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Review of some recent work on acceleration radiation

S. A. FULLING*
Department of Mathematics and Physics, Texas A&M University,
College Station, TX, 77843-3368, USA

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Some developments in quantum field theory in accelerated frames that may not have got the attention they deserve are reviewed here. They concern (1) elementary particles as radiating accelerated detectors; (2) sound calculations of mirror and black-hole radiation; (3) vacuum energy in the presence of accelerated boundaries; (4) exhaustive study of all accelerated stationary frames in flat space, resolving the issue of the effects of rotation. Some related outstanding questions are mentioned.

1. Elementary particles as radiating accelerated 'detectors'

Our subject begins with the famous calculation by Unruh [1] indicating that a uniformly accelerated two-level [2] system detects a thermal bath of 'Rindler' particles in the vacuum. (Rindler [3] particles are the quanta of the mathematically natural field quantization suggested by separation of variables in hyperbolic coordinates [4]. For more detail on these coordinates see section 3 below.) The physical consistency or plausibility of this conclusion was greatly strengthened by the later analysis of Unruh and Wald [5] showing that the accelerated system *emits* particles as described by an inertial observer.

It was probably Müller [6] who first showed that elementary particles can instantiate this scenario. For example, an accelerated proton can decay into a neutron, an electron and a neutrino—according to standard perturbative quantum field theory in the inertial frame. This point has been greatly developed and publicized by Matsas and Vanzella [7–9]. They presented more detailed calculations exhibiting consistent results in both frames (inertial and accelerated). In the accelerated frame's quantization, the proton absorbs a Rindler lepton to become a neutron. In the words of these authors, 'The Fulling–Davies–Unruh effect is mandatory for the consistency of quantum field theory'. (See also Barton and Calogeracos [10].)

The thermal effect of the vacuum on the accelerated system is now generally accepted. (It has been fairly argued, however, that 'detector' is a misleading term for

*Email: fulling@math.tamu.edu

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such a two-level system, which exhibits no permanent absorption of quanta beyond the approximation of first-order perturbation theory. A true detector should have many degrees of freedom, into which the absorption probability can dissipate.) On the other hand, a controversy continues over whether the accelerated system genuinely radiates [11–15]. The skeptics presumably do not challenge the correctness of the analyses in [6–10], but rather their relevance; the dispute seems to be essentially semantic. In its essence it goes back for some time [16–19]. A resolution requires agreement on three ingredients of the problem posed.

(1) Initial conditions: does one consider a stationary ('dressed') state of the interacting system, or the time evolution of the ground state for a system with either the field interaction or the acceleration initially switched off?
(2) Projections: does one look at the detector (to see that it is excited) before looking for radiation?
(3) Observables: does one look for radiated energy or for radiated particles?

With respect to (1) and (2), note that a truly stationary state would be a coherent superposition of decayed and undecayed states, with correspondingly different field contents. Whether that situation constitutes 'radiation' is largely a matter of 'terminology rather than physics' [18].

2. Sound calculations of mirror and black-hole radiation

The modulus of the Bogolubov coefficient that describes the thermal radiation from a Schwarzschild black hole is (in simplified units) [20, 21]

$$|\beta(\omega, \omega')|^2 \propto \frac{1}{2\pi\omega'} \frac{1}{\exp(\omega/T) - 1}.$$

This formula is almost universally accepted, because numerous independent lines of argument lead to essentially the same conclusion. The same formula arises in the analysis of a two-dimensional quantum field affected by a reflecting barrier that accelerates exponentially to a light-cone asymptote [22]. (After separation of variables in the three-dimensional black-hole problem, the origin of coordinates plays the role of the mirror.)

In more detail†, let the reflecting boundary follow the trajectory $z(t) = -\ln(\cosh t)$ in two-dimensional space–time, and consider normal modes for a scalar field in the region to its right. One complete set of normal modes has the form of (initially monochromatic) waves incident from the right and reflecting back to the left, in a manner that is complicated at late times when the mirror is moving. In the expansion of a quantized field with respect to these modes, the coefficients are annihilation and creation operators, a_ω and $a_\omega^\dagger$, for particles present at early times. Another set of modes includes (in addition to waves escaping to the left at late times) waves that are travelling to the right at late times in a monochromatic manner and,

†This summary is adapted from [26].

therefore, are given at early times by complicated left-moving waves destined to reflect off the moving mirror. The coefficients relative to these modes include annihilation operators $\overline{a}_\omega$ for particles present and moving rightward at late times. The two sets are related by a Bogolubov transformation

$$\overline{a}_\omega = \int_0^\infty \mathrm{d}\omega \Big(\alpha(\omega, \omega') a_{\omega'} + \beta^*(\omega, \omega') a^\dagger_{\omega'} \Big),$$

where α and β are distributional kernels which can be calculated in terms of improper integrals with very delicate convergence properties.

When Davies and I [22] analysed the mirror model, we adapted with minimal change the calculations and arguments that had recently been applied to the black hole, citing the relevant papers as justification: 'According to Hawking, this step is valid for large ω; according to DeWitt, it is valid if the Bogolubov coefficients are going to be used only in calculating physical quantities associated with late retarded times' [22]. This attempt to take the easy way out proved to be embarrassing. Later students [23, 24, 26, 27] have noticed that the original calculations involved approximations that are hard to justify. (Some of these are analogous to 'rotating wave' approximations in standard quantum optics, so they appeared plausible. Another is harder to understand; it may have been a calculational error on our part in [22].)

Calogeracos has now settled these issues definitively by careful calculations in a series of three papers, dealing successively with a well-behaved (asymptotically static) mirror [25], a runaway mirror (as studied in [22]) [26] and a collapsing black hole (which mathematically looks like a runaway mirror in Schwarzschild coordinates) [27]. The basic conclusions of the old papers are not changed.

3. 'Accelerated' vacuum energy

In recent years Saharian and co-workers have published a large number of calculations of Casimir-type energies and energy densities in various situations. Here I discuss only those papers in which the reflecting boundaries in question are accelerating [28–30].

The special-relativistic meaning of 'uniform acceleration' is that an entity follows a hyperbolic trajectory in space–time, so that its acceleration is always the same (and non-zero) in its instantaneous rest frame. Such a path can be embedded (as a curve $\xi = \text{constant}$) into a hyperbolic coordinate system (τ, ξ) defined in terms of the standard Minkowski coordinates by

$$t = \xi \sinh \tau, \qquad x = \xi \cosh \tau.$$

(Any spatial coordinates perpendicular to the acceleration are unchanged.) The new coordinates cover a 'wedge' occupying only 25% of the original space–time. They faithfully represent the perspective of a uniformly accelerating observer, who, by the principle of equivalence, perceives a uniform gravitational field in the ξ direction [3]. More precisely, the metric tensor (line element) in the new frame takes the form

$$\mathrm{d}s^2 = \xi^2\,\mathrm{d}\tau^2 - \mathrm{d}\xi^2 - (\text{perpendicular part}),$$

which is independent of the new time coordinate but has a non-trivial spatial dependence. The boundary of the wedge is exactly analogous to the *event horizon* of a black hole.

In section 1 we were concerned with an observer, detector or particle moving along one of the hyperbolic curves. Here, however, we are interested in making hyperbolas (or hyperbolic sheets in higher dimensions) into reflecting boundaries. Saharian *et al.* consider either one or two uniformly accelerated mirrors (or plates) in arbitrary dimension. For scalar and electromagnetic fields, they calculate vacuum energy density (by Abel–Plana summation) and total vacuum energy (by zeta-function renormalization) *relative to the empty Rindler background*. (That is, formal expressions for the field energy (for whose details we must refer to the literature) that would be divergent if evaluated naively are here rendered finite by subtraction of their values in the vacuum state of a uniformly accelerated observer in the absence of the hyperbolic boundaries.) The results defy easy summarization, but one can note the following main conclusions.

(a) The formulas are consistent in the sense that the correct results are always found in various limits (from two plates to a single plate, and from accelerating to inertial plates).

(b) In spatial dimension $d = 3$ one finds for a scalar field and a plate at Rindler spatial coordinate ξ_1 the total energy (per unit transverse area, and in a certain natural units)

$$E_{\mathrm{Dir}} = -\frac{0.00131}{\xi_1^2}, \quad E_{\mathrm{Neu}} = +\frac{0.00292}{\xi_1^2},$$

for the Dirichlet and Neumann boundary condition, respectively. (Which of these corresponds to ‘deceleration’ of the plate by interaction with the field, and which to ‘acceleration’? I differ with the authors on this physical point, which may be open to definitional dispute.)

(c) As $\xi_1 \to 0$, the plate energy density is logarithmically smaller than the Rindler background energy density [31–33], which is proportional to $\xi^{-(d+1)}$. (The definition of the background energy itself requires a renormalization procedure. One way of thinking of the latter is as a formal subtraction of the energy of the vacuum in the full Minkowski space. However, that subtraction is a more delicate matter than the ones that involve comparisons of states that are both constructed in the hyperbolic co-ordinates.)

Because the work of Saharian *et al.* takes that established value of the background energy density near the Rindler horizon as given, it does not cast as much light as one might have hoped on one of the remaining controversies in this area. The renormalized energy density in the Rindler vacuum is negative, intuitively because of the absence of the thermal Rindler quanta constituting the Minkowski vacuum (whose renormalized energy density is zero by definition). The same is true for the twin Rindler space on the other side of the horizon. But one would think that the total energy of any state in Minkowski space (other than the Minkowski vacuum) should be positive, because of the presence of quanta relative to the Minkowski vacuum (ground state).

This paradox was raised by Parentani [34], who proposed this resolution: if one calculates the energy density in two-dimensional Rindler space–time by means of regularization by an ultraviolet cutoff with parameter ϵ, then for non-zero values of ϵ one gets (in addition to some less important terms)

$$\rho(U) = \frac{\epsilon^2 - U^2}{(U^2 + \epsilon^2)^2},$$

where $U = t - x$. This expression is virtually identical to formulas that appear in recent literature concerning Casimir energy densities (near boundaries, not horizons) calculated with cutoffs motivated by a variety of mathematical or physical considerations [35–38]. As $\epsilon \to 0$, one recovers the known negative density proportional to $-U^{-2}$ [31, 32]. However, this limit is non-uniform. As U approaches 0 with fixed non-zero ϵ, $\rho(U)$ develops a large compensating positive spike near the horizon, proportional to $+\epsilon^{-2}$. (In fact $\int_0^\infty \rho(U)\ ,\mathrm{d}U$ equals 0.) Therefore, one can think of the positivity of total energy in the Rindler vacuum as being rescued by energy concentrated right on the horizon.

In my opinion there are several unsettled issues that must be faced before one can accept this argument.

(1) Is the double-Rindler vacuum too singular to make sense at all as a physical state in Minkowski space as a whole? (Note that this issue is irrelevant to the validity of the Unruh theory [1], since the latter concerns only *the ordinary Minkowski vacuum* as it affects an accelerated system [39].)
(2) Does the argument of positivity relative to the ground state really apply to states from inequivalent Fock spaces?
(3) What is the physical significance of calculations with a finite cutoff? Might not calculations in a different regularization scheme lead to a different conclusion?

4. All accelerated frames in flat space

A puzzle left over from the early 1980s is the question of the *rotational* analogue (if any) of the thermal effect of uniform *linear* acceleration. On the one hand, calculations by Letaw and Pfautsch and others [40–44] indicated that the natural vacuum in a rotating frame is the same as the ordinary Minkowski vacuum. On the other hand, physical calculations by Bell and Leinaas and others [45–47] showed that a rotating thermometer will get hot—and that this effect is actually *observed* in the polarization of electrons in storage rings.

Now Jan Ivar Korsbakken and his mentor, Jon Magne Leinaas, have written what I think will be the definitive paper on this topic [48]. Following and improving upon the work of Letaw and Pfautsch, they have catalogued and studied all accelerated trajectories in four-dimensional Minkowski space–time along which the dynamics is stationary (not necessarily static)—in other words, the acceleration is constant in the moving frame. These trajectories fall into families that correspond to symmetries (Killing vectors) that interpolate between linear acceleration and rotation. The authors clarify the physics by studying local Lorentz frames, not global coordinate systems (which are necessarily non-orthogonal when rotation is involved). They find that in general there are two coexisting effects.

(1) Existence of an event horizon implies that the Minkowski vacuum contains positive-energy quanta (relative to the natural quantization in the accelerated frame), which a detector can absorb. This is the familiar 'Rindler' thermal effect studied by Unruh and others in the case of purely linear acceleration.
(2) Existence of a static limit (ergosphere) implies that negative-energy modes exist, which a detector can emit. This is the effect that persists in purely rotational acceleration, where there is no change in the definition of the vacuum state.

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