1y} \frac{\partial B_y}{\partial x} + M_{1z} \frac{\partial B_z}{\partial x}$$

$$F_y = M_{1x} \frac{\partial B_x}{\partial y} + M_{1y} \frac{\partial B_y}{\partial y} + M_{1z} \frac{\partial B_z}{\partial y}$$

$$F_z = M_{1x} \frac{\partial B_x}{\partial z} + M_{1y} \frac{\partial B_y}{\partial z} + M_{1z} \frac{\partial B_z}{\partial z}$$

The force vectors can be expressed in spherical coordinates and the variable differentials can be redefined in terms of latitude and longitudinal angles  $\theta$  and  $\Phi$ . They will thus become,  $\partial R$ ,  $R\partial\theta$ , and  $R\partial\Phi\sin\theta$

Therefore, the force vectors will be

$$F_R = M_{1R} \frac{\partial B_R}{\partial R} + M_{1\theta} \frac{\partial B_\theta}{\partial R} + M_{1\Phi} \frac{\partial B_\Phi}{\partial R}$$

$$F_\theta = M_{1R} \frac{\partial B_R}{R\partial\theta} + M_{1\theta} \frac{\partial B_\theta}{R\partial\theta} + M_{1\Phi} \frac{\partial B_\Phi}{R\partial\theta}$$

$$F_\Phi = M_{1R} \frac{\partial B_R}{R\sin\theta\partial\Phi} + M_{1\theta} \frac{\partial B_\theta}{R\sin\theta\partial\Phi} + M_{1\Phi} \frac{\partial B_\Phi}{R\sin\theta\partial\Phi}$$

Only the dipole component  $B_D$  of the planetary magnetic induction,  $B_0$ , is considered. The substitution may result in subsequent errors where the non-dipole component if magnetic field is high, but the estimation is good enough for accurate measurements of the magnetic induction and the electrodynamic forces.

Since the magnetic dipole field of Earth does not contain the longitudinal component of the magnetic induction because the dipole field is axially symmetrical,  $B_\Phi$  is equal to zero.

Therefore, the other components of planetary magnetic dipole becomes,

$$B_{DR} = \frac{\mu_0 M_P 2\cos\theta}{4\pi R^3}$$

$$B_{D\theta} = \frac{\mu_0 M_P \sin\theta}{4\pi R^3}$$

The relation between the different components of the spacecraft's magnetic moment  $M_1$  can be shown as

$$\frac{M_{1R}}{M_{1\theta}} = \frac{2\cos\theta}{\sin\theta}$$

And thus,

$$M_{1R} = \frac{M_1 2\cos\theta}{\sqrt{3\cos^2\theta + 1}}$$

$$M_{1\theta} = \frac{M_1 \sin\theta}{\sqrt{3\cos^2\theta + 1}}$$

Since,  $B_\Phi = 0$ ;

The force vectors will be

$$F_R = M_{1R} \frac{\partial B_{DR}}{\partial R} + M_{1\theta} \frac{\partial B_{D\theta}}{\partial R}$$

$$F_\theta = M_{1R} \frac{\partial B_{DR}}{R\partial\theta} + M_{1\theta} \frac{\partial B_{D\theta}}{R\partial\theta}$$

Substituting the values of the spacecraft's magnetic dipole moment, the Thrust force vectors can be written as

$$F_R = -\frac{3\mu_0 M_E M_S \sqrt{(3(\cos\theta)^2 + 1)}}{4\pi R^4}$$

$$F_\theta = -\frac{3\mu_0 M_E M_S \sin 2\theta}{(8\pi R^4) \sqrt{(3(\cos\theta)^2 + 1)}}$$

Here, we can see that the dipole electromagnetic thrust vectors are inversely proportional to the height by the power of fourth. Therefore, it can be used more efficiently in lower earth orbits.

If some screen is added to the superconducting coil, the balance and equality of the forces in the opposite sections of the coil will be interfered. The cross force in the 'Partially Controlled'

system thus is the result of opposite and difference in unbalanced Ampere forces, and can be given as

$$F_c = F_2 - F_1$$

Where  $F_1$  and  $F_2$  are two opposite Ampere forces in the coil, and  $F_c$  is the cross force.

For a superconductor, running 40,000 A at a height of 250 km, the Ampere force will be equal to

$$F_1 = B_D \cdot I = 1.12 \text{ N}$$

## 2.5 *Orbital Mechanics and Spacecraft Simulation*

<sup>[12]</sup>A spacecraft can be considered as a projectile and once it is launched in orbit, the major force governing its motion is the gravitational force. Therefore for a circular orbit, the motion of the spacecraft can be explained directly and simply as that of any object traveling in circular motion. The velocity of the spacecraft is always tangential to its path of motion, and the acceleration of the spacecraft is always directed towards the center in the direction of the centripetal force. For spacecrafts, the centripetal force is equal to the gravitational force. In other cases of elliptical orbits, the acceleration of the spacecraft is directed towards the foci of the ellipse, in the same direction as the force. Also, there is a force in the direction of the motion in case of the elliptical orbits.

The tangential velocity with which the spacecraft travels is known as its orbital velocity. For an orbiting spacecraft of mass  $M_s$ , the centripetal force is equal to

$$F_c = \frac{(M_s V^2)}{R}$$

The gravitational force acting on the spacecraft is equal to

$$F_G = \frac{(GM_s \cdot M_E)}{R^2}$$

Where  $M_E$  is the mass of Earth or the body spacecraft is orbiting.

Since,  $F_G = F_c$ ,

$$\frac{(M_s V^2)}{R} = \frac{(GM_s \cdot M_E)}{R^2}$$

Or,

$$V = \sqrt{\frac{GM_E}{R}}$$

Where V is known as the orbital velocity of the spacecraft, R is the radius of the orbit and G is the gravitation constant with value  $6.673 \times 10^{-11}$ . It is interesting to note that the orbital velocity does not depend on the mass of the spacecraft.

The equation for the acceleration of gravity is equal to

$$g = \frac{GM_E}{R^2}$$

This is the centripetal acceleration for a spacecraft orbiting in a circular orbit, thus it can be rewritten as

$$a_c = \frac{GM_E}{R^2}$$

Also, the time period for an orbiting spacecraft can be given as

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_E}}$$

For simulation in any computer program, the problem statement can be considered as an initial value ODE problem. Under this condition, the spacecraft is given some initial conditions and its path is traced over a certain time period. During the simulation, the values for different parameters such as radius, velocity, acceleration, etc. are calculated at each time step, and they become the initial values for the next time step. Doing so gives us a very good approximation of the spacecraft conditions and behavior in real life.

The simulation can be performed by various methods, such as Euler method, Euler-Cromer method, Range Kutta and MATLAB's ODE45 method. We will be discussing Euler and Range Kutta- MATLAB's ODE45 method below.

### 2.5.1 Euler method

<sup>[13]</sup>The Euler method is the simplest and shortest simulation technique available. This kind of problem statement is known as the initial value problem. Here, firstly an initial values are assigned to the object and then its path is analyzed over a number of iterations. Let's define some initial values as  $x_i = x_1$  and  $v_i = v_1$ , which are the initial values of position and velocity respectively. The next value of x and v will, according to Euler's method, be

$$v_2 = v_1 + a \cdot dt$$

$$x_2 = x_1 + v_1 \cdot dt$$

Where,  $x_2$  and  $v_2$  are the new position and velocity after a time step  $dt$ , and  $a$  is the acceleration of the object.

Next step will be to update the values of initial position and velocity:

$$v_i = v_2$$

$$x_i = x_2$$

And thus the same process will be repeated for the same time steps for as many number of iterations we desire.

### 2.5.2 Orbit Energy Method

<sup>[23]</sup>For Continuous Thrust Orbit Transfer

Total orbital energy per unit mass,  $\varepsilon$

$$\varepsilon = \frac{-\mu}{2r}$$

Or

$$\frac{d\varepsilon}{dt} = \frac{-\mu}{2r^2} \frac{dr}{dt}$$

Where,  $\mu = G(m_1 + m_2)$  = Gravitational constant

If thrust acceleration,  $a = F/m$ , then

$$\frac{d\varepsilon}{dt} = a \cdot v_e$$

For low thrust, orbit is almost circular

$$v_e = \sqrt{\frac{\mu}{r}}$$

This gives,

$$\frac{d\varepsilon}{dt} = a \cdot v_e = a \cdot \sqrt{\frac{\mu}{r}}$$

Or

$$\frac{dr}{dt} = \frac{2a}{\sqrt{\mu}} r^{3/2}$$

The above equation can be integrated over a number of iterations to give the value of radius and altitude. The integration can be done normally over a number of time steps in a computer program, or the ode45 tool can be used in MATLAB.

The Runge Kutta or ode45 method is little advanced and complex than Euler method. The process treats the inputs as ordinary differential equations and integrates them over a number of iterations.

An ordinary differential equation or ODE contains one or more than one variables in it, with derivatives of atleast one dependent variable, with respect to a single independent variable, which is usually the time  $t$ . In an initial value problem, the equations are solved by initiating from an initial state. If  $x_i = x_0$  is the initial condition, the solution is found iteratively over its independent variable (time ( $t_0, t_f$ ) for instance).

A particular algorithm is applied to the results of the previous steps of the iterations, which gives a new solution,  $x_1$ , at the current time step,  $t_1$ . This process is repeated for the whole time vector  $t = [t_0, t_1, t_2, \dots, t_f]$  and returns the corresponding solutions in the form of final vector  $x = [x_0, x_1, x_2, \dots, x_f]$ .

As mentioned above, the ode45 method is little complex than Euler method, and it takes two MATLAB programs to execute the simulation. Being said that, the results obtained by this process is much more accurate and worthy than Euler method.

## ***2.6 Atmospheric Drag***

<sup>[9]</sup> The Drag is a force which acts in the opposite direction of the motion and tends to slower down the object. Above the atmosphere, the air density is pretty low and thin, but still it can generate a strong repulsion to the motion over a period of time.

The atmospheric drag has a significant impact on spacecrafts flying under 1200 km of altitude, and in Lower Earth Orbit. Thus the major forces that act on a spacecraft in space are the gravitational attraction and the opposing drag force.

The drag equation for a spacecraft can be given as

$$F_d = -\frac{1}{2} C_d \rho A V^2$$

Where,  $\rho$  is the density of the atmosphere at that altitude,  $C_d$  is the coefficient of drag,  $V$  is the spacecraft's orbital velocity, and  $A$  is the frontal area of the spacecraft exposed to drag. The  $C_d$  can be taken as 2- 2.2 for the Lower Earth Orbit.

The density of the atmosphere also changes as the altitude is varied, and the equation for the density can be given as

$$\rho = \rho_0 e^{\left(-\frac{\text{Altitude}}{H}\right)}$$

The equation assumes normal solar conditions and the altitude is broken down into the range of groups for better estimates.

The value of the nominal density ( $\rho_0$ ) and scale height (H) can be found from the table below:

Altitude $h_{ellp}$ (km)	Base Altitude $h_o$ (km)	Nominal Density $\rho_o$ (kg/m <sup>3</sup> )	Scale Height H (km)	Altitude $h_{ellp}$ (km)	Base Altitude $h_o$ (km)	Nominal Density $\rho_o$ (kg/m <sup>3</sup> )	Scale Height H (km)
0–25	0	1.225	7.249	150–180	150	$2.070 \times 10^{-9}$	22.523
25–30	25	$3.899 \times 10^{-2}$	6.349	180–200	180	$5.464 \times 10^{-10}$	29.740
30–40	30	$1.774 \times 10^{-2}$	6.682	200–250	200	$2.789 \times 10^{-10}$	37.105
40–50	40	$3.972 \times 10^{-3}$	7.554	250–300	250	$7.248 \times 10^{-11}$	45.546
50–60	50	$1.057 \times 10^{-3}$	8.382	300–350	300	$2.418 \times 10^{-11}$	53.628
60–70	60	$3.206 \times 10^{-4}$	7.714	350–400	350	$9.518 \times 10^{-12}$	53.298
70–80	70	$8.770 \times 10^{-5}$	6.549	400–450	400	$3.725 \times 10^{-12}$	58.515
80–90	80	$1.905 \times 10^{-5}$	5.799	450–500	450	$1.585 \times 10^{-12}$	60.828
90–100	90	$3.396 \times 10^{-6}$	5.382	500–600	500	$6.967 \times 10^{-13}$	63.822
100–110	100	$5.297 \times 10^{-7}$	5.877	600–700	600	$1.454 \times 10^{-13}$	71.835
110–120	110	$9.661 \times 10^{-8}$	7.263	700–800	700	$3.614 \times 10^{-14}$	88.667
120–130	120	$2.438 \times 10^{-8}$	9.473	800–900	800	$1.170 \times 10^{-14}$	124.64
130–140	130	$8.484 \times 10^{-9}$	12.636	900–1000	900	$5.245 \times 10^{-15}$	181.05
140–150	140	$3.845 \times 10^{-9}$	16.149	1000–	1000	$3.019 \times 10^{-15}$	268.00

Table 1. Nominal Densities and Scale Heights for different altitudes.

Thus, an estimated value of drag force and acceleration can be found from these equations and are used for simulating the drag forces and reverse acceleration faced by the spacecraft considered in this research.

### 3 Methods

Electrodynamics is a vast field and there are a lot of the ways by which an electromagnetic force can be generated. The main aim of the research is to find the best electromagnet that can produce



the maximum force we can get by interacting with Earth's magnetic field or any other geomagnetic field.

The methodology that is followed is based on the magnetic field interactions and the electromagnetic force generations. The proposed spacecraft propulsion system will be in the form of an electromagnet with a magnetic field of its own. This magnetic field will be generated as a particular amount of current is passed through the system. In doing so, the system will act as a magnetic dipole and will interact with Earth's magnetic field. Since, the Earth's magnetic field distribution also acts a magnetic dipole, the interaction between Earth's magnetic field and the propulsion system's magnetic field will be in the manner of dipole-dipole interaction and follow all its laws.

The electromagnets proposed by Pulatov<sup>[11]</sup> and Engelberger<sup>[10]</sup> are simple cases of an electromagnetic coils with current running through them. To produce a stronger and effective force, the electromagnets used in this research are normal solenoids and superconducting solenoids.

### ***3.1 Electromagnetic Propulsion System with Normal solenoids***

For generating a considerable amount of force with a much smaller system, solenoids with normal conducting wires are considered. The benefit of using a solenoid is that it doesn't take much space and size and can generate a vast & stronger magnetic field than compared to simple electromagnetic coils. The solenoid, through the coils of which when some amount of current is passed, will act as an electromagnet. It will behave as a bar magnet and will have the magnetic field lines in same orientation as of the same. Since bar magnets behave like a magnetic dipole, the interaction of the solenoid and Earth's magnetic field will be in the form of dipole-dipole interaction and will follow all its rules.

The system will have its own magnetic dipole moment  $M_S$ , which will be equal to

$$M_S = \frac{NIA}{L}$$

Where, N is the total number of turns in the solenoid, I is the current, A is the area of a single coil, and L is the length of the solenoid.

The magnetic dipole moment of the propulsion system will interact with the Earth's dipole moment  $M_E$ , and the result of this interaction will be a reaction force that will act as the thrust force that will propel the spacecraft. According to Pulatov<sup>[11]</sup>, the interaction force generated in such a manner will be equal to

$$F_r = -\frac{3\mu_0 M_E M_S \sqrt{(3(\cos\theta)^2 + 1)}}{4\pi R^4}$$

$$F_t = -\frac{3\mu_0 M_E M_S \sin 2\theta}{(8\pi R^4) \sqrt{(3(\cos\theta)^2 + 1)}}$$

Where,  $F_r$  is the radial force component of the force and  $F_t$  is the tangential force component of the Thrust force in radial and tangential directions respectively. These formulas are what is suggested by Valentine Pulatov in his research paper, but it can also be derived by taking the stepwise gradient of the potential energy of the solenoid. [3]

An iron core is used in the solenoid around which a number of coils are wound to form the system. The iron core used will have its own relative permeability and thus will increase the magnetization effect of the solenoid resulting in a higher intensity magnetic field and thus a stronger force. Permeability of a substance  $\mu$  is the product of the relative permeability of the substance  $\mu_r$  and permeability of free space  $\mu_0$  and is defined in the literature survey section, effect of magnetization on materials.

$$\mu = \mu_r \mu_0$$

Being a ferromagnetic material, an iron core can greatly increase the values of magnetic field flux density, as the atoms inside the material get aligned in the same direction as that of the magnetic moment when a current is applied. [1]

As we are using iron core in the solenoid, the  $\mu_0$  value in the above equations can thus be replaced by  $\mu$ .

Therefore, the component of Thrust forces can be given by

$$F_r = - \frac{3\mu M_E M_S \sqrt{(3(\cos\theta)^2 + 1)}}{4\pi R^4}$$

$$F_t = - \frac{3\mu M_E M_S \sin 2\theta}{(8\pi R^4) \sqrt{(3(\cos\theta)^2 + 1)}}$$

Also, the spacecraft considered is planned to be in the Lower Earth Orbit (LEO) and revolves Earth in a polar orbit. Therefore, the initial height,  $h$ , of the spacecraft is considered to be 250 km or 250,000m above the surface of the Earth. The radius of the Earth,  $R_e$ , is 6378245 m. Thus, the total distance of the spacecraft from center of Earth  $R = R_e + h = 6628245$  m.

Other values of the parameters that are used are,  $\mu_0 = 4\pi * 10^{-7}$ ,  $\mu_r = 5000$  for iron, total number of turns  $N = 10^7$ , current  $I = 10$  A, Area of the solenoid loop  $A = \pi r^2$ , with  $r$ , radius of loop = 2 m and length of solenoid  $L = 4$  m. The angles considered are the latitude angle  $\theta$  for different positions in the orbit.

The values of these forces are for latitude angles 0-90 degrees in a polar orbit. Since the magnetic field of the Earth is not constant, the force values varies at different positions in the orbit. Force  $F$  is the resultant force of radial force and tangential force, and is determined by the equation,

$$F = \sqrt{F_r^2 + F_t^2}$$

Using these values, we get the graphs in Figure 1, Figure 2, and Figure 3, from MATLAB™, for thrust force values in Newtons (N) for different values of  $\theta$  in degrees.

### 3.2 *Electromagnetic Propulsion System with Superconducting solenoids*

Type II superconducting wire can be used to form a solenoid to generate the desired values of force. The benefit of such a system will be zero resistance in wire, hence zero resistive heating in the propulsion system due to current. Also, since there is no resistance, the current can stay in the wire for a very long time because of no measurable dissipation thus giving the propulsion system a very long life.

The setback of such a system is that in order to obtain high values of forces, a large value of magnetic field is required, which will destroy the superconductivity of the material. The superconductivity of a material depends on 3 basic values as discussed before; critical temperature, critical magnetic field, and critical current density. [4] The critical magnetic field of a superconductor is temperature dependent and follows the following rule

$$B_c(T) = B_c(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$

Where,  $B_c(0)$  is the critical field at 0 K, and  $T_c$  is the critical temperature for a superconducting material. These superconducting fields exist in a range, and beyond that, superconductivity is lost and material behaves like a normal conductor.

Also, the critical current density follows the following rules for lower range,  $J_{c1}$ , and higher range,  $J_{c2}$ , in order to maintain superconductivity [6]

$$J_{c1} = \frac{2B_c}{a}$$

$$J_{c2} = \frac{B_c^2 b(1-b)V_f}{4\mu_0 \xi B}$$

Where,  $a$  is the area,  $b$  is the reduced induction,  $V_f$  is the volume fraction of particles,  $\xi$  is the coherence length and  $B$  is the applied field.

For  $MgB_2$ , the values for the critical field and critical current density falls in the range of 2- 25 T and  $10^5$ - $10^7$  A/cm<sup>2</sup> respectively, depending on the temperature. The critical temperature of  $MgB_2$  is 39K. [14]

The magnetic field of a solenoid is given by  $B = \mu NI$ , where  $NI = H$  is termed as Ampere-turns. For  $B = 25T$ ,  $H$  is found to be  $3.98 \times 10^3$  A-turns. The magnetic moment of satellite, for radius 2 m, will be thus  $5 \times 10^4$  Am<sup>2</sup>. Thus, the maximum value of  $F_r$  is found to be  $7 \times 10^{-3}$  N at North Pole, whereas the maximum value of  $F_t$  is found to be  $1.1 \times 10^{-3}$  N at around 61 degrees.

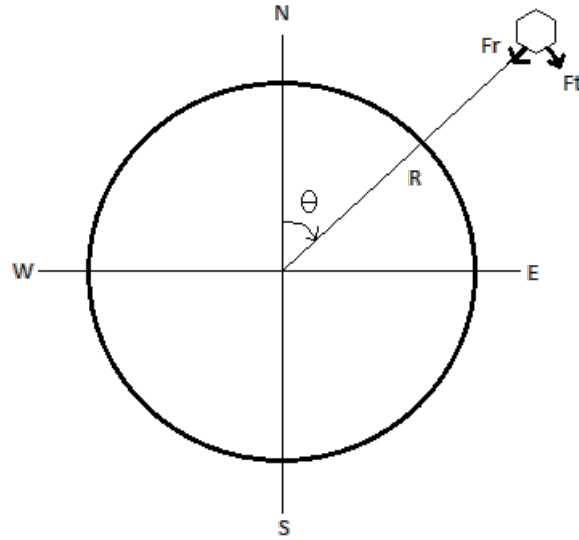


Figure 1. The position and attitude of spacecraft with respect to the Earth

### 3.3 Simulation

The mass of the system,  $m$ , is the summation of the mass of the solenoid coil, the refrigeration system, the spacecraft, and the iron core. The density of iron is  $7874 \text{ kg/m}^3$ . The mass of the iron core is calculated by multiplying the volume of the iron core to its density.

The acceleration will be the acceleration due to the propulsion system,  $a_t$ , acting in the tangential direction. This acceleration will be equal to

$$a_t = \frac{F}{m}$$

The drag force is also considered to play a vital role in the simulation. The drag force will vary at every altitude, as the radius will change. This varying drag force will provide a deceleration to the spacecraft in opposite direction. The total acceleration,  $a$ , the spacecraft will face in multiple directions are

$$a = a_t - a_d$$

Where  $a_d$  is the acceleration due to drag.

The drag force is calculated by:

$$F_d = -\frac{1}{2} C_d \rho A V^2$$

Where,  $\rho$  is the density of the atmosphere at that altitude,  $C_d$  is the coefficient of drag,  $V$  is the spacecraft's orbital velocity, and  $A$  is the frontal area of the spacecraft exposed to drag. The  $C_d$  is taken as 2.2 for the Lower Earth Orbit.

The density of the atmosphere also changes as the altitude is varied, and the density is calculated by:

$$\rho = \rho_0 e^{\left(-\frac{\text{Altitude}}{H}\right)}$$

The values of nominal density ( $\rho_0$ ) and scale height ( $H$ ) are taken from Table 1 for different altitude ranges.

Total orbital energy per unit mass,  $\epsilon$

$$\epsilon = \frac{-\mu}{2r}$$

Or

$$\frac{d\epsilon}{dt} = \frac{-\mu}{2r^2} \frac{dr}{dt}$$

Where,  $\mu = G(m_1 + m_2)$  = Gravitational constant

If thrust acceleration,  $a = F/m$ , then

$$\frac{d\epsilon}{dt} = a_t \cdot v_e$$

For low thrust, orbit is almost circular

$$v_e = \sqrt{\frac{\mu}{r}}$$

But the acceleration is variable and changes direction with the motion. So, if we assume the variable angle between the acceleration  $a_t$  and  $v_e$  to be  $\alpha$ , this gives

$$\frac{d\epsilon}{dt} = a_t \cdot v_e = a_t v_e \cos(\alpha)$$

This gives,

$$\frac{d\varepsilon}{dt} = a_t \cdot v_e = a \cdot \sqrt{\frac{\mu}{r}} \cos(\alpha)$$

Or

$$\frac{dr}{dt} = \frac{2a_t \cos(\alpha)}{\sqrt{\mu}} r^{3/2}$$

The angle  $\alpha$  can be found by the following formula

$$\alpha = \tan^{-1} \left( \frac{F_t}{F_r} \right)$$

Also, the change in angle with respect to time will be equal to the angular velocity,  $\omega$

$$\omega = \frac{d\theta}{dt}$$

Where,  $\theta$  is the latitude angle over which the force changes.

This angular velocity,  $\omega$ , for a circular orbit will also be equal to

$$\omega = \frac{v}{r}$$

Thus, all these formulas are written down in the form of a MATLAB code and run for a number of iterations to provide graphs for Force, Velocity, and Altitude change over an hour, a day, a month, and a year for a number of suitable time steps.

### ***3.4 Assumptions***

The following assumptions are taken for the better approximation of results for the electromagnetic propulsion system and the simulations:

- The orbit is assumed to be polar and circular, and initial altitude as Lower Earth Orbit (250 km).
- Since the problem is based in strictly polar orbit, the problem statement can be 2 dimensional.
- The propulsion system is assumed to be using a solenoid of small diameter so that a good understanding of its application can be achieved.
- The spacecraft is assumed to be in the orbit when the propulsion system is fired.

The results thus obtained for all the methods applied are in the form of graphs in the next section.

## 4 Results

Below the different results for the calculations can be seen:

### 4.1 *Electromagnetic Propulsion System with normal solenoid*

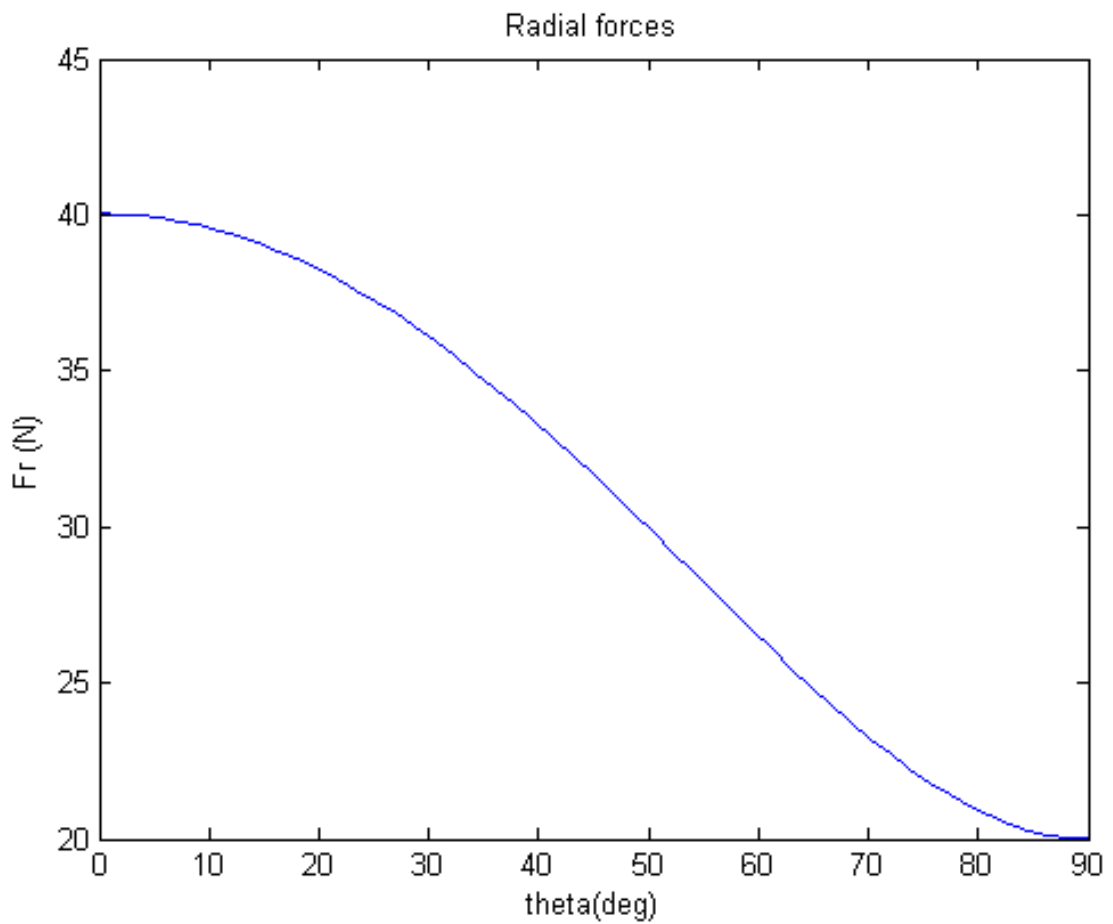


Figure 2. Radial forces,  $F_r$ , in Newtons, for superconducting solenoid, Generated in MATLAB™

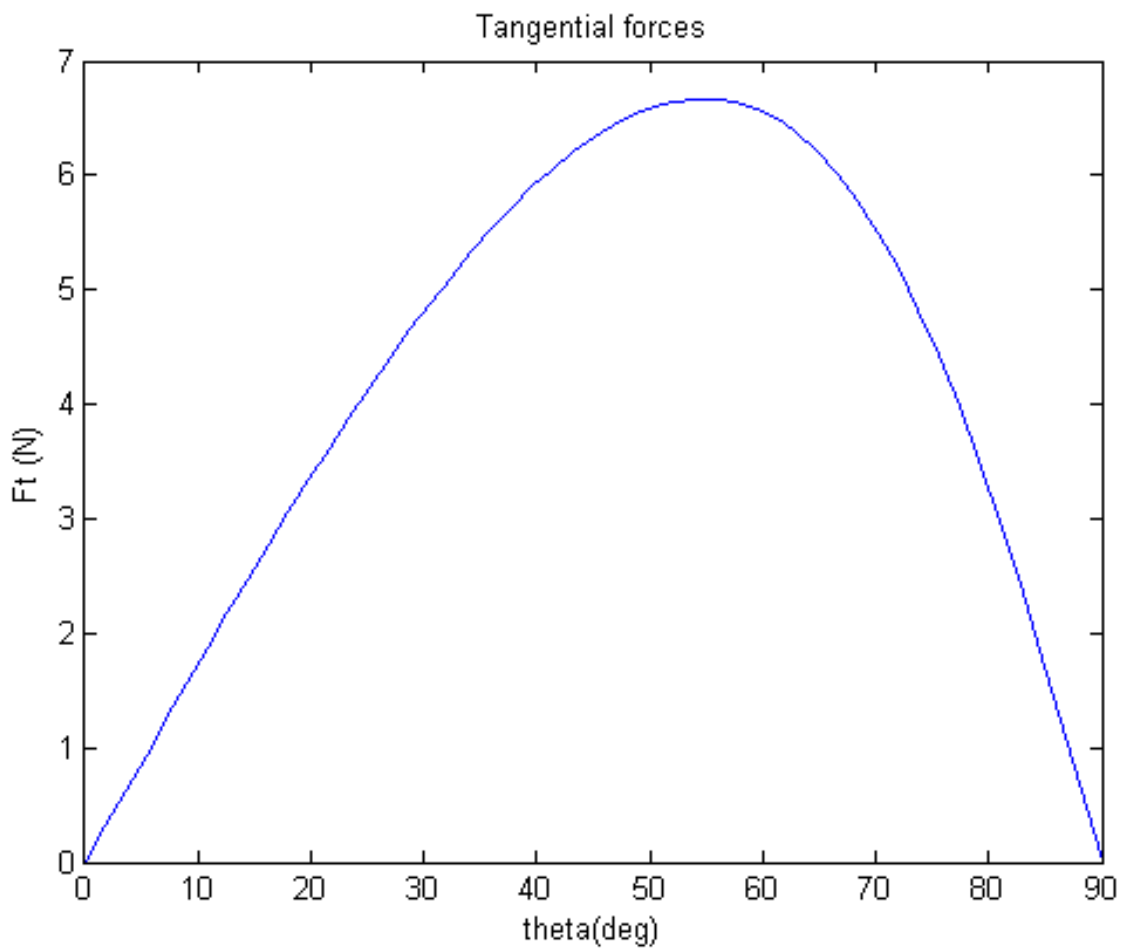


Figure 3. Tangential forces,  $F_t$ , in Newtons, for superconducting solenoid, Generated in MATLAB™



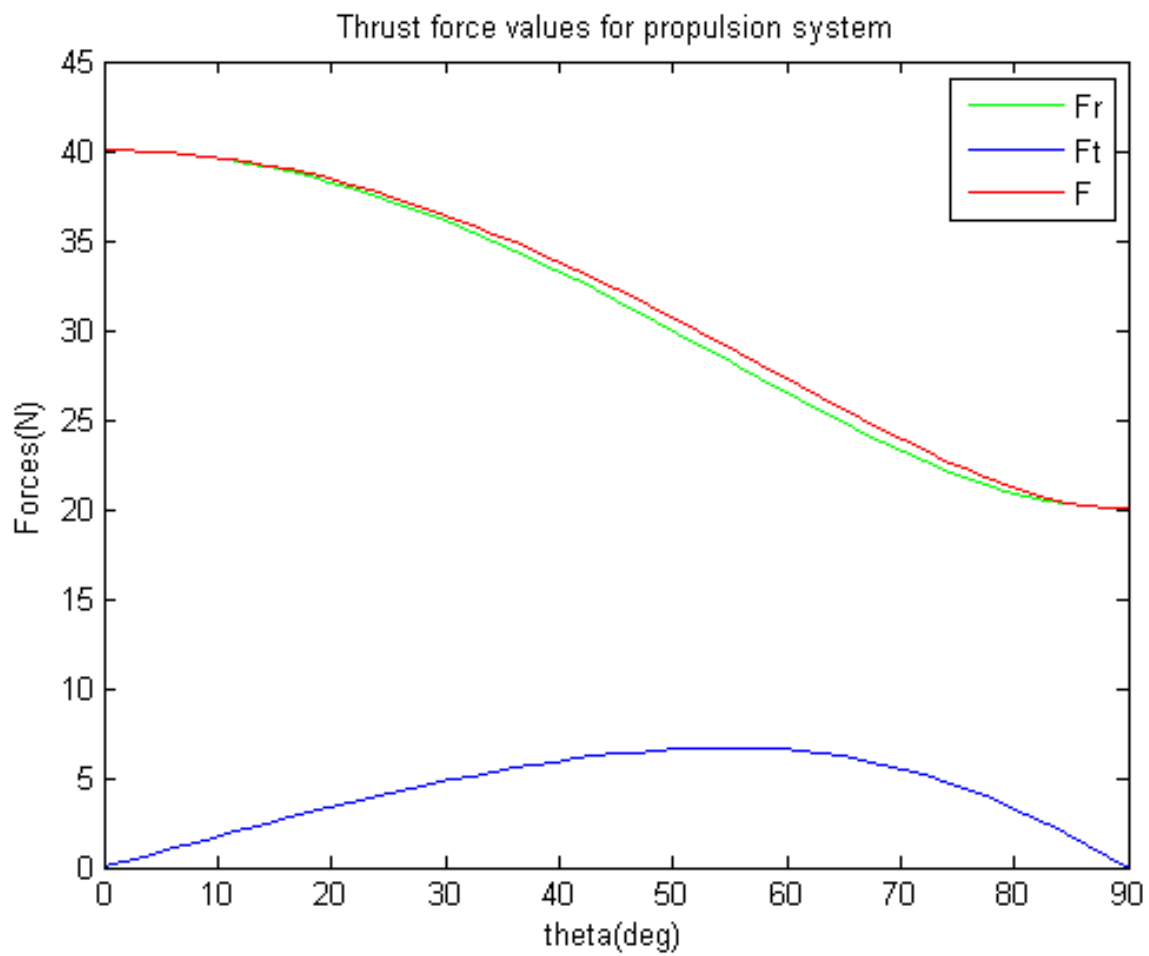


Figure 4. Resultant Force, F, in Newtons, for superconducting solenoid, Generated in MATLAB™

## 4.2 Electromagnetic Propulsion System with superconducting solenoid

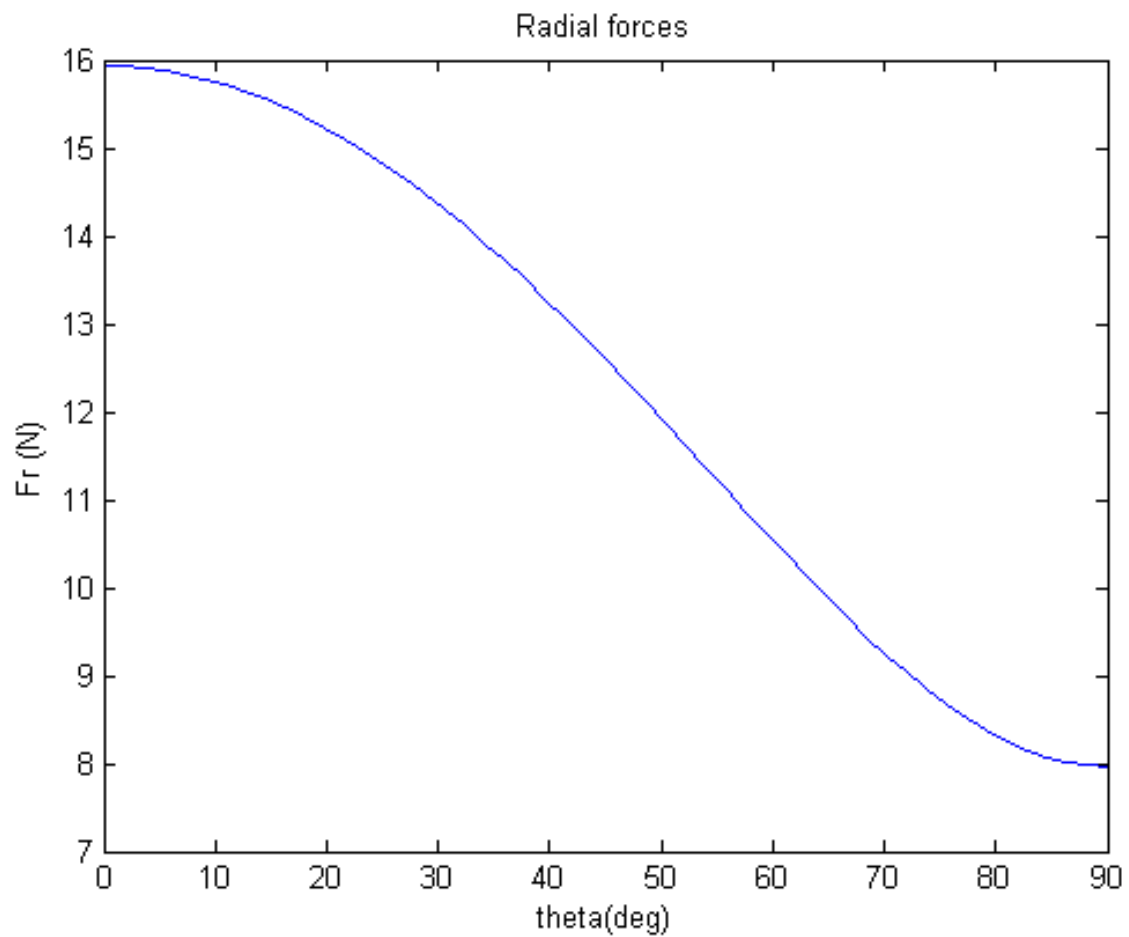


Figure 5. Radial forces,  $F_r$ , in Newtons, for superconducting solenoid, Generated in MATLAB™

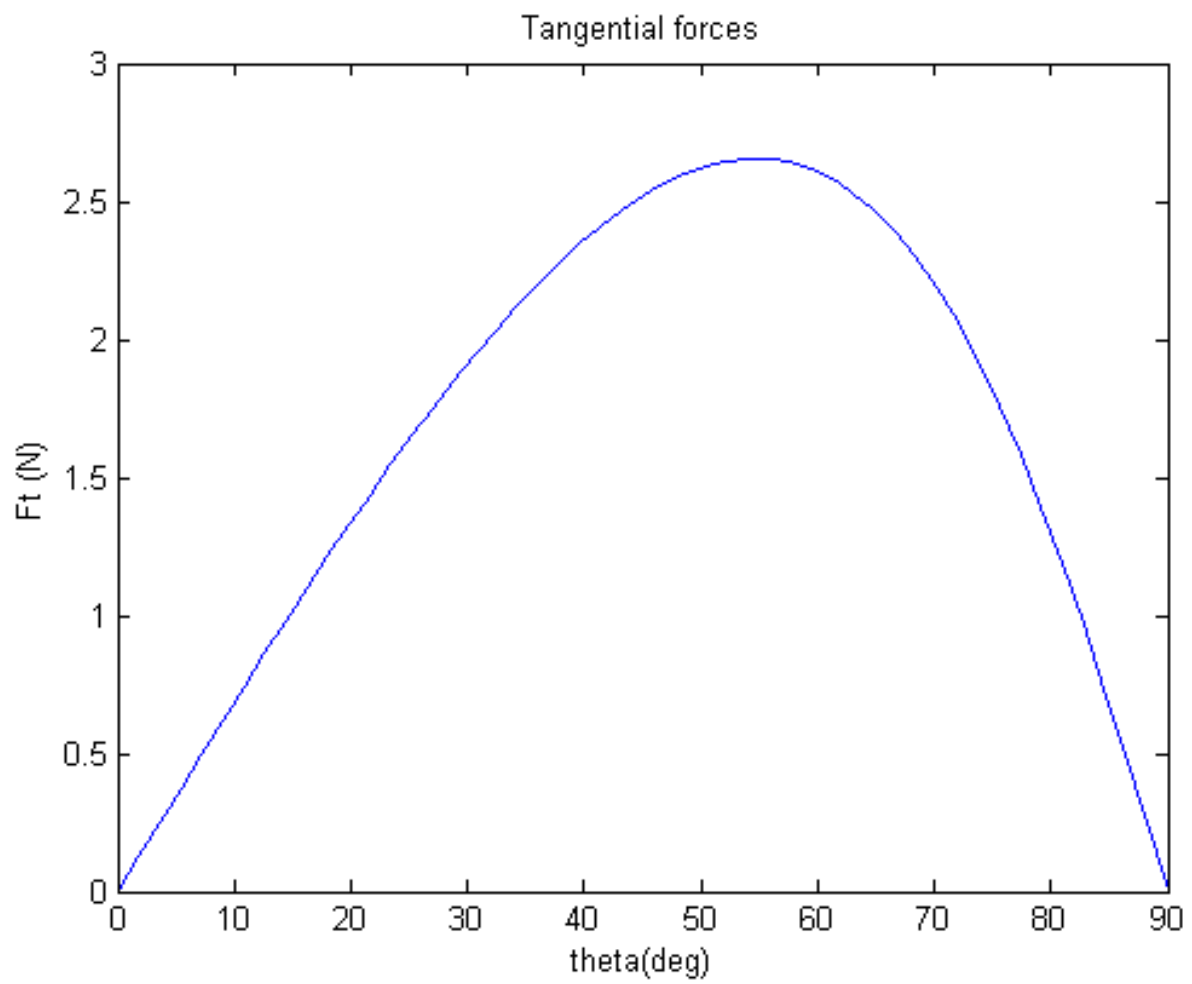


Figure 6. Tangential forces,  $F_t$ , in Newtons, for superconducting solenoid, Generated in MATLAB™

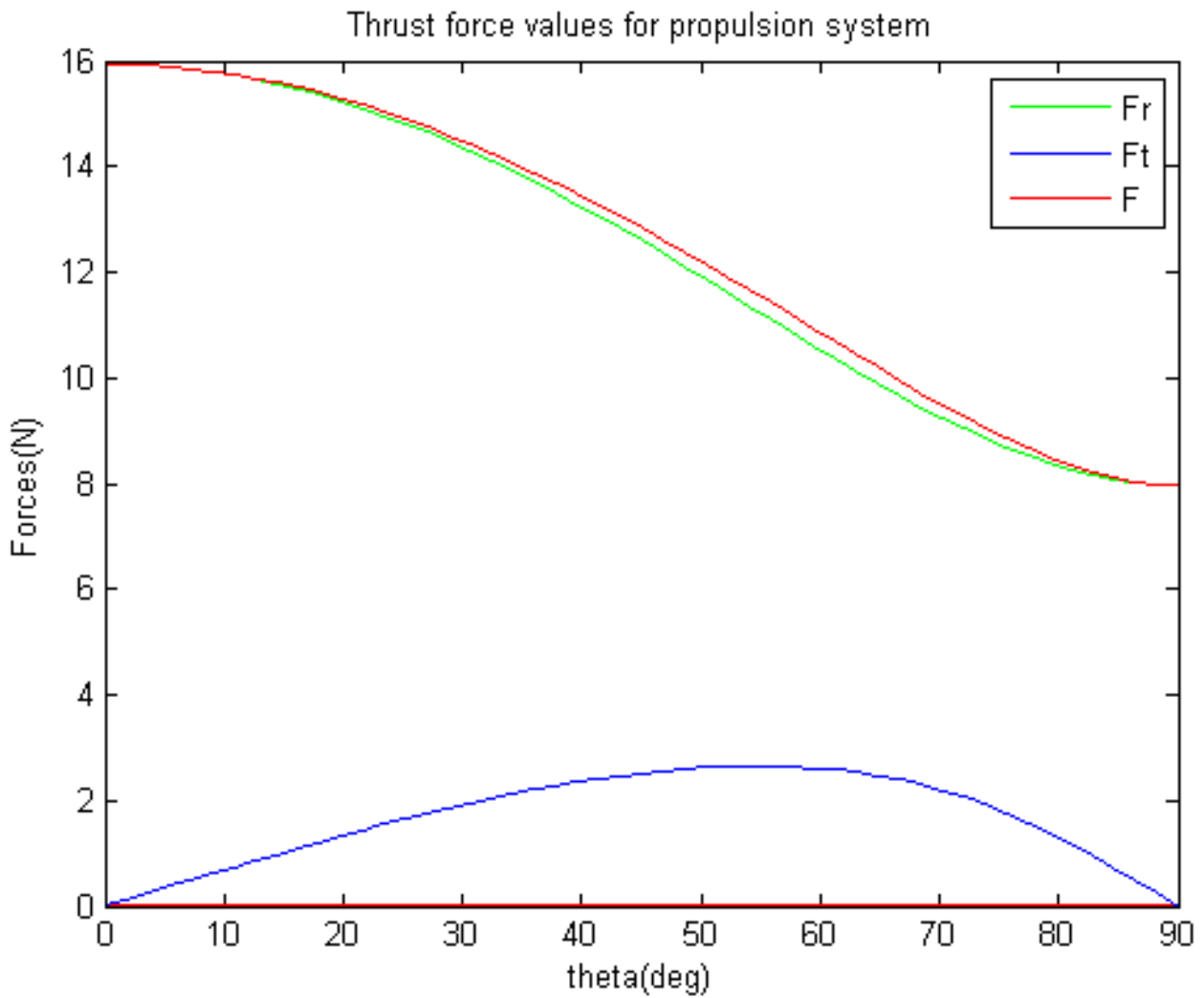


Figure 7. Resultant Force, F, in Newtons, for superconducting solenoid, Generated in MATLAB™

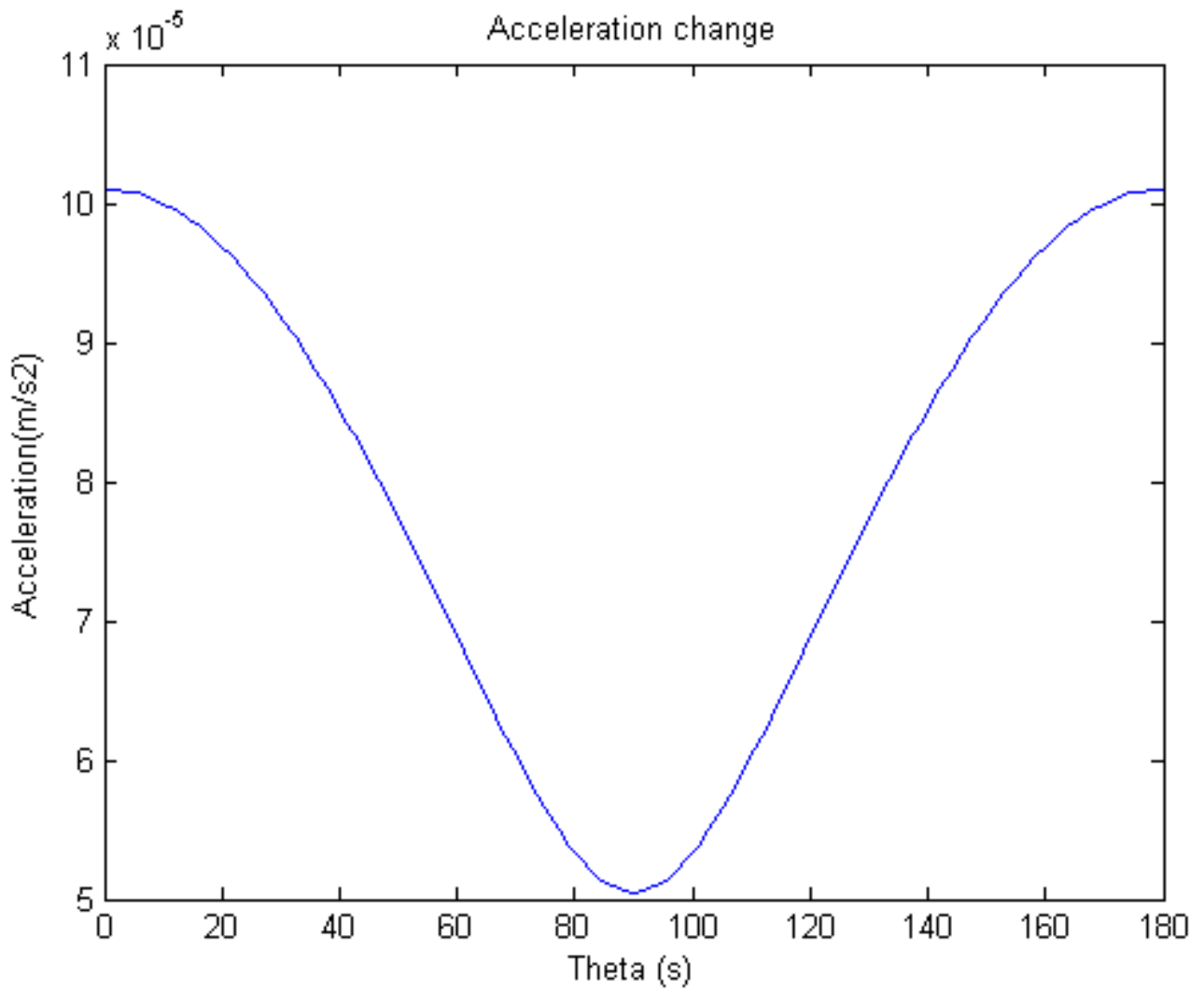


Figure 8. Change in acceleration

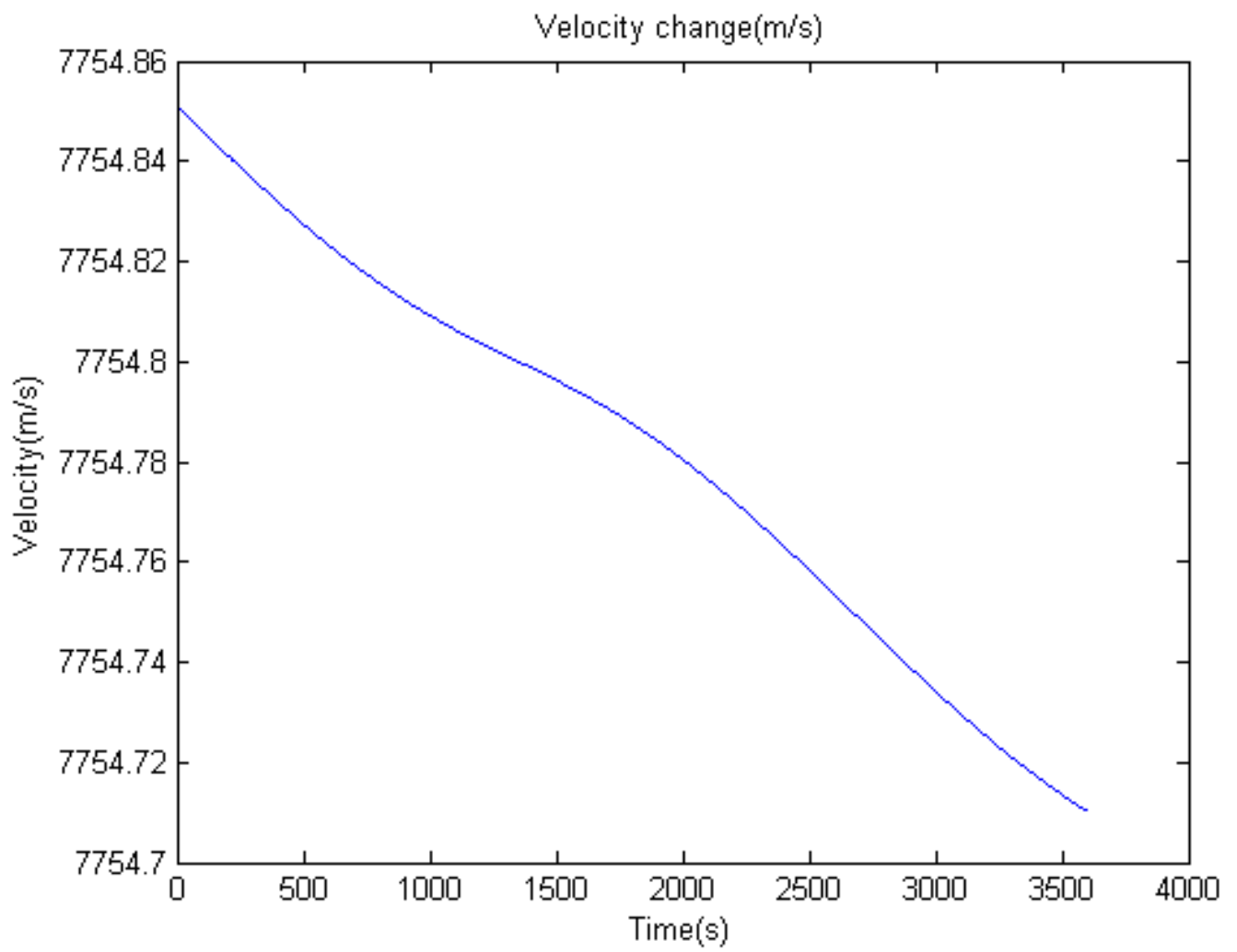


Figure 9. Velocity in an hour

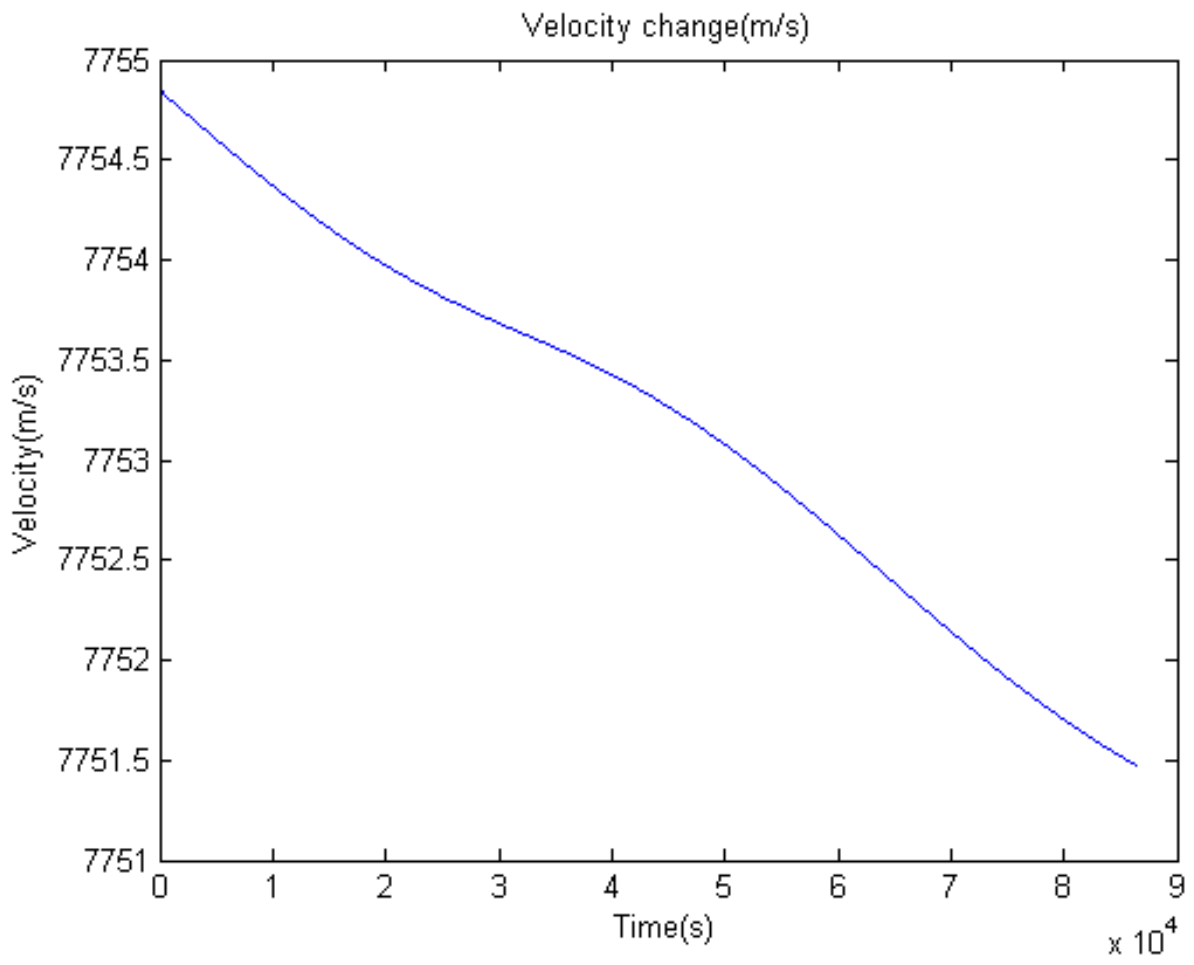


Figure 10. Velocity in a day

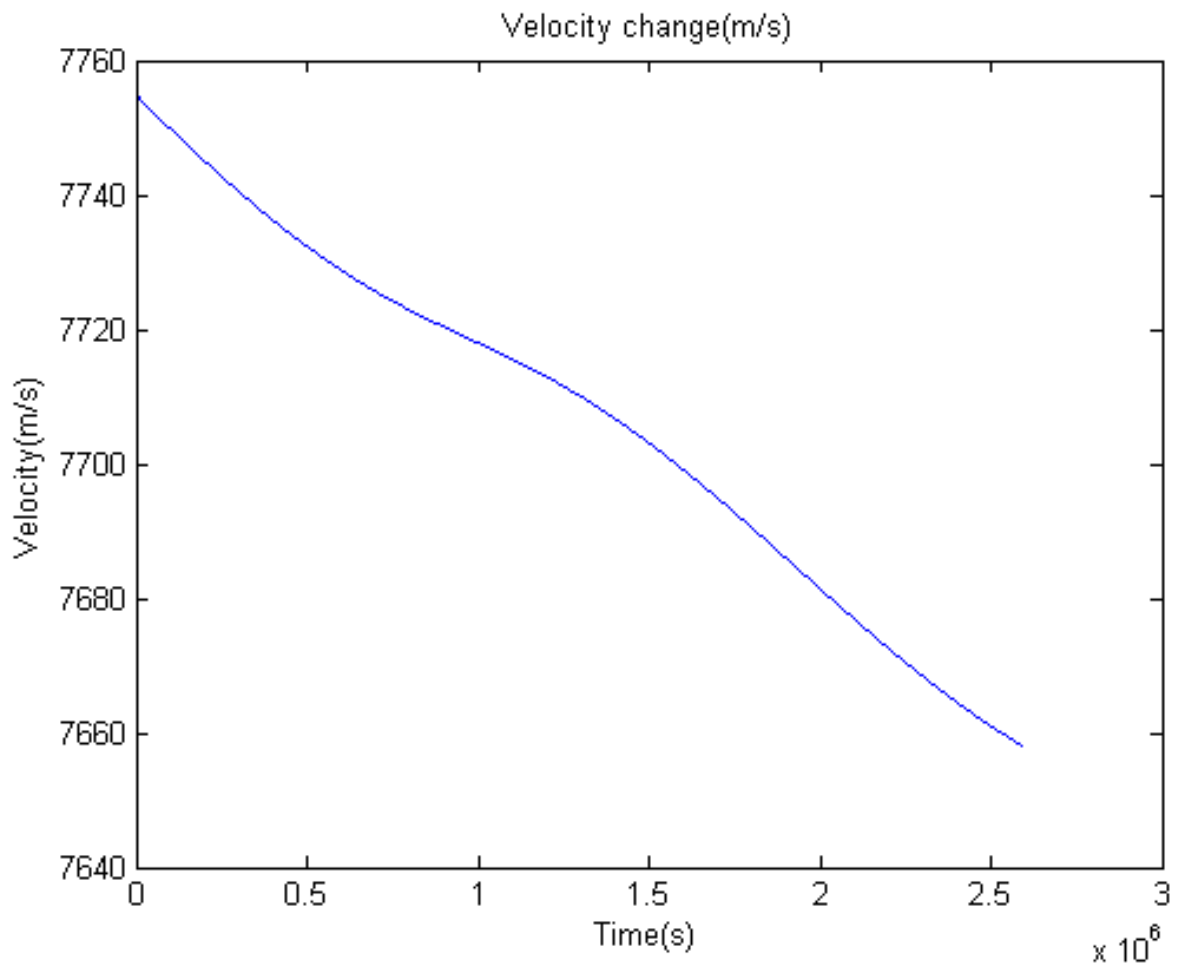


Figure 11. Velocity in a month



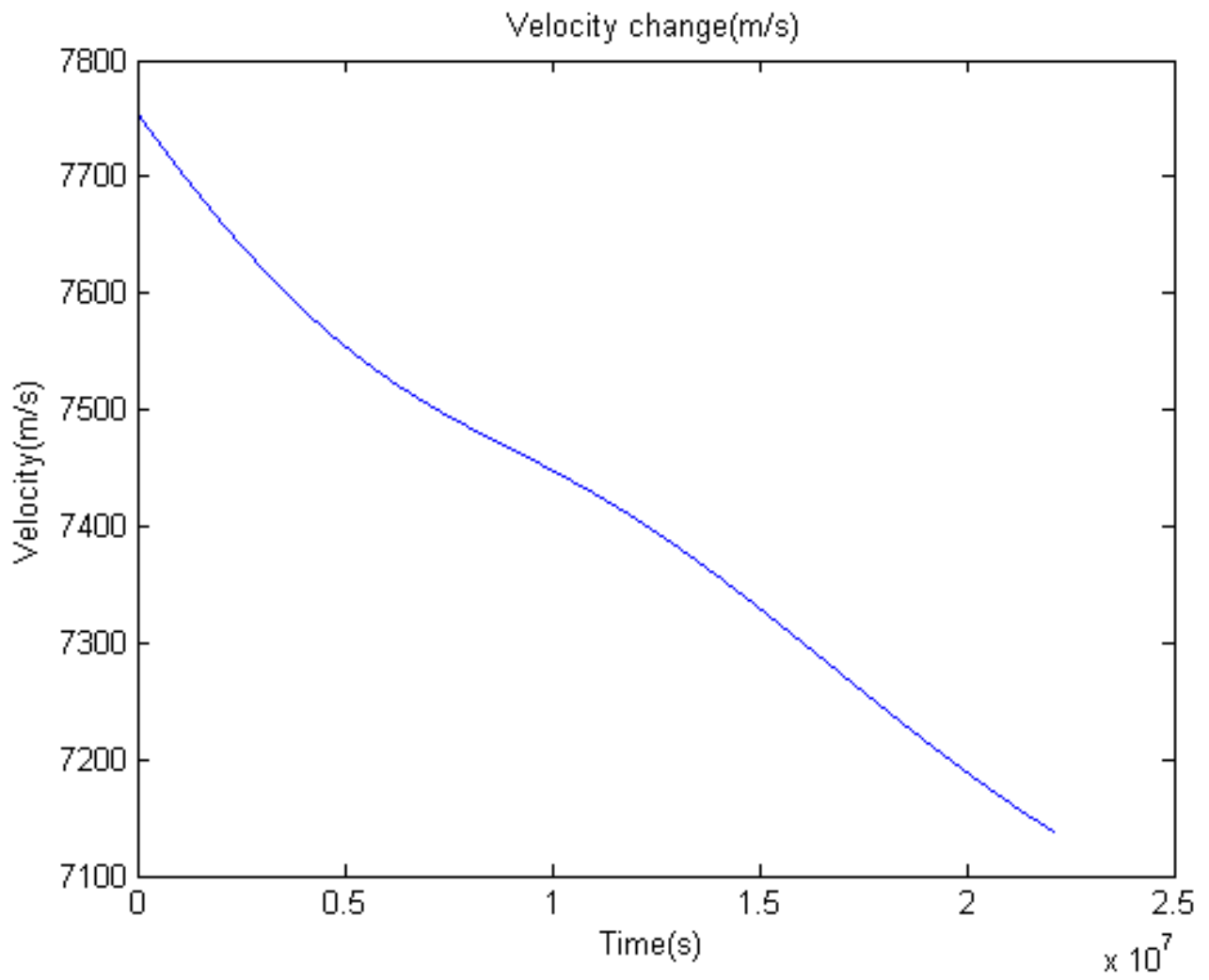


Figure 12. Velocity in a year

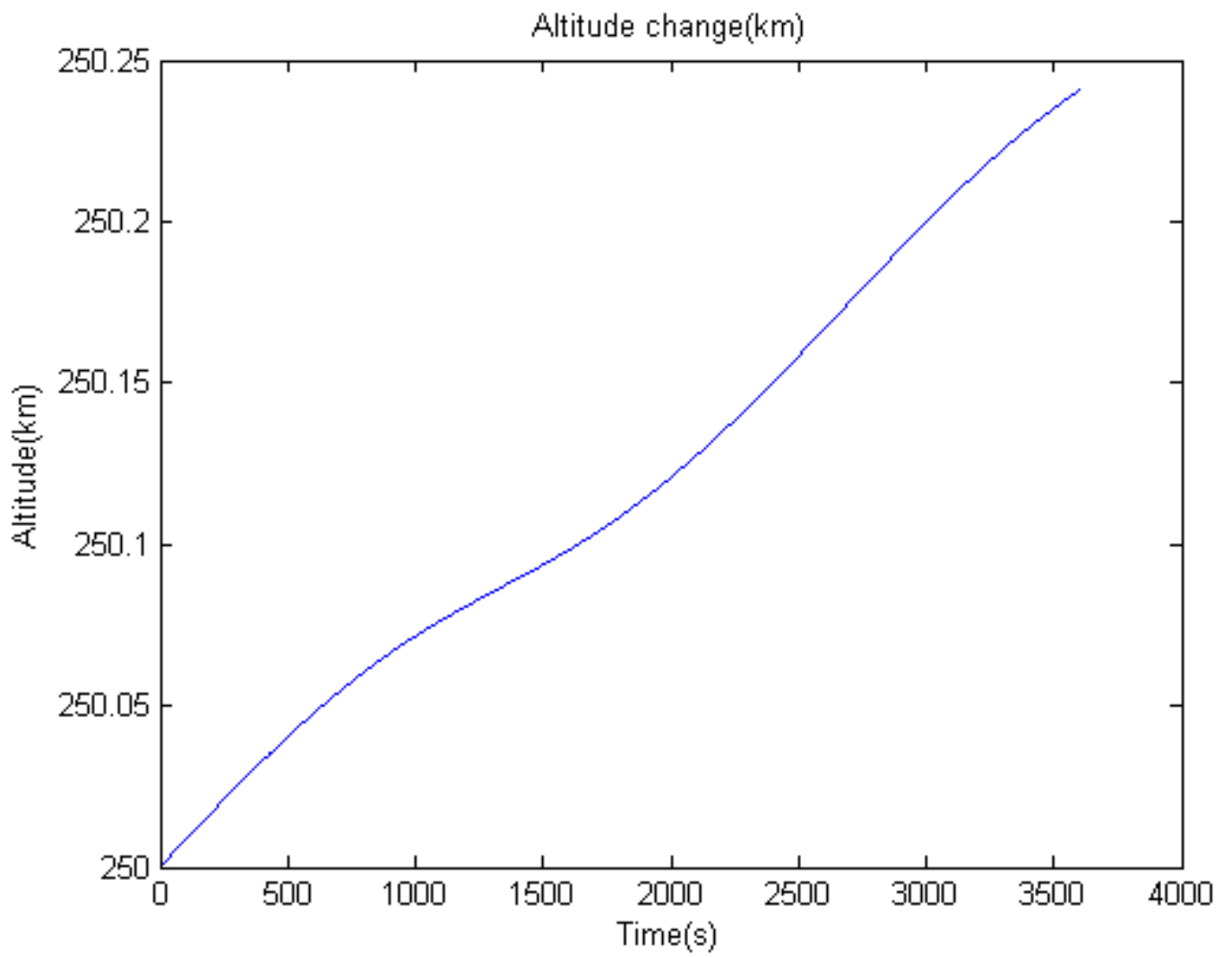


Figure 13. Change in altitude due to propulsion system in an hour

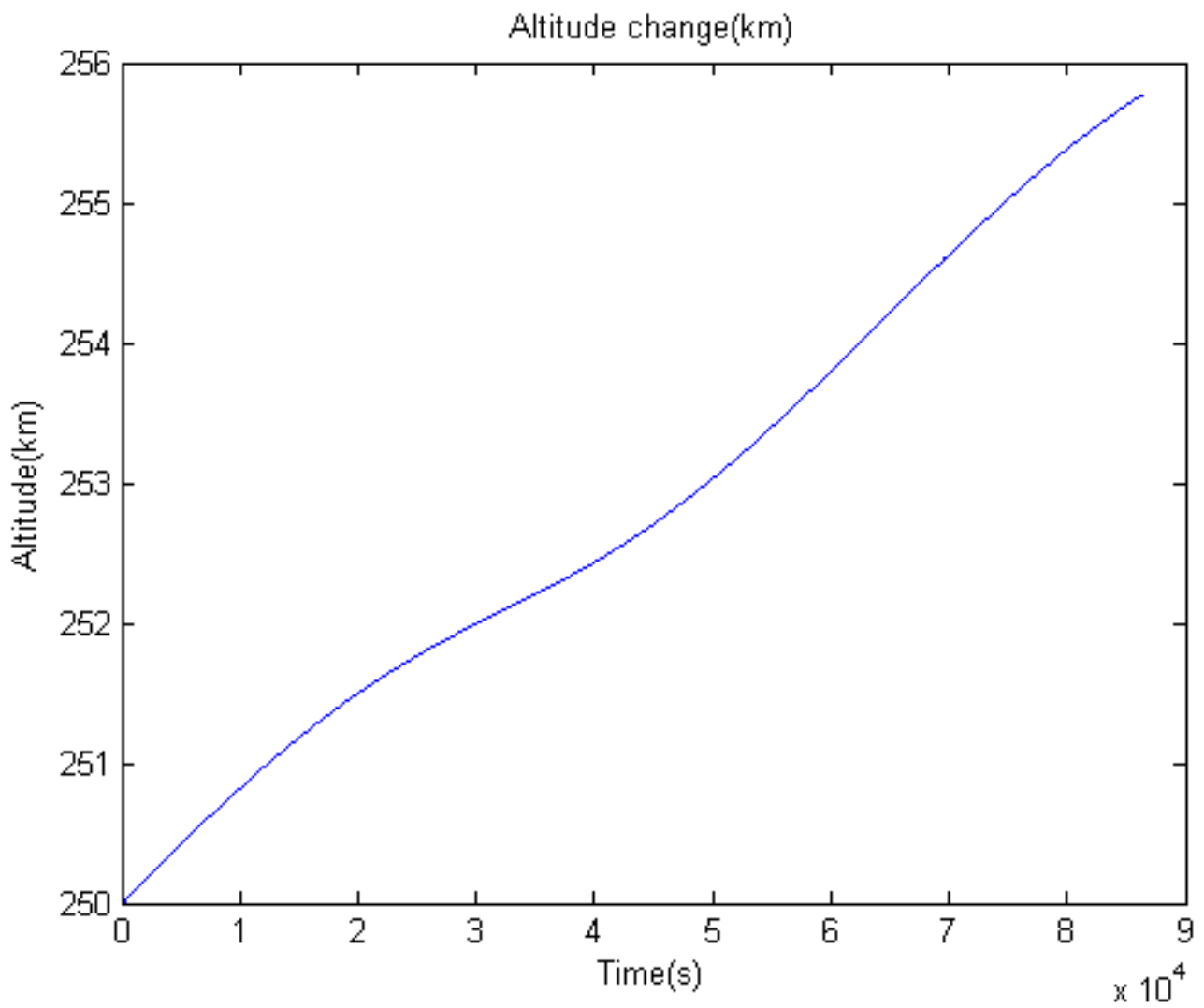


Figure 14. Change in altitude due to propulsion system in a day

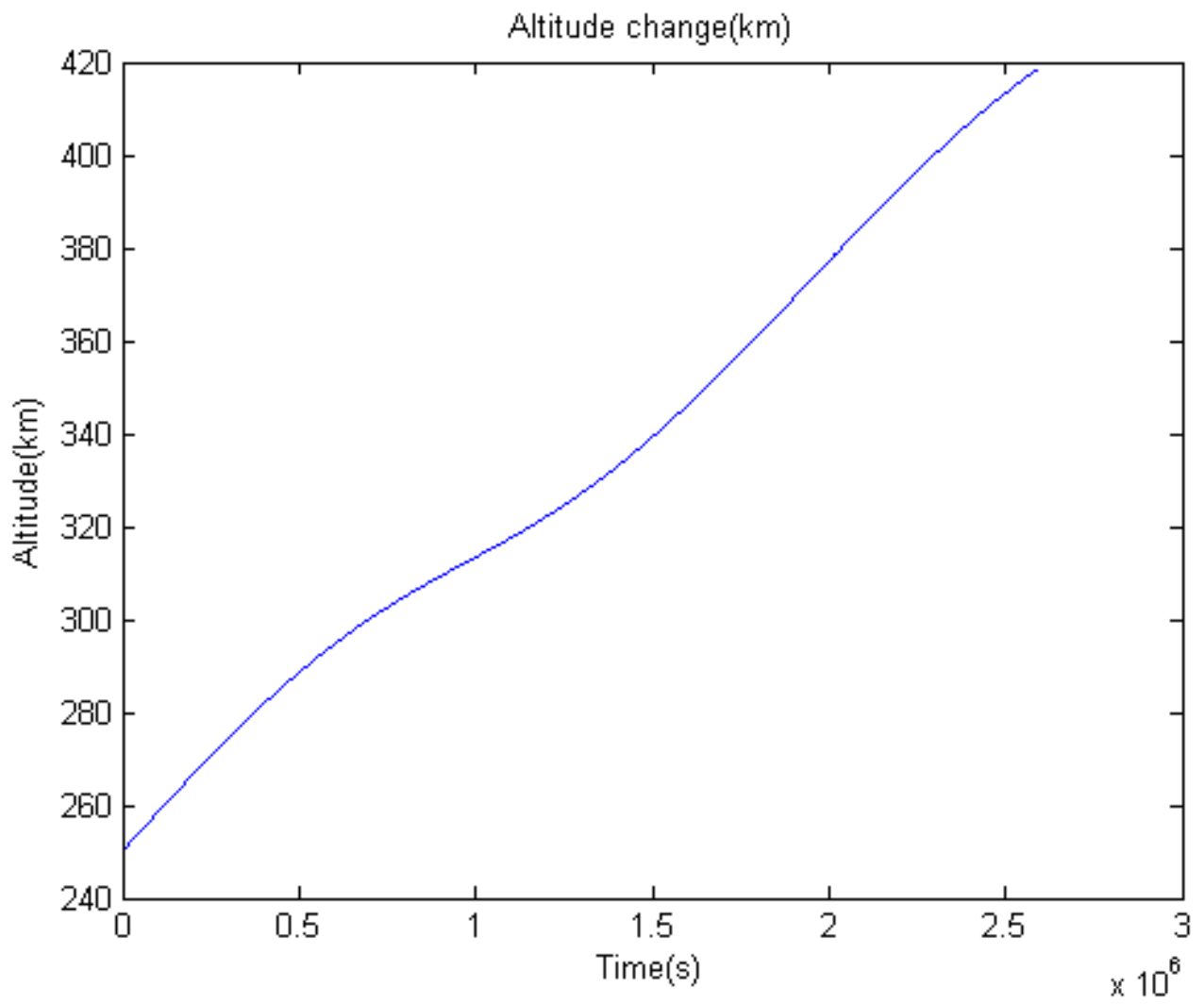


Figure 15. Change in altitude due to propulsion system in a month

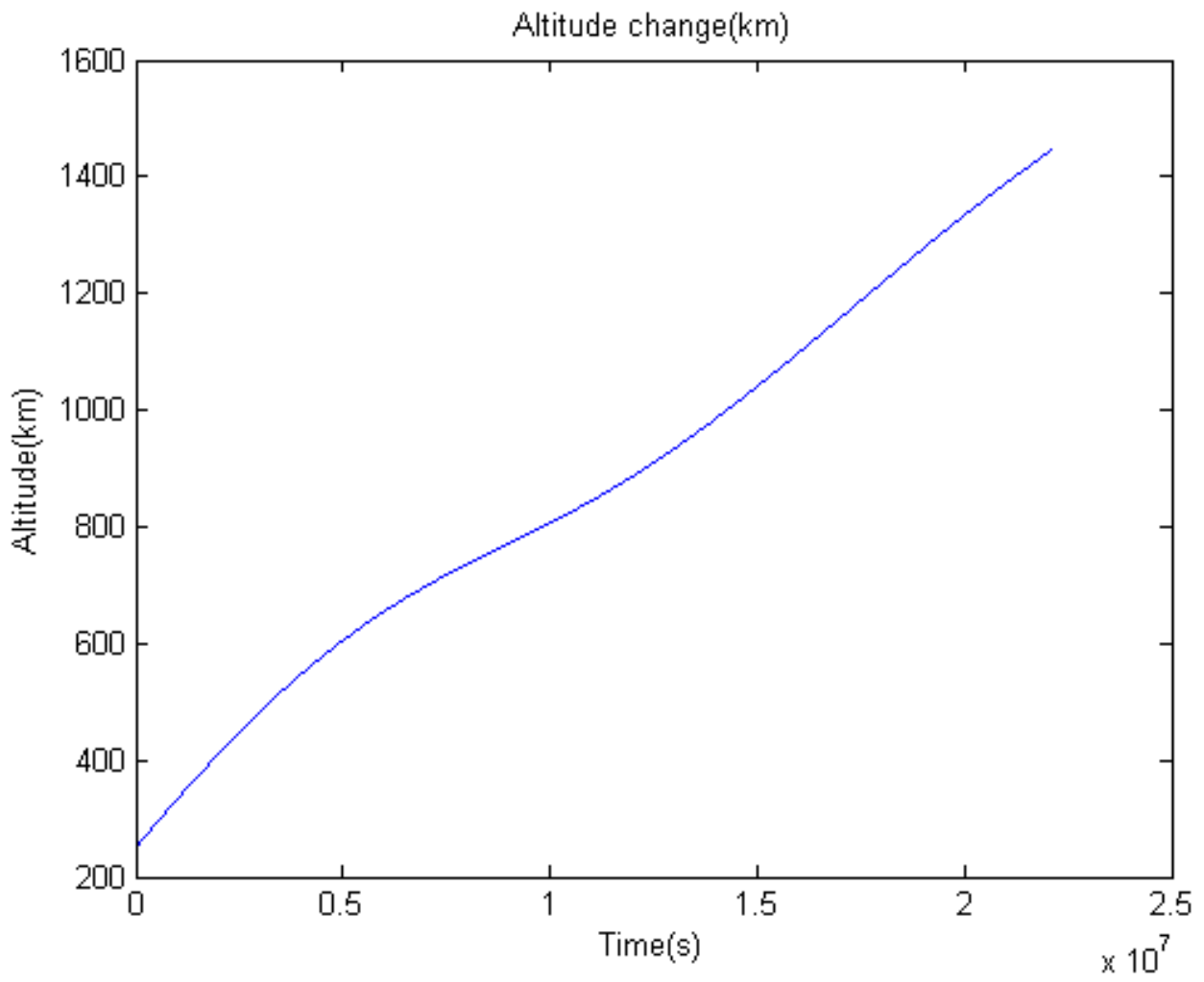


Figure 16. Change in altitude due to propulsion system in a year

## 5 Discussion

The results for the specified problem statement were found in the previous section with underlying assumptions. They are discussed in detail below.

### 5.1 Thrust Force

The thrust forces were found for the electromagnetic propulsion system in the radial and tangential directions. Their values can be seen from the graphs shown in figures 2 – 7. The radial force for electromagnetic propulsion system varies from 0 N to approximately 40 N in the first quadrant for different latitude angles. Similarly, the tangential force varies from 0 N to approximately 6.7 N.

The maximum value of the Force  $F$ , can be seen at the poles ( $\theta=0$  degrees), and minimum value at the equator ( $\theta=90$  degrees). It is because of the variable alignment of magnetic moment of the planet with the same of the spacecraft. Also, the direction of the force changes sign in alternate quadrants. This issue can be countered by manipulating the direction of the current and angle of solenoid with respect to the magnetic field lines, and is out of the scope for this research.

For a superconducting solenoid, the maximum thrust forces in the radial and tangential directions are 15.93 N and 2.65 N respectively. The maximum resultant forces for the normal electromagnetic propulsion system and superconducting propulsion system are 40.03 N and 16.67 N respectively. The difference in these values is because of the bigger size of the propulsion system with superconducting solenoids and lower critical magnetic field values of the superconductor used. Also, the bigger radius of the propulsion system is considered for the research to show that these kind of systems can be directly used for future large sized spaceships. From the graphs, it is found out that the forces obtained are comparable to the present day MPD thrusters and other electric engines.<sup>[15]-[17]</sup>, and is shown in Table (2).

<b><u>Type of Thrusters</u></b>	<b>Electromagnetic propulsion (normal solenoid)</b>	<b>Electromagnetic propulsion (superconducting solenoid)</b>	<b>Hall thrusters (Ion engine-Xe)</b>	<b>Dual-stage 4 grid DS4G (Xe)</b>	<b>Arcjet Thruster</b>	<b>Vasimir-VX Thruster</b>	<b>MPD Thrusters</b>
<b><u>Thrust generated</u></b>	40.03 N (max) $F_r= 40.03$ N $F_t= 6.67$ N	15.93 N (max) $F_r= 15.93$ N $F_t= 2.65$ N	0.5 N (max)	2.7 N	2 N	5 N	2.5-25 N

Table 2. Comparison of proposed Electromagnetic Propulsion Systems with other present day engines for spacecrafts.

The thrust values for the propulsion system is also bigger than the thrust force values of the electromagnetic propulsion system proposed by Engelberger and Pulatov for a much smaller size and lower current input values.

The Engelberger's system, which uses same technology, generates around 50 N of maximum thrust in the radial direction, but the size for such a system is massive (10000 km in radius), which is much bigger than the size of the electromagnetic system proposed. Also, the required current is 1000 A, which is 100 times larger than the proposed propulsion system.

Also, the current requirement to generate the Ampere cross force in Pulatov's system is 40000 A at the altitude of 250 km. The force generated by these values is 1.12 N, which is pretty low as compared to the proposed magnetic propulsion system.

## ***5.2 Acceleration and Velocity***

The values of acceleration over different values of latitude angles can be seen in the figure 8. The variable values of acceleration is due to the varying magnetic field. Since the mass of the propulsion system is very high, the values of acceleration are pretty low for each angle and in the range of  $10^{-5}$  m/s<sup>2</sup>

The velocity of the spacecraft is plotted in the figures 9-12. This velocity is the orbital velocity and accounts for both the orbital velocity due to the gravitational acceleration and the velocity due to the acceleration provided by the electromagnetic propulsion system. Its values are found by simulation in MATLAB using Euler method and the problem statement is initial value problem. The graphs of velocities are presented for an hour, a day, a month, and a year for initial velocity 0 m/s. It can be seen from the graphs that the velocity values of the spacecraft system is decreasing over the time, which makes sense because the orbital velocity is inversely proportional to the altitude of the spacecraft. Thus, as the altitude of the spacecraft increases over time due to the propulsion system, the velocity of the system will decrease.

$$v_e = \sqrt{\frac{\mu}{r}}$$

## ***5.3 Radius or Altitude of the spacecraft***

The values of the altitude changes is shown in figures 13- 16. The radius values shown are the altitude of the spacecraft above the surface of the Earth and shown in kms. Four diferent graphs are shown for the altitude changes in an hour, a day, a month, and a year. From the graphs it can be seen that the altitude of the spacecraft increases constantly throughout its life time. The opposing force that tries to decrease the altitude of the spacecraft is the drag force. The drag force also changes with altitude and gets less impactful as the altitude rises.

The change in the altitude is very low for each time step but the change is significant over a period of time. Thus, this kind of propulsion system is a viable option for orbit transfer and if the technology in theoretical physics is improved, the change will be much faster and instantaneous.

From here, we can also say that a certain altitude can be maintained for a spacecraft if the value of thrust is lowered down. For instance, say the radius and the length of the solenoid is lowered down to 0.001 m, 0.0001 m, and 2 m, the change in altitude can be seen as in the following graphs

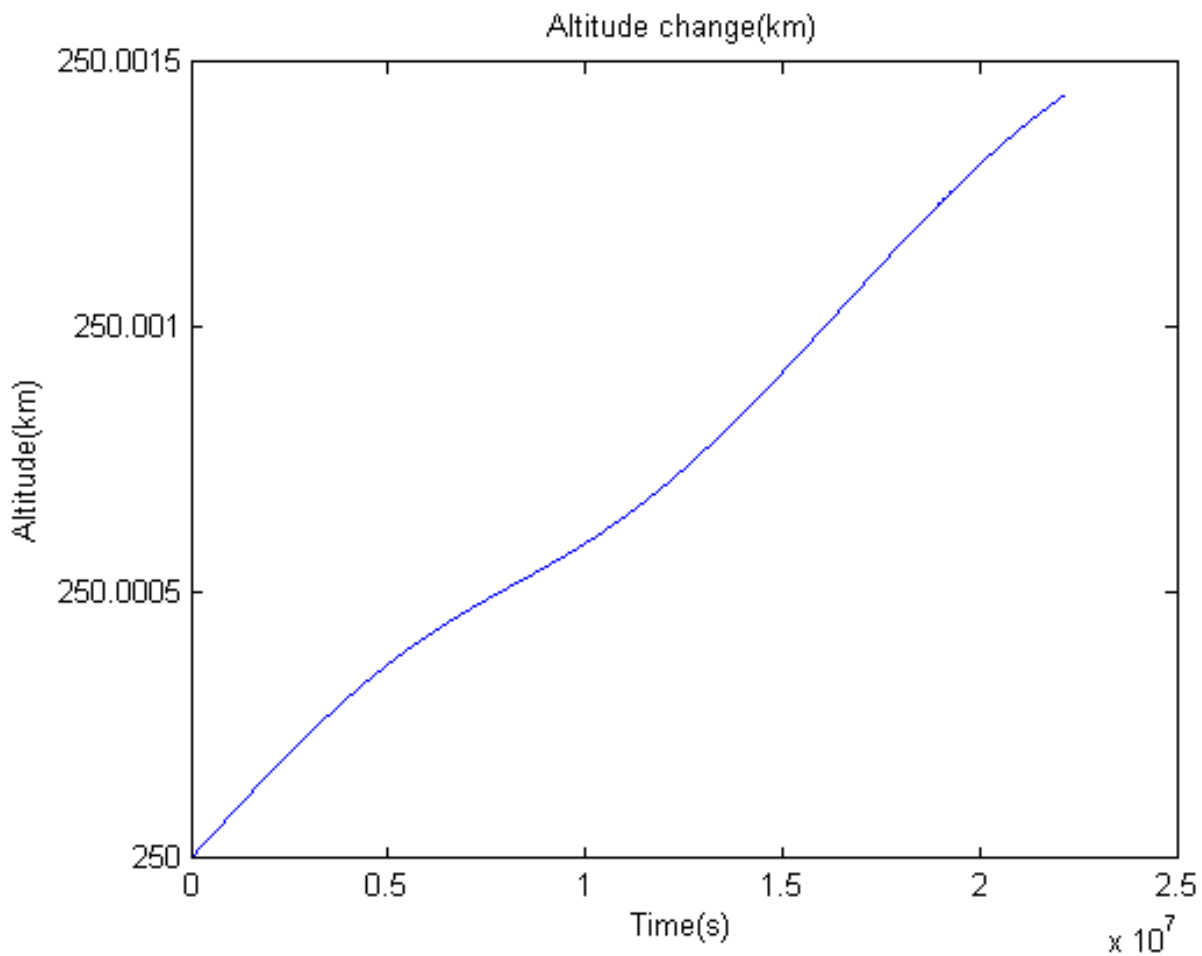


Figure 17. Altitude change over a year for solenoid; Radius  $r = 0.001\text{m}$ ,  $L = 2\text{ m}$



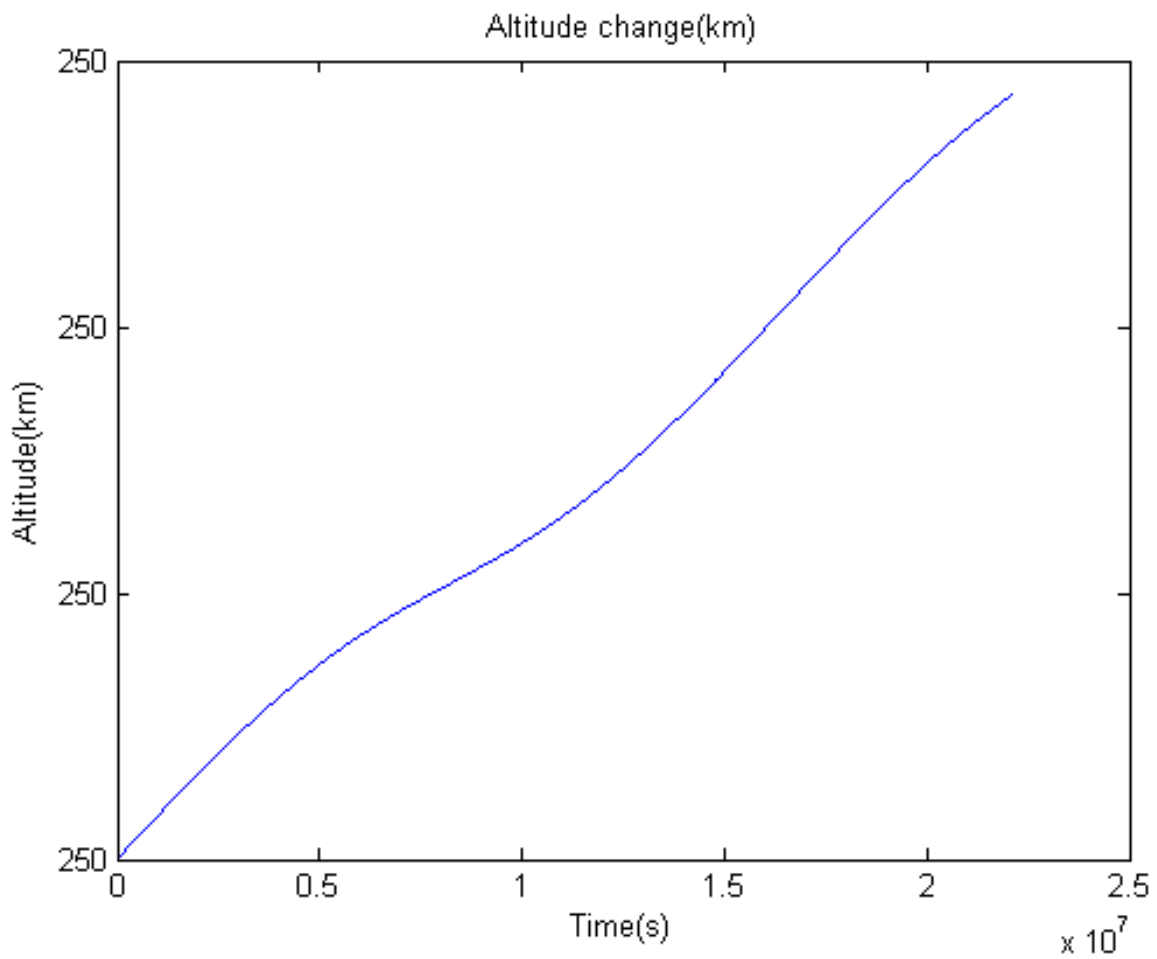


Figure 18. Altitude change over a year for solenoid; Radius  $r = 0.0001\text{m}$ ,  $L = 2\text{ m}$

The above graphs are for solenoid using an iron core. If we don't use iron core, the size of the propulsion system will increase a bit. With increase in size we also save on the number of turns that need to be applied to the solenoid.

The best configuration for such an application found yet is for a solenoid having radius 1m, length 2 m, number of turns  $10^3$ , and with no iron core.

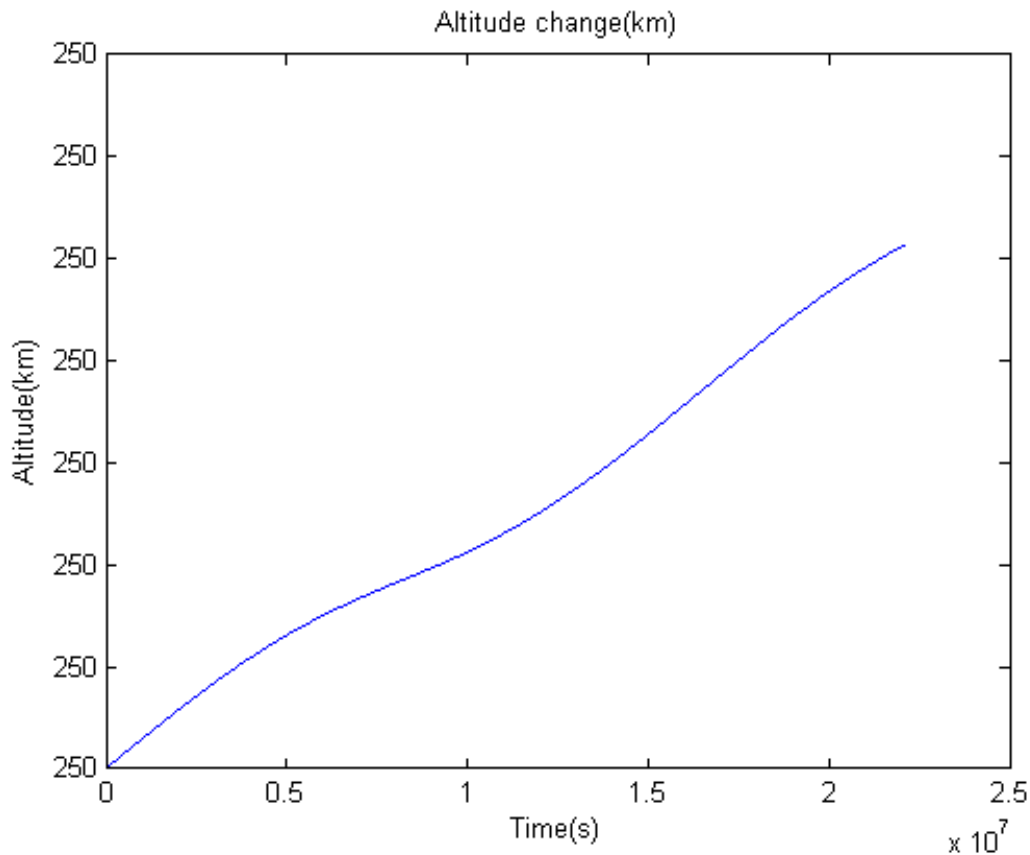


Figure 19. Altitude change over a year for solenoid; Radius  $r = 1\text{ m}$ ,  $L = 2\text{ m}$ , no iron core

Thus, from this point, we can also say that the proposed propulsion system can also be used for the purposes of stationkeeping as to keep the orbit of large sized spacecraft from decreasing.

#### 5.4 Challenges

The total number of suggested turns in the solenoid is very high and can be demotivating for some people. But very thin wires of thickness of radius in the range of  $0.0002\text{ cm}$  can be made easily in the present day technology. In 1960, 15 kilogauss of magnetic field was generated by a superconducting solenoid having  $10^4$  turns per cm ( $10^6$  turns/m).<sup>[18]</sup> Thus, with the modern technology and numerous engineering techniques, the proposed number of windings can easily be applied on the solenoids.

Resistive heating may occur in the conducting wire due to the high values of the magnetic field. The resistive heating in the wire is directly proportional to the square of the passed current, therefore the applied current should be as low as possible in order to minimize the resistive heating. The solar panels, being most common power source in space, also has very small power output,

that is why 10 A current is used for the calculations of thrust force, which can be easily obtained. The solar cells can be attached in series or parallel configuration as to generate higher voltage or current values. The other techniques through which the heating can be minimized are by using resistive conductors like copper, using pulsed currents, or integrating cooling techniques.

The general amount of power generated by a solar panel, measuring 20 x 44 inches, is 60 W. <sup>[19]</sup> The voltage generation is around 12 V. So the current generated by one panel will be

$$I = \frac{P}{v} = 5 A$$

Thus, for generating around 10 A, two solar panels of the size mentioned above will be required, connected in series, which are very easy to acquire and maintain.

High hoop stresses may also be generated if normal solenoids are used. The Lorentz force between the solenoid windings  $F = J \times B$  can burn the solenoid if high values of current is passed to generate magnetic field. The radial stresses and axial stresses will be formed at the circumference and at the end of the solenoid respectively. Thus, it will be a job for the electrical or mechanical designer to ensure that the radial stresses are compressive, instead of tensile, so that the hoop stresses are reduced. <sup>[20]</sup> Mechanical clamps and very strong conductors can be used further to provide strength to the system. <sup>[22]</sup>

The other important thing to keep in mind is the amount of magnetic field generated by the system. The magnetic field such generated can be very harmful for the human beings and can create a lot of interference for the onboard computer systems. One suggested way to counter this problem is by using superconducting body, surface, or shield for the spacecraft. One of the most important properties of the superconductors is that it can repel the magnetic field from its surface and the process is discussed earlier. The effect such produced is called as Meissner effect. With the surface of the spacecraft repelling the magnetic field from the propulsion system outside, the humans and the onboard systems will be safe and secured!

## 6 Conclusion and Recommendations

The thrust values generated by the proposed Electromagnetic Propulsion System are good and very comparable to the electric thrusters of present day. The applications of such a propulsion system are spacecraft propulsion, orbit transfer, and stationkeeping. The spacecraft equipped with such a propulsion system can maintain its attitude for a long time if superconducting solenoids are used!

The propulsion system has many advantages over its counterparts like long life, self-sufficiency, low power requirements, ecological purity, etc. It is also challenging to build such a system because of hoop stresses, large size, and constraints in applied magnetic field, applied current and temperature. These challenges can be encountered by employing modern engineering

techniques, and using various processes as suggested by some of the papers, through which a smaller size of solenoid can be made to generate same levels of magnetic field.

One such technique is to use rotating superconductors which can generate some level of magnetic field on its own. According to Hildebrandt <sup>[14]</sup>, the magnetic field of a superconducting shell depends linearly on its angular velocity,  $\omega$

$$B = -\left(\frac{2mc}{e}\right)\omega$$

The paper by Tajmar<sup>[8]</sup> also concentrates upon rotating superconductors and magnetic field generation by gravitomagnetic fields can be understood from his research. The paper by Feng<sup>[21]</sup> focuses on optimizing the force value, generated by the magnetic fields, considering it as a cost function, thus his mathematical model can be consulted to optimize the superconducting solenoid's design further for a much economical and fixed solution. The future and present day's ductile high temperature superconductors, which can work with liquid nitrogen, can be used instead of liquid helium to provide a cheaper and effective solution.

The field of the superconductivity is fresh and as more research will be conducted in theoretical physics and more contemporary superconductors are discovered, the more simpler, effective, and viable applications of superconductors are possible in the aerospace field.

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