ceed to use the most convenient gauge and coordinate system alone with no fear of loss of generality.

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[1]See, e.g., *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1987). The quantum Hall effect involves the much more complicated problem of electrons in a solid moving not only under the influence of a uniform magnetic field but also under the influence of their mutual interactions, the ion-core background, and a uniform applied electric field. The idea that it might be useful to understand electron motion in the quantum Hall effect problem from a wave-packet point of view provided the stimulus for the study of the much simpler system presented in the current article.

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Spatial geometry in a uniformly accelerating reference frame

Edward A. Desloge

Physics Department, Florida State University, Tallahassee, Florida 32306

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The meaning of spatial geometry in a reference frame is carefully analyzed. It is shown that in a uniformly accelerating reference frame spatial geometry is Euclidean if distance is measured with measuring rods and non-Euclidean if distance is measured with light signals. The distance function and the square of the line element associated with each mode of measurement are obtained.

I. INTRODUCTION

This article examines the nature of spatial geometry in a uniformly accelerating reference frame in flat space-time. Aside from its intrinsic interest, the subject provides an excellent pedagogical vehicle for illustrating a variety of geometrical concepts.

Spatial geometry in a reference frame depends on the nature of the frame and the manner in which the distance between two points in the frame is measured or defined. Our interest in this article will be in distance as measured using either measuring rods or clocks and light signals. These distances will be referred to as the rod and radar distances, respectively.

In an inertial frame, the rod and radar distances between two points are equal and the associated geometry is Euclidean. In a noninertial frame, the rod and radar distances between two points are not in general equal and the associated geometries are not in general Euclidean.

In an earlier article in this Journal,[1] we have discussed in detail the nature of a uniformly accelerating reference frame and demonstrated its utility in bridging the conceptual gap between special and general relativity. Only the one-dimensional uniformly accelerating reference frame was considered; hence, any discussions of the geometry in the frame were necessarily limited. In the present article, we extend our results to the three-dimensional uniformly accelerating reference frame and use it as a vehicle for illustrating the geometrical properties of noninertial frames.

Explicit expressions are obtained for the rod and radar distances between two points in a uniformly accelerating reference frame. We find (i) the two distances are not equal, (ii) the geometry associated with rod distance is Euclidean, and (iii) the geometry associated with radar distance is not Euclidean.

The non-Euclidean character of the geometry in a noninertial reference frame is usually illustrated by considering a rotating reference frame and noting that if measuring rods at rest in the frame are placed along the circumference of a circle centered on the axis of rotation, they will be contracted, whereas measuring rods placed along the radius of such a circle will not be contracted; hence, the usual Euclidean relationship between the circumference of a circle and its radius will not hold.[2] The uniformly accelerating reference frame provides an alternate and, in many ways, simpler vehicle for illustrating the non-Euclidean character of the geometry in a noninertial frame.

The fact that spatial geometry is dependent on the means used to measure distance is usually illustrated with rather contrived examples. If, for example, a differentially heated plate is measured with a rod with an appreciable coefficient of thermal expansion, the resulting geometry will be non-Euclidean and will depend on the value of the coefficient of thermal expansion of the rod.[3] In this example, it is easy to recognize the inadequacy of the measuring instrument. However, in the case of the uniformly accelerating reference frame, two highly acceptable methods of measuring distance result in one instance in a Euclidean geometry and in a second instance in a non-Euclidean geometry.

In summary, our objective in this article is (i) to define a uniformly accelerating reference frame in three dimensions; (ii) to show that the observers making up the frame are at rest with respect to one another and, hence, the frame is a rigid frame; (iii) to clarify the exact meaning of spatial geometry in a reference frame; (iv) to show that to obtain a satisfactory radar distance function it is necessary to employ clocks that keep coordinate time rather than proper time; (v) to obtain expressions for the rod and radar distances between two points in a uniformly accelerating reference frame; and (vi) to use the above results to illustrate the fact that spatial geometry in a noninertial frame depends not only on the nature of the frame but also on the mode of measuring distances.

II. DEFINITIONS AND NOTATION

A. Units

To simplify the mathematical expressions, we shall work in a system of units in which $c = g = 1$, where c is the speed of light and g is some conveniently chosen reference acceleration. Using such units is equivalent to writing all equations in terms of dimensionless quantities without specifically introducing new variables to designate the resulting quantities.[4] At any stage in the development, the equations can be written in dimensionless form, or equivalently, in terms of arbitrary units by dividing each quantity appearing in the expression by whatever combination of quantities c and g will make it dimensionless. In particular, length quantities are divided by c^2/g and time quantities by c/g.

B. Measurement

Geometry is based on measurement and measurement requires instruments with which to make the measurements. We assume we have available a large number of identically constructed and calibrated devices capable of measuring proper times and lengths. For convenience, we will refer to one of these devices as a metrosphere. Elsewhere[5] we have described in detail a hypothetical instrument whose operation is based on the properties of light signals and free particles and which has this capability. For the purposes of this article, however, it is not necessary to know how a metrosphere works. It is sufficient to know that it is capable of functioning as a standard rod or standard clock.

C. Observers

An observer is considered to be a hypothetical intelligent point particle,[6] equipped with a metrosphere and having the following properties: (i) negligible interaction with the surroundings, (ii) the ability to move arbitrarily consistent with the usual relativistic limits, and (iii) the ability to communicate with other observers by means of light signals. We assume that we have available an unlimited supply of observers.

D. Reference frames

We define a reference frame to be a spatially continuous set of observers moving in a specified manner.[7]

A particular event can be uniquely specified by designating (i) the observer in the frame on whose world line the event occurs; and (ii) the point in time, on the above world line, at which the event occurs. Which observer is present at the event determines the "where" of the event and the point on the observer's world line at which the event occurs determines the "when" of the event.

In an arbitrary reference frame neither the rod nor radar distance between two observers will in general be constant. In this article, we will consider only reference frames in which these distances are constants. In such reference frames all the observers are by definition at rest with respect to one another and the frame may be envisioned as a rigid three-dimensional structure.

E. Coordinatized reference frames

Given a frame of reference, numbers can be assigned to each of the observers in the frame and to each point on the world line of each of the observers.[8] Such an assignment constitutes a coordinate system for that frame. A coordinate system is not the same as a frame of reference. A frame of reference is a hypothetical physical entity. A coordinate system presupposes a frame of reference and is a particular mathematical entity associated with that frame.

In coordinatizing a frame, it is always possible to designate a particular observer by a set of three numbers (x_1,x_2,x_3) and a point on his world line by a single number t in such a way that the set of four numbers (x_1,x_2,x_3,t) varies in a continuous fashion along any curve in space-time. It is assumed that such a choice has been made.

Given a coordinatized reference frame, a particular event is specified by a particular set of values of the above four numbers. The set of numbers (x_1,x_2,x_3) designates the observer who is present at the event and the number t designates the time for that observer at which the event took place. Since the set of numbers (x_1,x_2,x_3) identifies a particular observer, it has a meaning independent of the number t; however, the number t represents a point on the world line of a particular observer and thus requires the prior specification of the set of values (x_1,x_2,x_3).

The notation $P(x_1,x_2,x_3)$ will be used in what follows to denote the observer P who is characterized by the set of numbers (x_1,x_2,x_3).

III. GEOMETRY

A. Spatial and space-time geometry

In this article, our interest is in the *spatial* geometry of reference frames. The results we obtain depend on both the nature of the reference frame and the manner in which the spatial distance between observers in the frame is measured.

In contrast to the above situation, the object of interest in space-time geometry is the four-dimensional manifold of events and rather than the distance between points, one considers the interval between events. The geometry of space-time is unique and, for the purposes of this article, will be assumed to be pseudo-Euclidean. This assumption confines us to special relativity. Although our results provide some insights into the subject of space-time geometry, no explicit use will be made in this article of the concepts of space-time geometry.

B. Distance functions

If in a particular reference frame it is possible to associate with every pair of observers (P', P'') a function $\rho(P',P'')$ having the properties

$$\rho(P',P'')\geqslant 0, \tag{1a}$$

$$\rho(P',P'')=0, \text{ if and only if } P'\equiv P'', \tag{1b}$$

$$\rho(P',P'')=\rho(P'',P'), \tag{2}$$

$$\rho(P',P''')\leqslant\rho(P',P'')+\rho(P'',P'''), \tag{3}$$

then the function is called a distance function.[9]

We will be interested in two approaches to the problem of establishing a distance relationship between arbitrary pairs of observers (P',P''). The first approach is based on the number of metrospheres or standard unit rods, each assumed to be locally at rest in the frame, required to connect P' and P''. The second approach is based on the time as measured by P' for a light signal to go from P' to P'' and back. As noted in Sec. I, a distance established by the first method will be called a rod distance and a distance established by the second method will be called a radar distance.

C. Riemannian geometry

Given a particular coordinatized reference frame, with spatial coordinates (x_1,x_2,x_3), for which there exists a distance function ρ, if it is possible to define a set of functions $g_{ij}(x_1,x_2,x_3)$, where i and j range between 1–3, such that the square of the distance $d\rho$ between the observer $P(x_1,x_2,x_3)$ and an arbitrary neighboring observer $P(x_1+dx_1,x_2+dx_2,x_3+dx_3)$ is given by

$$d\rho^2=\sum_i\sum_j g_{ij}\,dx_i\,dx_j, \tag{4}$$

then the spatial geometry of the frame for the given distance function is said to be Riemannian.[10] The quantity $d\rho$ is called the line element. The existence of a distance function does not automatically imply that the square of the line element can be written in the form given by Eq. (4).

D. Euclidean geometry

Given a coordinatized reference frame in which a distance function has been defined and which is such that the square of the line element can be written in the form given by Eq. (4), if it is possible to recoordinatize the frame in such a way that in terms of the new coordinates

$$d\rho^2=\sum_i dx_i^2, \tag{5}$$

that is, in such a way that $g_{ij}=\delta_{ij}$, where δ_{ij} is the Kronecker delta, then the spatial geometry of the frame for the given distance function is said to be Euclidean.[11]

IV. INERTIAL REFERENCE FRAMES

In flat space-time there exist frames of reference, called inertial frames, in which clocks can be synchronized and in which the geometry based either on rod or radar distance is Euclidean.

Let I be a particular inertial frame in which the clocks have been synchronized and the spatial coordinates (x,y,z) of the observers have been chosen in such a way that the square of the line element is given by $dx^2+dy^2+dz^2$.

If I' represents a similarly coordinatized inertial reference frame moving with respect to I in the positive x direction with speed V and if at $t=t'=0$ the origins and the axes of the two frames coincide, then the transformation between the two frames is given by the Lorentz transformation

$$x'=\Gamma(x-Vt),\quad y'=y,\quad z'=z,\quad t'=\Gamma(t-Vx), \tag{6a}$$

where

$$\Gamma\equiv(1-V^2)^{-1/2}. \tag{6b}$$

V. A UNIFORMLY ACCELERATING REFERENCE FRAME

A. Definition

We define the uniformly accelerating reference frame A as the frame made up of the set of observers $P(X,Y,Z)$ whose positions relative to the frame I are given by (see Ref. 1)

$$x=(X^2+t^2)^{1/2},\quad y=Y,\quad z=Z, \tag{7}$$

where $x\geqslant 0$, $X\geqslant 0$, and there are not restrictions on y, z, Y, and Z.

The velocity and acceleration of observer $P(X,Y,Z)$ relative to frame I can be obtained by taking the first and second derivatives of Eqs. (7) with respect to time t while holding X, Y, and Z constant. In doing this, we obtain

$$\dot{x}=t(X^2+t^2)^{-1/2},\quad \dot{y}=0,\quad \dot{z}=0, \tag{8}$$

$$\ddot{x}=X^2(X^2+t^2)^{-3/2},\quad \ddot{y}=0,\quad \ddot{z}=0. \tag{9}$$

Relative to frame I the observers in frame A are moving with different nonconstant velocities and accelerations. Nevertheless, it can be shown, and will be shown below, that the observers in K are at rest with respect to one another and each observer is accelerating at a constant rate relative to the instantaneously comoving inertial frame.

B. Comoving inertial frames

If Eqs. (7)–(9) are transformed from frame I to frame I' using the Lorentz transformation (6), then the resulting equations will be identical with those obtained by replacing x,y,z, and t in Eqs. (7)–(9) with x',y',z', and t', respectively. Hence Eqs. (7)–(9) are invariant in form under the Lorentz transformation (6).

From Eqs. (8), it follows that at $t=0$ frame A is at rest with respect to frame I. From the invariance of Eqs. (8) under the Lorentz transformation (6), it follows that at $t'=0$ frame A is at rest with respect to frame I'. Hence, as frame A moves it is always at rest with respect to one of the inertial frames that are moving with respect to I in the positive x direction. Furthermore, if these frames are coordinatized in the manner described in Sec. IV, then frame A will be at rest with respect to a particular one of these inertial frames at time zero in that frame. The frame with respect to which, at a given instant, A is at rest will be called the instantaneously comoving inertial frame.

From Eqs. (8) and the invariance of Eqs. (9) under the

Lorentz transformation (6) it follows that observer $P(X,Y,Z)$ is undergoing a constant acceleration with respect to the instantaneously comoving inertial frame and if the comoving inertial frame is coordinatized as described above, then the acceleration will be in the positive x direction and of magnitude $1/X$.

C. Proper time

Let us assume that at $t = 0$ in frame I each of the observers in frame A sets his metrosphere clock at zero. The time subsequently indicated by one of these clocks will be designated by τ. Since the metrosphere clocks measure proper time, this time will be called the proper time. If an observer P is moving with speed v with respect to the inertial frame I, then in time dt the passage of proper time is given by[12]

$$d\tau = (1 - v^2)^{1/2}\, dt \equiv [1 - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)]^{1/2}\, dt. \quad (10)$$

Substituting Eqs. (8) in Eq. (10) and solving the resulting equation we obtain, for the proper time of observer $P(X,Y,Z)$,

$$\tau = X \sinh^{-1}(t/X) = X \cosh^{-1}(x/X). \quad (11)$$

D. Coordinate time

For a number of reasons the proper time τ is not particularly convenient. First, clocks registering proper time cannot be synchronized. Second, as we shall subsequently show, although in an inertial frame one can define the radar distance between two observers P' and P″ as one-half the proper time measured by P' for a signal to make a round trip from P' to P″ and back, in a uniformly accelerating reference frame this definition will not satisfy the conditions required for a distance function. However, a time T that does not suffer these deficiencies can be defined as

$$T \equiv \tau/X. \quad (12)$$

The time T will be called coordinate time and we will assume that every observer in frame A has been equipped with an auxiliary clock that keeps coordinate time. The coordinate time T is the time designated by θ in Ref. 1. Some additional interesting properties of coordinate time are that (i) coordinate time and proper time coincide on the plane $X = 1$, (ii) the rate of a clock keeping coordinate time coincides with the rate of arrival of signals sent at unit proper time intervals by an observer in frame A located on the $X = 1$ plane, and (iii) the velocity with respect to I of an arbitrary observer in frame A is given by $(\dot{x},\dot{y},\dot{z}) = (\tanh T,0,0)$; hence, when frame A is at rest with respect to one of the comoving inertial frames, all the observers in A will note the same value of the coordinate time.

E. Signal transit time

The following theorem is central to our subsequent analysis of radar distance and is also of significance in analyzing the general properties of a uniformly accelerating reference frame.

Theorem: If in frame A a light signal is sent at coordinate time T_1 by an observer $P' \equiv P(X',Y',Z')$ and received at coordinate time T_2 by a second observer $P'' \equiv P(X'',Y'',Z'')$, then

$$T_2 - T_1 = K(P',P''), \quad (13a)$$

where

$$K(P',P'') = K(P'',P') = \cosh^{-1}\{1 + [(X'' - X')^2 + (Y'' - Y')^2 + (Z'' - Z')^2]/2X'X''\}. \quad (13b)$$

Proof: Let event 1 be the emission of the light signal by observer P' and event 2 the reception of the light signal by observer P''. The coordinates of events 1 and 2 in frame I are (x_1,y_1,z_1,t_1) and (x_2,y_2,z_2,t_2), respectively, and in frame A they are (X',Y',Z',T_1) and (X'',Y'',Z'',T_2), respectively. In frame I, the signal travels with unit speed along a straight line; hence,

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = (t_2 - t_1)^2. \quad (14)$$

However, from Eqs. (11) and (12), it follows that

$$x_1 = X' \cosh T_1, \quad y_1 = Y', \quad z_1 = Z', \quad t_1 = X' \sinh T_1, \quad (15)$$

$$x_2 = X'' \cosh T_2, \quad y_2 = Y'', \quad z_2 = Z'', \quad t_2 = X'' \sinh T_2. \quad (16)$$

Substituting Eqs. (15) and (16) in Eq. (14) and making use of the identities $\cosh^2 u - \sinh^2 u \equiv 1$ and $\cosh u \cosh v - \sinh u \sinh v \equiv \cosh(u - v)$, we obtain Eq. (13).

VI. ROD GEOMETRY IN A UNIFORMLY ACCELERATING REFERENCE FRAME

The uniformly accelerating reference frame A is always at rest with respect to one of the inertial frames $I, I', I'',...,$ and when it is at rest with respect to a particular one of these frames, for example, I, then the coordinates (X,Y,Z) coincide with the coordinates (x,y,z). It follows that the rod distance between two observers P' and P'' will be the same as the rod distance between the two observers in frame I with whom P' and P'' were, respectively, coincident at time $t = 0$. Thus the rod distance function between two observers P' and P'' in frame A is given by

$$\rho(P',P'') = [(X'' - X')^2 + (Y'' - Y')^2 + (Z'' - Z')^2]^{1/2}, \quad (17)$$

from which it follows that the square of the line element is given by

$$d\rho^2 = dX^2 + dY^2 + dZ^2. \quad (18)$$

It follows that the rod geometry in a uniformly accelerating reference frame is Euclidean.

VII. RADAR GEOMETRY IN A UNIFORMLY ACCELERATING REFERENCE FRAME

If an observer P' sends a signal to an observer P″ (event 1); observer P'' receives and returns the signal (event 2); and observer P' receives the return signal (event 3); then from the signal transit theorem it follows that

$$\tfrac{1}{2}(T_3 - T_1) = \tfrac{1}{2}[(T_3 - T_2) + (T_2 - T_1)] = \tfrac{1}{2}[K(P'',P') + K(P',P'')] = K(P',P'') \quad (19)$$

and consequently,

$$\tfrac{1}{2}(\tau_3 - \tau_1) = \tfrac{1}{2}(X'T_3 - X'T_1) = \tfrac{1}{2}X'(T_3 - T_1) = X'K(P',P''). \quad (20)$$

From Eq. (20) it follows that if one tries to use the proper time τ to set up a radar distance by defining the radar distance $\rho(P',P'')$ by the quantity $(\tau_3 - \tau_1)/2$, then

$\rho(P',P''))\neq\rho(P'',P')$; hence, condition (2) is not satisfied.

On the other hand, if one uses the coordinate time T to set up a radar distance by defining the radar distance $\rho(P',P'')$ by the quantity $(T_3 - T_1)/2$, then all of the required conditions for a distance function will be satisfied. Using $(T_3 - T_1)/2$ to define the radar distance between the two observers P' and P'', we obtain

$$\rho(P',P'') = \cosh^{-1}\{1 + [(X'' - X')^2 + (Y'' - Y')^2 + (Z'' - Z')^2]/2X'X''\}. \quad (21)$$

If we designate a particular observer on the plane $X = 1$ by O and the time that elapses on the clock of O when a signal traverses the closed path $OP'P''O$ as $\Delta\tau(OP'P''O)$, then it can be shown that the exact same distance function would have been obtained if we had defined $\rho(P'P'')$ by the relation $\Delta\tau(OP'P''O) - \frac{1}{2}\Delta\tau(OP'O) - \frac{1}{2}\Delta\tau(OP''O)$. Hence, the given distance function is also the distance function that would have been obtained if all distances were measured by means of light signals sent out by a *single* observer on the $X = 1$ plane.

From Eq. (21) it follows that the square of the line element is given by

$$d\rho^2 = (dX^2 + dY^2 + dZ^2)/X^2. \quad (22)$$

It can be shown by the usual techniques of metric differential geometry that frame A cannot be recoordinatized in such a way that $d\rho^2 = dx_1^2 + dx_2^2 + dx_3^2$; hence, the radar geometry will be non-Euclidean.

VIII. CONCLUSION

The most significant result obtained in the preceding analysis is the signal transit theorem (13). With it one is able not only to confirm the fact that the radar distance between any two observers in a three-dimensional uniformly accelerating frame remains constant, but also one is able to obtain an explicit expression for this distance. Given the distance function, the associated geometry can be determined.

The definition that we have used for a frame of reference is a definition that is more common in the Soviet Union than in the West.[13] It has the advantage of allowing a clear distinction to be made between space and time and, also, between a frame of reference and a coordinate system. The former distinction allows one to separate spatial geometry from space-time geometry. The latter distinction enables one to recognize the fact that spatial geometry is based on the choice of frame and the mode of distance measurement and not on the coordinates employed.

Our presentation has necessarily been abbreviated by limitations on the length of this article. We have, however, provided the reader with the necessary conceptual and analytical framework to pursue the subject in greater detail. We have used the material in a course and found it to be pedagogically quite helpful.

We have also tried in the present article to emphasize those aspects of the subject that are generally ignored in standard texts. There are a number of straightforward, but interesting extensions of the results, which we leave to the reader. As one example, we have simply stated that there is no recoordinatization of the uniformly accelerating reference frame A that would reduce the radar distance given by Eq. (22) to a Euclidean form. The proof of this is an interesting exercise in metric differential geometry. We have also not pursued the consequences of a geometry based on the square of the line element given by Eq. (22). Those consequences that do not explicitly depend on the spatial three-dimensionality of the frame have been considered in Ref. 1.

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