Classical Aberration And Obliquation

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Preface: A Tribute To Faraday

Paradise is a world without interpretations, a world where every mind agrees with every other mind about everything. Our world, however, is a world full of interpretations, where many minds disagree with many other minds about so many things. One of the peculiar aspects of our world is that, when properly exercised, our intuitive faculties permit us to catch a glimpse of paradise now and then, but such glimpses are beyond the reach of any mind for which intuition is a lost art. In the entire history of the subject matter of this monograph, perhaps no one has demonstrated the effectiveness of intuition as a living art in a more fruitful and a more engaging way than M. Faraday. His thoughts, work and interpretation of electromagnetic phenomena occupied the most capable thinkers in and beyond his generation, and to this day, they continue to inspire us as they have inspired J. C. Maxwell, H. Hertz and many others. We shall take it upon ourselves in this series of monographs to study certain problems raised by the work of H. Hertz regarding the optics of accelerated systems (with the understanding that Hertz's theory is Hertz's system of equations), because solutions to these problems appear to have many unexplored applications relevant to science and technology, and at the same time represent an unfinished chapter in the history of the investigations begun by Faraday.

Our first problem arises from the fact that a ray of light propagating relative to a moving observer can be associated with two directions in space. One is the true direction the ray would have if the observer were stationary, the other is the apparent direction the ray is actually observed to have. If the apparent direction is the same for two observers moving relatively to each other, then the true direction is likely to be different for the observers, and we may refer to this effect as aberration. Likewise, if the true direction is the same for the observers, then the apparent direction is likely to be different for the observers, and we may refer to this effect as obliquation. We shall show that while the wavefront of light is tilted in aberration, there is no such tilt in obliquation, and for an observer in accelerated translational, rotational or gravitational motion, we shall give a rigorous treatment of the apparent angular displacement of a light source due to obliquation, the rate of change of this displacement with the observer's velocity, the apparent path traced by the light source, the apparent geometry of a light ray from the source to the observer, and the apparent frequency of the ray. Readers with only a casual interest in the subject are advised that Part II may be omitted without an appreciable loss of continuity.

My thanks and gratitude go to Dr. T. E. Phipps Jr. for bringing the aberration problem in Hertz's electrodynamics to my attention many years ago. It gives me great pleasure to dedicate this series to all those whose minds are hardly ever at rest, and for whom, therefore, $\mathcal{P} - \mathcal{C} \triangleq \mathcal{N}$.

Part I: Foundations

It is true, however, that it is not customary to pull down all the houses of a town with the single design of rebuilding them differently, and thereby rendering the streets more handsome; but it often happens that a private individual takes down his own with the view of erecting it anew, and that people are even sometimes constrained to this when their houses are in danger of falling from age, or when the foundations are insecure.

René Descartes (1596-1650)

1 Introduction

Art 1. *Review of previous results.*

We have studied elsewhere [1] the problem of light propagation for accelerated observers in a stationary, homogeneous and isotropic medium using Hertz's version of Maxwell's theory. We showed in that work that a linearly polarized regular plane ray propagates relative to a translating, rotating or gravitating observer with a velocity $\boldsymbol{v}$ given by

$$\boldsymbol{v} = \mathbf{c}d + \mathbf{w} - \mathbf{u}, \quad \mathbf{c} = c\widehat{\boldsymbol{\kappa}} \tag{1.1}$$

where c is determined by the permittivity and the permeability of the medium in the usual way, $\mathbf{u}(\mathbf{r}, t)$ is the velocity of the observer at position $\mathbf{r}$ and time t, $\widehat{\boldsymbol{\kappa}}$ is the unit or normalized wave vector, and the quantities $d, \mathbf{w}$ depend on the wave vector, the wave polarization, and the observer's acceleration $\mathbf{a} = \dot{\mathbf{u}}$ in a fairly complicated way. It was shown, more precisely, that if $\widehat{\mathbf{p}}$ is a unit vector in the polarization direction and $\boldsymbol{\kappa}$ is the wave vector, then

$$d = \gamma \left\{ \frac{1 + \sqrt{1 + \vartheta^2}}{2} \right\}^{1/2}, \quad \mathbf{w} = \rho\mathbf{a} - \tau\boldsymbol{\kappa} + \mathbf{e} \tag{1.2a}$$

where (all square roots being nonnegative),

$$\rho = \pi/(4d\omega_o), \quad \pi = \vartheta/(1 + \vartheta^2)^{1/2}, \quad \vartheta = \alpha/(\gamma\omega_o)^2, \quad \alpha = \boldsymbol{\kappa} \cdot \mathbf{a}, \quad \omega_o = c\kappa \tag{1.2b}$$

and the quantities γ, $\mathbf{a}$, τ and $\mathbf{e}$ are given for each type of motion as follow. For an observer translating with acceleration $\mathbf{a}$,

$$\gamma = 1, \quad \mathbf{a} = \mathbf{a}(t), \quad \tau = 2\rho\alpha\kappa^{-2}, \quad \mathbf{e} = \mathbf{0} \tag{1.3a}$$

where

$$\mathbf{u} = \mathbf{u}(t), \quad \widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}} = 0. \tag{1.3b}$$

For an observer rotating with angular velocity $\boldsymbol{\Omega}(t)$,

$$\gamma = \left| 1 + \frac{s_0}{\eta\omega_o^2} \right|^{1/2} \neq 0, \quad \mathbf{a} = \boldsymbol{\Omega} \times \mathbf{u} + \boldsymbol{\Lambda} \times \mathbf{r} = (\boldsymbol{\Omega} \cdot \mathbf{r})\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda} \times \mathbf{r} \tag{1.4a}$$

$$\tau = 2(\rho\alpha - \eta s_2)\kappa^{-2}, \quad \mathbf{e} = s_2(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}} \tag{1.4b}$$

where

$$\begin{gathered} \mathbf{u} = \boldsymbol{\Omega} \times \mathbf{r}, \quad \eta = \boldsymbol{\kappa} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \neq 0, \quad \boldsymbol{\Lambda} = \dot{\boldsymbol{\Omega}} \\ \xi = 2(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\boldsymbol{\Omega} \cdot \boldsymbol{\kappa}) - \Omega^2(\boldsymbol{\kappa} \cdot \widehat{\mathbf{p}}), \quad \zeta = (\boldsymbol{\kappa} \cdot \widehat{\mathbf{p}})(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}) - (\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})(\boldsymbol{\Omega} \cdot \boldsymbol{\kappa}) \end{gathered} \tag{1.4c}$$

and

$$\begin{gathered} s_0 = \zeta - (\xi^2/\eta), \quad s_1 = \frac{2\rho\alpha - d\omega_o}{2\eta(s_0 + \eta\omega_o^2)}, \quad s_2 = s_0 s_1 \\ s_3 = \big\{\eta(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}}) + 4\xi(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})\big\}s_1, \quad s_4 = \big\{2\xi\Omega^2 + \eta(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})\big\}s_1. \end{gathered} \tag{1.4d}$$

For an observer gravitating with acceleration $\mathbf{a}$ due to gravity,

$$\gamma = \left|1 - \frac{\mho^2}{\kappa^2}\right|^{1/2}, \quad \mathbf{a} = \mathbf{a}(\mathbf{r}) = -q\mathbf{r}/r^3 \tag{1.5a}$$

$$\tau = (2\rho\alpha - n\mho^2 f_0)\kappa^{-2}, \quad \mathbf{e} = f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}} \tag{1.5b}$$

where, if $\widehat{\mathbf{r}}$ be a unit vector in the direction of $\mathbf{r}$,

$$\begin{gathered}\mathbf{u} = h^{-2}(\mathbf{z}\times\mathbf{h} + q\widehat{\mathbf{r}}\times\mathbf{h}), \quad \mathbf{h} = \mathbf{u}\times\mathbf{r}, \quad \mathbf{z} = \mathbf{h}\times\mathbf{u} - q\widehat{\mathbf{r}} \\ m = \boldsymbol{\kappa}\cdot(\mathbf{r}\times\mathbf{h}), \quad n = \widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h}) \neq 0, \quad \mho^2 = (m/n)(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}}) \neq \kappa^2\end{gathered} \tag{1.5c}$$

and ($\mathbf{h}$, $\mathbf{z}$, q being arbitrary constant quantities which determine the orbit of the observer according to Newton's theory),

$$f_0 = \frac{2\rho\alpha - d\omega_o}{n(\kappa^2 - \mho^2)}, \quad f_1 = \frac{(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})f_0}{2}, \quad f_2 = \frac{mf_0}{2}. \tag{1.5d}$$

For the three types of motion to be studied in this work, it is convenient to introduce the angles shown in Figure 1 as well as the quantities

$$\boldsymbol{\beta} = \mathbf{u}/c \neq \mathbf{0}, \quad \boldsymbol{\sigma} = \mathbf{a}/c, \quad \mu = \sigma/\omega_o, \quad \mathcal{Y} = cd - \kappa\tau. \tag{1.6}$$

We recall that the conditions for a linearly polarized plane ray to be regular (i.e., to propagate with a velocity that depends on c) are $\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}} = 0$ for a translating observer, $\eta \neq 0$ and $\gamma \neq 0$ for a rotating observer, and $\mho \neq \kappa$ for a gravitating observer [1]. When these conditions are not satisfied, the ray may either propagate with a velocity that is independent of c or cease to propagate altogether. Furthermore, for a gravitating observer, the ray propagates only if $n \neq 0$; otherwise, it degenerates into a nonpropagating mode[1].

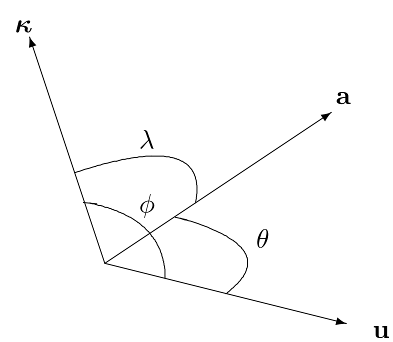


Figure 1: Common angular parameters. Rectilinear motion is defined to be such that the velocity $\mathbf{u}$ and the acceleration $\mathbf{a}$ are parallel ($\theta = 0, \phi = \lambda$), radial motion to be such that the velocity $\mathbf{u}$ and the wave vector $\boldsymbol{\kappa}$ are parallel ($\phi = 0, \theta = \lambda$), and coradial motion to be such that the acceleration $\mathbf{a}$ and the wave vector $\boldsymbol{\kappa}$ are parallel ($\lambda = 0, \theta = \phi$). Note that all three angles may change with time.

Art 2. *Scope of this work.*

Our purpose in this work is to apply the above results to the phenomenon of light aberration. This work is motivated in part by the need to correct the widespread misconception that light

[1]This implies for example that a gravitating observer cannot perceive the ray if the ray is linearly polarized along the position vector $\mathbf{r}$ or along a normal vector $\mathbf{h}$ to the plane of the observer's orbit. We note here that our vector $\mathbf{h}$ points to the south ecliptic pole instead of pointing to the more customary north ecliptic pole.

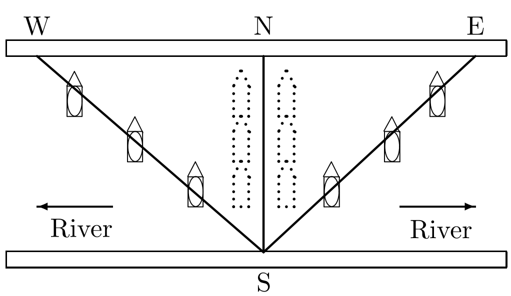
aberration is inconsistent with Hertz's electrodynamics [2, 3], with Fresnel's wave theory of light [4], or more generally, with classical physics [5, 6]. The practical motivations are to develop new imaging and visualization techniques for optical systems in accelerated motion [7, 8, 9, 10], to study the possibility of designing integrated accelerometers based on the effects of acceleration on obliquation or aberration [11, 12, 13], and to understand the corrections that must be applied in software for high-precision astrometric measurements due, for example, to the rotational and the orbital motion of the earth or a satellite [14, 15], if possible without using solar system barycentric velocity [16, 17, 18] or elliptic e-terms [19]. We illustrate the difference between aberration and obliquation in the next section by showing that what Bradley reported in 1729 was not aberration but obliquation. A rigorous treatment of obliquation for an observer in accelerated translational, rotational or gravitational motion is given in subsequent sections.

The treatment begins with a general theory of obliquation in Section §3. We apply the theory to a translating observer in Section §4, to a rotating observer in Section §5, and to a gravitating observer in Section §6. In each case, we study the apparent angular displacement of a light source due to obliquation, the rate of change of this angular displacement with the observer's velocity, the apparent path traced by the light source, the apparent geometry of a light ray from the source to the observer, and the apparent frequency of the ray. The remaining sections in the third part of the work illustrate the types of problems that can be solved by our methods and calculations with a number of interesting results that may be of practical importance.

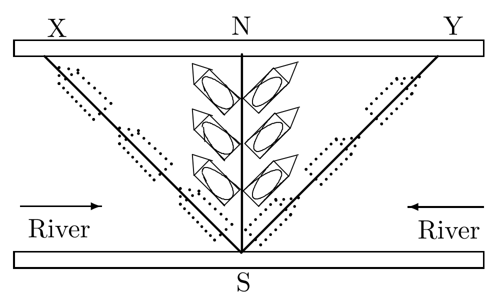
2 Bradley obliquation

Art 3. *The ferryman's problem.*

Consider a ferry travelling from south S to north N at right angles across a river as shown in Figure 2(a). If the river flows due east, the ferry will drift away from SN and will reach the other side of the river at some point E east of N; whereas, if the river flows due west, the ferry will



(a) Obliquation of a ferry. The ferry is aimed along SN (true direction) but drifts along SW or SE (apparent directions) due to the river. It is parallel or untilted to SN on arrival at W or E.



(b) Aberration of a ferry. The ferry is aimed along SX or SY (true directions) but drifts along SN (apparent direction) due to the river. It is inclined or tilted to SN on arrival at N.

Figure 2: Aberration and obliquation of a ferry.

reach the other side of the river at some point W west of N. But as the ferry drifts along SW or SE, it will be pointing in a direction parallel to SN. Similarly, as two light rays aimed in the same direction drift in different directions relative to two relatively moving observers, the normal to their wavefronts will be pointing in the same direction for both observers. We refer to this phenomenon as obliquation. Suppose now that the ferry is to arrive at N in spite of the river,

Figure 2(b). If the river flows due west, then one must aim the ferry at some point Y east of N so that the river will compel it to drift along SN, while if the river flows due east, one must aim the ferry at some point X west of N to compel it to drift along SN. In both cases, as the ferry drifts along SN, it will be pointing in a direction inclined to SN. Similarly, as two light rays aimed in different directions drift in the same direction relative to two relatively moving observers, the normal to their wavefronts will be pointing in different directions for both observers. We refer to this phenomenon as aberration.

Art 4. *Aberration versus obliquation.*

Several things follow from the above illustration. We see clearly that obliquation describes a situation in which one assumes the true direction of a light ray to be the same for two observers in relative motion, and consequently infers that the apparent direction of the ray must be different for the observers. Aberration on the other hand describes a situation in which one assumes the apparent direction of a light ray to be the same for two observers in relative motion, and consequently infers that the true direction of the ray must be different for the observers. We see further that while the wavefront of a light ray is not tilted in obliquation, a telescope that is intended to perceive the ray must be tilted from one direction (SW) to another (SE) when the motion of the observer is reversed. For aberration, on the other hand, we see that although the wavefront of a light ray is tilted, a telescope that is intended to perceive the ray must be pointed in the same direction (SN) when the motion of the observer is reversed. It follows from these

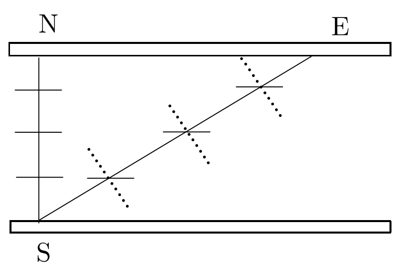


Figure 3: Tilted and untilted wavefronts. When a ray of light aimed along SN drifts relative to a moving observer along SE, its wavefronts remain perpendicular to SN. If the wavefronts were tilted, they would be perpendicular to SE as shown by the dotted lines.

considerations that Bradley's observations [20] correspond to obliquation instead of aberration. This conclusion is historically evident because in Bradley's time, there were no practical means of observing a tilt (illustrated in Figure 3) in the wavefront of light. Hence, as Fresnel [4, 21] pointed out, what Bradley observed was not aberration but obliquation [2].

Art 5. *Formulae for classical aberration and obliquation.*

Let us investigate the above conclusion with some rigour. We consider linearly polarized regular plane light rays from a star at points P and Q relative to an observer at points S and R as shown in Figure 4. If $\mathbf{u}_s, \mathbf{u}_r$ are respectively the velocity of the observer at S and R, and if we assume the observer to be translating with negligible acceleration, then by (1.1), (1.2) and (1.3), the velocity of the rays relative to the observer at these points will be

$$\boldsymbol{v}_s = \mathbf{c}_s - \mathbf{u}_s, \quad \mathbf{c}_s = c\widehat{\boldsymbol{\kappa}}_s \tag{2.1a}$$

$$\boldsymbol{v}_r = \mathbf{c}_r - \mathbf{u}_r, \quad \mathbf{c}_r = c\widehat{\boldsymbol{\kappa}}_r \tag{2.1b}$$

[2] What we call obliquation may also be called "ray aberration" while what we call aberration may also be called "wave aberration". In this terminology, Bradley's observation as well as all modern devices such as those used in adaptive optics measure ray aberration rather than wave aberration. The distinction is vital for any optical system in which the direction of a ray does not coincide with that of a wave normal, although both terms are often used interchangeably or even in different senses by others. We shall continue to use our preferred terminology for clarity.

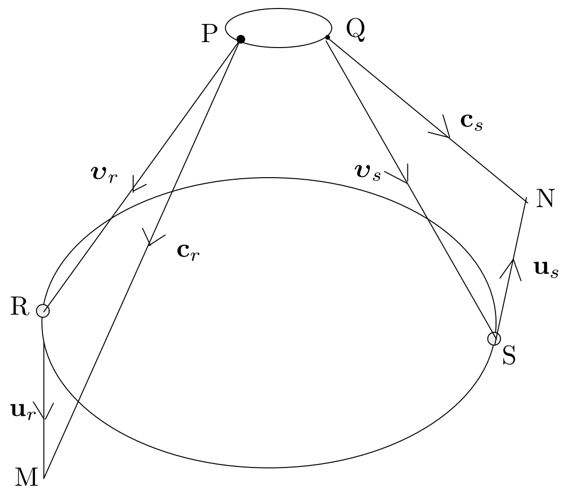


Figure 4: Aberration and obliquation of light. Light rays from a star at P that would have propagated along PM relative to an observer at R propagate instead along PR due to the observer's motion. Rays from the star at Q that would have propagated along QN if the observer were stationary at S propagate instead along QS due to the observer's motion. The angle between PR and QS describes obliquation while the angle between PM and QN describes aberration.

where $\mathbf{c}_s, \mathbf{c}_r$ are the velocities the rays would have if the observer were stationary at S and R respectively. The obliquation angle Φ for the star is the angle between $\boldsymbol{v}_s$ and $\boldsymbol{v}_r$, and is therefore given by

$$\tan\Phi = \frac{|\boldsymbol{v}_s \times \boldsymbol{v}_r|}{\boldsymbol{v}_s \cdot \boldsymbol{v}_r}. \tag{2.2}$$

If we refer to the directions of $\mathbf{c}_s, \mathbf{c}_r$ as true and to the directions of $\boldsymbol{v}_s, \boldsymbol{v}_r$ as apparent, then the angle defined by (2.2) represents a change in the apparent direction of the star. We can also define an aberration angle Ψ

$$\tan\Psi = \frac{|\mathbf{c}_s \times \mathbf{c}_r|}{\mathbf{c}_s \cdot \mathbf{c}_r} \tag{2.3}$$

which represents a change in the true direction of the star.

Art 6. *Obliquation of starlight viewed from the earth.*

As a concrete example, suppose that S and R correspond to the positions of the earth at a six months interval, when the earth must have reversed its direction of motion, so that

$$\mathbf{u}_s = -\mathbf{u}_r = -\mathbf{u} \text{ (say)}. \tag{2.4}$$

Suppose also that the true direction of the star remains unchanged, so that

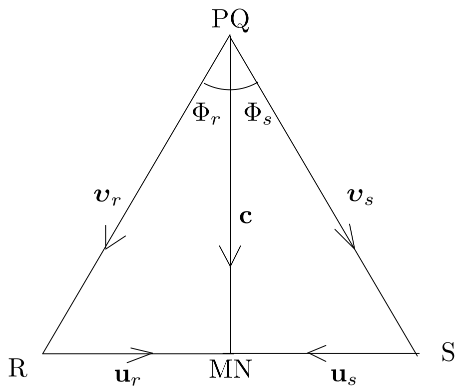
$$\mathbf{c}_s = \mathbf{c}_r = \mathbf{c} = c\widehat{\boldsymbol{\kappa}} \text{ (say)}. \tag{2.5}$$

Then by (2.1) one has $\boldsymbol{v}_s \times \boldsymbol{v}_r = (\mathbf{c}+\mathbf{u}) \times (\mathbf{c}-\mathbf{u}) = 2\mathbf{u} \times \mathbf{c}$ and $\boldsymbol{v}_s \cdot \boldsymbol{v}_r = (\mathbf{c}+\mathbf{u}) \cdot (\mathbf{c}-\mathbf{u}) = c^2 - u^2$. By (2.2), the obliquation angle for the star is

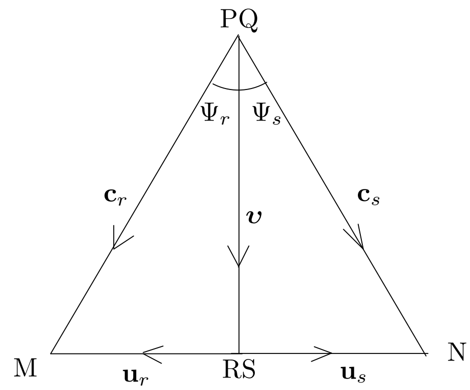
$$\tan\Phi = \frac{2\beta\sin\phi}{1-\beta^2} \tag{2.6a}$$

where ϕ is the angle between $\mathbf{c}$ and $\mathbf{u}$ (see Figure 1). In particular if the true direction of the star is at right angles to the earth's motion, so that $\phi = 90°$, then

$$\tan\Phi = \frac{2\beta}{1-\beta^2} \tag{2.6b}$$



(a) Transverse obliquation. The true direction of the star is assumed to be unchanged (PM ∥ QN) as the earth reverses its direction of motion. As a result, the apparent direction of the star is observed to have changed (PR ∠ QS).



(b) Transverse aberration. The apparent direction of the star is observed to be unchanged (PR ∥ QS) as the earth reverses its direction of motion. As a result, the true direction of the star is assumed to have changed (PM ∠ QN).

Figure 5: Transverse aberration and obliquation.

which agrees with Bradley's observations. The origin of the second order term in this equation is easy to explain [22]. As shown in Figure 5(a), when the observer is at positions S and R, the star is displaced from its true direction by angles Φ_s and Φ_r such that

$$\tan\Phi_s = \tan\Phi_r = \beta. \tag{2.6c}$$

Equation (2.6b) results from applying the trigonometric identity

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \tag{2.7}$$

with $A = \Phi_s, B = \Phi_r$.

Art 7. *Aberration of starlight viewed from the earth.*

Let us calculate the angle of aberration for the star. For this purpose the apparent direction of the star is kept unchanged, as described earlier, so that

$$\boldsymbol{v}_s = \boldsymbol{v}_r = \boldsymbol{v} \text{ (say)}. \tag{2.8}$$

From (2.1), (2.4) and (2.8), one has $\mathbf{c}_s \times \mathbf{c}_r = (\boldsymbol{v} - \mathbf{u}) \times (\boldsymbol{v} + \mathbf{u}) = 2\boldsymbol{v} \times \mathbf{u}$ and $\mathbf{c}_s \cdot \mathbf{c}_r = (\boldsymbol{v} - \mathbf{u}) \cdot (\boldsymbol{v} + \mathbf{u}) = v^2 - u^2$. Substituting into (2.3) gives

$$\tan\Psi = \frac{2(u/v)\sin\psi}{1 - (u/v)^2} \tag{2.9a}$$

where ψ is the angle between $\boldsymbol{v}$ and $\mathbf{u}$. Writing (2.1b) as $\boldsymbol{v} + \mathbf{u} = \mathbf{c}_r$ and squaring gives, after solving the resulting quadratic equation for v and using the fact that $|\mathbf{c}_s| = |\mathbf{c}_r| = c$ by (2.1),

$$u/v = \beta\left(-\beta\cos\psi + \sqrt{1 - \beta^2\sin^2\psi}\right)^{-1}. \tag{2.9b}$$

From (2.9a) and (2.9b), we finally get

$$\tan\Psi = \frac{2\beta\sin\psi\left(-\beta\cos\psi + \sqrt{1-\beta^2\sin^2\psi}\right)}{1+\beta^2(1-2\sin^2\psi) - 2\beta\cos\psi\sqrt{1-\beta^2\sin^2\psi}} \tag{2.10a}$$

as the aberration angle for the star. In particular if the apparent direction of the star is at right angles to the earth's motion, so that $\psi = 90°$, then

$$\tan\Psi = \frac{2\beta}{\sqrt{1-\beta^2}}. \tag{2.10b}$$

As shown in Figure 5(b), this corresponds to a displacement of the star from its true directions at S and R by angles Ψ_s and Ψ_r such that [23, 24]

$$\tan\Psi_s = \tan\Psi_r = \frac{\beta}{\sqrt{1-\beta^2}}. \tag{2.10c}$$

Art 8. *Bradley observed obliquation, not aberration.*

Equations (2.6) and (2.10) show that obliquation and aberration are not only distinct phenomena but also, generally speaking, are of different magnitudes. Moreover, while both (2.6c) and (2.10c) agree with Bradley's observation, (2.6c) describes a change in the apparent direction of a star while (2.10c) describes a change in the true direction of the star. Therefore, since Bradley's observations demonstrate a displacement in the apparent direction of a star, they correspond to (2.6c) rather than (2.10c). We shall confine our attention to phenomena related to obliquation in the remainder of this work.

Art 9. *Obliquation for accelerated observers.*

Equation (2.6c) is somewhat inaccurate because it requires the observer to be translating with a negligible acceleration. To obtain a more accurate equation that does not neglect the observer's acceleration, let us generalize (2.1) by using (1.1) to get

$$\boldsymbol{v}_s = \mathbf{c}_s d_s + \mathbf{w}_s - \mathbf{u}_s, \quad \boldsymbol{v}_r = \mathbf{c}_r d_r + \mathbf{w}_r - \mathbf{u}_r \tag{2.11}$$

and consider the situation when the motion of the observer at S and R is such that

$$\boldsymbol{\kappa}_s = \boldsymbol{\kappa}_r = \boldsymbol{\kappa} \text{ (say)}, \quad \mathbf{u}_s = -\mathbf{u}_r = -\mathbf{u} \text{ (say)}, \quad \mathbf{a}_s = -\mathbf{a}_r = -\mathbf{a} \text{ (say)}. \tag{2.12}$$

Equations (1.2) and (1.3) show that for a translating observer in this situation,

$$\mathbf{c}_s = \mathbf{c}_r = \mathbf{c} \text{ (say)}, \quad d_s = d_r = d \text{ (say)}, \quad \mathbf{w}_s = \mathbf{w}_r = \mathbf{w} \text{ (say)}. \tag{2.13}$$

Substituting (2.11), (2.12) and (2.13) into (2.2) gives

$$\tan\Phi = \frac{|(\mathbf{c}d + \mathbf{w} + \mathbf{u}) \times (\mathbf{c}d + \mathbf{w} - \mathbf{u})|}{(\mathbf{c}d + \mathbf{w} + \mathbf{u}) \cdot (\mathbf{c}d + \mathbf{w} - \mathbf{u})} \tag{2.14}$$

which can be simplified as follows. Let ϕ, θ and λ be the angles shown in Figure 1. Then, from (1.2) and (1.3),

$$\alpha = a\kappa\cos\lambda, \quad \mathbf{w} = \rho(\mathbf{a} - 2\widehat{\boldsymbol{\kappa}}a\cos\lambda) \tag{2.15}$$

and from these equations, we derive

$$\begin{aligned}\mathbf{w}\cdot\mathbf{w} = w^2 &= \rho^2(\mathbf{a} - 2\widehat{\boldsymbol{\kappa}}a\cos\lambda)\cdot(\mathbf{a} - 2\widehat{\boldsymbol{\kappa}}a\cos\lambda) \\ &= \rho^2(a^2 - 4(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{a})a\cos\lambda + 4a^2\cos^2\lambda) \\ &= \rho^2(a^2 - 4a^2\cos^2\lambda + 4a^2\cos^2\lambda) = \rho^2a^2\end{aligned} \tag{2.16a}$$

$$\begin{aligned}\mathbf{w}\cdot\mathbf{c} &= \rho(\mathbf{a}-2\widehat{\boldsymbol{\kappa}}a\cos\lambda)\cdot\mathbf{c}\\ &= \rho(ac\cos\lambda-2ac\cos\lambda) = -\rho ac\cos\lambda\end{aligned}\tag{2.16b}$$

$$\begin{aligned}\mathbf{w}\cdot\mathbf{u} &= \rho(\mathbf{a}-2\widehat{\boldsymbol{\kappa}}a\cos\lambda)\cdot\mathbf{u}\\ &= \rho(au\cos\theta-2(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})a\cos\lambda)\\ &= \rho(au\cos\theta-2au\cos\lambda\cos\phi)\\ &= \rho au(\cos\theta-2\cos\lambda\cos\phi).\end{aligned}\tag{2.16c}$$

Using these equations, the denominator of (2.14) becomes

$$\begin{aligned}(\mathbf{c}d+\mathbf{w}+\mathbf{u})\cdot(\mathbf{c}d+\mathbf{w}-\mathbf{u}) &= c^2d^2+2d\mathbf{w}\cdot\mathbf{c}+\mathbf{w}\cdot\mathbf{w}-u^2\\ &= c^2d^2+2d(-\rho ac\cos\lambda)+\rho^2a^2-u^2\\ &= c^2(d^2-\beta^2+\rho^2\sigma^2-2d\rho\sigma\cos\lambda).\end{aligned}\tag{2.17}$$

Expanding the numerator of (2.14) directly, we have

$$(\mathbf{c}d+\mathbf{w}+\mathbf{u})\times(\mathbf{c}d+\mathbf{w}-\mathbf{u}) = 2d\mathbf{u}\times\mathbf{c}+2\mathbf{u}\times\mathbf{w}.\tag{2.18a}$$

We square this equation to get, in view of (2.16) and (A.2),

$$\begin{aligned}(2d\mathbf{u}\times\mathbf{c}+2\mathbf{u}\times\mathbf{w})^2 &= 4(\mathbf{u}\times\mathbf{w}+d\mathbf{u}\times\mathbf{c})\cdot(\mathbf{u}\times\mathbf{w}+d\mathbf{u}\times\mathbf{c})\\ &= 4[(\mathbf{u}\times\mathbf{w})\cdot(\mathbf{u}\times\mathbf{w})+2d(\mathbf{u}\times\mathbf{w})\cdot(\mathbf{u}\times\mathbf{c})+d^2(\mathbf{u}\times\mathbf{c})\cdot(\mathbf{u}\times\mathbf{c})]\\ &= 4[\{u^2w^2-(\mathbf{u}\cdot\mathbf{w})^2\}+2d\{u^2(\mathbf{w}\cdot\mathbf{c})-(\mathbf{u}\cdot\mathbf{c})(\mathbf{u}\cdot\mathbf{w})\}+d^2\{u^2c^2-(\mathbf{u}\cdot\mathbf{c})^2\}]\\ &= 4[u^2w^2-(\mathbf{u}\cdot\mathbf{w})^2+2du^2(\mathbf{w}\cdot\mathbf{c})-2duc(\mathbf{u}\cdot\mathbf{w})\cos\phi+d^2u^2c^2\sin^2\phi]\\ &= 4[\rho^2a^2u^2-\rho^2a^2u^2(\cos\theta-2\cos\lambda\cos\phi)^2+2du^2(-\rho ac\cos\lambda)\\ &\quad -2d\rho acu^2(\cos\theta-2\cos\lambda\cos\phi)\cos\phi+d^2u^2c^2\sin^2\phi]\\ &= 4\beta^2c^4[\rho^2\sigma^2-2d\rho\sigma\cos\lambda-\rho^2\sigma^2(\cos\theta-2\cos\lambda\cos\phi)^2\\ &\quad -2d\rho\sigma(\cos\theta-2\cos\lambda\cos\phi)\cos\phi+d^2\sin^2\phi].\end{aligned}\tag{2.18b}$$

Substituting (2.17) and the square root of (2.18b) into (2.14) finally gives

$$\tan\Phi = \frac{2\beta\big(\varepsilon_a+d^2\sin^2\phi-\varepsilon_b^2-2d\varepsilon_b\cos\phi\big)^{1/2}}{\varepsilon_a+d^2-\beta^2}\tag{2.19a}$$

where

$$\varepsilon_a = \rho\sigma(\rho\sigma-2d\cos\lambda),\quad \varepsilon_b = \rho\sigma(\cos\theta-2\cos\lambda\cos\phi)\tag{2.19b}$$

and

$$\rho = \frac{\vartheta(1+\vartheta^2)^{-1/2}}{4d\omega_o},\quad d = \left\{\frac{1+\sqrt{1+\vartheta^2}}{2}\right\}^{1/2},\quad \vartheta = \mu\cos\lambda\tag{2.19c}$$

as the complete set of equations describing obliquation for an observer in accelerated translational motion.

Art 10. *Effects of acceleration on obliquation.*

We note that (2.19a) reduces to (2.6a) for a nonaccelerated observer ($a = 0$) or for an observer whose acceleration is perpendicular to the true direction of light ($\cos\lambda = 0$). We note also that the effect of acceleration on the obliquation angle is extremely small and is determined to a large extent by the quantity μ^2. For an observer moving with $a \approx 10\text{ms}^{-2}$ and for visible light with $\kappa \approx 10^7\text{m}^{-1}$ and $c \approx 3\times10^8\text{ms}^{-1}$, we get $\mu^2 \approx 10^{-46}$. This shows that for all practical purposes, translational acceleration has no effect on the obliquation of visible light. Despite the smallness of μ, there are three respects in which (2.19a) differs significantly from (2.6a) from a theoretical viewpoint. First, according to (2.19a), the obliquation angle is a function of wavelength, which means that the apparent direction of a star depends on the wavelength of light emitted by the star, or loosely speaking, on the spectral type of the star [25]. This dispersive effect does not exist for a nonaccelerated observer according to (2.6a). Second, (2.19a) implies that light from a star may be obliquated even when the true direction of the star is parallel to the observer's velocity ($\phi = 0$). A look at (2.6a) shows that this radial effect does not exist for a nonaccelerated observer. Third, (2.19a) shows that if the true direction of a star can be such that [3]

$$\varepsilon_a + d^2 \sin^2\phi - \varepsilon_b^2 - 2d\varepsilon_b \cos\phi = 0, \quad \varepsilon_a + d^2 - \beta^2 \neq 0, \tag{2.20}$$

then light from the star will not be obliquated. This means that the apparent direction of the star as observed on the earth will not change when the earth reverses its direction of motion. All these effects may be observed with reasonable certainty if an astronomical object with a measurable μ can be found. Such will be the case, for example, if the object emits waves with $\kappa \approx a/c^2$ which, for small values of a, implies that the waves must be infraradio waves. At present, the implied frequency of these waves is a great many orders of magnitude below the range of existing and proposed low frequency radio arrays such as LOFAR [26]. It is also well below the ionospheric cutoff frequency and the range of detectable LF radio waves.

3 Obliquation theory

Art 11. *Apparent direction to a light source.*

The method of quantifying obliquation described in the previous section is noninstantaneous because it depends on the velocity of light relative to an observer at two separate instants. To develop a method that depends only on the instantaneous velocity $\boldsymbol{v}$ of light relative to an observer, we note that one only needs to have a standard direction with respect to which the direction of $\boldsymbol{v}$ can be compared. We note further that for practical reasons, it is essential that this standard direction be observable while the observer is moving. This practical requirement rules out the possibility of using the true direction of light as a standard since this direction is not always observable by a moving observer. If we take the direction of the observer's velocity $\mathbf{u}$ as standard under the assumption that the observer can (in principle) determine this velocity by methods that assume nothing regarding the optical properties of light, then we may compare this direction with that of $\boldsymbol{v}$ at any instant by calculating the angle ψ defined by

$$\tan\psi = \frac{|\mathbf{u} \times \boldsymbol{v}|}{\mathbf{u} \cdot \boldsymbol{v}}. \tag{3.1}$$

This definition implies that at any instant, the apparent direction of light is inclined at angle ψ to the observer's velocity $\mathbf{u}$ at that instant. For this reason we shall call ψ the instantaneous angle of obliquation.

For the three types of motion that are of interest to us in this work, we can develop (3.1) by using (1.1) and (1.2) as follow. Let ϕ, θ, λ be the angles shown in Figure 1. Let also

$$\mathcal{B} = \mathbf{e}\cdot\mathbf{c}, \quad \mathcal{D} = \mathbf{e}\cdot\mathbf{u}, \quad \mathcal{A} = \mathbf{e}\cdot\mathbf{a}, \quad \mathcal{H} = \mathbf{e}\cdot\boldsymbol{\kappa}, \quad \mathcal{E} = \mathbf{e}\cdot\mathbf{e} \tag{3.2a}$$

$$\mathcal{L}_0 = \mathcal{D} - u\kappa\tau\cos\phi, \quad \mathcal{L}_1 = \rho a\cos\theta - \kappa\tau\cos\phi, \quad \mathcal{L}_2 = \mathcal{L}_1 + \rho a\cos\theta \tag{3.2b}$$

$$\mathcal{N}_1 = 2du^2(\mathcal{B} - \tau\omega_o), \quad \mathcal{N}_2 = 2duc\mathcal{L}_0\cos\phi, \quad \mathcal{N}_3 = u^2[\mathcal{E} + 2(\rho\mathcal{A} - \tau\mathcal{H})] \tag{3.2c}$$

$$\mathcal{N}_4 = u^2\kappa\tau(\kappa\tau - 2\rho a\cos\lambda), \quad \mathcal{N}_5 = u^2\kappa\tau\mathcal{L}_2\cos\phi, \quad \mathcal{N}_6 = \mathcal{D}(\mathcal{D} + 2u\mathcal{L}_1) \tag{3.2d}$$

[3]We shall sometimes refer to the true direction of light as the line of incidence or *loi*, and to the apparent direction of light as the line of sight or *los*. We emphasize that for an accelerated observer, the direction in which a light ray is incident differs from the direction in which the ray is sighted. Failure to grasp the full implications of this distinction has resulted in many errors being committed by those who are eager to find inadequacies in the classical treatment of the subject.

$$\mathcal{L} = \mathcal{L}_0/(\beta c^2) \tag{3.3a}$$
$$\mathcal{P} = [\mathcal{N}_1 + \mathcal{N}_3 + \mathcal{N}_4 - 2u^2(\mathcal{D} + u\mathcal{L}_1)]/(\beta^2 c^4) \tag{3.3b}$$
$$\mathcal{N} = (\mathcal{N}_1 - \mathcal{N}_2 + \mathcal{N}_3 + \mathcal{N}_4 + \mathcal{N}_5 - \mathcal{N}_6)/(\beta^2 c^4) \tag{3.3c}$$

$$\mathcal{G} = \mathcal{L} - \beta + d\cos\phi + \rho\sigma\cos\theta \tag{3.3d}$$
$$\mathcal{R} = [\mathcal{P} + d^2 + \beta^2 + \rho^2\sigma^2 + 2d(\rho\sigma\cos\lambda - \beta\cos\phi)]^{1/2} \tag{3.3e}$$
$$\mathcal{F} = [\mathcal{N} + d^2\sin^2\phi + \rho^2\sigma^2\sin^2\theta + 2d\rho\sigma(\cos\lambda - \cos\phi\cos\theta)]^{1/2}. \tag{3.3f}$$

Using (1.2a), we derive

$$\begin{aligned}\mathbf{c}\cdot\mathbf{w} &= \mathbf{c}\cdot[\rho\mathbf{a} - \tau\boldsymbol{\kappa} + \mathbf{e}]\\ &= c(\rho a\cos\lambda - \kappa\tau) + \mathcal{B}\end{aligned} \tag{3.4a}$$

$$\begin{aligned}\mathbf{u}\cdot\mathbf{w} &= \mathbf{u}\cdot[\rho\mathbf{a} - \tau\boldsymbol{\kappa} + \mathbf{e}]\\ &= u(\rho a\cos\theta - \kappa\tau\cos\phi) + \mathcal{D}\end{aligned} \tag{3.4b}$$

$$\begin{aligned}\mathbf{w}\cdot\mathbf{w} &= [\rho\mathbf{a} - \tau\boldsymbol{\kappa} + \mathbf{e}]\cdot[\rho\mathbf{a} - \tau\boldsymbol{\kappa} + \mathbf{e}]\\ &= (\rho\mathbf{a} - \tau\boldsymbol{\kappa})^2 + 2\mathbf{e}\cdot(\rho\mathbf{a} - \tau\boldsymbol{\kappa}) + \mathcal{E}\\ &= \rho^2a^2 - 2\rho a\kappa\tau\cos\lambda + \kappa^2\tau^2 + 2(\rho\mathcal{A} - \tau\mathcal{H}) + \mathcal{E}.\end{aligned} \tag{3.4c}$$

Similarly, using (1.1), we obtain[4]

$$\begin{aligned}\upsilon^2 &= (\mathbf{c}d + \mathbf{w} - \mathbf{u})\cdot(\mathbf{c}d + \mathbf{w} - \mathbf{u})\\ &= c^2d^2 + u^2 - 2d(\mathbf{u}\cdot\mathbf{c}) + 2d(\mathbf{w}\cdot\mathbf{c}) - 2\mathbf{w}\cdot\mathbf{u} + w^2\\ &= c^2d^2 + u^2 - 2duc\cos\phi + 2d[\mathcal{B} + c(\rho a\cos\lambda - \kappa\tau)] - 2[\mathcal{D} + u(\rho a\cos\theta - \kappa\tau\cos\phi)]\\ &\quad + \rho^2a^2 - 2\rho a\kappa\tau\cos\lambda + \kappa^2\tau^2 + 2(\rho\mathcal{A} - \tau\mathcal{H}) + \mathcal{E} \text{ by (3.4)}\\ &= c^2d^2 + u^2 + \rho^2a^2 + 2dc(\rho a\cos\lambda - u\cos\phi) + 2d(\mathcal{B} - c\kappa\tau) - 2(\mathcal{D} + u\mathcal{L}_1)\\ &\quad + \kappa\tau(\kappa\tau - 2\rho a\cos\lambda) + 2(\rho\mathcal{A} - \tau\mathcal{H}) + \mathcal{E} \text{ by (3.2b)}\\ &= c^2d^2 + u^2 + \rho^2a^2 + 2dc(\rho a\cos\lambda - u\cos\phi) + u^{-2}\mathcal{N}_1 - 2(\mathcal{D} + u\mathcal{L}_1)\\ &\quad + u^{-2}\mathcal{N}_4 + u^{-2}\mathcal{N}_3 \text{ by (3.2c) \& (3.2d)}\\ &= c^2[d^2 + \beta^2 + \rho^2\sigma^2 + 2d(\rho\sigma\cos\lambda - \beta\cos\phi) + \mathcal{P}] \text{ by (3.3b)}\\ \therefore\ \upsilon &= c\mathcal{R} \text{ by (3.3e).}\end{aligned} \tag{3.5a}$$

The denominator of (3.1) becomes

$$\begin{aligned}\mathbf{u}\cdot\boldsymbol{\upsilon} &= \mathbf{u}\cdot(\mathbf{c}d - \mathbf{u} + \mathbf{w}) \text{ by (1.1)}\\ &= duc\cos\phi - u^2 + u(\rho a\cos\theta - \kappa\tau\cos\phi) + \mathcal{D} \text{ by (3.4b)}\\ &= duc\cos\phi - u^2 + \rho au\cos\theta + (\mathcal{D} - u\kappa\tau\cos\phi)\\ &= duc\cos\phi - u^2 + \rho au\cos\theta + \mathcal{L}_0 \text{ by (3.2b)}\\ &= \beta c^2(d\cos\phi - \beta + \rho\sigma\cos\theta + \mathcal{L}) \text{ by (3.3a)}\\ &= \beta c^2\mathcal{G} \text{ by (3.3d).}\end{aligned} \tag{3.5b}$$

[4]Equation (3.5a) shows that one may regard $1/\mathcal{R}$ as an effective refractive index for the accelerating observer. This suggests an obvious and reasonably comprehensive formalism for solving some problems in classical kineoptics, including in particular, problems of kineoptic refraction or acceleration-induced "light bending". We shall suppose throughout this work that the case $\mathcal{R} = 0$ is to be excluded from general consideration.

Squaring the numerator of (3.1) using (1.1), we have

$$\begin{aligned}(\mathbf{u}\times\boldsymbol{v})^2 &= [\mathbf{u}\times(\mathbf{c}d+\mathbf{w}-\mathbf{u})]\cdot[\mathbf{u}\times(\mathbf{c}d+\mathbf{w}-\mathbf{u})]\\ &= d^2(\mathbf{u}\times\mathbf{c})\cdot(\mathbf{u}\times\mathbf{c})+2d(\mathbf{u}\times\mathbf{c})\cdot(\mathbf{u}\times\mathbf{w})+(\mathbf{u}\times\mathbf{w})\cdot(\mathbf{u}\times\mathbf{w})\\ &= d^2[u^2c^2-(\mathbf{u}\cdot\mathbf{c})^2]+2d[u^2(\mathbf{c}\cdot\mathbf{w})-(\mathbf{u}\cdot\mathbf{w})(\mathbf{u}\cdot\mathbf{c})]+[u^2w^2-(\mathbf{u}\cdot\mathbf{w})^2] \text{ by (A.2)}\\ &= d^2u^2c^2\sin^2\phi+2du^2(\mathbf{c}\cdot\mathbf{w})-2duc(\mathbf{u}\cdot\mathbf{w})\cos\phi+u^2w^2-(\mathbf{u}\cdot\mathbf{w})^2\end{aligned}$$

$$\begin{aligned}&= d^2u^2c^2\sin^2\phi+2du^2[c(\rho a\cos\lambda-\kappa\tau)+\mathcal{B}]-2duc[u(\rho a\cos\theta-\kappa\tau\cos\phi)+\mathcal{D}]\cos\phi\\ &\quad+u^2[\rho^2a^2-2\rho a\kappa\tau\cos\lambda+\kappa^2\tau^2+2(\rho\mathcal{A}-\tau\mathcal{H})+\mathcal{E}]\\ &\quad-[u(\rho a\cos\theta-\kappa\tau\cos\phi)+\mathcal{D}]^2 \text{ by (3.4)}\\ &= d^2u^2c^2\sin^2\phi+2d\rho acu^2\cos\lambda+2du^2(\mathcal{B}-\tau\omega_o)-2d\rho acu^2\cos\phi\cos\theta\\ &\quad-2duc(\mathcal{D}-u\kappa\tau\cos\phi)\cos\phi+\rho^2u^2a^2+u^2\kappa\tau(\kappa\tau-2\rho a\cos\lambda)+u^2[\mathcal{E}+2(\rho\mathcal{A}-\tau\mathcal{H})]\\ &\quad-\rho^2u^2a^2\cos^2\theta+u^2\kappa\tau(2\rho a\cos\theta-\kappa\tau\cos\phi)\cos\phi-\mathcal{D}[\mathcal{D}+2u(\rho a\cos\theta-\kappa\tau\cos\phi)]\end{aligned}$$

$$\begin{aligned}&= d^2u^2c^2\sin^2\phi+2d\rho acu^2\cos\lambda+\mathcal{N}_1-2d\rho acu^2\cos\phi\cos\theta-\mathcal{N}_2\\ &\quad+\rho^2u^2a^2+\mathcal{N}_4+\mathcal{N}_3-\rho^2u^2a^2\cos^2\theta+\mathcal{N}_5-\mathcal{N}_6 \text{ by (1.2b), (3.2c) \& (3.2d)}\\ &= d^2u^2c^2\sin^2\phi+2d\rho acu^2(\cos\lambda-\cos\phi\cos\theta)+\rho^2u^2a^2\sin^2\theta+\beta^2c^4\mathcal{N} \text{ by (3.3)}\\ &= \beta^2c^4[d^2\sin^2\phi+2d\rho\sigma(\cos\lambda-\cos\phi\cos\theta)+\rho^2\sigma^2\sin^2\theta+\mathcal{N}]\\ \therefore |\mathbf{u}\times\boldsymbol{v}| &= \beta c^2\mathcal{F} \text{ by (3.3f).}\end{aligned} \tag{3.5c}$$

We also have

$$\begin{aligned}u^2v^2 &= (\mathbf{u}\cdot\boldsymbol{v})^2+(\mathbf{u}\times\boldsymbol{v})^2 \text{ by (A.3)}\\ u^2c^2\mathcal{R}^2 &= \beta^2c^4\mathcal{G}^2+\beta^2c^4\mathcal{F}^2 \text{ by (3.5a), (3.5b), \& (3.5c)}\\ \therefore \mathcal{R}^2 &= \mathcal{F}^2+\mathcal{G}^2.\end{aligned} \tag{3.5d}$$

Substituting (3.5b) and (3.5c) into (3.1) leads to

$$\tan\psi=\mathcal{F}/\mathcal{G}=\frac{[\mathcal{N}+d^2\sin^2\phi+\rho^2\sigma^2\sin^2\theta+2d\rho\sigma(\cos\lambda-\cos\phi\cos\theta)]^{1/2}}{\mathcal{L}-\beta+d\cos\phi+\rho\sigma\cos\theta} \tag{3.6}$$

which, together with (3.3), allows ψ to be calculated for an observer in accelerated translational, rotational or gravitational motion.

Art 12. *Apparent drift of a light source.*

In general, we are interested not only in ψ but also in the rate at which ψ is changing with $\mathbf{u}$. To quantify this rate, we introduce a quantity $\mathfrak{X}$ defined by

$$\mathfrak{X}=\nabla_u\psi \tag{3.7a}$$

where the subscript on ∇ signifies that the gradient operation is to be performed with respect to $\mathbf{u}$. This definition implies that when the observer's velocity changes from $\mathbf{u}_1$ to $\mathbf{u}_2$, the corresponding change in ψ can be calculated from (3.7a) as

$$\psi=\int_{\mathbf{u}_1}^{\mathbf{u}_2}\mathfrak{X}\cdot\mathrm{d}\mathbf{u}. \tag{3.7b}$$

Moreover, the apparent motion or drift (defined as the total time derivative of ψ) per unit acceleration of the observer is given by

$$\Theta=\dot\psi/a,\quad \dot\psi=\mathfrak{X}\cdot\mathbf{a}. \tag{3.7c}$$

We shall refer to $\mathfrak{X}$ as the slope or gradient of obliquation, and to Θ as the variation of obliquation. To develop (3.7) for the three kinds of motion we are investigating, let

$$\begin{gathered}\nu_1 = \rho/\omega_o, \quad \nu_2 = (\omega_o \gamma)^{-2}, \quad \nu_3 = 2\alpha\nu_2\gamma^{-1}, \quad \nu_4 = (1+\vartheta^2)^{1/2}, \quad \nu_5 = (4d\omega_o\nu_4)^{-1} \\ \nu_6 = (2\alpha\nu_1 - d)/\gamma, \quad \nu_7 = \nu_5\left[\frac{\nu_2}{\nu_4^2} - \frac{\vartheta\nu_1}{d}\right], \quad \nu_8 = \nu_5\left[\frac{\nu_3}{\nu_4^2} - \frac{\vartheta\nu_6}{d}\right]\end{gathered} \tag{3.8a}$$

$$\begin{aligned}&\mathbf{g}_1 = (\boldsymbol{\kappa}\cdot\nabla_u)\,\mathbf{a}, \quad \mathbf{g}_2 = \nabla_u \times \mathbf{a}, \quad \mathbf{g}_3 = (\mathbf{u}\cdot\nabla_u)\,\mathbf{a}, \quad \mathbf{g}_4 = (\boldsymbol{\upsilon}\cdot\nabla_u)\,\mathbf{a}, \quad \mathbf{g}_5 = \nabla_u\gamma \\ &\mathbf{g}_6 = \mathbf{g}_1 + \boldsymbol{\kappa}\times\mathbf{g}_2, \quad \mathbf{g}_7 = \nu_2\mathbf{g}_6 - \nu_3\mathbf{g}_5, \quad \mathbf{g}_8 = \nu_1\mathbf{g}_6 - \nu_6\mathbf{g}_5, \quad \mathbf{g}_9 = \nu_7\mathbf{g}_6 - \nu_8\mathbf{g}_5\end{aligned} \tag{3.8b}$$

$$\begin{aligned}&\boldsymbol{\jmath}_1 = \nabla_u \times \mathbf{e}, \quad \boldsymbol{\jmath}_2 = (\mathbf{u}\cdot\nabla_u)\,\mathbf{e}, \quad \boldsymbol{\jmath}_3 = (\boldsymbol{\upsilon}\cdot\nabla_u)\,\mathbf{e}, \quad \boldsymbol{\jmath}_4 = \boldsymbol{\jmath}_1 + \rho\mathbf{g}_2, \quad \boldsymbol{\jmath}_5 = \boldsymbol{\jmath}_2 + \rho\mathbf{g}_3 \\ &\boldsymbol{\jmath}_6 = \boldsymbol{\jmath}_3 + \rho\mathbf{g}_4, \quad \boldsymbol{\jmath}_7 = \nabla_u\tau, \quad \boldsymbol{\jmath}_8 = (\mathbf{u}\times\boldsymbol{\upsilon})/|\mathbf{u}\times\boldsymbol{\upsilon}|, \quad \boldsymbol{\jmath}_9 = \boldsymbol{\jmath}_8 \times \mathbf{u}, \quad \boldsymbol{\jmath}_0 = \mathcal{G}\boldsymbol{\jmath}_9 - \mathcal{F}\mathbf{u}.\end{aligned} \tag{3.8c}$$

We derive

$$\begin{aligned}\nabla_u\alpha &= \nabla_u(\boldsymbol{\kappa}\cdot\mathbf{a}) \text{ by (1.2b)} \\ &= \boldsymbol{\kappa}\times(\nabla_u\times\mathbf{a}) + (\boldsymbol{\kappa}\cdot\nabla_u)\,\mathbf{a} + (\mathbf{a}\cdot\nabla_u)\,\boldsymbol{\kappa} + \mathbf{a}\times(\nabla_u\times\boldsymbol{\kappa}) \text{ by (A.19)} \\ &= \boldsymbol{\kappa}\times(\nabla_u\times\mathbf{a}) + (\boldsymbol{\kappa}\cdot\nabla_u)\,\mathbf{a} = \mathbf{g}_1 + \boldsymbol{\kappa}\times\mathbf{g}_2 \text{ by (3.8b)} \\ &= \mathbf{g}_6 \text{ by (3.8b)}\end{aligned} \tag{3.9a}$$

$$\begin{aligned}\nabla_u\vartheta &= \nabla_u(\alpha\gamma^{-2}\omega_o^{-2}) \text{ by (1.2b)} \\ &= \omega_o^{-2}\nabla_u(\alpha\gamma^{-2}) \\ &= \omega_o^{-2}(\alpha\nabla_u\gamma^{-2} + \gamma^{-2}\nabla_u\alpha) \text{ by (A.16)} \\ &= \omega_o^{-2}(-2\alpha\gamma^{-3}\nabla_u\gamma + \gamma^{-2}\nabla_u\alpha) \text{ by (A.15)} \\ &= \omega_o^{-2}\gamma^{-2}(\nabla_u\alpha - 2\alpha\gamma^{-1}\nabla_u\gamma) \\ &= \omega_o^{-2}\gamma^{-2}(\mathbf{g}_6 - 2\alpha\gamma^{-1}\mathbf{g}_5) \text{ by (3.9a) \& (3.8b)} \\ &= \mathbf{g}_7 \text{ by (3.8b)}\end{aligned} \tag{3.9b}$$

$$\begin{aligned}\nabla_u d &= \nabla_u\left[\gamma\left\{\frac{1+\sqrt{1+\vartheta^2}}{2}\right\}^{1/2}\right] \text{ by (1.2a)} \\ &= d\gamma^{-1}\nabla_u\gamma + \gamma\nabla_u\left\{\frac{1+\sqrt{1+\vartheta^2}}{2}\right\}^{1/2} \text{ by (A.16) \& (1.2a)} \\ &= d\gamma^{-1}\nabla_u\gamma + \frac{\gamma^2\vartheta(1+\vartheta^2)^{-1/2}}{4d}\nabla_u\vartheta \text{ by (A.15)} \\ &= d\gamma^{-1}\nabla_u\gamma + \rho\omega_o\gamma^2\nabla_u\vartheta \text{ by (1.2b)} \\ &= d\gamma^{-1}\mathbf{g}_5 + \rho\omega_o\gamma^2[\omega_o^{-2}\gamma^{-2}(\mathbf{g}_6 - 2\alpha\gamma^{-1}\mathbf{g}_5)] \text{ by (3.8) \& (3.9b)} \\ &= \omega_o^{-1}[\rho\mathbf{g}_6 - \gamma^{-1}(2\rho\alpha - d\omega_o)\mathbf{g}_5] \\ &= \mathbf{g}_8 \text{ by (3.8b)}\end{aligned} \tag{3.9c}$$

$$
\begin{aligned}
\nabla_u\rho &= (4\omega_o)^{-1}\nabla_u[d^{-1}\vartheta(1+\vartheta^2)^{-1/2}] \text{ by (1.2b)}\\
&= (4\omega_o)^{-1}\{\vartheta(1+\vartheta^2)^{-1/2}\nabla_u d^{-1} + d^{-1}\nabla_u[\vartheta(1+\vartheta^2)^{-1/2}]\} \text{ by (A.16)}\\
&= (4\omega_o)^{-1}[-d^{-2}\vartheta(1+\vartheta^2)^{-1/2}\nabla_u d + d^{-1}(1+\vartheta^2)^{-3/2}\nabla_u\vartheta] \text{ by (A.15)}\\
&= \left[\frac{(1+\vartheta^2)^{-1/2}}{4d\omega_o}\right]\left[-\frac{\vartheta\nabla_u d}{d} + \frac{\nabla_u\vartheta}{1+\vartheta^2}\right] = \nu_5\left[-\frac{\vartheta\nabla_u d}{d} + \frac{\nabla_u\vartheta}{\nu_4^2}\right] \text{ by (1.2b) \& (3.8a)}\\
&= \nu_5[-(\vartheta/d)\mathbf{g}_8 + (1/\nu_4^2)\mathbf{g}_7] \text{ by (3.9b) \& (3.9c)}\\
&= \nu_5[(\vartheta/d)(-\nu_1\mathbf{g}_6 + \nu_6\mathbf{g}_5) + (1/\nu_4^2)(\nu_2\mathbf{g}_6 - \nu_3\mathbf{g}_5)] \text{ by (3.8b)}\\
&= \nu_5[(\nu_2/\nu_4^2) - (\vartheta\nu_1/d)]\mathbf{g}_6 - \nu_5[(\nu_3/\nu_4^2) - (\vartheta\nu_6/d)]\mathbf{g}_5\\
&= \mathbf{g}_9 \text{ by (3.8a) \& (3.8b)} \qquad (3.9\text{d})
\end{aligned}
$$

$$
\begin{aligned}
\nabla_u\times\mathbf{w} &= \nabla_u\times(\rho\mathbf{a} - \tau\boldsymbol{\kappa} + \mathbf{e}) \text{ by (1.2a)}\\
&= \nabla_u\times\mathbf{e} + \nabla_u\times(\rho\mathbf{a}) - \nabla_u\times(\tau\boldsymbol{\kappa})\\
&= \nabla_u\times\mathbf{e} + \rho(\nabla_u\times\mathbf{a}) - \mathbf{a}\times\nabla_u\rho - \tau(\nabla_u\times\boldsymbol{\kappa}) + \boldsymbol{\kappa}\times\nabla_u\tau \text{ by (A.12)}\\
&= \boldsymbol{\jmath}_1 + \rho\mathbf{g}_2 - \mathbf{a}\times\mathbf{g}_9 + \boldsymbol{\kappa}\times\boldsymbol{\jmath}_7 \text{ by (3.8b) \& (3.8c)}\\
&= \boldsymbol{\jmath}_4 + \boldsymbol{\kappa}\times\boldsymbol{\jmath}_7 - \mathbf{a}\times\mathbf{g}_9 \text{ by (3.8c)} \qquad (3.10\text{a})
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{u}\cdot\nabla_u)\,\mathbf{w} &= (\mathbf{u}\cdot\nabla_u)\,(\rho\mathbf{a} - \tau\boldsymbol{\kappa} + \mathbf{e}) \text{ by (1.2a)}\\
&= (\mathbf{u}\cdot\nabla_u)\,\mathbf{e} + (\mathbf{u}\cdot\nabla_u)\,(\rho\mathbf{a}) - (\mathbf{u}\cdot\nabla_u)\,(\tau\boldsymbol{\kappa})\\
&= (\mathbf{u}\cdot\nabla_u)\,\mathbf{e} + \mathbf{a}(\mathbf{u}\cdot\nabla_u\rho) + \rho(\mathbf{u}\cdot\nabla_u)\,\mathbf{a} - \boldsymbol{\kappa}(\mathbf{u}\cdot\nabla_u\tau) - \tau(\mathbf{u}\cdot\nabla_u)\,\boldsymbol{\kappa} \text{ by (A.22)}\\
&= \boldsymbol{\jmath}_2 + \mathbf{a}(\mathbf{u}\cdot\mathbf{g}_9) + \rho\mathbf{g}_3 - \kappa(\mathbf{u}\cdot\boldsymbol{\jmath}_7) \text{ by (3.8b), (3.8c) \& (3.9d)}\\
&= \boldsymbol{\jmath}_5 + \mathbf{a}(\mathbf{u}\cdot\mathbf{g}_9) - \boldsymbol{\kappa}(\mathbf{u}\cdot\boldsymbol{\jmath}_7) \text{ by (3.8c)} \qquad (3.10\text{b})\\
\sim(\boldsymbol{\upsilon}\cdot\nabla_u)\,\mathbf{w} &= \boldsymbol{\jmath}_6 + \mathbf{a}(\boldsymbol{\upsilon}\cdot\mathbf{g}_9) - \boldsymbol{\kappa}(\boldsymbol{\upsilon}\cdot\boldsymbol{\jmath}_7) \qquad (3.10\text{c})
\end{aligned}
$$

$$
\begin{aligned}
\nabla_u\times\boldsymbol{\upsilon} &= \nabla_u\times(\mathbf{c}d + \mathbf{w} - \mathbf{u}) \text{ by (1.1)}\\
&= \nabla_u\times(\mathbf{c}d) + \nabla_u\times\mathbf{w} - \nabla_u\times\mathbf{u}\\
&= d(\nabla_u\times\mathbf{c}) - \mathbf{c}\times(\nabla_u d) + \nabla_u\times\mathbf{w} \text{ by (A.9) \& (A.12)}\\
&= \nabla_u\times\mathbf{w} - \mathbf{c}\times(\nabla_u d)\\
&= \boldsymbol{\jmath}_4 + \boldsymbol{\kappa}\times\boldsymbol{\jmath}_7 - \mathbf{a}\times\mathbf{g}_9 - \mathbf{c}\times\mathbf{g}_8 \text{ by (3.9c) \& (3.10a)} \qquad (3.11\text{a})
\end{aligned}
$$

$$
\begin{aligned}
(\mathbf{u}\cdot\nabla_u)\,\boldsymbol{\upsilon} &= (\mathbf{u}\cdot\nabla_u)\,(\mathbf{c}d + \mathbf{w} - \mathbf{u}) \text{ by (1.1)}\\
&= (\mathbf{u}\cdot\nabla_u)\,(\mathbf{c}d) + (\mathbf{u}\cdot\nabla_u)\,\mathbf{w} - (\mathbf{u}\cdot\nabla_u)\,\mathbf{u}\\
&= \mathbf{c}(\mathbf{u}\cdot\nabla_u d) + d(\mathbf{u}\cdot\nabla_u)\,\mathbf{c} + (\mathbf{u}\cdot\nabla_u)\,\mathbf{w} - \mathbf{u} \text{ by (A.21) \& (A.22)}\\
&= -\mathbf{u} + (\mathbf{u}\cdot\nabla_u)\,\mathbf{w} + \mathbf{c}(\mathbf{u}\cdot\mathbf{g}_8) \text{ by (3.9c)}\\
&= -\mathbf{u} + \boldsymbol{\jmath}_5 + \mathbf{a}(\mathbf{u}\cdot\mathbf{g}_9) - \boldsymbol{\kappa}(\mathbf{u}\cdot\boldsymbol{\jmath}_7) + \mathbf{c}(\mathbf{u}\cdot\mathbf{g}_8) \text{ by (3.10b)} \qquad (3.11\text{b})\\
\sim(\boldsymbol{\upsilon}\cdot\nabla_u)\,\boldsymbol{\upsilon} &= -\boldsymbol{\upsilon} + \boldsymbol{\jmath}_6 + \mathbf{a}(\boldsymbol{\upsilon}\cdot\mathbf{g}_9) - \boldsymbol{\kappa}(\boldsymbol{\upsilon}\cdot\boldsymbol{\jmath}_7) + \mathbf{c}(\boldsymbol{\upsilon}\cdot\mathbf{g}_8) \qquad (3.11\text{c})
\end{aligned}
$$

$$
\begin{aligned}
\nabla_u(\boldsymbol{\upsilon}\cdot\mathbf{u}) &= \boldsymbol{\upsilon} + (\mathbf{u}\cdot\nabla_u)\,\boldsymbol{\upsilon} + \mathbf{u}\times(\nabla_u\times\boldsymbol{\upsilon}) \text{ by (A.17)}\\
&= \boldsymbol{\upsilon} - \mathbf{u} + \boldsymbol{\jmath}_5 + \mathbf{a}(\mathbf{u}\cdot\mathbf{g}_9) - \boldsymbol{\kappa}(\mathbf{u}\cdot\boldsymbol{\jmath}_7) + \mathbf{c}(\mathbf{u}\cdot\mathbf{g}_8)\\
&\quad + \mathbf{u}\times(\boldsymbol{\jmath}_4 + \boldsymbol{\kappa}\times\boldsymbol{\jmath}_7 - \mathbf{a}\times\mathbf{g}_9 - \mathbf{c}\times\mathbf{g}_8) \text{ by (3.11)}\\
&= \boldsymbol{\upsilon} - \mathbf{u} + \boldsymbol{\jmath}_5 + \mathbf{g}_9(\mathbf{u}\cdot\mathbf{a}) - \boldsymbol{\jmath}_7(\mathbf{u}\cdot\boldsymbol{\kappa}) + \mathbf{g}_8(\mathbf{u}\cdot\mathbf{c}) + \mathbf{u}\times\boldsymbol{\jmath}_4 \text{ by (A.1)} \qquad (3.12\text{a})
\end{aligned}
$$

$$
\begin{aligned}
|\mathbf{u}\times\boldsymbol{v}|(\nabla_u|\mathbf{u}\times\boldsymbol{v}|) &= u^2 v(\nabla_u v) + v^2\mathbf{u} - (\boldsymbol{v}\cdot\mathbf{u})\nabla_u(\boldsymbol{v}\cdot\mathbf{u}) \text{ by (A.20)}\\
&= v^2\mathbf{u} + u^2[\boldsymbol{v}\times(\nabla_u\times\boldsymbol{v}) + (\boldsymbol{v}\cdot\nabla_u)\,\boldsymbol{v}] - (\boldsymbol{v}\cdot\mathbf{u})\nabla_u(\boldsymbol{v}\cdot\mathbf{u}) \text{ by (A.18)}\\
&= v^2\mathbf{u} + u^2\boldsymbol{v}\times[\boldsymbol{\jmath}_4 + \boldsymbol{\kappa}\times\boldsymbol{\jmath}_7 - \mathbf{a}\times\mathbf{g}_9 - \mathbf{c}\times\mathbf{g}_8]\\
&\quad + u^2[-\boldsymbol{v} + \boldsymbol{\jmath}_6 + \mathbf{a}(\boldsymbol{v}\cdot\mathbf{g}_9) - \boldsymbol{\kappa}(\boldsymbol{v}\cdot\boldsymbol{\jmath}_7) + \mathbf{c}(\boldsymbol{v}\cdot\mathbf{g}_8)]\\
&\quad - (\boldsymbol{v}\cdot\mathbf{u})[\boldsymbol{v} - \mathbf{u} + \boldsymbol{\jmath}_5 + \mathbf{g}_9(\mathbf{u}\cdot\mathbf{a}) - \boldsymbol{\jmath}_7(\mathbf{u}\cdot\boldsymbol{\kappa})\\
&\qquad + \mathbf{g}_8(\mathbf{u}\cdot\mathbf{c}) + \mathbf{u}\times\boldsymbol{\jmath}_4] \text{ by (3.11) \& (3.12a)}\\
&= v^2\mathbf{u} + u^2(\boldsymbol{v}\times\boldsymbol{\jmath}_4) + u^2[-\boldsymbol{v} + \boldsymbol{\jmath}_6 + \mathbf{g}_9(\boldsymbol{v}\cdot\mathbf{a}) - \boldsymbol{\jmath}_7(\boldsymbol{v}\cdot\boldsymbol{\kappa}) + \mathbf{g}_8(\boldsymbol{v}\cdot\mathbf{c})]\\
&\quad - (\boldsymbol{v}\cdot\mathbf{u})[\boldsymbol{v} - \mathbf{u} + \boldsymbol{\jmath}_5 + \mathbf{g}_9(\mathbf{u}\cdot\mathbf{a}) - \boldsymbol{\jmath}_7(\mathbf{u}\cdot\boldsymbol{\kappa}) + \mathbf{g}_8(\mathbf{u}\cdot\mathbf{c}) + \mathbf{u}\times\boldsymbol{\jmath}_4] \text{ by (A.1)}
\end{aligned}
$$

$$
\begin{aligned}
&= u^2\boldsymbol{\jmath}_6 - (\boldsymbol{v}\cdot\mathbf{u})\boldsymbol{\jmath}_5 + (v^2 + \boldsymbol{v}\cdot\mathbf{u})\mathbf{u} - (u^2 + \boldsymbol{v}\cdot\mathbf{u})\boldsymbol{v}\\
&\quad + [u^2\boldsymbol{v} - (\boldsymbol{v}\cdot\mathbf{u})\mathbf{u}]\times\boldsymbol{\jmath}_4 + [u^2(\boldsymbol{v}\cdot\mathbf{a}) - (\boldsymbol{v}\cdot\mathbf{u})(\mathbf{u}\cdot\mathbf{a})]\mathbf{g}_9\\
&\quad - [u^2(\boldsymbol{v}\cdot\boldsymbol{\kappa}) - (\boldsymbol{v}\cdot\mathbf{u})(\mathbf{u}\cdot\boldsymbol{\kappa})]\boldsymbol{\jmath}_7 + [u^2(\boldsymbol{v}\cdot\mathbf{c}) - (\boldsymbol{v}\cdot\mathbf{u})(\mathbf{u}\cdot\mathbf{c})]\mathbf{g}_8\\
&= u^2\boldsymbol{\jmath}_6 - (\boldsymbol{v}\cdot\mathbf{u})\boldsymbol{\jmath}_5 + (v^2 + \boldsymbol{v}\cdot\mathbf{u})\mathbf{u} - (u^2 + \boldsymbol{v}\cdot\mathbf{u})\boldsymbol{v}\\
&\quad - [\mathbf{u}\times(\mathbf{u}\times\boldsymbol{v})]\times\boldsymbol{\jmath}_4 + [(\mathbf{u}\times\boldsymbol{v})\cdot(\mathbf{u}\times\mathbf{a})]\mathbf{g}_9 \qquad (3.12b)\\
&\quad - [(\mathbf{u}\times\boldsymbol{v})\cdot(\mathbf{u}\times\boldsymbol{\kappa})]\boldsymbol{\jmath}_7 + [(\mathbf{u}\times\boldsymbol{v})\cdot(\mathbf{u}\times\mathbf{c})]\mathbf{g}_8 \text{ by (A.1) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
\nabla_u|\mathbf{u}\times\boldsymbol{v}| &= (\beta/\mathcal{F})\boldsymbol{\jmath}_6 - (\mathcal{G}/\mathcal{F})\boldsymbol{\jmath}_5 + [(\mathcal{R}^2 + \beta\mathcal{G})/(\beta\mathcal{F})]\mathbf{u} - [(\beta + \mathcal{G})/\mathcal{F}]\boldsymbol{v} + \boldsymbol{\jmath}_4\times(\mathbf{u}\times\boldsymbol{\jmath}_8)\\
&\quad + [\boldsymbol{\jmath}_8\cdot(\mathbf{u}\times\mathbf{a})]\mathbf{g}_9 - [\boldsymbol{\jmath}_8\cdot(\mathbf{u}\times\boldsymbol{\kappa})]\boldsymbol{\jmath}_7 + [\boldsymbol{\jmath}_8\cdot(\mathbf{u}\times\mathbf{c})]\mathbf{g}_8 \text{ by (3.12b), (3.5) \& (3.8c)}\\
&= (\beta/\mathcal{F})\boldsymbol{\jmath}_6 - (\mathcal{G}/\mathcal{F})\boldsymbol{\jmath}_5 + [(\mathcal{R}^2 + \beta\mathcal{G})/(\beta\mathcal{F})]\mathbf{u} - [(\beta + \mathcal{G})/\mathcal{F}]\boldsymbol{v}\\
&\quad - \boldsymbol{\jmath}_4\times\boldsymbol{\jmath}_9 + (\mathbf{a}\cdot\boldsymbol{\jmath}_9)\mathbf{g}_9 - (\boldsymbol{\kappa}\cdot\boldsymbol{\jmath}_9)\boldsymbol{\jmath}_7 + (\mathbf{c}\cdot\boldsymbol{\jmath}_9)\mathbf{g}_8 \text{ by (A.4) \& (3.8c)} \qquad (3.12c)
\end{aligned}
$$

$$
\begin{aligned}
\nabla_u\psi &= \nabla_u\left[\arctan\left(\frac{|\mathbf{u}\times\boldsymbol{v}|}{\mathbf{u}\cdot\boldsymbol{v}}\right)\right] \text{ by (3.1)}\\
&= \left[1 + \left(\frac{|\mathbf{u}\times\boldsymbol{v}|}{\mathbf{u}\cdot\boldsymbol{v}}\right)^2\right]^{-1}\nabla_u\left[\frac{|\mathbf{u}\times\boldsymbol{v}|}{\mathbf{u}\cdot\boldsymbol{v}}\right] \text{ by (A.15)}\\
&= \left[\frac{\mathbf{u}\cdot\boldsymbol{v}}{uv}\right]^2\nabla_u\left[\frac{|\mathbf{u}\times\boldsymbol{v}|}{\mathbf{u}\cdot\boldsymbol{v}}\right] \text{ by (A.3)}\\
&= \left[\frac{\mathbf{u}\cdot\boldsymbol{v}}{uv}\right]^2\left[\frac{\nabla_u|\mathbf{u}\times\boldsymbol{v}|}{\mathbf{u}\cdot\boldsymbol{v}} + |\mathbf{u}\times\boldsymbol{v}|\nabla_u(\mathbf{u}\cdot\boldsymbol{v})^{-1}\right] \text{ by (A.16)}\\
&= \left[\frac{\mathbf{u}\cdot\boldsymbol{v}}{uv}\right]^2\left[\frac{\nabla_u|\mathbf{u}\times\boldsymbol{v}|}{\mathbf{u}\cdot\boldsymbol{v}} - \frac{|\mathbf{u}\times\boldsymbol{v}|\nabla_u(\mathbf{u}\cdot\boldsymbol{v})}{(\mathbf{u}\cdot\boldsymbol{v})^2}\right] \text{ by (A.15)}\\
&= (uv)^{-2}[(\mathbf{u}\cdot\boldsymbol{v})\nabla_u|\mathbf{u}\times\boldsymbol{v}| - |\mathbf{u}\times\boldsymbol{v}|\nabla_u(\mathbf{u}\cdot\boldsymbol{v})]\\
&= \beta(u\mathcal{R})^{-2}[\mathcal{G}\nabla_u|\mathbf{u}\times\boldsymbol{v}| - \mathcal{F}\nabla_u(\mathbf{u}\cdot\boldsymbol{v})] \text{ by (3.5).} \qquad (3.12d)
\end{aligned}
$$

From (3.7a), (3.12) and (3.5d), we obtain

$$
\mathfrak{X} = \frac{\beta}{u^2\mathcal{R}^2}\left[\mathcal{X}_1\mathbf{u} - \mathcal{X}_2\boldsymbol{v} + \mathcal{X}_3\boldsymbol{\jmath}_6 - \mathcal{X}_4\boldsymbol{\jmath}_5 + (\mathbf{c}\cdot\boldsymbol{\jmath}_0)\mathbf{g}_8 - (\boldsymbol{\kappa}\cdot\boldsymbol{\jmath}_0)\boldsymbol{\jmath}_7 + (\mathbf{a}\cdot\boldsymbol{\jmath}_0)\mathbf{g}_9 + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4\right] \qquad (3.13a)
$$

$$
\mathcal{X}_1 = \frac{\mathcal{R}^2(\beta + \mathcal{G})}{\beta\mathcal{F}}, \quad \mathcal{X}_2 = \frac{\mathcal{R}^2 + \beta\mathcal{G}}{\mathcal{F}}, \quad \mathcal{X}_3 = \frac{\beta\mathcal{G}}{\mathcal{F}}, \quad \mathcal{X}_4 = \frac{\mathcal{R}^2}{\mathcal{F}}, \quad \mathcal{F} \neq 0 \qquad (3.13b)
$$

which, together with (3.3) and (3.8), allows $\mathfrak{X}$ (and hence Θ as well as $\dot{\psi}$) to be calculated for an observer in accelerated translational, rotational or gravitational motion.

Art 13. *Apparent path of a light source.*

If we draw several arrows from a fixed point in space to represent the ray velocity $\boldsymbol{\upsilon}$ in magnitude and direction at various instants, it is clear that the locus of the endpoints of these arrows will represent the apparent trajectory or path of the light source as viewed by the observer. By treating this locus or hodograph [27] as a curve in velocity space (which amounts to treating $\boldsymbol{\upsilon}$ as the position vector of the observer from the light source), the curvature $\mathbb{K}$ and torsion $\mathbb{T}$ of the apparent path of the light source can be calculated from the formulae

$$\mathbb{K} = \frac{|\dot{\boldsymbol{\upsilon}} \times \ddot{\boldsymbol{\upsilon}}|}{|\dot{\boldsymbol{\upsilon}}|^3}, \qquad \mathbb{T} = \frac{\dddot{\boldsymbol{\upsilon}} \cdot (\dot{\boldsymbol{\upsilon}} \times \ddot{\boldsymbol{\upsilon}})}{|\dot{\boldsymbol{\upsilon}} \times \ddot{\boldsymbol{\upsilon}}|^2}, \quad \mathbb{R} = 1/\mathbb{K} \tag{3.14a}$$

where $\mathbb{R}$ is the radius of curvature of the path for $\mathbb{K} \neq 0$. If the apparent path is smooth or regular, so that $\dot{\boldsymbol{\upsilon}}$ and $\ddot{\boldsymbol{\upsilon}}$ are nonzero everywhere on the path, then the curvature and torsion determine the path up to geometric congruence, or up to a rigid translation and rotation. It is easy to determine the path uniquely by evaluating $\boldsymbol{\upsilon}$ at an initial instant and by computing the Frenet frame at that instant [5]. The tangent vector $\boldsymbol{\ell}_t$, the normal vector $\boldsymbol{\ell}_n$, and the binormal vector $\boldsymbol{\ell}_b$ of the instantaneous Frenet frame at any point on the apparent path of the light source can be computed from the formulae

$$\boldsymbol{\ell}_t = \frac{\dot{\boldsymbol{\upsilon}}}{|\dot{\boldsymbol{\upsilon}}|}, \quad \boldsymbol{\ell}_b = \frac{\dot{\boldsymbol{\upsilon}} \times \ddot{\boldsymbol{\upsilon}}}{|\dot{\boldsymbol{\upsilon}} \times \ddot{\boldsymbol{\upsilon}}|}, \quad \boldsymbol{\ell}_n = \boldsymbol{\ell}_b \times \boldsymbol{\ell}_t, \quad \boldsymbol{\ell}_c = \mathbb{R}\,\boldsymbol{\ell}_n \tag{3.14b}$$

in which $\boldsymbol{\ell}_c$ gives the instantaneous center of curvature of the path, while according to (1.1), (1.2a) and (1.6), $\boldsymbol{\upsilon}$ can be evaluated at any instant from

$$\boldsymbol{\upsilon} = \mathfrak{y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e}. \tag{3.14c}$$

To develop the above equations for the three kinds of motion we are investigating, it is convenient to introduce the quantities

$$\begin{gathered} b_1 = \dot{\rho} - 1, \quad b_2 = 2\dot{\rho} - 1, \quad b_3 = 3\dot{\rho} - 1 \\ \dot{\mathfrak{y}} = c\dot{d} - \kappa\dot{\tau}, \quad \ddot{\mathfrak{y}} = c\ddot{d} - \kappa\ddot{\tau}, \quad \dddot{\mathfrak{y}} = c\dddot{d} - \kappa\dddot{\tau} \\ b_4 = \dddot{\rho}\dot{\mathfrak{y}} - \ddot{\mathfrak{y}}b_1, \quad b_5 = \dot{\mathfrak{y}}b_2 - \rho\ddot{\mathfrak{y}}, \quad b_6 = b_1 b_2 - \rho\ddot{\rho} \end{gathered} \tag{3.15a}$$

$$\begin{gathered} \boldsymbol{\mathfrak{J}}_a = \widehat{\boldsymbol{\kappa}} \times \mathbf{a}, \quad \boldsymbol{\mathfrak{J}}_b = \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_c = \widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_d = \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{e}}, \quad \boldsymbol{\mathfrak{J}}_e = \widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{e}} \\ \boldsymbol{\mathfrak{J}}_f = \mathbf{a} \times \dot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_g = \mathbf{a} \times \ddot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_h = \mathbf{a} \times \dot{\mathbf{e}}, \quad \boldsymbol{\mathfrak{J}}_i = \mathbf{a} \times \ddot{\mathbf{e}} \\ \boldsymbol{\mathfrak{J}}_j = \dot{\mathbf{a}} \times \ddot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_k = \dot{\mathbf{a}} \times \dot{\mathbf{e}}, \quad \boldsymbol{\mathfrak{J}}_l = \dot{\mathbf{a}} \times \ddot{\mathbf{e}}, \quad \boldsymbol{\mathfrak{J}}_n = \dot{\mathbf{e}} \times \ddot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_o = \dot{\mathbf{e}} \times \ddot{\mathbf{e}} \end{gathered} \tag{3.15b}$$

$$\begin{aligned} \boldsymbol{\mathfrak{J}}_p &= b_4\boldsymbol{\mathfrak{J}}_a + b_5\boldsymbol{\mathfrak{J}}_b + \rho\dot{\mathfrak{y}}\boldsymbol{\mathfrak{J}}_c - \ddot{\mathfrak{y}}\boldsymbol{\mathfrak{J}}_d + \dot{\mathfrak{y}}\boldsymbol{\mathfrak{J}}_e + b_6\boldsymbol{\mathfrak{J}}_f + \rho b_1\boldsymbol{\mathfrak{J}}_g \\ \boldsymbol{\mathfrak{J}}_q &= -\ddot{\rho}\boldsymbol{\mathfrak{J}}_h + b_1\boldsymbol{\mathfrak{J}}_i + \rho^2\boldsymbol{\mathfrak{J}}_j - b_2\boldsymbol{\mathfrak{J}}_k + \rho\boldsymbol{\mathfrak{J}}_l + \rho\boldsymbol{\mathfrak{J}}_n + \boldsymbol{\mathfrak{J}}_o \\ \boldsymbol{\mathfrak{J}}_r &= \dot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + b_1\mathbf{a} + \rho\dot{\mathbf{a}} + \dot{\mathbf{e}} \end{aligned} \tag{3.15c}$$

$$\begin{gathered} \aleph_1 = \widehat{\boldsymbol{\kappa}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q), \quad \aleph_2 = \mathbf{a} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q), \quad \aleph_3 = \dot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\ \aleph_4 = \ddot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q), \quad \aleph_5 = \dddot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q), \quad \aleph_6 = \dddot{\mathbf{e}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q). \end{gathered} \tag{3.15d}$$

Using primes to indicate the total time derivatives of complicated expressions for convenience, we derive

$$\begin{aligned} \dot{\boldsymbol{\upsilon}} &= (\mathfrak{y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e})' \text{ by (3.14c)} \\ &= \dot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \dot{\rho}\mathbf{a} + \rho\dot{\mathbf{a}} - \mathbf{a} + \dot{\mathbf{e}} \\ &= \dot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + b_1\mathbf{a} + \rho\dot{\mathbf{a}} + \dot{\mathbf{e}} \text{ by (3.15a)} \\ &= \boldsymbol{\mathfrak{J}}_r \text{ by (3.15c)} \end{aligned} \tag{3.16a}$$

[5] One can also determine the apparent path of the light source by deriving an equation for the hodograph of the ray velocity $\boldsymbol{\upsilon}$. This approach is not taken here because it is less formal and also because it requires the introduction of a coordinate system. Interested readers will find examples of this approach in [22, 28].

$$
\begin{aligned}
\ddot{\boldsymbol{\upsilon}} &= [\dot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + (\dot{\rho}-1)\mathbf{a} + \rho\dot{\mathbf{a}} + \dot{\mathbf{e}}]' \text{ by (3.16a), (3.15a) \& (3.15c)} \\
&= \ddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \ddot{\rho}\mathbf{a} + (\dot{\rho}-1)\dot{\mathbf{a}} + \dot{\rho}\dot{\mathbf{a}} + \rho\ddot{\mathbf{a}} + \ddot{\mathbf{e}} \\
&= \ddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \ddot{\rho}\mathbf{a} + b_2\dot{\mathbf{a}} + \rho\ddot{\mathbf{a}} + \ddot{\mathbf{e}} \text{ by (3.15a)}
\end{aligned}
\tag{3.16b}
$$

$$
\begin{aligned}
\dddot{\boldsymbol{\upsilon}} &= [\ddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \ddot{\rho}\mathbf{a} + (2\dot{\rho}-1)\dot{\mathbf{a}} + \rho\ddot{\mathbf{a}} + \ddot{\mathbf{e}}]' \text{ by (3.16b) \& (3.15a)} \\
&= \dddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \dddot{\rho}\mathbf{a} + \ddot{\rho}\dot{\mathbf{a}} + 2\ddot{\rho}\dot{\mathbf{a}} + (2\dot{\rho}-1)\ddot{\mathbf{a}} + \dot{\rho}\ddot{\mathbf{a}} + \rho\dddot{\mathbf{a}} + \dddot{\mathbf{e}} \\
&= \dddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \dddot{\rho}\mathbf{a} + 3\ddot{\rho}\dot{\mathbf{a}} + b_3\ddot{\mathbf{a}} + \rho\dddot{\mathbf{a}} + \dddot{\mathbf{e}} \text{ by (3.15a)}
\end{aligned}
\tag{3.16c}
$$

$$
\begin{aligned}
\dot{\boldsymbol{\upsilon}} \times \ddot{\boldsymbol{\upsilon}} &= [\dot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + b_1\mathbf{a} + \rho\dot{\mathbf{a}} + \dot{\mathbf{e}}] \times [\ddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \ddot{\rho}\mathbf{a} + b_2\dot{\mathbf{a}} + \rho\ddot{\mathbf{a}} + \ddot{\mathbf{e}}] \text{ by (3.16a), (3.16b) \& (3.15c)} \\
&= \dot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} \times [\ddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \ddot{\rho}\mathbf{a} + b_2\dot{\mathbf{a}} + \rho\ddot{\mathbf{a}} + \ddot{\mathbf{e}}] + b_1\mathbf{a} \times [\ddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \ddot{\rho}\mathbf{a} + b_2\dot{\mathbf{a}} + \rho\ddot{\mathbf{a}} + \ddot{\mathbf{e}}] \\
&\quad + \rho\dot{\mathbf{a}} \times [\ddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \ddot{\rho}\mathbf{a} + b_2\dot{\mathbf{a}} + \rho\ddot{\mathbf{a}} + \ddot{\mathbf{e}}] + \dot{\mathbf{e}} \times [\ddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \ddot{\rho}\mathbf{a} + b_2\dot{\mathbf{a}} + \rho\ddot{\mathbf{a}} + \ddot{\mathbf{e}}] \\
&= \ddot{\rho}\dot{\mathfrak{y}}(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \dot{\mathfrak{y}}b_2(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\dot{\mathfrak{y}}(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \dot{\mathfrak{y}}(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{e}}) + \ddot{\mathfrak{y}}b_1(\mathbf{a} \times \widehat{\boldsymbol{\kappa}}) + b_1b_2(\mathbf{a} \times \dot{\mathbf{a}}) \\
&\quad + \rho b_1(\mathbf{a} \times \ddot{\mathbf{a}}) + b_1(\mathbf{a} \times \ddot{\mathbf{e}}) + \rho\ddot{\mathfrak{y}}(\dot{\mathbf{a}} \times \widehat{\boldsymbol{\kappa}}) + \rho\ddot{\rho}(\dot{\mathbf{a}} \times \mathbf{a}) + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}}) + \rho(\dot{\mathbf{a}} \times \ddot{\mathbf{e}}) \\
&\quad + \ddot{\mathfrak{y}}(\dot{\mathbf{e}} \times \widehat{\boldsymbol{\kappa}}) + \ddot{\rho}(\dot{\mathbf{e}} \times \mathbf{a}) + b_2(\dot{\mathbf{e}} \times \dot{\mathbf{a}}) + \rho(\dot{\mathbf{e}} \times \ddot{\mathbf{a}}) + (\dot{\mathbf{e}} \times \ddot{\mathbf{e}})
\end{aligned}
$$

$$
\begin{aligned}
&= (\ddot{\rho}\dot{\mathfrak{y}} - \ddot{\mathfrak{y}}b_1)(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + (\dot{\mathfrak{y}}b_2 - \rho\ddot{\mathfrak{y}})(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\dot{\mathfrak{y}}(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) - \ddot{\mathfrak{y}}(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{e}}) + \dot{\mathfrak{y}}(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{e}}) \\
&\quad + (b_1b_2 - \rho\ddot{\rho})(\mathbf{a} \times \dot{\mathbf{a}}) + \rho b_1(\mathbf{a} \times \ddot{\mathbf{a}}) - \ddot{\rho}(\mathbf{a} \times \dot{\mathbf{e}}) + b_1(\mathbf{a} \times \ddot{\mathbf{e}}) + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}}) \\
&\quad - b_2(\dot{\mathbf{a}} \times \dot{\mathbf{e}}) + \rho(\dot{\mathbf{a}} \times \ddot{\mathbf{e}}) + \rho(\dot{\mathbf{e}} \times \ddot{\mathbf{a}}) + (\dot{\mathbf{e}} \times \ddot{\mathbf{e}}) \\
&= b_4\boldsymbol{\mathfrak{J}}_a + b_5\boldsymbol{\mathfrak{J}}_b + \rho\dot{\mathfrak{y}}\boldsymbol{\mathfrak{J}}_c - \ddot{\mathfrak{y}}\boldsymbol{\mathfrak{J}}_d + \dot{\mathfrak{y}}\boldsymbol{\mathfrak{J}}_e + b_6\boldsymbol{\mathfrak{J}}_f + \rho b_1\boldsymbol{\mathfrak{J}}_g - \ddot{\rho}\boldsymbol{\mathfrak{J}}_h + b_1\boldsymbol{\mathfrak{J}}_i \\
&\quad + \rho^2\boldsymbol{\mathfrak{J}}_j - b_2\boldsymbol{\mathfrak{J}}_k + \rho\boldsymbol{\mathfrak{J}}_l + \rho\boldsymbol{\mathfrak{J}}_n + \boldsymbol{\mathfrak{J}}_o \text{ by (3.15a) \& (3.15b)} \\
&= \boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q \text{ by (3.15c)}
\end{aligned}
\tag{3.16d}
$$

$$
\begin{aligned}
\dddot{\boldsymbol{\upsilon}} \cdot (\dot{\boldsymbol{\upsilon}} \times \ddot{\boldsymbol{\upsilon}}) &= (\dddot{\mathfrak{y}}\widehat{\boldsymbol{\kappa}} + \dddot{\rho}\mathbf{a} + 3\ddot{\rho}\dot{\mathbf{a}} + b_3\ddot{\mathbf{a}} + \rho\dddot{\mathbf{a}} + \dddot{\mathbf{e}}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.16c) \& (3.16d)} \\
&= \dddot{\mathfrak{y}}[\widehat{\boldsymbol{\kappa}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + \dddot{\rho}[\mathbf{a} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + 3\ddot{\rho}[\dot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] \\
&\quad + b_3[\ddot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + \rho[\dddot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + [\dddot{\mathbf{e}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] \\
&= \dddot{\mathfrak{y}}\aleph_1 + \dddot{\rho}\aleph_2 + 3\ddot{\rho}\aleph_3 + b_3\aleph_4 + \rho\aleph_5 + \aleph_6 \text{ by (3.15d).}
\end{aligned}
\tag{3.16e}
$$

From (1.2b), we have

$$
\dot{\alpha} = \boldsymbol{\kappa} \cdot \dot{\mathbf{a}}, \quad \ddot{\alpha} = \boldsymbol{\kappa} \cdot \ddot{\mathbf{a}}, \quad \dddot{\alpha} = \boldsymbol{\kappa} \cdot \dddot{\mathbf{a}}
\tag{3.17}
$$

so that

$$
\dot{\vartheta} = \frac{1}{\omega_o^2}\left[\frac{\alpha}{\gamma^2}\right]' = \frac{1}{\omega_o^2}\left[\frac{\dot{\alpha}}{\gamma^2} - \frac{2\alpha\dot{\gamma}}{\gamma^3}\right] = \frac{1}{\gamma^2\omega_o^2}\left[\dot{\alpha} - \frac{2\alpha\dot{\gamma}}{\gamma}\right]
\tag{3.18a}
$$

$$
\begin{aligned}
\ddot{\vartheta} &= \left\{\frac{1}{\gamma^2\omega_o^2}\left[\dot{\alpha} - \frac{2\alpha\dot{\gamma}}{\gamma}\right]\right\}' \text{ by (3.18a)} \\
&= \left[\frac{1}{\gamma^2\omega_o^2}\right]'\left[\dot{\alpha} - \frac{2\alpha\dot{\gamma}}{\gamma}\right] + \frac{1}{\gamma^2\omega_o^2}\left[\dot{\alpha} - \frac{2\alpha\dot{\gamma}}{\gamma}\right]' \\
&= \frac{-2\dot{\gamma}}{\gamma^2\omega_o^2}\left[\frac{\dot{\alpha}}{\gamma} - \frac{2\alpha\dot{\gamma}}{\gamma^2}\right] + \frac{1}{\gamma^2\omega_o^2}\left[\ddot{\alpha} - \frac{2\dot{\alpha}\dot{\gamma}}{\gamma} - \frac{2\alpha\ddot{\gamma}}{\gamma} + \frac{2\alpha\dot{\gamma}^2}{\gamma^2}\right] \\
&= \frac{1}{\gamma^2\omega_o^2}\left[\ddot{\alpha} - \frac{4\dot{\alpha}\dot{\gamma}}{\gamma} - \frac{2\alpha\ddot{\gamma}}{\gamma} + \frac{6\alpha\dot{\gamma}^2}{\gamma^2}\right]
\end{aligned}
\tag{3.18b}
$$

$$
\begin{aligned}
\dddot{\vartheta} &= \left\{\frac{1}{\gamma^2\omega_o^2}\left[\ddot{\alpha} - \frac{4\dot{\alpha}\dot{\gamma}}{\gamma} - \frac{2\alpha\ddot{\gamma}}{\gamma} + \frac{6\alpha\dot{\gamma}^2}{\gamma^2}\right]\right\}' \text{ by (3.18b)} \\
&= \frac{1}{\gamma^2\omega_o^2}\left[\ddot{\alpha} - \frac{4\dot{\alpha}\dot{\gamma}}{\gamma} - \frac{2\alpha\ddot{\gamma}}{\gamma} + \frac{6\alpha\dot{\gamma}^2}{\gamma^2}\right]' + \left[\frac{1}{\gamma^2\omega_o^2}\right]'\left[\ddot{\alpha} - \frac{4\dot{\alpha}\dot{\gamma}}{\gamma} - \frac{2\alpha\ddot{\gamma}}{\gamma} + \frac{6\alpha\dot{\gamma}^2}{\gamma^2}\right] \\
&= \frac{1}{\gamma^2\omega_o^2}\left[\dddot{\alpha} - \frac{4\ddot{\alpha}\dot{\gamma}}{\gamma} - \frac{4\dot{\alpha}\ddot{\gamma}}{\gamma} + \frac{4\dot{\alpha}\dot{\gamma}^2}{\gamma^2} - \frac{2\dot{\alpha}\ddot{\gamma}}{\gamma} - \frac{2\alpha\dddot{\gamma}}{\gamma} + \frac{2\alpha\dot{\gamma}\ddot{\gamma}}{\gamma^2} + \frac{6\dot{\alpha}\dot{\gamma}^2}{\gamma^2} + \frac{12\alpha\dot{\gamma}\ddot{\gamma}}{\gamma^2} - \frac{12\alpha\dot{\gamma}^3}{\gamma^3}\right] \\
&\quad + \frac{-2\dot{\gamma}}{\gamma^2\omega_o^2}\left[\frac{\ddot{\alpha}}{\gamma} - \frac{4\dot{\alpha}\dot{\gamma}}{\gamma^2} - \frac{2\alpha\ddot{\gamma}}{\gamma^2} + \frac{6\alpha\dot{\gamma}^2}{\gamma^3}\right] \\
&= \frac{1}{\gamma^2\omega_o^2}\left[\dddot{\alpha} - \frac{6\ddot{\alpha}\dot{\gamma}}{\gamma} - \frac{6\dot{\alpha}\ddot{\gamma}}{\gamma} + \frac{18\dot{\alpha}\dot{\gamma}^2}{\gamma^2} - \frac{2\alpha\dddot{\gamma}}{\gamma} + \frac{18\alpha\dot{\gamma}\ddot{\gamma}}{\gamma^2} - \frac{24\alpha\dot{\gamma}^3}{\gamma^3}\right] \qquad (3.18c)
\end{aligned}
$$

and

$$
\begin{aligned}
\dot{\pi} &= [\vartheta(1+\vartheta^2)^{-1/2}]' \text{ by (1.2b)} \\
&= \dot{\vartheta}(1+\vartheta^2)^{-1/2} + \vartheta[(1+\vartheta^2)^{-1/2}]' \\
&= \dot{\vartheta}(1+\vartheta^2)^{-1/2} - \vartheta^2\dot{\vartheta}(1+\vartheta^2)^{-3/2} \\
&= \dot{\vartheta}(1+\vartheta^2)^{-3/2} \qquad (3.19a)
\end{aligned}
$$

$$
\begin{aligned}
\ddot{\pi} &= [\dot{\vartheta}(1+\vartheta^2)^{-3/2}]' \text{ by (3.19a)} \\
&= \ddot{\vartheta}(1+\vartheta^2)^{-3/2} + \dot{\vartheta}[(1+\vartheta^2)^{-3/2}]' \\
&= \ddot{\vartheta}(1+\vartheta^2)^{-3/2} - 3\vartheta\dot{\vartheta}^2(1+\vartheta^2)^{-5/2} \\
&= (1+\vartheta^2)^{-5/2}[\ddot{\vartheta}(1+\vartheta^2) - 3\vartheta\dot{\vartheta}^2] \qquad (3.19b)
\end{aligned}
$$

$$
\begin{aligned}
\dddot{\pi} &= \{(1+\vartheta^2)^{-5/2}[\ddot{\vartheta}(1+\vartheta^2) - 3\vartheta\dot{\vartheta}^2]\}' \text{ by (3.19b)} \\
&= [(1+\vartheta^2)^{-5/2}]'[\ddot{\vartheta}(1+\vartheta^2) - 3\vartheta\dot{\vartheta}^2] + (1+\vartheta^2)^{-5/2}[\ddot{\vartheta}(1+\vartheta^2) - 3\vartheta\dot{\vartheta}^2]' \\
&= -5\vartheta\dot{\vartheta}(1+\vartheta^2)^{-7/2}[\ddot{\vartheta}(1+\vartheta^2) - 3\vartheta\dot{\vartheta}^2] + (1+\vartheta^2)^{-5/2}[\dddot{\vartheta}(1+\vartheta^2) - 4\vartheta\dot{\vartheta}\ddot{\vartheta} - 3\dot{\vartheta}^3] \\
&= (1+\vartheta^2)^{-7/2}\{-5\vartheta\dot{\vartheta}[\ddot{\vartheta}(1+\vartheta^2) - 3\vartheta\dot{\vartheta}^2] + (1+\vartheta^2)[\dddot{\vartheta}(1+\vartheta^2) - 3\dot{\vartheta}^3 - 4\vartheta\dot{\vartheta}\ddot{\vartheta}]\} \\
&= (1+\vartheta^2)^{-7/2}\{-5\vartheta\dot{\vartheta}\ddot{\vartheta} - 5\vartheta^3\dot{\vartheta}\ddot{\vartheta} + 15\vartheta^2\dot{\vartheta}^3 \\
&\qquad + \dddot{\vartheta}(1+\vartheta^2)^2 - 3\dot{\vartheta}^3 - 4\vartheta\dot{\vartheta}\ddot{\vartheta} - 3\vartheta^2\dot{\vartheta}^3 - 4\vartheta^3\dot{\vartheta}\ddot{\vartheta}\} \\
&= (1+\vartheta^2)^{-7/2}[\dddot{\vartheta}(1+\vartheta^2)^2 - 3\dot{\vartheta}^3 - 9\vartheta\dot{\vartheta}\ddot{\vartheta} + 12\vartheta^2\dot{\vartheta}^3 - 9\vartheta^3\dot{\vartheta}\ddot{\vartheta}] \\
&= (1+\vartheta^2)^{-7/2}[\dddot{\vartheta}(1+\vartheta^2)^2 - 9\vartheta\dot{\vartheta}\ddot{\vartheta}(1+\vartheta^2) - 3\dot{\vartheta}^3(1-4\vartheta^2)]. \qquad (3.19c)
\end{aligned}
$$

We have also that

$$
\begin{aligned}
\dot{d} &= \left[\gamma\left\{\frac{1+\sqrt{1+\vartheta^2}}{2}\right\}^{1/2}\right]' \text{ by (1.2a)} \\
&= \dot{\gamma}\left[\frac{1+\sqrt{1+\vartheta^2}}{2}\right]^{1/2} + \gamma\left[\frac{\vartheta\dot{\vartheta}(1+\vartheta^2)^{-1/2}}{4}\right]\left[\frac{1+\sqrt{1+\vartheta^2}}{2}\right]^{-1/2} \\
&= \dot{\gamma}(d/\gamma) + \gamma(\gamma/d)\left[\frac{\vartheta\dot{\vartheta}(1+\vartheta^2)^{-1/2}}{4}\right] \text{ by (1.2a)} \\
&= \frac{\dot{\gamma}d}{\gamma} + \frac{\pi\gamma^2\dot{\vartheta}}{4d} \text{ by (1.2b)} \qquad (3.20a)
\end{aligned}
$$

$$\begin{aligned}
\ddot{d} &= \left[\frac{\dot{\gamma}d}{\gamma} + \frac{\pi\gamma^2\dot{\vartheta}}{4d}\right]' \text{ by (3.20a)} \\
&= \frac{\ddot{\gamma}d}{\gamma} + \frac{\dot{\gamma}\dot{d}}{\gamma} - \frac{\dot{\gamma}^2 d}{\gamma^2} + \frac{1}{4}\left[\frac{\dot{\pi}\gamma^2\dot{\vartheta}}{d} + \frac{2\pi\gamma\dot{\gamma}\dot{\vartheta}}{d} + \frac{\pi\gamma^2\ddot{\vartheta}}{d} - \frac{\pi\gamma^2\dot{d}\dot{\vartheta}}{d^2}\right] \\
&= d\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2}\right] + \dot{d}\left[\frac{\dot{\gamma}}{\gamma} - \frac{\pi\gamma^2\dot{\vartheta}}{4d^2}\right] + \frac{\gamma}{4d}\left[\dot{\pi}\gamma\dot{\vartheta} + 2\pi\dot{\gamma}\dot{\vartheta} + \pi\gamma\ddot{\vartheta}\right] \qquad (3.20b)
\end{aligned}$$

$$\begin{aligned}
\dddot{d} &= \left\{d\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2}\right] + \dot{d}\left[\frac{\dot{\gamma}}{\gamma} - \frac{\pi\gamma^2\dot{\vartheta}}{4d^2}\right] + \frac{\gamma}{4d}\left[\dot{\pi}\gamma\dot{\vartheta} + 2\pi\dot{\gamma}\dot{\vartheta} + \pi\gamma\ddot{\vartheta}\right]\right\}' \text{ by (3.20b)} \\
&= \dot{d}\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2}\right] + d\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2}\right]' + \ddot{d}\left[\frac{\dot{\gamma}}{\gamma} - \frac{\pi\gamma^2\dot{\vartheta}}{4d^2}\right] + \dot{d}\left[\frac{\dot{\gamma}}{\gamma} - \frac{\pi\gamma^2\dot{\vartheta}}{4d^2}\right]' \\
&\quad + \left[\frac{\gamma}{4d}\right]'\left[\dot{\pi}\gamma\dot{\vartheta} + 2\pi\dot{\gamma}\dot{\vartheta} + \pi\gamma\ddot{\vartheta}\right] + \frac{\gamma}{4d}\left[\dot{\pi}\gamma\dot{\vartheta} + 2\pi\dot{\gamma}\dot{\vartheta} + \pi\gamma\ddot{\vartheta}\right]'
\end{aligned}$$

$$\begin{aligned}
&= d\left[\frac{\dddot{\gamma}}{\gamma} - \frac{\dot{\gamma}\ddot{\gamma}}{\gamma^2} - \frac{2\dot{\gamma}\ddot{\gamma}}{\gamma^2} + \frac{2\dot{\gamma}^3}{\gamma^3}\right] + \dot{d}\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2}\right] + \dot{d}\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2} - \frac{\dot{\pi}\gamma^2\dot{\vartheta}}{4d^2} - \frac{2\pi\gamma\dot{\gamma}\dot{\vartheta}}{4d^2} - \frac{\pi\gamma^2\ddot{\vartheta}}{4d^2} + \frac{2\pi\gamma^2\dot{d}\dot{\vartheta}}{4d^3}\right] \\
&\quad + \ddot{d}\left[\frac{\dot{\gamma}}{\gamma} - \frac{\pi\gamma^2\dot{\vartheta}}{4d^2}\right] + \left[\frac{\dot{\gamma}}{4d} - \frac{\gamma\dot{d}}{4d^2}\right]\left[\dot{\pi}\gamma\dot{\vartheta} + 2\pi\dot{\gamma}\dot{\vartheta} + \pi\gamma\ddot{\vartheta}\right] \\
&\quad + \frac{\gamma}{4d}\left[\ddot{\pi}\gamma\dot{\vartheta} + \dot{\pi}\dot{\gamma}\dot{\vartheta} + \dot{\pi}\gamma\ddot{\vartheta} + 2\dot{\pi}\dot{\gamma}\dot{\vartheta} + 2\pi\ddot{\gamma}\dot{\vartheta} + 2\pi\dot{\gamma}\ddot{\vartheta} + \dot{\pi}\gamma\ddot{\vartheta} + \pi\dot{\gamma}\ddot{\vartheta} + \pi\gamma\dddot{\vartheta}\right] \\
&= d\left[\frac{\dddot{\gamma}}{\gamma} - \frac{3\dot{\gamma}\ddot{\gamma}}{\gamma^2} + \frac{2\dot{\gamma}^3}{\gamma^3}\right] + 2\dot{d}\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2} - \frac{\dot{\pi}\gamma^2\dot{\vartheta}}{4d^2} - \frac{\pi\gamma\dot{\gamma}\dot{\vartheta}}{2d^2} - \frac{\pi\gamma^2\ddot{\vartheta}}{4d^2} + \frac{\pi\gamma^2\dot{d}\dot{\vartheta}}{4d^3}\right] + \ddot{d}\left[\frac{\dot{\gamma}}{\gamma} - \frac{\pi\gamma^2\dot{\vartheta}}{4d^2}\right] \\
&\quad + \frac{\dot{\gamma}}{4d}\left[\dot{\pi}\gamma\dot{\vartheta} + 2\pi\dot{\gamma}\dot{\vartheta} + \pi\gamma\ddot{\vartheta}\right] + \frac{\gamma}{4d}\left[\ddot{\pi}\gamma\dot{\vartheta} + 3\dot{\pi}\dot{\gamma}\dot{\vartheta} + 2\dot{\pi}\gamma\ddot{\vartheta} + 2\pi\ddot{\gamma}\dot{\vartheta} + 3\pi\dot{\gamma}\ddot{\vartheta} + \pi\gamma\dddot{\vartheta}\right]
\end{aligned}$$

(3.20c)

$$\begin{aligned}
\dot{\rho} &= \left[\frac{\pi}{4d\omega_o}\right]' \text{ by (1.2b)} \\
&= \frac{\dot{\pi}}{4d\omega_o} - \frac{\pi\dot{d}}{4d^2\omega_o} = \frac{d\dot{\pi} - \pi\dot{d}}{4\omega_o d^2} \qquad (3.21a) \\
\ddot{\rho} &= \left\{\frac{1}{4\omega_o d^2}\left[d\dot{\pi} - \pi\dot{d}\right]\right\}' \text{ by (3.21a)} \\
&= \left[\frac{1}{4\omega_o d^2}\right]'\left[d\dot{\pi} - \pi\dot{d}\right] + \frac{1}{4\omega_o d^2}\left[d\dot{\pi} - \pi\dot{d}\right]' = -\frac{\dot{d}(d\dot{\pi} - \pi\dot{d})}{2\omega_o d^3} + \frac{d\ddot{\pi} - \pi\ddot{d}}{4\omega_o d^2} \qquad (3.21b)
\end{aligned}$$

$$\begin{aligned}
\dddot{\rho} &= \left\{-\frac{\dot{d}}{2\omega_o d^3}\left[d\dot{\pi} - \pi\dot{d}\right] + \frac{1}{4\omega_o d^2}\left[d\ddot{\pi} - \pi\ddot{d}\right]\right\}' \text{ by (3.21b)} \\
&= \left[-\frac{\dot{d}}{2\omega_o d^3}\right]'\left[d\dot{\pi} - \pi\dot{d}\right] - \frac{\dot{d}}{2\omega_o d^3}\left[d\dot{\pi} - \pi\dot{d}\right]' + \left[\frac{1}{4\omega_o d^2}\right]'\left[d\ddot{\pi} - \pi\ddot{d}\right] + \frac{1}{4\omega_o d^2}\left[d\ddot{\pi} - \pi\ddot{d}\right]'
\end{aligned}$$

$$
\begin{aligned}
&= \left[-\frac{\ddot d}{2\omega_o d^3} + \frac{3\dot d^2}{2\omega_o d^4}\right]\left[d\dot\pi - \pi\dot d\right] - \frac{\dot d}{2\omega_o d^3}\left[d\ddot\pi - \pi\ddot d\right] - \frac{\dot d}{2\omega_o d^3}\left[d\ddot\pi - \pi\ddot d\right] \\
&\quad + \frac{1}{4\omega_o d^2}\left[\dot d\ddot\pi + d\dddot\pi - \dot\pi\ddot d - \pi\dddot d\right] \\
&= \frac{(3\dot d^2 - d\ddot d)(d\dot\pi - \pi\dot d)}{2\omega_o d^4} - \frac{\dot d(d\ddot\pi - \pi\ddot d)}{\omega_o d^3} + \frac{\dot d\ddot\pi + d\dddot\pi - \dot\pi\ddot d - \pi\dddot d}{4\omega_o d^2}.
\end{aligned}
\tag{3.21c}
$$

From (3.14) and (3.16), we get

$$
\mathbb{K} = \frac{|\boldsymbol{\mathcal{J}}_p + \boldsymbol{\mathcal{J}}_q|}{|\boldsymbol{\mathcal{J}}_r|^3}, \quad \mathbb{T} = \frac{\dddot{\mathfrak{y}}\aleph_1 + \dddot\rho\aleph_2 + 3\ddot\rho\aleph_3 + b_3\aleph_4 + \rho\aleph_5 + \aleph_6}{|\boldsymbol{\mathcal{J}}_p + \boldsymbol{\mathcal{J}}_q|^2} \tag{3.22a}
$$

$$
\boldsymbol{\ell}_t = \frac{\boldsymbol{\mathcal{J}}_r}{|\boldsymbol{\mathcal{J}}_r|}, \quad \boldsymbol{\ell}_b = \frac{\boldsymbol{\mathcal{J}}_p + \boldsymbol{\mathcal{J}}_q}{|\boldsymbol{\mathcal{J}}_p + \boldsymbol{\mathcal{J}}_q|}, \quad \boldsymbol{\ell}_n = \boldsymbol{\ell}_b \times \boldsymbol{\ell}_t \tag{3.22b}
$$

which, together with (3.15), is the complete set of equations describing the apparent path of the light source. To use these equations, one should first compute the quantities $\dot{\mathbf{a}}, \ddot{\mathbf{a}}, \dddot{\mathbf{a}}$ and $\dot\gamma, \ddot\gamma, \dddot\gamma$ for the type of observer's motion in question, followed by a computation of the quantities given by (3.17) through (3.21). Then $\dot\tau, \ddot\tau, \dddot\tau$ and $\dot{\mathbf{e}}, \ddot{\mathbf{e}}, \dddot{\mathbf{e}}$ as well as the quantities defined in (3.15) can be computed, after which (3.22) can finally be evaluated.

Art 14. *Apparent geometry of obliquated rays.*

Regarding a light ray as the curve described by a point moving with the ray velocity $\boldsymbol{\upsilon}$ in configuration space, we see that the curvature $\overline{\mathbb{K}}$, the torsion $\overline{\mathbb{T}}$ and the radius of curvature $\overline{\mathbb{R}}$ of the ray can be computed from the formulae

$$
\overline{\mathbb{K}} = \frac{|\boldsymbol{\upsilon} \times \dot{\boldsymbol{\upsilon}}|}{|\boldsymbol{\upsilon}|^3}, \qquad \overline{\mathbb{T}} = \frac{\ddot{\boldsymbol{\upsilon}} \cdot (\boldsymbol{\upsilon} \times \dot{\boldsymbol{\upsilon}})}{|\boldsymbol{\upsilon} \times \dot{\boldsymbol{\upsilon}}|^2}, \quad \overline{\mathbb{R}} = 1/\overline{\mathbb{K}} \tag{3.23a}
$$

while the tangent vector $\overline{\boldsymbol{\ell}}_t$, the normal vector $\overline{\boldsymbol{\ell}}_n$, and the binormal vector $\overline{\boldsymbol{\ell}}_b$ of the instantaneous Frenet frame at any point on the ray can be computed from the formulae

$$
\overline{\boldsymbol{\ell}}_t = \frac{\boldsymbol{\upsilon}}{|\boldsymbol{\upsilon}|}, \quad \overline{\boldsymbol{\ell}}_b = \frac{\boldsymbol{\upsilon} \times \dot{\boldsymbol{\upsilon}}}{|\boldsymbol{\upsilon} \times \dot{\boldsymbol{\upsilon}}|}, \quad \overline{\boldsymbol{\ell}}_n = \overline{\boldsymbol{\ell}}_b \times \overline{\boldsymbol{\ell}}_t, \quad \overline{\boldsymbol{\ell}}_c = \overline{\mathbb{R}}\,\overline{\boldsymbol{\ell}}_n \tag{3.23b}
$$

where $\overline{\boldsymbol{\ell}}_c$ gives the instantaneous center of curvature of the ray [6]. To develop these equations, we introduce

$$
\begin{gathered}
\boldsymbol{\mathcal{S}}_a = \widehat{\boldsymbol{\kappa}} \times \mathbf{u}, \quad \boldsymbol{\mathcal{S}}_b = \widehat{\boldsymbol{\kappa}} \times \mathbf{a}, \quad \boldsymbol{\mathcal{S}}_c = \widehat{\boldsymbol{\kappa}} \times \mathbf{e}, \quad \boldsymbol{\mathcal{S}}_d = \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}, \quad \boldsymbol{\mathcal{S}}_e = \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{e}} \\
\boldsymbol{\mathcal{S}}_f = \mathbf{a} \times \mathbf{u}, \quad \boldsymbol{\mathcal{S}}_g = \mathbf{a} \times \mathbf{e}, \quad \boldsymbol{\mathcal{S}}_h = \mathbf{a} \times \dot{\mathbf{a}}, \quad \boldsymbol{\mathcal{S}}_i = \mathbf{a} \times \dot{\mathbf{e}} \\
\boldsymbol{\mathcal{S}}_j = \mathbf{u} \times \dot{\mathbf{a}}, \quad \boldsymbol{\mathcal{S}}_k = \mathbf{u} \times \dot{\mathbf{e}}, \quad \boldsymbol{\mathcal{S}}_l = \mathbf{e} \times \dot{\mathbf{a}}, \quad \boldsymbol{\mathcal{S}}_m = \mathbf{e} \times \dot{\mathbf{e}}
\end{gathered}
\tag{3.24a}
$$

$$
\begin{gathered}
\boldsymbol{\mathcal{S}}_n = \rho\boldsymbol{\mathcal{S}}_d + \boldsymbol{\mathcal{S}}_e, \quad \boldsymbol{\mathcal{S}}_o = \rho\boldsymbol{\mathcal{S}}_h + \boldsymbol{\mathcal{S}}_i, \quad \boldsymbol{\mathcal{S}}_p = \boldsymbol{\mathcal{S}}_a - \boldsymbol{\mathcal{S}}_c \\
\boldsymbol{\mathcal{S}}_q = \boldsymbol{\mathcal{S}}_f - \boldsymbol{\mathcal{S}}_g, \quad \boldsymbol{\mathcal{S}}_r = \boldsymbol{\mathcal{S}}_l - \boldsymbol{\mathcal{S}}_j, \quad \boldsymbol{\mathcal{S}}_s = \boldsymbol{\mathcal{S}}_m - \boldsymbol{\mathcal{S}}_k \\
\boldsymbol{\mathcal{S}}_t = (\mathfrak{y}b_1 - \rho\dot{\mathfrak{y}})\boldsymbol{\mathcal{S}}_b + \mathfrak{y}\boldsymbol{\mathcal{S}}_n + \rho\boldsymbol{\mathcal{S}}_o, \quad \boldsymbol{\mathcal{S}}_u = \dot{\mathfrak{y}}\boldsymbol{\mathcal{S}}_p + b_1\boldsymbol{\mathcal{S}}_q + \rho\boldsymbol{\mathcal{S}}_r + \boldsymbol{\mathcal{S}}_s
\end{gathered}
\tag{3.24b}
$$

$$
\begin{gathered}
\Re_1 = \widehat{\boldsymbol{\kappa}} \cdot (\boldsymbol{\mathcal{S}}_t + \boldsymbol{\mathcal{S}}_u), \quad \Re_2 = \mathbf{a} \cdot (\boldsymbol{\mathcal{S}}_t + \boldsymbol{\mathcal{S}}_u) \\
\Re_3 = \dot{\mathbf{a}} \cdot (\boldsymbol{\mathcal{S}}_t + \boldsymbol{\mathcal{S}}_u), \quad \Re_4 = \ddot{\mathbf{a}} \cdot (\boldsymbol{\mathcal{S}}_t + \boldsymbol{\mathcal{S}}_u), \quad \Re_5 = \ddot{\mathbf{e}} \cdot (\boldsymbol{\mathcal{S}}_t + \boldsymbol{\mathcal{S}}_u)
\end{gathered}
\tag{3.24c}
$$

[6] Let $\dot{\mathbf{x}} = \boldsymbol{\upsilon}$. Then in order to determine the ray uniquely, one needs to evaluate $\mathbf{x}$ at an initial instant and to compute the Frenet frame at that instant. When this is not done, the ray is determined up to geometric congruence, which is good enough for our purposes here. We note that due diligence requires the position vector $\mathbf{x}$ of a point on the ray to be distinguished from the position vector $\mathbf{r}$ of the point of observation.

where b_1 is given by (3.15a), and we derive

$$
\begin{aligned}
\boldsymbol{\upsilon} \times \dot{\boldsymbol{\upsilon}} &= (\mathfrak{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e}) \times (\dot{\mathfrak{Y}}\widehat{\boldsymbol{\kappa}} + b_1\mathbf{a} + \rho\dot{\mathbf{a}} + \dot{\mathbf{e}}) \text{ by (3.14c) \& (3.16a)} \\
&= (\mathfrak{Y}b_1 - \rho\dot{\mathfrak{Y}})(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \rho\mathfrak{Y}(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \mathfrak{Y}(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{e}}) + \rho^2(\mathbf{a} \times \dot{\mathbf{a}}) + \rho(\mathbf{a} \times \dot{\mathbf{e}}) \\
&\quad - \dot{\mathfrak{Y}}(\mathbf{u} \times \widehat{\boldsymbol{\kappa}}) - b_1(\mathbf{u} \times \mathbf{a}) - \rho(\mathbf{u} \times \dot{\mathbf{a}}) - (\mathbf{u} \times \dot{\mathbf{e}}) + \dot{\mathfrak{Y}}(\mathbf{e} \times \widehat{\boldsymbol{\kappa}}) + b_1(\mathbf{e} \times \mathbf{a}) \\
&\quad + \rho(\mathbf{e} \times \dot{\mathbf{a}}) + (\mathbf{e} \times \dot{\mathbf{e}}) \\
&= (\mathfrak{Y}b_1 - \rho\dot{\mathfrak{Y}})\mathcal{S}_b + \mathfrak{Y}(\rho\mathcal{S}_d + \mathcal{S}_e) + \rho(\rho\mathcal{S}_h + \mathcal{S}_i) + \dot{\mathfrak{Y}}(\mathcal{S}_a - \mathcal{S}_c) + b_1(\mathcal{S}_f - \mathcal{S}_g) \\
&\qquad + \rho(\mathcal{S}_l - \mathcal{S}_j) + \mathcal{S}_m - \mathcal{S}_k \text{ by (3.24a)} \\
&= \mathcal{S}_t + \mathcal{S}_u \text{ by (3.24b)}
\end{aligned} \tag{3.25a}
$$

$$
\begin{aligned}
\ddot{\boldsymbol{\upsilon}} \cdot (\boldsymbol{\upsilon} \times \dot{\boldsymbol{\upsilon}}) &= (\ddot{\mathfrak{Y}}\widehat{\boldsymbol{\kappa}} + \ddot{\rho}\mathbf{a} + b_2\dot{\mathbf{a}} + \rho\ddot{\mathbf{a}} + \ddot{\mathbf{e}}) \cdot (\mathcal{S}_t + \mathcal{S}_u) \text{ by (3.16b) \& (3.25a)} \\
&= \ddot{\mathfrak{Y}}[\widehat{\boldsymbol{\kappa}} \cdot (\mathcal{S}_t + \mathcal{S}_u)] + \ddot{\rho}[\mathbf{a} \cdot (\mathcal{S}_t + \mathcal{S}_u)] + b_2[\dot{\mathbf{a}} \cdot (\mathcal{S}_t + \mathcal{S}_u)] \\
&\qquad + \rho[\ddot{\mathbf{a}} \cdot (\mathcal{S}_t + \mathcal{S}_u)] + \ddot{\mathbf{e}} \cdot (\mathcal{S}_t + \mathcal{S}_u) \\
&= \ddot{\mathfrak{Y}}\Re_1 + \ddot{\rho}\Re_2 + b_2\Re_3 + \rho\Re_4 + \Re_5 \text{ by (3.24c).}
\end{aligned} \tag{3.25b}
$$

Substituting (3.25), (3.14c) and (3.5a) into (3.23) yields

$$
\overline{\mathbb{K}} = \frac{|\mathcal{S}_t + \mathcal{S}_u|}{c^3\mathcal{R}^3}, \qquad \overline{\mathbb{T}} = \frac{\ddot{\mathfrak{Y}}\Re_1 + \ddot{\rho}\Re_2 + b_2\Re_3 + \rho\Re_4 + \Re_5}{|\mathcal{S}_t + \mathcal{S}_u|^2} \tag{3.26a}
$$

$$
\overline{\boldsymbol{\ell}}_t = \frac{\mathfrak{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e}}{c\mathcal{R}}, \quad \overline{\boldsymbol{\ell}}_b = \frac{\mathcal{S}_t + \mathcal{S}_u}{|\mathcal{S}_t + \mathcal{S}_u|}, \quad \overline{\boldsymbol{\ell}}_n = \overline{\boldsymbol{\ell}}_b \times \overline{\boldsymbol{\ell}}_t \tag{3.26b}
$$

as the complete set of equations describing the apparent geometry of obliquated rays.

Art 15. *Apparent frequency of obliquated rays.*

A change in the velocity of a light ray can be interpreted as a change in the frequency of the ray, and this change in *ray frequency* can be calculated as follows. From (1.2b) and (3.5a), we have

$$
c = \frac{\omega_o}{\kappa} = \frac{\upsilon}{\mathcal{R}} \tag{3.27}
$$

which leads to

$$
\omega' = \mathcal{R}\omega_o, \quad \omega' = \kappa\upsilon \tag{3.28}
$$

where the quantity ω' represents the apparent frequency of the ray. If we introduce the redshift

$$
\mathfrak{z} = \frac{\omega_o - \omega'}{\omega'} \tag{3.29}
$$

in order to facilitate comparison with other treatments, then from the foregoing equations, we shall obtain

$$
\frac{1}{1+\mathfrak{z}} = \frac{\omega'}{\omega_o} = \frac{\upsilon}{c} = \mathcal{R} \tag{3.30}
$$

which, with the help of (3.3), allows $\mathfrak{z}$ and ω' to be calculated for an observer in accelerated translational, rotational or gravitational motion [7].

[7] It may be worthwhile to emphasize here that all the quantities calculated in this work are due to the motion of the observer and not to any kind of motion of the light source. Kineoptic effects of moving light sources will be treated when we study the nature of the intrinsic redshifts advocated by our very own Galileo, the most eminent Dr. Halton C. Arp, and his coworkers.

Part II: Calculations

The investigations and calculations of astronomers have taught us much that is wonderful; but the most important lesson we have received from them is the discovery of the abyss of our ignorance in relation to the universe — an ignorance the magnitude of which reason, without the information thus derived, could never have conceived.

Immanuel Kant (1724-1804)

4 Translational obliquation

Art 16. *Apparent direction to a light source.*

To evaluate (3.6) for an observer in accelerated translation, we have from (1.2b) that $\alpha = \kappa a \cos\lambda$, whence $\kappa\tau = 2\rho a \cos\lambda$ by (1.3a). Moreover, since $\mathbf{e} = \mathbf{0}$ by (1.3a), we have in (3.2) that

$$\begin{aligned}
&\mathcal{B} = 0, \quad \mathcal{D} = 0, \quad \mathcal{A} = 0, \quad \mathcal{H} = 0, \quad \mathcal{E} = 0\\
&\mathcal{L}_0 = -2\rho a u \cos\lambda\cos\phi, \quad \mathcal{L}_1 = \rho a(\cos\theta - 2\cos\lambda\cos\phi), \quad \mathcal{L}_2 = 2\rho a(\cos\theta - \cos\lambda\cos\phi)\\
&\mathcal{N}_1 = -4d\rho a c u^2 \cos\lambda, \quad \mathcal{N}_2 = -4d\rho a c u^2 \cos\lambda\cos^2\phi, \quad \mathcal{N}_3 = 0, \quad \mathcal{N}_4 = 0\\
&\mathcal{N}_5 = 4\rho^2 a^2 u^2 \cos\lambda\cos\phi(\cos\theta - \cos\lambda\cos\phi), \quad \mathcal{N}_6 = 0
\end{aligned} \tag{4.1}$$

from which we obtain, by (3.3),

$$\begin{aligned}
\mathcal{L} &= -2\rho\sigma\cos\lambda\cos\phi\\
\mathcal{P} &= -2\rho\sigma[2d\cos\lambda + \beta(\cos\theta - 2\cos\lambda\cos\phi)]\\
\mathcal{N} &= 4\rho^2\sigma^2\cos\lambda\cos\phi(\cos\theta - \cos\lambda\cos\phi) - 4d\rho\sigma\cos\lambda\sin^2\phi
\end{aligned} \tag{4.2a}$$

$$\begin{aligned}
\mathcal{G} &= -\beta + d\cos\phi + \rho\sigma(\cos\theta - 2\cos\lambda\cos\phi)\\
\mathcal{R}^2 &= d^2 + \beta^2 - 2d\beta\cos\phi + \rho\sigma[\rho\sigma - 2d\cos\lambda - 2\beta(\cos\theta - 2\cos\lambda\cos\phi)]\\
\mathcal{F}^2 &= d^2\sin^2\phi + \rho^2\sigma^2[\sin^2\theta + 4\cos\lambda\cos\phi(\cos\theta - \cos\lambda\cos\phi)]\\
&\quad + 2d\rho\sigma(\cos\lambda\cos 2\phi - \cos\phi\cos\theta)
\end{aligned} \tag{4.2b}$$

and by (3.6),

$$\tan\psi = \mathcal{F}/\mathcal{G} \tag{4.2c}$$

while by (1.2), (1.3) and (1.6),

$$\begin{aligned}
&\rho = \frac{\pi}{4d\omega_o}, \quad \pi = \frac{\vartheta}{\sqrt{1+\vartheta^2}}, \quad d = \left\{\frac{1+\sqrt{1+\vartheta^2}}{2}\right\}^{1/2}\\
&\vartheta = \frac{\alpha}{\omega_o^2} = \mu\cos\lambda, \quad \mathcal{Y} = c(d - 2\rho\sigma\cos\lambda), \quad \boldsymbol{\kappa}\cdot\widehat{\mathbf{p}} = 0.
\end{aligned} \tag{4.2d}$$

Equation (4.2) gives a complete prescription for calculating ψ for a translating observer. For an observer in radial motion ($\phi = 0, \theta = \lambda$), we have from (4.2b),

$$\begin{aligned}
&\mathcal{F} = |\rho|\sigma\sin\lambda, \quad \mathcal{G} = d - \beta - \rho\sigma\cos\lambda\\
&\mathcal{R}^2 = \rho^2\sigma^2 + (d-\beta)(d - \beta - 2\rho\sigma\cos\lambda)
\end{aligned} \tag{4.3a}$$

whence (4.2c) becomes

$$\tan\psi = \frac{|\rho|\sigma\sin\lambda}{d - \beta - \rho\sigma\cos\lambda} \qquad (\phi = 0, \theta = \lambda). \tag{4.3b}$$

Similarly, for an observer in rectilinear motion ($\theta = 0, \phi = \lambda$), (4.2b) yields

$$\begin{aligned} \mathcal{F} &= |d - 2\rho\sigma\cos\lambda|\sin\lambda, \quad \mathcal{G} = d\cos\lambda - \beta - \rho\sigma\cos 2\lambda \\ \mathcal{R}^2 &= d^2 + \beta^2 - 2d\beta\cos\lambda + \rho\sigma(\rho\sigma - 2d\cos\lambda + 2\beta\cos 2\lambda) \end{aligned} \tag{4.4a}$$

which, when substituted into (4.2c), yields

$$\tan\psi = \frac{|d - 2\rho\sigma\cos\lambda|\sin\lambda}{d\cos\lambda - \beta - \rho\sigma\cos 2\lambda} \qquad (\theta = 0, \phi = \lambda). \tag{4.4b}$$

Finally, for an observer in coradial motion ($\lambda = 0, \theta = \phi$), (4.2b) gives

$$\begin{aligned} \mathcal{F} &= |d - \rho\sigma|\sin\phi, \quad \mathcal{G} = -\beta + (d - \rho\sigma)\cos\phi \\ \mathcal{R}^2 &= (d - \rho\sigma)^2 + \beta^2 - (2d + \rho\sigma)\beta\cos\phi \end{aligned} \tag{4.5a}$$

so that, from (4.2c), we get

$$\tan\psi = \frac{|d - \rho\sigma|\sin\phi}{-\beta + (d - \rho\sigma)\cos\phi} \qquad (\lambda = 0, \theta = \phi). \tag{4.5b}$$

These results establish once again that the effect of translational acceleration on obliquation is of second and higher orders in μ, which implies that it is of practical significance only in situations involving infraradio waves or ultrahigh accelerations or both.

Art 17. *Apparent drift of a light source.*

To evaluate (3.13) for a translating observer, it is necessary to know the acceleration $\mathbf{a}$ as a function of the velocity $\mathbf{u}$. In the simplest case when $\mathbf{a}$ is independent of $\mathbf{u}$, we have from (1.3a) and (3.8) that

$$\begin{gathered} \mathbf{g}_1 = \mathbf{0}, \quad \mathbf{g}_2 = \mathbf{0}, \quad \mathbf{g}_3 = \mathbf{0}, \quad \mathbf{g}_4 = \mathbf{0}, \quad \mathbf{g}_5 = \mathbf{0}, \quad \mathbf{g}_6 = \mathbf{0}, \quad \mathbf{g}_7 = \mathbf{0}, \quad \mathbf{g}_8 = \mathbf{0}, \quad \mathbf{g}_9 = \mathbf{0} \\ \boldsymbol{\jmath}_1 = \mathbf{0}, \quad \boldsymbol{\jmath}_2 = \mathbf{0}, \quad \boldsymbol{\jmath}_3 = \mathbf{0}, \quad \boldsymbol{\jmath}_4 = \mathbf{0}, \quad \boldsymbol{\jmath}_5 = \mathbf{0}, \quad \boldsymbol{\jmath}_6 = \mathbf{0} \end{gathered} \tag{4.6a}$$

$$\begin{aligned} \boldsymbol{\jmath}_7 &= \nabla_u \tau \text{ by (3.8b)} \\ &= \nabla_u(2\rho\alpha\kappa^{-2}) \text{ by (1.3a)} \\ &= 2\kappa^{-2}(\rho\nabla_u\alpha + \alpha\nabla_u\rho) \text{ by (A.16)} \\ &= 2\kappa^{-2}(\rho\mathbf{g}_6 + \alpha\mathbf{g}_9) \text{ by (3.9a) \& (3.9d)} \\ &= \mathbf{0} \text{ by (4.6a).} \end{aligned} \tag{4.6b}$$

Substituting (4.6) into (3.13) yields

$$\begin{aligned} \mathfrak{X} &= \beta(u\mathcal{R})^{-2}(\mathcal{X}_1\mathbf{u} - \mathcal{X}_2\boldsymbol{v}) \\ &= \beta(u\mathcal{R})^{-2}[\mathcal{X}_1\mathbf{u} - \mathcal{X}_2(\mathbf{c}d + \rho\mathbf{a} - \tau\boldsymbol{\kappa} - \mathbf{u})] \text{ by (1.1), (1.2a) \& (1.3a)} \\ &= \beta(u\mathcal{R})^{-2}[(\mathcal{X}_1 + \mathcal{X}_2)\mathbf{u} - \mathcal{X}_2\rho\mathbf{a} - \mathcal{X}_2(cd - \kappa\tau)\widehat{\boldsymbol{\kappa}}] \\ &= \beta(u\mathcal{R})^{-2}[(\mathcal{X}_1 + \mathcal{X}_2)\mathbf{u} - \mathcal{X}_2\rho\mathbf{a} - \mathcal{X}_2(cd - 2\rho a\cos\lambda)\widehat{\boldsymbol{\kappa}}] \text{ by (1.3a)} \end{aligned} \tag{4.7}$$

which, in view of the values of $\mathcal{X}_1$ and $\mathcal{X}_2$ from (3.13b), may be rewritten as

$$\mathfrak{X} = \imath_0\widehat{\boldsymbol{\kappa}} + \imath_1\mathbf{u} - \imath_2\mathbf{a} \tag{4.8a}$$

where, with $\mathcal{F}, \mathcal{G}, \mathcal{R}$ given by (4.2b) and $\mathcal{Y}, \rho, d$ given by (4.2d),

$$\imath_0 = -\imath_4\mathcal{Y}, \quad \imath_1 = \imath_3 + \imath_4, \quad \imath_2 = \imath_4\rho, \quad \imath_3 = \frac{\beta + \mathcal{G}}{\mathcal{F}u^2}, \quad \imath_4 = \frac{\beta(\mathcal{R}^2 + \beta\mathcal{G})}{\mathcal{F}\mathcal{R}^2u^2}. \tag{4.8b}$$

From (3.7c) and (4.8a), we get the variation of obliquation (or the apparent drift of the light source per unit acceleration of the observer) as

$$\Theta = \imath_0 \cos\lambda + \imath_1 u \cos\theta - \imath_2 a. \tag{4.9}$$

Art 18. *Apparent path of a light source.*

Equation (3.22) can be evaluated for a translating observer as follows. With ρ, π, d, ϑ and α given by (4.2d), we introduce

$$\begin{aligned}
&\varsigma_a = \boldsymbol{\kappa}\cdot\dot{\mathbf{a}}, \quad \varsigma_b = \boldsymbol{\kappa}\cdot\ddot{\mathbf{a}}, \quad \varsigma_c = \boldsymbol{\kappa}\cdot\dddot{\mathbf{a}}, \quad \varsigma_d = \mathbf{a}\cdot\dot{\mathbf{a}}, \quad \varsigma_e = \mathbf{a}\cdot\ddot{\mathbf{a}}, \quad \varsigma_f = \mathbf{a}\cdot\dddot{\mathbf{a}}\\
&\varsigma_g = \dot{\mathbf{a}}\cdot\dot{\mathbf{a}}, \quad \varsigma_h = \dot{\mathbf{a}}\cdot\ddot{\mathbf{a}}, \quad \varsigma_i = \dot{\mathbf{a}}\cdot\dddot{\mathbf{a}}, \quad \varsigma_j = \ddot{\mathbf{a}}\cdot\ddot{\mathbf{a}}, \quad \varsigma_k = \ddot{\mathbf{a}}\cdot\dddot{\mathbf{a}}, \quad \varsigma_l = \dddot{\mathbf{a}}\cdot\dddot{\mathbf{a}}\\
&\varsigma_m = \widehat{\boldsymbol{\kappa}}\cdot(\mathbf{a}\times\dot{\mathbf{a}}), \quad \varsigma_n = \widehat{\boldsymbol{\kappa}}\cdot(\mathbf{a}\times\ddot{\mathbf{a}}), \quad \varsigma_o = \widehat{\boldsymbol{\kappa}}\cdot(\dot{\mathbf{a}}\times\ddot{\mathbf{a}}), \quad \varsigma_p = \mathbf{a}\cdot(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})\\
&\qquad \varsigma_q = \dddot{\mathbf{a}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{a}), \quad \varsigma_r = \dddot{\mathbf{a}}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}}), \quad \varsigma_s = \dddot{\mathbf{a}}\cdot(\widehat{\boldsymbol{\kappa}}\times\ddot{\mathbf{a}})\\
&\qquad \varsigma_t = \dddot{\mathbf{a}}\cdot(\mathbf{a}\times\dot{\mathbf{a}}), \quad \varsigma_u = \dddot{\mathbf{a}}\cdot(\mathbf{a}\times\ddot{\mathbf{a}}), \quad \varsigma_v = \dddot{\mathbf{a}}\cdot(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})
\end{aligned} \tag{4.10a}$$

$$\begin{aligned}
&\varrho_a = (1+\vartheta^2)^{1/2}, \quad \varrho_b = 1-4\vartheta^2, \quad \varrho_c = \frac{1}{\varrho_a^3} - \frac{\pi^2}{4d^2}, \quad \varrho_d = -1 + \frac{\varsigma_a\varrho_c}{4d\omega_o^3}\\
&\varrho_e = \frac{1}{\varrho_a^5\omega_o^2}\left[\varsigma_b\varrho_a^2 - \frac{3\vartheta\varsigma_a^2}{\omega_o^2}\right], \quad \varrho_f = \frac{1}{\varrho_a^7\omega_o^2}\left[\varsigma_c\varrho_a^4 - \frac{3\varsigma_a}{\omega_o^2}\left(3\vartheta\varsigma_b\varrho_a^2 + \frac{\varrho_b\varsigma_a^2}{\omega_o^2}\right)\right]
\end{aligned} \tag{4.10b}$$

$$\begin{aligned}
\varrho_g &= \frac{1}{4d\omega_o^2}\left[\pi\varsigma_b + \frac{\varrho_c\varsigma_a^2}{\omega_o^2}\right]\\
\varrho_h &= \frac{1}{4d\omega_o^2}\left[\pi\left\{\varsigma_c - \frac{\varrho_g\varsigma_a}{d}\right\} + \varsigma_a\left\{\varrho_e + \frac{2\varsigma_b}{\omega_o^2\varrho_a^3}\right\} - \frac{\pi\varsigma_a}{2d^2\omega_o^2}\left\{\pi\varsigma_b + \frac{\varrho_c\varsigma_a^2}{\omega_o^2}\right\}\right]\\
\varrho_i &= \frac{1}{4d\omega_o^2}\left[\omega_o\left\{\varrho_e - \frac{\pi\varrho_g}{d}\right\} - \frac{\pi\varrho_c\varsigma_a^2}{2d^2\omega_o^3}\right]\\
\varrho_j &= \frac{1}{4d\omega_o^2}\left[\omega_o\left\{\varrho_f - \frac{\pi\varrho_h}{d}\right\} + \frac{3\varsigma_a}{d\omega_o}\left\{\frac{\pi}{4d}\left(\frac{\pi\varsigma_a^2\varrho_c}{2d^2\omega_o^4} - \varrho_e\right) - \varrho_g\left(\frac{1}{\varrho_a^3} - \frac{\pi^2}{2d^2}\right)\right\}\right]
\end{aligned} \tag{4.10c}$$

$$\begin{aligned}
\varrho_k &= \frac{\varsigma_a}{\kappa}\left[-2\rho + \frac{1}{4d\omega_o}\left\{\pi - \frac{2\alpha\varrho_c}{\omega_o^2}\right\}\right], \quad \varrho_l = \frac{1}{\kappa}\left[\omega_o\varrho_g - 2\left\{\rho\varsigma_b + \alpha\varrho_i + \frac{\varsigma_a^2\varrho_c}{2d\omega_o^3}\right\}\right]\\
\varrho_m &= \frac{1}{\kappa}\left[\omega_o\varrho_h - 2\left\{\rho\varsigma_c + \alpha\varrho_j + 3\varsigma_a\left(\varrho_i + \frac{\varsigma_b\varrho_c}{4d\omega_o^3}\right)\right\}\right]\\
\varrho_n &= \varrho_i\varrho_k - \varrho_l\varrho_d, \quad \varrho_o = \varrho_k(1+2\varrho_d) - \rho\varrho_l, \quad \varrho_p = \varrho_d(1+2\varrho_d) - \rho\varrho_i\\
\varrho_q &= \left[a^2\varrho_d^2 + \rho(\rho\varsigma_g + 2\varrho_d\varsigma_d) + \varrho_k\left\{\varrho_k + \frac{2(\alpha\varrho_d + \rho\varsigma_a)}{\kappa}\right\}\right]^{1/2}
\end{aligned} \tag{4.10d}$$

$$\begin{aligned}
\varrho_r = {}& \varrho_n(a^2 - \kappa^{-2}\alpha^2) + \varrho_o(\varsigma_d - \kappa^{-2}\alpha\varsigma_a) + \rho\varrho_k(\varsigma_e - \kappa^{-2}\alpha\varsigma_b) + \varrho_p\kappa^{-1}(\alpha\varsigma_d - \varsigma_a a^2)\\
&+ \rho\varrho_d\kappa^{-1}(\alpha\varsigma_e - \varsigma_b a^2) + \rho^2\kappa^{-1}(\varsigma_a\varsigma_e - \varsigma_b\varsigma_d)\\
\varrho_s = {}& \varrho_n(\varsigma_d - \kappa^{-2}\alpha\varsigma_a) + \varrho_o(\varsigma_g - \kappa^{-2}\varsigma_a^2) + \rho\varrho_k(\varsigma_h - \kappa^{-2}\varsigma_b\varsigma_a) + \varrho_p\kappa^{-1}(\alpha\varsigma_g - \varsigma_a\varsigma_d)\\
&+ \rho\varrho_d\kappa^{-1}(\alpha\varsigma_h - \varsigma_b\varsigma_d) + \rho^2\kappa^{-1}(\varsigma_a\varsigma_h - \varsigma_b\varsigma_g)\\
\varrho_t = {}& \varrho_n(\varsigma_e - \kappa^{-2}\alpha\varsigma_b) + \varrho_o(\varsigma_h - \kappa^{-2}\varsigma_a\varsigma_b) + \rho\varrho_k(\varsigma_j - \kappa^{-2}\varsigma_b^2) + \varrho_p\kappa^{-1}(\alpha\varsigma_h - \varsigma_a\varsigma_e)\\
&+ \rho\varrho_d\kappa^{-1}(\alpha\varsigma_j - \varsigma_b\varsigma_e) + \rho^2\kappa^{-1}(\varsigma_a\varsigma_j - \varsigma_b\varsigma_h)
\end{aligned} \tag{4.10e}$$

$$\begin{aligned}
\varrho_u &= \varrho_n\kappa^{-1}(\alpha\varsigma_d - a^2\varsigma_a) + \varrho_o\kappa^{-1}(\alpha\varsigma_g - \varsigma_d\varsigma_a) + \rho\varrho_k\kappa^{-1}(\alpha\varsigma_h - \varsigma_e\varsigma_a) + \varrho_p(a^2\varsigma_g - \varsigma_d^2) \\
&\quad + \rho\varrho_d(a^2\varsigma_h - \varsigma_e\varsigma_d) + \rho^2(\varsigma_d\varsigma_h - \varsigma_e\varsigma_g) \\
\varrho_v &= \varrho_n\kappa^{-1}(\alpha\varsigma_e - a^2\varsigma_b) + \varrho_o\kappa^{-1}(\alpha\varsigma_h - \varsigma_d\varsigma_b) + \rho\varrho_k\kappa^{-1}(\alpha\varsigma_j - \varsigma_e\varsigma_b) + \varrho_p(a^2\varsigma_h - \varsigma_d\varsigma_e) \\
&\quad + \rho\varrho_d(a^2\varsigma_j - \varsigma_e^2) + \rho^2(\varsigma_d\varsigma_j - \varsigma_e\varsigma_h) \\
\varrho_w &= \varrho_n\kappa^{-1}(\varsigma_a\varsigma_e - \varsigma_d\varsigma_b) + \varrho_o\kappa^{-1}(\varsigma_a\varsigma_h - \varsigma_g\varsigma_b) + \rho\varrho_k\kappa^{-1}(\varsigma_a\varsigma_j - \varsigma_h\varsigma_b) + \varrho_p(\varsigma_d\varsigma_h - \varsigma_g\varsigma_e) \\
&\quad + \rho\varrho_d(\varsigma_d\varsigma_j - \varsigma_h\varsigma_e) + \rho^2(\varsigma_g\varsigma_j - \varsigma_h^2)
\end{aligned} \tag{4.10f}$$

$$\begin{aligned}
\varrho_x &= [\varrho_n\varrho_r + \varrho_o\varrho_s + \rho\varrho_k\varrho_t + \varrho_p\varrho_u + \rho\varrho_d\varrho_v + \rho^2\varrho_w]^{1/2} \\
\varrho_y &= \varrho_m(\varrho_p\varsigma_m + \rho\varrho_d\varsigma_n + \rho^2\varsigma_o) + \varrho_j(-\varrho_o\varsigma_m - \rho\varrho_k\varsigma_n + \rho^2\varsigma_p) + 3\varrho_i(\varrho_n\varsigma_m - \rho\varrho_k\varsigma_o - \rho\varrho_d\varsigma_p) \\
&\quad + (2+3\varrho_d)(\varrho_n\varsigma_n + \varrho_o\varsigma_o + \varrho_p\varsigma_p) + \rho(\varrho_n\varsigma_q + \varrho_o\varsigma_r + \varrho_p\varsigma_t) + \rho^2(\varrho_k\varsigma_s + \varrho_d\varsigma_u + \rho\varsigma_v).
\end{aligned} \tag{4.10g}$$

Art 18a. *Development of equations* (3.17) *through* (3.21).

From (1.3a), (3.17), (3.18) and (4.10a), we have

$$\begin{gathered}
\dot\gamma = 0, \quad \ddot\gamma = 0, \quad \dddot\gamma = 0, \quad \dot\alpha = \varsigma_a, \quad \ddot\alpha = \varsigma_b, \quad \dddot\alpha = \varsigma_c \\
\dot\vartheta = \varsigma_a/\omega_o^2, \quad \ddot\vartheta = \varsigma_b/\omega_o^2, \quad \dddot\vartheta = \varsigma_c/\omega_o^2.
\end{gathered} \tag{4.11}$$

It is easy to see that

$$\begin{aligned}
\dot\pi &= \dot\vartheta(1+\vartheta^2)^{-3/2} \text{ by (3.19a)} \\
&= \varsigma_a/(\omega_o^2\varrho_a^3) \text{ by (4.10b) \& (4.11)}
\end{aligned} \tag{4.12a}$$

$$\begin{aligned}
\ddot\pi &= (1+\vartheta^2)^{-5/2}[\ddot\vartheta(1+\vartheta^2) - 3\vartheta\dot\vartheta^2] \text{ by (3.19b)} \\
&= \varrho_a^{-5}[\varsigma_b\omega_o^{-2}\varrho_a^2 - 3\vartheta(\varsigma_a\omega_o^{-2})^2] \text{ by (4.10b) \& (4.11)} \\
&= \varrho_e \text{ by (4.10b)}
\end{aligned} \tag{4.12b}$$

$$\begin{aligned}
\dddot\pi &= (1+\vartheta^2)^{-7/2}[\dddot\vartheta(1+\vartheta^2)^2 - 9\vartheta\dot\vartheta\ddot\vartheta(1+\vartheta^2) - 3\dot\vartheta^3(1-4\vartheta^2)] \text{ by (3.19c)} \\
&= \varrho_a^{-7}[(\varsigma_c\omega_o^{-2})\varrho_a^4 - 9\vartheta(\varsigma_a\omega_o^{-2})(\varsigma_b\omega_o^{-2})\varrho_a^2 - 3(\varsigma_a\omega_o^{-2})^3\varrho_b] \text{ by (4.10b) \& (4.11)} \\
&= \varrho_f \text{ by (4.10b)}
\end{aligned} \tag{4.12c}$$

$$\begin{aligned}
\dot d &= \frac{\dot\gamma d}{\gamma} + \frac{\pi\gamma^2\dot\vartheta}{4d} \text{ by (3.20a)} \\
&= \frac{\pi\varsigma_a}{4d\omega_o^2} \text{ by (4.11) \& (1.3a)}
\end{aligned} \tag{4.13a}$$

$$\begin{aligned}
\ddot d &= d\left[\frac{\ddot\gamma}{\gamma} - \frac{\dot\gamma^2}{\gamma^2}\right] + \dot d\left[\frac{\dot\gamma}{\gamma} - \frac{\pi\gamma^2\dot\vartheta}{4d^2}\right] + \frac{\gamma}{4d}\left[\dot\pi\gamma\dot\vartheta + 2\pi\dot\gamma\dot\vartheta + \pi\gamma\ddot\vartheta\right] \text{ by (3.20b)} \\
&= \dot d\left[-\frac{\pi\dot\vartheta}{4d^2}\right] + \frac{1}{4d}\left[\dot\pi\dot\vartheta + \pi\ddot\vartheta\right] \text{ by (4.11) \& (1.3a)} \\
&= \left[\frac{\pi\varsigma_a}{4d\omega_o^2}\right]\left[-\frac{\pi}{4d^2}\right]\left[\frac{\varsigma_a}{\omega_o^2}\right] + \frac{1}{4d}\left[\left\{\frac{\varsigma_a}{\omega_o^2\varrho_a^3}\right\}\left\{\frac{\varsigma_a}{\omega_o^2}\right\} + \pi\left\{\frac{\varsigma_b}{\omega_o^2}\right\}\right] \text{ by (4.11), (4.12a) \& (4.13a)} \\
&= -\frac{\pi^2\varsigma_a^2}{16d^3\omega_o^4} + \frac{1}{4d}\left[\frac{\varsigma_a^2}{\omega_o^4\varrho_a^3} + \frac{\pi\varsigma_b}{\omega_o^2}\right] = \frac{1}{4d\omega_o^2}\left[\pi\varsigma_b + \frac{\varsigma_a^2}{\omega_o^2}\left\{\frac{1}{\varrho_a^3} - \frac{\pi^2}{4d^2}\right\}\right] \\
&= \frac{1}{4d\omega_o^2}\left[\pi\varsigma_b + \frac{\varrho_c\varsigma_a^2}{\omega_o^2}\right] \text{ by (4.10b)} \\
&= \varrho_g \text{ by (4.10c)}
\end{aligned} \tag{4.13b}$$

$$\dddot{d} = d\left[\frac{\dddot{\gamma}}{\gamma} - \frac{3\dot{\gamma}\ddot{\gamma}}{\gamma^2} + \frac{2\dot{\gamma}^3}{\gamma^3}\right] + 2\dot{d}\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2} - \frac{\dot{\pi}\gamma^2\dot{\vartheta}}{4d^2} - \frac{\pi\gamma\dot{\gamma}\dot{\vartheta}}{2d^2} - \frac{\pi\gamma^2\ddot{\vartheta}}{4d^2} + \frac{\pi\gamma^2\dot{d}\dot{\vartheta}}{4d^3}\right] + \ddot{d}\left[\frac{\dot{\gamma}}{\gamma} - \frac{\pi\gamma^2\dot{\vartheta}}{4d^2}\right]$$
$$+ \frac{\dot{\gamma}}{4d}\left[\dot{\pi}\gamma\dot{\vartheta} + 2\pi\dot{\gamma}\dot{\vartheta} + \pi\gamma\ddot{\vartheta}\right] + \frac{\gamma}{4d}\left[\ddot{\pi}\gamma\dot{\vartheta} + 3\dot{\pi}\dot{\gamma}\dot{\vartheta} + 2\dot{\pi}\gamma\ddot{\vartheta} + 2\pi\ddot{\gamma}\dot{\vartheta} + 3\pi\dot{\gamma}\ddot{\vartheta} + \pi\gamma\dddot{\vartheta}\right] \text{ by (3.20c)}$$
$$= 2\dot{d}\left[-\frac{\dot{\pi}\dot{\vartheta}}{4d^2} - \frac{\pi\ddot{\vartheta}}{4d^2} + \frac{\pi\dot{d}\dot{\vartheta}}{4d^3}\right] + \ddot{d}\left[-\frac{\pi\dot{\vartheta}}{4d^2}\right] + \frac{1}{4d}\left[\ddot{\pi}\dot{\vartheta} + 2\dot{\pi}\ddot{\vartheta} + \pi\dddot{\vartheta}\right] \text{ by (1.3a) \& (4.11)}$$
$$= 2\dot{d}\left[-\frac{\dot{\pi}\varsigma_a}{4d^2\omega_o^2} - \frac{\pi\varsigma_b}{4d^2\omega_o^2} + \frac{\pi\dot{d}\varsigma_a}{4d^3\omega_o^2}\right] + \ddot{d}\left[-\frac{\pi\varsigma_a}{4d^2\omega_o^2}\right] + \frac{1}{4d}\left[\frac{\ddot{\pi}\varsigma_a}{\omega_o^2} + \frac{2\dot{\pi}\varsigma_b}{\omega_o^2} + \frac{\pi\varsigma_c}{\omega_o^2}\right] \text{ by (4.11)}$$

$$= 2\dot{d}\left[-\left\{\frac{\varsigma_a}{\omega_o^2\varrho_a^3}\right\}\left\{\frac{\varsigma_a}{4d^2\omega_o^2}\right\} - \frac{\pi\varsigma_b}{4d^2\omega_o^2} + \frac{\pi\dot{d}\varsigma_a}{4d^3\omega_o^2}\right] + \ddot{d}\left[-\frac{\pi\varsigma_a}{4d^2\omega_o^2}\right]$$
$$+ \frac{1}{4d}\left[\frac{\varrho_e\varsigma_a}{\omega_o^2} + \left\{\frac{\varsigma_a}{\omega_o^2\varrho_a^3}\right\}\left\{\frac{2\varsigma_b}{\omega_o^2}\right\} + \frac{\pi\varsigma_c}{\omega_o^2}\right] \text{ by (4.12)}$$
$$= \frac{\pi\varsigma_a}{2d\omega_o^2}\left[-\frac{\varsigma_a^2}{4d^2\omega_o^4\varrho_a^3} - \frac{\pi\varsigma_b}{4d^2\omega_o^2} + \left\{\frac{\pi\varsigma_a}{4d\omega_o^2}\right\}\left\{\frac{\pi\varsigma_a}{4d^3\omega_o^2}\right\}\right] - \frac{\pi\varrho_g\varsigma_a}{4d^2\omega_o^2} + \frac{1}{4d}\left[\frac{\varrho_e\varsigma_a}{\omega_o^2} + \frac{2\varsigma_a\varsigma_b}{\omega_o^4\varrho_a^3} + \frac{\pi\varsigma_c}{\omega_o^2}\right]$$
$$\text{by (4.13a) \& (4.13b)}$$
$$= -\frac{\pi\varsigma_a}{8d^3\omega_o^4}\left[\pi\varsigma_b + \frac{\varsigma_a^2}{\omega_o^2\varrho_a^3} - \frac{\pi^2\varsigma_a^2}{4d^2\omega_o^2}\right] + \frac{1}{4d\omega_o^2}\left[\varrho_e\varsigma_a + \pi\varsigma_c + \frac{2\varsigma_a\varsigma_b}{\omega_o^2\varrho_a^3} - \frac{\pi\varrho_g\varsigma_a}{d}\right]$$
$$= \frac{1}{4d\omega_o^2}\left[\pi\left\{\varsigma_c - \frac{\varrho_g\varsigma_a}{d}\right\} + \varsigma_a\left\{\varrho_e + \frac{2\varsigma_b}{\omega_o^2\varrho_a^3}\right\} - \frac{\pi\varsigma_a}{2d^2\omega_o^2}\left\{\pi\varsigma_b + \frac{\varsigma_a^2}{\omega_o^2}\left(\frac{1}{\varrho_a^3} - \frac{\pi^2}{4d^2}\right)\right\}\right]$$
$$= \varrho_h \text{ by (4.10b) \& (4.10c)} \tag{4.13c}$$

$$\dot{\rho} = \frac{d\dot{\pi} - \pi\dot{d}}{4\omega_o d^2} \text{ by (3.21a)}$$
$$= \frac{1}{4\omega_o d^2}\left[d\left\{\frac{\varsigma_a}{\omega_o^2\varrho_a^3}\right\} - \pi\left\{\frac{\pi\varsigma_a}{4d\omega_o^2}\right\}\right] \text{ by (4.12a) \& (4.13a)}$$
$$= \frac{\varsigma_a}{4d\omega_o^3}\left[\frac{1}{\varrho_a^3} - \frac{\pi^2}{4d^2}\right] = \frac{\varsigma_a\varrho_c}{4d\omega_o^3} \text{ by (4.10b)} \tag{4.14a}$$

$$\ddot{\rho} = -\frac{\dot{d}(d\dot{\pi} - \pi\dot{d})}{2\omega_o d^3} + \frac{d\ddot{\pi} - \pi\ddot{d}}{4\omega_o d^2} \text{ by (3.21b)}$$
$$= -\frac{1}{2\omega_o d^3}\left[\frac{\pi\varsigma_a}{4d\omega_o^2}\right]\left[d\left\{\frac{\varsigma_a}{\omega_o^2\varrho_a^3}\right\} - \pi\left\{\frac{\pi\varsigma_a}{4d\omega_o^2}\right\}\right] + \frac{d\varrho_e - \pi\varrho_g}{4\omega_o d^2} \text{ by (4.12) \& (4.13)}$$
$$= -\frac{\pi\varsigma_a^2}{8d^3\omega_o^5}\left[\frac{1}{\varrho_a^3} - \frac{\pi^2}{4d^2}\right] + \frac{1}{4d\omega_o}\left[\varrho_e - \frac{\pi\varrho_g}{d}\right]$$
$$= \frac{1}{4d\omega_o^2}\left[\omega_o\left\{\varrho_e - \frac{\pi\varrho_g}{d}\right\} - \frac{\pi\varsigma_a^2}{2d^2\omega_o^3}\left\{\frac{1}{\varrho_a^3} - \frac{\pi^2}{4d^2}\right\}\right]$$
$$= \varrho_i \text{ by (4.10b) \& (4.10c)} \tag{4.14b}$$

$$\begin{aligned}\dddot{\rho} &= \frac{(3\dot{d}^2 - d\ddot{d})(d\dot{\pi} - \pi\dot{d})}{2\omega_o d^4} - \frac{\dot{d}(d\ddot{\pi} - \pi\ddot{d})}{\omega_o d^3} + \frac{\dot{d}\ddot{\pi} + d\dddot{\pi} - \dot{\pi}\ddot{d} - \pi\dddot{d}}{4\omega_o d^2} \text{ by (3.21c)}\\
&= \frac{1}{2d^4\omega_o}\left[3\left\{\frac{\pi\varsigma_a}{4d\omega_o^2}\right\}^2 - d\varrho_g\right]\left[d\left\{\frac{\varsigma_a}{\omega_o^2\varrho_a^3}\right\} - \pi\left\{\frac{\pi\varsigma_a}{4d\omega_o^2}\right\}\right] - \frac{d\varrho_e - \pi\varrho_g}{d^3\omega_o}\left\{\frac{\pi\varsigma_a}{4d\omega_o^2}\right\}\\
&\quad + \frac{1}{4d^2\omega_o}\left[\varrho_e\left\{\frac{\pi\varsigma_a}{4d\omega_o^2}\right\} + d\varrho_f - \varrho_g\left\{\frac{\varsigma_a}{\omega_o^2\varrho_a^3}\right\} - \pi\varrho_h\right] \text{ by (4.12) \& (4.13)}\\
&= \frac{\varsigma_a}{2d^2\omega_o^3}\left[\frac{3\pi^2\varsigma_a^2}{16d^3\omega_o^4} - \varrho_g\right]\left[\frac{1}{\varrho_a^3} - \frac{\pi^2}{4d^2}\right] - \frac{\pi\varsigma_a}{4d^3\omega_o^3}\left[\varrho_e - \frac{\pi\varrho_g}{d}\right]\\
&\quad + \frac{1}{4d^2\omega_o}\left[\frac{\varsigma_a}{\omega_o^2}\left\{\frac{\pi\varrho_e}{4d} - \frac{\varrho_g}{\varrho_a^3}\right\} + d\varrho_f - \pi\varrho_h\right]\end{aligned}$$

$$\begin{aligned}&= \frac{\varsigma_a}{4d^2\omega_o^3}\left[2\varrho_c\left\{\frac{3\pi^2\varsigma_a^2}{16d^3\omega_o^4} - \varrho_g\right\} - \frac{\pi}{d}\left\{\varrho_e - \frac{\pi\varrho_g}{d}\right\} + \left\{\frac{\pi\varrho_e}{4d} - \frac{\varrho_g}{\varrho_a^3}\right\}\right]\\
&\quad + \frac{1}{4d\omega_o}\left\{\varrho_f - \frac{\pi\varrho_h}{d}\right\} \text{ by (4.10b)}\\
&= \frac{\varsigma_a}{4d^2\omega_o^3}\left[\frac{3\pi}{4d}\left\{\frac{\pi\varsigma_a^2\varrho_c}{2d^2\omega_o^4} - \varrho_e\right\} - \varrho_g\left\{2\varrho_c + \frac{1}{\varrho_a^3} - \frac{\pi^2}{d^2}\right\}\right] + \frac{1}{4d\omega_o}\left\{\varrho_f - \frac{\pi\varrho_h}{d}\right\}\\
&= \frac{1}{4d\omega_o^2}\left[\omega_o\left\{\varrho_f - \frac{\pi\varrho_h}{d}\right\} + \frac{\varsigma_a}{d\omega_o}\left\{\frac{3\pi}{4d}\left(\frac{\pi\varsigma_a^2\varrho_c}{2d^2\omega_o^4} - \varrho_e\right) - \varrho_g\left(2\varrho_c + \frac{1}{\varrho_a^3} - \frac{\pi^2}{d^2}\right)\right\}\right]\\
&= \frac{1}{4d\omega_o^2}\left[\omega_o\left\{\varrho_f - \frac{\pi\varrho_h}{d}\right\} + \frac{3\varsigma_a}{d\omega_o}\left\{\frac{\pi}{4d}\left(\frac{\pi\varsigma_a^2\varrho_c}{2d^2\omega_o^4} - \varrho_e\right) - \varrho_g\left(\frac{1}{\varrho_a^3} - \frac{\pi^2}{2d^2}\right)\right\}\right] \text{ by (4.10b)}\\
&= \varrho_j \text{ by (4.10c).}\end{aligned} \tag{4.14c}$$

Art 18b. *Derivatives of τ and* $\mathbf{e}$.

From (1.3a), we derive

$$\begin{aligned}&\dot{\mathbf{e}} = \mathbf{0}, \quad \ddot{\mathbf{e}} = \mathbf{0}, \quad \dddot{\mathbf{e}} = \mathbf{0}\\
&\dot{\tau} = (2\rho\alpha\kappa^{-2})' = 2\kappa^{-2}(\rho\dot{\alpha} + \dot{\rho}\alpha)\end{aligned} \tag{4.15a}$$

$$\begin{aligned}\ddot{\tau} &= 2\kappa^{-2}(\rho\dot{\alpha} + \dot{\rho}\alpha)' \text{ by (4.15a)}\\
&= 2\kappa^{-2}(\dot{\rho}\dot{\alpha} + \rho\ddot{\alpha} + \ddot{\rho}\alpha + \dot{\rho}\dot{\alpha})\\
&= 2\kappa^{-2}(\rho\ddot{\alpha} + 2\dot{\rho}\dot{\alpha} + \ddot{\rho}\alpha)\end{aligned} \tag{4.15b}$$

$$\begin{aligned}\dddot{\tau} &= 2\kappa^{-2}(\rho\ddot{\alpha} + 2\dot{\rho}\dot{\alpha} + \ddot{\rho}\alpha)' \text{ by (4.15b)}\\
&= 2\kappa^{-2}(\dot{\rho}\ddot{\alpha} + \rho\dddot{\alpha} + 2\ddot{\rho}\dot{\alpha} + 2\dot{\rho}\ddot{\alpha} + \dddot{\rho}\alpha + \ddot{\rho}\dot{\alpha})\\
&= 2\kappa^{-2}(\rho\dddot{\alpha} + 3\ddot{\rho}\dot{\alpha} + 3\dot{\rho}\ddot{\alpha} + \dddot{\rho}\alpha)\end{aligned} \tag{4.15c}$$

which leads to

$$\begin{aligned}\dot{\tau} &= 2\kappa^{-2}(\rho\dot{\alpha} + \dot{\rho}\alpha) \text{ by (4.15a)}\\
&= \frac{2}{\kappa^2}\left[\rho\varsigma_a + \alpha\left\{\frac{\varsigma_a\varrho_c}{4d\omega_o^3}\right\}\right] \text{ by (4.11) \& (4.14a)}\\
&= \frac{2\varsigma_a}{\kappa^2}\left[\rho + \frac{\alpha\varrho_c}{4d\omega_o^3}\right]\end{aligned} \tag{4.16a}$$

$$\begin{aligned}\ddot{\tau} &= 2\kappa^{-2}(\rho\ddot{\alpha} + 2\dot{\rho}\dot{\alpha} + \ddot{\rho}\alpha) \text{ by (4.15b)}\\ &= \frac{2}{\kappa^2}\left[\rho\varsigma_b + 2\varsigma_a\left\{\frac{\varsigma_a\varrho_c}{4d\omega_o^3}\right\} + \alpha\varrho_i\right] \text{ by (4.11) \& (4.14)}\\ &= \frac{2}{\kappa^2}\left[\rho\varsigma_b + \alpha\varrho_i + \frac{\varsigma_a^2\varrho_c}{2d\omega_o^3}\right] \qquad (4.16b)\end{aligned}$$

$$\begin{aligned}\dddot{\tau} &= 2\kappa^{-2}(\rho\dddot{\alpha} + 3\ddot{\rho}\dot{\alpha} + 3\dot{\rho}\ddot{\alpha} + \dddot{\rho}\alpha) \text{ by (4.15c)}\\ &= \frac{2}{\kappa^2}\left[\rho\varsigma_c + 3\varrho_i\varsigma_a + 3\varsigma_b\left\{\frac{\varsigma_a\varrho_c}{4d\omega_o^3}\right\} + \alpha\varrho_j\right] \text{ by (4.11) \& (4.14)}\\ &= \frac{2}{\kappa^2}\left[\rho\varsigma_c + \alpha\varrho_j + 3\varsigma_a\left\{\varrho_i + \frac{\varsigma_b\varrho_c}{4d\omega_o^3}\right\}\right]. \qquad (4.16c)\end{aligned}$$

Art 18c. *Development of equation* (3.15a).

We are now ready to evaluate the quantities defined by (3.15). We start with

$$\begin{aligned}\dot{y} &= c\dot{d} - \kappa\dot{\tau} \text{ by (3.15a)}\\ &= c\left\{\frac{\pi\varsigma_a}{4d\omega_o^2}\right\} - \kappa\left\{\frac{2\varsigma_a}{\kappa^2}\right\}\left\{\rho + \frac{\alpha\varrho_c}{4d\omega_o^3}\right\} \text{ by (4.13a) \& (4.16a)}\\ &= -\frac{2\rho\varsigma_a}{\kappa} + \frac{c\varsigma_a}{4d\omega_o^2}\left\{\pi - \frac{2\alpha\varrho_c}{\kappa c\omega_o}\right\}\\ &= \frac{\varsigma_a}{\kappa}\left[-2\rho + \frac{1}{4d\omega_o}\left\{\pi - \frac{2\alpha\varrho_c}{\omega_o^2}\right\}\right]\\ &= \varrho_k \text{ by (4.10d)} \qquad (4.17a)\end{aligned}$$

$$\begin{aligned}\ddot{y} &= c\ddot{d} - \kappa\ddot{\tau} \text{ by (3.15a)}\\ &= c\varrho_g - \kappa\left\{\frac{2}{\kappa^2}\right\}\left\{\rho\varsigma_b + \alpha\varrho_i + \frac{\varsigma_a^2\varrho_c}{2d\omega_o^3}\right\} \text{ by (4.13b) \& (4.16b)}\\ &= \frac{1}{\kappa}\left[\omega_o\varrho_g - 2\left\{\rho\varsigma_b + \alpha\varrho_i + \frac{\varsigma_a^2\varrho_c}{2d\omega_o^3}\right\}\right]\\ &= \varrho_l \text{ by (4.10d)} \qquad (4.17b)\end{aligned}$$

$$\begin{aligned}\dddot{y} &= c\dddot{d} - \kappa\dddot{\tau} \text{ by (3.15a)}\\ &= c\varrho_h - \kappa\left\{\frac{2}{\kappa^2}\right\}\left\{\rho\varsigma_c + \alpha\varrho_j + 3\varsigma_a\left\{\varrho_i + \frac{\varsigma_b\varrho_c}{4d\omega_o^3}\right\}\right\} \text{ by (4.13c) \& (4.16c)}\\ &= \frac{1}{\kappa}\left[\omega_o\varrho_h - 2\left\{\rho\varsigma_c + \alpha\varrho_j + 3\varsigma_a\left(\varrho_i + \frac{\varsigma_b\varrho_c}{4d\omega_o^3}\right)\right\}\right]\\ &= \varrho_m \text{ by (4.10d)} \qquad (4.17c)\end{aligned}$$

$$\begin{aligned}b_1 &= \dot{\rho} - 1 \text{ by (3.15a)}\\ &= \frac{\varsigma_a\varrho_c}{4d\omega_o^3} - 1 \text{ by (4.14a)}\\ &= \varrho_d \text{ by (4.10b)} \qquad (4.18a)\end{aligned}$$

$$\begin{aligned}b_2 &= 2\dot{\rho} - 1 = 1 + 2b_1 \text{ by (3.15a)}\\ &= 1 + 2\varrho_d \text{ by (4.18a)} \qquad (4.18b)\end{aligned}$$

$$\begin{aligned} b_3 &= 3\dot{\rho} - 1 = 2 + 3b_1 \text{ by (3.15a)} \\ &= 2 + 3\varrho_d \text{ by (4.18a)} \end{aligned} \tag{4.18c}$$

$$\begin{aligned} b_4 &= \ddot{\rho}\dot{\mathcal{Y}} - \ddot{\mathcal{Y}} b_1 \text{ by (3.15a)} \\ &= \varrho_i\varrho_k - \varrho_l\varrho_d \text{ by (4.14b), (4.17) \& (4.18a)} \\ &= \varrho_n \text{ by (4.10d)} \end{aligned} \tag{4.18d}$$

$$\begin{aligned} b_5 &= \dot{\mathcal{Y}} b_2 - \rho\ddot{\mathcal{Y}} \text{ by (3.15a)} \\ &= \varrho_k(1 + 2\varrho_d) - \rho\varrho_l \text{ by (4.17) \& (4.18b)} \\ &= \varrho_o \text{ by (4.10d)} \end{aligned} \tag{4.18e}$$

$$\begin{aligned} b_6 &= b_1 b_2 - \rho\ddot{\rho} \text{ by (3.15a)} \\ &= \varrho_d(1 + 2\varrho_d) - \rho\varrho_i \text{ by (4.18a), (4.18b) \& (4.14b)} \\ &= \varrho_p \text{ by (4.10d).} \end{aligned} \tag{4.18f}$$

Art 18d. *Development of equations* (3.15b) *and* (3.15c).

Substituting the derivatives of **e** from (4.15a) into (3.15b), we get

$$\begin{gathered} \boldsymbol{\mathfrak{J}}_a = \widehat{\boldsymbol{\kappa}} \times \mathbf{a}, \quad \boldsymbol{\mathfrak{J}}_b = \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_c = \widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_d = \mathbf{0}, \quad \boldsymbol{\mathfrak{J}}_e = \mathbf{0} \\ \boldsymbol{\mathfrak{J}}_f = \mathbf{a} \times \dot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_g = \mathbf{a} \times \ddot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_h = \mathbf{0}, \quad \boldsymbol{\mathfrak{J}}_i = \mathbf{0} \\ \boldsymbol{\mathfrak{J}}_j = \dot{\mathbf{a}} \times \ddot{\mathbf{a}}, \quad \boldsymbol{\mathfrak{J}}_k = \mathbf{0}, \quad \boldsymbol{\mathfrak{J}}_l = \mathbf{0}, \quad \boldsymbol{\mathfrak{J}}_n = \mathbf{0}, \quad \boldsymbol{\mathfrak{J}}_o = \mathbf{0} \end{gathered} \tag{4.19a}$$

from which we obtain, by (3.15c),

$$\boldsymbol{\mathfrak{J}}_p = b_4\boldsymbol{\mathfrak{J}}_a + b_5\boldsymbol{\mathfrak{J}}_b + \rho\dot{\mathcal{Y}}\boldsymbol{\mathfrak{J}}_c + b_6\boldsymbol{\mathfrak{J}}_f + \rho b_1\boldsymbol{\mathfrak{J}}_g, \quad \boldsymbol{\mathfrak{J}}_q = \rho^2\boldsymbol{\mathfrak{J}}_j, \quad \boldsymbol{\mathfrak{J}}_r = \dot{\mathcal{Y}}\widehat{\boldsymbol{\kappa}} + b_1\mathbf{a} + \rho\dot{\mathbf{a}}. \tag{4.19b}$$

Consequently, we have

$$\begin{aligned} \boldsymbol{\mathfrak{J}}_r &= \dot{\mathcal{Y}}\widehat{\boldsymbol{\kappa}} + b_1\mathbf{a} + \rho\dot{\mathbf{a}} \text{ by (4.19b)} \\ &= \varrho_k\widehat{\boldsymbol{\kappa}} + \varrho_d\mathbf{a} + \rho\dot{\mathbf{a}} \text{ by (4.17a) \& (4.18a)} \end{aligned} \tag{4.20a}$$

$$\begin{aligned} \boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q &= b_4\boldsymbol{\mathfrak{J}}_a + b_5\boldsymbol{\mathfrak{J}}_b + \rho\dot{\mathcal{Y}}\boldsymbol{\mathfrak{J}}_c + b_6\boldsymbol{\mathfrak{J}}_f + \rho b_1\boldsymbol{\mathfrak{J}}_g + \rho^2\boldsymbol{\mathfrak{J}}_j \text{ by (4.19b)} \\ &= b_4\boldsymbol{\mathfrak{J}}_a + b_5\boldsymbol{\mathfrak{J}}_b + \rho\varrho_k\boldsymbol{\mathfrak{J}}_c + b_6\boldsymbol{\mathfrak{J}}_f + \rho b_1\boldsymbol{\mathfrak{J}}_g + \rho^2\boldsymbol{\mathfrak{J}}_j \text{ by (4.17a)} \\ &= \varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) \\ &\quad + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}}) \text{ by (4.18) \& (4.19a)} \end{aligned} \tag{4.20b}$$

which in turn leads to

$$\begin{aligned} &(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\ &= (\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot [\varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\ &= \varrho_n[(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{a})] + \varrho_o[(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + \rho\varrho_k[(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}})] \\ &\quad + \varrho_p[(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] + \rho\varrho_d[(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\mathbf{a} \times \ddot{\mathbf{a}})] + \rho^2[(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\ &= \varrho_n[a^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})^2] + \varrho_o[(\mathbf{a} \cdot \dot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})] + \rho\varrho_k[(\mathbf{a} \cdot \ddot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \ddot{\mathbf{a}})(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})] \\ &\quad + \varrho_p[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})(\mathbf{a} \cdot \dot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})a^2] + \rho\varrho_d[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})(\mathbf{a} \cdot \ddot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \ddot{\mathbf{a}})a^2] \\ &\quad + \rho^2[(\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\mathbf{a} \cdot \ddot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \ddot{\mathbf{a}})(\mathbf{a} \cdot \dot{\mathbf{a}})] \text{ by (A.2)} \\ &= \varrho_n(a^2 - \kappa^{-2}\alpha^2) + \varrho_o(\varsigma_d - \kappa^{-2}\alpha\varsigma_a) + \rho\varrho_k(\varsigma_e - \kappa^{-2}\alpha\varsigma_b) + \varrho_p\kappa^{-1}(\alpha\varsigma_d - \varsigma_a a^2) \\ &\quad + \rho\varrho_d\kappa^{-1}(\alpha\varsigma_e - \varsigma_b a^2) + \rho^2\kappa^{-1}(\varsigma_a\varsigma_e - \varsigma_b\varsigma_d) \text{ by (4.10a) \& (1.2b)} \\ &= \varrho_r \text{ by (4.10e)} \end{aligned} \tag{4.21a}$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot [\varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\
&= \varrho_n[(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{a})] + \varrho_o[(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + \rho\varrho_k[(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}})] \\
&\quad + \varrho_p[(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] + \rho\varrho_d[(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot (\mathbf{a} \times \ddot{\mathbf{a}})] + \rho^2[(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\
&= \varrho_n[(\dot{\mathbf{a}} \cdot \mathbf{a}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})(\dot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] + \varrho_o[\dot{\mathbf{a}}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})^2] + \rho\varrho_k[(\dot{\mathbf{a}} \cdot \ddot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \ddot{\mathbf{a}})(\dot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \varrho_p[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})\dot{\mathbf{a}}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\dot{\mathbf{a}} \cdot \mathbf{a})] + \rho\varrho_d[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})(\dot{\mathbf{a}} \cdot \ddot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \ddot{\mathbf{a}})(\dot{\mathbf{a}} \cdot \mathbf{a})] \\
&\quad + \rho^2[(\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\dot{\mathbf{a}} \cdot \ddot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \ddot{\mathbf{a}})\dot{\mathbf{a}}^2] \text{ by (A.2)} \\
&= \varrho_n(\varsigma_d - \kappa^{-2}\alpha\varsigma_a) + \varrho_o(\varsigma_g - \kappa^{-2}\varsigma_a^2) + \rho\varrho_k(\varsigma_h - \kappa^{-2}\varsigma_b\varsigma_a) + \varrho_p\kappa^{-1}(\alpha\varsigma_g - \varsigma_a\varsigma_d) \\
&\quad + \rho\varrho_d\kappa^{-1}(\alpha\varsigma_h - \varsigma_b\varsigma_d) + \rho^2\kappa^{-1}(\varsigma_a\varsigma_h - \varsigma_b\varsigma_g) \text{ by (4.10a) \& (1.2b)} \\
&= \varrho_s \text{ by (4.10e)}
\end{aligned} \tag{4.21b}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= (\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) \cdot [\varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\
&= \varrho_n[(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{a})] + \varrho_o[(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + \rho\varrho_k[(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}})] \\
&\quad + \varrho_p[(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] + \rho\varrho_d[(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) \cdot (\mathbf{a} \times \ddot{\mathbf{a}})] + \rho^2[(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\
&= \varrho_n[(\ddot{\mathbf{a}} \cdot \mathbf{a}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})(\ddot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] + \varrho_o[(\ddot{\mathbf{a}} \cdot \dot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\ddot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] + \rho\varrho_k[\ddot{\mathbf{a}}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \ddot{\mathbf{a}})^2] \\
&\quad + \varrho_p[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})(\ddot{\mathbf{a}} \cdot \dot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\ddot{\mathbf{a}} \cdot \mathbf{a})] + \rho\varrho_d[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})\ddot{\mathbf{a}}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \ddot{\mathbf{a}})(\ddot{\mathbf{a}} \cdot \mathbf{a})] \\
&\quad + \rho^2[(\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})\ddot{\mathbf{a}}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \ddot{\mathbf{a}})(\ddot{\mathbf{a}} \cdot \dot{\mathbf{a}})] \text{ by (A.2)} \\
&= \varrho_n(\varsigma_e - \kappa^{-2}\alpha\varsigma_b) + \varrho_o(\varsigma_h - \kappa^{-2}\varsigma_a\varsigma_b) + \rho\varrho_k(\varsigma_j - \kappa^{-2}\varsigma_b^2) + \varrho_p\kappa^{-1}(\alpha\varsigma_h - \varsigma_a\varsigma_e) \\
&\quad + \rho\varrho_d\kappa^{-1}(\alpha\varsigma_j - \varsigma_b\varsigma_e) + \rho^2\kappa^{-1}(\varsigma_a\varsigma_j - \varsigma_b\varsigma_h) \text{ by (4.10a) \& (1.2b)} \\
&= \varrho_t \text{ by (4.10e)}
\end{aligned} \tag{4.21c}
$$

$$
\begin{aligned}
&(\mathbf{a} \times \dot{\mathbf{a}}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= (\mathbf{a} \times \dot{\mathbf{a}}) \cdot [\varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\
&= \varrho_n[(\mathbf{a} \times \dot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{a})] + \varrho_o[(\mathbf{a} \times \dot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + \rho\varrho_k[(\mathbf{a} \times \dot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}})] \\
&\quad + \varrho_p[(\mathbf{a} \times \dot{\mathbf{a}}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] + \rho\varrho_d[(\mathbf{a} \times \dot{\mathbf{a}}) \cdot (\mathbf{a} \times \ddot{\mathbf{a}})] + \rho^2[(\mathbf{a} \times \dot{\mathbf{a}}) \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\
&= \varrho_n[(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})(\dot{\mathbf{a}} \cdot \mathbf{a}) - a^2(\dot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] + \varrho_o[(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})\dot{\mathbf{a}}^2 - (\mathbf{a} \cdot \dot{\mathbf{a}})(\dot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] + \rho\varrho_k[(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})(\dot{\mathbf{a}} \cdot \ddot{\mathbf{a}}) - (\mathbf{a} \cdot \ddot{\mathbf{a}})(\dot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \varrho_p[a^2\dot{\mathbf{a}}^2 - (\mathbf{a} \cdot \dot{\mathbf{a}})^2] + \rho\varrho_d[a^2(\dot{\mathbf{a}} \cdot \ddot{\mathbf{a}}) - (\mathbf{a} \cdot \ddot{\mathbf{a}})(\dot{\mathbf{a}} \cdot \mathbf{a})] + \rho^2[(\mathbf{a} \cdot \dot{\mathbf{a}})(\dot{\mathbf{a}} \cdot \ddot{\mathbf{a}}) - (\mathbf{a} \cdot \ddot{\mathbf{a}})\dot{\mathbf{a}}^2] \text{ by (A.2)} \\
&= \varrho_n\kappa^{-1}(\alpha\varsigma_d - a^2\varsigma_a) + \varrho_o\kappa^{-1}(\alpha\varsigma_g - \varsigma_d\varsigma_a) + \rho\varrho_k\kappa^{-1}(\alpha\varsigma_h - \varsigma_e\varsigma_a) + \varrho_p(a^2\varsigma_g - \varsigma_d^2) \\
&\quad + \rho\varrho_d(a^2\varsigma_h - \varsigma_e\varsigma_d) + \rho^2(\varsigma_d\varsigma_h - \varsigma_e\varsigma_g) \text{ by (4.10a) \& (1.2b)} \\
&= \varrho_u \text{ by (4.10f)}
\end{aligned} \tag{4.21d}
$$

$$
\begin{aligned}
&(\mathbf{a} \times \ddot{\mathbf{a}}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= (\mathbf{a} \times \ddot{\mathbf{a}}) \cdot [\varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\
&= \varrho_n[(\mathbf{a} \times \ddot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{a})] + \varrho_o[(\mathbf{a} \times \ddot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + \rho\varrho_k[(\mathbf{a} \times \ddot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}})] \\
&\quad + \varrho_p[(\mathbf{a} \times \ddot{\mathbf{a}}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] + \rho\varrho_d[(\mathbf{a} \times \ddot{\mathbf{a}}) \cdot (\mathbf{a} \times \ddot{\mathbf{a}})] + \rho^2[(\mathbf{a} \times \ddot{\mathbf{a}}) \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\
&= \varrho_n[(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})(\ddot{\mathbf{a}} \cdot \mathbf{a}) - a^2(\ddot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] + \varrho_o[(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})(\ddot{\mathbf{a}} \cdot \dot{\mathbf{a}}) - (\mathbf{a} \cdot \dot{\mathbf{a}})(\ddot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] + \rho\varrho_k[(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})\ddot{\mathbf{a}}^2 - (\mathbf{a} \cdot \ddot{\mathbf{a}})(\ddot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \varrho_p[a^2(\ddot{\mathbf{a}} \cdot \dot{\mathbf{a}}) - (\mathbf{a} \cdot \dot{\mathbf{a}})(\ddot{\mathbf{a}} \cdot \mathbf{a})] + \rho\varrho_d[a^2\ddot{\mathbf{a}}^2 - (\mathbf{a} \cdot \ddot{\mathbf{a}})^2] + \rho^2[(\mathbf{a} \cdot \dot{\mathbf{a}})\ddot{\mathbf{a}}^2 - (\mathbf{a} \cdot \ddot{\mathbf{a}})(\ddot{\mathbf{a}} \cdot \dot{\mathbf{a}})] \text{ by (A.2)} \\
&= \varrho_n\kappa^{-1}(\alpha\varsigma_e - a^2\varsigma_b) + \varrho_o\kappa^{-1}(\alpha\varsigma_h - \varsigma_d\varsigma_b) + \rho\varrho_k\kappa^{-1}(\alpha\varsigma_j - \varsigma_e\varsigma_b) + \varrho_p(a^2\varsigma_h - \varsigma_d\varsigma_e) \\
&\quad + \rho\varrho_d(a^2\varsigma_j - \varsigma_e^2) + \rho^2(\varsigma_d\varsigma_j - \varsigma_e\varsigma_h) \text{ by (4.10a) \& (1.2b)} \\
&= \varrho_v \text{ by (4.10f)}
\end{aligned} \tag{4.21e}
$$

$$\begin{aligned}
&(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\\
&=(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})\cdot[\varrho_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{a})+\varrho_o(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}})+\rho\varrho_k(\widehat{\boldsymbol{\kappa}}\times\ddot{\mathbf{a}})+\varrho_p(\mathbf{a}\times\dot{\mathbf{a}})+\rho\varrho_d(\mathbf{a}\times\ddot{\mathbf{a}})+\rho^2(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})]\\
&=\varrho_n[(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{a})]+\varrho_o[(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}})]+\rho\varrho_k[(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})\cdot(\widehat{\boldsymbol{\kappa}}\times\ddot{\mathbf{a}})]\\
&\quad+\varrho_p[(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})\cdot(\mathbf{a}\times\dot{\mathbf{a}})]+\rho\varrho_d[(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})\cdot(\mathbf{a}\times\ddot{\mathbf{a}})]+\rho^2[(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})\cdot(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})]\\
&=\varrho_n[(\dot{\mathbf{a}}\cdot\widehat{\boldsymbol{\kappa}})(\ddot{\mathbf{a}}\cdot\mathbf{a})-(\dot{\mathbf{a}}\cdot\mathbf{a})(\ddot{\mathbf{a}}\cdot\widehat{\boldsymbol{\kappa}})]+\varrho_o[(\dot{\mathbf{a}}\cdot\widehat{\boldsymbol{\kappa}})(\ddot{\mathbf{a}}\cdot\dot{\mathbf{a}})-\dot{\mathbf{a}}^2(\ddot{\mathbf{a}}\cdot\widehat{\boldsymbol{\kappa}})]+\rho\varrho_k[(\dot{\mathbf{a}}\cdot\widehat{\boldsymbol{\kappa}})\ddot{\mathbf{a}}^2-(\dot{\mathbf{a}}\cdot\ddot{\mathbf{a}})(\ddot{\mathbf{a}}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\varrho_p[(\dot{\mathbf{a}}\cdot\mathbf{a})(\ddot{\mathbf{a}}\cdot\dot{\mathbf{a}})-\dot{\mathbf{a}}^2(\ddot{\mathbf{a}}\cdot\mathbf{a})]+\rho\varrho_d[(\dot{\mathbf{a}}\cdot\mathbf{a})\ddot{\mathbf{a}}^2-(\dot{\mathbf{a}}\cdot\ddot{\mathbf{a}})(\ddot{\mathbf{a}}\cdot\mathbf{a})]+\rho^2[\dot{\mathbf{a}}^2\ddot{\mathbf{a}}^2-(\dot{\mathbf{a}}\cdot\ddot{\mathbf{a}})^2]\text{ by (A.2)}\\
&=\varrho_n\kappa^{-1}(\varsigma_a\varsigma_e-\varsigma_d\varsigma_b)+\varrho_o\kappa^{-1}(\varsigma_a\varsigma_h-\varsigma_g\varsigma_b)+\rho\varrho_k\kappa^{-1}(\varsigma_a\varsigma_j-\varsigma_h\varsigma_b)+\varrho_p(\varsigma_d\varsigma_h-\varsigma_g\varsigma_e)\\
&\quad+\rho\varrho_d(\varsigma_d\varsigma_j-\varsigma_h\varsigma_e)+\rho^2(\varsigma_g\varsigma_j-\varsigma_h^2)\text{ by (4.10a) \& (1.2b)}\\
&=\varrho_w\text{ by (4.10f).}
\end{aligned}\tag{4.21f}$$

Using (4.20) and (4.21), we derive

$$\begin{aligned}
|\boldsymbol{\mathfrak{J}}_r|^2&=(\varrho_k\widehat{\boldsymbol{\kappa}}+\varrho_d\mathbf{a}+\rho\dot{\mathbf{a}})\cdot(\varrho_k\widehat{\boldsymbol{\kappa}}+\varrho_d\mathbf{a}+\rho\dot{\mathbf{a}})\text{ by (4.20a)}\\
&=\varrho_k^2+2\varrho_d\varrho_k(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{a})+2\rho\varrho_k(\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{a}})+\varrho_d^2(\mathbf{a}\cdot\mathbf{a})+2\rho\varrho_d(\mathbf{a}\cdot\dot{\mathbf{a}})+\rho^2(\dot{\mathbf{a}}\cdot\dot{\mathbf{a}})\\
&=\varrho_k^2+2\varrho_d\varrho_k\kappa^{-1}(\boldsymbol{\kappa}\cdot\mathbf{a})+2\rho\varrho_k\kappa^{-1}(\boldsymbol{\kappa}\cdot\dot{\mathbf{a}})+2\rho\varrho_d(\mathbf{a}\cdot\dot{\mathbf{a}})+\varrho_d^2a^2+\rho^2\dot{\mathbf{a}}^2\\
&=\varrho_k^2+2\alpha\varrho_d\varrho_k\kappa^{-1}+2\rho\varrho_k\varsigma_a\kappa^{-1}+2\rho\varrho_d\varsigma_d+\varrho_d^2a^2+\rho^2\varsigma_g\\
&=a^2\varrho_d^2+\rho(\rho\varsigma_g+2\varrho_d\varsigma_d)+\varrho_k[\varrho_k+2\kappa^{-1}(\alpha\varrho_d+\rho\varsigma_a)]\\
\therefore|\boldsymbol{\mathfrak{J}}_r|&=\varrho_q\text{ by (4.10d)}
\end{aligned}\tag{4.22a}$$

$$\begin{aligned}
|\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q|^2&=(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\\
&=(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot[\varrho_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{a})+\varrho_o(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}})+\rho\varrho_k(\widehat{\boldsymbol{\kappa}}\times\ddot{\mathbf{a}})+\varrho_p(\mathbf{a}\times\dot{\mathbf{a}})+\rho\varrho_d(\mathbf{a}\times\ddot{\mathbf{a}})\\
&\quad+\rho^2(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})]\text{ by (4.20b)}\\
&=\varrho_n[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{a})]+\varrho_o[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}})]+\rho\varrho_k[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\boldsymbol{\kappa}}\times\ddot{\mathbf{a}})]\\
&\quad+\varrho_p[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{a}\times\dot{\mathbf{a}})]+\rho\varrho_d[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{a}\times\ddot{\mathbf{a}})]+\rho^2[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})]\\
&=\varrho_n\varrho_r+\varrho_o\varrho_s+\rho\varrho_k\varrho_t+\varrho_p\varrho_u+\rho\varrho_d\varrho_v+\rho^2\varrho_w\\
\therefore|\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q|&=\varrho_x\text{ by (4.10g).}
\end{aligned}\tag{4.22b}$$

Art 18e. *Development of equation* (3.15d).

For the quantities defined in (3.15d), we derive

$$\begin{aligned}
\aleph_1&=\widehat{\boldsymbol{\kappa}}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\text{ by (3.15d)}\\
&=\widehat{\boldsymbol{\kappa}}\cdot[\varrho_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{a})+\varrho_o(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}})+\rho\varrho_k(\widehat{\boldsymbol{\kappa}}\times\ddot{\mathbf{a}})+\varrho_p(\mathbf{a}\times\dot{\mathbf{a}})+\rho\varrho_d(\mathbf{a}\times\ddot{\mathbf{a}})\\
&\quad+\rho^2(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})]\text{ by (4.20b)}\\
&=\widehat{\boldsymbol{\kappa}}\cdot[\varrho_p(\mathbf{a}\times\dot{\mathbf{a}})+\rho\varrho_d(\mathbf{a}\times\ddot{\mathbf{a}})+\rho^2(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})]\\
&=\varrho_p\varsigma_m+\rho\varrho_d\varsigma_n+\rho^2\varsigma_o\text{ by (4.10a)}
\end{aligned}\tag{4.23a}$$

$$\begin{aligned}
\aleph_2&=\mathbf{a}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\text{ by (3.15d)}\\
&=\mathbf{a}\cdot[\varrho_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{a})+\varrho_o(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}})+\rho\varrho_k(\widehat{\boldsymbol{\kappa}}\times\ddot{\mathbf{a}})+\varrho_p(\mathbf{a}\times\dot{\mathbf{a}})+\rho\varrho_d(\mathbf{a}\times\ddot{\mathbf{a}})\\
&\quad+\rho^2(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})]\text{ by (4.20b)}\\
&=-\varrho_o[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{a}\times\dot{\mathbf{a}})]-\rho\varrho_k[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{a}\times\ddot{\mathbf{a}})]+\rho^2[\mathbf{a}\cdot(\dot{\mathbf{a}}\times\ddot{\mathbf{a}})]\text{ by (A.4)}\\
&=-b_5\varsigma_m-\rho\varrho_k\varsigma_n+\rho^2\varsigma_p\text{ by (4.10a)}
\end{aligned}\tag{4.23b}$$

$$
\begin{aligned}
\aleph_3 &= \dot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)} \\
&= \dot{\mathbf{a}} \cdot [\varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) \\
&\quad + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \text{ by (4.20b)} \\
&= \varrho_n[\widehat{\boldsymbol{\kappa}} \cdot (\mathbf{a} \times \dot{\mathbf{a}})] - \rho\varrho_k[\widehat{\boldsymbol{\kappa}} \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] - \rho\varrho_d[\mathbf{a} \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \text{ by (A.4)} \\
&= \varrho_n\varsigma_m - \rho\varrho_k\varsigma_o - \rho\varrho_d\varsigma_p \text{ by (4.10a)}
\end{aligned} \tag{4.23c}
$$

$$
\begin{aligned}
\aleph_4 &= \ddot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)} \\
&= \ddot{\mathbf{a}} \cdot [\varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) \\
&\quad + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \text{ by (4.20b)} \\
&= \varrho_n[\widehat{\boldsymbol{\kappa}} \cdot (\mathbf{a} \times \ddot{\mathbf{a}})] + \varrho_o[\widehat{\boldsymbol{\kappa}} \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] + \varrho_p[\mathbf{a} \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \text{ by (A.4)} \\
&= \varrho_n\varsigma_n + \varrho_o\varsigma_o + \varrho_p\varsigma_p \text{ by (4.10a)}
\end{aligned} \tag{4.23d}
$$

$$
\begin{aligned}
\aleph_5 &= \dddot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)} \\
&= \dddot{\mathbf{a}} \cdot [\varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) \\
&\quad + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \text{ by (4.20b)} \\
&= \varrho_n\varsigma_q + \varrho_o\varsigma_r + \varrho_p\varsigma_t + \rho(\varrho_k\varsigma_s + \varrho_d\varsigma_u + \rho\varsigma_v) \text{ by (4.10a)}
\end{aligned} \tag{4.23e}
$$

$$
\begin{aligned}
\aleph_6 &= \dddot{\mathbf{e}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)} \\
&= 0 \text{ by (4.15a)}
\end{aligned} \tag{4.23f}
$$

$$
\begin{aligned}
&\dddot{\mathcal{Y}}\aleph_1 + \dddot{\rho}\aleph_2 + 3\ddot{\rho}\aleph_3 + b_3\aleph_4 + \rho\aleph_5 + \aleph_6 \\
&= \varrho_m\aleph_1 + \varrho_j\aleph_2 + 3\varrho_i\aleph_3 + (2 + 3\varrho_d)\aleph_4 + \rho\aleph_5 + \aleph_6 \text{ by (4.17c), (4.14) \& (4.18c)} \\
&= \varrho_m(\varrho_p\varsigma_m + \rho\varrho_d\varsigma_n + \rho^2\varsigma_o) + \varrho_j(-\varrho_o\varsigma_m - \rho\varrho_k\varsigma_n + \rho^2\varsigma_p) + 3\varrho_i(\varrho_n\varsigma_m - \rho\varrho_k\varsigma_o - \rho\varrho_d\varsigma_p) \\
&\quad + (2 + 3\varrho_d)(\varrho_n\varsigma_n + \varrho_o\varsigma_o + \varrho_p\varsigma_p) + \rho(\varrho_n\varsigma_q + \varrho_o\varsigma_r + \varrho_p\varsigma_t) \\
&\quad + \rho^2(\varrho_k\varsigma_s + \varrho_d\varsigma_u + \rho\varsigma_v) \text{ by (4.23)} \\
&= \varrho_y \text{ by (4.10g).}
\end{aligned} \tag{4.24}
$$

Art 18f. *Results of the computations.*

Substituting (4.20), (4.22) and (4.24) into (3.22), we finally get

$$
\begin{gathered}
\mathbb{K} = \frac{\varrho_x}{(\varrho_q)^3}, \quad \mathbb{T} = \frac{\varrho_y}{(\varrho_x)^2}, \quad \boldsymbol{\ell}_t = \frac{1}{\varrho_q}\left[\varrho_k\widehat{\boldsymbol{\kappa}} + \varrho_d\mathbf{a} + \rho\dot{\mathbf{a}}\right] \\
\boldsymbol{\ell}_b = \frac{1}{\varrho_x}\left[\varrho_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \varrho_o(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \varrho_p(\mathbf{a} \times \dot{\mathbf{a}}) + \rho\varrho_k(\widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}}) + \rho\varrho_d(\mathbf{a} \times \ddot{\mathbf{a}}) + \rho^2(\dot{\mathbf{a}} \times \ddot{\mathbf{a}})\right]
\end{gathered} \tag{4.25}
$$

as the set of equations that, together with (4.10), completely describes the apparent path of the light source.

Art 19. *Apparent geometry of obliquated rays.*

To evaluate (3.26) for a translating observer, we introduce, in addition to (4.10),

$$
\begin{gathered}
\mathfrak{H}_a = \widehat{\boldsymbol{\kappa}} \cdot (\mathbf{a} \times \mathbf{u}), \quad \mathfrak{H}_b = \widehat{\boldsymbol{\kappa}} \cdot (\dot{\mathbf{a}} \times \mathbf{u}), \quad \mathfrak{H}_c = \mathbf{a} \cdot (\dot{\mathbf{a}} \times \mathbf{u}), \quad \mathfrak{H}_d = \ddot{\mathbf{a}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) \\
\mathfrak{H}_e = \ddot{\mathbf{a}} \cdot (\mathbf{a} \times \mathbf{u}), \quad \mathfrak{H}_f = \ddot{\mathbf{a}} \cdot (\dot{\mathbf{a}} \times \mathbf{u}), \quad \mathfrak{H}_g = \widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}}, \quad \mathfrak{H}_h = \dot{\mathbf{a}} \cdot \mathbf{u}
\end{gathered} \tag{4.26a}
$$

$$
\begin{aligned}
&\mathfrak{K}_a = \mathcal{Y}\varrho_d - \rho\varrho_k, \quad \mathfrak{K}_b = \varrho_d\mathfrak{H}_a + \rho(\mathfrak{H}_b + \rho\varsigma_m), \quad \mathfrak{K}_c = \rho(\mathfrak{H}_c - \mathcal{Y}\varsigma_m) - \varrho_k\mathfrak{H}_a \\
&\mathfrak{K}_d = \mathfrak{K}_a\varsigma_m - \varrho_k\mathfrak{H}_b - \varrho_d\mathfrak{H}_c, \quad \mathfrak{K}_e = \mathfrak{K}_a\varsigma_n + \varrho_k\mathfrak{H}_d + \varrho_d\mathfrak{H}_e + \rho(\mathfrak{H}_f + \mathcal{Y}\varsigma_o + \rho\varsigma_p)
\end{aligned} \tag{4.26b}
$$

$$\begin{aligned}\mathfrak{K}_f &= a^2\mathfrak{K}_a^2\sin^2\lambda + 2\rho\mathfrak{Y}\mathfrak{K}_a(\varsigma_d - a\mathfrak{H}_g\cos\lambda) + 2a\rho^2\mathfrak{K}_a(\varsigma_d\cos\lambda - a\mathfrak{H}_g)\\ &\quad + \rho^2\mathfrak{Y}^2(\varsigma_g - \mathfrak{H}_g^2) + 2\rho^3\mathfrak{Y}(a\varsigma_g\cos\lambda - \mathfrak{H}_g\varsigma_d) + \rho^4(a^2\varsigma_g - \varsigma_d^2)\\ \mathfrak{K}_g &= \rho^2(u^2\varsigma_g - \mathfrak{H}_h^2) + u^2[\varrho_k^2\sin^2\phi + a^2\varrho_d^2\sin^2\theta + 2a\varrho_d\varrho_k(\cos\lambda - \cos\phi\cos\theta)]\\ &\quad + 2u\rho[\varrho_k(u\mathfrak{H}_g - \mathfrak{H}_h\cos\phi) + \varrho_d(u\varsigma_d - a\mathfrak{H}_h\cos\theta)]\end{aligned} \tag{4.26c}$$

$$\begin{aligned}\mathfrak{K}_h &= u\mathfrak{K}_a[\rho(a\mathfrak{H}_g\cos\theta - \varsigma_d\cos\phi) + a\varrho_k(\cos\theta - \cos\lambda\cos\phi) + a^2\varrho_d(\cos\lambda\cos\theta - \cos\phi)]\\ &\quad + \rho\mathfrak{Y}[\varrho_k(\mathfrak{H}_h - u\mathfrak{H}_g\cos\phi) + \varrho_d(a\mathfrak{H}_h\cos\lambda - u\varsigma_d\cos\phi)]\\ &\quad + a\rho^2[\varrho_k(\mathfrak{H}_h\cos\lambda - u\mathfrak{H}_g\cos\theta) + \varrho_d(a\mathfrak{H}_h - u\varsigma_d\cos\theta)]\\ &\quad + \rho^2[\mathfrak{Y}(\mathfrak{H}_g\mathfrak{H}_h - u\varsigma_g\cos\phi) + \rho(\varsigma_d\mathfrak{H}_h - ua\varsigma_g\cos\theta)].\end{aligned} \tag{4.26d}$$

Art 19a. *Development of equation* (3.24).

Using (1.3a) and (4.15a) in (3.24a), we have

$$\begin{gathered}\mathcal{S}_a = \widehat{\boldsymbol{\kappa}}\times\mathbf{u},\quad \mathcal{S}_b = \widehat{\boldsymbol{\kappa}}\times\mathbf{a},\quad \mathcal{S}_c = \mathbf{0},\quad \mathcal{S}_d = \widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}},\quad \mathcal{S}_e = \mathbf{0}\\ \mathcal{S}_f = \mathbf{a}\times\mathbf{u},\quad \mathcal{S}_g = \mathbf{0},\quad \mathcal{S}_h = \mathbf{a}\times\dot{\mathbf{a}},\quad \mathcal{S}_i = \mathbf{0}\\ \mathcal{S}_j = \mathbf{u}\times\dot{\mathbf{a}},\quad \mathcal{S}_k = \mathbf{0},\quad \mathcal{S}_l = \mathbf{0},\quad \mathcal{S}_m = \mathbf{0}\end{gathered} \tag{4.27a}$$

so that by (3.24b), (4.27a), (4.17a), (4.18a) and (4.26b),

$$\begin{aligned}&\mathcal{S}_n = \rho(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}}),\quad \mathcal{S}_o = \rho(\mathbf{a}\times\dot{\mathbf{a}}),\quad \mathcal{S}_p = \widehat{\boldsymbol{\kappa}}\times\mathbf{u},\quad \mathcal{S}_q = \mathbf{a}\times\mathbf{u},\quad \mathcal{S}_r = \dot{\mathbf{a}}\times\mathbf{u},\quad \mathcal{S}_s = \mathbf{0}\\ &\mathcal{S}_t = \mathfrak{K}_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{a}) + \mathfrak{Y}\rho(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}}) + \rho^2(\mathbf{a}\times\dot{\mathbf{a}}),\quad \mathcal{S}_u = \varrho_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + \varrho_d(\mathbf{a}\times\mathbf{u}) + \rho(\dot{\mathbf{a}}\times\mathbf{u})\end{aligned} \tag{4.27b}$$

from which we obtain

$$\mathcal{S}_t + \mathcal{S}_u = \varrho_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + \mathfrak{K}_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{a}) + \mathfrak{Y}\rho(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}}) + \rho^2(\mathbf{a}\times\dot{\mathbf{a}}) + \varrho_d(\mathbf{a}\times\mathbf{u}) + \rho(\dot{\mathbf{a}}\times\mathbf{u}). \tag{4.27c}$$

We then derive

$$\begin{aligned}\mathfrak{R}_1 &= \widehat{\boldsymbol{\kappa}}\cdot(\mathcal{S}_t + \mathcal{S}_u) \text{ by (3.24c)}\\ &= \widehat{\boldsymbol{\kappa}}\cdot[\varrho_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + \mathfrak{K}_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{a}) + \mathfrak{Y}\rho(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}}) + \rho^2(\mathbf{a}\times\dot{\mathbf{a}}) + \varrho_d(\mathbf{a}\times\mathbf{u}) + \rho(\dot{\mathbf{a}}\times\mathbf{u})] \text{ by (4.27c)}\\ &= \rho^2[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{a}\times\dot{\mathbf{a}})] + \varrho_d[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{a}\times\mathbf{u})] + \rho[\widehat{\boldsymbol{\kappa}}\cdot(\dot{\mathbf{a}}\times\mathbf{u})]\\ &= \rho^2\varsigma_m + \varrho_d\mathfrak{H}_a + \rho\mathfrak{H}_b \text{ by (4.10a) \& (4.26a)}\\ &= \mathfrak{K}_b \text{ by (4.26b)}\end{aligned} \tag{4.28a}$$

$$\begin{aligned}\mathfrak{R}_2 &= \mathbf{a}\cdot(\mathcal{S}_t + \mathcal{S}_u) \text{ by (3.24c)}\\ &= \mathbf{a}\cdot[\varrho_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + \mathfrak{K}_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{a}) + \mathfrak{Y}\rho(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}}) + \rho^2(\mathbf{a}\times\dot{\mathbf{a}}) + \varrho_d(\mathbf{a}\times\mathbf{u}) + \rho(\dot{\mathbf{a}}\times\mathbf{u})] \text{ by (4.27c)}\\ &= -\varrho_k[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{a}\times\mathbf{u})] - \mathfrak{Y}\rho[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{a}\times\dot{\mathbf{a}})] + \rho[\mathbf{a}\cdot(\dot{\mathbf{a}}\times\mathbf{u})] \text{ by (A.4)}\\ &= -\varrho_k\mathfrak{H}_a - \rho\mathfrak{Y}\varsigma_m + \rho\mathfrak{H}_c \text{ by (4.10a) \& (4.26a)}\\ &= \mathfrak{K}_c \text{ by (4.26b)}\end{aligned} \tag{4.28b}$$

$$\begin{aligned}\mathfrak{R}_3 &= \dot{\mathbf{a}}\cdot(\mathcal{S}_t + \mathcal{S}_u) \text{ by (3.24c)}\\ &= \dot{\mathbf{a}}\cdot[\varrho_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + \mathfrak{K}_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{a}) + \mathfrak{Y}\rho(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}}) + \rho^2(\mathbf{a}\times\dot{\mathbf{a}}) + \varrho_d(\mathbf{a}\times\mathbf{u}) + \rho(\dot{\mathbf{a}}\times\mathbf{u})] \text{ by (4.27c)}\\ &= -\varrho_k[\widehat{\boldsymbol{\kappa}}\cdot(\dot{\mathbf{a}}\times\mathbf{u})] + \mathfrak{K}_a[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{a}\times\dot{\mathbf{a}})] - \varrho_d[\mathbf{a}\cdot(\dot{\mathbf{a}}\times\mathbf{u})] \text{ by (A.4)}\\ &= -\varrho_k\mathfrak{H}_b + \mathfrak{K}_a\varsigma_m - \varrho_d\mathfrak{H}_c \text{ by (4.10a) \& (4.26a)}\\ &= \mathfrak{K}_d \text{ by (4.26b)}\end{aligned} \tag{4.28c}$$

$$\begin{aligned}
\Re_4 &= \ddot{\mathbf{a}} \cdot (\mathcal{S}_t + \mathcal{S}_u) \text{ by (3.24c)} \\
&= \ddot{\mathbf{a}} \cdot [\varrho_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) + \mathfrak{K}_a(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \mathcal{Y}\rho(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho^2(\mathbf{a} \times \dot{\mathbf{a}}) + \varrho_d(\mathbf{a} \times \mathbf{u}) + \rho(\dot{\mathbf{a}} \times \mathbf{u})] \text{ by (4.27c)} \\
&= \varrho_k[\ddot{\mathbf{a}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{u})] + \mathfrak{K}_a[\widehat{\boldsymbol{\kappa}} \cdot (\mathbf{a} \times \ddot{\mathbf{a}})] + \mathcal{Y}\rho[\widehat{\boldsymbol{\kappa}} \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] + \rho^2[\mathbf{a} \cdot (\dot{\mathbf{a}} \times \ddot{\mathbf{a}})] \\
&\quad + \varrho_d[\ddot{\mathbf{a}} \cdot (\mathbf{a} \times \mathbf{u})] + \rho[\ddot{\mathbf{a}} \cdot (\dot{\mathbf{a}} \times \mathbf{u})] \text{ by (A.4)} \\
&= \varrho_k \mathfrak{H}_d + \mathfrak{K}_a \varsigma_n + \rho \mathcal{Y} \varsigma_o + \rho^2 \varsigma_p + \varrho_d \mathfrak{H}_e + \rho \mathfrak{H}_f \text{ by (4.10a) \& (4.26a)} \\
&= \mathfrak{K}_e \text{ by (4.26b)}
\end{aligned} \tag{4.28d}$$

$$\begin{aligned}
\Re_5 &= \ddot{\mathbf{e}} \cdot (\mathcal{S}_t + \mathcal{S}_u) \text{ by (3.24c)} \\
&= 0 \text{ by (4.15a).}
\end{aligned} \tag{4.28e}$$

Art 19b. *Magnitude of the vector $\mathcal{S}_t + \mathcal{S}_u$.*

We also have

$$\begin{aligned}
|\mathcal{S}_t|^2 &= [\mathfrak{K}_a(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \mathcal{Y}\rho(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho^2(\mathbf{a} \times \dot{\mathbf{a}})] \cdot [\mathfrak{K}_a(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \mathcal{Y}\rho(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho^2(\mathbf{a} \times \dot{\mathbf{a}})] \text{ by (4.27b)} \\
&= \mathfrak{K}_a^2[(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{a})] + 2\rho\mathcal{Y}\mathfrak{K}_a[(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + 2\rho^2\mathfrak{K}_a[(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] \\
&\quad + \rho^2\mathcal{Y}^2[(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + 2\rho^3\mathcal{Y}[(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] + \rho^4[(\mathbf{a} \times \dot{\mathbf{a}}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] \\
&= \mathfrak{K}_a^2[\widehat{\boldsymbol{\kappa}}^2\mathbf{a}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})^2] + 2\rho\mathcal{Y}\mathfrak{K}_a[\widehat{\boldsymbol{\kappa}}^2(\mathbf{a} \cdot \dot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})] + 2\rho^2\mathfrak{K}_a[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})(\mathbf{a} \cdot \dot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})\mathbf{a}^2] \\
&\quad + \rho^2\mathcal{Y}^2[\widehat{\boldsymbol{\kappa}}^2\dot{\mathbf{a}}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})^2] + 2\rho^3\mathcal{Y}[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})\dot{\mathbf{a}}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\mathbf{a} \cdot \dot{\mathbf{a}})] + \rho^4[\mathbf{a}^2\dot{\mathbf{a}}^2 - (\mathbf{a} \cdot \dot{\mathbf{a}})^2] \text{ by (A.2)} \\
&= a^2\mathfrak{K}_a^2 \sin^2\lambda + 2\rho\mathcal{Y}\mathfrak{K}_a(\varsigma_d - a\mathfrak{H}_g \cos\lambda) + 2\rho^2\mathfrak{K}_a(a\varsigma_d \cos\lambda - \mathfrak{H}_g a^2) + \rho^2\mathcal{Y}^2(\varsigma_g - \mathfrak{H}_g^2) \\
&\quad + 2\rho^3\mathcal{Y}(a\varsigma_g \cos\lambda - \mathfrak{H}_g\varsigma_d) + \rho^4(a^2\varsigma_g - \varsigma_d^2) \text{ by (4.10a) \& (4.26a)} \\
&= \mathfrak{K}_f \text{ by (4.26c)}
\end{aligned} \tag{4.29a}$$

$$\begin{aligned}
|\mathcal{S}_u|^2 &= [\varrho_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) + \varrho_d(\mathbf{a} \times \mathbf{u}) + \rho(\dot{\mathbf{a}} \times \mathbf{u})] \cdot [\varrho_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) + \varrho_d(\mathbf{a} \times \mathbf{u}) + \rho(\dot{\mathbf{a}} \times \mathbf{u})] \text{ by (4.27b)} \\
&= \varrho_k^2[(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{u})] + 2\varrho_d\varrho_k[(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) \cdot (\mathbf{a} \times \mathbf{u})] + 2\rho\varrho_k[(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) \cdot (\dot{\mathbf{a}} \times \mathbf{u})] \\
&\quad + \varrho_d^2[(\mathbf{a} \times \mathbf{u}) \cdot (\mathbf{a} \times \mathbf{u})] + 2\rho\varrho_d[(\mathbf{a} \times \mathbf{u}) \cdot (\dot{\mathbf{a}} \times \mathbf{u})] + \rho^2[(\dot{\mathbf{a}} \times \mathbf{u}) \cdot (\dot{\mathbf{a}} \times \mathbf{u})] \\
&= \varrho_k^2[\widehat{\boldsymbol{\kappa}}^2\mathbf{u}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u})^2] + 2\varrho_d\varrho_k[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})\mathbf{u}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u})(\mathbf{a} \cdot \mathbf{u})] + 2\rho\varrho_k[(\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})\mathbf{u}^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u})(\dot{\mathbf{a}} \cdot \mathbf{u})] \\
&\quad + \varrho_d^2[\mathbf{a}^2\mathbf{u}^2 - (\mathbf{a} \cdot \mathbf{u})^2] + 2\rho\varrho_d[(\mathbf{a} \cdot \dot{\mathbf{a}})\mathbf{u}^2 - (\mathbf{a} \cdot \mathbf{u})(\dot{\mathbf{a}} \cdot \mathbf{u})] + \rho^2[\dot{\mathbf{a}}^2\mathbf{u}^2 - (\dot{\mathbf{a}} \cdot \mathbf{u})^2] \text{ by (A.2)} \\
&= u^2\varrho_k^2 \sin^2\phi + 2\varrho_d\varrho_k(u^2 a \cos\lambda - u^2 a \cos\phi \cos\theta) + 2\rho\varrho_k(u^2\mathfrak{H}_g - u\mathfrak{H}_h \cos\phi) \\
&\quad + a^2u^2\varrho_d^2 \sin^2\theta + 2\rho\varrho_d(\varsigma_d u^2 - au\mathfrak{H}_h \cos\theta) + \rho^2(u^2\varsigma_g - \mathfrak{H}_h^2) \text{ by (4.10a) \& (4.26a)} \\
&= \rho^2(u^2\varsigma_g - \mathfrak{H}_h^2) + u^2[\varrho_k^2 \sin^2\phi + a^2\varrho_d^2 \sin^2\theta + 2a\varrho_d\varrho_k(\cos\lambda - \cos\phi\cos\theta)] \\
&\quad + 2u\rho[\varrho_k(u\mathfrak{H}_g - \mathfrak{H}_h \cos\phi) + \varrho_d(u\varsigma_d - a\mathfrak{H}_h \cos\theta)] \\
&= \mathfrak{K}_g \text{ by (4.26c)}
\end{aligned} \tag{4.29b}$$

$$\begin{aligned}
\mathcal{S}_t \cdot \mathcal{S}_u &= [\mathfrak{K}_a(\widehat{\boldsymbol{\kappa}} \times \mathbf{a}) + \mathcal{Y}\rho(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}}) + \rho^2(\mathbf{a} \times \dot{\mathbf{a}})] \cdot [\varrho_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) + \varrho_d(\mathbf{a} \times \mathbf{u}) + \rho(\dot{\mathbf{a}} \times \mathbf{u})] \text{ by (4.27b)} \\
&= \mathfrak{K}_a\varrho_k[(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{a})] + \rho\mathcal{Y}\varrho_k[(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + \rho^2\varrho_k[(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] \\
&\quad + \mathfrak{K}_a\varrho_d[(\mathbf{a} \times \mathbf{u}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{a})] + \rho\mathcal{Y}\varrho_d[(\mathbf{a} \times \mathbf{u}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + \rho^2\varrho_d[(\mathbf{a} \times \mathbf{u}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] \\
&\quad + \mathfrak{K}_a\rho[(\dot{\mathbf{a}} \times \mathbf{u}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{a})] + \rho^2\mathcal{Y}[(\dot{\mathbf{a}} \times \mathbf{u}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}})] + \rho^3[(\dot{\mathbf{a}} \times \mathbf{u}) \cdot (\mathbf{a} \times \dot{\mathbf{a}})] \\
&= \mathfrak{K}_a\varrho_k[\widehat{\boldsymbol{\kappa}}^2(\mathbf{u} \cdot \mathbf{a}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u})] + \rho\mathcal{Y}\varrho_k[\widehat{\boldsymbol{\kappa}}^2(\mathbf{u} \cdot \dot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u})] \\
&\quad + \rho^2\varrho_k[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})(\mathbf{u} \cdot \dot{\mathbf{a}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{a}})(\mathbf{a} \cdot \mathbf{u})] + \mathfrak{K}_a\varrho_d[(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{u} \cdot \mathbf{a}) - \mathbf{a}^2(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u})] \\
&\quad + \rho\mathcal{Y}\varrho_d[(\mathbf{a} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{u} \cdot \dot{\mathbf{a}}) - (\mathbf{a} \cdot \dot{\mathbf{a}})(\mathbf{u} \cdot \widehat{\boldsymbol{\kappa}})] + \rho^2\varrho_d[\mathbf{a}^2(\mathbf{u} \cdot \dot{\mathbf{a}}) - (\mathbf{a} \cdot \dot{\mathbf{a}})(\mathbf{a} \cdot \mathbf{u})] \\
&\quad + \mathfrak{K}_a\rho[(\dot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{u} \cdot \mathbf{a}) - (\dot{\mathbf{a}} \cdot \mathbf{a})(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u})] + \rho^2\mathcal{Y}[(\dot{\mathbf{a}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{u} \cdot \dot{\mathbf{a}}) - \dot{\mathbf{a}}^2(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u})] \\
&\quad + \rho^3[(\dot{\mathbf{a}} \cdot \mathbf{a})(\mathbf{u} \cdot \dot{\mathbf{a}}) - \dot{\mathbf{a}}^2(\mathbf{a} \cdot \mathbf{u})] \text{ by (A.2)}
\end{aligned}$$

$$\begin{aligned}
&= ua\mathfrak{K}_a\varrho_k(\cos\theta - \cos\lambda\cos\phi) + \rho\mathfrak{Y}\varrho_k(\mathfrak{H}_h - u\mathfrak{H}_g\cos\phi) + a\rho^2\varrho_k(\mathfrak{H}_h\cos\lambda - u\mathfrak{H}_g\cos\theta)\\
&\quad + ua^2\mathfrak{K}_a\varrho_d(\cos\lambda\cos\theta - \cos\phi) + \rho\mathfrak{Y}\varrho_d(a\mathfrak{H}_h\cos\lambda - u\varsigma_d\cos\phi) + a\rho^2\varrho_d(a\mathfrak{H}_h - u\varsigma_d\cos\theta)\\
&\quad + u\rho\mathfrak{K}_a(a\mathfrak{H}_g\cos\theta - \varsigma_d\cos\phi) + \rho^2\mathfrak{Y}(\mathfrak{H}_g\mathfrak{H}_h - u\varsigma_g\cos\phi)\\
&\quad + \rho^3(\varsigma_d\mathfrak{H}_h - ua\varsigma_g\cos\theta) \text{ by (4.10a) \& (4.26a)}\\
&= u\mathfrak{K}_a[\rho(a\mathfrak{H}_g\cos\theta - \varsigma_d\cos\phi) + a\varrho_k(\cos\theta - \cos\lambda\cos\phi) + a^2\varrho_d(\cos\lambda\cos\theta - \cos\phi)]\\
&\quad + \rho\mathfrak{Y}[\varrho_k(\mathfrak{H}_h - u\mathfrak{H}_g\cos\phi) + \varrho_d(a\mathfrak{H}_h\cos\lambda - u\varsigma_d\cos\phi)]\\
&\quad + a\rho^2[\varrho_k(\mathfrak{H}_h\cos\lambda - u\mathfrak{H}_g\cos\theta) + \varrho_d(a\mathfrak{H}_h - u\varsigma_d\cos\theta)]\\
&\quad + \rho^2[\mathfrak{Y}(\mathfrak{H}_g\mathfrak{H}_h - u\varsigma_g\cos\phi) + \rho(\varsigma_d\mathfrak{H}_h - ua\varsigma_g\cos\theta)]\\
&= \mathfrak{K}_h \text{ by (4.26d)} \qquad (4.29c)
\end{aligned}$$

$$\begin{aligned}
|\mathbf{\mathfrak{S}}_t + \mathbf{\mathfrak{S}}_u|^2 &= |\mathbf{\mathfrak{S}}_t|^2 + 2(\mathbf{\mathfrak{S}}_t\cdot\mathbf{\mathfrak{S}}_u) + |\mathbf{\mathfrak{S}}_u|^2\\
&= \mathfrak{K}_f + 2\mathfrak{K}_h + \mathfrak{K}_g \text{ by (4.29)} \qquad (4.30a)
\end{aligned}$$

$$\begin{aligned}
&\ddot{\mathfrak{Y}}\mathfrak{R}_1 + \ddot{\rho}\mathfrak{R}_2 + b_2\mathfrak{R}_3 + \rho\mathfrak{R}_4 + \mathfrak{R}_5\\
&\quad = \ddot{\mathfrak{Y}}\mathfrak{K}_b + \ddot{\rho}\mathfrak{K}_c + b_2\mathfrak{K}_d + \rho\mathfrak{K}_e \text{ by (4.28)}\\
&\quad = \varrho_l\mathfrak{K}_b + \varrho_i\mathfrak{K}_c + \mathfrak{K}_d(1+2\varrho_d) + \rho\mathfrak{K}_e \text{ by (4.14b), (4.17b) \& (4.18b).} \qquad (4.30b)
\end{aligned}$$

Art 19c. *Results of the computations.*

Substituting (4.27c), (4.30) and (1.3a) into (3.26), we obtain the equations describing the apparent geometry of the rays as

$$\begin{aligned}
\overline{\mathbb{K}} = \frac{(\mathfrak{K}_f + 2\mathfrak{K}_h + \mathfrak{K}_g)^{1/2}}{c^3\mathcal{R}^3}, \quad \overline{\mathbb{T}} = \frac{\varrho_l\mathfrak{K}_b + \varrho_i\mathfrak{K}_c + \mathfrak{K}_d(1+2\varrho_d) + \rho\mathfrak{K}_e}{\mathfrak{K}_f + 2\mathfrak{K}_h + \mathfrak{K}_g}, \quad \overline{\boldsymbol{\ell}}_t = \frac{\mathfrak{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u}}{c\mathcal{R}}\\
\overline{\boldsymbol{\ell}}_b = \frac{\varrho_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + \mathfrak{K}_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{a}) + \varrho_d(\mathbf{a}\times\mathbf{u}) + \rho[(\dot{\mathbf{a}}\times\mathbf{u}) + \mathfrak{Y}(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}}) + \rho(\mathbf{a}\times\dot{\mathbf{a}})]}{(\mathfrak{K}_f + 2\mathfrak{K}_h + \mathfrak{K}_g)^{1/2}}
\end{aligned} \qquad (4.31)$$

where $\mathcal{R}$ is given by (4.2b), $\mathfrak{Y}$ is given by (4.2d), and all other quantities are given by (4.26) or (4.10).

5 Rotational obliquation

Art 20. *Apparent direction to a light source.*

To evaluate (3.6) for a rotating observer, it is convenient to introduce the quantities

$$\begin{aligned}
&\varepsilon_a = \boldsymbol{\Omega}\cdot\widehat{\boldsymbol{\kappa}}, \quad \varepsilon_b = \boldsymbol{\Omega}\cdot\mathbf{r}, \quad \varepsilon_c = \boldsymbol{\Omega}\cdot\widehat{\mathbf{p}}, \quad \varepsilon_d = \widehat{\boldsymbol{\kappa}}\cdot\mathbf{r}, \quad \varepsilon_e = \widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}}, \quad \varepsilon_f = \mathbf{r}\cdot\widehat{\mathbf{p}}\\
&\varepsilon_g = \boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}, \quad \varepsilon_h = \mathbf{r}\cdot\boldsymbol{\Lambda}, \quad \varepsilon_i = \widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda}, \quad \varepsilon_j = \widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) \neq 0, \quad \varepsilon_k = \widehat{\boldsymbol{\kappa}}\cdot(\boldsymbol{\Omega}\times\mathbf{r})\\
&\varepsilon_l = \widehat{\mathbf{p}}\cdot(\boldsymbol{\Omega}\times\mathbf{r}), \quad \varepsilon_m = \boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r}), \quad \varepsilon_n = \widehat{\boldsymbol{\kappa}}\cdot(\boldsymbol{\Lambda}\times\mathbf{r}), \quad \varepsilon_o = \widehat{\mathbf{p}}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})
\end{aligned} \qquad (5.1a)$$

$$\begin{aligned}
&\varphi_a = (\Omega^2 r^2 - \varepsilon_b^2)^{1/2} \neq 0, \quad \varphi_b = (\Omega^2\varphi_a^2 + 2\varepsilon_b\varepsilon_m + \Lambda^2 r^2 - \varepsilon_h^2)^{1/2}, \quad \varphi_c = r^2\varepsilon_g - \varepsilon_b\varepsilon_h\\
&\varphi_d = \varepsilon_a\varepsilon_b - \Omega^2\varepsilon_d + \varepsilon_n, \quad \varphi_e = \varepsilon_c\varepsilon_a - \Omega^2\varepsilon_e, \quad \varphi_f = \varepsilon_e\varepsilon_g - \varepsilon_i\varepsilon_a, \quad \varphi_g = \varphi_f - 4(\varphi_e^2/\varepsilon_j)\\
&\varphi_h = (2\rho\varphi_d - cd)/[2\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)], \quad \varphi_i = \varphi_h(\varepsilon_i\varepsilon_j + 8\varphi_e\varepsilon_c), \quad \varphi_j = \varphi_h(4\varphi_e\Omega^2 + \varepsilon_j\varepsilon_g)\\
&\varphi_k = 2(\rho\varphi_d - \varepsilon_j\varphi_g\varphi_h), \quad \varphi_l = \left|1 + [\varphi_g/(\varepsilon_j\omega_o^2)]\right|^{1/2} \neq 0, \quad \varphi_m = \varphi_g\varphi_h\varepsilon_j + \varphi_i\varepsilon_a - \varphi_j\varepsilon_e
\end{aligned} \qquad (5.1b)$$

$$\begin{aligned} \varphi_n &= \varphi_g\varphi_h(\varepsilon_c\varepsilon_b - \Omega^2\varepsilon_f) - \varphi_j\varepsilon_l, \quad \varphi_o = \varphi_g\varphi_h\varepsilon_j + \varphi_i\varepsilon_a - \varphi_j\varepsilon_e \\ \varphi_p &= \varphi_g\varphi_h(-\varepsilon_l\Omega^2 + \varepsilon_i\varepsilon_b - \varepsilon_f\varepsilon_g) + \varphi_i\varepsilon_m - \varphi_j(\varepsilon_c\varepsilon_b - \varepsilon_f\Omega^2 + \varepsilon_o) \\ \varphi_q &= [\varphi_g^2\varphi_h^2(\Omega^2 - \varepsilon_c^2) + \varphi_i^2\Omega^2 - 2\varphi_i\varphi_j\varepsilon_c + \varphi_j^2]^{1/2} \end{aligned} \tag{5.1c}$$

$$\begin{gathered} \varphi_r = [\varphi_n + 2(\rho\varphi_c - \varphi_k\varepsilon_k)]/\varphi_a^2, \quad \varphi_s = [\varphi_k(2\rho\varphi_c - \varphi_k\varepsilon_k) - 2cd(\varphi_n - \varphi_k\varepsilon_k)]/\varphi_a^2 \\ \varphi_t = \varepsilon_k/\varphi_a, \quad \varphi_u = \varphi_d/\varphi_b, \quad \varphi_v = \varphi_c/(\varphi_a\varphi_b). \end{gathered} \tag{5.1d}$$

Art 20a. *Development of equation* (1.4).

With the above quantities in view, we derive

$$\begin{aligned} u^2 &= (\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{r}) \text{ by (1.4c)} \\ &= \Omega^2 r^2 - (\mathbf{\Omega}\cdot\mathbf{r})^2 \text{ by (A.2)} \\ &= \Omega^2 r^2 - \varepsilon_b^2 \text{ by (5.1a)} \\ \therefore u &= \varphi_a \text{ by (5.1b)} \end{aligned} \tag{5.2a}$$

$$\begin{aligned} a^2 &= (\mathbf{\Omega}\times\mathbf{u} + \mathbf{\Lambda}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{u} + \mathbf{\Lambda}\times\mathbf{r}) \text{ by (1.4a)} \\ &= (\mathbf{\Omega}\times\mathbf{u})\cdot(\mathbf{\Omega}\times\mathbf{u}) + 2(\mathbf{\Omega}\times\mathbf{u})\cdot(\mathbf{\Lambda}\times\mathbf{r}) + (\mathbf{\Lambda}\times\mathbf{r})\cdot(\mathbf{\Lambda}\times\mathbf{r}) \\ &= [\Omega^2u^2 - (\mathbf{\Omega}\cdot\mathbf{u})^2] + 2[(\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{u}\cdot\mathbf{r}) - (\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\mathbf{u})] + [\Lambda^2r^2 - (\mathbf{\Lambda}\cdot\mathbf{r})^2] \text{ by (A.2)} \\ &= \Omega^2u^2 + 2(\mathbf{\Omega}\cdot\mathbf{r})[\mathbf{\Omega}\cdot(\mathbf{\Lambda}\times\mathbf{r})] + [\Lambda^2r^2 - (\mathbf{\Lambda}\cdot\mathbf{r})^2] \text{ by (1.4c) \& (A.4)} \\ &= \Omega^2\varphi_a^2 + 2\varepsilon_b\varepsilon_m + \Lambda^2r^2 - \varepsilon_h^2 \text{ by (5.1a) \& (5.2a)} \\ \therefore a &= \varphi_b \text{ by (5.1b)} \end{aligned} \tag{5.2b}$$

$$\begin{aligned} \mathbf{u}\cdot\mathbf{a} &= \mathbf{u}\cdot(\mathbf{\Omega}\times\mathbf{u} + \mathbf{\Lambda}\times\mathbf{r}) \text{ by (1.4a)} \\ &= (\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Lambda}\times\mathbf{r}) \text{ by (1.4c)} \\ &= r^2(\mathbf{\Omega}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\mathbf{r}) \text{ by (A.2)} \\ &= r^2\varepsilon_g - \varepsilon_b\varepsilon_h \text{ by (5.1a)} \\ &= \varphi_c \text{ by (5.1b)} \end{aligned} \tag{5.3a}$$

$$\begin{aligned} \boldsymbol{\kappa}\cdot\mathbf{a} &= \boldsymbol{\kappa}\cdot(\mathbf{\Omega}\times\mathbf{u} + \mathbf{\Lambda}\times\mathbf{r}) \text{ by (1.4a)} \\ &= \boldsymbol{\kappa}\cdot[\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})] + \boldsymbol{\kappa}\cdot(\mathbf{\Lambda}\times\mathbf{r}) \text{ by (1.4c)} \\ &= \boldsymbol{\kappa}\cdot[\mathbf{\Omega}(\mathbf{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}] + \boldsymbol{\kappa}\cdot(\mathbf{\Lambda}\times\mathbf{r}) \text{ by (A.1)} \\ &= (\boldsymbol{\kappa}\cdot\mathbf{\Omega})(\mathbf{\Omega}\cdot\mathbf{r}) - \Omega^2(\boldsymbol{\kappa}\cdot\mathbf{r}) + \boldsymbol{\kappa}\cdot(\mathbf{\Lambda}\times\mathbf{r}) \\ &= \kappa(\varepsilon_a\varepsilon_b - \Omega^2\varepsilon_d + \varepsilon_n) \text{ by (5.1a)} \\ &= \kappa\varphi_d \text{ by (5.1b)} \end{aligned} \tag{5.3b}$$

$$\begin{aligned} \alpha &= \boldsymbol{\kappa}\cdot\mathbf{a} \text{ by (1.2b)} \\ &= \kappa\varphi_d \text{ by (5.3b)} \end{aligned} \tag{5.4a}$$

$$\begin{aligned} \eta &= \kappa\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega}) \text{ by (1.4c)} \\ &= \kappa\varepsilon_j \text{ by (5.1a)} \end{aligned} \tag{5.4b}$$

$$\begin{aligned}\xi &= 2(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) - \Omega^2(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}}) \text{ by (1.4c)}\\ &= 2\kappa(\varepsilon_c\varepsilon_a - \Omega^2\varepsilon_e) \text{ by (5.1a)}\\ &= 2\kappa\varphi_e \text{ by (5.1b)}\end{aligned} \tag{5.4c}$$

$$\begin{aligned}\zeta &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\mathbf{\Lambda}) - (\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) \text{ by (1.4c)}\\ &= \kappa(\varepsilon_e\varepsilon_g - \varepsilon_i\varepsilon_a) \text{ by (5.1a)}\\ &= \kappa\varphi_f \text{ by (5.1b)}\end{aligned} \tag{5.4d}$$

$$\begin{aligned}s_0 &= \zeta - (\xi^2/\eta) \text{ by (1.4d)}\\ &= \kappa\varphi_f - [(2\kappa\varphi_e)^2/(\kappa\varepsilon_j)] \text{ by (5.4)}\\ &= \kappa[\varphi_f - 4(\varphi_e^2/\varepsilon_j)]\\ &= \kappa\varphi_g \text{ by (5.1b)}\end{aligned} \tag{5.5a}$$

$$\begin{aligned}s_1 &= (2\rho\alpha - d\omega_o)/[2\eta(s_0 + \eta\omega_o^2)] \text{ by (1.4d)}\\ &= (2\rho\kappa\varphi_d - d\omega_o)/[2(\kappa\varepsilon_j)(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)] \text{ by (5.4) \& (5.5a)}\\ &= (2\rho\varphi_d - cd)/[2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)]\\ &= \varphi_h/\kappa \text{ by (5.1b)}\end{aligned} \tag{5.5b}$$

$$\begin{aligned}s_2 &= s_0 s_1 \text{ by (1.4d)}\\ &= \varphi_g\varphi_h \text{ by (5.5a) \& (5.5b)}\end{aligned} \tag{5.5c}$$

$$\begin{aligned}s_3 &= [\eta(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) + 4\xi(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})]s_1 \text{ by (1.4d)}\\ &= [\kappa\varepsilon_j\varepsilon_i + 8\kappa\varphi_e\varepsilon_c](\varphi_h/\kappa) \text{ by (5.4), (5.1a) \& (5.5b)}\\ &= \varphi_h(\varepsilon_i\varepsilon_j + 8\varphi_e\varepsilon_c)\\ &= \varphi_i \text{ by (5.1b)}\end{aligned} \tag{5.5d}$$

$$\begin{aligned}s_4 &= [2\xi\Omega^2 + \eta(\mathbf{\Omega}\cdot\mathbf{\Lambda})]s_1 \text{ by (1.4d)}\\ &= [2(2\kappa\varphi_e)\Omega^2 + \kappa\varepsilon_j\varepsilon_g](\varphi_h/\kappa) \text{ by (5.4), (5.1a) \& (5.5b)}\\ &= \varphi_h(4\varphi_e\Omega^2 + \varepsilon_j\varepsilon_g)\\ &= \varphi_j \text{ by (5.1b)}\end{aligned} \tag{5.5e}$$

$$\begin{aligned}\kappa\tau &= 2(\rho\alpha - \eta s_2)/\kappa \text{ by (1.4b)}\\ &= 2(\rho\kappa\varphi_d - \kappa\varepsilon_j\varphi_g\varphi_h)/\kappa \text{ by (5.4) \& (5.5c)}\\ &= 2(\rho\varphi_d - \varepsilon_j\varphi_g\varphi_h)\\ &= \varphi_k \text{ by (5.1b)}\end{aligned} \tag{5.6a}$$

$$\begin{aligned}\gamma^2 &= \left|1 + [s_0/(\eta\omega_o^2)]\right| \text{ by (1.4a)}\\ &= \left|1 + [(\kappa\varphi_g)/(\kappa\varepsilon_j\omega_o^2)]\right| \text{ by (5.5a) \& (5.4b)}\\ &= \left|1 + [\varphi_g/(\varepsilon_j\omega_o^2)]\right|\\ \therefore \gamma &= \varphi_l \text{ by (5.1b).}\end{aligned} \tag{5.6b}$$

Art 20b. *Development of equation* (3.2).

The various quantities defined by (3.2) become

$$\begin{aligned}\mathcal{B} &= \mathbf{c}\cdot\mathbf{e} \text{ by (3.2a)}\\ &= c\widehat{\boldsymbol{\kappa}}\cdot[s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.1) \& (1.4b)}\\ &= cs_2[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + cs_3(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Omega}) - cs_4(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})\\ &= c(\varphi_g\varphi_h\varepsilon_j + \varphi_i\varepsilon_a - \varphi_j\varepsilon_e) \text{ by (5.1a) \& (5.5)}\\ &= c\varphi_m \text{ by (5.1b)}\end{aligned}\tag{5.7a}$$

$$\begin{aligned}\mathcal{D} &= \mathbf{u}\cdot\mathbf{e} \text{ by (3.2a)}\\ &= \mathbf{u}\cdot[s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.4b)}\\ &= s_2(\boldsymbol{\Omega}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3(\boldsymbol{\Omega}\times\mathbf{r})\cdot\boldsymbol{\Omega} - s_4(\boldsymbol{\Omega}\times\mathbf{r})\cdot\widehat{\mathbf{p}} \text{ by (1.4c)}\\ &= s_2[(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})(\boldsymbol{\Omega}\cdot\mathbf{r}) - \Omega^2(\widehat{\mathbf{p}}\cdot\mathbf{r})] - s_4[\widehat{\mathbf{p}}\cdot(\boldsymbol{\Omega}\times\mathbf{r})] \text{ by (A.2)}\\ &= \varphi_g\varphi_h(\varepsilon_c\varepsilon_b - \Omega^2\varepsilon_f) - \varphi_j\varepsilon_l \text{ by (5.1a) \& (5.5)}\\ &= \varphi_n \text{ by (5.1c)}\end{aligned}\tag{5.7b}$$

$$\begin{aligned}\mathcal{A} &= \mathbf{a}\cdot\mathbf{e} \text{ by (3.2a)}\\ &= \mathbf{a}\cdot[s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.4b)}\\ &= s_2[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\cdot(\boldsymbol{\Omega}\times\mathbf{u} + \boldsymbol{\Lambda}\times\mathbf{r})] + s_3[\boldsymbol{\Omega}\cdot(\boldsymbol{\Omega}\times\mathbf{u} + \boldsymbol{\Lambda}\times\mathbf{r})] - s_4[\widehat{\mathbf{p}}\cdot(\boldsymbol{\Omega}\times\mathbf{u} + \boldsymbol{\Lambda}\times\mathbf{r})]\\ &\qquad \text{by (1.4a)}\end{aligned}$$

$$\begin{aligned} &= s_2[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\cdot(\boldsymbol{\Omega}\times\mathbf{u}) + (\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\cdot(\boldsymbol{\Lambda}\times\mathbf{r})] + s_3[\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})]\\ &\qquad - s_4[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\cdot(\boldsymbol{\Omega}\times\mathbf{r}) + \widehat{\mathbf{p}}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})] \text{ by (A.4) \& (1.4c)}\\ &= s_2[(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})(\boldsymbol{\Omega}\cdot\mathbf{u}) - (\widehat{\mathbf{p}}\cdot\mathbf{u})\Omega^2 + (\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda})(\boldsymbol{\Omega}\cdot\mathbf{r}) - (\widehat{\mathbf{p}}\cdot\mathbf{r})(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega})] + s_3[\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})]\\ &\qquad - s_4[(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})(\boldsymbol{\Omega}\cdot\mathbf{r}) - (\widehat{\mathbf{p}}\cdot\mathbf{r})\Omega^2 + \widehat{\mathbf{p}}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})] \text{ by (A.2)}\\ &= \varphi_g\varphi_h(-\varepsilon_l\Omega^2 + \varepsilon_i\varepsilon_b - \varepsilon_f\varepsilon_g) + \varphi_i\varepsilon_m - \varphi_j(\varepsilon_c\varepsilon_b - \varepsilon_f\Omega^2 + \varepsilon_o) \text{ by (1.4c), (5.1a) \& (5.5)}\\ &= \varphi_p \text{ by (5.1c)}\end{aligned}\tag{5.7c}$$

$$\begin{aligned}\mathcal{H} &= \boldsymbol{\kappa}\cdot\mathbf{e} \text{ by (3.2a)}\\ &= \boldsymbol{\kappa}\cdot[s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.4b)}\\ &= s_2[\boldsymbol{\kappa}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + s_3(\boldsymbol{\kappa}\cdot\boldsymbol{\Omega}) - s_4(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})\\ &= \kappa(\varphi_g\varphi_h\varepsilon_j + \varphi_i\varepsilon_a - \varphi_j\varepsilon_e) \text{ by (5.1a) \& (5.5)}\\ &= \kappa\varphi_o \text{ by (5.1c)}\end{aligned}\tag{5.7d}$$

$$\begin{aligned}\mathcal{E} &= \mathbf{e}\cdot\mathbf{e} \text{ by (3.2a)}\\ &= [s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}]\cdot[s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.4b)}\\ &= s_2^2[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + 2s_2s_3[\boldsymbol{\Omega}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] - 2s_2s_4[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})]\\ &\qquad + s_3^2(\boldsymbol{\Omega}\cdot\boldsymbol{\Omega}) - 2s_3s_4(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega}) + s_4^2(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{p}})\\ &= s_2^2[\Omega^2 - (\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})^2] + s_3^2\Omega^2 - 2s_3s_4(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega}) + s_4^2 \text{ by (A.2)}\\ &= \varphi_g^2\varphi_h^2(\Omega^2 - \varepsilon_c^2) + \varphi_i^2\Omega^2 - 2\varphi_i\varphi_j\varepsilon_c + \varphi_j^2 \text{ by (5.1a) \& (5.5)}\\ &= \varphi_q^2 \text{ by (5.1c).}\end{aligned}\tag{5.7e}$$

Using (5.7) in (3.2), we have

$$\begin{aligned}\mathcal{L}_0 &= \mathcal{D} - u\kappa\tau\cos\phi \text{ by (3.2b)}\\ &= \varphi_n - \varphi_k(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u}) \text{ by (5.7b) \& (5.6a)}\\ &= \varphi_n - \varphi_k\varepsilon_k \text{ by (1.4c) \& (5.1a)}\end{aligned}\tag{5.8a}$$

$$\mathcal{L}_1 = \rho a\cos\theta - \kappa\tau\cos\phi \text{ by (3.2b)}\tag{5.8b}$$

$$\begin{aligned}\mathcal{L}_2 &= \mathcal{L}_1 + \rho a\cos\theta \text{ by (3.2b)}\\ &= 2\rho a\cos\theta - \kappa\tau\cos\phi \text{ by (5.8b)}\end{aligned}\tag{5.8c}$$

$$\begin{aligned}\mathcal{N}_1 &= 2du^2(\mathcal{B} - \tau\omega_o) \text{ by (3.2c)}\\ &= 2cdu^2(\varphi_m - \kappa\tau) \text{ by (5.7a) \& (1.2b)}\\ &= 2cdu^2(\varphi_m - \varphi_k) \text{ by (5.6a)}\end{aligned}\tag{5.9a}$$

$$\begin{aligned}\mathcal{N}_2 &= 2duc\mathcal{L}_0\cos\phi \text{ by (3.2c)}\\ &= 2duc(\varphi_n - \varphi_k\varepsilon_k)\cos\phi \text{ by (5.8a)}\\ &= 2dc(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\varphi_n - \varphi_k\varepsilon_k)\\ &= 2cd\varepsilon_k(\varphi_n - \varphi_k\varepsilon_k) \text{ by (1.4c) \& (5.1a)}\end{aligned}\tag{5.9b}$$

$$\begin{aligned}\mathcal{N}_3 &= u^2[\mathcal{E} + 2(\rho\mathcal{A} - \tau\mathcal{H})] \text{ by (3.2c)}\\ &= u^2(\varphi_q^2 + 2\rho\varphi_p - 2\kappa\tau\varphi_o) \text{ by (5.7)}\\ &= u^2(\varphi_q^2 + 2\rho\varphi_p - 2\varphi_k\varphi_o) \text{ by (5.6a)}\end{aligned}\tag{5.9c}$$

$$\begin{aligned}\mathcal{N}_4 &= u^2\kappa\tau(\kappa\tau - 2\rho a\cos\lambda) \text{ by (3.2d)}\\ &= u^2\varphi_k[\varphi_k - 2\rho(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{a})] \text{ by (5.6a)}\\ &= u^2\varphi_k(\varphi_k - 2\rho\varphi_d) \text{ by (5.3b)}\end{aligned}\tag{5.9d}$$

$$\begin{aligned}\mathcal{N}_5 &= u^2\kappa\tau\mathcal{L}_2\cos\phi \text{ by (3.2d)}\\ &= u^2\varphi_k(2\rho a\cos\theta - \varphi_k\cos\phi)\cos\phi \text{ by (5.8c) \& (5.6a)}\\ &= \varphi_k(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})[2\rho(\mathbf{u}\cdot\mathbf{a}) - \varphi_k(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})]\\ &= \varphi_k\varepsilon_k(2\rho\varphi_c - \varphi_k\varepsilon_k) \text{ by (1.4c), (5.1a) \& (5.3a)}\end{aligned}\tag{5.9e}$$

$$\begin{aligned}\mathcal{N}_6 &= \mathcal{D}(\mathcal{D} + 2u\mathcal{L}_1) \text{ by (3.2d)}\\ &= \varphi_n[\varphi_n + 2u(\rho a\cos\theta - \kappa\tau\cos\phi)] \text{ by (5.7b) \& (5.8b)}\\ &= \varphi_n[\varphi_n + 2\rho(\mathbf{u}\cdot\mathbf{a}) - 2\varphi_k(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})] \text{ by (5.6a)}\\ &= \varphi_n(\varphi_n + 2\rho\varphi_c - 2\varphi_k\varepsilon_k) \text{ by (1.4c), (5.1a) \& (5.3a)}\end{aligned}\tag{5.9f}$$

$$\begin{aligned}&\mathcal{N}_1 + \mathcal{N}_3 + \mathcal{N}_4 - 2u^2(\mathcal{D} + u\mathcal{L}_1)\\ &= 2cdu^2(\varphi_m - \varphi_k) + u^2(\varphi_q^2 + 2\rho\varphi_p - 2\varphi_k\varphi_o) + u^2\varphi_k(\varphi_k - 2\rho\varphi_d)\\ &\qquad - 2u^2[\varphi_n + u(\rho a\cos\theta - \kappa\tau\cos\phi)] \text{ by (5.9), (5.7b) \& (5.8b)}\\ &= u^2[\varphi_q^2 + 2\rho\varphi_p - 2\varphi_k\varphi_o + \varphi_k(\varphi_k - 2\rho\varphi_d) + 2cd(\varphi_m - \varphi_k)\\ &\qquad - 2\varphi_n - 2\rho(\mathbf{u}\cdot\mathbf{a}) + 2\varphi_k(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})] \text{ by (5.6a)}\\ &= u^2[\varphi_q^2 + 2\rho\varphi_p - 2\varphi_k\varphi_o + \varphi_k(\varphi_k - 2\rho\varphi_d) + 2cd(\varphi_m - \varphi_k)\\ &\qquad - 2\varphi_n - 2\rho\varphi_c + 2\varphi_k\varepsilon_k] \text{ by (5.3a), (1.4c) \& (5.1a)}\\ &= u^2(\varphi_q^2 + \varphi_k^2) - 2u^2[\varphi_n - \rho(\varphi_p - \varphi_c - \varphi_k\varphi_d) + \varphi_k(\varphi_o - \varepsilon_k) - cd(\varphi_m - \varphi_k)]\end{aligned}\tag{5.10a}$$

$$
\begin{aligned}
&\mathcal{N}_1 - \mathcal{N}_2 + \mathcal{N}_3 + \mathcal{N}_4 + \mathcal{N}_5 - \mathcal{N}_6 \\
&= 2cdu^2(\varphi_m - \varphi_k) - 2cd\varepsilon_k(\varphi_n - \varphi_k\varepsilon_k) + u^2(\varphi_q^2 + 2\rho\varphi_p - 2\varphi_k\varphi_o) \\
&\quad + u^2\varphi_k(\varphi_k - 2\rho\varphi_d) + \varphi_k\varepsilon_k(2\rho\varphi_c - \varphi_k\varepsilon_k) - \varphi_n(\varphi_n + 2\rho\varphi_c - 2\varphi_k\varepsilon_k) \text{ by (5.9)} \\
&= u^2(\varphi_q^2 + \varphi_k^2) - 2u^2[\varphi_k\varphi_o - \rho(\varphi_p - \varphi_k\varphi_d) - cd(\varphi_m - \varphi_k)] \\
&\quad + \varepsilon_k[\varphi_k(2\rho\varphi_c - \varphi_k\varepsilon_k) - 2cd(\varphi_n - \varphi_k\varepsilon_k)] - \varphi_n[\varphi_n + 2(\rho\varphi_c - \varphi_k\varepsilon_k)] \\
&= u^2(\varphi_q^2 + \varphi_k^2) - 2u^2[\varphi_k\varphi_o - \rho(\varphi_p - \varphi_k\varphi_d) - cd(\varphi_m - \varphi_k)] + u^2(\varepsilon_k\varphi_s - \varphi_n\varphi_r) \\
&\quad \text{by (5.1d) \& (5.2a).}
\end{aligned} \tag{5.10b}
$$

Using (5.1d) and the definition of the angles shown in Figure 1, we have

$$
\begin{gathered}
\cos\phi = (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u})/u = \varepsilon_k/\varphi_a = \varphi_t, \quad \cos\lambda = (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{a})/a = \varphi_d/\varphi_b = \varphi_u \\
\cos\theta = (\mathbf{u} \cdot \mathbf{a})/(ua) = \varphi_c/(\varphi_a\varphi_b) = \varphi_v.
\end{gathered} \tag{5.11}
$$

Art 20c. *Results of the computations.*

Substituting (5.2a), (5.8a), (5.10) and (5.11) into (3.3) yields

$$
\begin{aligned}
\mathcal{L}c &= (\varphi_n - \varphi_k\varepsilon_k)/\varphi_a \\
\mathcal{P}c^2 &= \varphi_q^2 + \varphi_k^2 - 2[\varphi_n - \rho(\varphi_p - \varphi_c - \varphi_k\varphi_d) + \varphi_k(\varphi_o - \varepsilon_k) - cd(\varphi_m - \varphi_k)] \\
\mathcal{N}c^2 &= \varphi_q^2 + \varphi_k^2 + \varepsilon_k\varphi_s - \varphi_n\varphi_r - 2[\varphi_k\varphi_o - \rho(\varphi_p - \varphi_k\varphi_d) - cd(\varphi_m - \varphi_k)]
\end{aligned} \tag{5.12a}
$$

$$
\begin{aligned}
\mathcal{G} &= \mathcal{L} - \beta + d\varphi_t + \rho\sigma\varphi_v \\
\mathcal{R}^2 &= \mathcal{P} + d^2 + \beta^2 + \rho^2\sigma^2 + 2d(\rho\sigma\varphi_u - \beta\varphi_t) \\
\mathcal{F}^2 &= \mathcal{N} + d^2(1 - \varphi_t^2) + \rho^2\sigma^2(1 - \varphi_v^2) + 2d\rho\sigma(\varphi_u - \varphi_t\varphi_v)
\end{aligned} \tag{5.12b}
$$

while from (5.12b) and (3.6), we get

$$
\tan\psi = \frac{[\mathcal{N} + d^2(1 - \varphi_t^2) + \rho^2\sigma^2(1 - \varphi_v^2) + 2d\rho\sigma(\varphi_u - \varphi_t\varphi_v)]^{1/2}}{\mathcal{L} - \beta + d\varphi_t + \rho\sigma\varphi_v} \tag{5.12c}
$$

and by (1.2), (1.6), (5.2), (5.6b) and (5.4a),

$$
\begin{gathered}
\rho = \frac{\pi}{4d\omega_o}, \quad \pi = \frac{\vartheta}{\sqrt{1 + \vartheta^2}}, \quad d = \gamma\left\{\frac{1 + \sqrt{1 + \vartheta^2}}{2}\right\}^{1/2} \\
\vartheta = \alpha/(\gamma\omega_o)^2, \quad \alpha = \kappa\varphi_d, \quad \gamma = \varphi_l, \quad \beta = \varphi_a/c, \quad \sigma = \varphi_b/c, \quad \mathcal{Y} = cd - \varphi_k.
\end{gathered} \tag{5.12d}
$$

Equations (5.1) and (5.12) give a complete prescription for calculating ψ for a rotating observer.

Art 21. *Apparent drift of a light source.*

To evaluate (3.13) for a rotating observer, let us introduce the following quantities in addition to those given by (5.1) and (5.12),

$$
\begin{aligned}
&\delta_a = \widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda}, \quad \delta_b = \widehat{\boldsymbol{\kappa}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}), \quad \delta_c = -\mathcal{Y}\varepsilon_d + \rho\varphi_a^2 - \varphi_g\varphi_h\varepsilon_l - \varphi_i\varepsilon_b + \varphi_j\varepsilon_f \\
&\delta_d = \mathcal{Y}\varepsilon_a + \rho\varepsilon_m + \varphi_i\Omega^2 - \varphi_j\varepsilon_c, \quad \delta_e = \varphi_j\varepsilon_j + \Omega^2(\varepsilon_d - \rho\varepsilon_k + \varphi_g\varphi_h\varepsilon_e) + \rho(\varepsilon_b\delta_a - \varepsilon_d\varepsilon_g) \\
&\delta_f = \delta_e - \varepsilon_a(\varepsilon_b + \varphi_g\varphi_h\varepsilon_c), \quad \delta_g = (1 + \vartheta^2)^{1/2}, \quad \delta_h = (d\gamma^{-2}\omega_o^{-1} - \rho\vartheta\delta_g^2)/(4d^2\omega_o^2\delta_g^3) \\
&\delta_i = (\rho + 2\kappa\varphi_d\delta_h)/[2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)], \quad \delta_j = \varepsilon_j\varepsilon_i + 8\varphi_e\varepsilon_c, \quad \delta_k = 4\varphi_e\Omega^2 + \varepsilon_j\varepsilon_g
\end{aligned} \tag{5.13a}
$$

$$
\begin{aligned}
&\delta_l = \kappa\delta_i(\varphi_g\varepsilon_j + \delta_j\varepsilon_a - \delta_k\varepsilon_e), \quad \delta_m = \kappa\delta_i(\delta_k\varepsilon_c - \delta_j\Omega^2), \quad \delta_n = \kappa\delta_i(\varepsilon_a\varepsilon_b - \Omega^2\varepsilon_d) \\
&\delta_o = \delta_l + 2\rho, \quad \delta_p = \delta_n\delta_j + \rho\varepsilon_b, \quad \delta_q = \rho(\Omega^2 - \rho\varepsilon_g), \quad \delta_r = \kappa\delta_i\delta_f\delta_j - \rho(\varepsilon_b + \varphi_g\varphi_h\varepsilon_c) \\
&\delta_s = \rho\varphi_g\varphi_h\Omega^2 - \kappa\delta_i\delta_f\delta_k, \quad \delta_t = \kappa\delta_i\delta_f\varphi_g + \rho\varphi_j, \quad \delta_u = 2[\varphi_d\delta_h - \varepsilon_j\varphi_g\delta_i + (\rho/\kappa)]
\end{aligned} \tag{5.13b}
$$

$$\begin{aligned}&\hbar_a = (\varepsilon_b\delta_c + r^2\delta_d)/(\beta c^2\mathcal{F}), \quad \hbar_b = (\Omega^2\delta_c + \varepsilon_b\delta_d)/(\beta c^2\mathcal{F}), \quad \hbar_c = \mathcal{G}(\hbar_a\varepsilon_a - \hbar_b\varepsilon_d) - \mathcal{F}\varepsilon_k\\ &\hbar_d = \mathcal{G}(\hbar_a\varepsilon_m + \hbar_b\varphi_a^2) - \mathcal{F}\varphi_c, \quad \hbar_e = \mathcal{F}(\delta_o\varepsilon_b + \delta_m\varepsilon_d), \quad \hbar_f = \mathcal{F}(\delta_o\Omega^2 + \varepsilon_a\delta_m).\end{aligned} \tag{5.13c}$$

Art 21a. *Development of equation* (3.8b).

We derive, with a view to the above quantities,

$$\begin{aligned}\boldsymbol{\Omega}\times\boldsymbol{\upsilon} &= \boldsymbol{\Omega}\times(\mathcal{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e}) \text{ by (3.14c)}\\ &= \mathcal{Y}(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}}) + \rho\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{u} + \boldsymbol{\Lambda}\times\mathbf{r}) - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})\\ &\quad + \boldsymbol{\Omega}\times[s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.4)}\\ &= \mathcal{Y}(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}}) + \rho\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{u}) + \rho\boldsymbol{\Omega}\times(\boldsymbol{\Lambda}\times\mathbf{r}) - \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})\\ &\quad + \varphi_g\varphi_h\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \varphi_i(\boldsymbol{\Omega}\times\boldsymbol{\Omega}) - \varphi_j(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) \text{ by (5.5)}\end{aligned}$$

$$\begin{aligned}&= \mathcal{Y}(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}}) + \rho[(\boldsymbol{\Omega}\cdot\mathbf{u})\boldsymbol{\Omega} - \Omega^2\mathbf{u}] + \rho[(\boldsymbol{\Omega}\cdot\mathbf{r})\boldsymbol{\Lambda} - (\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})\mathbf{r}] - [(\boldsymbol{\Omega}\cdot\mathbf{r})\boldsymbol{\Omega} - \Omega^2\mathbf{r}]\\ &\quad + \varphi_g\varphi_h[\Omega^2\widehat{\mathbf{p}} - (\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})\boldsymbol{\Omega}] - \varphi_j(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) \text{ by (A.1)}\\ &= \mathcal{Y}(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}}) - \rho\Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) + \rho(\varepsilon_b\boldsymbol{\Lambda} - \varepsilon_g\mathbf{r}) - \varepsilon_b\boldsymbol{\Omega} + \Omega^2\mathbf{r}\\ &\quad + \varphi_g\varphi_h(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\boldsymbol{\Omega}) - \varphi_j(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) \text{ by (1.4c) \& (5.1a)}\\ &= \mathcal{Y}(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}}) - \rho\Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) + \varphi_j(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \rho\varepsilon_b\boldsymbol{\Lambda} + \varphi_g\varphi_h\Omega^2\widehat{\mathbf{p}}\\ &\quad + (\Omega^2 - \rho\varepsilon_g)\mathbf{r} - (\varepsilon_b + \varphi_g\varphi_h\varepsilon_c)\boldsymbol{\Omega}\end{aligned} \tag{5.14a}$$

$$\begin{aligned}\mathbf{u}\times\boldsymbol{\upsilon} &= \mathbf{u}\times(\mathcal{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e}) \text{ by (3.14c)}\\ &= \mathcal{Y}(\mathbf{u}\times\widehat{\boldsymbol{\kappa}}) + \rho\mathbf{u}\times(\boldsymbol{\Omega}\times\mathbf{u} + \boldsymbol{\Lambda}\times\mathbf{r})\\ &\quad + \mathbf{u}\times[\varphi_g\varphi_h(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \varphi_i\boldsymbol{\Omega} - \varphi_j\widehat{\mathbf{p}}] \text{ by (1.4) \& (5.5)}\\ &= -\mathcal{Y}[\widehat{\boldsymbol{\kappa}}\times(\boldsymbol{\Omega}\times\mathbf{r})] + \rho[\mathbf{u}\times(\boldsymbol{\Omega}\times\mathbf{u})] + \rho[\mathbf{u}\times(\boldsymbol{\Lambda}\times\mathbf{r})]\\ &\quad + \varphi_g\varphi_h[\mathbf{u}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] - \varphi_i[\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})] + \varphi_j[\widehat{\mathbf{p}}\times(\boldsymbol{\Omega}\times\mathbf{r})] \text{ by (1.4c)}\\ &= -\mathcal{Y}[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})\boldsymbol{\Omega} - (\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Omega})\mathbf{r}] + \rho[u^2\boldsymbol{\Omega} - (\boldsymbol{\Omega}\cdot\mathbf{u})\mathbf{u}] + \rho[(\mathbf{r}\cdot\mathbf{u})\boldsymbol{\Lambda} - (\boldsymbol{\Lambda}\cdot\mathbf{u})\mathbf{r}]\\ &\quad + \varphi_g\varphi_h[(\boldsymbol{\Omega}\cdot\mathbf{u})\widehat{\mathbf{p}} - (\widehat{\mathbf{p}}\cdot\mathbf{u})\boldsymbol{\Omega}] - \varphi_i[(\mathbf{r}\cdot\boldsymbol{\Omega})\boldsymbol{\Omega} - \Omega^2\mathbf{r}] + \varphi_j[(\widehat{\mathbf{p}}\cdot\mathbf{r})\boldsymbol{\Omega} - (\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})\mathbf{r}] \text{ by (A.1)}\\ &= -\mathcal{Y}[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})\boldsymbol{\Omega} - (\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Omega})\mathbf{r}] + \rho u^2\boldsymbol{\Omega} + \rho[\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})]\mathbf{r} - \varphi_g\varphi_h[\widehat{\mathbf{p}}\cdot(\boldsymbol{\Omega}\times\mathbf{r})]\boldsymbol{\Omega}\\ &\quad - \varphi_i[(\mathbf{r}\cdot\boldsymbol{\Omega})\boldsymbol{\Omega} - \Omega^2\mathbf{r}] + \varphi_j[(\widehat{\mathbf{p}}\cdot\mathbf{r})\boldsymbol{\Omega} - (\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})\mathbf{r}] \text{ by (1.4c) \& (A.4)}\\ &= (-\mathcal{Y}\varepsilon_d + \rho u^2 - \varphi_g\varphi_h\varepsilon_l - \varphi_i\varepsilon_b + \varphi_j\varepsilon_f)\boldsymbol{\Omega} + (\mathcal{Y}\varepsilon_a + \rho\varepsilon_m + \varphi_i\Omega^2 - \varphi_j\varepsilon_c)\mathbf{r} \text{ by (5.1a)}\\ &= \delta_c\boldsymbol{\Omega} + \delta_d\mathbf{r} \text{ by (5.13a) \& (5.2a)}\end{aligned} \tag{5.14b}$$

$$\begin{aligned}\boldsymbol{\upsilon}\cdot(\boldsymbol{\kappa}\times\boldsymbol{\Omega}) &= \boldsymbol{\kappa}\cdot(\boldsymbol{\Omega}\times\boldsymbol{\upsilon}) \text{ by (A.4)}\\ &= \boldsymbol{\kappa}\cdot[\mathcal{Y}(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}}) - \rho\Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) + \varphi_j(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \rho\varepsilon_b\boldsymbol{\Lambda} + \varphi_g\varphi_h\Omega^2\widehat{\mathbf{p}}\\ &\quad + (\Omega^2 - \rho\varepsilon_g)\mathbf{r} - (\varepsilon_b + \varphi_g\varphi_h\varepsilon_c)\boldsymbol{\Omega}] \text{ by (5.14a)}\\ &= \kappa[-\rho\Omega^2\varepsilon_k + \varphi_j\varepsilon_j + \rho\varepsilon_b\delta_a + \varphi_g\varphi_h\Omega^2\varepsilon_e + \varepsilon_d(\Omega^2 - \rho\varepsilon_g) - \varepsilon_a(\varepsilon_b + \varphi_g\varphi_h\varepsilon_c)]\\ &\quad \text{by (5.1a) \& (5.13a)}\\ &= \kappa[\varphi_j\varepsilon_j + \Omega^2(\varepsilon_d - \rho\varepsilon_k + \varphi_g\varphi_h\varepsilon_e) + \rho(\varepsilon_b\delta_a - \varepsilon_d\varepsilon_g) - \varepsilon_a(\varepsilon_b + \varphi_g\varphi_h\varepsilon_c)]\\ &= \kappa\delta_f \text{ by (5.13a)}\end{aligned} \tag{5.14c}$$

$$\begin{aligned}\mathbf{g}_1 &= (\boldsymbol{\kappa}\cdot\nabla_u)\,\mathbf{a} \text{ by (3.8b)}\\ &= (\boldsymbol{\kappa}\cdot\nabla_u)\,(\boldsymbol{\Omega}\times\mathbf{u} + \boldsymbol{\Lambda}\times\mathbf{r}) \text{ by (1.4a)}\\ &= (\boldsymbol{\kappa}\cdot\nabla_u)\,(\boldsymbol{\Omega}\times\mathbf{u})\\ &= \boldsymbol{\Omega}\times[(\boldsymbol{\kappa}\cdot\nabla_u)\,\mathbf{u}] - \mathbf{u}\times[(\boldsymbol{\kappa}\cdot\nabla_u)\,\boldsymbol{\Omega}] \text{ by (A.23)}\\ &= \boldsymbol{\Omega}\times\boldsymbol{\kappa} \text{ by (A.21)}\end{aligned} \tag{5.15a}$$

$$\begin{aligned}
\mathbf{g}_2 &= \nabla_u \times \mathbf{a} \text{ by (3.8b)} \\
&= \nabla_u \times (\boldsymbol{\Omega} \times \mathbf{u} + \boldsymbol{\Lambda} \times \mathbf{r}) \text{ by (1.4a)} \\
&= \nabla_u \times (\boldsymbol{\Omega} \times \mathbf{u}) \\
&= \boldsymbol{\Omega}(\nabla_u \cdot \mathbf{u}) - (\boldsymbol{\Omega} \cdot \nabla_u)\,\mathbf{u} + (\mathbf{u} \cdot \nabla_u)\,\boldsymbol{\Omega} - \mathbf{u}(\nabla_u \cdot \boldsymbol{\Omega}) \text{ by (A.13)} \\
&= \boldsymbol{\Omega}(\nabla_u \cdot \mathbf{u}) - (\boldsymbol{\Omega} \cdot \nabla_u)\,\mathbf{u} \\
&= 2\boldsymbol{\Omega} \text{ by (A.6) \& (A.21)}
\end{aligned} \tag{5.15b}$$

$$\begin{aligned}
\mathbf{g}_3 &= (\mathbf{u} \cdot \nabla_u)\,\mathbf{a} \text{ by (3.8b)} \\
&= (\mathbf{u} \cdot \nabla_u)\,(\boldsymbol{\Omega} \times \mathbf{u} + \boldsymbol{\Lambda} \times \mathbf{r}) \text{ by (1.4a)} \\
&= (\mathbf{u} \cdot \nabla_u)\,(\boldsymbol{\Omega} \times \mathbf{u}) \\
&= \boldsymbol{\Omega} \times [(\mathbf{u} \cdot \nabla_u)\,\mathbf{u}] - \mathbf{u} \times [(\mathbf{u} \cdot \nabla_u)\,\boldsymbol{\Omega}] \text{ by (A.23)} \\
&= \boldsymbol{\Omega} \times \mathbf{u} \text{ by (A.21)}
\end{aligned} \tag{5.15c}$$

$$\sim \mathbf{g}_4 = (\boldsymbol{\upsilon} \cdot \nabla_u)\,\mathbf{a} = \boldsymbol{\Omega} \times \boldsymbol{\upsilon} \tag{5.15d}$$

$$\begin{aligned}
\mathbf{g}_5 &= \nabla_u \gamma \text{ by (3.8b)} \\
&= \nabla_u \left| 1 + \frac{s_0}{\eta \omega_o^2} \right|^{1/2} \text{ by (1.4a)} \\
&= 0 \text{ by (1.4c) \& (1.4d)}
\end{aligned} \tag{5.15e}$$

$$\begin{aligned}
\mathbf{g}_6 &= \mathbf{g}_1 + \boldsymbol{\kappa} \times \mathbf{g}_2 \text{ by (3.8b)} \\
&= \boldsymbol{\Omega} \times \boldsymbol{\kappa} + 2\boldsymbol{\kappa} \times \boldsymbol{\Omega} \text{ by (5.15a) \& (5.15b)} \\
&= \boldsymbol{\kappa} \times \boldsymbol{\Omega}
\end{aligned} \tag{5.15f}$$

$$\begin{aligned}
\mathbf{g}_7 &= \nu_2 \mathbf{g}_6 - \nu_3 \mathbf{g}_5 \text{ by (3.8b)} \\
&= \nu_2(\boldsymbol{\kappa} \times \boldsymbol{\Omega}) \text{ by (5.15f) \& (5.15e)}
\end{aligned} \tag{5.15g}$$

$$\begin{aligned}
\mathbf{g}_8 &= \nu_1 \mathbf{g}_6 - \nu_6 \mathbf{g}_5 \text{ by (3.8b)} \\
&= \nu_1(\boldsymbol{\kappa} \times \boldsymbol{\Omega}) \text{ by (5.15f) \& (5.15e)}
\end{aligned} \tag{5.15h}$$

$$\begin{aligned}
\mathbf{g}_9 &= \nu_7 \mathbf{g}_6 - \nu_8 \mathbf{g}_5 \text{ by (3.8b)} \\
&= \nu_7(\boldsymbol{\kappa} \times \boldsymbol{\Omega}) \text{ by (5.15f) \& (5.15e)}
\end{aligned} \tag{5.15i}$$

$$\begin{aligned}
\nu_7 &= \nu_5 \left[\frac{\nu_2}{\nu_4^2} - \frac{\vartheta \nu_1}{d} \right] \text{ by (3.8a)} \\
&= \frac{(1+\vartheta^2)^{-1/2}}{4d\omega_o} \left[\frac{1}{(1+\vartheta^2)\gamma^2\omega_o^2} - \frac{\vartheta\rho}{d\omega_o} \right] \text{ by (3.8a)} \\
&= \frac{(1+\vartheta^2)^{-3/2}}{4d\omega_o} \left[\frac{1}{\gamma^2\omega_o^2} - \frac{\rho\vartheta(1+\vartheta^2)}{d\omega_o} \right] = \frac{1}{4d^2\omega_o^2\delta_g^3} \left[\frac{d}{\gamma^2\omega_o} - \rho\vartheta\delta_g^2 \right] \text{ by (5.13a)} \\
&= \delta_h \text{ by (5.13a).}
\end{aligned} \tag{5.16}$$

Art 21b. *Development of equation* (3.8c).

The foregoing expressions lead to

$$\nabla_u s_0 = 0 \text{ by (1.4d) \& (1.4c)} \tag{5.17a}$$

$$
\begin{aligned}
\nabla_u s_1 &= \nabla_u \left[\frac{2\rho\alpha - d\omega_o}{2\eta(s_0 + \eta\omega_o^2)} \right] \text{ by (1.4d)} \\
&= \frac{2\eta(s_0 + \eta\omega_o^2)\nabla_u(2\rho\alpha - d\omega_o) - (2\rho\alpha - d\omega_o)\nabla_u[2\eta(s_0 + \eta\omega_o^2)]}{[2\eta(s_0 + \eta\omega_o^2)]^2} \text{ by (A.15) \& (A.16)} \\
&= [\nabla_u(2\rho\alpha - d\omega_o)]/[2\eta(s_0 + \eta\omega_o^2)] \text{ by (1.4d) \& (1.4c)} \\
&= [2\rho\nabla_u\alpha + 2\alpha\nabla_u\rho - \omega_o\nabla_u d]/[2\eta(s_0 + \eta\omega_o^2)] \text{ by (A.16)} \\
&= [2\rho\mathbf{g}_6 + 2\alpha\mathbf{g}_9 - \omega_o\mathbf{g}_8]/[2\eta(s_0 + \eta\omega_o^2)] \text{ by (3.9)} \\
&= [(2\rho + 2\alpha\nu_7 - \omega_o\nu_1)(\boldsymbol{\kappa} \times \boldsymbol{\Omega})]/[2\eta(s_0 + \eta\omega_o^2)] \text{ by (5.15)} \\
&= [(\rho + 2\kappa\varphi_d\delta_h)(\boldsymbol{\kappa} \times \boldsymbol{\Omega})]/[2\kappa\varepsilon_j(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)] \text{ by (3.8a), (5.16), (5.4) \& (5.5a)} \\
&= [(\rho + 2\kappa\varphi_d\delta_h)(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})]/[2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)] \\
&= \delta_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) \text{ by (5.13a)}
\end{aligned} \tag{5.17b}
$$

$$
\begin{aligned}
\nabla_u s_2 &= \nabla_u(s_0 s_1) \text{ by (1.4d)} \\
&= s_0 \nabla_u s_1 \text{ by (5.17a)} \\
&= \kappa\varphi_g\delta_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) \text{ by (5.5a) \& (5.17b)}
\end{aligned} \tag{5.17c}
$$

$$
\begin{aligned}
\nabla_u s_3 &= \nabla_u[\eta s_1(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}}) + 4\xi s_1(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})] \text{ by (1.4d)} \\
&= [\eta(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}}) + 4\xi(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})]\nabla_u s_1 \text{ by (1.4c)} \\
&= \kappa\delta_i(\varepsilon_j\varepsilon_i + 8\varphi_e\varepsilon_c)(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) \text{ by (5.1a), (5.4) \& (5.17b)} \\
&= \kappa\delta_j\delta_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) \text{ by (5.13a)}
\end{aligned} \tag{5.17d}
$$

$$
\begin{aligned}
\nabla_u s_4 &= \nabla_u[2\xi s_1\Omega^2 + \eta s_1(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})] \text{ by (1.4d)} \\
&= [2\xi\Omega^2 + \eta(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})]\nabla_u s_1 \text{ by (1.4c)} \\
&= \kappa\delta_i(4\varphi_e\Omega^2 + \varepsilon_j\varepsilon_g)(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) \text{ by (5.1a), (5.4) \& (5.17b)} \\
&= \kappa\delta_k\delta_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) \text{ by (5.13a)}
\end{aligned} \tag{5.17e}
$$

in consequence of which we have

$$
\begin{aligned}
\boldsymbol{\jmath}_1 &= \nabla_u \times \mathbf{e} \text{ by (3.8c)} \\
&= \nabla_u \times [s_2(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.4b)} \\
&= s_2(\nabla_u \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})) - (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\nabla_u s_2) + s_3(\nabla_u \times \boldsymbol{\Omega}) - \boldsymbol{\Omega} \times (\nabla_u s_3) \\
&\quad - s_4(\nabla_u \times \widehat{\mathbf{p}}) + \widehat{\mathbf{p}} \times (\nabla_u s_4) \text{ by (A.12)} \\
&= -(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\nabla_u s_2) - \boldsymbol{\Omega} \times (\nabla_u s_3) + \widehat{\mathbf{p}} \times (\nabla_u s_4) \\
&= -\kappa\varphi_g\delta_i(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \kappa\delta_j\delta_i\boldsymbol{\Omega} \times (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \kappa\delta_k\delta_i\widehat{\mathbf{p}} \times (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) \text{ by (5.17)} \\
&= -\kappa\varphi_g\delta_i[\widehat{\boldsymbol{\kappa}}\{\boldsymbol{\Omega} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})\} - \boldsymbol{\Omega}\{\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})\}] - \kappa\delta_j\delta_i[\widehat{\boldsymbol{\kappa}}\Omega^2 - \boldsymbol{\Omega}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})] \\
&\quad + \kappa\delta_k\delta_i[\widehat{\boldsymbol{\kappa}}(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}}) - \boldsymbol{\Omega}(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] \text{ by (A.1)} \\
&= \kappa\varphi_g\delta_i\varepsilon_j\boldsymbol{\Omega} - \kappa\delta_j\delta_i(\Omega^2\widehat{\boldsymbol{\kappa}} - \varepsilon_a\boldsymbol{\Omega}) + \kappa\delta_k\delta_i(\varepsilon_c\widehat{\boldsymbol{\kappa}} - \varepsilon_e\boldsymbol{\Omega}) \text{ by (5.1a)} \\
&= \kappa\delta_i(\varphi_g\varepsilon_j + \delta_j\varepsilon_a - \delta_k\varepsilon_e)\boldsymbol{\Omega} + \kappa\delta_i(\delta_k\varepsilon_c - \delta_j\Omega^2)\widehat{\boldsymbol{\kappa}} \\
&= \delta_l\boldsymbol{\Omega} + \delta_m\widehat{\boldsymbol{\kappa}} \text{ by (5.13b)}
\end{aligned} \tag{5.18a}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_2 &= (\mathbf{u} \cdot \nabla_u)\,\mathbf{e} \text{ by (3.8c)} \\
&= (\mathbf{u} \cdot \nabla_u)\,[s_2(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.4b)} \\
&= (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})(\mathbf{u} \cdot \nabla_u s_2) + s_2(\mathbf{u} \cdot \nabla_u)\,(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \boldsymbol{\Omega}(\mathbf{u} \cdot \nabla_u s_3) + s_3(\mathbf{u} \cdot \nabla_u)\,\boldsymbol{\Omega} - \widehat{\mathbf{p}}(\mathbf{u} \cdot \nabla_u s_4) \\
&\quad - s_4(\mathbf{u} \cdot \nabla_u)\,\widehat{\mathbf{p}} \text{ by (A.22)} \\
&= (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})(\mathbf{u} \cdot \nabla_u s_2) + \boldsymbol{\Omega}(\mathbf{u} \cdot \nabla_u s_3) - \widehat{\mathbf{p}}(\mathbf{u} \cdot \nabla_u s_4)
\end{aligned}
$$

$$
\begin{aligned}
&= \kappa\varphi_g\delta_i(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})[\mathbf{u}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})] + \kappa\delta_j\delta_i\boldsymbol{\Omega}[\mathbf{u}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})] - \kappa\delta_k\delta_i\widehat{\mathbf{p}}[\mathbf{u}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})] \text{ by (5.17)}\\
&= \kappa\delta_i[\varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \delta_j\boldsymbol{\Omega} - \delta_k\widehat{\mathbf{p}}][(\boldsymbol{\Omega}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})] \text{ by (1.4c)}\\
&= \kappa\delta_i[\varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \delta_j\boldsymbol{\Omega} - \delta_k\widehat{\mathbf{p}}][(\boldsymbol{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\boldsymbol{\Omega}) - \Omega^2(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})] \text{ by (A.2)}\\
&= \kappa\delta_i(\varepsilon_a\varepsilon_b - \Omega^2\varepsilon_d)[\varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \delta_j\boldsymbol{\Omega} - \delta_k\widehat{\mathbf{p}}] \text{ by (5.1a)}\\
&= \delta_n[\delta_j\boldsymbol{\Omega} - \delta_k\widehat{\mathbf{p}} + \varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] \text{ by (5.13b)}
\end{aligned}
\tag{5.18b}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_3 &= (\boldsymbol{\upsilon}\cdot\nabla_u)\,\mathbf{e} \text{ by (3.8c)}\\
&= (\boldsymbol{\upsilon}\cdot\nabla_u)\,[s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.4b)}\\
&= (\widehat{\mathbf{p}}\times\boldsymbol{\Omega})(\boldsymbol{\upsilon}\cdot\nabla_u s_2) + s_2(\boldsymbol{\upsilon}\cdot\nabla_u)\,(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \boldsymbol{\Omega}(\boldsymbol{\upsilon}\cdot\nabla_u s_3) + s_3(\boldsymbol{\upsilon}\cdot\nabla_u)\,\boldsymbol{\Omega}\\
&\qquad - \widehat{\mathbf{p}}(\boldsymbol{\upsilon}\cdot\nabla_u s_4) - s_4(\boldsymbol{\upsilon}\cdot\nabla_u)\,\widehat{\mathbf{p}} \text{ by (A.22)}\\
&= (\widehat{\mathbf{p}}\times\boldsymbol{\Omega})(\boldsymbol{\upsilon}\cdot\nabla_u s_2) + \boldsymbol{\Omega}(\boldsymbol{\upsilon}\cdot\nabla_u s_3) - \widehat{\mathbf{p}}(\boldsymbol{\upsilon}\cdot\nabla_u s_4)\\
&= \kappa\delta_i[\varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \delta_j\boldsymbol{\Omega} - \delta_k\widehat{\mathbf{p}}][\boldsymbol{\upsilon}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})] \text{ by (5.17)}\\
&= \kappa\delta_i\delta_f[\delta_j\boldsymbol{\Omega} - \delta_k\widehat{\mathbf{p}} + \varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] \text{ by (5.14c)}
\end{aligned}
\tag{5.18c}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_4 &= \boldsymbol{\jmath}_1 + \rho\mathbf{g}_2 \text{ by (3.8c)}\\
&= \delta_l\boldsymbol{\Omega} + \delta_m\widehat{\boldsymbol{\kappa}} + 2\rho\boldsymbol{\Omega} \text{ by (5.18a) \& (5.15b)}\\
&= (2\rho + \delta_l)\boldsymbol{\Omega} + \delta_m\widehat{\boldsymbol{\kappa}}\\
&= \delta_o\boldsymbol{\Omega} + \delta_m\widehat{\boldsymbol{\kappa}} \text{ by (5.13b)}
\end{aligned}
\tag{5.18d}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_5 &= \boldsymbol{\jmath}_2 + \rho\mathbf{g}_3 \text{ by (3.8c)}\\
&= \delta_n[\delta_j\boldsymbol{\Omega} - \delta_k\widehat{\mathbf{p}} + \varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \rho\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}) \text{ by (5.18b), (5.15c) \& (1.4c)}\\
&= \delta_n[\delta_j\boldsymbol{\Omega} - \delta_k\widehat{\mathbf{p}} + \varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \rho[\boldsymbol{\Omega}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}] \text{ by (A.1)}\\
&= \delta_n[\delta_j\boldsymbol{\Omega} - \delta_k\widehat{\mathbf{p}} + \varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \rho(\varepsilon_b\boldsymbol{\Omega} - \Omega^2\mathbf{r}) \text{ by (5.1a)}\\
&= -\rho\Omega^2\mathbf{r} + (\delta_n\delta_j + \rho\varepsilon_b)\boldsymbol{\Omega} - \delta_n\delta_k\widehat{\mathbf{p}} + \delta_n\varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\\
&= -\rho\Omega^2\mathbf{r} + \delta_p\boldsymbol{\Omega} - \delta_n\delta_k\widehat{\mathbf{p}} + \delta_n\varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) \text{ by (5.13b)}
\end{aligned}
\tag{5.18e}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_6 &= \boldsymbol{\jmath}_3 + \rho\mathbf{g}_4 \text{ by (3.8c)}\\
&= \kappa\delta_i\delta_f\delta_j\boldsymbol{\Omega} - \kappa\delta_i\delta_f\delta_k\widehat{\mathbf{p}} + \kappa\delta_i\delta_f\varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \rho(\boldsymbol{\Omega}\times\boldsymbol{\upsilon}) \text{ by (5.18c) \& (5.15d)}\\
&= \kappa\delta_i\delta_f\delta_j\boldsymbol{\Omega} - \kappa\delta_i\delta_f\delta_k\widehat{\mathbf{p}} + \kappa\delta_i\delta_f\varphi_g(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \rho[\mathcal{Y}(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}}) - \rho\Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) + \varphi_j(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\\
&\quad + \rho\varepsilon_b\boldsymbol{\Lambda} + \varphi_g\varphi_h\Omega^2\widehat{\mathbf{p}} + (\Omega^2 - \rho\varepsilon_g)\mathbf{r} - (\varepsilon_b + \varphi_g\varphi_h\varepsilon_c)\boldsymbol{\Omega}] \text{ by (5.14a)}\\
&= \rho^2\varepsilon_b\boldsymbol{\Lambda} + \rho(\Omega^2 - \rho\varepsilon_g)\mathbf{r} + [\kappa\delta_i\delta_f\delta_j - \rho(\varepsilon_b + \varphi_g\varphi_h\varepsilon_c)]\boldsymbol{\Omega} + (\rho\varphi_g\varphi_h\Omega^2 - \kappa\delta_i\delta_f\delta_k)\widehat{\mathbf{p}}\\
&\quad + (\kappa\delta_i\delta_f\varphi_g + \rho\varphi_j)(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \rho\mathcal{Y}(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}}) - \rho^2\Omega^2(\boldsymbol{\Omega}\times\mathbf{r})\\
&= \rho^2\varepsilon_b\boldsymbol{\Lambda} + \delta_q\mathbf{r} + \delta_r\boldsymbol{\Omega} + \delta_s\widehat{\mathbf{p}} + \delta_t(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \rho\mathcal{Y}(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}}) - \rho^2\Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) \text{ by (5.13b)}
\end{aligned}
\tag{5.18f}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_7 &= \nabla_u\tau \text{ by (3.8c)}\\
&= \nabla_u[2\kappa^{-2}(\rho\alpha - \eta s_2)] \text{ by (1.4b)}\\
&= 2\kappa^{-2}[\rho\nabla_u\alpha + \alpha\nabla_u\rho - \eta\nabla_u s_2] \text{ by (A.16) \& (1.4c)}\\
&= 2\kappa^{-2}[\rho\mathbf{g}_6 + \kappa\varphi_d\mathbf{g}_9 - \kappa^2\varepsilon_j\varphi_g\delta_i(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})] \text{ by (3.9), (5.4) \& (5.17c)}\\
&= 2\kappa^{-2}(\rho\kappa + \kappa^2\varphi_d\delta_h - \kappa^2\varepsilon_j\varphi_g\delta_i)(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) \text{ by (5.15f), (5.15i) \& (5.16)}\\
&= \delta_u(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) \text{ by (5.13b)}
\end{aligned}
\tag{5.18g}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_8 &= (\mathbf{u}\times\boldsymbol{v})/|\mathbf{u}\times\boldsymbol{v}| \text{ by (3.8c)}\\
&= (\beta c^2\mathcal{F})^{-1}(\mathbf{u}\times\boldsymbol{v}) \text{ by (3.5c)}\\
&= (\beta c^2\mathcal{F})^{-1}(\delta_c\boldsymbol{\Omega}+\delta_d\mathbf{r}) \text{ by (5.14b)}
\end{aligned}
\tag{5.18h}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_9 &= \boldsymbol{\jmath}_8\times\mathbf{u} \text{ by (3.8c)}\\
&= (\beta c^2\mathcal{F})^{-1}(\delta_c\boldsymbol{\Omega}+\delta_d\mathbf{r})\times(\boldsymbol{\Omega}\times\mathbf{r}) \text{ by (5.18h) \& (1.4c)}\\
&= (\beta c^2\mathcal{F})^{-1}[\delta_c(\boldsymbol{\Omega}\cdot\mathbf{r})\boldsymbol{\Omega}-\delta_c\Omega^2\mathbf{r}+\delta_d r^2\boldsymbol{\Omega}-\delta_d(\boldsymbol{\Omega}\cdot\mathbf{r})\mathbf{r}] \text{ by (A.1)}\\
&= (\beta c^2\mathcal{F})^{-1}[(\delta_c\varepsilon_b+r^2\delta_d)\boldsymbol{\Omega}-(\Omega^2\delta_c+\delta_d\varepsilon_b)\mathbf{r}] \text{ by (5.1a)}\\
&= \hbar_a\boldsymbol{\Omega}-\hbar_b\mathbf{r} \text{ by (5.13c)}
\end{aligned}
\tag{5.18i}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_0 &= \mathcal{G}\boldsymbol{\jmath}_9-\mathcal{F}\mathbf{u} \text{ by (3.8c)}\\
&= \mathcal{G}(\hbar_a\boldsymbol{\Omega}-\hbar_b\mathbf{r})-\mathcal{F}\mathbf{u} \text{ by (5.18i).}
\end{aligned}
\tag{5.18j}
$$

Furthermore, we derive

$$
\begin{aligned}
\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\jmath}_0 &= \widehat{\boldsymbol{\kappa}}\cdot[\mathcal{G}(\hbar_a\boldsymbol{\Omega}-\hbar_b\mathbf{r})-\mathcal{F}\mathbf{u}] \text{ by (5.18j)}\\
&= \mathcal{G}\hbar_a(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Omega})-\mathcal{G}\hbar_b(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})-\mathcal{F}(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})\\
&= \mathcal{G}(\hbar_a\varepsilon_a-\hbar_b\varepsilon_d)-\mathcal{F}\varepsilon_k \text{ by (5.1a) \& (1.4c)}\\
&= \hbar_c \text{ by (5.13c)}
\end{aligned}
\tag{5.19a}
$$

$$
\mathbf{c}\cdot\boldsymbol{\jmath}_0 = c(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\jmath}_0) = c\hbar_c, \quad \boldsymbol{\kappa}\cdot\boldsymbol{\jmath}_0 = \kappa(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\jmath}_0) = \kappa\hbar_c \text{ by (5.19a)}
\tag{5.19b}
$$

$$
\begin{aligned}
\mathbf{a}\cdot\boldsymbol{\jmath}_0 &= (\boldsymbol{\Omega}\times\mathbf{u}+\boldsymbol{\Lambda}\times\mathbf{r})\cdot(\mathcal{G}\hbar_a\boldsymbol{\Omega}-\mathcal{G}\hbar_b\mathbf{r}-\mathcal{F}\mathbf{u}) \text{ by (1.4a) \& (5.18j)}\\
&= \mathcal{G}\hbar_a[\boldsymbol{\Omega}\cdot(\boldsymbol{\Omega}\times\mathbf{u})+\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})]-\mathcal{G}\hbar_b[\mathbf{r}\cdot(\boldsymbol{\Omega}\times\mathbf{u})+\mathbf{r}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})]\\
&\qquad -\mathcal{F}[\mathbf{u}\cdot(\boldsymbol{\Omega}\times\mathbf{u})+\mathbf{u}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})]\\
&= \mathcal{G}\hbar_a\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})-\mathcal{G}\hbar_b\mathbf{r}\cdot[\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})]-\mathcal{F}[(\boldsymbol{\Omega}\times\mathbf{r})\cdot(\boldsymbol{\Lambda}\times\mathbf{r})] \text{ by (1.4c)}\\
&= \mathcal{G}\hbar_a\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})-\mathcal{G}\hbar_b\mathbf{r}\cdot[(\boldsymbol{\Omega}\cdot\mathbf{r})\boldsymbol{\Omega}-\Omega^2\mathbf{r}]-\mathcal{F}[(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})r^2-(\boldsymbol{\Omega}\cdot\mathbf{r})(\boldsymbol{\Lambda}\cdot\mathbf{r})] \text{ by (A.1) \& (A.2)}\\
&= \mathcal{G}[\hbar_a\varepsilon_m+\hbar_b(\Omega^2r^2-\varepsilon_b^2)]-\mathcal{F}(r^2\varepsilon_g-\varepsilon_b\varepsilon_h) \text{ by (5.1a)}\\
&= \mathcal{G}(\hbar_a\varepsilon_m+\hbar_b\varphi_a^2)-\mathcal{F}\varphi_c \text{ by (5.1b)}\\
&= \hbar_d \text{ by (5.13c)}
\end{aligned}
\tag{5.19c}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 &= (\mathcal{G}\hbar_a\boldsymbol{\Omega}-\mathcal{G}\hbar_b\mathbf{r}-\mathcal{F}\mathbf{u})\times(\delta_o\boldsymbol{\Omega}+\delta_m\widehat{\boldsymbol{\kappa}}) \text{ by (5.18j) \& (5.18d)}\\
&= \mathcal{G}\hbar_a\delta_o(\boldsymbol{\Omega}\times\boldsymbol{\Omega})+\mathcal{G}\hbar_a\delta_m(\boldsymbol{\Omega}\times\widehat{\boldsymbol{\kappa}})-\mathcal{G}\hbar_b\delta_o(\mathbf{r}\times\boldsymbol{\Omega})-\mathcal{G}\hbar_b\delta_m(\mathbf{r}\times\widehat{\boldsymbol{\kappa}})\\
&\qquad -\mathcal{F}\delta_o(\mathbf{u}\times\boldsymbol{\Omega})-\mathcal{F}\delta_m(\mathbf{u}\times\widehat{\boldsymbol{\kappa}})\\
&= -\mathcal{G}\hbar_a\delta_m(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})-\mathcal{G}\hbar_b\delta_o(\mathbf{r}\times\boldsymbol{\Omega})+\mathcal{G}\hbar_b\delta_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\qquad +\mathcal{F}\delta_o[\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})]+\mathcal{F}\delta_m[\widehat{\boldsymbol{\kappa}}\times(\boldsymbol{\Omega}\times\mathbf{r})] \text{ by (1.4c)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathcal{G}\hbar_a\delta_m(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})-\mathcal{G}\hbar_b\delta_o(\mathbf{r}\times\boldsymbol{\Omega})+\mathcal{G}\hbar_b\delta_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\qquad +\mathcal{F}\delta_o[(\boldsymbol{\Omega}\cdot\mathbf{r})\boldsymbol{\Omega}-\Omega^2\mathbf{r}]+\mathcal{F}\delta_m[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})\boldsymbol{\Omega}-(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Omega})\mathbf{r}] \text{ by (A.1)}\\
&= -\mathcal{G}\hbar_a\delta_m(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})-\mathcal{G}\hbar_b\delta_o(\mathbf{r}\times\boldsymbol{\Omega})+\mathcal{G}\hbar_b\delta_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\qquad +\mathcal{F}(\delta_o\varepsilon_b+\delta_m\varepsilon_d)\boldsymbol{\Omega}-\mathcal{F}(\delta_o\Omega^2+\varepsilon_a\delta_m)\mathbf{r} \text{ by (5.1a)}\\
&= \hbar_e\boldsymbol{\Omega}-\hbar_f\mathbf{r}-\mathcal{G}\hbar_a\delta_m(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})-\mathcal{G}\hbar_b\delta_o(\mathbf{r}\times\boldsymbol{\Omega})+\mathcal{G}\hbar_b\delta_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) \text{ by (5.13c)}
\end{aligned}
\tag{5.20}
$$

$$
\begin{aligned}
&\mathcal{X}_1\mathbf{u} - \mathcal{X}_2\boldsymbol{\upsilon} + \mathcal{X}_3\boldsymbol{\jmath}_6 - \mathcal{X}_4\boldsymbol{\jmath}_5 + (\mathbf{c}\cdot\boldsymbol{\jmath}_0)\mathbf{g}_8 - (\boldsymbol{\kappa}\cdot\boldsymbol{\jmath}_0)\boldsymbol{\jmath}_7 + (\mathbf{a}\cdot\boldsymbol{\jmath}_0)\mathbf{g}_9 + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4\\
&= \mathcal{X}_1\mathbf{u} - \mathcal{X}_2\boldsymbol{\upsilon} + \mathcal{X}_3\boldsymbol{\jmath}_6 - \mathcal{X}_4\boldsymbol{\jmath}_5 + c\hbar_c\mathbf{g}_8 - \kappa\hbar_c\boldsymbol{\jmath}_7 + \hbar_d\mathbf{g}_9 + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \text{ by (5.19)}\\
&= \mathcal{X}_1(\mathbf{\Omega}\times\mathbf{r}) - \mathcal{X}_2(\mathcal{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e}) + \mathcal{X}_3[\rho^2\varepsilon_b\mathbf{\Lambda} + \delta_q\mathbf{r} + \delta_r\mathbf{\Omega} + \delta_s\widehat{\mathbf{p}} + \delta_t(\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&\quad + \rho\mathcal{Y}(\mathbf{\Omega}\times\widehat{\boldsymbol{\kappa}}) - \rho^2\Omega^2(\mathbf{\Omega}\times\mathbf{r})] - \mathcal{X}_4[-\rho\Omega^2\mathbf{r} + \delta_p\mathbf{\Omega} - \delta_n\delta_k\widehat{\mathbf{p}} + \delta_n\varphi_g(\widehat{\mathbf{p}}\times\mathbf{\Omega})]\\
&\quad + c\hbar_c[\nu_1(\boldsymbol{\kappa}\times\mathbf{\Omega})] - \kappa\hbar_c[\delta_u(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})] + \hbar_d[\nu_7(\boldsymbol{\kappa}\times\mathbf{\Omega})] + \hbar_e\mathbf{\Omega} - \hbar_f\mathbf{r} - \mathcal{G}\hbar_a\delta_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})\\
&\quad - \mathcal{G}\hbar_b\delta_o(\mathbf{r}\times\mathbf{\Omega}) + \mathcal{G}\hbar_b\delta_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) \text{ by (1.4c), (3.14c), (5.18), (5.15) \& (5.20)}\\
&= -\mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + \rho^2\varepsilon_b\mathcal{X}_3\mathbf{\Lambda} + (\delta_s\mathcal{X}_3 + \delta_n\delta_k\mathcal{X}_4)\widehat{\mathbf{p}} + (\delta_q\mathcal{X}_3 + \rho\Omega^2\mathcal{X}_4 - \hbar_f)\mathbf{r} + (\delta_r\mathcal{X}_3 - \delta_p\mathcal{X}_4 + \hbar_e)\mathbf{\Omega}\\
&\quad + (c\kappa\hbar_c\nu_1 - \kappa\hbar_c\delta_u + \hbar_d\nu_7\kappa - \mathcal{G}\hbar_a\delta_m - \rho\mathcal{Y}\mathcal{X}_3)(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \mathcal{G}\hbar_b\delta_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\quad + (\delta_t\mathcal{X}_3 - \delta_n\varphi_g\mathcal{X}_4)(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + (\mathcal{X}_1 - \rho^2\Omega^2\mathcal{X}_3 + \mathcal{G}\hbar_b\delta_o)(\mathbf{\Omega}\times\mathbf{r})\\
&\quad - \mathcal{X}_2[\rho(\mathbf{\Omega}\times\mathbf{u} + \mathbf{\Lambda}\times\mathbf{r}) - (\mathbf{\Omega}\times\mathbf{r}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_i\mathbf{\Omega} - \varphi_j\widehat{\mathbf{p}}] \text{ by (1.4) \& (5.5)}\\
&\\
&= -\mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + \rho^2\varepsilon_b\mathcal{X}_3\mathbf{\Lambda} + (\delta_s\mathcal{X}_3 + \delta_n\delta_k\mathcal{X}_4 + \varphi_j\mathcal{X}_2)\widehat{\mathbf{p}} + (\delta_q\mathcal{X}_3 + \rho\Omega^2\mathcal{X}_4 - \hbar_f)\mathbf{r}\\
&\quad + (\delta_r\mathcal{X}_3 - \delta_p\mathcal{X}_4 + \hbar_e - \varphi_i\mathcal{X}_2)\mathbf{\Omega} + (c\kappa\hbar_c\nu_1 - \kappa\hbar_c\delta_u + \hbar_d\nu_7\kappa - \mathcal{G}\hbar_a\delta_m - \rho\mathcal{Y}\mathcal{X}_3)(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})\\
&\quad + \mathcal{G}\hbar_b\delta_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \rho\mathcal{X}_2(\mathbf{\Lambda}\times\mathbf{r}) + (\delta_t\mathcal{X}_3 - \delta_n\varphi_g\mathcal{X}_4 - \varphi_g\varphi_h\mathcal{X}_2)(\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&\quad + (\mathcal{X}_1 - \rho^2\Omega^2\mathcal{X}_3 + \mathcal{G}\hbar_b\delta_o + \mathcal{X}_2)(\mathbf{\Omega}\times\mathbf{r}) - \rho\mathcal{X}_2[(\mathbf{\Omega}\cdot\mathbf{r})\mathbf{\Omega} - \Omega^2\mathbf{r}] \text{ by (1.4c) \& (A.1)}\\
&= -\mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + \rho^2\varepsilon_b\mathcal{X}_3\mathbf{\Lambda} + (\varphi_j\mathcal{X}_2 + \delta_s\mathcal{X}_3 + \delta_n\delta_k\mathcal{X}_4)\widehat{\mathbf{p}} + [\rho\Omega^2(\mathcal{X}_2 + \mathcal{X}_4) + \delta_q\mathcal{X}_3 - \hbar_f]\mathbf{r}\\
&\quad + [\hbar_e - (\varphi_i + \rho\varepsilon_b)\mathcal{X}_2 + \delta_r\mathcal{X}_3 - \delta_p\mathcal{X}_4]\mathbf{\Omega} + [(\rho - \kappa\delta_u)\hbar_c + \kappa\hbar_d\delta_h - \mathcal{G}\hbar_a\delta_m - \rho\mathcal{Y}\mathcal{X}_3](\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})\\
&\quad + \mathcal{G}\hbar_b\delta_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \rho\mathcal{X}_2(\mathbf{\Lambda}\times\mathbf{r}) + [\delta_t\mathcal{X}_3 - (\varphi_h\mathcal{X}_2 + \delta_n\mathcal{X}_4)\varphi_g](\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&\quad + (\mathcal{G}\hbar_b\delta_o + \mathcal{X}_1 + \mathcal{X}_2 - \rho^2\Omega^2\mathcal{X}_3)(\mathbf{\Omega}\times\mathbf{r}) \text{ by (5.1a), (5.16) \& (3.8a).}
\end{aligned}
\tag{5.21}
$$

Art 21c. *Results of the computations.*

It follows from (5.21) and (3.13a) that

$$
\begin{aligned}
\mathfrak{X} = \frac{1}{\beta c^2\mathcal{R}^2}\Big[&\imath_0\widehat{\boldsymbol{\kappa}} + \imath_1\mathbf{\Lambda} + \imath_2\widehat{\mathbf{p}} + \imath_3\mathbf{r} + \imath_4\mathbf{\Omega} + \imath_5(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \imath_6(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&- \imath_7(\mathbf{\Lambda}\times\mathbf{r}) + \imath_8(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \imath_9(\mathbf{\Omega}\times\mathbf{r})\Big]
\end{aligned}
\tag{5.22a}
$$

where, with $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3, \mathcal{X}_4$ defined by (3.13b) wherein $\mathcal{R}, \mathcal{F}, \mathcal{G}, \beta$ are given by (5.12b) and (5.12d),

$$
\begin{aligned}
&\imath_0 = -\mathcal{Y}\mathcal{X}_2, \quad \imath_1 = \rho^2\varepsilon_b\mathcal{X}_3, \quad \imath_2 = \varphi_j\mathcal{X}_2 + \delta_s\mathcal{X}_3 + \delta_n\delta_k\mathcal{X}_4\\
&\imath_3 = \rho\Omega^2(\mathcal{X}_2 + \mathcal{X}_4) + \delta_q\mathcal{X}_3 - \hbar_f, \quad \imath_4 = \hbar_e - (\varphi_i + \rho\varepsilon_b)\mathcal{X}_2 + \delta_r\mathcal{X}_3 - \delta_p\mathcal{X}_4\\
&\imath_5 = (\rho - \kappa\delta_u)\hbar_c + \kappa\hbar_d\delta_h - \mathcal{G}\hbar_a\delta_m - \rho\mathcal{Y}\mathcal{X}_3, \quad \imath_6 = \mathcal{G}\hbar_b\delta_m, \quad \imath_7 = \rho\mathcal{X}_2\\
&\imath_8 = \delta_t\mathcal{X}_3 - (\varphi_h\mathcal{X}_2 + \delta_n\mathcal{X}_4)\varphi_g, \quad \imath_9 = \mathcal{G}\hbar_b\delta_o + \mathcal{X}_1 + \mathcal{X}_2 - \rho^2\Omega^2\mathcal{X}_3.
\end{aligned}
\tag{5.22b}
$$

Also, from (3.7c), we obtain

$$
\begin{aligned}
\Theta = (\mathfrak{X}\cdot\mathbf{a})/a = \frac{1}{\beta c^2\mathcal{R}^2\varphi_b}\Big[\varepsilon_b\mathbf{\Omega} - \Omega^2\mathbf{r} + \mathbf{\Lambda}\times\mathbf{r}\Big]\cdot\Big[&\imath_0\widehat{\boldsymbol{\kappa}} + \imath_1\mathbf{\Lambda} + \imath_2\widehat{\mathbf{p}} + \imath_3\mathbf{r} + \imath_4\mathbf{\Omega} + \imath_5(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})\\
+ \imath_6(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \imath_7(\mathbf{\Lambda}\times\mathbf{r}) + \imath_8(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \imath_9(\mathbf{\Omega}\times\mathbf{r})\Big] &\text{ by (1.4a), (5.2b), (5.1a) \& (5.22a)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\beta c^2 \mathcal{R}^2 \varphi_b}\Big\{\imath_0\varepsilon_b(\mathbf{\Omega}\cdot\widehat{\boldsymbol\kappa}) + \imath_1\varepsilon_b(\mathbf{\Omega}\cdot\mathbf{\Lambda}) + \imath_2\varepsilon_b(\mathbf{\Omega}\cdot\widehat{\mathbf{p}}) + \imath_3\varepsilon_b(\mathbf{\Omega}\cdot\mathbf{r}) + \imath_4\varepsilon_b\Omega^2 \\
&\quad + \imath_6\varepsilon_b[\mathbf{\Omega}\cdot(\widehat{\boldsymbol\kappa}\times\mathbf{r})] - \imath_7\varepsilon_b[\mathbf{\Omega}\cdot(\mathbf{\Lambda}\times\mathbf{r})] - \imath_0\Omega^2(\mathbf{r}\cdot\widehat{\boldsymbol\kappa}) - \imath_1\Omega^2(\mathbf{r}\cdot\mathbf{\Lambda}) - \imath_2\Omega^2(\mathbf{r}\cdot\widehat{\mathbf{p}}) - \imath_3\Omega^2 r^2 \\
&\quad - \imath_4\Omega^2(\mathbf{r}\cdot\mathbf{\Omega}) - \imath_5\Omega^2[\mathbf{r}\cdot(\widehat{\boldsymbol\kappa}\times\mathbf{\Omega})] - \imath_8\Omega^2[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + \imath_0[\widehat{\boldsymbol\kappa}\cdot(\mathbf{\Lambda}\times\mathbf{r})] + \imath_2[\widehat{\mathbf{p}}\cdot(\mathbf{\Lambda}\times\mathbf{r})] \\
&\quad + \imath_4[\mathbf{\Omega}\cdot(\mathbf{\Lambda}\times\mathbf{r})] + \imath_5[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\widehat{\boldsymbol\kappa}\times\mathbf{\Omega})] + \imath_6[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\widehat{\boldsymbol\kappa}\times\mathbf{r})] - \imath_7[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\mathbf{\Lambda}\times\mathbf{r})] \\
&\quad + \imath_8[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + \imath_9[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{r})]\Big\} \\
&= \frac{1}{\beta c^2 \mathcal{R}^2 \varphi_b}\Big\{\imath_0\varepsilon_b\varepsilon_a + \imath_1\varepsilon_b\varepsilon_g + \imath_2\varepsilon_b\varepsilon_c + \imath_3\varepsilon_b^2 + \imath_4\varepsilon_b\Omega^2 - \imath_6\varepsilon_b\varepsilon_k - \imath_7\varepsilon_b\varepsilon_m - \imath_0\Omega^2\varepsilon_d - \imath_1\Omega^2\varepsilon_h \\
&\quad - \imath_2\Omega^2\varepsilon_f - \imath_3\Omega^2 r^2 - \imath_4\Omega^2\varepsilon_b - \imath_5\Omega^2\varepsilon_k - \imath_8\Omega^2\varepsilon_l + \imath_0\varepsilon_n + \imath_2\varepsilon_o + \imath_4\varepsilon_m \\
&\quad + \imath_5[(\mathbf{\Lambda}\cdot\widehat{\boldsymbol\kappa})(\mathbf{r}\cdot\mathbf{\Omega}) - (\mathbf{\Lambda}\cdot\mathbf{\Omega})(\widehat{\boldsymbol\kappa}\cdot\mathbf{r})] + \imath_6[r^2(\mathbf{\Lambda}\cdot\widehat{\boldsymbol\kappa}) - (\mathbf{\Lambda}\cdot\mathbf{r})(\mathbf{r}\cdot\widehat{\boldsymbol\kappa})] - \imath_7[\Lambda^2 r^2 - (\mathbf{\Lambda}\cdot\mathbf{r})^2] \\
&\quad + \imath_8[(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Omega}) - (\mathbf{\Lambda}\cdot\mathbf{\Omega})(\widehat{\mathbf{p}}\cdot\mathbf{r})] + \imath_9[r^2(\mathbf{\Lambda}\cdot\mathbf{\Omega}) - (\mathbf{\Lambda}\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{\Omega})]\Big\} \text{ by (5.1a), (A.4) \& (A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\beta c^2 \mathcal{R}^2 \varphi_b}\Big\{\imath_0\varepsilon_b\varepsilon_a + \imath_1\varepsilon_b\varepsilon_g + \imath_2\varepsilon_b\varepsilon_c + \imath_3\varepsilon_b^2 + \imath_4\varepsilon_b\Omega^2 - \imath_6\varepsilon_b\varepsilon_k - \imath_7\varepsilon_b\varepsilon_m - \imath_0\Omega^2\varepsilon_d - \imath_1\Omega^2\varepsilon_h \\
&\quad - \imath_2\Omega^2\varepsilon_f - \imath_3\Omega^2 r^2 - \imath_4\Omega^2\varepsilon_b - \imath_5\Omega^2\varepsilon_k - \imath_8\Omega^2\varepsilon_l + \imath_0\varepsilon_n + \imath_2\varepsilon_o + \imath_4\varepsilon_m + \imath_5(\delta_a\varepsilon_b - \varepsilon_g\varepsilon_d) \\
&\quad + \imath_6(r^2\delta_a - \varepsilon_h\varepsilon_d) - \imath_7(\Lambda^2 r^2 - \varepsilon_h^2) + \imath_8(\varepsilon_i\varepsilon_b - \varepsilon_g\varepsilon_f) + \imath_9(r^2\varepsilon_g - \varepsilon_h\varepsilon_b)\Big\} \text{ by (5.1a) \& (5.13a)} \\
&= \frac{1}{\beta c^2 \mathcal{R}^2 \varphi_b}\Big\{\imath_0(\varepsilon_n + \varepsilon_b\varepsilon_a - \Omega^2\varepsilon_d) + \imath_1(\varepsilon_b\varepsilon_g - \Omega^2\varepsilon_h) + \imath_2(\varepsilon_o + \varepsilon_b\varepsilon_c - \Omega^2\varepsilon_f) \\
&\quad + \imath_3(\varepsilon_b^2 - \Omega^2 r^2) + \imath_4\varepsilon_m - \imath_5(\delta_a\varepsilon_b - \varepsilon_g\varepsilon_d + \Omega^2\varepsilon_k) - \imath_6(\varepsilon_b\varepsilon_k - \varepsilon_h\varepsilon_d + r^2\delta_a) \qquad (5.22c) \\
&\quad - \imath_7(\varepsilon_b\varepsilon_m - \Lambda^2 r^2 - \varepsilon_h^2) - \imath_8(\Omega^2\varepsilon_l + \varepsilon_i\varepsilon_b - \varepsilon_g\varepsilon_f) + \imath_9(r^2\varepsilon_g - \varepsilon_h\varepsilon_b)\Big\}.
\end{aligned}
$$

Equations (5.22) and (5.13) completely determine $\mathfrak{X}$ and Θ (as well as the drift $\dot\psi$) for a rotating observer.

Art 22. *Apparent path of a light source.*

Equation (3.22) can be evaluated for a rotating observer as follows. Let us introduce the following quantities in addition to those defined by (5.1), (5.12) and (5.13),

$$
\begin{gathered}
\varsigma_a = \dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}, \quad \varsigma_b = \dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol\kappa}, \quad \varsigma_c = \dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega}, \quad \varsigma_d = \dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}, \quad \varsigma_e = \dot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}}, \quad \varsigma_f = \ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}} \\
\varsigma_g = \ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol\kappa}, \quad \varsigma_h = \ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega}, \quad \varsigma_i = \ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}, \quad \varsigma_j = \ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}}, \quad \varsigma_k = \ddot{\mathbf{\Lambda}}\cdot\ddot{\mathbf{\Lambda}}, \quad \varsigma_l = \dddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}} \\
\varsigma_m = \dddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega}, \quad \varsigma_n = \mathbf{r}\cdot\dot{\mathbf{\Lambda}}, \quad \varsigma_o = \mathbf{r}\cdot\ddot{\mathbf{\Lambda}}, \quad \varsigma_p = \dddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol\kappa} \qquad (5.23a) \\
\varsigma_q = \dddot{\mathbf{\Lambda}}\cdot\mathbf{r}, \quad \varsigma_r = \dddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}, \quad \varsigma_s = \dddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}}
\end{gathered}
$$

$$
\begin{gathered}
\mathfrak{d}_a = \widehat{\boldsymbol\kappa}\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}), \quad \mathfrak{d}_b = \widehat{\boldsymbol\kappa}\cdot(\widehat{\mathbf{p}}\times\ddot{\mathbf{\Lambda}}), \quad \mathfrak{d}_c = \widehat{\boldsymbol\kappa}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r}), \quad \mathfrak{d}_d = \widehat{\boldsymbol\kappa}\cdot(\ddot{\mathbf{\Lambda}}\times\mathbf{r}) \\
\mathfrak{d}_e = \widehat{\boldsymbol\kappa}\cdot(\mathbf{\Lambda}\times\mathbf{\Omega}), \quad \mathfrak{d}_f = \widehat{\boldsymbol\kappa}\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}), \quad \mathfrak{d}_g = \widehat{\boldsymbol\kappa}\cdot(\dddot{\mathbf{\Lambda}}\times\mathbf{r}), \quad \mathfrak{d}_h = \dot{\mathbf{\Lambda}}\cdot(\mathbf{\Lambda}\times\mathbf{r}) \\
\mathfrak{d}_i = \ddot{\mathbf{\Lambda}}\cdot(\mathbf{\Lambda}\times\mathbf{r}), \quad \mathfrak{d}_j = \ddot{\mathbf{\Lambda}}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r}), \quad \mathfrak{d}_k = \mathbf{\Lambda}\cdot(\mathbf{\Omega}\times\mathbf{r}), \quad \mathfrak{d}_l = \dot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\mathbf{r}) \qquad (5.23b) \\
\mathfrak{d}_m = \ddot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\mathbf{r}), \quad \mathfrak{d}_n = \widehat{\mathbf{p}}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r}), \quad \mathfrak{d}_o = \widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\mathbf{\Lambda}), \quad \mathfrak{d}_p = \widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) \\
\mathfrak{d}_q = \widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\ddot{\mathbf{\Lambda}}), \quad \mathfrak{d}_r = \widehat{\mathbf{p}}\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}), \quad \mathfrak{d}_s = \widehat{\mathbf{p}}\cdot(\mathbf{\Lambda}\times\ddot{\mathbf{\Lambda}}), \quad \mathfrak{d}_t = \widehat{\boldsymbol\kappa}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})
\end{gathered}
$$

$$\begin{aligned}
&\mathfrak{d}_u = \widehat{\boldsymbol{\kappa}}\cdot(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}}),\quad \mathfrak{d}_v = \boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}}),\quad \mathfrak{d}_w = \ddot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}),\quad \mathfrak{d}_x = \ddot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})\\
&\mathfrak{d}_y = \ddot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}}),\quad \mathfrak{d}_z = \ddot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}),\quad \mathfrak{p}_a = \ddot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}}),\quad \mathfrak{p}_b = \ddot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\mathfrak{p}_c = \ddot{\boldsymbol{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}}),\quad \mathfrak{p}_d = \ddot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}}),\quad \mathfrak{p}_e = \boldsymbol{\Lambda}\cdot(\dddot{\boldsymbol{\Lambda}}\times\mathbf{r}),\quad \mathfrak{p}_f = \widehat{\mathbf{p}}\cdot(\dddot{\boldsymbol{\Lambda}}\times\mathbf{r})\\
&\qquad \mathfrak{p}_g = \boldsymbol{\Omega}\cdot(\dddot{\boldsymbol{\Lambda}}\times\mathbf{r}),\quad \mathfrak{p}_h = \dot{\boldsymbol{\Lambda}}\cdot(\dddot{\boldsymbol{\Lambda}}\times\mathbf{r}),\quad \mathfrak{p}_i = \ddot{\boldsymbol{\Lambda}}\cdot(\dddot{\boldsymbol{\Lambda}}\times\mathbf{r})
\end{aligned}\tag{5.23c}$$

$$\begin{aligned}
&\varrho_a = \varepsilon_i\varepsilon_a + \varepsilon_c\delta_a - \varepsilon_g\varepsilon_e,\quad \varrho_b = \varsigma_a\varepsilon_a + 2\varepsilon_i\delta_a + \varepsilon_c\varsigma_b - \varepsilon_e(\Lambda^2+\varsigma_c),\quad \varrho_c = \varepsilon_e(3\varsigma_d+\varsigma_h)\\
&\varrho_d = \varsigma_f\varepsilon_a + 3\varsigma_a\delta_a + 3\varepsilon_i\varsigma_b + \varepsilon_c\varsigma_g - \varrho_c,\quad \varrho_e = \varepsilon_e(\Lambda^2+\varsigma_c) - \varsigma_a\varepsilon_a - \varepsilon_i\delta_a\\
&\varrho_f = \varrho_c - \varsigma_f\varepsilon_a - 2\varsigma_a\delta_a - \varepsilon_i\varsigma_b,\quad \varrho_g = \varepsilon_e(3\varsigma_e + 4\varsigma_i + \varsigma_m) - \varsigma_l\varepsilon_a - 3\varsigma_f\delta_a - 3\varsigma_a\varsigma_b - \varepsilon_i\varsigma_g\\
&\varrho_h = 2\varrho_a - \delta_b(\varphi_e/\varepsilon_j),\quad \varrho_i = \varrho_e - 4\varrho_h(\varphi_e/\varepsilon_j),\quad \varrho_j = \varrho_a^2 + \varphi_e\varrho_b,\quad \varrho_k = \varphi_e\mathfrak{d}_a + \delta_b\varrho_h
\end{aligned}\tag{5.23d}$$

$$\begin{aligned}
&\varrho_l = 2\gamma\omega_o^2\varepsilon_j,\quad \varrho_m = \varrho_i - \delta_b(\varphi_g/\varepsilon_j),\quad \varrho_n = \varrho_f - \frac{8}{\varepsilon_j}\left[\varrho_j - \frac{\varphi_e}{2\varepsilon_j}\left\{\varrho_k + \delta_b\varrho_h\right\}\right]\\
&\varrho_o = \varrho_g - \frac{8}{\varepsilon_j}\left[\varphi_e\varrho_d + 3\varrho_a\varrho_b - \frac{1}{2\varepsilon_j}\left\{\varphi_e^2\mathfrak{d}_b + 6\delta_b\varrho_j + 6\varphi_e\varrho_a\mathfrak{d}_a\right\} + \frac{3\varrho_k\varphi_e\delta_b}{\varepsilon_j^2}\right]\\
&\varrho_p = \pm\varrho_m/\varrho_l,\quad \varrho_q = \frac{\varrho_p}{\gamma} + \frac{2\delta_b}{\varepsilon_j},\quad \varrho_r = \pm\frac{1}{\varrho_l}\left\{\varrho_n - \varrho_m\varrho_q - \frac{\varphi_g\mathfrak{d}_a}{\varepsilon_j}\right\}\\
&\varrho_s = \pm\frac{1}{\varrho_l}\left[\varrho_o - \frac{1}{\varepsilon_j}\left\{\varphi_g\mathfrak{d}_b + 3\varrho_n\delta_b + 3\varrho_i\mathfrak{d}_a\right\} + \frac{6\delta_b}{\varepsilon_j^2}\left\{\varrho_i\delta_b + \varphi_g\left(\mathfrak{d}_a - \frac{\delta_b^2}{\varepsilon_j}\right)\right\}\right.\\
&\qquad\left. - \frac{1}{\gamma}\left\{\varrho_i\varrho_r + 2\varrho_n\varrho_p\right\} + \frac{\varphi_g}{\gamma\varepsilon_j}\left\{2\mathfrak{d}_a\varrho_p + \delta_b\varrho_r\right\} + \frac{2\varrho_m\varrho_p\varrho_q}{\gamma}\right]
\end{aligned}\tag{5.23e}$$

$$\begin{aligned}
&\varrho_t = 3\varsigma_n - \Omega^2\varepsilon_b,\quad \varrho_u = \Omega^4 - 4\varsigma_c - 3\Lambda^2,\quad \varrho_v = 4\varsigma_o - 4\Omega^2\varepsilon_h - 5\varepsilon_b\varepsilon_g\\
&\varrho_w = 2\Omega^2\varepsilon_g - \varsigma_h - 2\varsigma_d,\quad \varrho_x = \mathfrak{d}_c + \varepsilon_b\delta_a + 2\varepsilon_a\varepsilon_h - 3\varepsilon_d\varepsilon_g - \Omega^2\varepsilon_k\\
&\varrho_y = \mathfrak{d}_d + \varepsilon_a\varrho_t + \varepsilon_d\varrho_u + \varepsilon_b(\varsigma_b + 2\mathfrak{d}_e) + 3(\varepsilon_h\delta_a - \Omega^2\varepsilon_n - \varepsilon_g\varepsilon_k)\\
&\varrho_z = \mathfrak{d}_g + \varepsilon_a\varrho_v + \varepsilon_k\varrho_u + 5\varepsilon_d\varrho_w + \varepsilon_b(\varsigma_g - 5\mathfrak{d}_f) + \delta_a(\varrho_t + 3\varsigma_n) + \varepsilon_h(4\varsigma_b + 5\mathfrak{d}_e)\\
&\qquad - 6(\Omega^2\mathfrak{d}_c + 2\varepsilon_g\varepsilon_n)
\end{aligned}\tag{5.23f}$$

$$\begin{aligned}
&\mathfrak{x}_a = \frac{\kappa}{\gamma^2\omega_o^2}\left\{\varrho_x - \frac{2\varphi_d\varrho_p}{\gamma}\right\},\quad \mathfrak{x}_b = \frac{\kappa}{\gamma^2\omega_o^2}\left\{\varrho_y - \frac{4\varrho_x\varrho_p}{\gamma} - \frac{2\varphi_d\varrho_r}{\gamma} + \frac{6\varphi_d\varrho_p^2}{\gamma^2}\right\}\\
&\mathfrak{x}_c = \frac{\kappa}{\gamma^2\omega_o^2}\left\{\varrho_z - \frac{6\varrho_y\varrho_p}{\gamma} - \frac{6\varrho_x\varrho_r}{\gamma} + \frac{18\varrho_x\varrho_p^2}{\gamma^2} - \frac{2\varphi_d\varrho_s}{\gamma} + \frac{18\varphi_d\varrho_p\varrho_r}{\gamma^2} - \frac{24\varphi_d\varrho_p^3}{\gamma^3}\right\}\\
&\mathfrak{x}_d = \delta_g^{-5}(\mathfrak{x}_b\delta_g^2 - 3\vartheta\mathfrak{x}_a^2),\quad \mathfrak{x}_e = \delta_g^{-7}[\mathfrak{x}_c\delta_g^4 - 9\vartheta\mathfrak{x}_a\mathfrak{x}_b\delta_g^2 - 3\mathfrak{x}_a^3(1-4\vartheta^2)]
\end{aligned}\tag{5.23g}$$

$$\begin{aligned}
&\mathfrak{x}_f = \frac{d\varrho_p}{\gamma} + \frac{\pi\gamma^2\mathfrak{x}_a}{4d}\\
&\mathfrak{x}_g = d\left\{\frac{\varrho_r}{\gamma} - \frac{\varrho_p^2}{\gamma^2}\right\} + \mathfrak{x}_f\left\{\frac{\varrho_p}{\gamma} - \frac{\pi\gamma^2\mathfrak{x}_a}{4d^2}\right\} + \frac{\gamma}{4d}\left\{\frac{\gamma\mathfrak{x}_a^2}{\delta_g^3} + 2\pi\varrho_p\mathfrak{x}_a + \pi\gamma\mathfrak{x}_b\right\}\\
&\mathfrak{x}_h = d\left\{\frac{\varrho_s}{\gamma} - \frac{3\varrho_p\varrho_r}{\gamma^2} + \frac{2\varrho_p^3}{\gamma^3}\right\} + \mathfrak{x}_g\left\{\frac{\varrho_p}{\gamma} - \frac{\pi\gamma^2\mathfrak{x}_a}{4d^2}\right\} + \frac{\varrho_p}{4d}\left\{\frac{\gamma\mathfrak{x}_a^2}{\delta_g^3} + 2\pi\varrho_p\mathfrak{x}_a + \pi\gamma\mathfrak{x}_b\right\}\\
&\qquad + \frac{\gamma}{4d}\left\{\gamma\mathfrak{x}_a\mathfrak{x}_d + \frac{3\mathfrak{x}_a^2\varrho_p}{\delta_g^3} + \frac{2\gamma\mathfrak{x}_a\mathfrak{x}_b}{\delta_g^3} + 2\pi\varrho_r\mathfrak{x}_a + 3\pi\varrho_p\mathfrak{x}_b + \pi\gamma\mathfrak{x}_c\right\}\\
&\qquad + 2\mathfrak{x}_f\left\{\frac{\varrho_r}{\gamma} - \frac{\varrho_p^2}{\gamma^2} - \frac{\gamma^2\mathfrak{x}_a^2}{4d^2\delta_g^3} - \frac{\pi\gamma\varrho_p\mathfrak{x}_a}{2d^2} - \frac{\pi\gamma^2\mathfrak{x}_b}{4d^2} + \frac{\pi\gamma^2\mathfrak{x}_f\mathfrak{x}_a}{4d^3}\right\}
\end{aligned}\tag{5.23h}$$

$$\mathfrak{x}_i = \frac{1}{4\omega_o d^2}\left\{\frac{d\mathfrak{x}_a}{\delta_g^3} - \pi\mathfrak{x}_f\right\}, \quad \mathfrak{x}_j = \frac{1}{4\omega_o d^2}\left[d\mathfrak{x}_d - \pi\mathfrak{x}_g - \frac{2\mathfrak{x}_f}{d}\left\{\frac{d\mathfrak{x}_a}{\delta_g^3} - \pi\mathfrak{x}_f\right\}\right]$$
$$\mathfrak{x}_k = \frac{1}{4\omega_o d^2}\left[\mathfrak{x}_f\mathfrak{x}_d + d\mathfrak{x}_e - \pi\mathfrak{x}_h - \frac{\mathfrak{x}_a\mathfrak{x}_g}{\delta_g^3} + \frac{2(3\mathfrak{x}_f^2 - d\mathfrak{x}_g)}{d^2}\left\{\frac{d\mathfrak{x}_a}{\delta_g^3} - \pi\mathfrak{x}_f\right\} - \frac{4\mathfrak{x}_f}{d}\left\{d\mathfrak{x}_d - \pi\mathfrak{x}_g\right\}\right] \tag{5.23i}$$

$$\begin{aligned}
\mathfrak{x}_l &= \frac{2\varphi_d\mathfrak{x}_i + 2\rho\varrho_x - c\mathfrak{x}_f - 2\varrho_h(\delta_b\varphi_g + \varepsilon_j\varrho_i + 2\varepsilon_j\delta_b\omega_o^2)}{2\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} \\
\mathfrak{x}_m &= \frac{2\varphi_d\mathfrak{x}_j + 4\varrho_x\mathfrak{x}_i + 2\rho\varrho_y - c\mathfrak{x}_g}{2\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} - \frac{4\mathfrak{x}_l(\delta_b\varphi_g + \varepsilon_j\varrho_i + 2\varepsilon_j\delta_b\omega_o^2)}{2\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} \\
&\quad - \frac{2\varphi_h(\varphi_g\mathfrak{d}_a + \varepsilon_j\varrho_n + 2\delta_b\varrho_i + 2\delta_b^2\omega_o^2 + 2\varepsilon_j\mathfrak{d}_a\omega_o^2)}{2\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} \\
\mathfrak{x}_n &= \frac{2\varphi_d\mathfrak{x}_k + 2\rho\varrho_z + 6\varrho_x\mathfrak{x}_j + 6\varrho_y\mathfrak{x}_i - c\mathfrak{x}_h}{2\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} - \frac{6\mathfrak{x}_m(\delta_b\varphi_g + \varepsilon_j\varrho_i + 2\delta_b\varepsilon_j\omega_o^2)}{2\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} \\
&\quad - \frac{2\varphi_h(\varphi_g\mathfrak{d}_b + \varepsilon_j\varrho_o + 3\varrho_i\mathfrak{d}_a + 3\delta_b\varrho_n + 6\delta_b\mathfrak{d}_a\omega_o^2 + 2\varepsilon_j\mathfrak{d}_b\omega_o^2)}{2\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} \\
&\quad - \frac{6\mathfrak{x}_l(\varphi_g\mathfrak{d}_a + \varepsilon_j\varrho_n + 2\delta_b\varrho_i + 2\delta_b^2\omega_o^2 + 2\varepsilon_j\mathfrak{d}_a\omega_o^2)}{2\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)}
\end{aligned} \tag{5.23j}$$

$$\begin{aligned}
&\mathfrak{x}_o = \varphi_g\mathfrak{x}_l + \varrho_i\varphi_h, \quad \mathfrak{x}_p = 2\varrho_i\mathfrak{x}_l + \varphi_g\mathfrak{x}_m + \varrho_n\varphi_h, \quad \mathfrak{x}_q = 3(\varrho_n\mathfrak{x}_l + \varrho_i\mathfrak{x}_m) + \varphi_g\mathfrak{x}_n + \varrho_o\varphi_h \\
&\mathfrak{x}_r = \varepsilon_j(\varepsilon_i\mathfrak{x}_l + \varphi_h\varsigma_a) + \varepsilon_i\varphi_h(\delta_b + 8\varphi_e) + 8\varepsilon_c(\varphi_e\mathfrak{x}_l + \varrho_a\varphi_h) \\
&\mathfrak{x}_s = \varepsilon_j(\varphi_h\varsigma_f + \varepsilon_i\mathfrak{x}_m) + \varepsilon_i(\mathfrak{d}_a\varphi_h + 2\delta_b\mathfrak{x}_l + 16\varrho_a\varphi_h + 16\varphi_e\mathfrak{x}_l) \\
&\qquad + 2\varsigma_a(\varepsilon_j\mathfrak{x}_l + \delta_b\varphi_h + 4\varphi_e\varphi_h) + 8\varepsilon_c(\varphi_e\mathfrak{x}_m + \varrho_b\varphi_h + 2\varrho_a\mathfrak{x}_l) \\
&\mathfrak{x}_t = \varepsilon_j(\varphi_h\varsigma_l + \varepsilon_i\mathfrak{x}_n) + \varepsilon_i(\mathfrak{d}_b\varphi_h + 3\mathfrak{d}_a\mathfrak{x}_l + 3\delta_b\mathfrak{x}_m + 24\varrho_b\varphi_h + 48\varrho_a\mathfrak{x}_l + 24\varphi_e\mathfrak{x}_m) \\
&\qquad + 3\varsigma_a(\varepsilon_j\mathfrak{x}_m + \mathfrak{d}_a\varphi_h + 2\delta_b\mathfrak{x}_l + 8\varrho_a\varphi_h + 8\varphi_e\mathfrak{x}_l) + \varsigma_f(3\varepsilon_j\mathfrak{x}_l + 3\delta_b\varphi_h + 8\varphi_e\varphi_h) \\
&\qquad + 8\varepsilon_c(\varphi_e\mathfrak{x}_n + \varrho_d\varphi_h + 3\varrho_b\mathfrak{x}_l + 3\varrho_a\mathfrak{x}_m)
\end{aligned} \tag{5.23k}$$

$$\begin{aligned}
\mathfrak{x}_u &= 4\Omega^2(\varrho_a\varphi_h + \varphi_e\mathfrak{x}_l) + \varepsilon_g(\varepsilon_j\mathfrak{x}_l + \delta_b\varphi_h + 8\varphi_e\varphi_h) + \varepsilon_j\varphi_h(\Lambda^2 + \varsigma_c) \\
\mathfrak{x}_v &= 4\Omega^2(\varrho_b\varphi_h + \varphi_e\mathfrak{x}_m + 2\varrho_a\mathfrak{x}_l) + \varepsilon_g(\varepsilon_j\mathfrak{x}_m + \mathfrak{d}_a\varphi_h + 2\delta_b\mathfrak{x}_l + 16\varrho_a\varphi_h + 16\varphi_e\mathfrak{x}_l) \\
&\quad + 2(\Lambda^2 + \varsigma_c)(\varepsilon_j\mathfrak{x}_l + \delta_b\varphi_h + 4\varphi_e\varphi_h) + \varepsilon_j\varphi_h(3\varsigma_d + \varsigma_h) \\
\mathfrak{x}_w &= 4\Omega^2(\varrho_d\varphi_h + \varphi_e\mathfrak{x}_n + 3\varrho_b\mathfrak{x}_l + 3\varrho_a\mathfrak{x}_m) + \varepsilon_j\varphi_h(4\varsigma_i + 3\varsigma_e + \varsigma_m) \\
&\quad + \varepsilon_g(\varepsilon_j\mathfrak{x}_n + \mathfrak{d}_b\varphi_h + 3\mathfrak{d}_a\mathfrak{x}_l + 3\delta_b\mathfrak{x}_m + 24\varrho_b\varphi_h + 48\varrho_a\mathfrak{x}_l + 24\varphi_e\mathfrak{x}_m) \\
&\quad + 3(\Lambda^2 + \varsigma_c)(\varepsilon_j\mathfrak{x}_m + \mathfrak{d}_a\varphi_h + 2\delta_b\mathfrak{x}_l + 8\varrho_a\varphi_h + 8\varphi_e\mathfrak{x}_l) \\
&\quad + (\varsigma_h + 3\varsigma_d)(3\varepsilon_j\mathfrak{x}_l + 3\delta_b\varphi_h + 8\varphi_e\varphi_h)
\end{aligned} \tag{5.23l}$$

$$\begin{aligned}
\mathfrak{x}_x &= \varrho_d\mathfrak{x}_i + \rho\varrho_x - \delta_b\varphi_g\varphi_h - \varepsilon_j\mathfrak{x}_o \\
\mathfrak{x}_y &= \mathfrak{x}_j\varrho_d + 2\varrho_x\mathfrak{x}_i + \rho\varrho_y - \mathfrak{d}_a\varphi_g\varphi_h - 2\delta_b\mathfrak{x}_o - \varepsilon_j\mathfrak{x}_p \\
\mathfrak{x}_z &= \mathfrak{x}_k\varphi_d + 3\mathfrak{x}_j\varrho_x + 3\mathfrak{x}_i\varrho_y + \rho\varrho_z - \mathfrak{d}_b\varphi_g\varphi_h - 3\mathfrak{d}_a\mathfrak{x}_o - 3\delta_b\mathfrak{x}_p - \varepsilon_j\mathfrak{x}_q
\end{aligned} \tag{5.23m}$$

$$\begin{aligned}
&\mathfrak{y}_a = \Omega^2\varepsilon_a - \varsigma_b, \quad \mathfrak{y}_b = 2\varepsilon_a\varepsilon_b - 3\Omega^2\varepsilon_d, \quad \mathfrak{y}_c = 2\varepsilon_b\delta_a + 3\varepsilon_d\varepsilon_g, \quad \mathfrak{y}_d = 3\Omega^2\delta_a + 3\varepsilon_a\varepsilon_g - \varsigma_g \\
&\mathfrak{y}_e = \varepsilon_a\mathfrak{x}_o + \varphi_g\varphi_h\delta_a, \quad \mathfrak{y}_f = \mathfrak{x}_p\varepsilon_a + 2\mathfrak{x}_o\delta_a + \varphi_g\varphi_h\varsigma_b, \quad \mathfrak{y}_g = 2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b \\
&\mathfrak{y}_h = \Omega^2(\varsigma_n + \varepsilon_m) + \varepsilon_b(\Lambda^2 - \varsigma_c) - \varepsilon_g\varepsilon_h - \mathfrak{d}_h, \quad \mathfrak{y}_i = \varepsilon_b\varrho_u + \Omega^2\varrho_t \\
&\mathfrak{y}_j = 3\Omega^2\varphi_a^2 - 3\varepsilon_h^2 - \varepsilon_b\varsigma_n - \varrho_t\varepsilon_b - \varrho_u r^2 + 2\varepsilon_b\varepsilon_m, \quad \mathfrak{y}_k = 3r^2\Omega^2\varepsilon_g + 2\Omega^2\varepsilon_b\varepsilon_h - 5\varepsilon_g\varepsilon_b^2 \\
&\mathfrak{y}_l = 3\Omega^2\varepsilon_b\varepsilon_g - \varepsilon_b\varsigma_h - 3\Omega^4\varepsilon_h + \Omega^2\varsigma_o + 3\varepsilon_h\Lambda^2 + \varepsilon_b\varsigma_d + \varrho_t\varepsilon_g + \varrho_u\varepsilon_h + 3\varepsilon_g\varepsilon_m - \mathfrak{d}_i \\
&\mathfrak{y}_m = \Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o, \quad \mathfrak{y}_n = \mathfrak{x}_r\varepsilon_g + \varphi_i\Lambda^2 - \mathfrak{x}_u\varepsilon_i, \quad \mathfrak{y}_o = \varphi_g\varphi_h(\varepsilon_b\varepsilon_g - \varepsilon_h\Omega^2) + \mathfrak{x}_o\varepsilon_m \\
&\mathfrak{y}_p = \mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h + \varphi_g\varphi_h\mathfrak{y}_m, \quad \mathfrak{y}_q = \mathfrak{x}_s\varepsilon_g + 2\mathfrak{x}_r\Lambda^2 + \varphi_i\varsigma_d - \mathfrak{x}_v\varepsilon_i
\end{aligned} \tag{5.23n}$$

$$
\begin{aligned}
&\mathfrak{y}_r = \mathfrak{x}_p\varepsilon_m + 2\mathfrak{x}_o(\varepsilon_b\varepsilon_g - \Omega^2\varepsilon_h) + \varphi_g\varphi_h(\varepsilon_b\varsigma_c - \Omega^2\varsigma_n + \mathfrak{d}_h) \\
&\mathfrak{y}_s = \mathfrak{x}_v\varepsilon_f - \mathfrak{x}_s\varepsilon_b - 2\mathfrak{x}_r\varepsilon_h - \varphi_i\varsigma_n + 2\mathfrak{x}_o\mathfrak{y}_m, \quad \mathfrak{y}_t = 6\varepsilon_h^2 - \varrho_t\varepsilon_b, \quad \mathfrak{y}_u = 2\varrho_u\varepsilon_h + 3\varrho_t\varepsilon_g \\
&\mathfrak{y}_v = 9\varepsilon_h\varepsilon_g + \varrho_u\varepsilon_b, \quad \mathfrak{y}_w = 5\Omega^2\varphi_c + 4\varepsilon_g\varphi_a^2 - 2\varepsilon_b\mathfrak{d}_l, \quad \mathfrak{y}_x = 3\varepsilon_h^2 + \varepsilon_b\varsigma_n + \varrho_t\varepsilon_b + \varrho_u r^2 \\
&\mathfrak{y}_y = 7\varepsilon_g\varphi_c + \Omega^2\varepsilon_b(\varrho_t + 2\mathfrak{d}_k + \varsigma_n) + \Omega^2(r^2\varrho_u + 3\varepsilon_h^2) + 2\varepsilon_b(\mathfrak{d}_h - \Lambda^2\varepsilon_b) + 2r^2\varepsilon_g^2 \\
&\mathfrak{y}_z = \Omega^4(3\varepsilon_m - \varrho_t) + \Omega^2(\mathfrak{d}_m - 3\mathfrak{d}_h) + \Omega^2\varepsilon_b(3\Lambda^2 - \varrho_u - \varsigma_c) + \varepsilon_b(\dot{\mathbf{\Lambda}}^2 - 6\varepsilon_g^2 - \varsigma_i) \\
&\qquad + \varepsilon_h(3\varsigma_d - 2\varsigma_h) + 3\varepsilon_g(\varsigma_o - \mathfrak{d}_l) + \varrho_t\varsigma_c + \varrho_u\varsigma_n - \mathfrak{d}_j
\end{aligned} \tag{5.23o}
$$

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\begin{aligned}
&\varkappa_a = 2\varphi_i\varepsilon_h - \mathfrak{x}_r\varepsilon_b, \quad \varkappa_b = \mathfrak{x}_r(\varsigma_c - \Omega^4) + \varphi_i(\varsigma_d - \Omega^2\varepsilon_g) + \mathfrak{x}_u(\Omega^2\varepsilon_c - \varsigma_a) \\
&\varkappa_c = \mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h, \quad \varkappa_d = -2\varepsilon_c\varepsilon_h + 3\varepsilon_f\varepsilon_g - \varepsilon_b\varepsilon_i - \mathfrak{d}_n + \Omega^2\varepsilon_l \\
&\varkappa_e = \Omega^2(2\varepsilon_h\mathfrak{x}_o + \varepsilon_m\varphi_g\varphi_h) - \mathfrak{x}_o(\mathfrak{d}_l + 2\varepsilon_b\varepsilon_g) + \varphi_g\varphi_h(\Lambda^2\varepsilon_b - \mathfrak{d}_h - \varepsilon_g\varepsilon_h) \\
&\varkappa_f = \Omega^2(\varepsilon_l\mathfrak{x}_o - \varepsilon_f\mathfrak{x}_u + \varepsilon_b\mathfrak{x}_r + \varepsilon_h\varphi_i) + \mathfrak{x}_o(3\varepsilon_f\varepsilon_g - 2\varepsilon_c\varepsilon_h - \varepsilon_b\varepsilon_i - \mathfrak{d}_n) \\
&\varkappa_g = 4\mathfrak{x}_r\varepsilon_h - \mathfrak{x}_s\varepsilon_b, \quad \varkappa_h = \Omega^2\varepsilon_l - 2\varepsilon_h\varepsilon_c + 3\varepsilon_g\varepsilon_f - \varepsilon_b\varepsilon_i - \mathfrak{d}_n \\
&\varkappa_i = 2\mathfrak{x}_o(\varepsilon_b\Lambda^2 - \varepsilon_g\varepsilon_h - \mathfrak{d}_h) + \varphi_g\varphi_h(-\Omega^2\mathfrak{d}_l + 2\varepsilon_h\varsigma_c - 3\varepsilon_g\varsigma_n + \varepsilon_b\varsigma_d) \\
&\qquad + 2\Omega^2(\mathfrak{x}_p\varepsilon_h + \mathfrak{x}_o\varepsilon_m) - \mathfrak{x}_p(2\varepsilon_g\varepsilon_b + \mathfrak{d}_l) \\
&\varkappa_j = \Omega^2(\mathfrak{x}_s\varepsilon_b + 2\mathfrak{x}_r\varepsilon_h + \varphi_i\varsigma_n - \mathfrak{x}_v\varepsilon_f + \mathfrak{x}_p\varepsilon_l) + \mathfrak{x}_p(3\varepsilon_g\varepsilon_f - 2\varepsilon_h\varepsilon_c - \varepsilon_b\varepsilon_i - \mathfrak{d}_n) \\
&\varkappa_k = \varphi_g\varphi_h(3\varepsilon_g\varepsilon_f - 2\varepsilon_h\varepsilon_c - \varepsilon_b\varepsilon_i - \mathfrak{d}_n + \Omega^2\varepsilon_l) - \mathfrak{x}_s\varepsilon_b - 2\mathfrak{x}_r\varepsilon_h - \varphi_i\varsigma_n + \mathfrak{x}_v\varepsilon_f \\
&\varkappa_l = \mathfrak{x}_s(\varsigma_c - \Omega^4) + 2\mathfrak{x}_r(\varsigma_d - \Omega^2\varepsilon_g) + \varphi_i(\dot{\mathbf{\Lambda}}^2 - \Omega^2\varsigma_c) + \mathfrak{x}_v(\Omega^2\varepsilon_c - \varsigma_a)
\end{aligned} \tag{5.23p}
$$

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\begin{aligned}
&\varkappa_m = 3\varepsilon_h\mathfrak{x}_r - \varrho_t\varphi_i, \quad \varkappa_n = \mathfrak{x}_r\varepsilon_b + \varphi_i\varepsilon_h - \mathfrak{x}_u\varepsilon_f + \mathfrak{x}_o\varepsilon_l + \varphi_g\varphi_h\varepsilon_o \\
&\varkappa_o = -\mathfrak{x}_o(3\varepsilon_h\varepsilon_g + \varepsilon_b\varsigma_c + \varrho_t\Omega^2 + \varrho_u\varepsilon_b) + \varphi_g\varphi_h(-3\varepsilon_h\Lambda^2 - \varepsilon_b\varsigma_d - \varrho_t\varepsilon_g - \varrho_u\varepsilon_h) \\
&\varkappa_p = 3\Omega^2(2\mathfrak{x}_r\varepsilon_g + \varphi_i\Lambda^2 - \mathfrak{x}_u\varepsilon_i + \mathfrak{x}_o\mathfrak{d}_o) + \varphi_i(3\varepsilon_g^2 - \varsigma_i) + \mathfrak{x}_u(\varsigma_f - 3\varepsilon_g\varepsilon_c) \\
&\qquad - \varphi_g\varphi_h(3\varepsilon_g\mathfrak{d}_o + \mathfrak{d}_s) - \mathfrak{x}_o\mathfrak{d}_q - \mathfrak{x}_r\varsigma_h \\
&\varkappa_q = \mathfrak{x}_o(3\varepsilon_h\varepsilon_i + \varepsilon_b\varsigma_a + \varrho_t\varepsilon_c + \varrho_u\varepsilon_f) + 2\varepsilon_b(\mathfrak{x}_u\varepsilon_i - \mathfrak{x}_o\mathfrak{d}_o - \mathfrak{x}_r\varepsilon_g - \varphi_i\Lambda^2) \\
&\qquad + 3\varepsilon_g(\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_o\varepsilon_l - \varphi_g\varphi_h\varepsilon_o - \varepsilon_b\mathfrak{x}_r - \varphi_i\varepsilon_h) \\
&\varkappa_r = \varphi_g\varphi_h(3\varepsilon_h\varepsilon_i + \varepsilon_b\varsigma_a + \varrho_t\varepsilon_c + \varrho_u\varepsilon_f) - 3\Omega^2(\mathfrak{x}_r\varepsilon_b + \varphi_i\varepsilon_h - \mathfrak{x}_u\varepsilon_f + \mathfrak{x}_o\varepsilon_l + \varphi_g\varphi_h\varepsilon_o) \\
&\qquad + 2\varepsilon_b(\varphi_i\varepsilon_g + \mathfrak{x}_r\Omega^2 - \mathfrak{x}_u\varepsilon_c - \varphi_g\varphi_h\mathfrak{d}_o) \\
&\varkappa_s = 2\mathfrak{x}_r^2 - \mathfrak{x}_s\varphi_i, \quad \varkappa_t = \mathfrak{x}_s\mathfrak{x}_u - \mathfrak{x}_v\mathfrak{x}_r, \quad \varkappa_u = \mathfrak{x}_v\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_u, \quad \varkappa_v = \mathfrak{x}_u - \varepsilon_c\mathfrak{x}_r - \varepsilon_i\varphi_i \\
&\varkappa_w = \Omega^2(\mathfrak{x}_p\mathfrak{x}_r - \mathfrak{x}_s\mathfrak{x}_o) + \varepsilon_g(\mathfrak{x}_p\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_o) + 2\mathfrak{x}_o^2\mathfrak{d}_o - \varsigma_c\varphi_i\mathfrak{x}_o + \varepsilon_c(\mathfrak{x}_v\mathfrak{x}_o - \mathfrak{x}_p\mathfrak{x}_u) \\
&\qquad + \varepsilon_g(2\mathfrak{x}_o\mathfrak{x}_r - \mathfrak{x}_s\varphi_g\varphi_h) + 2\Lambda^2(\mathfrak{x}_o\varphi_i - \mathfrak{x}_r\varphi_g\varphi_h) + \varepsilon_i(\mathfrak{x}_v\varphi_g\varphi_h - 2\mathfrak{x}_o\mathfrak{x}_u) \\
&\qquad + \varphi_g\varphi_h(-\varsigma_a\mathfrak{x}_u + \varsigma_c\mathfrak{x}_r + \mathfrak{x}_o\mathfrak{d}_p - \mathfrak{x}_p\mathfrak{d}_o + \varphi_g\varphi_h\mathfrak{d}_r) \\
&\varkappa_x = \varphi_g\varphi_h(\varsigma_a\varphi_i - \mathfrak{x}_v) - \varepsilon_c(2\mathfrak{x}_o\mathfrak{x}_r - \mathfrak{x}_s\varphi_g\varphi_h) - 2\varepsilon_i(\mathfrak{x}_o\varphi_i - \mathfrak{x}_r\varphi_g\varphi_h) + 2\mathfrak{x}_o\mathfrak{x}_u) \\
&\varkappa_y = \mathfrak{x}_o(\varsigma_a\varphi_i - \mathfrak{x}_v) - \varepsilon_c(\mathfrak{x}_p\mathfrak{x}_r - \mathfrak{x}_s\mathfrak{x}_o) - \varepsilon_i(\mathfrak{x}_p\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_o) + \mathfrak{x}_p\mathfrak{x}_u)
\end{aligned} \tag{5.23q}
$$

$$
\begin{aligned}
&\mathfrak{v}_a = \mathfrak{x}_i - 1, \quad \mathfrak{v}_b = c\mathfrak{x}_f - 2\mathfrak{x}_x, \quad \mathfrak{v}_c = \varepsilon_b\mathfrak{v}_a + 2\rho\varepsilon_h + \mathfrak{x}_r, \quad \mathfrak{v}_d = \Omega^2\mathfrak{v}_a + 3\rho\varepsilon_g \\
&\mathfrak{v}_e = \rho\varepsilon_b + \varphi_i, \quad \mathfrak{v}_f = c\mathfrak{x}_g - 2\mathfrak{x}_y, \quad \mathfrak{v}_g = c\mathfrak{x}_h - 2\mathfrak{x}_z, \quad \mathfrak{v}_h = 2\mathfrak{x}_i - 1, \quad \mathfrak{v}_i = 3\mathfrak{x}_i - 1 \\
&\mathfrak{v}_j = \mathfrak{x}_j\mathfrak{v}_b - \mathfrak{v}_f\mathfrak{v}_a, \quad \mathfrak{v}_k = \mathfrak{v}_b\mathfrak{v}_h - \rho\mathfrak{v}_f, \quad \mathfrak{v}_l = \mathfrak{v}_a\mathfrak{v}_h - \rho\mathfrak{x}_j
\end{aligned} \tag{5.23r}
$$

$$
\begin{aligned}
&\mathfrak{v}_m = \mathfrak{v}_j\varepsilon_b + 2\mathfrak{v}_k\varepsilon_h\varrho_t - \mathfrak{v}_f\mathfrak{x}_r + \mathfrak{v}_b(\rho + \mathfrak{x}_s), \quad \mathfrak{v}_n = (\rho\mathfrak{v}_b\varrho_u - \mathfrak{v}_j\Omega^2 - 3\mathfrak{v}_k\varepsilon_g)\\
&\mathfrak{v}_o = \mathfrak{v}_k\varepsilon_b - \mathfrak{v}_f\varphi_i + \mathfrak{v}_b(3\rho\varepsilon_h + 2\mathfrak{x}_r), \quad \mathfrak{v}_p = \mathfrak{v}_f\mathfrak{x}_u - \mathfrak{v}_b\mathfrak{x}_v\\
&\mathfrak{v}_q = \mathfrak{v}_l\mathfrak{y}_g + \mathfrak{v}_a(\rho\mathfrak{y}_i + \mathfrak{x}_s\Omega^2) + \rho(\rho\mathfrak{y}_u + 3\mathfrak{x}_s\varepsilon_g) + \mathfrak{x}_r(\rho\varrho_u - \mathfrak{x}_j\Omega^2 - 3\mathfrak{v}_h\varepsilon_g)\\
&\mathfrak{v}_r = \varkappa_s - \mathfrak{v}_h\varkappa_a + \varepsilon_b(\mathfrak{v}_l\varepsilon_b - \mathfrak{x}_j\varphi_i + \mathfrak{v}_a(3\rho\varepsilon_h + 2\mathfrak{x}_r)) + \rho(\varkappa_g + \varkappa_m + \rho\mathfrak{y}_t)\\
&\mathfrak{v}_s = \rho\varepsilon_b(\mathfrak{x}_r + \mathfrak{v}_a\varepsilon_b + 2\rho\varepsilon_h) + \varphi_i(\mathfrak{x}_r + \mathfrak{v}_a\varepsilon_b + 2\rho\varepsilon_h)\\
&\mathfrak{v}_t = \varkappa_t + \rho\varrho_t\mathfrak{x}_u + \varepsilon_b(\mathfrak{x}_j\mathfrak{x}_u - \mathfrak{v}_a\mathfrak{x}_v) + 2\varepsilon_h(\mathfrak{v}_h\mathfrak{x}_u - \rho\mathfrak{x}_v)
\end{aligned}
\tag{5.23s}
$$

$$
\begin{aligned}
\mathfrak{v}_u &= \varphi_i(3\mathfrak{v}_h\varepsilon_g + \mathfrak{x}_j\Omega^2) - \Omega^2(\mathfrak{v}_l\varepsilon_b + \mathfrak{v}_a(3\rho\varepsilon_h + 2\mathfrak{x}_r)) - \rho(\rho\mathfrak{y}_v + 6\mathfrak{x}_r\varepsilon_g + \varrho_u\varphi_i)\\
\mathfrak{v}_v &= \rho\varrho_u\mathfrak{x}_u + \Omega^2(\mathfrak{v}_a\mathfrak{x}_v - \mathfrak{x}_j\mathfrak{x}_u) + 3\varepsilon_g(\rho\mathfrak{x}_v - \mathfrak{v}_h\mathfrak{x}_u), \quad \mathfrak{v}_w = \varkappa_u - \mathfrak{v}_h\mathfrak{x}_u\varepsilon_b + \rho(\mathfrak{x}_v\varepsilon_b - 3\varepsilon_h\mathfrak{x}_u)\\
\mathfrak{v}_x &= \varkappa_x - \mathfrak{x}_j\mathfrak{y}_p + 3\mathfrak{v}_l\varphi_c + \mathfrak{v}_j\varepsilon_d + \mathfrak{v}_b(\rho\mathfrak{y}_b - 2\mathfrak{x}_o\varepsilon_e) + \mathfrak{v}_a(\mathfrak{y}_s + \rho\mathfrak{y}_j)\\
&\quad + \rho(\varkappa_r + \rho\mathfrak{y}_w + 2\mathfrak{x}_o\varkappa_h) + \varphi_g\varphi_h(\mathfrak{v}_f\varepsilon_e - \mathfrak{v}_h\varkappa_d)\\
\mathfrak{v}_y &= \mathfrak{v}_k\mathfrak{y}_a - \mathfrak{v}_j\delta_a + \mathfrak{v}_l\mathfrak{y}_h - \mathfrak{x}_j\mathfrak{y}_n + \mathfrak{v}_a\mathfrak{y}_q - \mathfrak{v}_h\varkappa_b + \rho(\varkappa_l + \varkappa_p + \rho\mathfrak{y}_z + \mathfrak{v}_b\mathfrak{y}_d + \mathfrak{v}_a\mathfrak{y}_l)\\
\mathfrak{v}_z &= \varkappa_w - \mathfrak{x}_j\mathfrak{y}_o + \mathfrak{v}_a\mathfrak{y}_r + \mathfrak{v}_b\mathfrak{y}_f - \mathfrak{v}_f\mathfrak{y}_e - \mathfrak{v}_h\varkappa_e + \rho(\varkappa_i + \varkappa_o)
\end{aligned}
\tag{5.23t}
$$

$$
\begin{aligned}
\mathfrak{o}_a &= \varkappa_y - \mathfrak{v}_h\varkappa_f + \Omega^2(\mathfrak{v}_l\varphi_a^2 - \mathfrak{v}_k\varepsilon_d) + \mathfrak{x}_o(\mathfrak{v}_f\varepsilon_e - \mathfrak{x}_j\mathfrak{y}_m) + \rho(\mathfrak{v}_a\mathfrak{y}_k - \mathfrak{v}_b\mathfrak{y}_c + \varkappa_j + \varkappa_q + \rho\mathfrak{y}_y)\\
&\quad + \mathfrak{x}_p(\mathfrak{v}_a\mathfrak{y}_m - \mathfrak{v}_b\varepsilon_e)\\
\mathfrak{o}_b &= \mathfrak{v}_k\varepsilon_d - \mathfrak{v}_l\varphi_a^2 - \mathfrak{v}_h\varkappa_c + \varphi_g\varphi_h(\varkappa_v - \mathfrak{v}_b\varepsilon_e + \mathfrak{v}_a\mathfrak{y}_m) + \rho(\varkappa_k - \rho\mathfrak{y}_x)\\
\mathfrak{o}_c &= \rho(\mathfrak{v}_b\varepsilon_d - \mathfrak{v}_a\varphi_a^2 - 3\rho\varphi_c + \varkappa_n)\\
\mathfrak{o}_d &= \mathfrak{v}_x\Lambda^2 + \mathfrak{v}_y\varepsilon_h + \mathfrak{v}_z\varepsilon_i + \mathfrak{o}_a\varepsilon_g + \mathfrak{o}_b\varsigma_d + \mathfrak{o}_c\varsigma_i - \mathfrak{v}_m\mathfrak{d}_e - \mathfrak{v}_n\varepsilon_n - \mathfrak{v}_b\mathfrak{v}_e\mathfrak{d}_u + \mathfrak{v}_p\delta_b - \mathfrak{v}_q\varepsilon_m\\
&\quad - \mathfrak{v}_s\mathfrak{d}_v - \mathfrak{v}_t\mathfrak{d}_o - \mathfrak{v}_e\mathfrak{v}_d\mathfrak{d}_h + \mathfrak{v}_v\varepsilon_o + \mathfrak{x}_u\mathfrak{v}_e\mathfrak{d}_r\\
\mathfrak{o}_e &= \mathfrak{v}_x\varepsilon_h + \mathfrak{v}_y r^2 + \mathfrak{v}_z\varepsilon_f + \mathfrak{o}_a\varepsilon_b + \mathfrak{o}_b\varsigma_n + \mathfrak{o}_c\varsigma_o + \mathfrak{v}_m\varepsilon_k + \mathfrak{v}_o\varepsilon_n + \mathfrak{v}_b\mathfrak{v}_e\mathfrak{d}_c - \mathfrak{v}_p\mathfrak{d}_t + \mathfrak{v}_r\varepsilon_m\\
&\quad - \mathfrak{v}_s\mathfrak{d}_l - \mathfrak{v}_t\varepsilon_l - \mathfrak{x}_u\mathfrak{v}_e\mathfrak{d}_n + \mathfrak{v}_w\varepsilon_o - \mathfrak{v}_e^2\mathfrak{d}_h
\end{aligned}
\tag{5.24a}
$$

$$
\begin{aligned}
\mathfrak{o}_f &= \mathfrak{v}_x\varepsilon_i + \mathfrak{v}_y\varepsilon_f + \mathfrak{v}_z + \mathfrak{o}_a\varepsilon_c + \mathfrak{o}_b\varsigma_a + \mathfrak{o}_c\varsigma_f - \mathfrak{v}_m\varepsilon_j + \mathfrak{v}_n\mathfrak{d}_t - \mathfrak{v}_o\delta_b - \mathfrak{v}_b\mathfrak{v}_e\mathfrak{d}_a + \mathfrak{v}_q\varepsilon_l\\
&\quad + \mathfrak{v}_r\mathfrak{d}_o + \mathfrak{v}_s\mathfrak{d}_p + \mathfrak{v}_e\mathfrak{v}_d\mathfrak{d}_n - \mathfrak{v}_u\varepsilon_o + \mathfrak{v}_e^2\mathfrak{d}_r\\
\mathfrak{o}_g &= \mathfrak{v}_x\varepsilon_g + \mathfrak{v}_y\varepsilon_b + \mathfrak{v}_z\varepsilon_c + \mathfrak{o}_a\Omega^2 + \mathfrak{o}_b\varsigma_c + \mathfrak{o}_c\varsigma_h - \mathfrak{v}_n\varepsilon_k + \mathfrak{v}_o\mathfrak{d}_e - \mathfrak{v}_b\mathfrak{v}_e\mathfrak{d}_f + \mathfrak{v}_p\varepsilon_j - \mathfrak{v}_e\mathfrak{v}_d\mathfrak{d}_l\\
&\quad - \mathfrak{v}_u\varepsilon_m + \mathfrak{v}_v\varepsilon_l + \mathfrak{x}_u\mathfrak{v}_e\mathfrak{d}_p - \mathfrak{v}_w\mathfrak{d}_o + \mathfrak{v}_e^2\mathfrak{d}_v\\
\mathfrak{o}_h &= \mathfrak{v}_x\varsigma_d + \mathfrak{v}_y\varsigma_n + \mathfrak{v}_z\varsigma_a + \mathfrak{o}_a\varsigma_c + \mathfrak{o}_b\varsigma_e + \mathfrak{o}_c\varsigma_j + \mathfrak{v}_m\mathfrak{d}_f - \mathfrak{v}_n\mathfrak{d}_c + \mathfrak{v}_o\mathfrak{d}_u + \mathfrak{v}_p\mathfrak{d}_a + \mathfrak{v}_q\mathfrak{d}_l\\
&\quad + \mathfrak{v}_r\mathfrak{d}_v - \mathfrak{v}_t\mathfrak{d}_p - \mathfrak{v}_u\mathfrak{d}_h + \mathfrak{v}_v\mathfrak{d}_n + \mathfrak{v}_w\mathfrak{d}_r\\
\mathfrak{o}_i &= \mathfrak{v}_x\varsigma_i + \mathfrak{v}_y\varsigma_o + \mathfrak{v}_z\varsigma_f + \mathfrak{o}_a\varsigma_h + \mathfrak{o}_b\varsigma_j + \mathfrak{o}_c\varsigma_k + \mathfrak{v}_m\mathfrak{d}_w - \mathfrak{v}_n\mathfrak{d}_d + \mathfrak{v}_o\mathfrak{d}_x + \mathfrak{v}_b\mathfrak{v}_e\mathfrak{d}_y + \mathfrak{v}_p\mathfrak{d}_b\\
&\quad + \mathfrak{v}_q\mathfrak{d}_m + \mathfrak{v}_r\mathfrak{d}_z + \mathfrak{v}_s\mathfrak{p}_a - \mathfrak{v}_t\mathfrak{d}_q + \mathfrak{v}_e\mathfrak{v}_d\mathfrak{d}_j - \mathfrak{v}_u\mathfrak{d}_i + \mathfrak{v}_v\mathfrak{p}_b - \mathfrak{x}_u\mathfrak{v}_e\mathfrak{p}_c + \mathfrak{v}_w\mathfrak{d}_s + \mathfrak{v}_e^2\mathfrak{p}_d
\end{aligned}
\tag{5.24b}
$$

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\begin{aligned}
\mathfrak{o}_j &= -\mathfrak{v}_x\mathfrak{d}_e + \mathfrak{v}_y\varepsilon_k - \mathfrak{v}_z\varepsilon_j + \mathfrak{o}_b\mathfrak{d}_f + \mathfrak{o}_c\mathfrak{d}_w + \mathfrak{v}_m(\Omega^2 - \varepsilon_a^2) + \mathfrak{v}_n(\varepsilon_b - \varepsilon_d\varepsilon_a) + \mathfrak{v}_o(\varepsilon_g - \delta_a\varepsilon_a)\\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_c - \varsigma_b\varepsilon_a) + \mathfrak{v}_p(\varepsilon_c - \varepsilon_e\varepsilon_a) + \mathfrak{v}_q(\varepsilon_a\varepsilon_b - \varepsilon_d\Omega^2) + \mathfrak{v}_r(\varepsilon_a\varepsilon_g - \delta_a\Omega^2)\\
&\quad + \mathfrak{v}_s(\varepsilon_a\varsigma_c - \varsigma_b\Omega^2) + \mathfrak{v}_t(\varepsilon_a\varepsilon_c - \varepsilon_e\Omega^2) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_d\varsigma_c - \varsigma_b\varepsilon_b) + \mathfrak{v}_u(\varepsilon_d\varepsilon_g - \delta_a\varepsilon_b)\\
&\quad + \mathfrak{v}_v(\varepsilon_d\varepsilon_c - \varepsilon_e\varepsilon_b) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_e\varsigma_c - \varsigma_b\varepsilon_c) + \mathfrak{v}_w(\varepsilon_e\varepsilon_g - \delta_a\varepsilon_c) + \mathfrak{v}_e^2(\delta_a\varsigma_c - \varsigma_b\varepsilon_g)\\
\mathfrak{o}_k &= -\mathfrak{v}_x\varepsilon_n + \mathfrak{v}_z\mathfrak{d}_t - \mathfrak{o}_a\varepsilon_k - \mathfrak{o}_b\mathfrak{d}_c - \mathfrak{o}_c\mathfrak{d}_d + \mathfrak{v}_m(\varepsilon_b - \varepsilon_a\varepsilon_d) + \mathfrak{v}_n(r^2 - \varepsilon_d^2) + \mathfrak{v}_o(\varepsilon_h - \delta_a\varepsilon_d)\\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_n - \varsigma_b\varepsilon_d) + \mathfrak{v}_p(\varepsilon_f - \varepsilon_e\varepsilon_d) + \mathfrak{v}_q(\varepsilon_a r^2 - \varepsilon_d\varepsilon_b) + \mathfrak{v}_r(\varepsilon_a\varepsilon_h - \delta_a\varepsilon_b)\\
&\quad + \mathfrak{v}_s(\varepsilon_a\varsigma_n - \varsigma_b\varepsilon_b) + \mathfrak{v}_t(\varepsilon_a\varepsilon_f - \varepsilon_e\varepsilon_b) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_d\varsigma_n - \varsigma_b r^2) + \mathfrak{v}_u(\varepsilon_d\varepsilon_h - \delta_a r^2)\\
&\quad + \mathfrak{v}_v(\varepsilon_d\varepsilon_f - \varepsilon_e r^2) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_e\varsigma_n - \varsigma_b\varepsilon_f) + \mathfrak{v}_w(\varepsilon_e\varepsilon_h - \delta_a\varepsilon_f) + \mathfrak{v}_e^2(\delta_a\varsigma_n - \varsigma_b\varepsilon_h)
\end{aligned}
\tag{5.24c}
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\begin{aligned}
\mathfrak{o}_l &= \mathfrak{v}_y\varepsilon_n - \mathfrak{v}_z\delta_b + \mathfrak{o}_a\mathfrak{d}_e + \mathfrak{o}_b\mathfrak{d}_u + \mathfrak{o}_c\mathfrak{d}_x + \mathfrak{v}_m(\varepsilon_g - \varepsilon_a\delta_a) + \mathfrak{v}_n(\varepsilon_h - \varepsilon_d\delta_a) + \mathfrak{v}_o(\Lambda^2 - \delta_a^2) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_d - \varsigma_b\delta_a) + \mathfrak{v}_p(\varepsilon_i - \varepsilon_e\delta_a) + \mathfrak{v}_q(\varepsilon_a\varepsilon_h - \varepsilon_d\varepsilon_g) + \mathfrak{v}_r(\varepsilon_a\Lambda^2 - \delta_a\varepsilon_g) \\
&\quad + \mathfrak{v}_s(\varepsilon_a\varsigma_d - \varsigma_b\varepsilon_g) + \mathfrak{v}_t(\varepsilon_a\varepsilon_i - \varepsilon_e\varepsilon_g) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_d\varsigma_d - \varsigma_b\varepsilon_h) + \mathfrak{v}_u(\varepsilon_d\Lambda^2 - \delta_a\varepsilon_h) \\
&\quad + \mathfrak{v}_v(\varepsilon_d\varepsilon_i - \varepsilon_e\varepsilon_h) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_e\varsigma_d - \varsigma_b\varepsilon_i) + \mathfrak{v}_w(\varepsilon_e\Lambda^2 - \delta_a\varepsilon_i) + \mathfrak{v}_e^2(\delta_a\varsigma_d - \varsigma_b\Lambda^2) \\
\mathfrak{o}_m &= -\mathfrak{v}_x\mathfrak{d}_u + \mathfrak{v}_y\mathfrak{d}_c - \mathfrak{v}_z\mathfrak{d}_a - \mathfrak{o}_a\mathfrak{d}_f + \mathfrak{o}_c\mathfrak{d}_y + \mathfrak{v}_m(\varsigma_c - \varepsilon_a\varsigma_b) + \mathfrak{v}_n(\varsigma_n - \varepsilon_d\varsigma_b) \\
&\quad + \mathfrak{v}_o(\varsigma_d - \delta_a\varsigma_b) + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_e - \varsigma_b^2) + \mathfrak{v}_p(\varsigma_a - \varepsilon_e\varsigma_b) + \mathfrak{v}_q(\varepsilon_a\varsigma_n - \varepsilon_d\varsigma_c) + \mathfrak{v}_r(\varepsilon_a\varsigma_d - \delta_a\varsigma_c) \\
&\quad + \mathfrak{v}_s(\varepsilon_a\varsigma_e - \varsigma_b\varsigma_c) + \mathfrak{v}_t(\varepsilon_a\varsigma_a - \varepsilon_e\varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_d\varsigma_e - \varsigma_b\varsigma_n) + \mathfrak{v}_u(\varepsilon_d\varsigma_d - \delta_a\varsigma_n) \\
&\quad + \mathfrak{v}_v(\varepsilon_d\varsigma_a - \varepsilon_e\varsigma_n) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_e\varsigma_e - \varsigma_b\varsigma_a) + \mathfrak{v}_w(\varepsilon_e\varsigma_d - \delta_a\varsigma_a) + \mathfrak{v}_e^2(\delta_a\varsigma_e - \varsigma_b\varsigma_d)
\end{aligned}
\tag{5.24d}
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\begin{aligned}
\mathfrak{o}_n &= \mathfrak{v}_x\delta_b - \mathfrak{v}_y\mathfrak{d}_t + \mathfrak{o}_a\varepsilon_j + \mathfrak{o}_b\mathfrak{d}_a + \mathfrak{o}_c\mathfrak{d}_b + \mathfrak{v}_m(\varepsilon_c - \varepsilon_a\varepsilon_e) + \mathfrak{v}_n(\varepsilon_f - \varepsilon_d\varepsilon_e) + \mathfrak{v}_o(\varepsilon_i - \delta_a\varepsilon_e) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_a - \varsigma_b\varepsilon_e) + \mathfrak{v}_p(1 - \varepsilon_e^2) + \mathfrak{v}_q(\varepsilon_a\varepsilon_f - \varepsilon_d\varepsilon_c) + \mathfrak{v}_r(\varepsilon_a\varepsilon_i - \delta_a\varepsilon_c) + \mathfrak{v}_s(\varepsilon_a\varsigma_a - \varsigma_b\varepsilon_c) \\
&\quad + \mathfrak{v}_t(\varepsilon_a - \varepsilon_e\varepsilon_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_d\varsigma_a - \varsigma_b\varepsilon_f) + \mathfrak{v}_u(\varepsilon_d\varepsilon_i - \delta_a\varepsilon_f) + \mathfrak{v}_v(\varepsilon_d - \varepsilon_e\varepsilon_f) \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_e\varsigma_a - \varsigma_b) + \mathfrak{v}_w(\varepsilon_e\varepsilon_i - \delta_a) + \mathfrak{v}_e^2(\delta_a\varsigma_a - \varsigma_b\varepsilon_i) \\
\mathfrak{o}_o &= -\mathfrak{v}_x\varepsilon_m + \mathfrak{v}_z\varepsilon_l + \mathfrak{o}_b\mathfrak{d}_l + \mathfrak{o}_c\mathfrak{d}_m + \mathfrak{v}_m(\varepsilon_a\varepsilon_b - \Omega^2\varepsilon_d) + \mathfrak{v}_n(\varepsilon_a r^2 - \varepsilon_b\varepsilon_d) \\
&\quad + \mathfrak{v}_o(\varepsilon_a\varepsilon_h - \varepsilon_g\varepsilon_d) + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_a\varsigma_n - \varsigma_c\varepsilon_d) + \mathfrak{v}_p(\varepsilon_a\varepsilon_f - \varepsilon_c\varepsilon_d) + \mathfrak{v}_q(\Omega^2 r^2 - \varepsilon_b^2) \\
&\quad + \mathfrak{v}_r(\Omega^2\varepsilon_h - \varepsilon_g\varepsilon_b) + \mathfrak{v}_s(\Omega^2\varsigma_n - \varsigma_c\varepsilon_b) + \mathfrak{v}_t(\Omega^2\varepsilon_f - \varepsilon_c\varepsilon_b) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_b\varsigma_n - \varsigma_c r^2) \\
&\quad + \mathfrak{v}_u(\varepsilon_b\varepsilon_h - \varepsilon_g r^2) + \mathfrak{v}_v(\varepsilon_b\varepsilon_f - \varepsilon_c r^2) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_c\varsigma_n - \varsigma_c\varepsilon_f) + \mathfrak{v}_w(\varepsilon_c\varepsilon_h - \varepsilon_g\varepsilon_f) \\
&\quad + \mathfrak{v}_e^2(\varepsilon_g\varsigma_n - \varsigma_c\varepsilon_h)
\end{aligned}
\tag{5.24e}
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\begin{aligned}
\mathfrak{o}_p &= \mathfrak{v}_y\varepsilon_m + \mathfrak{v}_z\mathfrak{d}_o + \mathfrak{o}_b\mathfrak{d}_v + \mathfrak{o}_c\mathfrak{d}_z + \mathfrak{v}_m(\varepsilon_a\varepsilon_g - \Omega^2\delta_a) + \mathfrak{v}_n(\varepsilon_a\varepsilon_h - \varepsilon_b\delta_a) \\
&\quad + \mathfrak{v}_o(\varepsilon_a\Lambda^2 - \varepsilon_g\delta_a) + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_a\varsigma_d - \varsigma_c\delta_a) + \mathfrak{v}_p(\varepsilon_a\varepsilon_i - \varepsilon_c\delta_a) + \mathfrak{v}_q(\Omega^2\varepsilon_h - \varepsilon_b\varepsilon_g) \\
&\quad + \mathfrak{v}_r(\Omega^2\Lambda^2 - \varepsilon_g^2) + \mathfrak{v}_s(\Omega^2\varsigma_d - \varsigma_c\varepsilon_g) + \mathfrak{v}_t(\Omega^2\varepsilon_i - \varepsilon_c\varepsilon_g) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_b\varsigma_d - \varsigma_c\varepsilon_h) \\
&\quad + \mathfrak{v}_u(\varepsilon_b\Lambda^2 - \varepsilon_g\varepsilon_h) + \mathfrak{v}_v(\varepsilon_b\varepsilon_i - \varepsilon_c\varepsilon_h) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_c\varsigma_d - \varsigma_c\varepsilon_i) + \mathfrak{v}_w(\varepsilon_c\Lambda^2 - \varepsilon_g\varepsilon_i) \\
&\quad + \mathfrak{v}_e^2(\varepsilon_g\varsigma_d - \varsigma_c\Lambda^2)
\end{aligned}
\tag{5.24f}
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\begin{aligned}
\mathfrak{o}_q &= -\mathfrak{v}_x\mathfrak{d}_v - \mathfrak{v}_y\mathfrak{d}_l + \mathfrak{v}_z\mathfrak{d}_p + \mathfrak{o}_c\mathfrak{p}_a + \mathfrak{v}_m(\varepsilon_a\varsigma_c - \Omega^2\varsigma_b) + \mathfrak{v}_n(\varepsilon_a\varsigma_n - \varepsilon_b\varsigma_b) + \mathfrak{v}_o(\varepsilon_a\varsigma_d - \varepsilon_g\varsigma_b) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_a\varsigma_e - \varsigma_c\varsigma_b) + \mathfrak{v}_p(\varepsilon_a\varsigma_a - \varepsilon_c\varsigma_b) + \mathfrak{v}_q(\Omega^2\varsigma_n - \varepsilon_b\varsigma_c) + \mathfrak{v}_r(\Omega^2\varsigma_d - \varepsilon_g\varsigma_c) \\
&\quad + \mathfrak{v}_s(\Omega^2\varsigma_e - \varsigma_c^2) + \mathfrak{v}_t(\Omega^2\varsigma_a - \varepsilon_c\varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_b\varsigma_e - \varsigma_c\varsigma_n) + \mathfrak{v}_u(\varepsilon_b\varsigma_d - \varepsilon_g\varsigma_n) \\
&\quad + \mathfrak{v}_v(\varepsilon_b\varsigma_a - \varepsilon_c\varsigma_n) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_c\varsigma_e - \varsigma_c\varsigma_a) + \mathfrak{v}_w(\varepsilon_c\varsigma_d - \varepsilon_g\varsigma_a) + \mathfrak{v}_e^2(\varepsilon_g\varsigma_e - \varsigma_c\varsigma_d)
\end{aligned}
\tag{5.24g}
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\begin{aligned}
\mathfrak{o}_r &= -\mathfrak{v}_x\mathfrak{d}_o - \mathfrak{v}_y\varepsilon_l - \mathfrak{o}_b\mathfrak{d}_p - \mathfrak{o}_c\mathfrak{d}_q + \mathfrak{v}_m(\varepsilon_a\varepsilon_c - \Omega^2\varepsilon_e) + \mathfrak{v}_n(\varepsilon_a\varepsilon_f - \varepsilon_b\varepsilon_e) + \mathfrak{v}_o(\varepsilon_a\varepsilon_i - \varepsilon_g\varepsilon_e) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_a\varsigma_a - \varsigma_c\varepsilon_e) + \mathfrak{v}_p(\varepsilon_a - \varepsilon_c\varepsilon_e) + \mathfrak{v}_q(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c) + \mathfrak{v}_r(\Omega^2\varepsilon_i - \varepsilon_g\varepsilon_c) \\
&\quad + \mathfrak{v}_s(\Omega^2\varsigma_a - \varsigma_c\varepsilon_c) + \mathfrak{v}_t(\Omega^2 - \varepsilon_c^2) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_b\varsigma_a - \varsigma_c\varepsilon_f) + \mathfrak{v}_u(\varepsilon_b\varepsilon_i - \varepsilon_g\varepsilon_f) \\
&\quad + \mathfrak{v}_v(\varepsilon_b - \varepsilon_c\varepsilon_f) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_c\varsigma_a - \varsigma_c) + \mathfrak{v}_w(\varepsilon_c\varepsilon_i - \varepsilon_g) + \mathfrak{v}_e^2(\varepsilon_g\varsigma_a - \varsigma_c\varepsilon_i) \\
\mathfrak{o}_s &= \mathfrak{v}_x\mathfrak{d}_h - \mathfrak{v}_z\mathfrak{d}_n + \mathfrak{o}_a\mathfrak{d}_l - \mathfrak{o}_c\mathfrak{d}_j + \mathfrak{v}_m(\varepsilon_d\varsigma_c - \varepsilon_b\varsigma_b) + \mathfrak{v}_n(\varepsilon_d\varsigma_n - r^2\varsigma_b) + \mathfrak{v}_o(\varepsilon_d\varsigma_d - \varepsilon_h\varsigma_b) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_d\varsigma_e - \varsigma_n\varsigma_b) + \mathfrak{v}_p(\varepsilon_d\varsigma_a - \varepsilon_f\varsigma_b) + \mathfrak{v}_q(\varepsilon_b\varsigma_n - r^2\varsigma_c) + \mathfrak{v}_r(\varepsilon_b\varsigma_d - \varepsilon_h\varsigma_c) \\
&\quad + \mathfrak{v}_s(\varepsilon_b\varsigma_e - \varsigma_n\varsigma_c) + \mathfrak{v}_t(\varepsilon_b\varsigma_a - \varepsilon_f\varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(r^2\varsigma_e - \varsigma_n^2) + \mathfrak{v}_u(r^2\varsigma_d - \varepsilon_h\varsigma_n) \\
&\quad + \mathfrak{v}_v(r^2\varsigma_a - \varepsilon_f\varsigma_n) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_f\varsigma_e - \varsigma_n\varsigma_a) + \mathfrak{v}_w(\varepsilon_f\varsigma_d - \varepsilon_h\varsigma_a) + \mathfrak{v}_e^2(\varepsilon_h\varsigma_e - \varsigma_n\varsigma_d)
\end{aligned}
\tag{5.24h}
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\begin{aligned}
\mathfrak{o}_t &= -\mathfrak{v}_z\varepsilon_o - \mathfrak{o}_a\varepsilon_m - \mathfrak{o}_b\mathfrak{d}_h - \mathfrak{o}_c\mathfrak{d}_i + \mathfrak{v}_m(\varepsilon_d\varepsilon_g - \varepsilon_b\delta_a) + \mathfrak{v}_n(\varepsilon_d\varepsilon_h - r^2\delta_a) \\
&\quad + \mathfrak{v}_o(\varepsilon_d\Lambda^2 - \varepsilon_h\delta_a) + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_d\varsigma_d - \varsigma_n\delta_a) + \mathfrak{v}_p(\varepsilon_d\varepsilon_i - \varepsilon_f\delta_a) + \mathfrak{v}_q(\varepsilon_b\varepsilon_h - r^2\varepsilon_g) \\
&\quad + \mathfrak{v}_r(\varepsilon_b\Lambda^2 - \varepsilon_h\varepsilon_g) + \mathfrak{v}_s(\varepsilon_b\varsigma_d - \varsigma_n\varepsilon_g) + \mathfrak{v}_t(\varepsilon_b\varepsilon_i - \varepsilon_f\varepsilon_g) - \mathfrak{v}_e\mathfrak{v}_d(r^2\varsigma_d - \varsigma_n\varepsilon_h) \\
&\quad + \mathfrak{v}_u(r^2\Lambda^2 - \varepsilon_h^2) + \mathfrak{v}_v(r^2\varepsilon_i - \varepsilon_f\varepsilon_h) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_f\varsigma_d - \varsigma_n\varepsilon_i) + \mathfrak{v}_w(\varepsilon_f\Lambda^2 - \varepsilon_h\varepsilon_i) \\
&\quad + \mathfrak{v}_e^2(\varepsilon_h\varsigma_d - \varsigma_n\Lambda^2) \qquad (5.24i) \\
\mathfrak{o}_u &= \mathfrak{v}_x\varepsilon_o + \mathfrak{o}_a\varepsilon_l + \mathfrak{o}_b\mathfrak{d}_n + \mathfrak{o}_c\mathfrak{p}_b + \mathfrak{v}_m(\varepsilon_d\varepsilon_c - \varepsilon_b\varepsilon_e) + \mathfrak{v}_n(\varepsilon_d\varepsilon_f - r^2\varepsilon_e) + \mathfrak{v}_o(\varepsilon_d\varepsilon_i - \varepsilon_h\varepsilon_e) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_d\varsigma_a - \varsigma_n\varepsilon_e) + \mathfrak{v}_p(\varepsilon_d - \varepsilon_f\varepsilon_e) + \mathfrak{v}_q(\varepsilon_b\varepsilon_f - r^2\varepsilon_c) + \mathfrak{v}_r(\varepsilon_b\varepsilon_i - \varepsilon_h\varepsilon_c) \\
&\quad + \mathfrak{v}_s(\varepsilon_b\varsigma_a - \varsigma_n\varepsilon_c) + \mathfrak{v}_t(\varepsilon_b - \varepsilon_f\varepsilon_c) - \mathfrak{v}_e\mathfrak{v}_d(r^2\varsigma_a - \varsigma_n\varepsilon_f) + \mathfrak{v}_u(r^2\varepsilon_i - \varepsilon_h\varepsilon_f) \\
&\quad + \mathfrak{v}_v(r^2 - \varepsilon_f^2) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_f\varsigma_a - \varsigma_n) + \mathfrak{v}_w(\varepsilon_f\varepsilon_i - \varepsilon_h) + \mathfrak{v}_e^2(\varepsilon_h\varsigma_a - \varsigma_n\varepsilon_i)
\end{aligned}
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\begin{aligned}
\mathfrak{o}_v &= -\mathfrak{v}_x\mathfrak{d}_r + \mathfrak{v}_y\mathfrak{d}_n - \mathfrak{o}_a\mathfrak{d}_p + \mathfrak{o}_c\mathfrak{p}_c + \mathfrak{v}_m(\varepsilon_e\varsigma_c - \varepsilon_c\varsigma_b) + \mathfrak{v}_n(\varepsilon_e\varsigma_n - \varepsilon_f\varsigma_b) + \mathfrak{v}_o(\varepsilon_e\varsigma_d - \varepsilon_i\varsigma_b) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_e\varsigma_e - \varsigma_a\varsigma_b) + \mathfrak{v}_p(\varepsilon_e\varsigma_a - \varsigma_b) + \mathfrak{v}_q(\varepsilon_c\varsigma_n - \varepsilon_f\varsigma_c) + \mathfrak{v}_r(\varepsilon_c\varsigma_d - \varepsilon_i\varsigma_c) \\
&\quad + \mathfrak{v}_s(\varepsilon_c\varsigma_e - \varsigma_a\varsigma_c) + \mathfrak{v}_t(\varepsilon_c\varsigma_a - \varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_f\varsigma_e - \varsigma_a\varsigma_n) + \mathfrak{v}_u(\varepsilon_f\varsigma_d - \varepsilon_i\varsigma_n) \\
&\quad + \mathfrak{v}_v(\varepsilon_f\varsigma_a - \varsigma_n) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_e - \varsigma_a^2) + \mathfrak{v}_w(\varsigma_d - \varepsilon_i\varsigma_a) + \mathfrak{v}_e^2(\varepsilon_i\varsigma_e - \varsigma_a\varsigma_d) \qquad (5.24j) \\
\mathfrak{o}_w &= \mathfrak{v}_y\varepsilon_o - \mathfrak{o}_a\mathfrak{d}_o + \mathfrak{o}_b\mathfrak{d}_r + \mathfrak{o}_c\mathfrak{d}_s + \mathfrak{v}_m(\varepsilon_e\varepsilon_g - \varepsilon_c\delta_a) + \mathfrak{v}_n(\varepsilon_e\varepsilon_h - \varepsilon_f\delta_a) + \mathfrak{v}_o(\varepsilon_e\Lambda^2 - \varepsilon_i\delta_a) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_e\varsigma_d - \varsigma_a\delta_a) + \mathfrak{v}_p(\varepsilon_e\varepsilon_i - \delta_a) + \mathfrak{v}_q(\varepsilon_c\varepsilon_h - \varepsilon_f\varepsilon_g) + \mathfrak{v}_r(\varepsilon_c\Lambda^2 - \varepsilon_i\varepsilon_g) \\
&\quad + \mathfrak{v}_s(\varepsilon_c\varsigma_d - \varsigma_a\varepsilon_g) + \mathfrak{v}_t(\varepsilon_c\varepsilon_i - \varepsilon_g) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_f\varsigma_d - \varsigma_a\varepsilon_h) + \mathfrak{v}_u(\varepsilon_f\Lambda^2 - \varepsilon_i\varepsilon_h) \\
&\quad + \mathfrak{v}_v(\varepsilon_f\varepsilon_i - \varepsilon_h) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_d - \varsigma_a\varepsilon_i) + \mathfrak{v}_w(\Lambda^2 - \varepsilon_i^2) + \mathfrak{v}_e^2(\varepsilon_i\varsigma_d - \varsigma_a\Lambda^2)
\end{aligned}
$$

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\begin{aligned}
\mathfrak{o}_x &= -\mathfrak{v}_y\mathfrak{d}_h + \mathfrak{v}_z\mathfrak{d}_r + \mathfrak{o}_a\mathfrak{d}_v + \mathfrak{o}_c\mathfrak{p}_d + \mathfrak{v}_m(\delta_a\varsigma_c - \varepsilon_g\varsigma_b) + \mathfrak{v}_n(\delta_a\varsigma_n - \varepsilon_h\varsigma_b) + \mathfrak{v}_o(\delta_a\varsigma_d - \Lambda^2\varsigma_b) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\delta_a\varsigma_e - \varsigma_d\varsigma_b) + \mathfrak{v}_p(\delta_a\varsigma_a - \varepsilon_i\varsigma_b) + \mathfrak{v}_q(\varepsilon_g\varsigma_n - \varepsilon_h\varsigma_c) + \mathfrak{v}_r(\varepsilon_g\varsigma_d - \Lambda^2\varsigma_c) \\
&\quad + \mathfrak{v}_s(\varepsilon_g\varsigma_e - \varsigma_d\varsigma_c) + \mathfrak{v}_t(\varepsilon_g\varsigma_a - \varepsilon_i\varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_h\varsigma_e - \varsigma_d\varsigma_n) + \mathfrak{v}_u(\varepsilon_h\varsigma_d - \Lambda^2\varsigma_n) \\
&\quad + \mathfrak{v}_v(\varepsilon_h\varsigma_a - \varepsilon_i\varsigma_n) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_i\varsigma_e - \varsigma_d\varsigma_a) + \mathfrak{v}_w(\varepsilon_i\varsigma_d - \Lambda^2\varsigma_a) + \mathfrak{v}_e^2(\Lambda^2\varsigma_e - \varsigma_d^2) \qquad (5.24k) \\
\mathfrak{o}_y &= \mathfrak{v}_x\delta_a + \mathfrak{v}_y\varepsilon_d + \mathfrak{v}_z\varepsilon_e + \mathfrak{o}_a\varepsilon_a + \mathfrak{o}_b\varsigma_b + \mathfrak{o}_c\varsigma_g + \mathfrak{v}_q\varepsilon_k - \mathfrak{v}_r\mathfrak{d}_e + \mathfrak{v}_s\mathfrak{d}_f - \mathfrak{v}_t\varepsilon_j + \mathfrak{v}_e\mathfrak{v}_d\mathfrak{d}_c \\
&\quad - \mathfrak{v}_u\varepsilon_n + \mathfrak{v}_v\mathfrak{d}_t - \mathfrak{x}_u\mathfrak{v}_e\mathfrak{d}_a + \mathfrak{v}_w\delta_b + \mathfrak{v}_e^2\mathfrak{d}_u
\end{aligned}
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$$
\begin{aligned}
\mathfrak{o}_z &= \mathfrak{v}_x\mathfrak{d}_s - \mathfrak{v}_y\mathfrak{p}_b + \mathfrak{o}_a\mathfrak{d}_q + \mathfrak{o}_b\mathfrak{p}_c + \mathfrak{v}_m(\varsigma_g\varepsilon_c - \varsigma_h\varepsilon_e) + \mathfrak{v}_n(\varsigma_g\varepsilon_f - \varsigma_o\varepsilon_e) + \mathfrak{v}_o(\varsigma_g\varepsilon_i - \varsigma_i\varepsilon_e) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_g\varsigma_a - \varsigma_j\varepsilon_e) + \mathfrak{v}_p(\varsigma_g - \varsigma_f\varepsilon_e) + \mathfrak{v}_q(\varsigma_h\varepsilon_f - \varsigma_o\varepsilon_c) + \mathfrak{v}_r(\varsigma_h\varepsilon_i - \varsigma_i\varepsilon_c) \\
&\quad + \mathfrak{v}_s(\varsigma_h\varsigma_a - \varsigma_j\varepsilon_c) + \mathfrak{v}_t(\varsigma_h - \varsigma_f\varepsilon_c) - \mathfrak{v}_e\mathfrak{v}_d(\varsigma_o\varsigma_a - \varsigma_j\varepsilon_f) + \mathfrak{v}_u(\varsigma_o\varepsilon_i - \varsigma_i\varepsilon_f) \qquad (5.24l) \\
&\quad + \mathfrak{v}_v(\varsigma_o - \varsigma_f\varepsilon_f) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_f\varsigma_a - \varsigma_j) + \mathfrak{v}_w(\varsigma_f\varepsilon_i - \varsigma_i) + \mathfrak{v}_e^2(\varsigma_i\varsigma_a - \varsigma_j\varepsilon_i)
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{f}_a &= -\mathfrak{v}_x\mathfrak{d}_i + \mathfrak{v}_z\mathfrak{p}_b - \mathfrak{o}_a\mathfrak{d}_m - \mathfrak{o}_b\mathfrak{d}_j + \mathfrak{v}_m(\varsigma_g\varepsilon_b - \varsigma_h\varepsilon_d) + \mathfrak{v}_n(\varsigma_g r^2 - \varsigma_o\varepsilon_d) + \mathfrak{v}_o(\varsigma_g\varepsilon_h - \varsigma_i\varepsilon_d) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_g\varsigma_n - \varsigma_j\varepsilon_d) + \mathfrak{v}_p(\varsigma_g\varepsilon_f - \varsigma_f\varepsilon_d) + \mathfrak{v}_q(\varsigma_h r^2 - \varsigma_o\varepsilon_b) + \mathfrak{v}_r(\varsigma_h\varepsilon_h - \varsigma_i\varepsilon_b) \\
&\quad + \mathfrak{v}_s(\varsigma_h\varsigma_n - \varsigma_j\varepsilon_b) + \mathfrak{v}_t(\varsigma_h\varepsilon_f - \varsigma_f\varepsilon_b) - \mathfrak{v}_e\mathfrak{v}_d(\varsigma_o\varsigma_n - \varsigma_j r^2) + \mathfrak{v}_u(\varsigma_o\varepsilon_h - \varsigma_i r^2) \\
&\quad + \mathfrak{v}_v(\varsigma_o\varepsilon_f - \varsigma_f r^2) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_f\varsigma_n - \varsigma_j\varepsilon_f) + \mathfrak{v}_w(\varsigma_f\varepsilon_h - \varsigma_i\varepsilon_f) + \mathfrak{v}_e^2(\varsigma_i\varsigma_n - \varsigma_j\varepsilon_h) \\
\mathfrak{f}_b &= \mathfrak{v}_x\mathfrak{p}_e + \mathfrak{v}_z\mathfrak{p}_f + \mathfrak{o}_a\mathfrak{p}_g + \mathfrak{o}_b\mathfrak{p}_h + \mathfrak{o}_c\mathfrak{p}_i + \mathfrak{v}_m(\varsigma_p\varepsilon_b - \varsigma_m\varepsilon_d) + \mathfrak{v}_n(\varsigma_p r^2 - \varsigma_q\varepsilon_d) \qquad (5.24m) \\
&\quad + \mathfrak{v}_o(\varsigma_p\varepsilon_h - \varsigma_r\varepsilon_d) + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_p\varsigma_n - \varsigma_s\varepsilon_d) + \mathfrak{v}_p(\varsigma_p\varepsilon_f - \varsigma_l\varepsilon_d) + \mathfrak{v}_q(\varsigma_m r^2 - \varsigma_q\varepsilon_b) \\
&\quad + \mathfrak{v}_r(\varsigma_m\varepsilon_h - \varsigma_r\varepsilon_b) + \mathfrak{v}_s(\varsigma_m\varsigma_n - \varsigma_s\varepsilon_b) + \mathfrak{v}_t(\varsigma_m\varepsilon_f - \varsigma_l\varepsilon_b) - \mathfrak{v}_e\mathfrak{v}_d(\varsigma_q\varsigma_n - \varsigma_s r^2) \\
&\quad + \mathfrak{v}_u(\varsigma_q\varepsilon_h - \varsigma_r r^2) + \mathfrak{v}_v(\varsigma_q\varepsilon_f - \varsigma_l r^2) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_l\varsigma_n - \varsigma_s\varepsilon_f) + \mathfrak{v}_w(\varsigma_l\varepsilon_h - \varsigma_r\varepsilon_f) \\
&\quad + \mathfrak{v}_e^2(\varsigma_r\varsigma_n - \varsigma_s\varepsilon_h)
\end{aligned}
$$

$$\begin{aligned}\mathfrak{f}_c = [&\mathfrak{v}_x\mathfrak{o}_d + \mathfrak{v}_y\mathfrak{o}_e + \mathfrak{v}_z\mathfrak{o}_f + \mathfrak{o}_a\mathfrak{o}_g + \mathfrak{o}_b\mathfrak{o}_h + \mathfrak{o}_c\mathfrak{o}_i + \mathfrak{v}_m\mathfrak{o}_j + \mathfrak{v}_n\mathfrak{o}_k + \mathfrak{v}_o\mathfrak{o}_l + \mathfrak{v}_b\mathfrak{v}_e\mathfrak{o}_m + \mathfrak{v}_p\mathfrak{o}_n \\ &+ \mathfrak{v}_q\mathfrak{o}_o + \mathfrak{v}_r\mathfrak{o}_p + \mathfrak{v}_s\mathfrak{o}_q + \mathfrak{v}_t\mathfrak{o}_r - \mathfrak{v}_e\mathfrak{v}_d\mathfrak{o}_s + \mathfrak{v}_u\mathfrak{o}_t + \mathfrak{v}_v\mathfrak{o}_u - \mathfrak{x}_u\mathfrak{v}_e\mathfrak{o}_v + \mathfrak{v}_w\mathfrak{o}_w + \mathfrak{v}_e^2\mathfrak{o}_x]^{1/2}\end{aligned} \tag{5.24n}$$

$$\begin{aligned}\mathfrak{f}_d = &\,\mathfrak{v}_b^2 + 2\mathfrak{v}_c\mathfrak{v}_b\varepsilon_a - 2\mathfrak{v}_d\mathfrak{v}_b\varepsilon_d + 2\mathfrak{v}_e\mathfrak{v}_b\delta_a - 2\mathfrak{x}_u\mathfrak{v}_b\varepsilon_e + 2\mathfrak{v}_a\mathfrak{v}_b\varepsilon_n + 2\rho\mathfrak{v}_b\mathfrak{d}_c - 2\rho\Omega^2\mathfrak{v}_b\varepsilon_k \\ &+ 2\mathfrak{x}_o\mathfrak{v}_b\varepsilon_j + 2\varphi_g\varphi_h\mathfrak{v}_b\delta_b + \mathfrak{v}_c^2\Omega^2 - 2\mathfrak{v}_d\mathfrak{v}_c\varepsilon_b + 2\mathfrak{v}_e\mathfrak{v}_c\varepsilon_g - 2\mathfrak{x}_u\mathfrak{v}_c\varepsilon_c + 2\mathfrak{v}_a\mathfrak{v}_c\varepsilon_m \\ &- 2\rho\mathfrak{v}_c\mathfrak{d}_l - 2\varphi_g\varphi_h\mathfrak{v}_c\mathfrak{d}_o + \mathfrak{v}_d^2 r^2 - 2\mathfrak{v}_e\mathfrak{v}_d\varepsilon_h + 2\mathfrak{x}_u\mathfrak{v}_d\varepsilon_f - 2\mathfrak{x}_o\mathfrak{v}_d\varepsilon_l - 2\mathfrak{v}_d\varphi_g\varphi_h\varepsilon_o \\ &+ \mathfrak{v}_e^2\Lambda^2 - 2\mathfrak{x}_u\mathfrak{v}_e\varepsilon_i - 2\rho\mathfrak{v}_e\mathfrak{d}_h + 2\rho\Omega^2\mathfrak{v}_e\varepsilon_m + 2\mathfrak{x}_o\mathfrak{v}_e\mathfrak{d}_o + \mathfrak{x}_u^2 \\ &- 2\mathfrak{v}_a\mathfrak{x}_u\varepsilon_o - 2\rho\mathfrak{x}_u\mathfrak{d}_n + 2\rho\Omega^2\mathfrak{x}_u\varepsilon_l \\ \mathfrak{f}_e = [&\mathfrak{f}_d + \mathfrak{v}_a^2(\Lambda^2r^2 - \varepsilon_h^2) + 2\rho\mathfrak{v}_a(\varsigma_d r^2 - \varepsilon_h\varsigma_n) - 2\rho\Omega^2\mathfrak{v}_a(\varepsilon_g r^2 - \varepsilon_h\varepsilon_b) \\ &+ 2\mathfrak{x}_o\mathfrak{v}_a(\varepsilon_i\varepsilon_b - \varepsilon_g\varepsilon_f) + 2\varphi_g\varphi_h\mathfrak{v}_a(\varepsilon_i\varepsilon_h - \Lambda^2\varepsilon_f) - 2\rho^2\Omega^2(\varsigma_c r^2 - \varepsilon_b\varsigma_n) \\ &+ \rho^2\Omega^4(\Omega^2r^2 - \varepsilon_b^2) - 2\mathfrak{x}_o\rho\Omega^2(\varepsilon_c\varepsilon_b - \Omega^2\varepsilon_f) - 2\varphi_g\varphi_h\rho\Omega^2(\varepsilon_c\varepsilon_h - \varepsilon_g\varepsilon_f) \\ &+ \rho^2(\varsigma_e r^2 - \varsigma_n^2) + 2\mathfrak{x}_o\rho(\varsigma_a\varepsilon_b - \varsigma_c\varepsilon_f) + 2\varphi_g\varphi_h\rho(\varsigma_a\varepsilon_h - \varsigma_d\varepsilon_f) \\ &+ \mathfrak{x}_o^2(\Omega^2 - \varepsilon_c^2) + 2\varphi_g\varphi_h\mathfrak{x}_o(\varepsilon_g - \varepsilon_i\varepsilon_c) + \varphi_g^2\varphi_h^2(\Lambda^2 - \varepsilon_i^2)]^{1/2}\end{aligned} \tag{5.24o}$$

$$\begin{aligned}\mathfrak{f}_f = &\,\mathfrak{v}_g\mathfrak{o}_y + \mathfrak{x}_k(\varepsilon_b\mathfrak{o}_g - \Omega^2\mathfrak{o}_e - \mathfrak{o}_t) + 3\mathfrak{x}_j(2\varepsilon_h\mathfrak{o}_g - 3\varepsilon_g\mathfrak{o}_e - \mathfrak{o}_s - \Omega^2\mathfrak{o}_o + \varepsilon_b\mathfrak{o}_d) \\ &+ \mathfrak{v}_i(3\varepsilon_h\mathfrak{o}_d + \varepsilon_b\mathfrak{o}_h + \varrho_t\mathfrak{o}_g + \varrho_u\mathfrak{o}_e - 2\varepsilon_b\mathfrak{o}_p + 3\Omega^2\mathfrak{o}_t - 3\varepsilon_g\mathfrak{o}_o + \mathfrak{f}_a) \\ &- \mathfrak{x}_q\mathfrak{o}_r + 3\mathfrak{x}_p\mathfrak{o}_w + 3\mathfrak{x}_o\mathfrak{o}_v - \varphi_g\varphi_h\mathfrak{o}_z + \mathfrak{x}_t\mathfrak{o}_g + 3\mathfrak{x}_s\mathfrak{o}_d + 3\mathfrak{x}_r\mathfrak{o}_h + \varphi_i\mathfrak{o}_i - \mathfrak{x}_w\mathfrak{o}_f \\ &+ \rho[(\varrho_t + 3\varsigma_n)\mathfrak{o}_d + 4\varepsilon_h\mathfrak{o}_h + \varepsilon_b\mathfrak{o}_i + \varrho_v\mathfrak{o}_g + 5\varrho_w\mathfrak{o}_e - 5\varepsilon_b\mathfrak{o}_q + 6\Omega^2\mathfrak{o}_s - 5\varepsilon_h\mathfrak{o}_p \\ &\quad + 12\varepsilon_g\mathfrak{o}_t + \varrho_u\mathfrak{o}_o + \mathfrak{f}_b].\end{aligned} \tag{5.24p}$$

Art 22a. *Development of equation* (1.4).

Bearing the foregoing quantities in mind, we derive

$$\begin{aligned}\dot{\xi} &= [2(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) - (\mathbf{\Omega}\cdot\mathbf{\Omega})(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})]' \text{ by (1.4c)} \\ &= 2(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) + 2(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\boldsymbol{\kappa}) - 2(\mathbf{\Omega}\cdot\mathbf{\Lambda})(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}}) \\ &= 2\kappa(\varepsilon_i\varepsilon_a + \varepsilon_c\delta_a - \varepsilon_g\varepsilon_e) \text{ by (5.1a) \& (5.13a)} \\ &= 2\kappa\varrho_a \text{ by (5.23d)}\end{aligned} \tag{5.25a}$$

$$\begin{aligned}\ddot{\xi} &= [2(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) + 2(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\boldsymbol{\kappa}) - 2(\mathbf{\Omega}\cdot\mathbf{\Lambda})(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})]' \text{ by (5.25a)} \\ &= 2(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) + 2(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\boldsymbol{\kappa}) + 2(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\boldsymbol{\kappa}) + 2(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\boldsymbol{\kappa}) \\ &\quad - 2(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(\mathbf{\Lambda}\cdot\mathbf{\Lambda}) + (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})] \\ &= 2(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) + 4(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\boldsymbol{\kappa}) + 2(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\boldsymbol{\kappa}) - 2(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(\mathbf{\Lambda}\cdot\mathbf{\Lambda}) + (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})] \\ &= 2\kappa[\varsigma_a\varepsilon_a + 2\varepsilon_i\delta_a + \varepsilon_c\varsigma_b - \varepsilon_e(\Lambda^2 + \varsigma_c)] \text{ by (5.1a), (5.13a) \& (5.23a)} \\ &= 2\kappa\varrho_b \text{ by (5.23d)}\end{aligned} \tag{5.25b}$$

$$\begin{aligned}\dddot{\xi} &= [2(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) + 4(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\boldsymbol{\kappa}) + 2(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\boldsymbol{\kappa}) \\ &\qquad - 2(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\mathbf{\Lambda}) - 2(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})]' \text{ by (5.25b)} \\ &= 2(\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) + 2(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\boldsymbol{\kappa}) + 4(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\boldsymbol{\kappa}) + 4(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\boldsymbol{\kappa}) \\ &\quad + 2(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\boldsymbol{\kappa}) + 2(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\ddot{\mathbf{\Lambda}}\cdot\boldsymbol{\kappa}) - 4(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - 2(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) + (\mathbf{\Omega}\cdot\ddot{\mathbf{\Lambda}})] \\ &= 2(\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\boldsymbol{\kappa}) + 6(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\boldsymbol{\kappa}) + 6(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\boldsymbol{\kappa}) \\ &\quad + 2(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\ddot{\mathbf{\Lambda}}\cdot\boldsymbol{\kappa}) - 6(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - 2(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\mathbf{\Omega}\cdot\ddot{\mathbf{\Lambda}}) \\ &= 2\kappa[\varsigma_f\varepsilon_a + 3\varsigma_a\delta_a + 3\varepsilon_i\varsigma_b + \varepsilon_c\varsigma_g - \varepsilon_e(3\varsigma_d + \varsigma_h)] \text{ by (5.1a), (5.13a) \& (5.23a)} \\ &= 2\kappa(\varsigma_f\varepsilon_a + 3\varsigma_a\delta_a + 3\varepsilon_i\varsigma_b + \varepsilon_c\varsigma_g - \varrho_c) = 2\kappa\varrho_d \text{ by (5.23d)}\end{aligned} \tag{5.25c}$$

$$\begin{aligned}\dot{\zeta} &= [(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}) - (\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\kappa})]' \text{ by } (1.4\text{c})\\ &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(\boldsymbol{\Lambda}\cdot\boldsymbol{\Lambda}) + (\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})] - (\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\kappa}) - (\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\kappa})\\ &= \kappa[\varepsilon_e(\Lambda^2+\varsigma_c) - \varsigma_a\varepsilon_a - \varepsilon_i\delta_a] \text{ by } (5.1\text{a}), (5.13\text{a}) \ \& \ (5.23\text{a})\\ &= \kappa\varrho_e \text{ by } (5.23\text{d}) \end{aligned} \tag{5.26a}$$

$$\begin{aligned}\ddot{\zeta} &= [(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\Lambda}) + (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}}) - (\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\kappa}) - (\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\kappa})]' \text{ by } (5.26\text{a})\\ &= 2(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) + (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) + (\boldsymbol{\Omega}\cdot\ddot{\boldsymbol{\Lambda}})] - (\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\kappa})\\ &\quad - (\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\kappa}) - (\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\kappa}) - (\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\kappa})\\ &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[3(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) + (\boldsymbol{\Omega}\cdot\ddot{\boldsymbol{\Lambda}})] - (\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\kappa}) - 2(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\kappa}) - (\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\kappa})\\ &= \kappa[\varepsilon_e(3\varsigma_d+\varsigma_h) - \varsigma_f\varepsilon_a - 2\varsigma_a\delta_a - \varepsilon_i\varsigma_b] \text{ by } (5.1\text{a}), (5.13\text{a}) \ \& \ (5.23\text{a})\\ &= \kappa(\varrho_c - \varsigma_f\varepsilon_a - 2\varsigma_a\delta_a - \varepsilon_i\varsigma_b) = \kappa\varrho_f \text{ by } (5.23\text{d}) \end{aligned} \tag{5.26b}$$

$$\begin{aligned}\dddot{\zeta} &= [3(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) + (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\ddot{\boldsymbol{\Lambda}}) - (\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\kappa})\\ &\qquad - 2(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\kappa}) - (\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\kappa})]' \text{ by } (5.26\text{b})\\ &= 3(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(\dot{\boldsymbol{\Lambda}}\cdot\dot{\boldsymbol{\Lambda}}) + (\boldsymbol{\Lambda}\cdot\ddot{\boldsymbol{\Lambda}})] + (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(\boldsymbol{\Lambda}\cdot\ddot{\boldsymbol{\Lambda}}) + (\boldsymbol{\Omega}\cdot\dddot{\boldsymbol{\Lambda}})] - (\dddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\kappa}) - (\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\kappa})\\ &\quad - 2(\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\kappa}) - 2(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\kappa}) - (\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\kappa}) - (\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})(\ddot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\kappa})\\[1em] &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[3(\dot{\boldsymbol{\Lambda}}\cdot\dot{\boldsymbol{\Lambda}}) + 4(\boldsymbol{\Lambda}\cdot\ddot{\boldsymbol{\Lambda}}) + (\boldsymbol{\Omega}\cdot\dddot{\boldsymbol{\Lambda}})] - (\dddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\kappa})\\ &\quad - 3(\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\kappa}) - 3(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\kappa}) - (\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})(\ddot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\kappa})\\ &= \kappa[\varepsilon_e(3\varsigma_e + 4\varsigma_i + \varsigma_m) - \varsigma_l\varepsilon_a - 3\varsigma_f\delta_a - 3\varsigma_a\varsigma_b - \varepsilon_i\varsigma_g] \text{ by } (5.1\text{a}), (5.13\text{a}) \ \& \ (5.23\text{a})\\ &= \kappa\varrho_g \text{ by } (5.23\text{d}). \end{aligned} \tag{5.26c}$$

From (1.4c), (5.13a) and (5.23b), we have

$$\dot{\eta} = \boldsymbol{\kappa}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda}) = \kappa\delta_b, \quad \ddot{\eta} = \boldsymbol{\kappa}\cdot(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}}) = \kappa\mathfrak{d}_a, \quad \dddot{\eta} = \boldsymbol{\kappa}\cdot(\widehat{\mathbf{p}}\times\ddot{\boldsymbol{\Lambda}}) = \kappa\mathfrak{d}_b \tag{5.27}$$

whilst from (1.4d), we derive

$$\begin{aligned}\dot{s_0} &= \left[\zeta - \frac{\xi^2}{\eta}\right]' = \dot{\zeta} - \frac{2\xi\dot{\xi}}{\eta} + \frac{\xi^2\dot{\eta}}{\eta^2} = \dot{\zeta} + \frac{\xi}{\eta}\left[-2\dot{\xi} + \frac{\xi\dot{\eta}}{\eta}\right]\\ &= \kappa\varrho_e + \frac{2\kappa\varphi_e}{\kappa\varepsilon_j}\left[-4\kappa\varrho_a + \frac{2\kappa^2\varphi_e\delta_b}{\kappa\varepsilon_j}\right] \text{ by } (5.4), (5.25\text{a}), (5.26\text{a}) \ \& \ (5.27)\\ &= \kappa\left[\varrho_e - \frac{4\varphi_e}{\varepsilon_j}\left\{2\varrho_a - \frac{\delta_b\varphi_e}{\varepsilon_j}\right\}\right] = \kappa\left\{\varrho_e - \frac{4\varrho_h\varphi_e}{\varepsilon_j}\right\} = \kappa\varrho_i \text{ by } (5.23\text{d}) \end{aligned} \tag{5.28a}$$

$$\begin{aligned}\ddot{s_0} &= \left[\dot{\zeta} - \frac{2\xi\dot{\xi}}{\eta} + \frac{\xi^2\dot{\eta}}{\eta^2}\right]' \text{ by } (5.28\text{a})\\ &= \ddot{\zeta} - 2\left[\frac{\dot{\xi}^2}{\eta} + \frac{\xi\ddot{\xi}}{\eta} - \frac{\xi\dot{\xi}\dot{\eta}}{\eta^2}\right] + \frac{2\xi\dot{\xi}\dot{\eta}}{\eta^2} + \frac{\xi^2\ddot{\eta}}{\eta^2} - \frac{2\xi^2\dot{\eta}^2}{\eta^3}\\ &= \ddot{\zeta} - \frac{2\dot{\xi}^2}{\eta} - \frac{2\xi\ddot{\xi}}{\eta} + \frac{\xi^2\ddot{\eta}}{\eta^2} + \frac{4\xi\dot{\xi}\dot{\eta}}{\eta^2} - \frac{2\xi^2\dot{\eta}^2}{\eta^3} = \ddot{\zeta} + \frac{1}{\eta}\left[-2\dot{\xi}^2 - 2\xi\ddot{\xi} + \frac{\xi}{\eta}\left\{\xi\ddot{\eta} + 4\dot{\xi}\dot{\eta} - \frac{2\xi\dot{\eta}^2}{\eta}\right\}\right] \end{aligned}$$

$$= \kappa\varrho_f + \frac{1}{\kappa\varepsilon_j}\left[-8\kappa^2\varrho_a^2 - 8\kappa^2\varphi_e\varrho_b + \frac{2\kappa\varphi_e}{\kappa\varepsilon_j}\left\{2\kappa^2\varphi_e\mathfrak{d}_a + 8\kappa^2\varrho_a\delta_b - \frac{4\kappa^3\varphi_e\delta_b^2}{\kappa\varepsilon_j}\right\}\right]$$

by (5.4), (5.25), (5.26b) & (5.27)

$$= \kappa\varrho_f - \frac{8\kappa}{\varepsilon_j}\left[\varrho_a^2 + \varphi_e\varrho_b - \frac{\varphi_e}{2\varepsilon_j}\left\{\varphi_e\mathfrak{d}_a + 2\delta_b\left(2\varrho_a - \frac{\varphi_e\delta_b}{\varepsilon_j}\right)\right\}\right]$$

$$= \kappa\varrho_f - \frac{8\kappa}{\varepsilon_j}\left[\varrho_j - \frac{\varphi_e}{2\varepsilon_j}\left\{\varrho_k + \delta_b\varrho_h\right\}\right] = \kappa\varrho_n \text{ by (5.23d) \& (5.23e)} \tag{5.28b}$$

$$\dddot{s}_0 = \left[\ddot{\zeta} - \frac{2\dot{\xi}^2}{\eta} - \frac{2\xi\ddot{\xi}}{\eta} + \frac{4\xi\dot{\xi}\dot{\eta}}{\eta^2} + \frac{\xi^2\ddot{\eta}}{\eta^2} - \frac{2\xi^2\dot{\eta}^2}{\eta^3}\right]' \text{ by (5.28b)}$$

$$= \dddot{\zeta} - 2\left[\frac{2\dot{\xi}\ddot{\xi}}{\eta} - \frac{\dot{\xi}^2\dot{\eta}}{\eta^2}\right] - 2\left[\frac{\dot{\xi}\ddot{\xi}}{\eta} + \frac{\xi\dddot{\xi}}{\eta} - \frac{\xi\ddot{\xi}\dot{\eta}}{\eta^2}\right] + 4\left[\frac{\dot{\xi}^2\dot{\eta}}{\eta^2} + \frac{\xi\ddot{\xi}\dot{\eta}}{\eta^2} + \frac{\xi\dot{\xi}\ddot{\eta}}{\eta^2} - \frac{2\xi\dot{\xi}\dot{\eta}^2}{\eta^3}\right]$$

$$+ \left[\frac{2\xi\dot{\xi}\ddot{\eta}}{\eta^2} + \frac{\xi^2\dddot{\eta}}{\eta^2} - \frac{2\xi^2\ddot{\eta}\dot{\eta}}{\eta^3}\right] - 2\left[\frac{2\xi\dot{\xi}\dot{\eta}^2}{\eta^3} + \frac{2\xi^2\dot{\eta}\ddot{\eta}}{\eta^3} - \frac{3\xi^2\dot{\eta}^3}{\eta^4}\right]$$

$$= \dddot{\zeta} - \frac{2\xi\dddot{\xi}}{\eta} - \frac{6\dot{\xi}\ddot{\xi}}{\eta} + \frac{\xi^2\dddot{\eta}}{\eta^2} + \frac{6\dot{\xi}^2\dot{\eta}}{\eta^2} + \frac{6\xi\ddot{\xi}\dot{\eta}}{\eta^2} + \frac{6\xi\dot{\xi}\ddot{\eta}}{\eta^2} - \frac{12\xi\dot{\xi}\dot{\eta}^2}{\eta^3} - \frac{6\xi^2\dot{\eta}\ddot{\eta}}{\eta^3} + \frac{6\xi^2\dot{\eta}^3}{\eta^4}$$

$$= \dddot{\zeta} - \frac{1}{\eta}\left[2\xi\dddot{\xi} + 6\dot{\xi}\ddot{\xi} - \frac{1}{\eta}\left\{\xi^2\dddot{\eta} + 6\dot{\xi}^2\dot{\eta} + 6\xi\ddot{\xi}\dot{\eta} + 6\xi\dot{\xi}\ddot{\eta}\right\} + \frac{\xi}{\eta^2}\left\{12\dot{\xi}\dot{\eta}^2 + 6\xi\dot{\eta}\ddot{\eta} - \frac{6\xi\dot{\eta}^3}{\eta}\right\}\right]$$

$$= \kappa\varrho_g - \frac{1}{\kappa\varepsilon_j}\left[8\kappa^2\varphi_e\varrho_d + 24\kappa^2\varrho_a\varrho_b - \frac{1}{\kappa\varepsilon_j}\left\{4\kappa^3\varphi_e^2\mathfrak{d}_b + 24\kappa^3\varrho_a^2\delta_b + 24\kappa^3\varphi_e\varrho_b\delta_b + 24\kappa^3\varphi_e\varrho_a\mathfrak{d}_a\right\}\right.$$

$$\left. + \frac{2\kappa\varphi_e}{\kappa^2\varepsilon_j^2}\left\{24\kappa^3\varrho_a\delta_b^2 + 12\kappa^3\varphi_e\delta_b\mathfrak{d}_a - \frac{12\kappa^4\varphi_e\delta_b^3}{\kappa\varepsilon_j}\right\}\right] \text{ by (5.4), (5.25), (5.26c) \& (5.27)}$$

$$= \kappa\varrho_g - \frac{8\kappa}{\varepsilon_j}\left[\varphi_e\varrho_d + 3\varrho_a\varrho_b - \frac{1}{2\varepsilon_j}\left\{\varphi_e^2\mathfrak{d}_b + 6\delta_b(\varrho_a^2 + \varphi_e\varrho_b) + 6\varphi_e\varrho_a\mathfrak{d}_a\right\}\right.$$

$$\left. + \frac{3\varphi_e\delta_b}{\varepsilon_j^2}\left\{\varphi_e\mathfrak{d}_a + \delta_b\left(2\varrho_a - \frac{\varphi_e\delta_b}{\varepsilon_j}\right)\right\}\right]$$

$$= \kappa\varrho_g - \frac{8\kappa}{\varepsilon_j}\left[\varphi_e\varrho_d + 3\varrho_a\varrho_b - \frac{1}{2\varepsilon_j}\left\{\varphi_e^2\mathfrak{d}_b + 6\delta_b\varrho_j + 6\varphi_e\varrho_a\mathfrak{d}_a\right\} + \frac{3\varrho_k\varphi_e\delta_b}{\varepsilon_j^2}\right] \text{ by (5.23d)}$$

$$= \kappa\varrho_o \text{ by (5.23e).} \tag{5.28c}$$

Art 22b. *Derivatives of* **a** *and* γ.

From (1.4a), we obtain[8]

$$\dot{\gamma} = \left[\left|1 + \frac{s_0}{\eta\omega_o^2}\right|^{1/2}\right]' = \pm\frac{1}{2}\left|1 + \frac{s_0}{\eta\omega_o^2}\right|^{-1/2}\left[\frac{s_0}{\eta\omega_o^2}\right]' = \pm\frac{1}{2\gamma}\left[\frac{s_0}{\eta\omega_o^2}\right]'$$

$$= \pm\frac{1}{2\gamma\omega_o^2}\left[\frac{\dot{s}_0}{\eta} - \frac{s_0\dot{\eta}}{\eta^2}\right] = \pm\frac{1}{2\gamma\eta\omega_o^2}\left[\dot{s}_0 - \frac{s_0\dot{\eta}}{\eta}\right]$$

$$= \pm\frac{1}{2\gamma\kappa\varepsilon_j\omega_o^2}\left[\kappa\varrho_i - \frac{\kappa^2\varphi_g\delta_b}{\kappa\varepsilon_j}\right] \text{ by (5.4), (5.5a), (5.28a) \& (5.27)}$$

$$= \pm\frac{1}{2\gamma\varepsilon_j\omega_o^2}\left[\varrho_i - \frac{\varphi_g\delta_b}{\varepsilon_j}\right] = \pm(\varrho_m/\varrho_l) = \varrho_p \text{ by (5.23e)} \tag{5.29a}$$

[8]In the expressions for the derivatives of γ, we are to take the positive sign when $s_0 > -\eta\omega_o^2$ and the negative sign when $s_0 < -\eta\omega_o^2$. The case $s_0 = -\eta\omega_o^2$ is forbidden since γ is nonzero.

$$\begin{aligned}
\ddot{\gamma} &= \pm\frac{1}{2\omega_o^2}\left[\frac{\dot{s_0}}{\gamma\eta} - \frac{s_0\dot{\eta}}{\gamma\eta^2}\right]' \text{ by (5.29a)}\\
&= \pm\frac{1}{2\omega_o^2}\left[\frac{\ddot{s_0}}{\gamma\eta} - \frac{\dot{s_0}\dot{\gamma}}{\gamma^2\eta} - \frac{\dot{s_0}\dot{\eta}}{\gamma\eta^2} - \frac{\dot{s_0}\dot{\eta}}{\gamma\eta^2} - \frac{s_0\ddot{\eta}}{\gamma\eta^2} + \frac{s_0\dot{\eta}\dot{\gamma}}{\gamma^2\eta^2} + \frac{2s_0\dot{\eta}^2}{\gamma\eta^3}\right]\\
&= \pm\frac{1}{2\gamma\eta\omega_o^2}\left[\ddot{s_0} - \dot{s_0}\left\{\frac{\dot{\gamma}}{\gamma} + \frac{2\dot{\eta}}{\eta}\right\} - \frac{s_0}{\eta}\left\{\ddot{\eta} - \frac{\dot{\eta}\dot{\gamma}}{\gamma} - \frac{2\dot{\eta}^2}{\eta}\right\}\right]\\
&= \pm\frac{1}{2\gamma\kappa\varepsilon_j\omega_o^2}\left[\kappa\varrho_n - \kappa\varrho_i\left\{\frac{\varrho_p}{\gamma} + \frac{2\kappa\delta_b}{\kappa\varepsilon_j}\right\} - \frac{\kappa\varphi_g}{\kappa\varepsilon_j}\left\{\kappa\mathfrak{d}_a - \frac{\kappa\delta_b\varrho_p}{\gamma} - \frac{2\kappa^2\delta_b^2}{\kappa\varepsilon_j}\right\}\right]\\
&\qquad \text{by (5.4), (5.5a), (5.28), (5.29a) \& (5.27)}
\end{aligned}$$

$$\begin{aligned}
&= \pm\frac{1}{2\gamma\varepsilon_j\omega_o^2}\left[\varrho_n - \frac{\varphi_g\mathfrak{d}_a}{\varepsilon_j} - \varrho_i\left\{\frac{\varrho_p}{\gamma} + \frac{2\delta_b}{\varepsilon_j}\right\} + \frac{\delta_b\varphi_g}{\varepsilon_j}\left\{\frac{\varrho_p}{\gamma} + \frac{2\delta_b}{\varepsilon_j}\right\}\right]\\
&= \pm\frac{1}{\varrho_l}\left[\varrho_n - \frac{\varphi_g\mathfrak{d}_a}{\varepsilon_j} - \left\{\varrho_i - \frac{\delta_b\varphi_g}{\varepsilon_j}\right\}\left\{\frac{\varrho_p}{\gamma} + \frac{2\delta_b}{\varepsilon_j}\right\}\right]\\
&= \frac{1}{\varrho_l}\left\{\varrho_n - \varrho_m\varrho_q - \frac{\varphi_g\mathfrak{d}_a}{\varepsilon_j}\right\} = \varrho_r \text{ by (5.23e)} \qquad (5.29b)
\end{aligned}$$

$$\begin{aligned}
\dddot{\gamma} &= \pm\frac{1}{2\omega_o^2}\left[\frac{\ddot{s_0}}{\gamma\eta} - \frac{\dot{s_0}\dot{\gamma}}{\gamma^2\eta} - \frac{2\dot{s_0}\dot{\eta}}{\gamma\eta^2} - \frac{s_0\ddot{\eta}}{\gamma\eta^2} + \frac{s_0\dot{\eta}\dot{\gamma}}{\gamma^2\eta^2} + \frac{2s_0\dot{\eta}^2}{\gamma\eta^3}\right]' \text{ by (5.29b)}\\
&= \pm\frac{1}{2\omega_o^2}\left[\left\{\frac{\dddot{s_0}}{\gamma\eta} - \frac{\ddot{s_0}\dot{\gamma}}{\gamma^2\eta} - \frac{\ddot{s_0}\dot{\eta}}{\gamma\eta^2}\right\} - \left\{\frac{\ddot{s_0}\dot{\gamma}}{\gamma^2\eta} + \frac{\dot{s_0}\ddot{\gamma}}{\gamma^2\eta} - \frac{2\dot{s_0}\dot{\gamma}^2}{\gamma^3\eta} - \frac{\dot{s_0}\dot{\gamma}\dot{\eta}}{\gamma^2\eta^2}\right\}\right.\\
&\quad - 2\left\{\frac{\ddot{s_0}\dot{\eta}}{\gamma\eta^2} + \frac{\dot{s_0}\ddot{\eta}}{\gamma\eta^2} - \frac{\dot{s_0}\dot{\eta}\dot{\gamma}}{\gamma^2\eta^2} - \frac{2\dot{s_0}\dot{\eta}^2}{\gamma\eta^3}\right\} - \left\{\frac{\dot{s_0}\ddot{\eta}}{\gamma\eta^2} + \frac{s_0\dddot{\eta}}{\gamma\eta^2} - \frac{s_0\ddot{\eta}\dot{\gamma}}{\gamma^2\eta^2} - \frac{2s_0\ddot{\eta}\dot{\eta}}{\gamma\eta^3}\right\}\\
&\quad \left. + \left\{\frac{\dot{s_0}\dot{\eta}\dot{\gamma}}{\gamma^2\eta^2} + \frac{s_0\ddot{\eta}\dot{\gamma}}{\gamma^2\eta^2} + \frac{s_0\dot{\eta}\ddot{\gamma}}{\gamma^2\eta^2} - \frac{2s_0\dot{\eta}\dot{\gamma}^2}{\gamma^3\eta^2} - \frac{2s_0\dot{\eta}^2\dot{\gamma}}{\gamma^2\eta^3}\right\} + 2\left\{\frac{\dot{s_0}\dot{\eta}^2}{\gamma\eta^3} + \frac{2s_0\dot{\eta}\ddot{\eta}}{\gamma\eta^3} - \frac{s_0\dot{\eta}^2\dot{\gamma}}{\gamma^2\eta^3} - \frac{3s_0\dot{\eta}^3}{\gamma\eta^4}\right\}\right]
\end{aligned}$$

$$\begin{aligned}
&= \pm\frac{1}{2\gamma\eta\omega_o^2}\left[\dddot{s_0} - \frac{s_0\dddot{\eta}}{\eta} - \frac{3\ddot{s_0}\dot{\eta}}{\eta} - \frac{3\dot{s_0}\ddot{\eta}}{\eta} + \frac{6\dot{s_0}\dot{\eta}^2}{\eta^2} + \frac{6s_0\ddot{\eta}\dot{\eta}}{\eta^2} - \frac{6s_0\dot{\eta}^3}{\eta^3}\right.\\
&\qquad \left. - \frac{\dot{s_0}\ddot{\gamma}}{\gamma} - \frac{2\ddot{s_0}\dot{\gamma}}{\gamma} + \frac{2s_0\ddot{\eta}\dot{\gamma}}{\gamma\eta} + \frac{s_0\dot{\eta}\ddot{\gamma}}{\gamma\eta} + \frac{4\dot{s_0}\dot{\gamma}\dot{\eta}}{\gamma\eta} - \frac{4s_0\dot{\eta}^2\dot{\gamma}}{\gamma\eta^2} + \frac{2\dot{s_0}\dot{\gamma}^2}{\gamma^2} - \frac{2s_0\dot{\eta}\dot{\gamma}^2}{\gamma^2\eta}\right]\\
&= \pm\frac{1}{2\gamma\eta\omega_o^2}\left[\dddot{s_0} - \frac{1}{\eta}\left\{s_0\dddot{\eta} + 3\ddot{s_0}\dot{\eta} + 3\dot{s_0}\ddot{\eta}\right\} + \frac{6\dot{\eta}}{\eta^2}\left\{\dot{s_0}\dot{\eta} + s_0\left(\ddot{\eta} - \frac{\dot{\eta}^2}{\eta}\right)\right\}\right.\\
&\qquad \left. - \frac{1}{\gamma}\left\{\dot{s_0}\ddot{\gamma} + 2\ddot{s_0}\dot{\gamma}\right\} + \frac{s_0}{\gamma\eta}\left\{2\ddot{\eta}\dot{\gamma} + \dot{\eta}\ddot{\gamma}\right\} + \frac{4\dot{\gamma}\dot{\eta}}{\gamma\eta}\left\{\dot{s_0} - \frac{s_0\dot{\eta}}{\eta}\right\} + \frac{2\dot{\gamma}^2}{\gamma^2}\left\{\dot{s_0} - \frac{s_0\dot{\eta}}{\eta}\right\}\right]
\end{aligned}$$

$$\begin{aligned}
&= \pm\frac{1}{2\gamma\eta\omega_o^2}\left[\dddot{s_0} - \frac{1}{\eta}\left\{s_0\dddot{\eta} + 3\ddot{s_0}\dot{\eta} + 3\dot{s_0}\ddot{\eta}\right\} + \frac{6\dot{\eta}}{\eta^2}\left\{\dot{s_0}\dot{\eta} + s_0\left(\ddot{\eta} - \frac{\dot{\eta}^2}{\eta}\right)\right\}\right.\\
&\qquad \left. - \frac{1}{\gamma}\left\{\dot{s_0}\ddot{\gamma} + 2\ddot{s_0}\dot{\gamma}\right\} + \frac{s_0}{\gamma\eta}\left\{2\ddot{\eta}\dot{\gamma} + \dot{\eta}\ddot{\gamma}\right\} + \frac{2\dot{\gamma}}{\gamma}\left\{\frac{2\dot{\eta}}{\eta} + \frac{\dot{\gamma}}{\gamma}\right\}\left\{\dot{s_0} - \frac{s_0\dot{\eta}}{\eta}\right\}\right]\\
&= \pm\frac{1}{2\gamma\kappa\varepsilon_j\omega_o^2}\left[\kappa\varrho_o - \frac{1}{\kappa\varepsilon_j}\left\{\kappa^2\varphi_g\mathfrak{d}_b + 3\kappa^2\varrho_n\delta_b + 3\kappa^2\varrho_i\mathfrak{d}_a\right\} + \frac{6\kappa\delta_b}{\kappa^2\varepsilon_j^2}\left\{\kappa^2\varrho_i\delta_b + \kappa\varphi_g\left(\kappa\mathfrak{d}_a - \frac{\kappa^2\delta_b^2}{\kappa\varepsilon_j}\right)\right\}\right.\\
&\qquad \left. - \frac{1}{\gamma}\left\{\kappa\varrho_i\varrho_r + 2\kappa\varrho_n\varrho_p\right\} + \frac{\kappa\varphi_g}{\gamma\kappa\varepsilon_j}\left\{2\kappa\mathfrak{d}_a\varrho_p + \kappa\delta_b\varrho_r\right\} + \frac{2\varrho_p}{\gamma}\left\{\frac{2\kappa\delta_b}{\kappa\varepsilon_j} + \frac{\varrho_p}{\gamma}\right\}\left\{\kappa\varrho_i - \frac{\kappa^2\varphi_g\delta_b}{\kappa\varepsilon_j}\right\}\right]\\
&\qquad \text{by (5.4b), (5.5a), (5.28), (5.27), (5.29a) \& (5.29b)}
\end{aligned}$$

$$
\begin{aligned}
&= \pm\frac{1}{\varrho_l}\Bigg[\varrho_o - \frac{1}{\varepsilon_j}\Big\{\varphi_g\mathfrak{d}_b + 3\varrho_n\delta_b + 3\varrho_i\mathfrak{d}_a\Big\} + \frac{6\delta_b}{\varepsilon_j^2}\Big\{\varrho_i\delta_b + \varphi_g\Big(\mathfrak{d}_a - \frac{\delta_b^2}{\varepsilon_j}\Big)\Big\} \\
&\quad - \frac{1}{\gamma}\Big\{\varrho_i\varrho_r + 2\varrho_n\varrho_p\Big\} + \frac{\varphi_g}{\gamma\varepsilon_j}\Big\{2\mathfrak{d}_a\varrho_p + \delta_b\varrho_r\Big\} + \frac{2\varrho_m\varrho_p\varrho_q}{\gamma}\Bigg] = \varrho_s \text{ by (5.23e).}
\end{aligned} \tag{5.29c}
$$

Moreover, from (1.4a), we derive

$$
\begin{aligned}
\dot{\mathbf{a}} &= (\boldsymbol{\Omega}\times\mathbf{u} + \boldsymbol{\Lambda}\times\mathbf{r})' = \boldsymbol{\Lambda}\times\mathbf{u} + \boldsymbol{\Omega}\times\mathbf{a} + \dot{\boldsymbol{\Lambda}}\times\mathbf{r} + \boldsymbol{\Lambda}\times\mathbf{u} = 2\boldsymbol{\Lambda}\times\mathbf{u} + \dot{\boldsymbol{\Lambda}}\times\mathbf{r} + \boldsymbol{\Omega}\times\mathbf{a} \\
&= 2\boldsymbol{\Lambda}\times(\boldsymbol{\Omega}\times\mathbf{r}) + \dot{\boldsymbol{\Lambda}}\times\mathbf{r} + \boldsymbol{\Omega}\times[(\boldsymbol{\Omega}\cdot\mathbf{r})\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda}\times\mathbf{r}] \text{ by (1.4)} \\
&= 2[\boldsymbol{\Omega}(\boldsymbol{\Lambda}\cdot\mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})] + \dot{\boldsymbol{\Lambda}}\times\mathbf{r} - \Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) + \boldsymbol{\Lambda}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}) \text{ by (A.1)} \\
&= 2\varepsilon_h\boldsymbol{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\boldsymbol{\Lambda}}\times\mathbf{r} - \Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b\boldsymbol{\Lambda} \text{ by (5.1a)}
\end{aligned} \tag{5.30a}
$$

$$
\begin{aligned}
\ddot{\mathbf{a}} &= (2\boldsymbol{\Lambda}\times\mathbf{u} + \dot{\boldsymbol{\Lambda}}\times\mathbf{r} + \boldsymbol{\Omega}\times\mathbf{a})' \text{ by (5.30a)} \\
&= 2\dot{\boldsymbol{\Lambda}}\times\mathbf{u} + 2\boldsymbol{\Lambda}\times\mathbf{a} + \ddot{\boldsymbol{\Lambda}}\times\mathbf{r} + \dot{\boldsymbol{\Lambda}}\times\mathbf{u} + \boldsymbol{\Lambda}\times\mathbf{a} + \boldsymbol{\Omega}\times\dot{\mathbf{a}} \\
&= 3\dot{\boldsymbol{\Lambda}}\times\mathbf{u} + 3\boldsymbol{\Lambda}\times\mathbf{a} + \ddot{\boldsymbol{\Lambda}}\times\mathbf{r} + \boldsymbol{\Omega}\times\dot{\mathbf{a}} \\
&= 3\dot{\boldsymbol{\Lambda}}\times(\boldsymbol{\Omega}\times\mathbf{r}) + 3\boldsymbol{\Lambda}\times[(\boldsymbol{\Omega}\cdot\mathbf{r})\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda}\times\mathbf{r}] + \ddot{\boldsymbol{\Lambda}}\times\mathbf{r} \\
&\quad + \boldsymbol{\Omega}\times[2\varepsilon_h\boldsymbol{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\boldsymbol{\Lambda}}\times\mathbf{r} - \Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b\boldsymbol{\Lambda}] \text{ by (1.4) \& (5.30a)} \\
&= 3\dot{\boldsymbol{\Lambda}}\times(\boldsymbol{\Omega}\times\mathbf{r}) + 3\varepsilon_b(\boldsymbol{\Lambda}\times\boldsymbol{\Omega}) - 3\Omega^2(\boldsymbol{\Lambda}\times\mathbf{r}) + 3[\boldsymbol{\Lambda}\times(\boldsymbol{\Lambda}\times\mathbf{r})] + \ddot{\boldsymbol{\Lambda}}\times\mathbf{r} \\
&\quad - 3\varepsilon_g(\boldsymbol{\Omega}\times\mathbf{r}) + \boldsymbol{\Omega}\times(\dot{\boldsymbol{\Lambda}}\times\mathbf{r}) - \Omega^2[\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})] + \varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) \text{ by (5.1a)}
\end{aligned}
$$

$$
\begin{aligned}
&= 3[\boldsymbol{\Omega}(\dot{\boldsymbol{\Lambda}}\cdot\mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})] + 3\varepsilon_b(\boldsymbol{\Lambda}\times\boldsymbol{\Omega}) - 3\Omega^2(\boldsymbol{\Lambda}\times\mathbf{r}) + 3[\boldsymbol{\Lambda}(\boldsymbol{\Lambda}\cdot\mathbf{r}) - \Lambda^2\mathbf{r}] + \ddot{\boldsymbol{\Lambda}}\times\mathbf{r} \\
&\quad - 3\varepsilon_g(\boldsymbol{\Omega}\times\mathbf{r}) + [\dot{\boldsymbol{\Lambda}}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Omega})] - \Omega^2[\boldsymbol{\Omega}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}] + \varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) \text{ by (A.1)} \\
&= 3\varsigma_n\boldsymbol{\Omega} - 3\varsigma_c\mathbf{r} + 3\varepsilon_b(\boldsymbol{\Lambda}\times\boldsymbol{\Omega}) - 3\Omega^2(\boldsymbol{\Lambda}\times\mathbf{r}) + 3\varepsilon_h\boldsymbol{\Lambda} - 3\Lambda^2\mathbf{r} + \ddot{\boldsymbol{\Lambda}}\times\mathbf{r} \\
&\quad - 3\varepsilon_g(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b\dot{\boldsymbol{\Lambda}} - \varsigma_c\mathbf{r} - \Omega^2\varepsilon_b\boldsymbol{\Omega} + \Omega^4\mathbf{r} + \varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) \text{ by (5.1a) \& (5.23a)} \\
&= 3\varepsilon_h\boldsymbol{\Lambda} + \varepsilon_b\dot{\boldsymbol{\Lambda}} + (3\varsigma_n - \Omega^2\varepsilon_b)\boldsymbol{\Omega} + (\Omega^4 - 4\varsigma_c - 3\Lambda^2)\mathbf{r} + 2\varepsilon_b(\boldsymbol{\Lambda}\times\boldsymbol{\Omega}) \\
&\quad - 3\Omega^2(\boldsymbol{\Lambda}\times\mathbf{r}) - 3\varepsilon_g(\boldsymbol{\Omega}\times\mathbf{r}) + (\ddot{\boldsymbol{\Lambda}}\times\mathbf{r})
\end{aligned}
$$

$$
\begin{aligned}
&= 3\varepsilon_h\boldsymbol{\Lambda} + \varepsilon_b\dot{\boldsymbol{\Lambda}} + \varrho_t\boldsymbol{\Omega} + \varrho_u\mathbf{r} + 2\varepsilon_b(\boldsymbol{\Lambda}\times\boldsymbol{\Omega}) - 3\Omega^2(\boldsymbol{\Lambda}\times\mathbf{r}) - 3\varepsilon_g(\boldsymbol{\Omega}\times\mathbf{r}) \\
&\quad + (\ddot{\boldsymbol{\Lambda}}\times\mathbf{r}) \text{ by (5.23f)}
\end{aligned} \tag{5.30b}
$$

$$
\begin{aligned}
\dddot{\mathbf{a}} &= (3\dot{\boldsymbol{\Lambda}}\times\mathbf{u} + 3\boldsymbol{\Lambda}\times\mathbf{a} + \ddot{\boldsymbol{\Lambda}}\times\mathbf{r} + \boldsymbol{\Omega}\times\dot{\mathbf{a}})' \text{ by (5.30b)} \\
&= 3\ddot{\boldsymbol{\Lambda}}\times\mathbf{u} + 3\dot{\boldsymbol{\Lambda}}\times\mathbf{a} + 3\dot{\boldsymbol{\Lambda}}\times\mathbf{a} + 3\boldsymbol{\Lambda}\times\dot{\mathbf{a}} + \dddot{\boldsymbol{\Lambda}}\times\mathbf{r} + \ddot{\boldsymbol{\Lambda}}\times\mathbf{u} + \boldsymbol{\Lambda}\times\dot{\mathbf{a}} + \boldsymbol{\Omega}\times\ddot{\mathbf{a}} \\
&= 4\ddot{\boldsymbol{\Lambda}}\times\mathbf{u} + 6\dot{\boldsymbol{\Lambda}}\times\mathbf{a} + 4\boldsymbol{\Lambda}\times\dot{\mathbf{a}} + \dddot{\boldsymbol{\Lambda}}\times\mathbf{r} + \boldsymbol{\Omega}\times\ddot{\mathbf{a}}
\end{aligned}
$$

$$
\begin{aligned}
&= \dddot{\boldsymbol{\Lambda}}\times\mathbf{r} + 4\ddot{\boldsymbol{\Lambda}}\times(\boldsymbol{\Omega}\times\mathbf{r}) + 6\dot{\boldsymbol{\Lambda}}\times[(\boldsymbol{\Omega}\cdot\mathbf{r})\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda}\times\mathbf{r}] \\
&\quad + 4\boldsymbol{\Lambda}\times[2\varepsilon_h\boldsymbol{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\boldsymbol{\Lambda}}\times\mathbf{r} - \Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b\boldsymbol{\Lambda}] \\
&\quad + \boldsymbol{\Omega}\times[3\varepsilon_h\boldsymbol{\Lambda} + \varepsilon_b\dot{\boldsymbol{\Lambda}} + \varrho_t\boldsymbol{\Omega} + \varrho_u\mathbf{r} + 2\varepsilon_b(\boldsymbol{\Lambda}\times\boldsymbol{\Omega})] \\
&\quad + \boldsymbol{\Omega}\times[-3\Omega^2(\boldsymbol{\Lambda}\times\mathbf{r}) - 3\varepsilon_g(\boldsymbol{\Omega}\times\mathbf{r}) + (\ddot{\boldsymbol{\Lambda}}\times\mathbf{r})] \text{ by (1.4), (5.30a) \& (5.30b)} \\
&= \dddot{\boldsymbol{\Lambda}}\times\mathbf{r} + 4\ddot{\boldsymbol{\Lambda}}\times(\boldsymbol{\Omega}\times\mathbf{r}) + 6\varepsilon_b(\dot{\boldsymbol{\Lambda}}\times\boldsymbol{\Omega}) - 6\Omega^2(\dot{\boldsymbol{\Lambda}}\times\mathbf{r}) + 6[\dot{\boldsymbol{\Lambda}}\times(\boldsymbol{\Lambda}\times\mathbf{r})] \\
&\quad + 8\varepsilon_h(\boldsymbol{\Lambda}\times\boldsymbol{\Omega}) - 12\varepsilon_g(\boldsymbol{\Lambda}\times\mathbf{r}) + 4[\boldsymbol{\Lambda}\times(\dot{\boldsymbol{\Lambda}}\times\mathbf{r})] - 4\Omega^2[\boldsymbol{\Lambda}\times(\boldsymbol{\Omega}\times\mathbf{r})] + 4\varepsilon_b(\boldsymbol{\Lambda}\times\boldsymbol{\Lambda}) \\
&\quad + 3\varepsilon_h(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) + \varepsilon_b(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}}) + \varrho_t(\boldsymbol{\Omega}\times\boldsymbol{\Omega}) + \varrho_u(\boldsymbol{\Omega}\times\mathbf{r}) + 2\varepsilon_b[\boldsymbol{\Omega}\times(\boldsymbol{\Lambda}\times\boldsymbol{\Omega})] \\
&\quad - 3\Omega^2[\boldsymbol{\Omega}\times(\boldsymbol{\Lambda}\times\mathbf{r})] - 3\varepsilon_g[\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})] + \boldsymbol{\Omega}\times(\ddot{\boldsymbol{\Lambda}}\times\mathbf{r}) \text{ by (5.1a)}
\end{aligned}
$$

$$
\begin{aligned}
&= \dddot{\boldsymbol{\Lambda}} \times \mathbf{r} + 4[\boldsymbol{\Omega}(\ddot{\boldsymbol{\Lambda}} \cdot \mathbf{r}) - \mathbf{r}(\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})] + 6\varepsilon_b(\dot{\boldsymbol{\Lambda}} \times \boldsymbol{\Omega}) - 6\Omega^2(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) + 6[\boldsymbol{\Lambda}(\dot{\boldsymbol{\Lambda}} \cdot \mathbf{r}) - \mathbf{r}(\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda})] \\
&\quad + 5\varepsilon_h(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) - 12\varepsilon_g(\boldsymbol{\Lambda} \times \mathbf{r}) + 4[\dot{\boldsymbol{\Lambda}}(\boldsymbol{\Lambda} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Lambda} \cdot \dot{\boldsymbol{\Lambda}})] - 4\Omega^2[\boldsymbol{\Omega}(\boldsymbol{\Lambda} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega})] \\
&\quad + \varepsilon_b(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \varrho_u(\boldsymbol{\Omega} \times \mathbf{r}) + 2\varepsilon_b[\Omega^2\boldsymbol{\Lambda} - \boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})] - 3\Omega^2[\boldsymbol{\Lambda}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})] \\
&\quad - 3\varepsilon_g[\boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \Omega^2\mathbf{r}] + [\ddot{\boldsymbol{\Lambda}}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \mathbf{r}(\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})] \text{ by (A.1)} \\
&= \dddot{\boldsymbol{\Lambda}} \times \mathbf{r} + 4(\varsigma_o\boldsymbol{\Omega} - \varsigma_h\mathbf{r}) + 6\varepsilon_b(\dot{\boldsymbol{\Lambda}} \times \boldsymbol{\Omega}) - 6\Omega^2(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) + 6(\varsigma_n\boldsymbol{\Lambda} - \varsigma_d\mathbf{r}) + 5\varepsilon_h(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) \\
&\quad - 12\varepsilon_g(\boldsymbol{\Lambda} \times \mathbf{r}) + 4(\varepsilon_h\dot{\boldsymbol{\Lambda}} - \varsigma_d\mathbf{r}) - 4\Omega^2(\varepsilon_h\boldsymbol{\Omega} - \varepsilon_g\mathbf{r}) + \varepsilon_b(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \varrho_u(\boldsymbol{\Omega} \times \mathbf{r}) \\
&\quad + 2\varepsilon_b(\Omega^2\boldsymbol{\Lambda} - \varepsilon_g\boldsymbol{\Omega}) - 3\Omega^2(\varepsilon_b\boldsymbol{\Lambda} - \varepsilon_g\mathbf{r}) - 3\varepsilon_g(\varepsilon_b\boldsymbol{\Omega} - \Omega^2\mathbf{r}) + (\varepsilon_b\ddot{\boldsymbol{\Lambda}} - \varsigma_h\mathbf{r}) \text{ by (5.1a) \& (5.23a)}
\end{aligned}
$$

$$
\begin{aligned}
&= (6\varsigma_n - \Omega^2\varepsilon_b)\boldsymbol{\Lambda} + 4\varepsilon_h\dot{\boldsymbol{\Lambda}} + \varepsilon_b\ddot{\boldsymbol{\Lambda}} + (4\varsigma_o - 4\Omega^2\varepsilon_h - 5\varepsilon_b\varepsilon_g)\boldsymbol{\Omega} + 5(2\Omega^2\varepsilon_g - \varsigma_h - 2\varsigma_d)\mathbf{r} \\
&\quad - 5\varepsilon_b(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) - 6\Omega^2(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) + 5\varepsilon_h(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) - 12\varepsilon_g(\boldsymbol{\Lambda} \times \mathbf{r}) + \varrho_u(\boldsymbol{\Omega} \times \mathbf{r}) + \dddot{\boldsymbol{\Lambda}} \times \mathbf{r} \\
&= (\varrho_t + 3\varsigma_n)\boldsymbol{\Lambda} + 4\varepsilon_h\dot{\boldsymbol{\Lambda}} + \varepsilon_b\ddot{\boldsymbol{\Lambda}} + \varrho_v\boldsymbol{\Omega} + 5\varrho_w\mathbf{r} - 5\varepsilon_b(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) - 6\Omega^2(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \\
&\quad + 5\varepsilon_h(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) - 12\varepsilon_g(\boldsymbol{\Lambda} \times \mathbf{r}) + \varrho_u(\boldsymbol{\Omega} \times \mathbf{r}) + \dddot{\boldsymbol{\Lambda}} \times \mathbf{r} \text{ by (5.23f).}
\end{aligned} \tag{5.30c}
$$

Art 22c. *Development of equations* (3.17) *through* (3.21).

Equations (3.17) through (3.21) evaluate as

$$
\begin{aligned}
\dot{\alpha} &= \boldsymbol{\kappa} \cdot \dot{\mathbf{a}} \text{ by (3.17)} \\
&= \boldsymbol{\kappa} \cdot [2\varepsilon_h\boldsymbol{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\boldsymbol{\Lambda}} \times \mathbf{r} - \Omega^2(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b\boldsymbol{\Lambda}] \text{ by (5.30a)} \\
&= \kappa(2\varepsilon_a\varepsilon_h - 3\varepsilon_d\varepsilon_g + \mathfrak{d}_c - \Omega^2\varepsilon_k + \delta_a\varepsilon_b) \text{ by (5.1a), (5.13a) \& (5.23b)} \\
&= \kappa(\mathfrak{d}_c + \varepsilon_b\delta_a + 2\varepsilon_a\varepsilon_h - 3\varepsilon_d\varepsilon_g - \Omega^2\varepsilon_k) = \kappa\varrho_x \text{ by (5.23f)}
\end{aligned} \tag{5.31a}
$$

$$
\begin{aligned}
\ddot{\alpha} &= \boldsymbol{\kappa} \cdot \ddot{\mathbf{a}} \text{ by (3.17)} \\
&= \boldsymbol{\kappa} \cdot [3\varepsilon_h\boldsymbol{\Lambda} + \varepsilon_b\dot{\boldsymbol{\Lambda}} + \varrho_t\boldsymbol{\Omega} + \varrho_u\mathbf{r} + 2\varepsilon_b(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) - 3\Omega^2(\boldsymbol{\Lambda} \times \mathbf{r}) - 3\varepsilon_g(\boldsymbol{\Omega} \times \mathbf{r}) \\
&\qquad + (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] \text{ by (5.30b)} \\
&= \kappa[3\varepsilon_h\delta_a + \varepsilon_b\varsigma_b + \varrho_t\varepsilon_a + \varrho_u\varepsilon_d + 2\varepsilon_b\mathfrak{d}_e - 3\Omega^2\varepsilon_n - 3\varepsilon_g\varepsilon_k + \mathfrak{d}_d] \\
&\qquad \text{by (5.1a), (5.13a), (5.23a) \& (5.23b)} \\
&= \kappa[\mathfrak{d}_d + \varepsilon_a\varrho_t + \varepsilon_d\varrho_u + \varepsilon_b(\varsigma_b + 2\mathfrak{d}_e) + 3(\varepsilon_h\delta_a - \Omega^2\varepsilon_n - \varepsilon_g\varepsilon_k)] = \kappa\varrho_y \text{ by (5.23f)}
\end{aligned} \tag{5.31b}
$$

$$
\begin{aligned}
\dddot{\alpha} &= \boldsymbol{\kappa} \cdot \dddot{\mathbf{a}} \text{ by (3.17)} \\
&= \boldsymbol{\kappa} \cdot [(\varrho_t + 3\varsigma_n)\boldsymbol{\Lambda} + 4\varepsilon_h\dot{\boldsymbol{\Lambda}} + \varepsilon_b\ddot{\boldsymbol{\Lambda}} + \varrho_v\boldsymbol{\Omega} + 5\varrho_w\mathbf{r} - 5\varepsilon_b(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) \\
&\qquad - 6\Omega^2(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) + 5\varepsilon_h(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) - 12\varepsilon_g(\boldsymbol{\Lambda} \times \mathbf{r}) + \varrho_u(\boldsymbol{\Omega} \times \mathbf{r}) + \dddot{\boldsymbol{\Lambda}} \times \mathbf{r}] \text{ by (5.30c)}
\end{aligned}
$$

$$
\begin{aligned}
&= \kappa[\delta_a(\varrho_t + 3\varsigma_n) + 4\varepsilon_h\varsigma_b + \varepsilon_b\varsigma_g + \varepsilon_a\varrho_v + 5\varepsilon_d\varrho_w - 5\varepsilon_b\mathfrak{d}_f - 6\Omega^2\mathfrak{d}_c + 5\varepsilon_h\mathfrak{d}_e \\
&\qquad - 12\varepsilon_g\varepsilon_n + \varrho_u\varepsilon_k + \mathfrak{d}_g] \text{ by (5.1a), (5.13a), (5.23a) \& (5.23b)} \\
&= \kappa[\mathfrak{d}_g + \varepsilon_a\varrho_v + \varepsilon_k\varrho_u + 5\varepsilon_d\varrho_w + \varepsilon_b(\varsigma_g - 5\mathfrak{d}_f) + \delta_a(\varrho_t + 3\varsigma_n) \\
&\qquad + \varepsilon_h(4\varsigma_b + 5\mathfrak{d}_e) - 6(\Omega^2\mathfrak{d}_c + 2\varepsilon_g\varepsilon_n)] = \kappa\varrho_z \text{ by (5.23f)}
\end{aligned} \tag{5.31c}
$$

$$
\begin{aligned}
\dot{\vartheta} &= \frac{1}{\gamma^2\omega_o^2}\left[\dot{\alpha} - \frac{2\alpha\dot{\gamma}}{\gamma}\right] \text{ by (3.18a)} \\
&= \frac{1}{\gamma^2\omega_o^2}\left[\kappa\varrho_x - \frac{2\kappa\varphi_d\varrho_p}{\gamma}\right] \text{ by (5.4a), (5.29a), \& (5.31a)} \\
&= \mathfrak{x}_a \text{ by (5.23g)}
\end{aligned} \tag{5.32a}
$$

$$\begin{aligned}
\ddot{\vartheta} &= \frac{1}{\gamma^2\omega_o^2}\left[\ddot{\alpha} - \frac{4\dot{\alpha}\dot{\gamma}}{\gamma} - \frac{2\alpha\ddot{\gamma}}{\gamma} + \frac{6\alpha\dot{\gamma}^2}{\gamma^2}\right] \text{ by (3.18b)} \\
&= \frac{1}{\gamma^2\omega_o^2}\left[\kappa\varrho_y - \frac{4\kappa\varrho_x\varrho_p}{\gamma} - \frac{2\kappa\varphi_d\varrho_r}{\gamma} + \frac{6\kappa\varphi_d\varrho_p^2}{\gamma^2}\right] \text{ by (5.4a), (5.29) \& (5.31)} \\
&= \mathfrak{x}_b \text{ by (5.23g)}
\end{aligned} \tag{5.32b}$$

$$\begin{aligned}
\dddot{\vartheta} &= \frac{1}{\gamma^2\omega_o^2}\left[\dddot{\alpha} - \frac{6\ddot{\alpha}\dot{\gamma}}{\gamma} - \frac{6\dot{\alpha}\ddot{\gamma}}{\gamma} + \frac{18\dot{\alpha}\dot{\gamma}^2}{\gamma^2} - \frac{2\alpha\dddot{\gamma}}{\gamma} + \frac{18\alpha\dot{\gamma}\ddot{\gamma}}{\gamma^2} - \frac{24\alpha\dot{\gamma}^3}{\gamma^3}\right] \text{ by (3.18c)} \\
&= \frac{1}{\gamma^2\omega_o^2}\left[\kappa\varrho_z - \frac{6\kappa\varrho_y\varrho_p}{\gamma} - \frac{6\kappa\varrho_x\varrho_r}{\gamma} + \frac{18\kappa\varrho_x\varrho_p^2}{\gamma^2} - \frac{2\kappa\varphi_d\varrho_s}{\gamma} + \frac{18\kappa\varphi_d\varrho_p\varrho_r}{\gamma^2} - \frac{24\kappa\varphi_d\varrho_p^3}{\gamma^3}\right] \\
&\qquad \text{by (5.4a), (5.29) \& (5.31)} \\
&= \mathfrak{x}_c \text{ by (5.23g)}
\end{aligned} \tag{5.32c}$$

$$\begin{aligned}
\dot{\pi} &= \dot{\vartheta}(1+\vartheta^2)^{-3/2} \text{ by (3.19a)} \\
&= \mathfrak{x}_a\delta_g^{-3} \text{ by (5.32a) \& (5.13a)}
\end{aligned} \tag{5.33a}$$

$$\begin{aligned}
\ddot{\pi} &= (1+\vartheta^2)^{-5/2}[\ddot{\vartheta}(1+\vartheta^2) - 3\vartheta\dot{\vartheta}^2] \text{ by (3.19b)} \\
&= \delta_g^{-5}(\mathfrak{x}_b\delta_g^2 - 3\vartheta\mathfrak{x}_a^2) \text{ by (5.32) \& (5.13a)} \\
&= \mathfrak{x}_d \text{ by (5.23g)}
\end{aligned} \tag{5.33b}$$

$$\begin{aligned}
\dddot{\pi} &= (1+\vartheta^2)^{-7/2}[\dddot{\vartheta}(1+\vartheta^2)^2 - 9\vartheta\dot{\vartheta}\ddot{\vartheta}(1+\vartheta^2) - 3\dot{\vartheta}^3(1-4\vartheta^2)] \text{ by (3.19c)} \\
&= \delta_g^{-7}[\mathfrak{x}_c\delta_g^4 - 9\vartheta\mathfrak{x}_a\mathfrak{x}_b\delta_g^2 - 3\mathfrak{x}_a^3(1-4\vartheta^2)] \text{ by (5.32) \& (5.13a)} \\
&= \mathfrak{x}_e \text{ by (5.23g)}
\end{aligned} \tag{5.33c}$$

$$\begin{aligned}
\dot{d} &= \frac{\dot{\gamma}d}{\gamma} + \frac{\pi\gamma^2\dot{\vartheta}}{4d} \text{ by (3.20a)} \\
&= \frac{\varrho_p d}{\gamma} + \frac{\pi\gamma^2\mathfrak{x}_a}{4d} \text{ by (5.29a) \& (5.32a)} \\
&= \mathfrak{x}_f \text{ by (5.23h)}
\end{aligned} \tag{5.34a}$$

$$\begin{aligned}
\ddot{d} &= d\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2}\right] + \dot{d}\left[\frac{\dot{\gamma}}{\gamma} - \frac{\pi\gamma^2\dot{\vartheta}}{4d^2}\right] + \frac{\gamma}{4d}\left[\dot{\pi}\gamma\dot{\vartheta} + 2\pi\dot{\gamma}\dot{\vartheta} + \pi\gamma\ddot{\vartheta}\right] \text{ by (3.20b)} \\
&= d\left[\frac{\varrho_r}{\gamma} - \frac{\varrho_p^2}{\gamma^2}\right] + \mathfrak{x}_f\left[\frac{\varrho_p}{\gamma} - \frac{\pi\gamma^2\mathfrak{x}_a}{4d^2}\right] + \frac{\gamma}{4d}\left[\frac{\gamma\mathfrak{x}_a^2}{\delta_g^3} + 2\pi\varrho_p\mathfrak{x}_a + \pi\gamma\mathfrak{x}_b\right] \\
&\qquad \text{by (5.29), (5.32), (5.33a) \& (5.34a)} \\
&= \mathfrak{x}_g \text{ by (5.23h)}
\end{aligned} \tag{5.34b}$$

$$
\begin{aligned}
\dddot{d} &= d\left[\frac{\dddot{\gamma}}{\gamma} - \frac{3\dot{\gamma}\ddot{\gamma}}{\gamma^2} + \frac{2\dot{\gamma}^3}{\gamma^3}\right] + 2\dot{d}\left[\frac{\ddot{\gamma}}{\gamma} - \frac{\dot{\gamma}^2}{\gamma^2} - \frac{\dot{\pi}\gamma^2\dot{\vartheta}}{4d^2} - \frac{\pi\gamma\dot{\gamma}\dot{\vartheta}}{2d^2} - \frac{\pi\gamma^2\ddot{\vartheta}}{4d^2} + \frac{\pi\gamma^2\dot{d}\dot{\vartheta}}{4d^3}\right] + \ddot{d}\left[\frac{\dot{\gamma}}{\gamma} - \frac{\pi\gamma^2\dot{\vartheta}}{4d^2}\right] \\
&\quad + \frac{\dot{\gamma}}{4d}\left[\dot{\pi}\gamma\dot{\vartheta} + 2\pi\dot{\gamma}\dot{\vartheta} + \pi\gamma\ddot{\vartheta}\right] + \frac{\gamma}{4d}\left[\ddot{\pi}\gamma\dot{\vartheta} + 3\dot{\pi}\dot{\gamma}\dot{\vartheta} + 2\dot{\pi}\gamma\ddot{\vartheta} + 2\pi\ddot{\gamma}\dot{\vartheta} + 3\pi\dot{\gamma}\ddot{\vartheta} + \pi\gamma\dddot{\vartheta}\right] \text{ by (3.20c)} \\
&= d\left[\frac{\varrho_s}{\gamma} - \frac{3\varrho_p\varrho_r}{\gamma^2} + \frac{2\varrho_p^3}{\gamma^3}\right] + 2\mathfrak{x}_f\left[\frac{\varrho_r}{\gamma} - \frac{\varrho_p^2}{\gamma^2} - \frac{\gamma^2\mathfrak{x}_a^2}{4d^2\delta_g^3} - \frac{\pi\gamma\varrho_p\mathfrak{x}_a}{2d^2} - \frac{\pi\gamma^2\mathfrak{x}_b}{4d^2} + \frac{\pi\gamma^2\mathfrak{x}_f\mathfrak{x}_a}{4d^3}\right] \\
&\quad + \mathfrak{x}_g\left[\frac{\varrho_p}{\gamma} - \frac{\pi\gamma^2\mathfrak{x}_a}{4d^2}\right] + \frac{\varrho_p}{4d}\left[\frac{\gamma\mathfrak{x}_a^2}{\delta_g^3} + 2\pi\varrho_p\mathfrak{x}_a + \pi\gamma\mathfrak{x}_b\right] + \frac{\gamma}{4d}\left[\gamma\mathfrak{x}_a\mathfrak{x}_d + \frac{3\mathfrak{x}_a^2\varrho_p}{\delta_g^3} + \frac{2\gamma\mathfrak{x}_a\mathfrak{x}_b}{\delta_g^3}\right. \\
&\quad \left. + 2\pi\varrho_r\mathfrak{x}_a + 3\pi\varrho_p\mathfrak{x}_b + \pi\gamma\mathfrak{x}_c\right] \text{ by (5.29), (5.32), (5.33), (5.34a) \& (5.34b)} \\
&= \mathfrak{x}_h \text{ by (5.23h)}
\end{aligned} \tag{5.34c}
$$

$$
\begin{aligned}
\dot{\rho} &= \frac{d\dot{\pi} - \pi\dot{d}}{4\omega_o d^2} \text{ by (3.21a)} \\
&= \frac{1}{4\omega_o d^2}\left[\frac{d\mathfrak{x}_a}{\delta_g^3} - \pi\mathfrak{x}_f\right] \text{ by (5.33a) \& (5.34a)} \\
&= \mathfrak{x}_i \text{ by (5.23i)}
\end{aligned} \tag{5.35a}
$$

$$
\begin{aligned}
\ddot{\rho} &= -\frac{\dot{d}(d\dot{\pi} - \pi\dot{d})}{2\omega_o d^3} + \frac{d\ddot{\pi} - \pi\ddot{d}}{4\omega_o d^2} \text{ by (3.21b)} \\
&= -\frac{\mathfrak{x}_f}{2\omega_o d^3}\left[\frac{d\mathfrak{x}_a}{\delta_g^3} - \pi\mathfrak{x}_f\right] + \frac{1}{4\omega_o d^2}\left[d\mathfrak{x}_d - \pi\mathfrak{x}_g\right] \text{ by (5.33) \& (5.34)} \\
&= \frac{1}{4\omega_o d^2}\left[d\mathfrak{x}_d - \pi\mathfrak{x}_g - \frac{2\mathfrak{x}_f}{d}\left\{\frac{d\mathfrak{x}_a}{\delta_g^3} - \pi\mathfrak{x}_f\right\}\right] \\
&= \mathfrak{x}_j \text{ by (5.23i)}
\end{aligned} \tag{5.35b}
$$

$$
\begin{aligned}
\dddot{\rho} &= \frac{(3\dot{d}^2 - d\ddot{d})(d\dot{\pi} - \pi\dot{d})}{2\omega_o d^4} - \frac{\dot{d}(d\ddot{\pi} - \pi\ddot{d})}{\omega_o d^3} + \frac{\dot{d}\ddot{\pi} + d\dddot{\pi} - \dot{\pi}\ddot{d} - \pi\dddot{d}}{4\omega_o d^2} \text{ by (3.21c)} \\
&= \frac{3\mathfrak{x}_f^2 - d\mathfrak{x}_g}{2\omega_o d^4}\left[\frac{d\mathfrak{x}_a}{\delta_g^3} - \pi\mathfrak{x}_f\right] - \frac{\mathfrak{x}_f}{\omega_o d^3}\left[d\mathfrak{x}_d - \pi\mathfrak{x}_g\right] \\
&\quad + \frac{1}{4\omega_o d^2}\left[\mathfrak{x}_f\mathfrak{x}_d + d\mathfrak{x}_e - \frac{\mathfrak{x}_a\mathfrak{x}_g}{\delta_g^3} - \pi\mathfrak{x}_h\right] \text{ by (5.33) \& (5.34)} \\
&= \frac{1}{4\omega_o d^2}\left[\mathfrak{x}_f\mathfrak{x}_d + d\mathfrak{x}_e - \pi\mathfrak{x}_h - \frac{\mathfrak{x}_a\mathfrak{x}_g}{\delta_g^3} + \frac{2(3\mathfrak{x}_f^2 - d\mathfrak{x}_g)}{d^2}\left\{\frac{d\mathfrak{x}_a}{\delta_g^3} - \pi\mathfrak{x}_f\right\} - \frac{4\mathfrak{x}_f}{d}\left\{d\mathfrak{x}_d - \pi\mathfrak{x}_g\right\}\right] \\
&= \mathfrak{x}_k \text{ by (5.23i).}
\end{aligned} \tag{5.35c}
$$

Art 22d. *Derivatives of* τ *and* $\mathbf{e}$.

To compute the derivatives of τ and $\mathbf{e}$, we first derive

$$\begin{aligned}
\dot{s_1} &= \left[\frac{2\rho\alpha - d\omega_o}{2\eta(s_0+\eta\omega_o^2)}\right]' \text{ by (1.4d)} \\
&= \frac{2\eta(s_0+\eta\omega_o^2)(2\rho\alpha - d\omega_o)' - (2\rho\alpha - d\omega_o)[2\eta(s_0+\eta\omega_o^2)]'}{4\eta^2(s_0+\eta\omega_o^2)^2} \\
&= \frac{2\dot{\rho}\alpha + 2\rho\dot{\alpha} - \dot{d}\omega_o - s_1[2\dot{\eta}(s_0+\eta\omega_o^2) + 2\eta(\dot{s_0}+\dot{\eta}\omega_o^2)]}{2\eta(s_0+\eta\omega_o^2)} \text{ by (1.4d)} \\
&= \frac{2\dot{\rho}\alpha + 2\rho\dot{\alpha} - \dot{d}\omega_o - 2s_1(\dot{\eta}s_0 + \eta\dot{s_0} + 2\eta\dot{\eta}\omega_o^2)}{2\eta(s_0+\eta\omega_o^2)} \\
&= \frac{2\kappa\varphi_d\mathfrak{x}_i + 2\rho\kappa\varrho_x - \mathfrak{x}_f\omega_o - 2(\varrho_h/\kappa)(\kappa^2\delta_b\varphi_g + \kappa^2\varepsilon_j\varrho_i + 2\kappa^2\varepsilon_j\delta_b\omega_o^2)}{2\kappa\varepsilon_j(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)} \\
&\qquad \text{by (5.4), (5.5), (5.27), (5.28a), (5.31a), (5.34a) \& (5.35a)} \\
&= \frac{2\varphi_d\mathfrak{x}_i + 2\rho\varrho_x - c\mathfrak{x}_f - 2\varrho_h(\delta_b\varphi_g + \varepsilon_j\varrho_i + 2\varepsilon_j\delta_b\omega_o^2)}{2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} = \mathfrak{x}_l/\kappa \text{ by (5.23j)}
\end{aligned} \tag{5.36a}$$

$$\begin{aligned}
\ddot{s_1} &= \left[\frac{2\dot{\rho}\alpha + 2\rho\dot{\alpha} - \dot{d}\omega_o - 2s_1(\dot{\eta}s_0 + \eta\dot{s_0} + 2\eta\dot{\eta}\omega_o^2)}{2\eta(s_0+\eta\omega_o^2)}\right]' \text{ by (5.36a)} \\
&= \frac{2\eta(s_0+\eta\omega_o^2)[2\dot{\rho}\alpha + 2\rho\dot{\alpha} - \dot{d}\omega_o - 2s_1(\dot{\eta}s_0 + \eta\dot{s_0} + 2\eta\dot{\eta}\omega_o^2)]'}{4\eta^2(s_0+\eta\omega_o^2)^2} \\
&\qquad - \frac{[2\dot{\rho}\alpha + 2\rho\dot{\alpha} - \dot{d}\omega_o - 2s_1(\dot{\eta}s_0 + \eta\dot{s_0} + 2\eta\dot{\eta}\omega_o^2)][2\eta(s_0+\eta\omega_o^2)]'}{4\eta^2(s_0+\eta\omega_o^2)^2} \\
&= \frac{[2\dot{\rho}\alpha + 2\rho\dot{\alpha} - \dot{d}\omega_o]'}{2\eta(s_0+\eta\omega_o^2)} - \frac{[2s_1(\dot{\eta}s_0 + \eta\dot{s_0} + 2\eta\dot{\eta}\omega_o^2)]'}{2\eta(s_0+\eta\omega_o^2)} - \frac{\dot{s_1}[2\eta(s_0+\eta\omega_o^2)]'}{2\eta(s_0+\eta\omega_o^2)} \text{ by (5.36a)} \\
&= \frac{2\ddot{\rho}\alpha + 2\dot{\rho}\dot{\alpha} + 2\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} - \ddot{d}\omega_o}{2\eta(s_0+\eta\omega_o^2)} - \frac{2\dot{s_1}(\dot{\eta}s_0 + \eta\dot{s_0} + 2\eta\dot{\eta}\omega_o^2)}{2\eta(s_0+\eta\omega_o^2)} \\
&\qquad - \frac{2s_1(\ddot{\eta}s_0 + \dot{\eta}\dot{s_0} + \dot{\eta}\dot{s_0} + \eta\ddot{s_0} + 2\dot{\eta}^2\omega_o^2 + 2\eta\ddot{\eta}\omega_o^2)}{2\eta(s_0+\eta\omega_o^2)} - \frac{2\dot{s_1}(\dot{\eta}s_0 + \eta\dot{s_0} + 2\eta\dot{\eta}\omega_o^2)}{2\eta(s_0+\eta\omega_o^2)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} - \ddot{d}\omega_o}{2\eta(s_0+\eta\omega_o^2)} - \frac{4\dot{s_1}(\dot{\eta}s_0 + \eta\dot{s_0} + 2\eta\dot{\eta}\omega_o^2)}{2\eta(s_0+\eta\omega_o^2)} \\
&\qquad - \frac{2s_1(\ddot{\eta}s_0 + \eta\ddot{s_0} + 2\dot{\eta}\dot{s_0} + 2\dot{\eta}^2\omega_o^2 + 2\eta\ddot{\eta}\omega_o^2)}{2\eta(s_0+\eta\omega_o^2)} \\
&= \frac{2\kappa\varphi_d\mathfrak{x}_j + 4\kappa\varrho_x\mathfrak{x}_i + 2\rho\kappa\varrho_y - \mathfrak{x}_g\omega_o}{2\kappa\varepsilon_j(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)} - \frac{4(\mathfrak{x}_l/\kappa)(\kappa^2\delta_b\varphi_g + \kappa^2\varepsilon_j\varrho_i + 2\kappa^2\varepsilon_j\delta_b\omega_o^2)}{2\kappa\varepsilon_j(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)} \\
&\qquad - \frac{2(\varphi_h/\kappa)(\kappa^2\varphi_g\mathfrak{d}_a + \kappa^2\varepsilon_j\varrho_n + 2\kappa^2\delta_b\varrho_i + 2\kappa^2\delta_b^2\omega_o^2 + 2\kappa^2\varepsilon_j\mathfrak{d}_a\omega_o^2)}{2\kappa\varepsilon_j(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)} \\
&\qquad \text{by (5.4), (5.5), (5.27), (5.28), (5.31), (5.34b), (5.35) \& (5.36a)} \\
&= \frac{2\varphi_d\mathfrak{x}_j + 4\varrho_x\mathfrak{x}_i + 2\rho\varrho_y - c\mathfrak{x}_g}{2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} - \frac{4\mathfrak{x}_l(\delta_b\varphi_g + \varepsilon_j\varrho_i + 2\varepsilon_j\delta_b\omega_o^2)}{2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} \\
&\qquad - \frac{2\varphi_h(\varphi_g\mathfrak{d}_a + \varepsilon_j\varrho_n + 2\delta_b\varrho_i + 2\delta_b^2\omega_o^2 + 2\varepsilon_j\mathfrak{d}_a\omega_o^2)}{2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} = \mathfrak{x}_m/\kappa \text{ by (5.23j)}
\end{aligned} \tag{5.36b}$$

$$
\begin{aligned}
\dddot{s}_1 &= \left[\frac{2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} - \ddot{d}\omega_o}{2\eta(s_0 + \eta\omega_o^2)}\right]' - \left[\frac{4\dot{s}_1(\dot{\eta}s_0 + \eta\dot{s}_0 + 2\eta\dot{\eta}\omega_o^2)}{2\eta(s_0 + \eta\omega_o^2)}\right]' \\
&\quad - \left[\frac{2s_1(\ddot{\eta}s_0 + \eta\ddot{s}_0 + 2\dot{\eta}\dot{s}_0 + 2\dot{\eta}^2\omega_o^2 + 2\eta\ddot{\eta}\omega_o^2)}{2\eta(s_0 + \eta\omega_o^2)}\right]' \text{ by (5.36b)} \\
&= \frac{2\eta(s_0 + \eta\omega_o^2)[2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} - \ddot{d}\omega_o]' - [2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} - \ddot{d}\omega_o][2\eta(s_0 + \eta\omega_o^2)]'}{4\eta^2(s_0 + \eta\omega_o^2)^2} \\
&\quad - \frac{2\eta(s_0 + \eta\omega_o^2)[4\dot{s}_1(\dot{\eta}s_0 + \eta\dot{s}_0 + 2\eta\dot{\eta}\omega_o^2)]' - [4\dot{s}_1(\dot{\eta}s_0 + \eta\dot{s}_0 + 2\eta\dot{\eta}\omega_o^2)][2\eta(s_0 + \eta\omega_o^2)]'}{4\eta^2(s_0 + \eta\omega_o^2)^2} \\
&\quad - \frac{2\eta(s_0 + \eta\omega_o^2)[2s_1(\ddot{\eta}s_0 + \eta\ddot{s}_0 + 2\dot{\eta}\dot{s}_0 + 2\dot{\eta}^2\omega_o^2 + 2\eta\ddot{\eta}\omega_o^2)]'}{4\eta^2(s_0 + \eta\omega_o^2)^2} \\
&\quad + \frac{[2s_1(\ddot{\eta}s_0 + \eta\ddot{s}_0 + 2\dot{\eta}\dot{s}_0 + 2\dot{\eta}^2\omega_o^2 + 2\eta\ddot{\eta}\omega_o^2)][2\eta(s_0 + \eta\omega_o^2)]'}{4\eta^2(s_0 + \eta\omega_o^2)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{[2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} - \ddot{d}\omega_o]'}{2\eta(s_0 + \eta\omega_o^2)} - \frac{\ddot{s}_1[2\eta(s_0 + \eta\omega_o^2)]'}{2\eta(s_0 + \eta\omega_o^2)} - \frac{[4\dot{s}_1(\dot{\eta}s_0 + \eta\dot{s}_0 + 2\eta\dot{\eta}\omega_o^2)]'}{2\eta(s_0 + \eta\omega_o^2)} \\
&\quad - \frac{[2s_1(\ddot{\eta}s_0 + \eta\ddot{s}_0 + 2\dot{\eta}\dot{s}_0 + 2\dot{\eta}^2\omega_o^2 + 2\eta\ddot{\eta}\omega_o^2)]'}{2\eta(s_0 + \eta\omega_o^2)} \text{ by (5.36b)} \\
&= \frac{2\dddot{\rho}\alpha + 2\ddot{\rho}\dot{\alpha} + 4\ddot{\rho}\dot{\alpha} + 4\dot{\rho}\ddot{\alpha} + 2\dot{\rho}\ddot{\alpha} + 2\rho\dddot{\alpha} - \dddot{d}\omega_o}{2\eta(s_0 + \eta\omega_o^2)} - \frac{2\ddot{s}_1(\dot{\eta}s_0 + \eta\dot{s}_0 + 2\eta\dot{\eta}\omega_o^2)}{2\eta(s_0 + \eta\omega_o^2)} \\
&\quad - \frac{4\ddot{s}_1(\dot{\eta}s_0 + \eta\dot{s}_0 + 2\eta\dot{\eta}\omega_o^2) + 4\dot{s}_1(\ddot{\eta}s_0 + \dot{\eta}\dot{s}_0 + \dot{\eta}\dot{s}_0 + \eta\ddot{s}_0 + 2\dot{\eta}^2\omega_o^2 + 2\eta\ddot{\eta}\omega_o^2)}{2\eta(s_0 + \eta\omega_o^2)} \\
&\quad - \frac{2s_1(\dddot{\eta}s_0 + \ddot{\eta}\dot{s}_0 + \dot{\eta}\ddot{s}_0 + \eta\dddot{s}_0 + 2\ddot{\eta}\dot{s}_0 + 2\dot{\eta}\ddot{s}_0 + 4\dot{\eta}\ddot{\eta}\omega_o^2 + 2\dot{\eta}\ddot{\eta}\omega_o^2 + 2\eta\dddot{\eta}\omega_o^2)}{2\eta(s_0 + \eta\omega_o^2)} \\
&\quad - \frac{2\dot{s}_1(\ddot{\eta}s_0 + \eta\ddot{s}_0 + 2\dot{\eta}\dot{s}_0 + 2\dot{\eta}^2\omega_o^2 + 2\eta\ddot{\eta}\omega_o^2)}{2\eta(s_0 + \eta\omega_o^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\dddot{\rho}\alpha + 2\rho\dddot{\alpha} + 6\ddot{\rho}\dot{\alpha} + 6\dot{\rho}\ddot{\alpha} - \dddot{d}\omega_o}{2\eta(s_0 + \eta\omega_o^2)} - \frac{6\ddot{s}_1(\dot{\eta}s_0 + \eta\dot{s}_0 + 2\eta\dot{\eta}\omega_o^2)}{2\eta(s_0 + \eta\omega_o^2)} \\
&\quad - \frac{2s_1(\dddot{\eta}s_0 + \eta\dddot{s}_0 + 3\ddot{\eta}\dot{s}_0 + 3\dot{\eta}\ddot{s}_0 + 6\dot{\eta}\ddot{\eta}\omega_o^2 + 2\eta\dddot{\eta}\omega_o^2)}{2\eta(s_0 + \eta\omega_o^2)} - \frac{6\dot{s}_1(\ddot{\eta}s_0 + \eta\ddot{s}_0 + 2\dot{\eta}\dot{s}_0 + 2\dot{\eta}^2\omega_o^2 + 2\eta\ddot{\eta}\omega_o^2)}{2\eta(s_0 + \eta\omega_o^2)} \\
&= \frac{2\kappa\varphi_d\mathfrak{x}_k + 2\rho\kappa\varrho_z + 6\kappa\varrho_x\mathfrak{x}_j + 6\kappa\varrho_y\mathfrak{x}_i - \mathfrak{x}_h\omega_o}{2\kappa\varepsilon_j(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)} - \frac{6(\mathfrak{x}_m/\kappa)(\kappa^2\delta_b\varphi_g + \kappa^2\varepsilon_j\varrho_i + 2\kappa^2\delta_b\varepsilon_j\omega_o^2)}{2\kappa\varepsilon_j(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)} \\
&\quad - \frac{2(\varphi_h/\kappa)(\kappa^2\varphi_g\mathfrak{d}_b + \kappa^2\varepsilon_j\varrho_o + 3\kappa^2\varrho_i\mathfrak{d}_a + 3\kappa^2\delta_b\varrho_n + 6\kappa^2\delta_b\mathfrak{d}_a\omega_o^2 + 2\kappa^2\varepsilon_j\mathfrak{d}_b\omega_o^2)}{2\kappa\varepsilon_j(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)} \\
&\quad - \frac{6(\mathfrak{x}_l/\kappa)(\kappa^2\varphi_g\mathfrak{d}_a + \kappa^2\varepsilon_j\varrho_n + 2\kappa^2\delta_b\varrho_i + 2\kappa^2\delta_b^2\omega_o^2 + 2\kappa^2\varepsilon_j\mathfrak{d}_a\omega_o^2)}{2\kappa\varepsilon_j(\kappa\varphi_g + \kappa\varepsilon_j\omega_o^2)} \\
&\quad \text{by (5.4), (5.5), (5.27), (5.28), (5.31), (5.34c), (5.35), (5.36a) \& (5.36b)} \\
&= \frac{2\varphi_d\mathfrak{x}_k + 2\rho\varrho_z + 6\varrho_x\mathfrak{x}_j + 6\varrho_y\mathfrak{x}_i - c\mathfrak{x}_h}{2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} - \frac{6\mathfrak{x}_m(\delta_b\varphi_g + \varepsilon_j\varrho_i + 2\delta_b\varepsilon_j\omega_o^2)}{2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} \\
&\quad - \frac{2\varphi_h(\varphi_g\mathfrak{d}_b + \varepsilon_j\varrho_o + 3\varrho_i\mathfrak{d}_a + 3\delta_b\varrho_n + 6\delta_b\mathfrak{d}_a\omega_o^2 + 2\varepsilon_j\mathfrak{d}_b\omega_o^2)}{2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} \qquad (5.36c) \\
&\quad - \frac{6\mathfrak{x}_l(\varphi_g\mathfrak{d}_a + \varepsilon_j\varrho_n + 2\delta_b\varrho_i + 2\delta_b^2\omega_o^2 + 2\varepsilon_j\mathfrak{d}_a\omega_o^2)}{2\kappa\varepsilon_j(\varphi_g + \varepsilon_j\omega_o^2)} = \mathfrak{x}_n/\kappa \text{ by (5.23j)}
\end{aligned}
$$

$$\begin{aligned}\dot{s_2} &= [s_0 s_1]' \text{ by (1.4d)}\\ &= s_0\dot{s_1} + \dot{s_0}s_1 = (\kappa\varphi_g)(\mathfrak{x}_l/\kappa) + (\kappa\varrho_i)(\varphi_h/\kappa) \text{ by (5.5), (5.28a) \& (5.36a)}\\ &= \varphi_g\mathfrak{x}_l + \varrho_i\varphi_h = \mathfrak{x}_o \text{ by (5.23k)}\end{aligned} \tag{5.37a}$$

$$\begin{aligned}\ddot{s_2} &= [s_0\dot{s_1} + \dot{s_0}s_1]' \text{ by (5.37a)}\\ &= \dot{s_0}\dot{s_1} + s_0\ddot{s_1} + \ddot{s_0}s_1 + \dot{s_0}\dot{s_1} = 2\dot{s_0}\dot{s_1} + s_0\ddot{s_1} + \ddot{s_0}s_1\\ &= 2(\kappa\varrho_i)(\mathfrak{x}_l/\kappa) + (\kappa\varphi_g)(\mathfrak{x}_m/\kappa) + (\kappa\varrho_n)(\varphi_h/\kappa) \text{ by (5.5), (5.28) \& (5.36)}\\ &= 2\varrho_i\mathfrak{x}_l + \varphi_g\mathfrak{x}_m + \varrho_n\varphi_h = \mathfrak{x}_p \text{ by (5.23k)}\end{aligned} \tag{5.37b}$$

$$\begin{aligned}\dddot{s_2} &= [2\dot{s_0}\dot{s_1} + s_0\ddot{s_1} + \ddot{s_0}s_1]' \text{ by (5.37b)}\\ &= 2\ddot{s_0}\dot{s_1} + 2\dot{s_0}\ddot{s_1} + \dot{s_0}\ddot{s_1} + s_0\dddot{s_1} + \dddot{s_0}s_1 + \ddot{s_0}\dot{s_1} = 3\ddot{s_0}\dot{s_1} + 3\dot{s_0}\ddot{s_1} + s_0\dddot{s_1} + \dddot{s_0}s_1\\ &= 3(\kappa\varrho_n)(\mathfrak{x}_l/\kappa) + 3(\kappa\varrho_i)(\mathfrak{x}_m/\kappa) + (\kappa\varphi_g)(\mathfrak{x}_n/\kappa) + (\kappa\varrho_o)(\varphi_h/\kappa) \text{ by (5.5), (5.28) \& (5.36)}\\ &= 3(\varrho_n\mathfrak{x}_l + \varrho_i\mathfrak{x}_m) + \varphi_g\mathfrak{x}_n + \varrho_o\varphi_h = \mathfrak{x}_q \text{ by (5.23k)}\end{aligned} \tag{5.37c}$$

$$\begin{aligned}\dot{s_3} &= [\{\eta(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) + 4\xi(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})\}s_1]' \text{ by (1.4d)}\\ &= \dot{s_1}[\eta(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) + 4\xi(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})] + s_1[\dot{\eta}(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) + \eta(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) + 4\dot{\xi}(\mathbf{\Omega}\cdot\widehat{\mathbf{p}}) + 4\xi(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})]\\ &= \dot{s_1}(\eta\varepsilon_i + 4\xi\varepsilon_c) + s_1(\dot{\eta}\varepsilon_i + \eta\varsigma_a + 4\dot{\xi}\varepsilon_c + 4\xi\varepsilon_i) \text{ by (5.1a) \& (5.23a)}\\ &= (\eta\dot{s_1} + \dot{\eta}s_1 + 4\xi s_1)\varepsilon_i + (4\xi\dot{s_1} + 4\dot{\xi}s_1)\varepsilon_c + \eta s_1\varsigma_a\\ &= [\kappa\varepsilon_j(\mathfrak{x}_l/\kappa) + \kappa\delta_b(\varphi_h/\kappa) + 8\kappa\varphi_e(\varphi_h/\kappa)]\varepsilon_i + [8\kappa\varphi_e(\mathfrak{x}_l/\kappa) + 8\kappa\varrho_a(\varphi_h/\kappa)]\varepsilon_c\\ &\qquad + \kappa\varepsilon_j(\varphi_h/\kappa)\varsigma_a \text{ by (5.4), (5.5b), (5.25a), (5.27) \& (5.36a)}\\ &= \varepsilon_j(\varepsilon_i\mathfrak{x}_l + \varphi_h\varsigma_a) + \varepsilon_i\varphi_h(\delta_b + 8\varphi_e) + 8\varepsilon_c(\varphi_e\mathfrak{x}_l + \varrho_a\varphi_h)\\ &= \mathfrak{x}_r \text{ by (5.23k)}\end{aligned} \tag{5.38a}$$

$$\begin{aligned}\ddot{s_3} &= [(\eta\dot{s_1} + \dot{\eta}s_1 + 4\xi s_1)(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) + (4\xi\dot{s_1} + 4\dot{\xi}s_1)(\mathbf{\Omega}\cdot\widehat{\mathbf{p}}) + \eta s_1(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})]' \text{ by (5.38a)}\\ &= (\dot{\eta}\dot{s_1} + \eta\ddot{s_1} + \ddot{\eta}s_1 + \dot{\eta}\dot{s_1} + 4\dot{\xi}s_1 + 4\xi\dot{s_1})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) + (\eta\dot{s_1} + \dot{\eta}s_1 + 4\xi s_1)(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})\\ &\quad + (4\dot{\xi}\dot{s_1} + 4\xi\ddot{s_1} + 4\ddot{\xi}s_1 + 4\dot{\xi}\dot{s_1})(\mathbf{\Omega}\cdot\widehat{\mathbf{p}}) + (4\xi\dot{s_1} + 4\dot{\xi}s_1)(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})\\ &\quad + (\dot{\eta}s_1 + \eta\dot{s_1})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) + \eta s_1(\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})\\ &= (\eta\ddot{s_1} + \ddot{\eta}s_1 + 2\dot{\eta}\dot{s_1} + 8\dot{\xi}s_1 + 8\xi\dot{s_1})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) + (2\eta\dot{s_1} + 2\dot{\eta}s_1 + 4\xi s_1)(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})\\ &\quad + (4\xi\ddot{s_1} + 4\ddot{\xi}s_1 + 8\dot{\xi}\dot{s_1})(\mathbf{\Omega}\cdot\widehat{\mathbf{p}}) + \eta s_1(\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})\\ &= [\kappa\varepsilon_j(\mathfrak{x}_m/\kappa) + \kappa\mathfrak{d}_a(\varphi_h/\kappa) + 2\kappa\delta_b(\mathfrak{x}_l/\kappa) + 16\kappa\varrho_a(\varphi_h/\kappa) + 16\kappa\varphi_e(\mathfrak{x}_l/\kappa)]\varepsilon_i\\ &\quad + [2\kappa\varepsilon_j(\mathfrak{x}_l/\kappa) + 2\kappa\delta_b(\varphi_h/\kappa) + 8\kappa\varphi_e(\varphi_h/\kappa)]\varsigma_a + [8\kappa\varphi_e(\mathfrak{x}_m/\kappa) + 8\kappa\varrho_b(\varphi_h/\kappa) + 16\kappa\varrho_a(\mathfrak{x}_l/\kappa)]\varepsilon_c\\ &\quad + \kappa\varepsilon_j(\varphi_h/\kappa)\varsigma_f \text{ by (5.1a), (5.4), (5.5b), (5.23a), (5.25), (5.27) \& (5.36)}\\ &= \varepsilon_j(\varphi_h\varsigma_f + \varepsilon_i\mathfrak{x}_m) + \varepsilon_i(\mathfrak{d}_a\varphi_h + 2\delta_b\mathfrak{x}_l + 16\varrho_a\varphi_h + 16\varphi_e\mathfrak{x}_l)\\ &\quad + 2\varsigma_a(\varepsilon_j\mathfrak{x}_l + \delta_b\varphi_h + 4\varphi_e\varphi_h) + 8\varepsilon_c(\varphi_e\mathfrak{x}_m + \varrho_b\varphi_h + 2\varrho_a\mathfrak{x}_l)\\ &= \mathfrak{x}_s \text{ by (5.23k)}\end{aligned} \tag{5.38b}$$

$$
\begin{aligned}
\dddot{s}_3 &= [(\eta\ddot{s}_1 + \ddot{\eta}s_1 + 2\dot{\eta}\dot{s}_1 + 8\dot{\xi}s_1 + 8\xi\dot{s}_1)(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}}) + (2\eta\dot{s}_1 + 2\dot{\eta}s_1 + 4\xi s_1)(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) \\
&\quad + (4\xi\ddot{s}_1 + 4\ddot{\xi}s_1 + 8\dot{\xi}\dot{s}_1)(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}}) + \eta s_1(\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})]' \text{ by (5.38b)} \\
&= (\dot{\eta}\ddot{s}_1 + \eta\dddot{s}_1 + \dddot{\eta}\, s_1 + \ddot{\eta}\dot{s}_1 + 2\ddot{\eta}\dot{s}_1 + 2\dot{\eta}\ddot{s}_1 + 8\ddot{\xi}s_1 + 8\dot{\xi}\dot{s}_1 + 8\dot{\xi}\dot{s}_1 + 8\xi\ddot{s}_1)(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}}) \\
&\quad + (\eta\ddot{s}_1 + \ddot{\eta}s_1 + 2\dot{\eta}\dot{s}_1 + 8\dot{\xi}s_1 + 8\xi\dot{s}_1)(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) + (2\dot{\eta}\dot{s}_1 + 2\eta\ddot{s}_1 + 2\ddot{\eta}s_1 + 2\dot{\eta}\dot{s}_1 + 4\dot{\xi}s_1 + 4\xi\dot{s}_1)(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) \\
&\quad + (2\eta\dot{s}_1 + 2\dot{\eta}s_1 + 4\xi s_1)(\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) + (4\dot{\xi}\ddot{s}_1 + 4\xi\dddot{s}_1 + 4\dddot{\xi}\, s_1 + 4\ddot{\xi}\dot{s}_1 + 8\ddot{\xi}\dot{s}_1 + 8\dot{\xi}\ddot{s}_1)(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}}) \\
&\quad + (4\xi\ddot{s}_1 + 4\ddot{\xi}s_1 + 8\dot{\xi}\dot{s}_1)(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}}) + (\dot{\eta}s_1 + \eta\dot{s}_1)(\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) + \eta s_1(\dddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) \\
&= (\eta\dddot{s}_1 + \dddot{\eta}\, s_1 + 3\ddot{\eta}\dot{s}_1 + 3\dot{\eta}\ddot{s}_1 + 12\ddot{\xi}s_1 + 24\dot{\xi}\dot{s}_1 + 12\xi\ddot{s}_1)(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}}) \\
&\quad + (3\eta\ddot{s}_1 + 3\ddot{\eta}s_1 + 6\dot{\eta}\dot{s}_1 + 12\dot{\xi}s_1 + 12\xi\dot{s}_1)(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) + (3\eta\dot{s}_1 + 3\dot{\eta}s_1 + 4\xi s_1)(\ddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) \\
&\quad + (4\xi\dddot{s}_1 + 4\dddot{\xi}\, s_1 + 12\ddot{\xi}\dot{s}_1 + 12\dot{\xi}\ddot{s}_1)(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}}) + \eta s_1(\dddot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})
\end{aligned}
$$

$$
\begin{aligned}
&= [\kappa\varepsilon_j(\mathfrak{x}_n/\kappa) + \kappa\mathfrak{d}_b(\varphi_h/\kappa) + 3\kappa\mathfrak{d}_a(\mathfrak{x}_l/\kappa) + 3\kappa\delta_b(\mathfrak{x}_m/\kappa) + 24\kappa\varrho_b(\varphi_h/\kappa) + 48\kappa\varrho_a(\mathfrak{x}_l/\kappa) \\
&\quad + 24\kappa\varphi_e(\mathfrak{x}_m/\kappa)]\varepsilon_i + [3\kappa\varepsilon_j(\mathfrak{x}_m/\kappa) + 3\kappa\mathfrak{d}_a(\varphi_h/\kappa) + 6\kappa\delta_b(\mathfrak{x}_l/\kappa) + 24\kappa\varrho_a(\varphi_h/\kappa) \\
&\quad + 24\kappa\varphi_e(\mathfrak{x}_l/\kappa)]\varsigma_a + [3\kappa\varepsilon_j(\mathfrak{x}_l/\kappa) + 3\kappa\delta_b(\varphi_h/\kappa) + 8\kappa\varphi_e(\varphi_h/\kappa)]\varsigma_f \\
&\quad + [8\kappa\varphi_e(\mathfrak{x}_n/\kappa) + 8\kappa\varrho_d(\varphi_h/\kappa) + 24\kappa\varrho_b(\mathfrak{x}_l/\kappa) + 24\kappa\varrho_a(\mathfrak{x}_m/\kappa)]\varepsilon_c + \kappa\varepsilon_j(\varphi_h/\kappa)\varsigma_l \\
&\quad \text{by (5.1a), (5.4), (5.5b), (5.23a), (5.25), (5.27) \& (5.36)} \\
&= \varepsilon_j(\varphi_h\varsigma_l + \varepsilon_i\mathfrak{x}_n) + \varepsilon_i(\mathfrak{d}_b\varphi_h + 3\mathfrak{d}_a\mathfrak{x}_l + 3\delta_b\mathfrak{x}_m + 24\varrho_b\varphi_h + 48\varrho_a\mathfrak{x}_l + 24\varphi_e\mathfrak{x}_m) \\
&\quad + 3\varsigma_a(\varepsilon_j\mathfrak{x}_m + \mathfrak{d}_a\varphi_h + 2\delta_b\mathfrak{x}_l + 8\varrho_a\varphi_h + 8\varphi_e\mathfrak{x}_l) + \varsigma_f(3\varepsilon_j\mathfrak{x}_l + 3\delta_b\varphi_h + 8\varphi_e\varphi_h) \\
&\quad + 8\varepsilon_c(\varphi_e\mathfrak{x}_n + \varrho_d\varphi_h + 3\varrho_b\mathfrak{x}_l + 3\varrho_a\mathfrak{x}_m) \\
&= \mathfrak{x}_t \text{ by (5.23k)}
\end{aligned}
\tag{5.38c}
$$

$$
\begin{aligned}
\dot{s}_4 &= [\{2\xi\Omega^2 + \eta(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})\}s_1]' \text{ by (1.4d)} \\
&= [2\dot{\xi}\Omega^2 + 4\xi\Omega\dot{\Omega} + \dot{\eta}(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}) + \eta(\boldsymbol{\Lambda}\cdot\boldsymbol{\Lambda}) + \eta(\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})]s_1 + [2\xi\Omega^2 + \eta(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})]\dot{s}_1 \\
&= 2\Omega^2(\dot{\xi}s_1 + \xi\dot{s}_1) + (\eta\dot{s}_1 + \dot{\eta}s_1 + 4\xi s_1)(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}) + \eta s_1[(\boldsymbol{\Lambda}\cdot\boldsymbol{\Lambda}) + (\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})] \\
&= 2\Omega^2[2\kappa\varrho_a(\varphi_h/\kappa) + 2\kappa\varphi_e(\mathfrak{x}_l/\kappa)] + [\kappa\varepsilon_j(\mathfrak{x}_l/\kappa) + \kappa\delta_b(\varphi_h/\kappa) + 8\kappa\varphi_e(\varphi_h/\kappa)]\varepsilon_g \\
&\quad + \kappa\varepsilon_j(\varphi_h/\kappa)(\Lambda^2 + \varsigma_c) \text{ by (5.1a), (5.4), (5.5b), (5.23a), (5.25a), (5.27) \& (5.36a)} \\
&= 4\Omega^2(\varrho_a\varphi_h + \varphi_e\mathfrak{x}_l) + \varepsilon_g(\varepsilon_j\mathfrak{x}_l + \delta_b\varphi_h + 8\varphi_e\varphi_h) + \varepsilon_j\varphi_h(\Lambda^2 + \varsigma_c) \\
&= \mathfrak{x}_u \text{ by (5.23l)}
\end{aligned}
\tag{5.39a}
$$

$$
\begin{aligned}
\ddot{s}_4 &= [2\Omega^2(\dot{\xi}s_1 + \xi\dot{s}_1) + (\eta\dot{s}_1 + \dot{\eta}s_1 + 4\xi s_1)(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}) + \eta s_1(\boldsymbol{\Lambda}\cdot\boldsymbol{\Lambda}) + \eta s_1(\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})]' \text{ by (5.39a)} \\
&= 4\Omega\dot{\Omega}(\dot{\xi}s_1 + \xi\dot{s}_1) + 2\Omega^2(\ddot{\xi}s_1 + \dot{\xi}\dot{s}_1 + \dot{\xi}\dot{s}_1 + \xi\ddot{s}_1) \\
&\quad + (\dot{\eta}\dot{s}_1 + \eta\ddot{s}_1 + \ddot{\eta}s_1 + \dot{\eta}\dot{s}_1 + 4\dot{\xi}s_1 + 4\xi\dot{s}_1)(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}) + (\eta\dot{s}_1 + \dot{\eta}s_1 + 4\xi s_1)[(\boldsymbol{\Lambda}\cdot\boldsymbol{\Lambda}) + (\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})] \\
&\quad + (\dot{\eta}s_1 + \eta\dot{s}_1)(\boldsymbol{\Lambda}\cdot\boldsymbol{\Lambda}) + 2\eta s_1(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Lambda}) + (\dot{\eta}s_1 + \eta\dot{s}_1)(\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}}) + \eta s_1[(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) + (\boldsymbol{\Omega}\cdot\ddot{\boldsymbol{\Lambda}})] \\
&= 2\Omega^2(\ddot{\xi}s_1 + \xi\ddot{s}_1 + 2\dot{\xi}\dot{s}_1) + (\eta\ddot{s}_1 + \ddot{\eta}s_1 + 2\dot{\eta}\dot{s}_1 + 8\dot{\xi}s_1 + 8\xi\dot{s}_1)(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}) \\
&\quad + (2\eta\dot{s}_1 + 2\dot{\eta}s_1 + 4\xi s_1)[(\boldsymbol{\Lambda}\cdot\boldsymbol{\Lambda}) + (\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})] + \eta s_1[3(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Lambda}) + (\boldsymbol{\Omega}\cdot\ddot{\boldsymbol{\Lambda}})] \\
&= 2\Omega^2[2\kappa\varrho_b(\varphi_h/\kappa) + 2\kappa\varphi_e(\mathfrak{x}_m/\kappa) + 4\kappa\varrho_a(\mathfrak{x}_l/\kappa)] + [\kappa\varepsilon_j(\mathfrak{x}_m/\kappa) + \kappa\mathfrak{d}_a(\varphi_h/\kappa) + 2\kappa\delta_b(\mathfrak{x}_l/\kappa) \\
&\quad + 16\kappa\varrho_a(\varphi_h/\kappa) + 16\kappa\varphi_e(\mathfrak{x}_l/\kappa)]\varepsilon_g + [2\kappa\varepsilon_j(\mathfrak{x}_l/\kappa) + 2\kappa\delta_b(\varphi_h/\kappa) + 8\kappa\varphi_e(\varphi_h/\kappa)](\Lambda^2 + \varsigma_c) \\
&\quad + \kappa\varepsilon_j(\varphi_h/\kappa)(3\varsigma_d + \varsigma_h) \text{ by (5.1a), (5.4), (5.5b), (5.23a), (5.25), (5.27) \& (5.36)} \\
&= 4\Omega^2(\varrho_b\varphi_h + \varphi_e\mathfrak{x}_m + 2\varrho_a\mathfrak{x}_l) + \varepsilon_g(\varepsilon_j\mathfrak{x}_m + \mathfrak{d}_a\varphi_h + 2\delta_b\mathfrak{x}_l + 16\varrho_a\varphi_h + 16\varphi_e\mathfrak{x}_l) \\
&\quad + 2(\Lambda^2 + \varsigma_c)(\varepsilon_j\mathfrak{x}_l + \delta_b\varphi_h + 4\varphi_e\varphi_h) + \varepsilon_j\varphi_h(3\varsigma_d + \varsigma_h) \\
&= \mathfrak{x}_v \text{ by (5.23l)}
\end{aligned}
\tag{5.39b}
$$

$$
\begin{aligned}
\dddot{s}_4 &= [2\Omega^2(\ddot{\xi}s_1 + \xi\ddot{s}_1 + 2\dot{\xi}\dot{s}_1) + (\eta\ddot{s}_1 + \ddot{\eta}s_1 + 2\dot{\eta}\dot{s}_1 + 8\dot{\xi}s_1 + 8\xi\dot{s}_1)(\mathbf{\Omega}\cdot\mathbf{\Lambda}) \\
&\quad + (2\eta\dot{s}_1 + 2\dot{\eta}s_1 + 4\xi s_1)(\mathbf{\Lambda}\cdot\mathbf{\Lambda}) + (2\eta\dot{s}_1 + 2\dot{\eta}s_1 + 4\xi s_1)(\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}}) \\
&\quad + 3\eta s_1(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) + \eta s_1(\mathbf{\Omega}\cdot\ddot{\mathbf{\Lambda}})]' \text{ by } (5.39\text{b}) \\
&= 4\Omega\dot{\Omega}(\ddot{\xi}s_1 + \xi\ddot{s}_1 + 2\dot{\xi}\dot{s}_1) + 2\Omega^2(\dddot{\xi}s_1 + \ddot{\xi}\dot{s}_1 + \dot{\xi}\ddot{s}_1 + \xi\dddot{s}_1 + 2\ddot{\xi}\dot{s}_1 + 2\dot{\xi}\ddot{s}_1) \\
&\quad + (\dot{\eta}\ddot{s}_1 + \eta\dddot{s}_1 + \dddot{\eta}s_1 + \ddot{\eta}\dot{s}_1 + 2\ddot{\eta}\dot{s}_1 + 2\dot{\eta}\ddot{s}_1 + 8\ddot{\xi}s_1 + 8\dot{\xi}\dot{s}_1 + 8\dot{\xi}\dot{s}_1 + 8\xi\ddot{s}_1)(\mathbf{\Omega}\cdot\mathbf{\Lambda}) \\
&\quad + (\eta\ddot{s}_1 + \ddot{\eta}s_1 + 2\dot{\eta}\dot{s}_1 + 8\dot{\xi}s_1 + 8\xi\dot{s}_1)[(\mathbf{\Lambda}\cdot\mathbf{\Lambda}) + (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})] \\
&\quad + (2\dot{\eta}\dot{s}_1 + 2\eta\ddot{s}_1 + 2\ddot{\eta}s_1 + 2\dot{\eta}\dot{s}_1 + 4\dot{\xi}s_1 + 4\xi\dot{s}_1)(\mathbf{\Lambda}\cdot\mathbf{\Lambda}) + 2(2\eta\dot{s}_1 + 2\dot{\eta}s_1 + 4\xi s_1)(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) \\
&\quad + (2\dot{\eta}\dot{s}_1 + 2\eta\ddot{s}_1 + 2\ddot{\eta}s_1 + 2\dot{\eta}\dot{s}_1 + 4\dot{\xi}s_1 + 4\xi\dot{s}_1)(\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}}) \\
&\quad + (2\eta\dot{s}_1 + 2\dot{\eta}s_1 + 4\xi s_1)[(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) + (\mathbf{\Omega}\cdot\ddot{\mathbf{\Lambda}})] + 3(\dot{\eta}s_1 + \eta\dot{s}_1)(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) \\
&\quad + 3\eta s_1[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) + (\dot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})] + (\dot{\eta}s_1 + \eta\dot{s}_1)(\mathbf{\Omega}\cdot\ddot{\mathbf{\Lambda}}) + \eta s_1[(\mathbf{\Lambda}\cdot\ddot{\mathbf{\Lambda}}) + (\mathbf{\Omega}\cdot\dddot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= 2\Omega^2(\dddot{\xi}s_1 + \xi\dddot{s}_1 + 3\ddot{\xi}\dot{s}_1 + 3\dot{\xi}\ddot{s}_1) + \eta s_1[4(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) + 3(\dot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}}) + (\mathbf{\Omega}\cdot\dddot{\mathbf{\Lambda}})] \\
&\quad + (\eta\dddot{s}_1 + \dddot{\eta}s_1 + 3\ddot{\eta}\dot{s}_1 + 3\dot{\eta}\ddot{s}_1 + 12\ddot{\xi}s_1 + 24\dot{\xi}\dot{s}_1 + 12\xi\ddot{s}_1)(\mathbf{\Omega}\cdot\mathbf{\Lambda}) \\
&\quad + (3\eta\ddot{s}_1 + 3\ddot{\eta}s_1 + 6\dot{\eta}\dot{s}_1 + 12\dot{\xi}s_1 + 12\xi\dot{s}_1)[(\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}}) + (\mathbf{\Lambda}\cdot\mathbf{\Lambda})] \\
&\quad + (3\eta\dot{s}_1 + 3\dot{\eta}s_1 + 4\xi s_1)[(\mathbf{\Omega}\cdot\ddot{\mathbf{\Lambda}}) + 3(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})] \\
&= 2\Omega^2[2\kappa\varrho_d(\varphi_h/\kappa) + 2\kappa\varphi_e(\mathfrak{x}_n/\kappa) + 6\kappa\varrho_b(\mathfrak{x}_l/\kappa) + 6\kappa\varrho_a(\mathfrak{x}_m/\kappa)] + \kappa\varepsilon_j(\varphi_h/\kappa)(4\varsigma_i + 3\varsigma_e + \varsigma_m) \\
&\quad + [\kappa\varepsilon_j(\mathfrak{x}_n/\kappa) + \kappa\mathfrak{d}_b(\varphi_h/\kappa) + 3\kappa\mathfrak{d}_a(\mathfrak{x}_l/\kappa) + 3\kappa\delta_b(\mathfrak{x}_m/\kappa) + 24\kappa\varrho_b(\varphi_h/\kappa) + 48\kappa\varrho_a(\mathfrak{x}_l/\kappa) \\
&\quad + 24\kappa\varphi_e(\mathfrak{x}_m/\kappa)]\varepsilon_g + [3\kappa\varepsilon_j(\mathfrak{x}_m/\kappa) + 3\kappa\mathfrak{d}_a(\varphi_h/\kappa) + 6\kappa\delta_b(\mathfrak{x}_l/\kappa) + 24\kappa\varrho_a(\varphi_h/\kappa) \\
&\quad + 24\kappa\varphi_e(\mathfrak{x}_l/\kappa)](\Lambda^2 + \varsigma_c) + [3\kappa\varepsilon_j(\mathfrak{x}_l/\kappa) + 3\kappa\delta_b(\varphi_h/\kappa) + 8\kappa\varphi_e(\varphi_h/\kappa)](\varsigma_h + 3\varsigma_d) \\
&\quad \text{by } (5.1\text{a}), (5.4), (5.5\text{b}), (5.23\text{a}), (5.25), (5.27) \,\&\, (5.36) \\
&= 4\Omega^2(\varrho_d\varphi_h + \varphi_e\mathfrak{x}_n + 3\varrho_b\mathfrak{x}_l + 3\varrho_a\mathfrak{x}_m) + \varepsilon_j\varphi_h(4\varsigma_i + 3\varsigma_e + \varsigma_m) \\
&\quad + \varepsilon_g(\varepsilon_j\mathfrak{x}_n + \mathfrak{d}_b\varphi_h + 3\mathfrak{d}_a\mathfrak{x}_l + 3\delta_b\mathfrak{x}_m + 24\varrho_b\varphi_h + 48\varrho_a\mathfrak{x}_l + 24\varphi_e\mathfrak{x}_m) \\
&\quad + 3(\Lambda^2 + \varsigma_c)(\varepsilon_j\mathfrak{x}_m + \mathfrak{d}_a\varphi_h + 2\delta_b\mathfrak{x}_l + 8\varrho_a\varphi_h + 8\varphi_e\mathfrak{x}_l) \\
&\quad + (\varsigma_h + 3\varsigma_d)(3\varepsilon_j\mathfrak{x}_l + 3\delta_b\varphi_h + 8\varphi_e\varphi_h) \\
&= \mathfrak{x}_w \text{ by } (5.23\text{l})
\end{aligned}
\tag{5.39c}
$$

in consequence of which the derivatives of τ and $\mathbf{e}$ are computed as

$$
\begin{aligned}
\dot{\mathbf{e}} &= [s_2(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + s_3\mathbf{\Omega} - s_4\widehat{\mathbf{p}}]' \text{ by } (1.4\text{b}) \\
&= \dot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + s_2(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \dot{s}_3\mathbf{\Omega} + s_3\mathbf{\Lambda} - \dot{s}_4\widehat{\mathbf{p}} \\
&= \mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{x}_r\mathbf{\Omega} + \varphi_i\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}} \\
&\quad \text{by } (5.5), (5.37\text{a}), (5.38\text{a}) \,\&\, (5.39\text{a})
\end{aligned}
\tag{5.40a}
$$

$$
\begin{aligned}
\ddot{\mathbf{e}} &= [\dot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + s_2(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \dot{s}_3\mathbf{\Omega} + s_3\mathbf{\Lambda} - \dot{s}_4\widehat{\mathbf{p}}]' \text{ by } (5.40\text{a}) \\
&= \ddot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \dot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \dot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + s_2(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \ddot{s}_3\mathbf{\Omega} + \dot{s}_3\mathbf{\Lambda} + \dot{s}_3\mathbf{\Lambda} + s_3\dot{\mathbf{\Lambda}} - \ddot{s}_4\widehat{\mathbf{p}} \\
&= \ddot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + 2\dot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + s_2(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \ddot{s}_3\mathbf{\Omega} + 2\dot{s}_3\mathbf{\Lambda} + s_3\dot{\mathbf{\Lambda}} - \ddot{s}_4\widehat{\mathbf{p}} \\
&= \mathfrak{x}_p(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + 2\mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{x}_s\mathbf{\Omega} + 2\mathfrak{x}_r\mathbf{\Lambda} + \varphi_i\dot{\mathbf{\Lambda}} - \mathfrak{x}_v\widehat{\mathbf{p}} \\
&\quad \text{by } (5.5), (5.37), (5.38) \,\&\, (5.39\text{b})
\end{aligned}
\tag{5.40b}
$$

$$
\begin{aligned}
\dddot{\mathbf{e}} &= [\ddot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + 2\dot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + s_2(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \ddot{s}_3\mathbf{\Omega} + 2\dot{s}_3\mathbf{\Lambda} + s_3\dot{\mathbf{\Lambda}} - \ddot{s}_4\widehat{\mathbf{p}}]' \text{ by } (5.40\text{b}) \\
&= \dddot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \ddot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + 2\ddot{s}_2(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + 2\dot{s}_2(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \dot{s}_2(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + s_2(\widehat{\mathbf{p}}\times\ddot{\mathbf{\Lambda}}) \\
&\quad + \dddot{s}_3\mathbf{\Omega} + \ddot{s}_3\mathbf{\Lambda} + 2\ddot{s}_3\mathbf{\Lambda} + 2\dot{s}_3\dot{\mathbf{\Lambda}} + \dot{s}_3\dot{\mathbf{\Lambda}} + s_3\ddot{\mathbf{\Lambda}} - \dddot{s}_4\widehat{\mathbf{p}}
\end{aligned}
$$

$$\begin{aligned}
&= \dddot{s}_2(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + 3\ddot{s}_2(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + 3\dot{s}_2(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + s_2(\widehat{\mathbf{p}} \times \ddot{\boldsymbol{\Lambda}}) + \dddot{s}_3\boldsymbol{\Omega} + 3\ddot{s}_3\boldsymbol{\Lambda} + 3\dot{s}_3\dot{\boldsymbol{\Lambda}} \\
&\quad + s_3\ddot{\boldsymbol{\Lambda}} - \dddot{s}_4\widehat{\mathbf{p}} \\
&= \mathfrak{x}_q(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + 3\mathfrak{x}_p(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + 3\mathfrak{x}_o(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \varphi_g\varphi_h(\widehat{\mathbf{p}} \times \ddot{\boldsymbol{\Lambda}}) + \mathfrak{x}_t\boldsymbol{\Omega} + 3\mathfrak{x}_s\boldsymbol{\Lambda} + 3\mathfrak{x}_r\dot{\boldsymbol{\Lambda}} \\
&\quad + \varphi_i\ddot{\boldsymbol{\Lambda}} - \mathfrak{x}_w\widehat{\mathbf{p}} \text{ by } (5.5), (5.37), (5.38) \;\&\; (5.39\text{c}) \qquad (5.40\text{c})
\end{aligned}$$

$$\begin{aligned}
\dot{\tau} &= 2\kappa^{-2}[\rho\alpha - \eta s_2]' \text{ by } (1.4\text{b}) \\
&= 2\kappa^{-2}(\dot{\rho}\alpha + \rho\dot{\alpha} - \dot{\eta}s_2 - \eta\dot{s}_2) \\
&= 2\kappa^{-2}(\kappa\varrho_d\mathfrak{x}_i + \rho\kappa\varrho_x - \kappa\delta_b\varphi_g\varphi_h - \kappa\varepsilon_j\mathfrak{x}_o) \\
&\qquad \text{by } (5.4), (5.5\text{c}), (5.27), (5.31\text{a}), (5.35\text{a}) \;\&\; (5.37\text{a}) \\
&= 2\kappa^{-1}(\varrho_d\mathfrak{x}_i + \rho\varrho_x - \delta_b\varphi_g\varphi_h - \varepsilon_j\mathfrak{x}_o) \\
&= 2(\mathfrak{x}_x/\kappa) \text{ by } (5.23\text{m}) \qquad (5.41\text{a})
\end{aligned}$$

$$\begin{aligned}
\ddot{\tau} &= 2\kappa^{-2}[\dot{\rho}\alpha + \rho\dot{\alpha} - \dot{\eta}s_2 - \eta\dot{s}_2]' \text{ by } (5.41\text{a}) \\
&= 2\kappa^{-2}(\ddot{\rho}\alpha + \dot{\rho}\dot{\alpha} + \dot{\rho}\dot{\alpha} + \rho\ddot{\alpha} - \ddot{\eta}s_2 - \dot{\eta}\dot{s}_2 - \dot{\eta}\dot{s}_2 - \eta\ddot{s}_2) \\
&= 2\kappa^{-2}(\ddot{\rho}\alpha + 2\dot{\rho}\dot{\alpha} + \rho\ddot{\alpha} - \ddot{\eta}s_2 - 2\dot{\eta}\dot{s}_2 - \eta\ddot{s}_2) \\
&= 2\kappa^{-2}(\mathfrak{x}_j\kappa\varrho_d + 2\mathfrak{x}_i\kappa\varrho_x + \rho\kappa\varrho_y - \kappa\mathfrak{d}_a\varphi_g\varphi_h - 2\kappa\delta_b\mathfrak{x}_o - \kappa\varepsilon_j\mathfrak{x}_p) \\
&\qquad \text{by } (5.4), (5.5\text{c}), (5.27), (5.31), (5.35) \;\&\; (5.37) \\
&= 2\kappa^{-1}(\mathfrak{x}_j\varrho_d + 2\varrho_x\mathfrak{x}_i + \rho\varrho_y - \mathfrak{d}_a\varphi_g\varphi_h - 2\delta_b\mathfrak{x}_o - \varepsilon_j\mathfrak{x}_p) \\
&= 2(\mathfrak{x}_y/\kappa) \text{ by } (5.23\text{m}) \qquad (5.41\text{b})
\end{aligned}$$

$$\begin{aligned}
\dddot{\tau} &= 2\kappa^{-2}[\ddot{\rho}\alpha + 2\dot{\rho}\dot{\alpha} + \rho\ddot{\alpha} - \ddot{\eta}s_2 - 2\dot{\eta}\dot{s}_2 - \eta\ddot{s}_2]' \text{ by } (5.41\text{b}) \\
&= 2\kappa^{-2}(\dddot{\rho}\alpha + \ddot{\rho}\dot{\alpha} + 2\ddot{\rho}\dot{\alpha} + 2\dot{\rho}\ddot{\alpha} + \dot{\rho}\ddot{\alpha} + \rho\dddot{\alpha} - \dddot{\eta}s_2 - \ddot{\eta}\dot{s}_2 - 2\ddot{\eta}\dot{s}_2 - 2\dot{\eta}\ddot{s}_2 - \dot{\eta}\ddot{s}_2 - \eta\dddot{s}_2) \\
&= 2\kappa^{-2}(\dddot{\rho}\alpha + 3\ddot{\rho}\dot{\alpha} + 3\dot{\rho}\ddot{\alpha} + \rho\dddot{\alpha} - \dddot{\eta}s_2 - 3\ddot{\eta}\dot{s}_2 - 3\dot{\eta}\ddot{s}_2 - \eta\dddot{s}_2) \\
&= 2\kappa^{-2}(\mathfrak{x}_k\kappa\varphi_d + 3\mathfrak{x}_j\kappa\varrho_x + 3\mathfrak{x}_i\kappa\varrho_y + \rho\kappa\varrho_z - \kappa\mathfrak{d}_b\varphi_g\varphi_h - 3\kappa\mathfrak{d}_a\mathfrak{x}_o - 3\kappa\delta_b\mathfrak{x}_p - \kappa\varepsilon_j\mathfrak{x}_q) \\
&\qquad \text{by } (5.4), (5.5\text{c}), (5.27), (5.31), (5.35) \;\&\; (5.37) \\
&= 2\kappa^{-1}(\mathfrak{x}_k\varphi_d + 3\mathfrak{x}_j\varrho_x + 3\mathfrak{x}_i\varrho_y + \rho\varrho_z - \mathfrak{d}_b\varphi_g\varphi_h - 3\mathfrak{d}_a\mathfrak{x}_o - 3\delta_b\mathfrak{x}_p - \varepsilon_j\mathfrak{x}_q) \\
&= 2(\mathfrak{x}_z/\kappa) \text{ by } (5.23\text{m}). \qquad (5.41\text{c})
\end{aligned}$$

Art 22e. *Development of equation* (3.15a).

The quantities defined by (3.15a) evaluate as

$$\begin{aligned}
\dot{\mathfrak{y}} &= c\dot{d} - \kappa\dot{\tau} \text{ by } (3.15\text{a}) \\
&= c\mathfrak{x}_f - 2\mathfrak{x}_x \text{ by } (5.34\text{a}) \;\&\; (5.41\text{a}) \qquad (5.42\text{a})
\end{aligned}$$

$$\begin{aligned}
\ddot{\mathfrak{y}} &= c\ddot{d} - \kappa\ddot{\tau} \text{ by } (3.15\text{a}) \\
&= c\mathfrak{x}_g - 2\mathfrak{x}_y \text{ by } (5.34\text{b}) \;\&\; (5.41\text{b}) \qquad (5.42\text{b})
\end{aligned}$$

$$\begin{aligned}
\dddot{\mathfrak{y}} &= c\dddot{d} - \kappa\dddot{\tau} \text{ by } (3.15\text{a}) \\
&= c\mathfrak{x}_h - 2\mathfrak{x}_z \text{ by } (5.34\text{c}) \;\&\; (5.41\text{c}) \qquad (5.42\text{c})
\end{aligned}$$

$$\begin{aligned}
b_1 &= \dot{\rho} - 1 \text{ by } (3.15\text{a}) \\
&= \mathfrak{x}_i - 1 \text{ by } (5.35\text{a}) \qquad (5.43\text{a})
\end{aligned}$$

$$\begin{aligned} b_2 &= 2\dot\rho - 1 \text{ by (3.15a)} \\ &= 2\mathfrak{x}_i - 1 \text{ by (5.35a)} \end{aligned} \tag{5.43b}$$

$$\begin{aligned} b_3 &= 3\dot\rho - 1 \text{ by (3.15a)} \\ &= 3\mathfrak{x}_i - 1 \text{ by (5.35a)} \end{aligned} \tag{5.43c}$$

$$\begin{aligned} b_4 &= \ddot\rho\dot{\mathfrak{Y}} - \ddot{\mathfrak{Y}} b_1 \text{ by (3.15a)} \\ &= \mathfrak{x}_j(c\mathfrak{x}_f - 2\mathfrak{x}_x) - (c\mathfrak{x}_g - 2\mathfrak{x}_y)(\mathfrak{x}_i - 1) \text{ by (5.35b), (5.42) \& (5.43a)} \end{aligned} \tag{5.43d}$$

$$\begin{aligned} b_5 &= \dot{\mathfrak{Y}} b_2 - \rho\ddot{\mathfrak{Y}} \text{ by (3.15a)} \\ &= (c\mathfrak{x}_f - 2\mathfrak{x}_x)(2\mathfrak{x}_i - 1) - \rho(c\mathfrak{x}_g - 2\mathfrak{x}_y) \text{ by (5.42) \& (5.43b)} \end{aligned} \tag{5.43e}$$

$$\begin{aligned} b_6 &= b_1 b_2 - \rho\ddot\rho \text{ by (3.15a)} \\ &= (\mathfrak{x}_i - 1)(2\mathfrak{x}_i - 1) - \rho\mathfrak{x}_j \text{ by (5.43a), (5.43b) \& (5.35b).} \end{aligned} \tag{5.43f}$$

Art 22f. *Development of equation* (3.15b).

The quantities defined by (3.15b) become

$$\begin{aligned} \mathfrak{J}_a &= \widehat{\boldsymbol\kappa} \times \mathbf{a} \text{ by (3.15b)} \\ &= \widehat{\boldsymbol\kappa} \times [(\boldsymbol\Omega \cdot \mathbf{r})\boldsymbol\Omega - \Omega^2\mathbf{r} + \boldsymbol\Lambda \times \mathbf{r}] \text{ by (1.4a)} \\ &= \varepsilon_b(\widehat{\boldsymbol\kappa} \times \boldsymbol\Omega) - \Omega^2(\widehat{\boldsymbol\kappa} \times \mathbf{r}) + \boldsymbol\Lambda(\widehat{\boldsymbol\kappa} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol\kappa} \cdot \boldsymbol\Lambda) \text{ by (5.1a) \& (A.1)} \\ &= \varepsilon_b(\widehat{\boldsymbol\kappa} \times \boldsymbol\Omega) - \Omega^2(\widehat{\boldsymbol\kappa} \times \mathbf{r}) + \varepsilon_d\boldsymbol\Lambda - \delta_a\mathbf{r} \text{ by (5.1a) \& (5.13a)} \end{aligned} \tag{5.44a}$$

$$\begin{aligned} \mathfrak{J}_b &= \widehat{\boldsymbol\kappa} \times \dot{\mathbf{a}} \text{ by (3.15b)} \\ &= \widehat{\boldsymbol\kappa} \times [2\varepsilon_h\boldsymbol\Omega - 3\varepsilon_g\mathbf{r} + \dot{\boldsymbol\Lambda} \times \mathbf{r} - \Omega^2(\boldsymbol\Omega \times \mathbf{r}) + \varepsilon_b\boldsymbol\Lambda] \text{ by (5.30a)} \\ &= \widehat{\boldsymbol\kappa} \times [2\varepsilon_h\boldsymbol\Omega - 3\varepsilon_g\mathbf{r} + \varepsilon_b\boldsymbol\Lambda + \dot{\boldsymbol\Lambda} \times \mathbf{r} - \Omega^2(\boldsymbol\Omega \times \mathbf{r})] \\ &= 2\varepsilon_h(\widehat{\boldsymbol\kappa} \times \boldsymbol\Omega) - 3\varepsilon_g(\widehat{\boldsymbol\kappa} \times \mathbf{r}) + \varepsilon_b(\widehat{\boldsymbol\kappa} \times \boldsymbol\Lambda) + \dot{\boldsymbol\Lambda}(\widehat{\boldsymbol\kappa} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol\kappa} \cdot \dot{\boldsymbol\Lambda}) \\ &\quad - \Omega^2[\boldsymbol\Omega(\widehat{\boldsymbol\kappa} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol\kappa} \cdot \boldsymbol\Omega)] \text{ by (A.1)} \end{aligned}$$

$$\begin{aligned} &= 2\varepsilon_h(\widehat{\boldsymbol\kappa} \times \boldsymbol\Omega) - 3\varepsilon_g(\widehat{\boldsymbol\kappa} \times \mathbf{r}) + \varepsilon_b(\widehat{\boldsymbol\kappa} \times \boldsymbol\Lambda) + \varepsilon_d\dot{\boldsymbol\Lambda} - \varsigma_b\mathbf{r} - \Omega^2(\varepsilon_d\boldsymbol\Omega - \varepsilon_a\mathbf{r}) \\ &\quad \text{by (5.1a) \& (5.23a)} \\ &= 2\varepsilon_h(\widehat{\boldsymbol\kappa} \times \boldsymbol\Omega) - 3\varepsilon_g(\widehat{\boldsymbol\kappa} \times \mathbf{r}) + \varepsilon_b(\widehat{\boldsymbol\kappa} \times \boldsymbol\Lambda) + \varepsilon_d\dot{\boldsymbol\Lambda} - \varepsilon_d\Omega^2\boldsymbol\Omega + (\Omega^2\varepsilon_a - \varsigma_b)\mathbf{r} \\ &= 2\varepsilon_h(\widehat{\boldsymbol\kappa} \times \boldsymbol\Omega) - 3\varepsilon_g(\widehat{\boldsymbol\kappa} \times \mathbf{r}) + \varepsilon_b(\widehat{\boldsymbol\kappa} \times \boldsymbol\Lambda) + \varepsilon_d\dot{\boldsymbol\Lambda} - \varepsilon_d\Omega^2\boldsymbol\Omega + \mathfrak{y}_a\mathbf{r} \text{ by (5.23n)} \end{aligned} \tag{5.44b}$$

$$\begin{aligned} \mathfrak{J}_c &= \widehat{\boldsymbol\kappa} \times \ddot{\mathbf{a}} \text{ by (3.15b)} \\ &= \widehat{\boldsymbol\kappa} \times [3\varepsilon_h\boldsymbol\Lambda + \varepsilon_b\dot{\boldsymbol\Lambda} + \varrho_t\boldsymbol\Omega + \varrho_u\mathbf{r} + 2\varepsilon_b(\boldsymbol\Lambda \times \boldsymbol\Omega) - 3\Omega^2(\boldsymbol\Lambda \times \mathbf{r}) \\ &\quad - 3\varepsilon_g(\boldsymbol\Omega \times \mathbf{r}) + (\ddot{\boldsymbol\Lambda} \times \mathbf{r})] \text{ by (5.30b)} \\ &= 3\varepsilon_h(\widehat{\boldsymbol\kappa} \times \boldsymbol\Lambda) + \varepsilon_b(\widehat{\boldsymbol\kappa} \times \dot{\boldsymbol\Lambda}) + \varrho_t(\widehat{\boldsymbol\kappa} \times \boldsymbol\Omega) + \varrho_u(\widehat{\boldsymbol\kappa} \times \mathbf{r}) + 2\varepsilon_b[\widehat{\boldsymbol\kappa} \times (\boldsymbol\Lambda \times \boldsymbol\Omega)] \\ &\quad - 3\Omega^2[\widehat{\boldsymbol\kappa} \times (\boldsymbol\Lambda \times \mathbf{r})] - 3\varepsilon_g[\widehat{\boldsymbol\kappa} \times (\boldsymbol\Omega \times \mathbf{r})] + \widehat{\boldsymbol\kappa} \times (\ddot{\boldsymbol\Lambda} \times \mathbf{r}) \\ &= 3\varepsilon_h(\widehat{\boldsymbol\kappa} \times \boldsymbol\Lambda) + \varepsilon_b(\widehat{\boldsymbol\kappa} \times \dot{\boldsymbol\Lambda}) + \varrho_t(\widehat{\boldsymbol\kappa} \times \boldsymbol\Omega) + \varrho_u(\widehat{\boldsymbol\kappa} \times \mathbf{r}) + 2\varepsilon_b[\boldsymbol\Lambda(\widehat{\boldsymbol\kappa} \cdot \boldsymbol\Omega) - \boldsymbol\Omega(\widehat{\boldsymbol\kappa} \cdot \boldsymbol\Lambda)] \\ &\quad - 3\Omega^2[\boldsymbol\Lambda(\widehat{\boldsymbol\kappa} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol\kappa} \cdot \boldsymbol\Lambda)] - 3\varepsilon_g[\boldsymbol\Omega(\widehat{\boldsymbol\kappa} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol\kappa} \cdot \boldsymbol\Omega)] + \ddot{\boldsymbol\Lambda}(\widehat{\boldsymbol\kappa} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol\kappa} \cdot \ddot{\boldsymbol\Lambda}) \text{ by (A.1)} \end{aligned}$$

$$
\begin{aligned}
&= 3\varepsilon_h(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \varrho_t(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varrho_u(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + 2\varepsilon_b(\varepsilon_a\boldsymbol{\Lambda} - \delta_a\boldsymbol{\Omega}) \\
&\quad - 3\Omega^2(\varepsilon_d\boldsymbol{\Lambda} - \delta_a\mathbf{r}) - 3\varepsilon_g(\varepsilon_d\boldsymbol{\Omega} - \varepsilon_a\mathbf{r}) + \varepsilon_d\ddot{\boldsymbol{\Lambda}} - \varsigma_g\mathbf{r} \text{ by (5.1a), (5.13a) \& (5.23a)} \\
&= 3\varepsilon_h(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \varrho_t(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varrho_u(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + 2\varepsilon_a\varepsilon_b\boldsymbol{\Lambda} - 2\varepsilon_b\delta_a\boldsymbol{\Omega} \\
&\quad - 3\Omega^2\varepsilon_d\boldsymbol{\Lambda} + 3\Omega^2\delta_a\mathbf{r} - 3\varepsilon_d\varepsilon_g\boldsymbol{\Omega} + 3\varepsilon_a\varepsilon_g\mathbf{r} + \varepsilon_d\ddot{\boldsymbol{\Lambda}} - \varsigma_g\mathbf{r} \\
&= 3\varepsilon_h(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \varrho_t(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varrho_u(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + (2\varepsilon_a\varepsilon_b - 3\Omega^2\varepsilon_d)\boldsymbol{\Lambda} \\
&\quad - (2\varepsilon_b\delta_a + 3\varepsilon_d\varepsilon_g)\boldsymbol{\Omega} + (3\Omega^2\delta_a + 3\varepsilon_a\varepsilon_g - \varsigma_g)\mathbf{r} + \varepsilon_d\ddot{\boldsymbol{\Lambda}} \\
&= 3\varepsilon_h(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \varrho_t(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varrho_u(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\qquad + \mathfrak{y}_b\boldsymbol{\Lambda} - \mathfrak{y}_c\boldsymbol{\Omega} + \mathfrak{y}_d\mathbf{r} + \varepsilon_d\ddot{\boldsymbol{\Lambda}} \text{ by (5.23n)}
\end{aligned} \tag{5.44c}
$$

$$
\begin{aligned}
\boldsymbol{\mathcal{J}}_d &= \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{e}} \text{ by (3.15b)} \\
&= \widehat{\boldsymbol{\kappa}} \times [\mathfrak{x}_o(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{x}_r\boldsymbol{\Omega} + \varphi_i\boldsymbol{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}}] \text{ by (5.40a)} \\
&= \widehat{\boldsymbol{\kappa}} \times [\mathfrak{x}_r\boldsymbol{\Omega} + \varphi_i\boldsymbol{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}} + \mathfrak{x}_o(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
&= \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{x}_o[\widehat{\boldsymbol{\kappa}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \varphi_g\varphi_h[\widehat{\boldsymbol{\kappa}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
&= \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{x}_o[\widehat{\mathbf{p}}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] \\
&\qquad + \varphi_g\varphi_h[\widehat{\mathbf{p}}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] \text{ by (A.1)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{x}_o(\varepsilon_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Omega}) + \varphi_g\varphi_h(\delta_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Lambda}) \\
&\qquad \text{by (5.1a) \& (5.13a)} \\
&= \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{x}_o\varepsilon_a\widehat{\mathbf{p}} - \mathfrak{x}_o\varepsilon_e\boldsymbol{\Omega} + \varphi_g\varphi_h\delta_a\widehat{\mathbf{p}} - \varphi_g\varphi_h\varepsilon_e\boldsymbol{\Lambda} \\
&= \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + (\varepsilon_a\mathfrak{x}_o + \varphi_g\varphi_h\delta_a)\widehat{\mathbf{p}} - \varepsilon_e\mathfrak{x}_o\boldsymbol{\Omega} - \varepsilon_e\varphi_g\varphi_h\boldsymbol{\Lambda} \\
&= \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{y}_e\widehat{\mathbf{p}} - \varepsilon_e\mathfrak{x}_o\boldsymbol{\Omega} - \varepsilon_e\varphi_g\varphi_h\boldsymbol{\Lambda} \text{ by (5.23n)}
\end{aligned} \tag{5.44d}
$$

$$
\begin{aligned}
\boldsymbol{\mathcal{J}}_e &= \widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{e}} \text{ by (3.15b)} \\
&= \widehat{\boldsymbol{\kappa}} \times [\mathfrak{x}_p(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + 2\mathfrak{x}_o(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \varphi_g\varphi_h(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{x}_s\boldsymbol{\Omega} + 2\mathfrak{x}_r\boldsymbol{\Lambda} + \varphi_i\dot{\boldsymbol{\Lambda}} - \mathfrak{x}_v\widehat{\mathbf{p}}] \text{ by (5.40b)} \\
&= \widehat{\boldsymbol{\kappa}} \times [\mathfrak{x}_s\boldsymbol{\Omega} + 2\mathfrak{x}_r\boldsymbol{\Lambda} + \varphi_i\dot{\boldsymbol{\Lambda}} - \mathfrak{x}_v\widehat{\mathbf{p}} + \mathfrak{x}_p(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + 2\mathfrak{x}_o(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \varphi_g\varphi_h(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&= \mathfrak{x}_s(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + 2\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) - \mathfrak{x}_v(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{x}_p[\widehat{\boldsymbol{\kappa}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] \\
&\quad + 2\mathfrak{x}_o[\widehat{\boldsymbol{\kappa}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \varphi_g\varphi_h[\widehat{\boldsymbol{\kappa}} \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&= \mathfrak{x}_s(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + 2\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) - \mathfrak{x}_v(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{x}_p[\widehat{\mathbf{p}}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] \\
&\quad + 2\mathfrak{x}_o[\widehat{\mathbf{p}}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] + \varphi_g\varphi_h[\widehat{\mathbf{p}}(\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}}) - \dot{\boldsymbol{\Lambda}}(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] \text{ by (A.1)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{x}_s(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + 2\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) - \mathfrak{x}_v(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{x}_p(\varepsilon_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Omega}) \\
&\quad + 2\mathfrak{x}_o(\delta_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Lambda}) + \varphi_g\varphi_h(\varsigma_b\widehat{\mathbf{p}} - \varepsilon_e\dot{\boldsymbol{\Lambda}}) \text{ by (5.1a), (5.13a) \& (5.23a)} \\
&= \mathfrak{x}_s(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + 2\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) - \mathfrak{x}_v(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{x}_p\varepsilon_a\widehat{\mathbf{p}} - \mathfrak{x}_p\varepsilon_e\boldsymbol{\Omega} \\
&\quad + 2\mathfrak{x}_o\delta_a\widehat{\mathbf{p}} - 2\mathfrak{x}_o\varepsilon_e\boldsymbol{\Lambda} + \varphi_g\varphi_h\varsigma_b\widehat{\mathbf{p}} - \varphi_g\varphi_h\varepsilon_e\dot{\boldsymbol{\Lambda}} \\
&= \mathfrak{x}_s(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + 2\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) - \mathfrak{x}_v(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + (\mathfrak{x}_p\varepsilon_a + 2\mathfrak{x}_o\delta_a + \varphi_g\varphi_h\varsigma_b)\widehat{\mathbf{p}} \\
&\quad - \mathfrak{x}_p\varepsilon_e\boldsymbol{\Omega} - 2\mathfrak{x}_o\varepsilon_e\boldsymbol{\Lambda} - \varphi_g\varphi_h\varepsilon_e\dot{\boldsymbol{\Lambda}} \\
&= \mathfrak{x}_s(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + 2\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) - \mathfrak{x}_v(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&\qquad + \mathfrak{y}_f\widehat{\mathbf{p}} - \mathfrak{x}_p\varepsilon_e\boldsymbol{\Omega} - 2\mathfrak{x}_o\varepsilon_e\boldsymbol{\Lambda} - \varphi_g\varphi_h\varepsilon_e\dot{\boldsymbol{\Lambda}} \text{ by (5.23n)}
\end{aligned} \tag{5.44e}
$$

$$
\begin{aligned}
\mathfrak{J}_f &= \mathbf{a} \times \dot{\mathbf{a}} \text{ by (3.15b)} \\
&= [(\boldsymbol{\Omega} \cdot \mathbf{r})\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda} \times \mathbf{r}] \times [2\varepsilon_h\boldsymbol{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\boldsymbol{\Lambda}} \times \mathbf{r} - \Omega^2(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b\boldsymbol{\Lambda}] \text{ by (1.4a) \& (5.30a)} \\
&= [\varepsilon_b\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda} \times \mathbf{r}] \times [2\varepsilon_h\boldsymbol{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\boldsymbol{\Lambda}} \times \mathbf{r} - \Omega^2(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b\boldsymbol{\Lambda}] \text{ by (5.1a)} \\
&= -3\varepsilon_g\varepsilon_b(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b[\boldsymbol{\Omega} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] - \Omega^2\varepsilon_b[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - 2\varepsilon_h\Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) \\
&\quad - \Omega^2[\mathbf{r} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \Omega^4[\mathbf{r} \times (\boldsymbol{\Omega} \times \mathbf{r})] - \varepsilon_b\Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) - 2\varepsilon_h[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&\quad + 3\varepsilon_g[\mathbf{r} \times (\boldsymbol{\Lambda} \times \mathbf{r})] + (\boldsymbol{\Lambda} \times \mathbf{r}) \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) - \Omega^2[(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\boldsymbol{\Omega} \times \mathbf{r})] - \varepsilon_b[\boldsymbol{\Lambda} \times (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) + \varepsilon_b[\boldsymbol{\Omega} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&\quad - \Omega^2\varepsilon_b[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] - \Omega^2[\mathbf{r} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \Omega^4[\mathbf{r} \times (\boldsymbol{\Omega} \times \mathbf{r})] - 2\varepsilon_h[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&\quad + 3\varepsilon_g[\mathbf{r} \times (\boldsymbol{\Lambda} \times \mathbf{r})] + (\boldsymbol{\Lambda} \times \mathbf{r}) \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) - \Omega^2[(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\boldsymbol{\Omega} \times \mathbf{r})] - \varepsilon_b[\boldsymbol{\Lambda} \times (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) + \varepsilon_b[\dot{\boldsymbol{\Lambda}}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega} \cdot \dot{\boldsymbol{\Lambda}})] \\
&\quad - \Omega^2\varepsilon_b[\boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \Omega^2\mathbf{r}] - \Omega^2[r^2\dot{\boldsymbol{\Lambda}} - \mathbf{r}(\mathbf{r} \cdot \dot{\boldsymbol{\Lambda}})] + \Omega^4[r^2\boldsymbol{\Omega} - \mathbf{r}(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad - 2\varepsilon_h[\boldsymbol{\Lambda}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})] + 3\varepsilon_g[r^2\boldsymbol{\Lambda} - \mathbf{r}(\mathbf{r} \cdot \boldsymbol{\Lambda})] + \dot{\boldsymbol{\Lambda}}[\mathbf{r} \cdot (\boldsymbol{\Lambda} \times \mathbf{r})] - \mathbf{r}[\dot{\boldsymbol{\Lambda}} \cdot (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&\quad - \Omega^2\boldsymbol{\Omega}[\mathbf{r} \cdot (\boldsymbol{\Lambda} \times \mathbf{r})] + \Omega^2\mathbf{r}[\boldsymbol{\Omega} \cdot (\boldsymbol{\Lambda} \times \mathbf{r})] - \varepsilon_b[\boldsymbol{\Lambda}(\boldsymbol{\Lambda} \cdot \mathbf{r}) - \Lambda^2\mathbf{r}] \text{ by (A.1) \& (A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) + \varepsilon_b(\varepsilon_b\dot{\boldsymbol{\Lambda}} - \varsigma_c\mathbf{r}) - \Omega^2\varepsilon_b(\varepsilon_b\boldsymbol{\Omega} - \Omega^2\mathbf{r}) \\
&\quad - \Omega^2(r^2\dot{\boldsymbol{\Lambda}} - \varsigma_n\mathbf{r}) + \Omega^4(r^2\boldsymbol{\Omega} - \varepsilon_b\mathbf{r}) - 2\varepsilon_h(\varepsilon_b\boldsymbol{\Lambda} - \varepsilon_g\mathbf{r}) + 3\varepsilon_g(r^2\boldsymbol{\Lambda} - \varepsilon_h\mathbf{r}) \\
&\quad - \mathfrak{d}_h\mathbf{r} + \Omega^2\varepsilon_m\mathbf{r} - \varepsilon_b(\varepsilon_h\boldsymbol{\Lambda} - \Lambda^2\mathbf{r}) \text{ by (5.1a), (5.23a), (5.23b) \& (A.4)} \\
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) + \varepsilon_b^2\dot{\boldsymbol{\Lambda}} - \varepsilon_b\varsigma_c\mathbf{r} - \Omega^2\varepsilon_b^2\boldsymbol{\Omega} + \varepsilon_b\Omega^4\mathbf{r} \\
&\quad - \Omega^2r^2\dot{\boldsymbol{\Lambda}} + \Omega^2\varsigma_n\mathbf{r} + \Omega^4r^2\boldsymbol{\Omega} - \Omega^4\varepsilon_b\mathbf{r} - 2\varepsilon_h\varepsilon_b\boldsymbol{\Lambda} + 2\varepsilon_h\varepsilon_g\mathbf{r} + 3\varepsilon_gr^2\boldsymbol{\Lambda} - 3\varepsilon_g\varepsilon_h\mathbf{r} \\
&\quad - \mathfrak{d}_h\mathbf{r} + \Omega^2\varepsilon_m\mathbf{r} - \varepsilon_b\varepsilon_h\boldsymbol{\Lambda} + \varepsilon_b\Lambda^2\mathbf{r} \\
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) + \varepsilon_b^2\dot{\boldsymbol{\Lambda}} - \Omega^2r^2\dot{\boldsymbol{\Lambda}} - \varepsilon_b\varsigma_c\mathbf{r} + \varepsilon_b\Omega^4\mathbf{r} \\
&\quad + \Omega^2\varsigma_n\mathbf{r} - \Omega^4\varepsilon_b\mathbf{r} + 2\varepsilon_h\varepsilon_g\mathbf{r} - 3\varepsilon_g\varepsilon_h\mathbf{r} - \mathfrak{d}_h\mathbf{r} + \Omega^2\varepsilon_m\mathbf{r} + \varepsilon_b\Lambda^2\mathbf{r} \\
&\quad - \Omega^2\varepsilon_b^2\boldsymbol{\Omega} + \Omega^4r^2\boldsymbol{\Omega} - 2\varepsilon_h\varepsilon_b\boldsymbol{\Lambda} + 3\varepsilon_gr^2\boldsymbol{\Lambda} - \varepsilon_b\varepsilon_h\boldsymbol{\Lambda} \\
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) - (\Omega^2r^2 - \varepsilon_b^2)\dot{\boldsymbol{\Lambda}} \\
&\quad + [\Omega^2(\varsigma_n + \varepsilon_m) + \varepsilon_b(\Lambda^2 - \varsigma_c) - \varepsilon_g\varepsilon_h - \mathfrak{d}_h]\mathbf{r} + \Omega^2(\Omega^2r^2 - \varepsilon_b^2)\boldsymbol{\Omega} + 3(\varepsilon_gr^2 - \varepsilon_h\varepsilon_b)\boldsymbol{\Lambda} \\
&= \mathfrak{y}_g(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) - \varphi_a^2\dot{\boldsymbol{\Lambda}} + \mathfrak{y}_h\mathbf{r} + \Omega^2\varphi_a^2\boldsymbol{\Omega} + 3\varphi_c\boldsymbol{\Lambda} \\
&\qquad \text{by (5.1b) \& (5.23n)}
\end{aligned}
\tag{5.44f}
$$

$$
\begin{aligned}
\mathfrak{J}_g &= \mathbf{a} \times \ddot{\mathbf{a}} \text{ by (3.15b)} \\
&= [(\boldsymbol{\Omega} \cdot \mathbf{r})\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda} \times \mathbf{r}] \times [3\varepsilon_h\boldsymbol{\Lambda} + \varepsilon_b\dot{\boldsymbol{\Lambda}} + \varrho_t\boldsymbol{\Omega} + \varrho_u\mathbf{r} \\
&\quad + 2\varepsilon_b(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) - 3\Omega^2(\boldsymbol{\Lambda} \times \mathbf{r}) - 3\varepsilon_g(\boldsymbol{\Omega} \times \mathbf{r}) + (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] \text{ by (5.30b) \& (1.4a)} \\
&= 3\varepsilon_h\varepsilon_b(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \varepsilon_b^2(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \varepsilon_b\varrho_u(\boldsymbol{\Omega} \times \mathbf{r}) + 2\varepsilon_b^2[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] \\
&\quad - 3\Omega^2\varepsilon_b[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \mathbf{r})] - 3\varepsilon_g\varepsilon_b[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] + \varepsilon_b[\boldsymbol{\Omega} \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] - 3\varepsilon_h\Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad - \varepsilon_b\Omega^2(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) - \Omega^2\varrho_t(\mathbf{r} \times \boldsymbol{\Omega}) - 2\varepsilon_b\Omega^2[\mathbf{r} \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] + 3\Omega^4[\mathbf{r} \times (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&\quad + 3\varepsilon_g\Omega^2[\mathbf{r} \times (\boldsymbol{\Omega} \times \mathbf{r})] - \Omega^2[\mathbf{r} \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] - 3\varepsilon_h[\boldsymbol{\Lambda} \times (\boldsymbol{\Lambda} \times \mathbf{r})] - \varepsilon_b[\dot{\boldsymbol{\Lambda}} \times (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&\quad - \varrho_t[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \mathbf{r})] - \varrho_u[\mathbf{r} \times (\boldsymbol{\Lambda} \times \mathbf{r})] + 2\varepsilon_b[(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] \\
&\quad - 3\varepsilon_g[(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\boldsymbol{\Omega} \times \mathbf{r})] + [(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] \text{ by (5.1a)}
\end{aligned}
$$

$$
\begin{aligned}
&= 3\varepsilon_h\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\varepsilon_b\varrho_u + \Omega^2\varrho_t)(\mathbf{\Omega}\times\mathbf{r}) - 3\varepsilon_h\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}})\\
&\quad + 2\varepsilon_b^2[\Omega^2\mathbf{\Lambda} - \mathbf{\Omega}(\mathbf{\Omega}\cdot\mathbf{\Lambda})] - 3\Omega^2\varepsilon_b[\mathbf{\Lambda}(\mathbf{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Omega}\cdot\mathbf{\Lambda})] - 3\varepsilon_g\varepsilon_b[\mathbf{\Omega}(\mathbf{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}]\\
&\quad + \varepsilon_b[\ddot{\mathbf{\Lambda}}(\mathbf{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Omega}\cdot\ddot{\mathbf{\Lambda}})] - 2\varepsilon_b\Omega^2[\mathbf{\Lambda}(\mathbf{r}\cdot\mathbf{\Omega}) - \mathbf{\Omega}(\mathbf{\Lambda}\cdot\mathbf{r})] + 3\Omega^4[r^2\mathbf{\Lambda} - \mathbf{r}(\mathbf{r}\cdot\mathbf{\Lambda})]\\
&\quad + 3\varepsilon_g\Omega^2[r^2\mathbf{\Omega} - \mathbf{r}(\mathbf{\Omega}\cdot\mathbf{r})] - \Omega^2[r^2\ddot{\mathbf{\Lambda}} - \mathbf{r}(\mathbf{r}\cdot\ddot{\mathbf{\Lambda}})] - 3\varepsilon_h[\mathbf{\Lambda}(\mathbf{\Lambda}\cdot\mathbf{r}) - \Lambda^2\mathbf{r}]\\
&\quad - \varepsilon_b[\mathbf{\Lambda}(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - \mathbf{r}(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})] - \varrho_t[\mathbf{\Lambda}(\mathbf{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Omega}\cdot\mathbf{\Lambda})] - \varrho_u[r^2\mathbf{\Lambda} - \mathbf{r}(\mathbf{r}\cdot\mathbf{\Lambda})]\\
&\quad + 2\varepsilon_b[\mathbf{\Lambda}(\mathbf{\Omega}\cdot(\mathbf{\Lambda}\times\mathbf{r})) - \mathbf{\Omega}(\mathbf{\Lambda}\cdot(\mathbf{\Lambda}\times\mathbf{r}))] - 3\varepsilon_g[\mathbf{\Omega}(\mathbf{r}\cdot(\mathbf{\Lambda}\times\mathbf{r})) - \mathbf{r}(\mathbf{\Omega}\cdot(\mathbf{\Lambda}\times\mathbf{r}))]\\
&\quad + [\ddot{\mathbf{\Lambda}}(\mathbf{r}\cdot(\mathbf{\Lambda}\times\mathbf{r})) - \mathbf{r}(\ddot{\mathbf{\Lambda}}\cdot(\mathbf{\Lambda}\times\mathbf{r}))] \text{ by (A.1) \& (A.5)}\\
&= 3\varepsilon_h\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\varepsilon_b\varrho_u + \Omega^2\varrho_t)(\mathbf{\Omega}\times\mathbf{r}) - 3\varepsilon_h\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + 2\varepsilon_b^2(\Omega^2\mathbf{\Lambda} - \varepsilon_g\mathbf{\Omega}) - 3\Omega^2\varepsilon_b(\varepsilon_b\mathbf{\Lambda} - \varepsilon_g\mathbf{r}) - 3\varepsilon_g\varepsilon_b(\varepsilon_b\mathbf{\Omega} - \Omega^2\mathbf{r})\\
&\quad + \varepsilon_b(\varepsilon_b\ddot{\mathbf{\Lambda}} - \varsigma_h\mathbf{r}) - 2\varepsilon_b\Omega^2(\varepsilon_b\mathbf{\Lambda} - \varepsilon_h\mathbf{\Omega}) + 3\Omega^4(r^2\mathbf{\Lambda} - \varepsilon_h\mathbf{r}) + 3\varepsilon_g\Omega^2(r^2\mathbf{\Omega} - \varepsilon_b\mathbf{r})\\
&\quad - \Omega^2(r^2\ddot{\mathbf{\Lambda}} - \varsigma_o\mathbf{r}) - 3\varepsilon_h(\varepsilon_h\mathbf{\Lambda} - \Lambda^2\mathbf{r}) - \varepsilon_b(\varsigma_n\mathbf{\Lambda} - \varsigma_d\mathbf{r}) - \varrho_t(\varepsilon_b\mathbf{\Lambda} - \varepsilon_g\mathbf{r})\\
&\quad - \varrho_u(r^2\mathbf{\Lambda} - \varepsilon_h\mathbf{r}) + 2\varepsilon_b\varepsilon_m\mathbf{\Lambda} + 3\varepsilon_g\varepsilon_m\mathbf{r} - \mathfrak{d}_i\mathbf{r} \text{ by (5.1a), (5.23a) \& (5.23b)}
\end{aligned}
$$

$$
\begin{aligned}
&= 3\varepsilon_h\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\varepsilon_b\varrho_u + \Omega^2\varrho_t)(\mathbf{\Omega}\times\mathbf{r}) - 3\varepsilon_h\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + 2\varepsilon_b^2\Omega^2\mathbf{\Lambda} - 2\varepsilon_b^2\varepsilon_g\mathbf{\Omega} - 3\Omega^2\varepsilon_b^2\mathbf{\Lambda} + 3\Omega^2\varepsilon_b\varepsilon_g\mathbf{r} - 3\varepsilon_g\varepsilon_b^2\mathbf{\Omega} + 3\varepsilon_g\varepsilon_b\Omega^2\mathbf{r}\\
&\quad + \varepsilon_b^2\ddot{\mathbf{\Lambda}} - \varepsilon_b\varsigma_h\mathbf{r} - 2\varepsilon_b^2\Omega^2\mathbf{\Lambda} + 2\varepsilon_b\varepsilon_h\Omega^2\mathbf{\Omega} + 3\Omega^4r^2\mathbf{\Lambda} - 3\Omega^4\varepsilon_h\mathbf{r} + 3\varepsilon_g\Omega^2r^2\mathbf{\Omega} - 3\varepsilon_g\varepsilon_b\Omega^2\mathbf{r}\\
&\quad - \Omega^2r^2\ddot{\mathbf{\Lambda}} + \Omega^2\varsigma_o\mathbf{r} - 3\varepsilon_h^2\mathbf{\Lambda} + 3\varepsilon_h\Lambda^2\mathbf{r} - \varepsilon_b\varsigma_n\mathbf{\Lambda} + \varepsilon_b\varsigma_d\mathbf{r} - \varrho_t\varepsilon_b\mathbf{\Lambda} + \varrho_t\varepsilon_g\mathbf{r}\\
&\quad - \varrho_ur^2\mathbf{\Lambda} + \varrho_u\varepsilon_h\mathbf{r} + 2\varepsilon_b\varepsilon_m\mathbf{\Lambda} + 3\varepsilon_g\varepsilon_m\mathbf{r} - \mathfrak{d}_i\mathbf{r}\\
&= 3\varepsilon_h\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\varepsilon_b\varrho_u + \Omega^2\varrho_t)(\mathbf{\Omega}\times\mathbf{r}) - 3\varepsilon_h\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}})\\
&\quad + (2\varepsilon_b^2\Omega^2 - 3\Omega^2\varepsilon_b^2 - 2\varepsilon_b^2\Omega^2 + 3\Omega^4r^2 - 3\varepsilon_h^2 - \varepsilon_b\varsigma_n - \varrho_t\varepsilon_b - \varrho_ur^2 + 2\varepsilon_b\varepsilon_m)\mathbf{\Lambda}\\
&\quad + (3\Omega^2\varepsilon_b\varepsilon_g + 3\varepsilon_g\varepsilon_b\Omega^2 - \varepsilon_b\varsigma_h - 3\Omega^4\varepsilon_h - 3\varepsilon_g\varepsilon_b\Omega^2 + \Omega^2\varsigma_o + 3\varepsilon_h\Lambda^2 + \varepsilon_b\varsigma_d + \varrho_t\varepsilon_g\\
&\quad + \varrho_u\varepsilon_h + 3\varepsilon_g\varepsilon_m - \mathfrak{d}_i)\mathbf{r} + (-2\varepsilon_b^2\varepsilon_g - 3\varepsilon_g\varepsilon_b^2 + 2\varepsilon_b\varepsilon_h\Omega^2 + 3\varepsilon_g\Omega^2r^2)\mathbf{\Omega} + (\varepsilon_b^2 - \Omega^2r^2)\ddot{\mathbf{\Lambda}}
\end{aligned}
$$

$$
\begin{aligned}
&= 3\varepsilon_h\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\varepsilon_b\varrho_u + \Omega^2\varrho_t)(\mathbf{\Omega}\times\mathbf{r}) - 3\varepsilon_h\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + [3\Omega^2(\Omega^2r^2 - \varepsilon_b^2) - 3\varepsilon_h^2 - \varepsilon_b\varsigma_n - \varrho_t\varepsilon_b - \varrho_ur^2 + 2\varepsilon_b\varepsilon_m]\mathbf{\Lambda}\\
&\quad + (3\Omega^2\varepsilon_b\varepsilon_g - \varepsilon_b\varsigma_h - 3\Omega^4\varepsilon_h + \Omega^2\varsigma_o + 3\varepsilon_h\Lambda^2 + \varepsilon_b\varsigma_d + \varrho_t\varepsilon_g + \varrho_u\varepsilon_h + 3\varepsilon_g\varepsilon_m - \mathfrak{d}_i)\mathbf{r}\\
&\quad + (3r^2\Omega^2\varepsilon_g + 2\Omega^2\varepsilon_b\varepsilon_h - 5\varepsilon_g\varepsilon_b^2)\mathbf{\Omega} - (\Omega^2r^2 - \varepsilon_b^2)\ddot{\mathbf{\Lambda}}
\end{aligned}
$$

$$
\begin{aligned}
&= 3\varepsilon_h\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\varepsilon_b\varrho_u + \Omega^2\varrho_t)(\mathbf{\Omega}\times\mathbf{r}) - 3\varepsilon_h\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + (3\Omega^2\varphi_a^2 - 3\varepsilon_h^2 - \varepsilon_b\varsigma_n - \varrho_t\varepsilon_b - \varrho_ur^2 + 2\varepsilon_b\varepsilon_m)\mathbf{\Lambda}\\
&\quad + (3\Omega^2\varepsilon_b\varepsilon_g - \varepsilon_b\varsigma_h - 3\Omega^4\varepsilon_h + \Omega^2\varsigma_o + 3\varepsilon_h\Lambda^2 + \varepsilon_b\varsigma_d + \varrho_t\varepsilon_g + \varrho_u\varepsilon_h + 3\varepsilon_g\varepsilon_m - \mathfrak{d}_i)\mathbf{r}\\
&\quad + (3r^2\Omega^2\varepsilon_g + 2\Omega^2\varepsilon_b\varepsilon_h - 5\varepsilon_g\varepsilon_b^2)\mathbf{\Omega} - \varphi_a^2\ddot{\mathbf{\Lambda}} \text{ by (5.1b)}\\
&= 3\varepsilon_h\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{y}_i(\mathbf{\Omega}\times\mathbf{r}) - 3\varepsilon_h\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{y}_j\mathbf{\Lambda} + \mathfrak{y}_l\mathbf{r} + \mathfrak{y}_k\mathbf{\Omega} - \varphi_a^2\ddot{\mathbf{\Lambda}} \text{ by (5.23n)}
\end{aligned}
\tag{5.44g}
$$

$$
\begin{aligned}
\mathfrak{J}_h &= \mathbf{a}\times\dot{\mathbf{e}} \text{ by (3.15b)}\\
&= [(\mathbf{\Omega}\cdot\mathbf{r})\mathbf{\Omega} - \Omega^2\mathbf{r} + \mathbf{\Lambda}\times\mathbf{r}]\times[\mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{x}_r\mathbf{\Omega} + \varphi_i\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}}]\\
&\qquad \text{by (1.4a) \& (5.40a)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{x}_o\varepsilon_b[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \varphi_g\varphi_h\varepsilon_b[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] + \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{x}_o\Omega^2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] - \varphi_g\varphi_h\Omega^2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) \\
&\quad + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_o[(\boldsymbol{\Lambda}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \varphi_g\varphi_h[(\boldsymbol{\Lambda}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] \\
&\quad - \mathfrak{x}_r[\boldsymbol{\Omega}\times(\boldsymbol{\Lambda}\times\mathbf{r})] - \varphi_i[\boldsymbol{\Lambda}\times(\boldsymbol{\Lambda}\times\mathbf{r})] + \mathfrak{x}_u[\widehat{\mathbf{p}}\times(\boldsymbol{\Lambda}\times\mathbf{r})] \text{ by (5.1a)} \\
&= \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + \mathfrak{x}_o\varepsilon_b[\Omega^2\widehat{\mathbf{p}} - \boldsymbol{\Omega}(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})] + \varphi_g\varphi_h\varepsilon_b[\widehat{\mathbf{p}}(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})] - \mathfrak{x}_o\Omega^2[\widehat{\mathbf{p}}(\mathbf{r}\cdot\boldsymbol{\Omega}) - \boldsymbol{\Omega}(\widehat{\mathbf{p}}\cdot\mathbf{r})] \\
&\quad - \varphi_g\varphi_h\Omega^2[\widehat{\mathbf{p}}(\mathbf{r}\cdot\boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\mathbf{r}\cdot\widehat{\mathbf{p}})] + \mathfrak{x}_o[\widehat{\mathbf{p}}(\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})) - \boldsymbol{\Omega}(\widehat{\mathbf{p}}\cdot(\boldsymbol{\Lambda}\times\mathbf{r}))] \\
&\quad + \varphi_g\varphi_h[\widehat{\mathbf{p}}(\boldsymbol{\Lambda}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})) - \boldsymbol{\Lambda}(\widehat{\mathbf{p}}\cdot(\boldsymbol{\Lambda}\times\mathbf{r}))] - \mathfrak{x}_r[\boldsymbol{\Lambda}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})] \\
&\quad - \varphi_i[\boldsymbol{\Lambda}(\boldsymbol{\Lambda}\cdot\mathbf{r}) - \Lambda^2\mathbf{r}] + \mathfrak{x}_u[\boldsymbol{\Lambda}(\widehat{\mathbf{p}}\cdot\mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda})] \text{ by (A.1) \& (A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&= \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + \mathfrak{x}_o\varepsilon_b(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\boldsymbol{\Omega}) + \varphi_g\varphi_h\varepsilon_b(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_c\boldsymbol{\Lambda}) - \mathfrak{x}_o\Omega^2(\varepsilon_b\widehat{\mathbf{p}} - \varepsilon_f\boldsymbol{\Omega}) \\
&\quad - \varphi_g\varphi_h\Omega^2(\varepsilon_h\widehat{\mathbf{p}} - \varepsilon_f\boldsymbol{\Lambda}) + \mathfrak{x}_o(\varepsilon_m\widehat{\mathbf{p}} - \varepsilon_o\boldsymbol{\Omega}) - \varphi_g\varphi_h\varepsilon_o\boldsymbol{\Lambda} - \mathfrak{x}_r(\varepsilon_b\boldsymbol{\Lambda} - \varepsilon_g\mathbf{r}) \\
&\quad - \varphi_i(\varepsilon_h\boldsymbol{\Lambda} - \Lambda^2\mathbf{r}) + \mathfrak{x}_u(\varepsilon_f\boldsymbol{\Lambda} - \varepsilon_i\mathbf{r}) \text{ by (5.1a)} \\
&= \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + \mathfrak{x}_o\varepsilon_b\Omega^2\widehat{\mathbf{p}} - \mathfrak{x}_o\varepsilon_b\varepsilon_c\boldsymbol{\Omega} + \varphi_g\varphi_h\varepsilon_b\varepsilon_g\widehat{\mathbf{p}} - \varphi_g\varphi_h\varepsilon_b\varepsilon_c\boldsymbol{\Lambda} - \mathfrak{x}_o\Omega^2\varepsilon_b\widehat{\mathbf{p}} + \mathfrak{x}_o\Omega^2\varepsilon_f\boldsymbol{\Omega} \\
&\quad - \varphi_g\varphi_h\Omega^2\varepsilon_h\widehat{\mathbf{p}} + \varphi_g\varphi_h\Omega^2\varepsilon_f\boldsymbol{\Lambda} + \mathfrak{x}_o\varepsilon_m\widehat{\mathbf{p}} - \mathfrak{x}_o\varepsilon_o\boldsymbol{\Omega} - \varphi_g\varphi_h\varepsilon_o\boldsymbol{\Lambda} - \mathfrak{x}_r\varepsilon_b\boldsymbol{\Lambda} + \mathfrak{x}_r\varepsilon_g\mathbf{r} \\
&\quad - \varphi_i\varepsilon_h\boldsymbol{\Lambda} + \varphi_i\Lambda^2\mathbf{r} + \mathfrak{x}_u\varepsilon_f\boldsymbol{\Lambda} - \mathfrak{x}_u\varepsilon_i\mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + (\mathfrak{x}_o\varepsilon_b\Omega^2 + \varphi_g\varphi_h\varepsilon_b\varepsilon_g - \mathfrak{x}_o\Omega^2\varepsilon_b - \varphi_g\varphi_h\Omega^2\varepsilon_h + \mathfrak{x}_o\varepsilon_m)\widehat{\mathbf{p}} + (-\mathfrak{x}_o\varepsilon_b\varepsilon_c + \mathfrak{x}_o\Omega^2\varepsilon_f - \mathfrak{x}_o\varepsilon_o)\boldsymbol{\Omega} \\
&\quad + (-\varphi_g\varphi_h\varepsilon_b\varepsilon_c + \varphi_g\varphi_h\Omega^2\varepsilon_f - \varphi_g\varphi_h\varepsilon_o - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h + \mathfrak{x}_u\varepsilon_f)\boldsymbol{\Lambda} + (\mathfrak{x}_r\varepsilon_g + \varphi_i\Lambda^2 - \mathfrak{x}_u\varepsilon_i)\mathbf{r} \\
&= \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + \mathfrak{x}_o(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)\boldsymbol{\Omega} + (\mathfrak{x}_r\varepsilon_g + \varphi_i\Lambda^2 - \mathfrak{x}_u\varepsilon_i)\mathbf{r} + [\varphi_g\varphi_h(\varepsilon_b\varepsilon_g - \varepsilon_h\Omega^2) + \mathfrak{x}_o\varepsilon_m]\widehat{\mathbf{p}} \\
&\quad + [\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h + \varphi_g\varphi_h(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)]\boldsymbol{\Lambda} \\
&= \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + \mathfrak{x}_o\mathfrak{y}_m\boldsymbol{\Omega} + \mathfrak{y}_n\mathbf{r} + \mathfrak{y}_o\widehat{\mathbf{p}} + \mathfrak{y}_p\boldsymbol{\Lambda} \text{ by (5.23n)}
\end{aligned}
\tag{5.44h}
$$

$$
\begin{aligned}
\mathfrak{J}_i &= \mathbf{a}\times\ddot{\mathbf{e}} \text{ by (3.15b)} \\
&= [(\boldsymbol{\Omega}\cdot\mathbf{r})\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda}\times\mathbf{r}]\times[\mathfrak{x}_p(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + 2\mathfrak{x}_o(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{x}_s\boldsymbol{\Omega} \\
&\qquad + 2\mathfrak{x}_r\boldsymbol{\Lambda} + \varphi_i\dot{\boldsymbol{\Lambda}} - \mathfrak{x}_v\widehat{\mathbf{p}}] \text{ by (1.4a) \& (5.40b)} \\
&= \mathfrak{x}_p\varepsilon_b[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + 2\mathfrak{x}_o\varepsilon_b[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] + \varphi_g\varphi_h\varepsilon_b[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})] + 2\mathfrak{x}_r\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) \\
&\quad + \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}}) - \mathfrak{x}_v\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_p\Omega^2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] - 2\mathfrak{x}_o\Omega^2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] \\
&\quad - \varphi_g\varphi_h\Omega^2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})] - \mathfrak{x}_s\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - 2\mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) - \varphi_i\Omega^2(\mathbf{r}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{x}_v\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + \mathfrak{x}_p[(\boldsymbol{\Lambda}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + 2\mathfrak{x}_o[(\boldsymbol{\Lambda}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] + \varphi_g\varphi_h[(\boldsymbol{\Lambda}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})] \\
&\quad - \mathfrak{x}_s[\boldsymbol{\Omega}\times(\boldsymbol{\Lambda}\times\mathbf{r})] - 2\mathfrak{x}_r[\boldsymbol{\Lambda}\times(\boldsymbol{\Lambda}\times\mathbf{r})] - \varphi_i[\dot{\boldsymbol{\Lambda}}\times(\boldsymbol{\Lambda}\times\mathbf{r})] + \mathfrak{x}_v[\widehat{\mathbf{p}}\times(\boldsymbol{\Lambda}\times\mathbf{r})] \text{ by (5.1a)}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathfrak{x}_r\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\varepsilon_b(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_s\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - 2\mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varphi_i\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{x}_v\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_p\varepsilon_b[\Omega^2\widehat{\mathbf{p}} - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})] + 2\mathfrak{x}_o\varepsilon_b[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot\mathbf{\Lambda}) - \mathbf{\Lambda}(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})]\\
&\quad + \varphi_g\varphi_h\varepsilon_b[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}}) - \dot{\mathbf{\Lambda}}(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})] - \mathfrak{x}_p\Omega^2[\widehat{\mathbf{p}}(\mathbf{r}\cdot\mathbf{\Omega}) - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot\mathbf{r})] - 2\mathfrak{x}_o\Omega^2[\widehat{\mathbf{p}}(\mathbf{r}\cdot\mathbf{\Lambda}) - \mathbf{\Lambda}(\mathbf{r}\cdot\widehat{\mathbf{p}})]\\
&\quad - \varphi_g\varphi_h\Omega^2[\widehat{\mathbf{p}}(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - \dot{\mathbf{\Lambda}}(\widehat{\mathbf{p}}\cdot\mathbf{r})] + \mathfrak{x}_p[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot(\mathbf{\Lambda}\times\mathbf{r})) - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot(\mathbf{\Lambda}\times\mathbf{r}))]\\
&\quad + 2\mathfrak{x}_o[\widehat{\mathbf{p}}(\mathbf{\Lambda}\cdot(\mathbf{\Lambda}\times\mathbf{r})) - \mathbf{\Lambda}(\widehat{\mathbf{p}}\cdot(\mathbf{\Lambda}\times\mathbf{r}))] + \varphi_g\varphi_h[\widehat{\mathbf{p}}(\dot{\mathbf{\Lambda}}\cdot(\mathbf{\Lambda}\times\mathbf{r})) - \dot{\mathbf{\Lambda}}(\widehat{\mathbf{p}}\cdot(\mathbf{\Lambda}\times\mathbf{r}))]\\
&\quad - \mathfrak{x}_s[\mathbf{\Lambda}(\mathbf{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Omega}\cdot\mathbf{\Lambda})] - 2\mathfrak{x}_r[\mathbf{\Lambda}(\mathbf{\Lambda}\cdot\mathbf{r}) - \Lambda^2\mathbf{r}] - \varphi_i[\mathbf{\Lambda}(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}})]\\
&\quad + \mathfrak{x}_v[\mathbf{\Lambda}(\widehat{\mathbf{p}}\cdot\mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})] \text{ by (A.1) \& (A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathfrak{x}_r\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\varepsilon_b(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_s\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - 2\mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varphi_i\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{x}_v\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_p\varepsilon_b(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Omega}) + 2\mathfrak{x}_o\varepsilon_b(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Lambda})\\
&\quad + \varphi_g\varphi_h\varepsilon_b(\varsigma_c\widehat{\mathbf{p}} - \varepsilon_c\dot{\mathbf{\Lambda}}) - \mathfrak{x}_p\Omega^2(\varepsilon_b\widehat{\mathbf{p}} - \varepsilon_f\mathbf{\Omega}) - 2\mathfrak{x}_o\Omega^2(\varepsilon_h\widehat{\mathbf{p}} - \varepsilon_f\mathbf{\Lambda})\\
&\quad - \varphi_g\varphi_h\Omega^2(\varsigma_n\widehat{\mathbf{p}} - \varepsilon_f\dot{\mathbf{\Lambda}}) + \mathfrak{x}_p(\varepsilon_m\widehat{\mathbf{p}} - \varepsilon_o\mathbf{\Omega}) - 2\mathfrak{x}_o\varepsilon_o\mathbf{\Lambda} + \varphi_g\varphi_h(\mathfrak{d}_h\widehat{\mathbf{p}} - \varepsilon_o\dot{\mathbf{\Lambda}})\\
&\quad - \mathfrak{x}_s(\varepsilon_b\mathbf{\Lambda} - \varepsilon_g\mathbf{r}) - 2\mathfrak{x}_r(\varepsilon_h\mathbf{\Lambda} - \Lambda^2\mathbf{r}) - \varphi_i(\varsigma_n\mathbf{\Lambda} - \varsigma_d\mathbf{r})\\
&\quad + \mathfrak{x}_v(\varepsilon_f\mathbf{\Lambda} - \varepsilon_i\mathbf{r}) \text{ by (5.1a), (5.23a) \& (5.23b)}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathfrak{x}_r\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\varepsilon_b(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_s\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - 2\mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varphi_i\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{x}_v\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_p\varepsilon_b\Omega^2\widehat{\mathbf{p}} - \mathfrak{x}_p\varepsilon_b\varepsilon_c\mathbf{\Omega} + 2\mathfrak{x}_o\varepsilon_b\varepsilon_g\widehat{\mathbf{p}} - 2\mathfrak{x}_o\varepsilon_b\varepsilon_c\mathbf{\Lambda}\\
&\quad + \varphi_g\varphi_h\varepsilon_b\varsigma_c\widehat{\mathbf{p}} - \varphi_g\varphi_h\varepsilon_b\varepsilon_c\dot{\mathbf{\Lambda}} - \mathfrak{x}_p\Omega^2\varepsilon_b\widehat{\mathbf{p}} + \mathfrak{x}_p\Omega^2\varepsilon_f\mathbf{\Omega} - 2\mathfrak{x}_o\Omega^2\varepsilon_h\widehat{\mathbf{p}} + 2\mathfrak{x}_o\Omega^2\varepsilon_f\mathbf{\Lambda}\\
&\quad - \varphi_g\varphi_h\Omega^2\varsigma_n\widehat{\mathbf{p}} + \varphi_g\varphi_h\Omega^2\varepsilon_f\dot{\mathbf{\Lambda}} + \mathfrak{x}_p\varepsilon_m\widehat{\mathbf{p}} - \mathfrak{x}_p\varepsilon_o\mathbf{\Omega} - 2\mathfrak{x}_o\varepsilon_o\mathbf{\Lambda} + \varphi_g\varphi_h\mathfrak{d}_h\widehat{\mathbf{p}} - \varphi_g\varphi_h\varepsilon_o\dot{\mathbf{\Lambda}}\\
&\quad - \mathfrak{x}_s\varepsilon_b\mathbf{\Lambda} + \mathfrak{x}_s\varepsilon_g\mathbf{r} - 2\mathfrak{x}_r\varepsilon_h\mathbf{\Lambda} + 2\mathfrak{x}_r\Lambda^2\mathbf{r} - \varphi_i\varsigma_n\mathbf{\Lambda} + \varphi_i\varsigma_d\mathbf{r} + \mathfrak{x}_v\varepsilon_f\mathbf{\Lambda} - \mathfrak{x}_v\varepsilon_i\mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathfrak{x}_r\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\varepsilon_b(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_s\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - 2\mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) - \varphi_i\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}})\\
&\quad + \mathfrak{x}_v\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + (-\mathfrak{x}_p\varepsilon_b\varepsilon_c + \mathfrak{x}_p\Omega^2\varepsilon_f - \mathfrak{x}_p\varepsilon_o)\mathbf{\Omega} + (-\varphi_g\varphi_h\varepsilon_b\varepsilon_c + \varphi_g\varphi_h\Omega^2\varepsilon_f - \varphi_g\varphi_h\varepsilon_o)\dot{\mathbf{\Lambda}}\\
&\quad + (\mathfrak{x}_p\varepsilon_b\Omega^2 + 2\mathfrak{x}_o\varepsilon_b\varepsilon_g + \varphi_g\varphi_h\varepsilon_b\varsigma_c - \mathfrak{x}_p\Omega^2\varepsilon_b - 2\mathfrak{x}_o\Omega^2\varepsilon_h - \varphi_g\varphi_h\Omega^2\varsigma_n + \mathfrak{x}_p\varepsilon_m + \varphi_g\varphi_h\mathfrak{d}_h)\widehat{\mathbf{p}}\\
&\quad + (\mathfrak{x}_s\varepsilon_g + 2\mathfrak{x}_r\Lambda^2 + \varphi_i\varsigma_d - \mathfrak{x}_v\varepsilon_i)\mathbf{r} + (-2\mathfrak{x}_o\varepsilon_b\varepsilon_c + 2\mathfrak{x}_o\Omega^2\varepsilon_f - 2\mathfrak{x}_o\varepsilon_o - \mathfrak{x}_s\varepsilon_b - 2\mathfrak{x}_r\varepsilon_h - \varphi_i\varsigma_n + \mathfrak{x}_v\varepsilon_f)\mathbf{\Lambda}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathfrak{x}_r\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\varepsilon_b(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_s\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - 2\mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varphi_i\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{x}_v\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_p(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)\mathbf{\Omega} + \varphi_g\varphi_h(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)\dot{\mathbf{\Lambda}}\\
&\quad + (\mathfrak{x}_s\varepsilon_g + 2\mathfrak{x}_r\Lambda^2 + \varphi_i\varsigma_d - \mathfrak{x}_v\varepsilon_i)\mathbf{r} + [\mathfrak{x}_p\varepsilon_m + 2\mathfrak{x}_o(\varepsilon_b\varepsilon_g - \Omega^2\varepsilon_h) + \varphi_g\varphi_h(\varepsilon_b\varsigma_c - \Omega^2\varsigma_n + \mathfrak{d}_h)]\widehat{\mathbf{p}}\\
&\quad + [\mathfrak{x}_v\varepsilon_f - \mathfrak{x}_s\varepsilon_b - 2\mathfrak{x}_r\varepsilon_h - \varphi_i\varsigma_n + 2\mathfrak{x}_o(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)]\mathbf{\Lambda}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathfrak{x}_r\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\varepsilon_b(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_s\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - 2\mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad - \varphi_i\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{x}_v\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_p\mathfrak{y}_m\mathbf{\Omega} + \varphi_g\varphi_h\mathfrak{y}_m\dot{\mathbf{\Lambda}} + \mathfrak{y}_q\mathbf{r} + \mathfrak{y}_r\widehat{\mathbf{p}} + \mathfrak{y}_s\mathbf{\Lambda} \qquad (5.44\text{i})\\
&\quad \text{by (5.23n) \& (5.23o)}
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{J}_j &= \dot{\mathbf{a}}\times\ddot{\mathbf{a}} \text{ by (3.15b)}\\
&= [2\varepsilon_h\mathbf{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\mathbf{\Lambda}}\times\mathbf{r} - \Omega^2(\mathbf{\Omega}\times\mathbf{r}) + \varepsilon_b\mathbf{\Lambda}]\times[3\varepsilon_h\mathbf{\Lambda} + \varepsilon_b\dot{\mathbf{\Lambda}} + \varrho_t\mathbf{\Omega} + \varrho_u\mathbf{r}\\
&\quad + 2\varepsilon_b(\mathbf{\Lambda}\times\mathbf{\Omega}) - 3\Omega^2(\mathbf{\Lambda}\times\mathbf{r}) - 3\varepsilon_g(\mathbf{\Omega}\times\mathbf{r}) + (\ddot{\mathbf{\Lambda}}\times\mathbf{r})] \text{ by (5.30a) \& (5.30b)}
\end{aligned}
$$

$$
\begin{aligned}
&= 6\varepsilon_h^2(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + 2\varrho_u\varepsilon_h(\mathbf{\Omega}\times\mathbf{r}) + 4\varepsilon_b\varepsilon_h[\mathbf{\Omega}\times(\mathbf{\Lambda}\times\mathbf{\Omega})] - 6\Omega^2\varepsilon_h[\mathbf{\Omega}\times(\mathbf{\Lambda}\times\mathbf{r})]\\
&\quad - 6\varepsilon_g\varepsilon_h[\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})] + 2\varepsilon_h[\mathbf{\Omega}\times(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] - 9\varepsilon_h\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda}) - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) - 3\varrho_t\varepsilon_g(\mathbf{r}\times\mathbf{\Omega})\\
&\quad - 6\varepsilon_b\varepsilon_g[\mathbf{r}\times(\mathbf{\Lambda}\times\mathbf{\Omega})] + 9\Omega^2\varepsilon_g[\mathbf{r}\times(\mathbf{\Lambda}\times\mathbf{r})] + 9\varepsilon_g^2[\mathbf{r}\times(\mathbf{\Omega}\times\mathbf{r})] - 3\varepsilon_g[\mathbf{r}\times(\ddot{\mathbf{\Lambda}}\times\mathbf{r})]\\
&\quad - 3\varepsilon_h[\mathbf{\Lambda}\times(\dot{\mathbf{\Lambda}}\times\mathbf{r})] - \varepsilon_b[\dot{\mathbf{\Lambda}}\times(\dot{\mathbf{\Lambda}}\times\mathbf{r})] - \varrho_t[\mathbf{\Omega}\times(\dot{\mathbf{\Lambda}}\times\mathbf{r})] - \varrho_u[\mathbf{r}\times(\dot{\mathbf{\Lambda}}\times\mathbf{r})]\\
&\quad + 2\varepsilon_b[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\times(\mathbf{\Lambda}\times\mathbf{\Omega})] - 3\Omega^2[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\times(\mathbf{\Lambda}\times\mathbf{r})] - 3\varepsilon_g[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\times(\mathbf{\Omega}\times\mathbf{r})]\\
&\quad + [(\dot{\mathbf{\Lambda}}\times\mathbf{r})\times(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] + 3\varepsilon_h\Omega^2[\mathbf{\Lambda}\times(\mathbf{\Omega}\times\mathbf{r})] + \varepsilon_b\Omega^2[\dot{\mathbf{\Lambda}}\times(\mathbf{\Omega}\times\mathbf{r})] + \varrho_t\Omega^2[\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})]\\
&\quad + \varrho_u\Omega^2[\mathbf{r}\times(\mathbf{\Omega}\times\mathbf{r})] - 2\varepsilon_b\Omega^2[(\mathbf{\Omega}\times\mathbf{r})\times(\mathbf{\Lambda}\times\mathbf{\Omega})] + 3\Omega^4[(\mathbf{\Omega}\times\mathbf{r})\times(\mathbf{\Lambda}\times\mathbf{r})]\\
&\quad - \Omega^2[(\mathbf{\Omega}\times\mathbf{r})\times(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + \varrho_t\varepsilon_b(\mathbf{\Lambda}\times\mathbf{\Omega}) + \varrho_u\varepsilon_b(\mathbf{\Lambda}\times\mathbf{r}) + 2\varepsilon_b^2[\mathbf{\Lambda}\times(\mathbf{\Lambda}\times\mathbf{\Omega})]\\
&\quad - 3\Omega^2\varepsilon_b[\mathbf{\Lambda}\times(\mathbf{\Lambda}\times\mathbf{r})] - 3\varepsilon_g\varepsilon_b[\mathbf{\Lambda}\times(\mathbf{\Omega}\times\mathbf{r})] + \varepsilon_b[\mathbf{\Lambda}\times(\ddot{\mathbf{\Lambda}}\times\mathbf{r})]
\end{aligned}
$$

$$
\begin{aligned}
&= (6\varepsilon_h^2 - \varrho_t\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (2\varrho_u\varepsilon_h + 3\varrho_t\varepsilon_g)(\mathbf{\Omega}\times\mathbf{r}) + (9\varepsilon_h\varepsilon_g + \varrho_u\varepsilon_b)(\mathbf{\Lambda}\times\mathbf{r})\\
&\quad - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + 4\varepsilon_b\varepsilon_h[\Omega^2\mathbf{\Lambda} - \mathbf{\Omega}(\mathbf{\Omega}\cdot\mathbf{\Lambda})] - 6\Omega^2\varepsilon_h[\mathbf{\Lambda}(\mathbf{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Omega}\cdot\mathbf{\Lambda})]\\
&\quad + (\varrho_t\Omega^2 - 6\varepsilon_g\varepsilon_h)[\mathbf{\Omega}(\mathbf{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}] + 2\varepsilon_h[\ddot{\mathbf{\Lambda}}(\mathbf{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Omega}\cdot\ddot{\mathbf{\Lambda}})] - 6\varepsilon_b\varepsilon_g[\mathbf{\Lambda}(\mathbf{r}\cdot\mathbf{\Omega}) - \mathbf{\Omega}(\mathbf{r}\cdot\mathbf{\Lambda})]\\
&\quad + 9\Omega^2\varepsilon_g[r^2\mathbf{\Lambda} - \mathbf{r}(\mathbf{r}\cdot\mathbf{\Lambda})] + (9\varepsilon_g^2 + \varrho_u\Omega^2)[r^2\mathbf{\Omega} - \mathbf{r}(\mathbf{r}\cdot\mathbf{\Omega})] - 3\varepsilon_g[r^2\ddot{\mathbf{\Lambda}} - \mathbf{r}(\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})]\\
&\quad - 3\varepsilon_h[\dot{\mathbf{\Lambda}}(\mathbf{\Lambda}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}})] - \varepsilon_b[\dot{\mathbf{\Lambda}}(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - \dot{\mathbf{\Lambda}}^2\mathbf{r}] - \varrho_t[\dot{\mathbf{\Lambda}}(\mathbf{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})] - \varrho_u[r^2\dot{\mathbf{\Lambda}} - \mathbf{r}(\mathbf{r}\cdot\dot{\mathbf{\Lambda}})]\\
&\quad + 3(\varepsilon_h\Omega^2 - \varepsilon_g\varepsilon_b)[\mathbf{\Omega}(\mathbf{\Lambda}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Lambda}\cdot\mathbf{\Omega})] + \varepsilon_b\Omega^2[\mathbf{\Omega}(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - \mathbf{r}(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] + 2\varepsilon_b^2[\mathbf{\Lambda}(\mathbf{\Lambda}\cdot\mathbf{\Omega}) - \mathbf{\Lambda}^2\mathbf{\Omega}]\\
&\quad - 3\Omega^2\varepsilon_b[\mathbf{\Lambda}(\mathbf{\Lambda}\cdot\mathbf{r}) - \mathbf{\Lambda}^2\mathbf{r}] + \varepsilon_b[\ddot{\mathbf{\Lambda}}(\mathbf{\Lambda}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Lambda}\cdot\ddot{\mathbf{\Lambda}})] + 2\varepsilon_b[\mathbf{\Lambda}(\mathbf{\Omega}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})) - \mathbf{\Omega}(\mathbf{\Lambda}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r}))]\\
&\quad - 3\Omega^2[\mathbf{\Lambda}(\mathbf{r}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})) - \mathbf{r}(\mathbf{\Lambda}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r}))] - 3\varepsilon_g[\mathbf{\Omega}(\mathbf{r}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})) - \mathbf{r}(\mathbf{\Omega}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r}))]\\
&\quad + [\ddot{\mathbf{\Lambda}}(\mathbf{r}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})) - \mathbf{r}(\ddot{\mathbf{\Lambda}}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r}))] - 2\varepsilon_b\Omega^2[\mathbf{\Lambda}(\mathbf{\Omega}\cdot(\mathbf{\Omega}\times\mathbf{r})) - \mathbf{\Omega}(\mathbf{\Lambda}\cdot(\mathbf{\Omega}\times\mathbf{r}))]\\
&\quad + 3\Omega^4[\mathbf{\Lambda}(\mathbf{r}\cdot(\mathbf{\Omega}\times\mathbf{r})) - \mathbf{r}(\mathbf{\Lambda}\cdot(\mathbf{\Omega}\times\mathbf{r}))] - \Omega^2[\ddot{\mathbf{\Lambda}}(\mathbf{r}\cdot(\mathbf{\Omega}\times\mathbf{r})) - \mathbf{r}(\ddot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\mathbf{r}))] \text{ by (A.1) \& (A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&= (6\varepsilon_h^2 - \varrho_t\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (2\varrho_u\varepsilon_h + 3\varrho_t\varepsilon_g)(\mathbf{\Omega}\times\mathbf{r}) + (9\varepsilon_h\varepsilon_g + \varrho_u\varepsilon_b)(\mathbf{\Lambda}\times\mathbf{r})\\
&\quad - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + 4\varepsilon_b\varepsilon_h(\Omega^2\mathbf{\Lambda} - \varepsilon_g\mathbf{\Omega}) - 6\Omega^2\varepsilon_h(\varepsilon_b\mathbf{\Lambda} - \varepsilon_g\mathbf{r})\\
&\quad + (\varrho_t\Omega^2 - 6\varepsilon_g\varepsilon_h)(\varepsilon_b\mathbf{\Omega} - \Omega^2\mathbf{r}) + 2\varepsilon_h(\varepsilon_b\ddot{\mathbf{\Lambda}} - \varsigma_h\mathbf{r}) - 6\varepsilon_b\varepsilon_g(\varepsilon_b\mathbf{\Lambda} - \varepsilon_h\mathbf{\Omega}) + 9\Omega^2\varepsilon_g(r^2\mathbf{\Lambda} - \varepsilon_h\mathbf{r})\\
&\quad + (9\varepsilon_g^2 + \varrho_u\Omega^2)(r^2\mathbf{\Omega} - \varepsilon_b\mathbf{r}) - 3\varepsilon_g(r^2\ddot{\mathbf{\Lambda}} - \varsigma_o\mathbf{r}) - 3\varepsilon_h(\varepsilon_h\dot{\mathbf{\Lambda}} - \varsigma_d\mathbf{r}) - \varepsilon_b(\varsigma_n\dot{\mathbf{\Lambda}} - \dot{\mathbf{\Lambda}}^2\mathbf{r})\\
&\quad - \varrho_t(\varepsilon_b\dot{\mathbf{\Lambda}} - \varsigma_c\mathbf{r}) - \varrho_u(r^2\dot{\mathbf{\Lambda}} - \varsigma_n\mathbf{r}) + 3(\varepsilon_h\Omega^2 - \varepsilon_g\varepsilon_b)(\varepsilon_h\mathbf{\Omega} - \varepsilon_g\mathbf{r}) + \varepsilon_b\Omega^2(\varsigma_n\mathbf{\Omega} - \varsigma_c\mathbf{r})\\
&\quad + 2\varepsilon_b^2(\varepsilon_g\mathbf{\Lambda} - \mathbf{\Lambda}^2\mathbf{\Omega}) - 3\Omega^2\varepsilon_b(\varepsilon_h\mathbf{\Lambda} - \mathbf{\Lambda}^2\mathbf{r}) + \varepsilon_b(\varepsilon_h\ddot{\mathbf{\Lambda}} - \varsigma_i\mathbf{r}) + 2\varepsilon_b(-\mathfrak{d}_l\mathbf{\Lambda} + \mathfrak{d}_h\mathbf{\Omega}) - 3\Omega^2\mathfrak{d}_h\mathbf{r}\\
&\quad - 3\varepsilon_g\mathfrak{d}_l\mathbf{r} - \mathfrak{d}_j\mathbf{r} + 2\varepsilon_b\Omega^2\mathfrak{d}_k\mathbf{\Omega} + 3\Omega^4\varepsilon_m\mathbf{r} + \Omega^2\mathfrak{d}_m\mathbf{r} \text{ by (5.1a), (5.23a), (5.23b) \& (A.4)}
\end{aligned}
$$

$$
\begin{aligned}
&= (6\varepsilon_h^2 - \varrho_t\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (2\varrho_u\varepsilon_h + 3\varrho_t\varepsilon_g)(\mathbf{\Omega}\times\mathbf{r}) + (9\varepsilon_h\varepsilon_g + \varrho_u\varepsilon_b)(\mathbf{\Lambda}\times\mathbf{r})\\
&\quad - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + 4\varepsilon_b\varepsilon_h\Omega^2\mathbf{\Lambda} - 4\varepsilon_b\varepsilon_h\varepsilon_g\mathbf{\Omega} - 6\Omega^2\varepsilon_h\varepsilon_b\mathbf{\Lambda} + 6\Omega^2\varepsilon_h\varepsilon_g\mathbf{r}\\
&\quad + \varepsilon_b(\varrho_t\Omega^2 - 6\varepsilon_g\varepsilon_h)\mathbf{\Omega} - \Omega^2(\varrho_t\Omega^2 - 6\varepsilon_g\varepsilon_h)\mathbf{r} + 2\varepsilon_h\varepsilon_b\ddot{\mathbf{\Lambda}} - 2\varepsilon_h\varsigma_h\mathbf{r} - 6\varepsilon_b^2\varepsilon_g\mathbf{\Lambda} + 6\varepsilon_b\varepsilon_g\varepsilon_h\mathbf{\Omega}\\
&\quad + 9\Omega^2\varepsilon_g r^2\mathbf{\Lambda} - 9\Omega^2\varepsilon_g\varepsilon_h\mathbf{r} + r^2(9\varepsilon_g^2 + \varrho_u\Omega^2)\mathbf{\Omega} - \varepsilon_b(9\varepsilon_g^2 + \varrho_u\Omega^2)\mathbf{r} - 3\varepsilon_g r^2\ddot{\mathbf{\Lambda}} + 3\varepsilon_g\varsigma_o\mathbf{r} - 3\varepsilon_h^2\dot{\mathbf{\Lambda}}\\
&\quad + 3\varepsilon_h\varsigma_d\mathbf{r} - \varepsilon_b\varsigma_n\dot{\mathbf{\Lambda}} + \varepsilon_b\dot{\mathbf{\Lambda}}^2\mathbf{r} - \varrho_t\varepsilon_b\dot{\mathbf{\Lambda}} + \varrho_t\varsigma_c\mathbf{r} - \varrho_u r^2\dot{\mathbf{\Lambda}} + \varrho_u\varsigma_n\mathbf{r} + 3\varepsilon_h(\varepsilon_h\Omega^2 - \varepsilon_g\varepsilon_b)\mathbf{\Omega}\\
&\quad - 3\varepsilon_g(\varepsilon_h\Omega^2 - \varepsilon_g\varepsilon_b)\mathbf{r} + \varepsilon_b\Omega^2\varsigma_n\mathbf{\Omega} - \varepsilon_b\Omega^2\varsigma_c\mathbf{r} + 2\varepsilon_b^2\varepsilon_g\mathbf{\Lambda} - 2\varepsilon_b^2\mathbf{\Lambda}^2\mathbf{\Omega} - 3\Omega^2\varepsilon_b\varepsilon_h\mathbf{\Lambda} + 3\Omega^2\varepsilon_b\mathbf{\Lambda}^2\mathbf{r}\\
&\quad + \varepsilon_b\varepsilon_h\ddot{\mathbf{\Lambda}} - \varepsilon_b\varsigma_i\mathbf{r} - 2\varepsilon_b\mathfrak{d}_l\mathbf{\Lambda} + 2\varepsilon_b\mathfrak{d}_h\mathbf{\Omega} - 3\Omega^2\mathfrak{d}_h\mathbf{r} - 3\varepsilon_g\mathfrak{d}_l\mathbf{r} - \mathfrak{d}_j\mathbf{r} + 2\varepsilon_b\Omega^2\mathfrak{d}_k\mathbf{\Omega} + 3\Omega^4\varepsilon_m\mathbf{r} + \Omega^2\mathfrak{d}_m\mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= (6\varepsilon_h^2 - \varrho_t\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (2\varrho_u\varepsilon_h + 3\varrho_t\varepsilon_g)(\mathbf{\Omega}\times\mathbf{r}) + (9\varepsilon_h\varepsilon_g + \varrho_u\varepsilon_b)(\mathbf{\Lambda}\times\mathbf{r}) \\
&\quad - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + (-3\varepsilon_h^2 - \varepsilon_b\varsigma_n - \varrho_t\varepsilon_b - \varrho_u r^2)\dot{\mathbf{\Lambda}} + (2\varepsilon_h\varepsilon_b - 3\varepsilon_g r^2 + \varepsilon_b\varepsilon_h)\ddot{\mathbf{\Lambda}} \\
&\quad + (4\varepsilon_b\varepsilon_h\Omega^2 - 6\Omega^2\varepsilon_h\varepsilon_b - 6\varepsilon_b^2\varepsilon_g + 9\Omega^2\varepsilon_g r^2 + 2\varepsilon_b^2\varepsilon_g - 3\Omega^2\varepsilon_b\varepsilon_h - 2\varepsilon_b\mathfrak{d}_l)\mathbf{\Lambda} \\
&\quad + [-4\varepsilon_b\varepsilon_h\varepsilon_g + \varepsilon_b(\varrho_t\Omega^2 - 6\varepsilon_g\varepsilon_h) + 6\varepsilon_b\varepsilon_g\varepsilon_h + r^2(9\varepsilon_g^2 + \varrho_u\Omega^2) + 3\varepsilon_h(\varepsilon_h\Omega^2 - \varepsilon_g\varepsilon_b) \\
&\qquad + \varepsilon_b\Omega^2\varsigma_n - 2\varepsilon_b^2\mathbf{\Lambda}^2 + 2\varepsilon_b\mathfrak{d}_h + 2\varepsilon_b\Omega^2\mathfrak{d}_k]\mathbf{\Omega} \\
&\quad + [6\Omega^2\varepsilon_h\varepsilon_g - \Omega^2(\varrho_t\Omega^2 - 6\varepsilon_g\varepsilon_h) - 2\varepsilon_h\varsigma_h - 9\Omega^2\varepsilon_g\varepsilon_h - \varepsilon_b(9\varepsilon_g^2 + \varrho_u\Omega^2) + 3\varepsilon_g\varsigma_o + \varepsilon_b\dot{\mathbf{\Lambda}}^2 + \varrho_t\varsigma_c \\
&\qquad + \varrho_u\varsigma_n - 3\varepsilon_g(\varepsilon_h\Omega^2 - \varepsilon_g\varepsilon_b) - \varepsilon_b\Omega^2\varsigma_c + 3\Omega^2\varepsilon_b\mathbf{\Lambda}^2 - \varepsilon_b\varsigma_i - 3\Omega^2\mathfrak{d}_h - 3\varepsilon_g\mathfrak{d}_l - \mathfrak{d}_j \\
&\qquad + 3\Omega^4\varepsilon_m + \Omega^2\mathfrak{d}_m + 3\varepsilon_h\varsigma_d]\mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= (6\varepsilon_h^2 - \varrho_t\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (2\varrho_u\varepsilon_h + 3\varrho_t\varepsilon_g)(\mathbf{\Omega}\times\mathbf{r}) + (9\varepsilon_h\varepsilon_g + \varrho_u\varepsilon_b)(\mathbf{\Lambda}\times\mathbf{r}) \\
&\quad - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) - (3\varepsilon_h^2 + \varepsilon_b\varsigma_n + \varrho_t\varepsilon_b + \varrho_u r^2)\dot{\mathbf{\Lambda}} + 3(\varepsilon_h\varepsilon_b - \varepsilon_g r^2)\ddot{\mathbf{\Lambda}} \\
&\quad + [5\Omega^2(r^2\varepsilon_g - \varepsilon_b\varepsilon_h) + 4\varepsilon_g(r^2\Omega^2 - \varepsilon_b^2) - 2\varepsilon_b\mathfrak{d}_l]\mathbf{\Lambda} \\
&\quad + [7\varepsilon_g(r^2\varepsilon_g - \varepsilon_b\varepsilon_h) + \Omega^2\varepsilon_b(\varrho_t + 2\mathfrak{d}_k + \varsigma_n) + \Omega^2(r^2\varrho_u + 3\varepsilon_h^2) + 2\varepsilon_b(\mathfrak{d}_h - \Lambda^2\varepsilon_b) + 2r^2\varepsilon_g^2]\mathbf{\Omega} \\
&\quad + [\Omega^4(3\varepsilon_m - \varrho_t) + \Omega^2(\mathfrak{d}_m - 3\mathfrak{d}_h) + \Omega^2\varepsilon_b(3\Lambda^2 - \varrho_u - \varsigma_c) + \varepsilon_b(\dot{\mathbf{\Lambda}}^2 - 6\varepsilon_g^2 - \varsigma_i) \\
&\qquad + \varepsilon_h(3\varsigma_d - 2\varsigma_h) + 3\varepsilon_g(\varsigma_o - \mathfrak{d}_l) + \varrho_t\varsigma_c + \varrho_u\varsigma_n - \mathfrak{d}_j]\mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= (6\varepsilon_h^2 - \varrho_t\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (2\varrho_u\varepsilon_h + 3\varrho_t\varepsilon_g)(\mathbf{\Omega}\times\mathbf{r}) + (9\varepsilon_h\varepsilon_g + \varrho_u\varepsilon_b)(\mathbf{\Lambda}\times\mathbf{r}) \\
&\quad - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) - (3\varepsilon_h^2 + \varepsilon_b\varsigma_n + \varrho_t\varepsilon_b + \varrho_u r^2)\dot{\mathbf{\Lambda}} - 3(\varepsilon_g r^2 - \varepsilon_h\varepsilon_b)\ddot{\mathbf{\Lambda}} \\
&\quad + [5\Omega^2(r^2\varepsilon_g - \varepsilon_b\varepsilon_h) + 4\varepsilon_g(r^2\Omega^2 - \varepsilon_b^2) - 2\varepsilon_b\mathfrak{d}_l]\mathbf{\Lambda} \\
&\quad + [7\varepsilon_g(r^2\varepsilon_g - \varepsilon_b\varepsilon_h) + \Omega^2\varepsilon_b(\varrho_t + 2\mathfrak{d}_k + \varsigma_n) + \Omega^2(r^2\varrho_u + 3\varepsilon_h^2) + 2\varepsilon_b(\mathfrak{d}_h - \Lambda^2\varepsilon_b) + 2r^2\varepsilon_g^2]\mathbf{\Omega} \\
&\quad + [\Omega^4(3\varepsilon_m - \varrho_t) + \Omega^2(\mathfrak{d}_m - 3\mathfrak{d}_h) + \Omega^2\varepsilon_b(3\Lambda^2 - \varrho_u - \varsigma_c) + \varepsilon_b(\dot{\mathbf{\Lambda}}^2 - 6\varepsilon_g^2 - \varsigma_i) \\
&\qquad + \varepsilon_h(3\varsigma_d - 2\varsigma_h) + 3\varepsilon_g(\varsigma_o - \mathfrak{d}_l) + \varrho_t\varsigma_c + \varrho_u\varsigma_n - \mathfrak{d}_j]\mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= (6\varepsilon_h^2 - \varrho_t\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (2\varrho_u\varepsilon_h + 3\varrho_t\varepsilon_g)(\mathbf{\Omega}\times\mathbf{r}) + (9\varepsilon_h\varepsilon_g + \varrho_u\varepsilon_b)(\mathbf{\Lambda}\times\mathbf{r}) \\
&\quad - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + (5\Omega^2\varphi_c + 4\varepsilon_g\varphi_a^2 - 2\varepsilon_b\mathfrak{d}_l)\mathbf{\Lambda} - (3\varepsilon_h^2 + \varepsilon_b\varsigma_n + \varrho_t\varepsilon_b + \varrho_u r^2)\dot{\mathbf{\Lambda}} \\
&\quad - 3\varphi_c\ddot{\mathbf{\Lambda}} + [7\varepsilon_g\varphi_c + \Omega^2\varepsilon_b(\varrho_t + 2\mathfrak{d}_k + \varsigma_n) + \Omega^2(r^2\varrho_u + 3\varepsilon_h^2) + 2\varepsilon_b(\mathfrak{d}_h - \Lambda^2\varepsilon_b) + 2r^2\varepsilon_g^2]\mathbf{\Omega} \\
&\quad + [\Omega^4(3\varepsilon_m - \varrho_t) + \Omega^2(\mathfrak{d}_m - 3\mathfrak{d}_h) + \Omega^2\varepsilon_b(3\Lambda^2 - \varrho_u - \varsigma_c) + \varepsilon_b(\dot{\mathbf{\Lambda}}^2 - 6\varepsilon_g^2 - \varsigma_i) \\
&\qquad + \varepsilon_h(3\varsigma_d - 2\varsigma_h) + 3\varepsilon_g(\varsigma_o - \mathfrak{d}_l) + \varrho_t\varsigma_c + \varrho_u\varsigma_n - \mathfrak{d}_j]\mathbf{r} \text{ by (5.1b)} \\
&= \mathfrak{y}_t(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{y}_u(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{y}_v(\mathbf{\Lambda}\times\mathbf{r}) - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) \\
&\qquad + \mathfrak{y}_w\mathbf{\Lambda} - \mathfrak{y}_x\dot{\mathbf{\Lambda}} - 3\varphi_c\ddot{\mathbf{\Lambda}} + \mathfrak{y}_y\mathbf{\Omega} + \mathfrak{y}_z\mathbf{r} \text{ by (5.23o)}
\end{aligned}
\tag{5.44j}
$$

$$
\begin{aligned}
\mathfrak{J}_k &= \dot{\mathbf{a}}\times\dot{\mathbf{e}} \text{ by (3.15b)} \\
&= [2\varepsilon_h\mathbf{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\mathbf{\Lambda}}\times\mathbf{r} - \Omega^2(\mathbf{\Omega}\times\mathbf{r}) + \varepsilon_b\mathbf{\Lambda}]\times[\mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{x}_r\mathbf{\Omega} + \varphi_i\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}}] \\
&\qquad \text{by (5.30a) \& (5.40a)} \\
&= 2\mathfrak{x}_o\varepsilon_h[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + 2\varphi_g\varphi_h\varepsilon_h[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + 2\varphi_i\varepsilon_h(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_u\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - 3\mathfrak{x}_o\varepsilon_g[\mathbf{r}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] - 3\varphi_g\varphi_h\varepsilon_g[\mathbf{r}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] - 3\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda}) \\
&\quad + 3\mathfrak{x}_u\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_o[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + \varphi_g\varphi_h[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] - \mathfrak{x}_r[\mathbf{\Omega}\times(\dot{\mathbf{\Lambda}}\times\mathbf{r})] \\
&\quad - \varphi_i[\mathbf{\Lambda}\times(\dot{\mathbf{\Lambda}}\times\mathbf{r})] + \mathfrak{x}_u[\widehat{\mathbf{p}}\times(\dot{\mathbf{\Lambda}}\times\mathbf{r})] - \mathfrak{x}_o\Omega^2[(\mathbf{\Omega}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] - \varphi_g\varphi_h\Omega^2[(\mathbf{\Omega}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{x}_r\Omega^2[\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})] + \varphi_i\Omega^2[\mathbf{\Lambda}\times(\mathbf{\Omega}\times\mathbf{r})] - \mathfrak{x}_u\Omega^2[\widehat{\mathbf{p}}\times(\mathbf{\Omega}\times\mathbf{r})] + \mathfrak{x}_o\varepsilon_b[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] \\
&\quad + \varphi_g\varphi_h\varepsilon_b[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{x}_r\varepsilon_b(\mathbf{\Lambda}\times\mathbf{\Omega}) - \mathfrak{x}_u\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}})
\end{aligned}
$$

$$= (2\varphi_i\varepsilon_h - \mathfrak{x}_r\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_u\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda})$$
$$+ 3\mathfrak{x}_u\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + 2\mathfrak{x}_o\varepsilon_h[\Omega^2\widehat{\mathbf{p}} - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})] + 2\varphi_g\varphi_h\varepsilon_h[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot\mathbf{\Lambda}) - \mathbf{\Lambda}(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})]$$
$$- 3\mathfrak{x}_o\varepsilon_g[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot\mathbf{r}) - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot\mathbf{r})] - 3\varphi_g\varphi_h\varepsilon_g[\widehat{\mathbf{p}}(\mathbf{r}\cdot\mathbf{\Lambda}) - \mathbf{\Lambda}(\mathbf{r}\cdot\widehat{\mathbf{p}})] - \mathfrak{x}_r[\dot{\mathbf{\Lambda}}(\mathbf{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})]$$
$$- \varphi_i[\dot{\mathbf{\Lambda}}(\mathbf{\Lambda}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}})] + \mathfrak{x}_u[\dot{\mathbf{\Lambda}}(\widehat{\mathbf{p}}\cdot\mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}})] + \mathfrak{x}_r\Omega^2[\mathbf{\Omega}(\mathbf{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}]$$
$$+ \varphi_i\Omega^2[\mathbf{\Omega}(\mathbf{\Lambda}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Omega}\cdot\mathbf{\Lambda})] - \mathfrak{x}_u\Omega^2[\mathbf{\Omega}(\widehat{\mathbf{p}}\cdot\mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})] + \mathfrak{x}_o\varepsilon_b[\widehat{\mathbf{p}}(\mathbf{\Lambda}\cdot\mathbf{\Omega}) - \mathbf{\Omega}(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})]$$
$$+ \varphi_g\varphi_h\varepsilon_b[\Lambda^2\widehat{\mathbf{p}} - \mathbf{\Lambda}(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})] + \mathfrak{x}_o[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})) - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r}))]$$
$$+ \varphi_g\varphi_h[\widehat{\mathbf{p}}(\mathbf{\Lambda}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})) - \mathbf{\Lambda}(\widehat{\mathbf{p}}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r}))] - \mathfrak{x}_o\Omega^2[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot(\mathbf{\Omega}\times\mathbf{r})) - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\mathbf{r}))]$$
$$- \varphi_g\varphi_h\Omega^2[\widehat{\mathbf{p}}(\mathbf{\Lambda}\cdot(\mathbf{\Omega}\times\mathbf{r})) - \mathbf{\Lambda}(\widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\mathbf{r}))] \text{ by (A.1) \& (A.5)}$$

$$= (2\varphi_i\varepsilon_h - \mathfrak{x}_r\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_u\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda})$$
$$+ 3\mathfrak{x}_u\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + 2\mathfrak{x}_o\varepsilon_h(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Omega}) + 2\varphi_g\varphi_h\varepsilon_h(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Lambda})$$
$$- 3\mathfrak{x}_o\varepsilon_g(\varepsilon_b\widehat{\mathbf{p}} - \varepsilon_f\mathbf{\Omega}) - 3\varphi_g\varphi_h\varepsilon_g(\varepsilon_h\widehat{\mathbf{p}} - \varepsilon_f\mathbf{\Lambda}) - \mathfrak{x}_r(\varepsilon_b\dot{\mathbf{\Lambda}} - \varsigma_c\mathbf{r}) - \varphi_i(\varepsilon_h\dot{\mathbf{\Lambda}} - \varsigma_d\mathbf{r})$$
$$+ \mathfrak{x}_u(\varepsilon_f\dot{\mathbf{\Lambda}} - \varsigma_a\mathbf{r}) + \mathfrak{x}_r\Omega^2(\varepsilon_b\mathbf{\Omega} - \Omega^2\mathbf{r}) + \varphi_i\Omega^2(\varepsilon_h\mathbf{\Omega} - \varepsilon_g\mathbf{r}) - \mathfrak{x}_u\Omega^2(\varepsilon_f\mathbf{\Omega} - \varepsilon_c\mathbf{r})$$
$$+ \mathfrak{x}_o\varepsilon_b(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_i\mathbf{\Omega}) + \varphi_g\varphi_h\varepsilon_b(\Lambda^2\widehat{\mathbf{p}} - \varepsilon_i\mathbf{\Lambda}) + \mathfrak{x}_o(-\mathfrak{d}_l\widehat{\mathbf{p}} - \mathfrak{d}_n\mathbf{\Omega}) + \varphi_g\varphi_h(-\mathfrak{d}_h\widehat{\mathbf{p}} - \mathfrak{d}_n\mathbf{\Lambda})$$
$$+ \varepsilon_l\mathfrak{x}_o\Omega^2\mathbf{\Omega} - \varphi_g\varphi_h\Omega^2(-\varepsilon_m\widehat{\mathbf{p}} - \varepsilon_l\mathbf{\Lambda}) \text{ by (5.1a), (5.23a) \& (5.23b)}$$

$$= (2\varphi_i\varepsilon_h - \mathfrak{x}_r\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_u\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda})$$
$$+ 3\mathfrak{x}_u\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + 2\mathfrak{x}_o\varepsilon_h\Omega^2\widehat{\mathbf{p}} - 2\mathfrak{x}_o\varepsilon_h\varepsilon_c\mathbf{\Omega} + 2\varphi_g\varphi_h\varepsilon_h\varepsilon_g\widehat{\mathbf{p}} - 2\varphi_g\varphi_h\varepsilon_h\varepsilon_c\mathbf{\Lambda}$$
$$- 3\mathfrak{x}_o\varepsilon_g\varepsilon_b\widehat{\mathbf{p}} + 3\mathfrak{x}_o\varepsilon_g\varepsilon_f\mathbf{\Omega} - 3\varphi_g\varphi_h\varepsilon_g\varepsilon_h\widehat{\mathbf{p}} + 3\varphi_g\varphi_h\varepsilon_g\varepsilon_f\mathbf{\Lambda} - \mathfrak{x}_r\varepsilon_b\dot{\mathbf{\Lambda}} + \mathfrak{x}_r\varsigma_c\mathbf{r} - \varphi_i\varepsilon_h\dot{\mathbf{\Lambda}} + \varphi_i\varsigma_d\mathbf{r}$$
$$+ \mathfrak{x}_u\varepsilon_f\dot{\mathbf{\Lambda}} - \mathfrak{x}_u\varsigma_a\mathbf{r} + \mathfrak{x}_r\Omega^2\varepsilon_b\mathbf{\Omega} - \mathfrak{x}_r\Omega^4\mathbf{r} + \varphi_i\Omega^2\varepsilon_h\mathbf{\Omega} - \varphi_i\Omega^2\varepsilon_g\mathbf{r} - \mathfrak{x}_u\Omega^2\varepsilon_f\mathbf{\Omega} + \mathfrak{x}_u\Omega^2\varepsilon_c\mathbf{r}$$
$$+ \mathfrak{x}_o\varepsilon_b\varepsilon_g\widehat{\mathbf{p}} - \mathfrak{x}_o\varepsilon_b\varepsilon_i\mathbf{\Omega} + \varphi_g\varphi_h\varepsilon_b\Lambda^2\widehat{\mathbf{p}} - \varphi_g\varphi_h\varepsilon_b\varepsilon_i\mathbf{\Lambda} - \mathfrak{x}_o\mathfrak{d}_l\widehat{\mathbf{p}} - \mathfrak{x}_o\mathfrak{d}_n\mathbf{\Omega} - \varphi_g\varphi_h\mathfrak{d}_h\widehat{\mathbf{p}} - \varphi_g\varphi_h\mathfrak{d}_n\mathbf{\Lambda}$$
$$+ \varepsilon_l\mathfrak{x}_o\Omega^2\mathbf{\Omega} + \varphi_g\varphi_h\Omega^2\varepsilon_m\widehat{\mathbf{p}} + \varphi_g\varphi_h\Omega^2\varepsilon_l\mathbf{\Lambda}$$

$$= (2\varphi_i\varepsilon_h - \mathfrak{x}_r\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_u\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda}) + 3\mathfrak{x}_u\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}})$$
$$- \mathfrak{x}_u\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + (-\mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h + \mathfrak{x}_u\varepsilon_f)\dot{\mathbf{\Lambda}} + (\mathfrak{x}_r\varsigma_c + \varphi_i\varsigma_d - \mathfrak{x}_u\varsigma_a - \mathfrak{x}_r\Omega^4 - \varphi_i\Omega^2\varepsilon_g + \mathfrak{x}_u\Omega^2\varepsilon_c)\mathbf{r}$$
$$+ (2\mathfrak{x}_o\varepsilon_h\Omega^2 + 2\varphi_g\varphi_h\varepsilon_h\varepsilon_g - 3\mathfrak{x}_o\varepsilon_g\varepsilon_b - 3\varphi_g\varphi_h\varepsilon_g\varepsilon_h + \mathfrak{x}_o\varepsilon_b\varepsilon_g + \varphi_g\varphi_h\varepsilon_b\Lambda^2 - \mathfrak{x}_o\mathfrak{d}_l - \varphi_g\varphi_h\mathfrak{d}_h$$
$$+ \varphi_g\varphi_h\Omega^2\varepsilon_m)\widehat{\mathbf{p}} + (-2\mathfrak{x}_o\varepsilon_h\varepsilon_c + 3\mathfrak{x}_o\varepsilon_g\varepsilon_f + \mathfrak{x}_r\Omega^2\varepsilon_b + \varphi_i\Omega^2\varepsilon_h - \mathfrak{x}_u\Omega^2\varepsilon_f - \mathfrak{x}_o\varepsilon_b\varepsilon_i - \mathfrak{x}_o\mathfrak{d}_n + \varepsilon_l\mathfrak{x}_o\Omega^2)\mathbf{\Omega}$$
$$+ (-2\mathfrak{x}_o\varepsilon_h\varepsilon_c + 3\mathfrak{x}_o\varepsilon_g\varepsilon_f + \mathfrak{x}_r\Omega^2\varepsilon_b + \varphi_i\Omega^2\varepsilon_h - \mathfrak{x}_u\Omega^2\varepsilon_f - \mathfrak{x}_o\varepsilon_b\varepsilon_i - \mathfrak{x}_o\mathfrak{d}_n + \varepsilon_l\mathfrak{x}_o\Omega^2$$
$$- 2\varphi_g\varphi_h\varepsilon_h\varepsilon_c + 3\varphi_g\varphi_h\varepsilon_g\varepsilon_f - \varphi_g\varphi_h\varepsilon_b\varepsilon_i - \varphi_g\varphi_h\mathfrak{d}_n + \varphi_g\varphi_h\Omega^2\varepsilon_l)\mathbf{\Lambda}$$

$$= (2\varphi_i\varepsilon_h - \mathfrak{x}_r\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_u\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda})$$
$$+ 3\mathfrak{x}_u\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + [\mathfrak{x}_r(\varsigma_c - \Omega^4) + \varphi_i(\varsigma_d - \Omega^2\varepsilon_g) + \mathfrak{x}_u(\Omega^2\varepsilon_c - \varsigma_a)]\mathbf{r}$$
$$+ [\Omega^2(2\varepsilon_h\mathfrak{x}_o + \varepsilon_m\varphi_g\varphi_h) - \mathfrak{x}_o(\mathfrak{d}_l + 2\varepsilon_b\varepsilon_g) + \varphi_g\varphi_h(\Lambda^2\varepsilon_b - \mathfrak{d}_h - \varepsilon_g\varepsilon_h)]\widehat{\mathbf{p}}$$
$$+ [\Omega^2(\varepsilon_l\mathfrak{x}_o - \varepsilon_f\mathfrak{x}_u + \varepsilon_b\mathfrak{x}_r + \varepsilon_h\varphi_i) + \mathfrak{x}_o(3\varepsilon_f\varepsilon_g - 2\varepsilon_c\varepsilon_h - \varepsilon_b\varepsilon_i - \mathfrak{d}_n)]\mathbf{\Omega}$$
$$+ \varphi_g\varphi_h(-2\varepsilon_c\varepsilon_h + 3\varepsilon_f\varepsilon_g - \varepsilon_b\varepsilon_i - \mathfrak{d}_n + \Omega^2\varepsilon_l)\mathbf{\Lambda} + (\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h)\dot{\mathbf{\Lambda}}$$
$$= \varkappa_a(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_u\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda}) + 3\mathfrak{x}_u\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}})$$
$$- \mathfrak{x}_u\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + \varkappa_b\mathbf{r} + \varkappa_e\widehat{\mathbf{p}} + \varkappa_f\mathbf{\Omega} + \varphi_g\varphi_h\varkappa_d\mathbf{\Lambda} + \varkappa_c\dot{\mathbf{\Lambda}} \text{ by (5.23p)} \tag{5.44k}$$

$$
\begin{aligned}
\mathfrak{J}_l &= \dot{\mathbf{a}} \times \ddot{\mathbf{e}} \text{ by (3.15b)} \\
&= [2\varepsilon_h \boldsymbol{\Omega} - 3\varepsilon_g \mathbf{r} + \dot{\boldsymbol{\Lambda}} \times \mathbf{r} - \Omega^2(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b \boldsymbol{\Lambda}] \times [\mathfrak{x}_p(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + 2\mathfrak{x}_o(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \varphi_g \varphi_h(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) \\
&\qquad + \mathfrak{x}_s \boldsymbol{\Omega} + 2\mathfrak{x}_r \boldsymbol{\Lambda} + \varphi_i \dot{\boldsymbol{\Lambda}} - \mathfrak{x}_v \widehat{\mathbf{p}}] \text{ by (5.30a) \& (5.40b)} \\
&= 2\mathfrak{x}_p \varepsilon_h[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + 4\mathfrak{x}_o \varepsilon_h[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + 2\varphi_g \varphi_h \varepsilon_h[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&\quad + 2\mathfrak{x}_s \varepsilon_h(\boldsymbol{\Omega} \times \boldsymbol{\Omega}) + 4\mathfrak{x}_r \varepsilon_h(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + 2\varphi_i \varepsilon_h(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) - 2\mathfrak{x}_v \varepsilon_h(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - 3\mathfrak{x}_p \varepsilon_g[\mathbf{r} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - 6\mathfrak{x}_o \varepsilon_g[\mathbf{r} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] - 3\varphi_g \varphi_h \varepsilon_g[\mathbf{r} \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&\quad - 3\mathfrak{x}_s \varepsilon_g(\mathbf{r} \times \boldsymbol{\Omega}) - 6\mathfrak{x}_r \varepsilon_g(\mathbf{r} \times \boldsymbol{\Lambda}) - 3\varphi_i \varepsilon_g(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + 3\mathfrak{x}_v \varepsilon_g(\mathbf{r} \times \widehat{\mathbf{p}}) \\
&\quad + \mathfrak{x}_p[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + 2\mathfrak{x}_o[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \varphi_g \varphi_h[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&\quad - \mathfrak{x}_s[\boldsymbol{\Omega} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] - 2\mathfrak{x}_r[\boldsymbol{\Lambda} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] - \varphi_i[\dot{\boldsymbol{\Lambda}} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{x}_v[\widehat{\mathbf{p}} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&\quad - \mathfrak{x}_p \Omega^2[(\boldsymbol{\Omega} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - 2\mathfrak{x}_o \Omega^2[(\boldsymbol{\Omega} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] - \varphi_g \varphi_h \Omega^2[(\boldsymbol{\Omega} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&\quad + \mathfrak{x}_s \Omega^2[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] + 2\mathfrak{x}_r \Omega^2[\boldsymbol{\Lambda} \times (\boldsymbol{\Omega} \times \mathbf{r})] + \varphi_i \Omega^2[\dot{\boldsymbol{\Lambda}} \times (\boldsymbol{\Omega} \times \mathbf{r})] - \mathfrak{x}_v \Omega^2[\widehat{\mathbf{p}} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{x}_p \varepsilon_b[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + 2\mathfrak{x}_o \varepsilon_b[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \varphi_g \varphi_h \varepsilon_b[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{x}_s \varepsilon_b(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) \\
&\quad + 2\mathfrak{x}_r \varepsilon_b(\boldsymbol{\Lambda} \times \boldsymbol{\Lambda}) + \varphi_i \varepsilon_b(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}}) - \mathfrak{x}_v \varepsilon_b(\boldsymbol{\Lambda} \times \widehat{\mathbf{p}})
\end{aligned}
$$

$$
\begin{aligned}
&= (4\mathfrak{x}_r \varepsilon_h - \mathfrak{x}_s \varepsilon_b)(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - 2\mathfrak{x}_v \varepsilon_h(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) - 3\mathfrak{x}_s \varepsilon_g(\mathbf{r} \times \boldsymbol{\Omega}) - 6\mathfrak{x}_r \varepsilon_g(\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + 3\mathfrak{x}_v \varepsilon_g(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_v \varepsilon_b(\boldsymbol{\Lambda} \times \widehat{\mathbf{p}}) + 2\varphi_i \varepsilon_h(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) - 3\varphi_i \varepsilon_g(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \varphi_i \varepsilon_b(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + 2\mathfrak{x}_p \varepsilon_h[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + 4\mathfrak{x}_o \varepsilon_h[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + 2\varphi_g \varphi_h \varepsilon_h[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&\quad - 3\mathfrak{x}_p \varepsilon_g[\mathbf{r} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - 6\mathfrak{x}_o \varepsilon_g[\mathbf{r} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] - 3\varphi_g \varphi_h \varepsilon_g[\mathbf{r} \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&\quad - \mathfrak{x}_s[\boldsymbol{\Omega} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] - 2\mathfrak{x}_r[\boldsymbol{\Lambda} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] - \varphi_i[\dot{\boldsymbol{\Lambda}} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{x}_v[\widehat{\mathbf{p}} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&\quad + \mathfrak{x}_s \Omega^2[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] + 2\mathfrak{x}_r \Omega^2[\boldsymbol{\Lambda} \times (\boldsymbol{\Omega} \times \mathbf{r})] + \varphi_i \Omega^2[\dot{\boldsymbol{\Lambda}} \times (\boldsymbol{\Omega} \times \mathbf{r})] - \mathfrak{x}_v \Omega^2[\widehat{\mathbf{p}} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{x}_p \varepsilon_b[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + 2\mathfrak{x}_o \varepsilon_b[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \varphi_g \varphi_h \varepsilon_b[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&\quad + \mathfrak{x}_p[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + 2\mathfrak{x}_o[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \varphi_g \varphi_h[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&\quad - \mathfrak{x}_p \Omega^2[(\boldsymbol{\Omega} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - 2\mathfrak{x}_o \Omega^2[(\boldsymbol{\Omega} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] - \varphi_g \varphi_h \Omega^2[(\boldsymbol{\Omega} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= (4\mathfrak{x}_r \varepsilon_h - \mathfrak{x}_s \varepsilon_b)(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - 2\mathfrak{x}_v \varepsilon_h(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) - 3\mathfrak{x}_s \varepsilon_g(\mathbf{r} \times \boldsymbol{\Omega}) - 6\mathfrak{x}_r \varepsilon_g(\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + 3\mathfrak{x}_v \varepsilon_g(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_v \varepsilon_b(\boldsymbol{\Lambda} \times \widehat{\mathbf{p}}) + 2\varphi_i \varepsilon_h(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) - 3\varphi_i \varepsilon_g(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \varphi_i \varepsilon_b(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + 2\mathfrak{x}_p \varepsilon_h[\Omega^2 \widehat{\mathbf{p}} - \boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})] + 4\mathfrak{x}_o \varepsilon_h[\widehat{\mathbf{p}}(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})] + 2\varphi_g \varphi_h \varepsilon_h[\widehat{\mathbf{p}}(\boldsymbol{\Omega} \cdot \dot{\boldsymbol{\Lambda}}) - \dot{\boldsymbol{\Lambda}}(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})] \\
&\quad - 3\mathfrak{x}_p \varepsilon_g[\widehat{\mathbf{p}}(\mathbf{r} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\mathbf{r} \cdot \widehat{\mathbf{p}})] - 6\mathfrak{x}_o \varepsilon_g[\widehat{\mathbf{p}}(\mathbf{r} \cdot \boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\mathbf{r} \cdot \widehat{\mathbf{p}})] - 3\varphi_g \varphi_h \varepsilon_g[\widehat{\mathbf{p}}(\mathbf{r} \cdot \dot{\boldsymbol{\Lambda}}) - \dot{\boldsymbol{\Lambda}}(\mathbf{r} \cdot \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_s[\dot{\boldsymbol{\Lambda}}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega} \cdot \dot{\boldsymbol{\Lambda}})] - 2\mathfrak{x}_r[\dot{\boldsymbol{\Lambda}}(\boldsymbol{\Lambda} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Lambda} \cdot \dot{\boldsymbol{\Lambda}})] - \varphi_i[\dot{\boldsymbol{\Lambda}}(\dot{\boldsymbol{\Lambda}} \cdot \mathbf{r}) - \dot{\boldsymbol{\Lambda}}^2 \mathbf{r}] \\
&\quad + \mathfrak{x}_v[\dot{\boldsymbol{\Lambda}}(\widehat{\mathbf{p}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}})] + \mathfrak{x}_s \Omega^2[\boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \Omega^2 \mathbf{r}] + 2\mathfrak{x}_r \Omega^2[\boldsymbol{\Omega}(\boldsymbol{\Lambda} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega})] \\
&\quad + \varphi_i \Omega^2[\boldsymbol{\Omega}(\dot{\boldsymbol{\Lambda}} \cdot \mathbf{r}) - \mathbf{r}(\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})] - \mathfrak{x}_v \Omega^2[\boldsymbol{\Omega}(\widehat{\mathbf{p}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] + \mathfrak{x}_p \varepsilon_b[\widehat{\mathbf{p}}(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] \\
&\quad + 2\mathfrak{x}_o \varepsilon_b[\Lambda^2 \widehat{\mathbf{p}} - \boldsymbol{\Lambda}(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] + \varphi_g \varphi_h \varepsilon_b[\widehat{\mathbf{p}}(\boldsymbol{\Lambda} \cdot \dot{\boldsymbol{\Lambda}}) - \dot{\boldsymbol{\Lambda}}(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] + \mathfrak{x}_p[\widehat{\mathbf{p}}(\boldsymbol{\Omega} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})) - \boldsymbol{\Omega}(\widehat{\mathbf{p}} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r}))] \\
&\quad + 2\mathfrak{x}_o[\widehat{\mathbf{p}}(\boldsymbol{\Lambda} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})) - \boldsymbol{\Lambda}(\widehat{\mathbf{p}} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r}))] + \varphi_g \varphi_h[\widehat{\mathbf{p}}(\dot{\boldsymbol{\Lambda}} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})) - \dot{\boldsymbol{\Lambda}}(\widehat{\mathbf{p}} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r}))] \\
&\quad - \mathfrak{x}_p \Omega^2[\widehat{\mathbf{p}}(\boldsymbol{\Omega} \cdot (\boldsymbol{\Omega} \times \mathbf{r})) - \boldsymbol{\Omega}(\widehat{\mathbf{p}} \cdot (\boldsymbol{\Omega} \times \mathbf{r}))] - 2\mathfrak{x}_o \Omega^2[\widehat{\mathbf{p}}(\boldsymbol{\Lambda} \cdot (\boldsymbol{\Omega} \times \mathbf{r})) - \boldsymbol{\Lambda}(\widehat{\mathbf{p}} \cdot (\boldsymbol{\Omega} \times \mathbf{r}))] \\
&\quad - \varphi_g \varphi_h \Omega^2[\widehat{\mathbf{p}}(\dot{\boldsymbol{\Lambda}} \cdot (\boldsymbol{\Omega} \times \mathbf{r})) - \dot{\boldsymbol{\Lambda}}(\widehat{\mathbf{p}} \cdot (\boldsymbol{\Omega} \times \mathbf{r}))] \text{ by (A.1) \& (A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&= (4\mathfrak{x}_r\varepsilon_h - \mathfrak{x}_s\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_v\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_s\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 6\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad + 3\mathfrak{x}_v\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + 2\varphi_i\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varphi_i\varepsilon_b(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\\
&\quad + 2\mathfrak{x}_p\varepsilon_h(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Omega}) + 4\mathfrak{x}_o\varepsilon_h(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Lambda}) + 2\varphi_g\varphi_h\varepsilon_h(\varsigma_c\widehat{\mathbf{p}} - \varepsilon_c\dot{\mathbf{\Lambda}}) - 3\mathfrak{x}_p\varepsilon_g(\varepsilon_b\widehat{\mathbf{p}} - \varepsilon_f\mathbf{\Omega})\\
&\quad - 6\mathfrak{x}_o\varepsilon_g(\varepsilon_h\widehat{\mathbf{p}} - \varepsilon_f\mathbf{\Lambda}) - 3\varphi_g\varphi_h\varepsilon_g(\varsigma_n\widehat{\mathbf{p}} - \varepsilon_f\dot{\mathbf{\Lambda}}) - \mathfrak{x}_s(\varepsilon_b\dot{\mathbf{\Lambda}} - \varsigma_c\mathbf{r}) - 2\mathfrak{x}_r(\varepsilon_h\dot{\mathbf{\Lambda}} - \varsigma_d\mathbf{r}) - \varphi_i(\varsigma_n\dot{\mathbf{\Lambda}} - \dot{\mathbf{\Lambda}}^2\mathbf{r})\\
&\quad + \mathfrak{x}_v(\varepsilon_f\dot{\mathbf{\Lambda}} - \varsigma_a\mathbf{r}) + \mathfrak{x}_s\Omega^2(\varepsilon_b\mathbf{\Omega} - \Omega^2\mathbf{r}) + 2\mathfrak{x}_r\Omega^2(\varepsilon_h\mathbf{\Omega} - \varepsilon_g\mathbf{r}) + \varphi_i\Omega^2(\varsigma_n\mathbf{\Omega} - \varsigma_c\mathbf{r}) - \mathfrak{x}_v\Omega^2(\varepsilon_f\mathbf{\Omega} - \varepsilon_c\mathbf{r})\\
&\quad + \mathfrak{x}_p\varepsilon_b(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_i\mathbf{\Omega}) + 2\mathfrak{x}_o\varepsilon_b(\Lambda^2\widehat{\mathbf{p}} - \varepsilon_i\mathbf{\Lambda}) + \varphi_g\varphi_h\varepsilon_b(\varsigma_d\widehat{\mathbf{p}} - \varepsilon_i\dot{\mathbf{\Lambda}}) + \mathfrak{x}_p(-\mathfrak{d}_l\widehat{\mathbf{p}} - \mathfrak{d}_n\mathbf{\Omega})\\
&\quad + 2\mathfrak{x}_o(-\mathfrak{d}_h\widehat{\mathbf{p}} - \mathfrak{d}_n\mathbf{\Lambda}) + \varphi_g\varphi_h(-\mathfrak{d}_n\dot{\mathbf{\Lambda}}) - \mathfrak{x}_p\Omega^2(-\varepsilon_l\mathbf{\Omega}) - 2\mathfrak{x}_o\Omega^2(-\varepsilon_m\widehat{\mathbf{p}} - \varepsilon_l\mathbf{\Lambda})\\
&\quad - \varphi_g\varphi_h\Omega^2(\mathfrak{d}_l\widehat{\mathbf{p}} - \varepsilon_l\dot{\mathbf{\Lambda}}) \text{ by (5.1a), (5.23a) \& (5.23b)}
\end{aligned}
$$

$$
\begin{aligned}
&= (4\mathfrak{x}_r\varepsilon_h - \mathfrak{x}_s\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_v\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_s\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 6\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad + 3\mathfrak{x}_v\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + 2\varphi_i\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varphi_i\varepsilon_b(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\\
&\quad + 2\mathfrak{x}_p\varepsilon_h\Omega^2\widehat{\mathbf{p}} - 2\mathfrak{x}_p\varepsilon_h\varepsilon_c\mathbf{\Omega} + 4\mathfrak{x}_o\varepsilon_h\varepsilon_g\widehat{\mathbf{p}} - 4\mathfrak{x}_o\varepsilon_h\varepsilon_c\mathbf{\Lambda} + 2\varphi_g\varphi_h\varepsilon_h\varsigma_c\widehat{\mathbf{p}} - 2\varphi_g\varphi_h\varepsilon_h\varepsilon_c\dot{\mathbf{\Lambda}}\\
&\quad - 3\mathfrak{x}_p\varepsilon_g\varepsilon_b\widehat{\mathbf{p}} + 3\mathfrak{x}_p\varepsilon_g\varepsilon_f\mathbf{\Omega} - 6\mathfrak{x}_o\varepsilon_g\varepsilon_h\widehat{\mathbf{p}} + 6\mathfrak{x}_o\varepsilon_g\varepsilon_f\mathbf{\Lambda} - 3\varphi_g\varphi_h\varepsilon_g\varsigma_n\widehat{\mathbf{p}} + 3\varphi_g\varphi_h\varepsilon_g\varepsilon_f\dot{\mathbf{\Lambda}}\\
&\quad - \mathfrak{x}_s\varepsilon_b\dot{\mathbf{\Lambda}} + \mathfrak{x}_s\varsigma_c\mathbf{r} - 2\mathfrak{x}_r\varepsilon_h\dot{\mathbf{\Lambda}} + 2\mathfrak{x}_r\varsigma_d\mathbf{r} - \varphi_i\varsigma_n\dot{\mathbf{\Lambda}} + \varphi_i\dot{\mathbf{\Lambda}}^2\mathbf{r} + \mathfrak{x}_v\varepsilon_f\dot{\mathbf{\Lambda}} - \mathfrak{x}_v\varsigma_a\mathbf{r} + \mathfrak{x}_s\Omega^2\varepsilon_b\mathbf{\Omega} - \mathfrak{x}_s\Omega^4\mathbf{r}\\
&\quad + 2\mathfrak{x}_r\Omega^2\varepsilon_h\mathbf{\Omega} - 2\mathfrak{x}_r\Omega^2\varepsilon_g\mathbf{r} + \varphi_i\Omega^2\varsigma_n\mathbf{\Omega} - \varphi_i\Omega^2\varsigma_c\mathbf{r} - \mathfrak{x}_v\Omega^2\varepsilon_f\mathbf{\Omega} + \mathfrak{x}_v\Omega^2\varepsilon_c\mathbf{r} + \mathfrak{x}_p\varepsilon_b\varepsilon_g\widehat{\mathbf{p}} - \mathfrak{x}_p\varepsilon_b\varepsilon_i\mathbf{\Omega}\\
&\quad + 2\mathfrak{x}_o\varepsilon_b\Lambda^2\widehat{\mathbf{p}} - 2\mathfrak{x}_o\varepsilon_b\varepsilon_i\mathbf{\Lambda} + \varphi_g\varphi_h\varepsilon_b\varsigma_d\widehat{\mathbf{p}} - \varphi_g\varphi_h\varepsilon_b\varepsilon_i\dot{\mathbf{\Lambda}} - \mathfrak{x}_p\mathfrak{d}_l\widehat{\mathbf{p}} - \mathfrak{x}_p\mathfrak{d}_n\mathbf{\Omega} - 2\mathfrak{x}_o\mathfrak{d}_h\widehat{\mathbf{p}} - 2\mathfrak{x}_o\mathfrak{d}_n\mathbf{\Lambda}\\
&\quad - \varphi_g\varphi_h\mathfrak{d}_n\dot{\mathbf{\Lambda}} + \mathfrak{x}_p\Omega^2\varepsilon_l\mathbf{\Omega} + 2\mathfrak{x}_o\Omega^2\varepsilon_m\widehat{\mathbf{p}} + 2\mathfrak{x}_o\Omega^2\varepsilon_l\mathbf{\Lambda} - \varphi_g\varphi_h\Omega^2\mathfrak{d}_l\widehat{\mathbf{p}} + \varphi_g\varphi_h\Omega^2\varepsilon_l\dot{\mathbf{\Lambda}}
\end{aligned}
$$

$$
\begin{aligned}
&= (4\mathfrak{x}_r\varepsilon_h - \mathfrak{x}_s\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_v\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_s\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 6\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad + 3\mathfrak{x}_v\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + 2\varphi_i\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varphi_i\varepsilon_b(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\\
&\quad + [2\mathfrak{x}_p\varepsilon_h\Omega^2 + 4\mathfrak{x}_o\varepsilon_h\varepsilon_g + 2\varphi_g\varphi_h\varepsilon_h\varsigma_c - 3\mathfrak{x}_p\varepsilon_g\varepsilon_b - 6\mathfrak{x}_o\varepsilon_g\varepsilon_h - 3\varphi_g\varphi_h\varepsilon_g\varsigma_n + \mathfrak{x}_p\varepsilon_b\varepsilon_g + 2\mathfrak{x}_o\varepsilon_b\Lambda^2\\
&\qquad + \varphi_g\varphi_h\varepsilon_b\varsigma_d - 2\mathfrak{x}_o\mathfrak{d}_h - \mathfrak{x}_p\mathfrak{d}_l + 2\mathfrak{x}_o\Omega^2\varepsilon_m - \varphi_g\varphi_h\Omega^2\mathfrak{d}_l]\widehat{\mathbf{p}}\\
&\quad + [-4\mathfrak{x}_o\varepsilon_h\varepsilon_c + 6\mathfrak{x}_o\varepsilon_g\varepsilon_f - 2\mathfrak{x}_o\varepsilon_b\varepsilon_i - 2\mathfrak{x}_o\mathfrak{d}_n + 2\mathfrak{x}_o\Omega^2\varepsilon_l]\mathbf{\Lambda}\\
&\quad + [\mathfrak{x}_s\varsigma_c + 2\mathfrak{x}_r\varsigma_d + \varphi_i\dot{\mathbf{\Lambda}}^2 - \mathfrak{x}_v\varsigma_a - \mathfrak{x}_s\Omega^4 - 2\mathfrak{x}_r\Omega^2\varepsilon_g - \varphi_i\Omega^2\varsigma_c + \mathfrak{x}_v\Omega^2\varepsilon_c]\mathbf{r}\\
&\quad + [-2\mathfrak{x}_p\varepsilon_h\varepsilon_c + 3\mathfrak{x}_p\varepsilon_g\varepsilon_f + \mathfrak{x}_s\Omega^2\varepsilon_b + 2\mathfrak{x}_r\Omega^2\varepsilon_h + \varphi_i\Omega^2\varsigma_n - \mathfrak{x}_v\Omega^2\varepsilon_f - \mathfrak{x}_p\varepsilon_b\varepsilon_i - \mathfrak{x}_p\mathfrak{d}_n + \mathfrak{x}_p\Omega^2\varepsilon_l]\mathbf{\Omega}\\
&\quad + [-2\varphi_g\varphi_h\varepsilon_h\varepsilon_c + 3\varphi_g\varphi_h\varepsilon_g\varepsilon_f - \mathfrak{x}_s\varepsilon_b - 2\mathfrak{x}_r\varepsilon_h - \varphi_i\varsigma_n + \mathfrak{x}_v\varepsilon_f - \varphi_g\varphi_h\varepsilon_b\varepsilon_i - \varphi_g\varphi_h\mathfrak{d}_n + \varphi_g\varphi_h\Omega^2\varepsilon_l]\dot{\mathbf{\Lambda}}
\end{aligned}
$$

$$
\begin{aligned}
&= (4\mathfrak{x}_r\varepsilon_h - \mathfrak{x}_s\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_v\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_s\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 6\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad + 3\mathfrak{x}_v\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + 2\varphi_i\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varphi_i\varepsilon_b(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\\
&\quad + [2\Omega^2(\mathfrak{x}_p\varepsilon_h + \mathfrak{x}_o\varepsilon_m) + 2\mathfrak{x}_o(\varepsilon_b\Lambda^2 - \varepsilon_g\varepsilon_h - \mathfrak{d}_h) + \varphi_g\varphi_h(-\Omega^2\mathfrak{d}_l + 2\varepsilon_h\varsigma_c - 3\varepsilon_g\varsigma_n + \varepsilon_b\varsigma_d)\\
&\qquad - \mathfrak{x}_p(2\varepsilon_g\varepsilon_b + \mathfrak{d}_l)]\widehat{\mathbf{p}} + 2\mathfrak{x}_o[\Omega^2\varepsilon_l - 2\varepsilon_h\varepsilon_c + 3\varepsilon_g\varepsilon_f - \varepsilon_b\varepsilon_i - \mathfrak{d}_n]\mathbf{\Lambda}\\
&\quad + [\Omega^2(\mathfrak{x}_s\varepsilon_b + 2\mathfrak{x}_r\varepsilon_h + \varphi_i\varsigma_n - \mathfrak{x}_v\varepsilon_f + \mathfrak{x}_p\varepsilon_l) + \mathfrak{x}_p(3\varepsilon_g\varepsilon_f - 2\varepsilon_h\varepsilon_c - \varepsilon_b\varepsilon_i - \mathfrak{d}_n)]\mathbf{\Omega}\\
&\quad + [\varphi_g\varphi_h(3\varepsilon_g\varepsilon_f - 2\varepsilon_h\varepsilon_c - \varepsilon_b\varepsilon_i - \mathfrak{d}_n + \Omega^2\varepsilon_l) - \mathfrak{x}_s\varepsilon_b - 2\mathfrak{x}_r\varepsilon_h - \varphi_i\varsigma_n + \mathfrak{x}_v\varepsilon_f]\dot{\mathbf{\Lambda}}\\
&\quad + [\mathfrak{x}_s(\varsigma_c - \Omega^4) + 2\mathfrak{x}_r(\varsigma_d - \Omega^2\varepsilon_g) + \varphi_i(\dot{\mathbf{\Lambda}}^2 - \Omega^2\varsigma_c) + \mathfrak{x}_v(\Omega^2\varepsilon_c - \varsigma_a)]\mathbf{r}\\
&= \varkappa_g(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_v\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_s\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 6\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad + 3\mathfrak{x}_v\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + 2\varphi_i\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varphi_i\varepsilon_b(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) \qquad (5.44\text{l})\\
&\quad + \varkappa_i\widehat{\mathbf{p}} + 2\mathfrak{x}_o\varkappa_h\mathbf{\Lambda} + \varkappa_j\mathbf{\Omega} + \varkappa_k\dot{\mathbf{\Lambda}} + \varkappa_l\mathbf{r} \text{ by (5.23p)}
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{J}_n &= \dot{\mathbf{e}} \times \ddot{\mathbf{a}} \text{ by (3.15b)} \\
&= [\mathfrak{r}_o(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{r}_r\boldsymbol{\Omega} + \varphi_i\boldsymbol{\Lambda} - \mathfrak{r}_u\widehat{\mathbf{p}}] \times [3\varepsilon_h\boldsymbol{\Lambda} + \varepsilon_b\dot{\boldsymbol{\Lambda}} + \varrho_t\boldsymbol{\Omega} + \varrho_u\mathbf{r} \\
&\quad + 2\varepsilon_b(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) - 3\Omega^2(\boldsymbol{\Lambda} \times \mathbf{r}) - 3\varepsilon_g(\boldsymbol{\Omega} \times \mathbf{r}) + (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] \text{ by (5.30b) \& (5.40a)} \\
&= -3\varepsilon_h\mathfrak{r}_o[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - \varepsilon_b\mathfrak{r}_o[\dot{\boldsymbol{\Lambda}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - \varrho_t\mathfrak{r}_o[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - \varrho_u\mathfrak{r}_o[\mathbf{r} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] \\
&\quad + 2\varepsilon_b\mathfrak{r}_o[(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] - 3\Omega^2\mathfrak{r}_o[(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\boldsymbol{\Lambda} \times \mathbf{r})] - 3\varepsilon_g\mathfrak{r}_o[(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{r}_o[(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] - 3\varepsilon_h\varphi_g\varphi_h[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] - \varepsilon_b\varphi_g\varphi_h[\dot{\boldsymbol{\Lambda}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
&\quad - \varrho_t\varphi_g\varphi_h[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] - \varrho_u\varphi_g\varphi_h[\mathbf{r} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + 2\varepsilon_b\varphi_g\varphi_h[(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] \\
&\quad - 3\Omega^2\varphi_g\varphi_h[(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) \times (\boldsymbol{\Lambda} \times \mathbf{r})] - 3\varepsilon_g\varphi_g\varphi_h[(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) \times (\boldsymbol{\Omega} \times \mathbf{r})] + \varphi_g\varphi_h[(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&\quad + 3\varepsilon_h\mathfrak{r}_r(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \varepsilon_b\mathfrak{r}_r(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \varrho_u\mathfrak{r}_r(\boldsymbol{\Omega} \times \mathbf{r}) + 2\varepsilon_b\mathfrak{r}_r[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] \\
&\quad - 3\Omega^2\mathfrak{r}_r[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \mathbf{r})] - 3\varepsilon_g\mathfrak{r}_r[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] + \mathfrak{r}_r[\boldsymbol{\Omega} \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \varepsilon_b\varphi_i(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}}) + \varrho_t\varphi_i(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) \\
&\quad + \varrho_u\varphi_i(\boldsymbol{\Lambda} \times \mathbf{r}) + 2\varepsilon_b\varphi_i[\boldsymbol{\Lambda} \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] - 3\Omega^2\varphi_i[\boldsymbol{\Lambda} \times (\boldsymbol{\Lambda} \times \mathbf{r})] - 3\varepsilon_g\varphi_i[\boldsymbol{\Lambda} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \varphi_i[\boldsymbol{\Lambda} \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] - 3\varepsilon_h\mathfrak{r}_u(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) - \varepsilon_b\mathfrak{r}_u(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) - \varrho_t\mathfrak{r}_u(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) - \varrho_u\mathfrak{r}_u(\widehat{\mathbf{p}} \times \mathbf{r}) \\
&\quad - 2\varepsilon_b\mathfrak{r}_u[\widehat{\mathbf{p}} \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] + 3\Omega^2\mathfrak{r}_u[\widehat{\mathbf{p}} \times (\boldsymbol{\Lambda} \times \mathbf{r})] + 3\varepsilon_g\mathfrak{r}_u[\widehat{\mathbf{p}} \times (\boldsymbol{\Omega} \times \mathbf{r})] - \mathfrak{r}_u[\widehat{\mathbf{p}} \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})]
\end{aligned}
$$

$$
\begin{aligned}
&= (3\varepsilon_h\mathfrak{r}_r - \varrho_t\varphi_i)(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \varrho_u\mathfrak{r}_r(\boldsymbol{\Omega} \times \mathbf{r}) + \varrho_u\varphi_i(\boldsymbol{\Lambda} \times \mathbf{r}) - \varrho_t\mathfrak{r}_u(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \\
&\quad - \varrho_u\mathfrak{r}_u(\widehat{\mathbf{p}} \times \mathbf{r}) - 3\varepsilon_h\mathfrak{r}_u(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) - \varepsilon_b\mathfrak{r}_u(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \varepsilon_b\mathfrak{r}_r(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \varepsilon_b\varphi_i(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad - 3\varepsilon_h\mathfrak{r}_o[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - \varepsilon_b\mathfrak{r}_o[\dot{\boldsymbol{\Lambda}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - \varrho_t\mathfrak{r}_o[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] - \varrho_u\mathfrak{r}_o[\mathbf{r} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] \\
&\quad - 3\varepsilon_h\varphi_g\varphi_h[\boldsymbol{\Lambda} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] - \varepsilon_b\varphi_g\varphi_h[\dot{\boldsymbol{\Lambda}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] - \varrho_t\varphi_g\varphi_h[\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
&\quad - \varrho_u\varphi_g\varphi_h[\mathbf{r} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + 2\varepsilon_b\mathfrak{r}_r[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] - 3\Omega^2\mathfrak{r}_r[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \mathbf{r})] - 3\varepsilon_g\mathfrak{r}_r[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{r}_r[\boldsymbol{\Omega} \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] + 2\varepsilon_b\varphi_i[\boldsymbol{\Lambda} \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] - 3\Omega^2\varphi_i[\boldsymbol{\Lambda} \times (\boldsymbol{\Lambda} \times \mathbf{r})] - 3\varepsilon_g\varphi_i[\boldsymbol{\Lambda} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \varphi_i[\boldsymbol{\Lambda} \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] - 2\varepsilon_b\mathfrak{r}_u[\widehat{\mathbf{p}} \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] + 3\Omega^2\mathfrak{r}_u[\widehat{\mathbf{p}} \times (\boldsymbol{\Lambda} \times \mathbf{r})] + 3\varepsilon_g\mathfrak{r}_u[\widehat{\mathbf{p}} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad - \mathfrak{r}_u[\widehat{\mathbf{p}} \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] + 2\varepsilon_b\mathfrak{r}_o[(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] - 3\Omega^2\mathfrak{r}_o[(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&\quad - 3\varepsilon_g\mathfrak{r}_o[(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\boldsymbol{\Omega} \times \mathbf{r})] + \mathfrak{r}_o[(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] + 2\varepsilon_b\varphi_g\varphi_h[(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) \times (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] \\
&\quad - 3\Omega^2\varphi_g\varphi_h[(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) \times (\boldsymbol{\Lambda} \times \mathbf{r})] - 3\varepsilon_g\varphi_g\varphi_h[(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) \times (\boldsymbol{\Omega} \times \mathbf{r})] + \varphi_g\varphi_h[(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) \times (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})]
\end{aligned}
$$

$$
\begin{aligned}
&= (3\varepsilon_h\mathfrak{r}_r - \varrho_t\varphi_i)(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \varrho_u\mathfrak{r}_r(\boldsymbol{\Omega} \times \mathbf{r}) + \varrho_u\varphi_i(\boldsymbol{\Lambda} \times \mathbf{r}) - \varrho_t\mathfrak{r}_u(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \\
&\quad - \varrho_u\mathfrak{r}_u(\widehat{\mathbf{p}} \times \mathbf{r}) - 3\varepsilon_h\mathfrak{r}_u(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) - \varepsilon_b\mathfrak{r}_u(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \varepsilon_b\mathfrak{r}_r(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \varepsilon_b\varphi_i(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad - 3\varepsilon_h\mathfrak{r}_o[\widehat{\mathbf{p}}(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] - \varepsilon_b\mathfrak{r}_o[\widehat{\mathbf{p}}(\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}})] - \varrho_t\mathfrak{r}_o[\Omega^2\widehat{\mathbf{p}} - \boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})] \\
&\quad - \varrho_u\mathfrak{r}_o[\widehat{\mathbf{p}}(\mathbf{r} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\mathbf{r} \cdot \widehat{\mathbf{p}})] - 3\varepsilon_h\varphi_g\varphi_h[\Lambda^2\widehat{\mathbf{p}} - \boldsymbol{\Lambda}(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] - \varepsilon_b\varphi_g\varphi_h[\widehat{\mathbf{p}}(\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}})] \\
&\quad - \varrho_t\varphi_g\varphi_h[\widehat{\mathbf{p}}(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})] - \varrho_u\varphi_g\varphi_h[\widehat{\mathbf{p}}(\mathbf{r} \cdot \boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\mathbf{r} \cdot \widehat{\mathbf{p}})] + 2\varepsilon_b\mathfrak{r}_r[\Omega^2\boldsymbol{\Lambda} - \boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})] \\
&\quad - 3\Omega^2\mathfrak{r}_r[\boldsymbol{\Lambda}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})] - 3\varepsilon_g\mathfrak{r}_r[\boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \Omega^2\mathbf{r}] + \mathfrak{r}_r[\ddot{\boldsymbol{\Lambda}}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega} \cdot \ddot{\boldsymbol{\Lambda}})] \\
&\quad + 2\varepsilon_b\varphi_i[\boldsymbol{\Lambda}(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega}) - \Lambda^2\boldsymbol{\Omega}] - 3\Omega^2\varphi_i[\boldsymbol{\Lambda}(\boldsymbol{\Lambda} \cdot \mathbf{r}) - \Lambda^2\mathbf{r}] - 3\varepsilon_g\varphi_i[\boldsymbol{\Omega}(\boldsymbol{\Lambda} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega})] \\
&\quad + \varphi_i[\ddot{\boldsymbol{\Lambda}}(\boldsymbol{\Lambda} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Lambda} \cdot \ddot{\boldsymbol{\Lambda}})] - 2\varepsilon_b\mathfrak{r}_u[\boldsymbol{\Lambda}(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda})] + 3\Omega^2\mathfrak{r}_u[\boldsymbol{\Lambda}(\widehat{\mathbf{p}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda})] \\
&\quad + 3\varepsilon_g\mathfrak{r}_u[\boldsymbol{\Omega}(\widehat{\mathbf{p}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] - \mathfrak{r}_u[\ddot{\boldsymbol{\Lambda}}(\widehat{\mathbf{p}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}} \cdot \ddot{\boldsymbol{\Lambda}})] + 2\varepsilon_b\mathfrak{r}_o[\boldsymbol{\Lambda}(\boldsymbol{\Omega} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})) - \boldsymbol{\Omega}(\boldsymbol{\Lambda} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}))] \\
&\quad - 3\Omega^2\mathfrak{r}_o[\boldsymbol{\Lambda}(\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})) - \mathbf{r}(\boldsymbol{\Lambda} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}))] - 3\varepsilon_g\mathfrak{r}_o[\boldsymbol{\Omega}(\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})) - \mathbf{r}(\boldsymbol{\Omega} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}))] \\
&\quad + \mathfrak{r}_o[\ddot{\boldsymbol{\Lambda}}(\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})) - \mathbf{r}(\ddot{\boldsymbol{\Lambda}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}))] + 2\varepsilon_b\varphi_g\varphi_h[\boldsymbol{\Lambda}(\boldsymbol{\Omega} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})) - \boldsymbol{\Omega}(\boldsymbol{\Lambda} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}))] \\
&\quad - 3\Omega^2\varphi_g\varphi_h[\boldsymbol{\Lambda}(\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})) - \mathbf{r}(\boldsymbol{\Lambda} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}))] - 3\varepsilon_g\varphi_g\varphi_h[\boldsymbol{\Omega}(\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})) - \mathbf{r}(\boldsymbol{\Omega} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}))] \\
&\quad + \varphi_g\varphi_h[\ddot{\boldsymbol{\Lambda}}(\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})) - \mathbf{r}(\ddot{\boldsymbol{\Lambda}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}))] \text{ by (A.1) \& (A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&= (3\varepsilon_h\mathfrak{x}_r - \varrho_t\varphi_i)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varrho_u\mathfrak{x}_r(\mathbf{\Omega}\times\mathbf{r}) + \varrho_u\varphi_i(\mathbf{\Lambda}\times\mathbf{r}) - \varrho_t\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&\quad - \varrho_u\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{r}) - 3\varepsilon_h\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) - \varepsilon_b\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\varphi_i(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\\
&\quad - 3\varepsilon_h\mathfrak{x}_o(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_i\mathbf{\Omega}) - \varepsilon_b\mathfrak{x}_o(\varsigma_c\widehat{\mathbf{p}} - \varsigma_a\mathbf{\Omega}) - \varrho_t\mathfrak{x}_o(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Omega}) - \varrho_u\mathfrak{x}_o(\varepsilon_b\widehat{\mathbf{p}} - \varepsilon_f\mathbf{\Omega})\\
&\quad - 3\varepsilon_h\varphi_g\varphi_h(\Lambda^2\widehat{\mathbf{p}} - \varepsilon_i\mathbf{\Lambda}) - \varepsilon_b\varphi_g\varphi_h(\varsigma_d\widehat{\mathbf{p}} - \varsigma_a\mathbf{\Lambda}) - \varrho_t\varphi_g\varphi_h(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Lambda}) - \varrho_u\varphi_g\varphi_h(\varepsilon_h\widehat{\mathbf{p}} - \varepsilon_f\mathbf{\Lambda})\\
&\quad + 2\varepsilon_b\mathfrak{x}_r(\Omega^2\mathbf{\Lambda} - \varepsilon_g\mathbf{\Omega}) - 3\Omega^2\mathfrak{x}_r(\varepsilon_b\mathbf{\Lambda} - \varepsilon_g\mathbf{r}) - 3\varepsilon_g\mathfrak{x}_r(\varepsilon_b\mathbf{\Omega} - \Omega^2\mathbf{r}) + \mathfrak{x}_r(\varepsilon_b\ddot{\mathbf{\Lambda}} - \varsigma_h\mathbf{r})\\
&\quad + 2\varepsilon_b\varphi_i(\varepsilon_g\mathbf{\Lambda} - \Lambda^2\mathbf{\Omega}) - 3\Omega^2\varphi_i(\varepsilon_h\mathbf{\Lambda} - \Lambda^2\mathbf{r}) - 3\varepsilon_g\varphi_i(\varepsilon_h\mathbf{\Omega} - \varepsilon_g\mathbf{r}) + \varphi_i(\varepsilon_h\ddot{\mathbf{\Lambda}} - \varsigma_i\mathbf{r})\\
&\quad - 2\varepsilon_b\mathfrak{x}_u(\varepsilon_c\mathbf{\Lambda} - \varepsilon_i\mathbf{\Omega}) + 3\Omega^2\mathfrak{x}_u(\varepsilon_f\mathbf{\Lambda} - \varepsilon_i\mathbf{r}) + 3\varepsilon_g\mathfrak{x}_u(\varepsilon_f\mathbf{\Omega} - \varepsilon_c\mathbf{r}) - \mathfrak{x}_u(\varepsilon_f\ddot{\mathbf{\Lambda}} - \varsigma_f\mathbf{r})\\
&\quad - 2\varepsilon_b\mathfrak{x}_o\mathfrak{d}_o\mathbf{\Omega} - 3\Omega^2\mathfrak{x}_o(\varepsilon_l\mathbf{\Lambda} - \mathfrak{d}_o\mathbf{r}) - 3\varepsilon_g\mathfrak{x}_o\varepsilon_l\mathbf{\Omega} + \mathfrak{x}_o(\varepsilon_l\ddot{\mathbf{\Lambda}} - \mathfrak{d}_q\mathbf{r}) - 2\varepsilon_b\varphi_g\varphi_h\mathfrak{d}_o\mathbf{\Lambda}\\
&\quad - 3\Omega^2\varphi_g\varphi_h\varepsilon_o\mathbf{\Lambda} - 3\varepsilon_g\varphi_g\varphi_h(\varepsilon_o\mathbf{\Omega} + \mathfrak{d}_o\mathbf{r}) + \varphi_g\varphi_h(\varepsilon_o\ddot{\mathbf{\Lambda}} - \mathfrak{d}_s\mathbf{r}) \text{ by (5.1a), (5.23a), (5.23b) \& (A.4)}
\end{aligned}
$$

$$
\begin{aligned}
&= (3\varepsilon_h\mathfrak{x}_r - \varrho_t\varphi_i)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varrho_u\mathfrak{x}_r(\mathbf{\Omega}\times\mathbf{r}) + \varrho_u\varphi_i(\mathbf{\Lambda}\times\mathbf{r}) - \varrho_t\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&\quad - \varrho_u\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{r}) - 3\varepsilon_h\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) - \varepsilon_b\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\varphi_i(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\\
&\quad - 3\varepsilon_h\mathfrak{x}_o\varepsilon_g\widehat{\mathbf{p}} + 3\varepsilon_h\mathfrak{x}_o\varepsilon_i\mathbf{\Omega} - \varepsilon_b\mathfrak{x}_o\varsigma_c\widehat{\mathbf{p}} + \varepsilon_b\mathfrak{x}_o\varsigma_a\mathbf{\Omega} - \varrho_t\mathfrak{x}_o\Omega^2\widehat{\mathbf{p}} + \varrho_t\mathfrak{x}_o\varepsilon_c\mathbf{\Omega} - \varrho_u\mathfrak{x}_o\varepsilon_b\widehat{\mathbf{p}} + \varrho_u\mathfrak{x}_o\varepsilon_f\mathbf{\Omega}\\
&\quad - 3\varepsilon_h\varphi_g\varphi_h\Lambda^2\widehat{\mathbf{p}} + 3\varepsilon_h\varphi_g\varphi_h\varepsilon_i\mathbf{\Lambda} - \varepsilon_b\varphi_g\varphi_h\varsigma_d\widehat{\mathbf{p}} + \varepsilon_b\varphi_g\varphi_h\varsigma_a\mathbf{\Lambda} - \varrho_t\varphi_g\varphi_h\varepsilon_g\widehat{\mathbf{p}} + \varrho_t\varphi_g\varphi_h\varepsilon_c\mathbf{\Lambda}\\
&\quad - \varrho_u\varphi_g\varphi_h\varepsilon_h\widehat{\mathbf{p}} + \varrho_u\varphi_g\varphi_h\varepsilon_f\mathbf{\Lambda} + 2\varepsilon_b\mathfrak{x}_r\Omega^2\mathbf{\Lambda} - 2\varepsilon_b\mathfrak{x}_r\varepsilon_g\mathbf{\Omega} - 3\Omega^2\mathfrak{x}_r\varepsilon_b\mathbf{\Lambda} + 3\Omega^2\mathfrak{x}_r\varepsilon_g\mathbf{r}\\
&\quad - 3\varepsilon_g\mathfrak{x}_r\varepsilon_b\mathbf{\Omega} + 3\varepsilon_g\mathfrak{x}_r\Omega^2\mathbf{r} + \mathfrak{x}_r\varepsilon_b\ddot{\mathbf{\Lambda}} - \mathfrak{x}_r\varsigma_h\mathbf{r} + 2\varepsilon_b\varphi_i\varepsilon_g\mathbf{\Lambda} - 2\varepsilon_b\varphi_i\Lambda^2\mathbf{\Omega} - 3\Omega^2\varphi_i\varepsilon_h\mathbf{\Lambda} + 3\Omega^2\varphi_i\Lambda^2\mathbf{r}\\
&\quad - 3\varepsilon_g\varphi_i\varepsilon_h\mathbf{\Omega} + 3\varepsilon_g\varphi_i\varepsilon_g\mathbf{r} + \varphi_i\varepsilon_h\ddot{\mathbf{\Lambda}} - \varphi_i\varsigma_i\mathbf{r} - 2\varepsilon_b\mathfrak{x}_u\varepsilon_c\mathbf{\Lambda} + 2\varepsilon_b\mathfrak{x}_u\varepsilon_i\mathbf{\Omega} + 3\Omega^2\mathfrak{x}_u\varepsilon_f\mathbf{\Lambda} - 3\Omega^2\mathfrak{x}_u\varepsilon_i\mathbf{r}\\
&\quad + 3\varepsilon_g\mathfrak{x}_u\varepsilon_f\mathbf{\Omega} - 3\varepsilon_g\mathfrak{x}_u\varepsilon_c\mathbf{r} - \mathfrak{x}_u\varepsilon_f\ddot{\mathbf{\Lambda}} + \mathfrak{x}_u\varsigma_f\mathbf{r} - 2\varepsilon_b\mathfrak{x}_o\mathfrak{d}_o\mathbf{\Omega} - 3\Omega^2\mathfrak{x}_o\varepsilon_l\mathbf{\Lambda} + 3\Omega^2\mathfrak{x}_o\mathfrak{d}_o\mathbf{r}\\
&\quad - 3\varepsilon_g\mathfrak{x}_o\varepsilon_l\mathbf{\Omega} + \mathfrak{x}_o\varepsilon_l\ddot{\mathbf{\Lambda}} - \mathfrak{x}_o\mathfrak{d}_q\mathbf{r} - 2\varepsilon_b\varphi_g\varphi_h\mathfrak{d}_o\mathbf{\Lambda} - 3\Omega^2\varphi_g\varphi_h\varepsilon_o\mathbf{\Lambda}\\
&\quad - 3\varepsilon_g\varphi_g\varphi_h\varepsilon_o\mathbf{\Omega} - 3\varepsilon_g\varphi_g\varphi_h\mathfrak{d}_o\mathbf{r} + \varphi_g\varphi_h\varepsilon_o\ddot{\mathbf{\Lambda}} - \varphi_g\varphi_h\mathfrak{d}_s\mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= (3\varepsilon_h\mathfrak{x}_r - \varrho_t\varphi_i)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varrho_u\mathfrak{x}_r(\mathbf{\Omega}\times\mathbf{r}) + \varrho_u\varphi_i(\mathbf{\Lambda}\times\mathbf{r}) - \varrho_t\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&\quad - \varrho_u\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{r}) - 3\varepsilon_h\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) - \varepsilon_b\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\varphi_i(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\\
&\quad + [-3\varepsilon_h\mathfrak{x}_o\varepsilon_g - \varepsilon_b\mathfrak{x}_o\varsigma_c - \varrho_t\mathfrak{x}_o\Omega^2 - \varrho_u\mathfrak{x}_o\varepsilon_b - 3\varepsilon_h\varphi_g\varphi_h\Lambda^2 - \varepsilon_b\varphi_g\varphi_h\varsigma_d - \varrho_t\varphi_g\varphi_h\varepsilon_g - \varrho_u\varphi_g\varphi_h\varepsilon_h]\widehat{\mathbf{p}}\\
&\quad + [3\Omega^2\mathfrak{x}_r\varepsilon_g + 3\varepsilon_g\mathfrak{x}_r\Omega^2 - \mathfrak{x}_r\varsigma_h + 3\Omega^2\varphi_i\Lambda^2 + 3\varepsilon_g\varphi_i\varepsilon_g - \varphi_i\varsigma_i - 3\Omega^2\mathfrak{x}_u\varepsilon_i - 3\varepsilon_g\mathfrak{x}_u\varepsilon_c\\
&\qquad + \mathfrak{x}_u\varsigma_f + 3\Omega^2\mathfrak{x}_o\mathfrak{d}_o - \mathfrak{x}_o\mathfrak{d}_q - 3\varepsilon_g\varphi_g\varphi_h\mathfrak{d}_o - \varphi_g\varphi_h\mathfrak{d}_s]\mathbf{r} + [\mathfrak{x}_r\varepsilon_b + \varphi_i\varepsilon_h - \mathfrak{x}_u\varepsilon_f + \mathfrak{x}_o\varepsilon_l + \varphi_g\varphi_h\varepsilon_o]\ddot{\mathbf{\Lambda}}\\
&\quad + [3\varepsilon_h\mathfrak{x}_o\varepsilon_i + \varepsilon_b\mathfrak{x}_o\varsigma_a + \varrho_t\mathfrak{x}_o\varepsilon_c + \varrho_u\mathfrak{x}_o\varepsilon_f - 2\varepsilon_b\mathfrak{x}_r\varepsilon_g - 3\varepsilon_g\mathfrak{x}_r\varepsilon_b - 2\varepsilon_b\varphi_i\Lambda^2 - 3\varepsilon_g\varphi_i\varepsilon_h\\
&\qquad + 2\varepsilon_b\mathfrak{x}_u\varepsilon_i + 3\varepsilon_g\mathfrak{x}_u\varepsilon_f - 2\varepsilon_b\mathfrak{x}_o\mathfrak{d}_o - 3\varepsilon_g\mathfrak{x}_o\varepsilon_l - 3\varepsilon_g\varphi_g\varphi_h\varepsilon_o]\mathbf{\Omega}\\
&\quad + [3\varepsilon_h\varphi_g\varphi_h\varepsilon_i + \varepsilon_b\varphi_g\varphi_h\varsigma_a + \varrho_t\varphi_g\varphi_h\varepsilon_c + \varrho_u\varphi_g\varphi_h\varepsilon_f + 2\varepsilon_b\mathfrak{x}_r\Omega^2 - 3\Omega^2\mathfrak{x}_r\varepsilon_b + 2\varepsilon_b\varphi_i\varepsilon_g\\
&\qquad - 3\Omega^2\varphi_i\varepsilon_h - 2\varepsilon_b\mathfrak{x}_u\varepsilon_c + 3\Omega^2\mathfrak{x}_u\varepsilon_f - 3\Omega^2\mathfrak{x}_o\varepsilon_l - 2\varepsilon_b\varphi_g\varphi_h\mathfrak{d}_o - 3\Omega^2\varphi_g\varphi_h\varepsilon_o]\mathbf{\Lambda}
\end{aligned}
$$

$$
\begin{aligned}
&= (3\varepsilon_h\mathfrak{x}_r - \varrho_t\varphi_i)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varrho_u\mathfrak{x}_r(\mathbf{\Omega}\times\mathbf{r}) + \varrho_u\varphi_i(\mathbf{\Lambda}\times\mathbf{r}) - \varrho_t\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&\quad - \varrho_u\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{r}) - 3\varepsilon_h\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) - \varepsilon_b\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\varphi_i(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\\
&\quad + [-\mathfrak{x}_o(3\varepsilon_h\varepsilon_g + \varepsilon_b\varsigma_c + \varrho_t\Omega^2 + \varrho_u\varepsilon_b) + \varphi_g\varphi_h(-3\varepsilon_h\Lambda^2 - \varepsilon_b\varsigma_d - \varrho_t\varepsilon_g - \varrho_u\varepsilon_h)]\widehat{\mathbf{p}}\\
&\quad + [3\Omega^2(2\mathfrak{x}_r\varepsilon_g + \varphi_i\Lambda^2 - \mathfrak{x}_u\varepsilon_i + \mathfrak{x}_o\mathfrak{d}_o) + \varphi_i(3\varepsilon_g^2 - \varsigma_i) + \mathfrak{x}_u(\varsigma_f - 3\varepsilon_g\varepsilon_c)\\
&\qquad - \varphi_g\varphi_h(3\varepsilon_g\mathfrak{d}_o + \mathfrak{d}_s) - \mathfrak{x}_o\mathfrak{d}_q - \mathfrak{x}_r\varsigma_h]\mathbf{r} + [\mathfrak{x}_r\varepsilon_b + \varphi_i\varepsilon_h - \mathfrak{x}_u\varepsilon_f + \mathfrak{x}_o\varepsilon_l + \varphi_g\varphi_h\varepsilon_o]\ddot{\mathbf{\Lambda}}\\
&\quad + [\mathfrak{x}_o(3\varepsilon_h\varepsilon_i + \varepsilon_b\varsigma_a + \varrho_t\varepsilon_c + \varrho_u\varepsilon_f) + 2\varepsilon_b(\mathfrak{x}_u\varepsilon_i - \mathfrak{x}_o\mathfrak{d}_o - \mathfrak{x}_r\varepsilon_g - \varphi_i\Lambda^2)\\
&\qquad + 3\varepsilon_g(\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_o\varepsilon_l - \varphi_g\varphi_h\varepsilon_o - \varepsilon_b\mathfrak{x}_r - \varphi_i\varepsilon_h)]\mathbf{\Omega}\\
&\quad + [\varphi_g\varphi_h(3\varepsilon_h\varepsilon_i + \varepsilon_b\varsigma_a + \varrho_t\varepsilon_c + \varrho_u\varepsilon_f) - 3\Omega^2(\mathfrak{x}_r\varepsilon_b + \varphi_i\varepsilon_h - \mathfrak{x}_u\varepsilon_f + \mathfrak{x}_o\varepsilon_l + \varphi_g\varphi_h\varepsilon_o)\\
&\qquad + 2\varepsilon_b(\varphi_i\varepsilon_g + \mathfrak{x}_r\Omega^2 - \mathfrak{x}_u\varepsilon_c - \varphi_g\varphi_h\mathfrak{d}_o)]\mathbf{\Lambda}
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_m(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varrho_u\mathfrak{x}_r(\mathbf{\Omega}\times\mathbf{r}) + \varrho_u\varphi_i(\mathbf{\Lambda}\times\mathbf{r}) - \varrho_t\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&\quad - \varrho_u\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{r}) - 3\varepsilon_h\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) - \varepsilon_b\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\varphi_i(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) \qquad (5.44\text{m})\\
&\quad + \varkappa_o\widehat{\mathbf{p}} + \varkappa_p\mathbf{r} + \varkappa_q\mathbf{\Omega} + \varkappa_r\mathbf{\Lambda} + \varkappa_n\ddot{\mathbf{\Lambda}} \text{ by (5.23q)}
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{J}_o &= \dot{\mathbf{e}}\times\ddot{\mathbf{e}} \text{ by (3.15b)}\\
&= [\mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{x}_r\mathbf{\Omega} + \varphi_i\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}}]\times[\mathfrak{x}_p(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + 2\mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\\
&\qquad + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{x}_s\mathbf{\Omega} + 2\mathfrak{x}_r\mathbf{\Lambda} + \varphi_i\dot{\mathbf{\Lambda}} - \mathfrak{x}_v\widehat{\mathbf{p}}] \text{ by (5.40)}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathfrak{x}_o^2[(\widehat{\mathbf{p}}\times\mathbf{\Omega})\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \varphi_g\varphi_h\mathfrak{x}_o[(\widehat{\mathbf{p}}\times\mathbf{\Omega})\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] - \mathfrak{x}_s\mathfrak{x}_o[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] - 2\mathfrak{x}_r\mathfrak{x}_o[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})]\\
&\quad - \varphi_i\mathfrak{x}_o[\dot{\mathbf{\Lambda}}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + \mathfrak{x}_v\mathfrak{x}_o[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + \mathfrak{x}_p\varphi_g\varphi_h[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + \varphi_g^2\varphi_h^2[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})]\\
&\quad - \mathfrak{x}_s\varphi_g\varphi_h[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] - 2\mathfrak{x}_r\varphi_g\varphi_h[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] - \varphi_i\varphi_g\varphi_h[\dot{\mathbf{\Lambda}}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{x}_v\varphi_g\varphi_h[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&\quad + \mathfrak{x}_p\mathfrak{x}_r[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + 2\mathfrak{x}_o\mathfrak{x}_r[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \varphi_g\varphi_h\mathfrak{x}_r[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + 2\mathfrak{x}_r^2(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\\
&\quad - \mathfrak{x}_v\mathfrak{x}_r(\mathbf{\Omega}\times\widehat{\mathbf{p}}) + \mathfrak{x}_p\varphi_i[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + 2\mathfrak{x}_o\varphi_i[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \varphi_g\varphi_h\varphi_i[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})]\\
&\quad + \mathfrak{x}_s\varphi_i(\mathbf{\Lambda}\times\mathbf{\Omega}) + \varphi_i^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) - \mathfrak{x}_v\varphi_i(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) - \mathfrak{x}_p\mathfrak{x}_u[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] - 2\mathfrak{x}_o\mathfrak{x}_u[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&\quad - \varphi_g\varphi_h\mathfrak{x}_u[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] - \mathfrak{x}_s\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Omega}) - 2\mathfrak{x}_r\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) - \varphi_i\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})
\end{aligned}
$$

$$
\begin{aligned}
&= (2\mathfrak{x}_r^2 - \mathfrak{x}_s\varphi_i)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\mathfrak{x}_s\mathfrak{x}_u - \mathfrak{x}_v\mathfrak{x}_r)(\mathbf{\Omega}\times\widehat{\mathbf{p}}) + (\mathfrak{x}_v\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_u)(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\\
&\quad - \varphi_i\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varphi_i^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + (\mathfrak{x}_p\mathfrak{x}_r - \mathfrak{x}_s\mathfrak{x}_o)[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + (\mathfrak{x}_p\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_o)[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})]\\
&\quad - \varphi_i\mathfrak{x}_o[\dot{\mathbf{\Lambda}}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + (\mathfrak{x}_v\mathfrak{x}_o - \mathfrak{x}_p\mathfrak{x}_u)[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + (2\mathfrak{x}_o\mathfrak{x}_r - \mathfrak{x}_s\varphi_g\varphi_h)[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&\quad + 2(\mathfrak{x}_o\varphi_i - \mathfrak{x}_r\varphi_g\varphi_h)[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] - \varphi_i\varphi_g\varphi_h[\dot{\mathbf{\Lambda}}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + (\mathfrak{x}_v\varphi_g\varphi_h - 2\mathfrak{x}_o\mathfrak{x}_u)[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&\quad + \varphi_g\varphi_h\mathfrak{x}_r[\mathbf{\Omega}\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \varphi_g\varphi_h\varphi_i[\mathbf{\Lambda}\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] - \varphi_g\varphi_h\mathfrak{x}_u[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})]\\
&\quad + 2\mathfrak{x}_o^2[(\widehat{\mathbf{p}}\times\mathbf{\Omega})\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \varphi_g\varphi_h\mathfrak{x}_o[(\widehat{\mathbf{p}}\times\mathbf{\Omega})\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{x}_p\varphi_g\varphi_h[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})]\\
&\quad + \varphi_g^2\varphi_h^2[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\times(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= (2\mathfrak{x}_r^2 - \mathfrak{x}_s\varphi_i)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\mathfrak{x}_s\mathfrak{x}_u - \mathfrak{x}_v\mathfrak{x}_r)(\mathbf{\Omega}\times\widehat{\mathbf{p}}) + (\mathfrak{x}_v\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_u)(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) - \varphi_i\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})\\
&\quad + \varphi_i^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + (\mathfrak{x}_p\mathfrak{x}_r - \mathfrak{x}_s\mathfrak{x}_o)[\Omega^2\widehat{\mathbf{p}} - \mathbf{\Omega}(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})] + (\mathfrak{x}_p\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_o)[\widehat{\mathbf{p}}(\mathbf{\Lambda}\cdot\mathbf{\Omega}) - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})]\\
&\quad - \varphi_i\mathfrak{x}_o[\widehat{\mathbf{p}}(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega}) - \mathbf{\Omega}(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})] + (\mathfrak{x}_v\mathfrak{x}_o - \mathfrak{x}_p\mathfrak{x}_u)[\widehat{\mathbf{p}}(\widehat{\mathbf{p}}\cdot\mathbf{\Omega}) - \mathbf{\Omega}] + (2\mathfrak{x}_o\mathfrak{x}_r - \mathfrak{x}_s\varphi_g\varphi_h)[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot\mathbf{\Lambda}) - \mathbf{\Lambda}(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})]\\
&\quad + 2(\mathfrak{x}_o\varphi_i - \mathfrak{x}_r\varphi_g\varphi_h)[\Lambda^2\widehat{\mathbf{p}} - \mathbf{\Lambda}(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})] - \varphi_i\varphi_g\varphi_h[\widehat{\mathbf{p}}(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - \mathbf{\Lambda}(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})]\\
&\quad + (\mathfrak{x}_v\varphi_g\varphi_h - 2\mathfrak{x}_o\mathfrak{x}_u)[\widehat{\mathbf{p}}(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda}) - \mathbf{\Lambda}] + \varphi_g\varphi_h\mathfrak{x}_r[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}}) - \dot{\mathbf{\Lambda}}(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})] + \varphi_g\varphi_h\varphi_i[\widehat{\mathbf{p}}(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - \dot{\mathbf{\Lambda}}(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})]\\
&\quad - \varphi_g\varphi_h\mathfrak{x}_u[\widehat{\mathbf{p}}(\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}}) - \dot{\mathbf{\Lambda}}] + 2\mathfrak{x}_o^2[\widehat{\mathbf{p}}(\mathbf{\Lambda}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})) - \mathbf{\Lambda}(\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega}))]\\
&\quad + \varphi_g\varphi_h\mathfrak{x}_o[\widehat{\mathbf{p}}(\dot{\mathbf{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})) - \dot{\mathbf{\Lambda}}(\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega}))] + \mathfrak{x}_p\varphi_g\varphi_h[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})) - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda}))]\\
&\quad + \varphi_g^2\varphi_h^2[\widehat{\mathbf{p}}(\dot{\mathbf{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})) - \dot{\mathbf{\Lambda}}(\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda}))] \text{ by (A.1) \& (A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&= (2\mathfrak{x}_r^2 - \mathfrak{x}_s\varphi_i)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\mathfrak{x}_s\mathfrak{x}_u - \mathfrak{x}_v\mathfrak{x}_r)(\mathbf{\Omega}\times\widehat{\mathbf{p}}) + (\mathfrak{x}_v\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_u)(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\\
&\quad - \varphi_i\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varphi_i^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + (\mathfrak{x}_p\mathfrak{x}_r - \mathfrak{x}_s\mathfrak{x}_o)(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Omega}) + (\mathfrak{x}_p\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_o)(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_i\mathbf{\Omega})\\
&\quad - \varphi_i\mathfrak{x}_o(\varsigma_c\widehat{\mathbf{p}} - \varsigma_a\mathbf{\Omega}) + (\mathfrak{x}_v\mathfrak{x}_o - \mathfrak{x}_p\mathfrak{x}_u)(\varepsilon_c\widehat{\mathbf{p}} - \mathbf{\Omega}) + (2\mathfrak{x}_o\mathfrak{x}_r - \mathfrak{x}_s\varphi_g\varphi_h)(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_c\mathbf{\Lambda})\\
&\quad + 2(\mathfrak{x}_o\varphi_i - \mathfrak{x}_r\varphi_g\varphi_h)(\Lambda^2\widehat{\mathbf{p}} - \varepsilon_i\mathbf{\Lambda}) - \varphi_i\varphi_g\varphi_h(\varsigma_d\widehat{\mathbf{p}} - \varsigma_a\mathbf{\Lambda}) + (\mathfrak{x}_v\varphi_g\varphi_h - 2\mathfrak{x}_o\mathfrak{x}_u)(\varepsilon_i\widehat{\mathbf{p}} - \mathbf{\Lambda})\\
&\quad + \varphi_g\varphi_h\mathfrak{x}_r(\varsigma_c\widehat{\mathbf{p}} - \varepsilon_c\dot{\mathbf{\Lambda}}) + \varphi_g\varphi_h\varphi_i(\varsigma_d\widehat{\mathbf{p}} - \varepsilon_i\dot{\mathbf{\Lambda}}) - \varphi_g\varphi_h\mathfrak{x}_u(\varsigma_a\widehat{\mathbf{p}} - \dot{\mathbf{\Lambda}}) + 2\mathfrak{x}_o^2\mathfrak{d}_o\widehat{\mathbf{p}} + \varphi_g\varphi_h\mathfrak{x}_o\mathfrak{d}_p\widehat{\mathbf{p}}\\
&\quad - \mathfrak{x}_p\varphi_g\varphi_h\mathfrak{d}_o\widehat{\mathbf{p}} + \varphi_g^2\varphi_h^2\mathfrak{d}_r\widehat{\mathbf{p}} \text{ by (5.1a), (5.23a) \& (5.23b)}
\end{aligned}
$$

$$
\begin{aligned}
&= (2\mathfrak{x}_r^2 - \mathfrak{x}_s\varphi_i)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\mathfrak{x}_s\mathfrak{x}_u - \mathfrak{x}_v\mathfrak{x}_r)(\mathbf{\Omega}\times\widehat{\mathbf{p}}) + (\mathfrak{x}_v\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_u)(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\\
&\quad - \varphi_i\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varphi_i^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + \Omega^2(\mathfrak{x}_p\mathfrak{x}_r - \mathfrak{x}_s\mathfrak{x}_o)\widehat{\mathbf{p}} - \varepsilon_c(\mathfrak{x}_p\mathfrak{x}_r - \mathfrak{x}_s\mathfrak{x}_o)\mathbf{\Omega} + \varepsilon_g(\mathfrak{x}_p\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_o)\widehat{\mathbf{p}}\\
&\quad - \varepsilon_i(\mathfrak{x}_p\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_o)\mathbf{\Omega} - \varsigma_c\varphi_i\mathfrak{x}_o\widehat{\mathbf{p}} + \varsigma_a\varphi_i\mathfrak{x}_o\mathbf{\Omega} + \varepsilon_c(\mathfrak{x}_v\mathfrak{x}_o - \mathfrak{x}_p\mathfrak{x}_u)\widehat{\mathbf{p}} - (\mathfrak{x}_v\mathfrak{x}_o - \mathfrak{x}_p\mathfrak{x}_u)\mathbf{\Omega}\\
&\quad + \varepsilon_g(2\mathfrak{x}_o\mathfrak{x}_r - \mathfrak{x}_s\varphi_g\varphi_h)\widehat{\mathbf{p}} - \varepsilon_c(2\mathfrak{x}_o\mathfrak{x}_r - \mathfrak{x}_s\varphi_g\varphi_h)\mathbf{\Lambda} + 2\Lambda^2(\mathfrak{x}_o\varphi_i - \mathfrak{x}_r\varphi_g\varphi_h)\widehat{\mathbf{p}} - 2\varepsilon_i(\mathfrak{x}_o\varphi_i - \mathfrak{x}_r\varphi_g\varphi_h)\mathbf{\Lambda}\\
&\quad - \varsigma_d\varphi_i\varphi_g\varphi_h\widehat{\mathbf{p}} + \varsigma_a\varphi_i\varphi_g\varphi_h\mathbf{\Lambda} + \varepsilon_i(\mathfrak{x}_v\varphi_g\varphi_h - 2\mathfrak{x}_o\mathfrak{x}_u)\widehat{\mathbf{p}} - (\mathfrak{x}_v\varphi_g\varphi_h - 2\mathfrak{x}_o\mathfrak{x}_u)\mathbf{\Lambda}\\
&\quad + \varphi_g\varphi_h\mathfrak{x}_r(\varsigma_c\widehat{\mathbf{p}} - \varepsilon_c\dot{\mathbf{\Lambda}}) + \varsigma_d\varphi_g\varphi_h\varphi_i\widehat{\mathbf{p}} - \varepsilon_i\varphi_g\varphi_h\varphi_i\dot{\mathbf{\Lambda}} - \varsigma_a\varphi_g\varphi_h\mathfrak{x}_u\widehat{\mathbf{p}} + \varphi_g\varphi_h\mathfrak{x}_u\dot{\mathbf{\Lambda}}\\
&\quad + 2\mathfrak{x}_o^2\mathfrak{d}_o\widehat{\mathbf{p}} + \varphi_g\varphi_h\mathfrak{x}_o\mathfrak{d}_p\widehat{\mathbf{p}} - \mathfrak{x}_p\varphi_g\varphi_h\mathfrak{d}_o\widehat{\mathbf{p}} + \varphi_g^2\varphi_h^2\mathfrak{d}_r\widehat{\mathbf{p}}
\end{aligned}
$$

$$
\begin{aligned}
&= (2\mathfrak{x}_r^2 - \mathfrak{x}_s\varphi_i)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + (\mathfrak{x}_s\mathfrak{x}_u - \mathfrak{x}_v\mathfrak{x}_r)(\mathbf{\Omega}\times\widehat{\mathbf{p}}) + (\mathfrak{x}_v\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_u)(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\\
&\quad - \varphi_i\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varphi_i^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + [\Omega^2(\mathfrak{x}_p\mathfrak{x}_r - \mathfrak{x}_s\mathfrak{x}_o) + \varepsilon_g(\mathfrak{x}_p\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_o) - \varsigma_c\varphi_i\mathfrak{x}_o\\
&\quad + \varepsilon_c(\mathfrak{x}_v\mathfrak{x}_o - \mathfrak{x}_p\mathfrak{x}_u) + \varepsilon_g(2\mathfrak{x}_o\mathfrak{x}_r - \mathfrak{x}_s\varphi_g\varphi_h) + 2\Lambda^2(\mathfrak{x}_o\varphi_i - \mathfrak{x}_r\varphi_g\varphi_h) - \varsigma_d\varphi_i\varphi_g\varphi_h\\
&\qquad + \varepsilon_i(\mathfrak{x}_v\varphi_g\varphi_h - 2\mathfrak{x}_o\mathfrak{x}_u) + \varsigma_d\varphi_g\varphi_h\varphi_i - \varsigma_a\varphi_g\varphi_h\mathfrak{x}_u + \varsigma_c\varphi_g\varphi_h\mathfrak{x}_r + 2\mathfrak{x}_o^2\mathfrak{d}_o + \varphi_g\varphi_h\mathfrak{x}_o\mathfrak{d}_p\\
&\qquad - \mathfrak{x}_p\varphi_g\varphi_h\mathfrak{d}_o + \varphi_g^2\varphi_h^2\mathfrak{d}_r]\widehat{\mathbf{p}} + [\varphi_g\varphi_h\mathfrak{x}_u - \varphi_g\varphi_h\varepsilon_c\mathfrak{x}_r - \varphi_g\varphi_h\varepsilon_i\varphi_i]\dot{\mathbf{\Lambda}}\\
&\quad + [\varsigma_a\varphi_i\varphi_g\varphi_h - \varepsilon_c(2\mathfrak{x}_o\mathfrak{x}_r - \mathfrak{x}_s\varphi_g\varphi_h) - 2\varepsilon_i(\mathfrak{x}_o\varphi_i - \mathfrak{x}_r\varphi_g\varphi_h) - (\mathfrak{x}_v\varphi_g\varphi_h - 2\mathfrak{x}_o\mathfrak{x}_u)]\mathbf{\Lambda}\\
&\quad + [\varsigma_a\varphi_i\mathfrak{x}_o - \varepsilon_c(\mathfrak{x}_p\mathfrak{x}_r - \mathfrak{x}_s\mathfrak{x}_o) - \varepsilon_i(\mathfrak{x}_p\varphi_i - 2\mathfrak{x}_r\mathfrak{x}_o) - (\mathfrak{x}_v\mathfrak{x}_o - \mathfrak{x}_p\mathfrak{x}_u)]\mathbf{\Omega}\\
&= \varkappa_s(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \varkappa_t(\mathbf{\Omega}\times\widehat{\mathbf{p}}) + \varkappa_u(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) - \varphi_i\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varphi_i^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\\
&\quad + \varkappa_w\widehat{\mathbf{p}} + \varphi_g\varphi_h\varkappa_v\dot{\mathbf{\Lambda}} + \varkappa_x\mathbf{\Lambda} + \varkappa_y\mathbf{\Omega} \text{ by (5.23q).}
\end{aligned}
\tag{5.44n}
$$

Art 22g. *Development of equation* (3.15c).

Having the foregoing derivations in view, we have that

$$
\begin{aligned}
\mathfrak{J}_p &= b_4\mathfrak{J}_a + b_5\mathfrak{J}_b + \rho\dot{\mathfrak{y}}\mathfrak{J}_c - \ddot{\mathfrak{y}}\mathfrak{J}_d + \dot{\mathfrak{y}}\mathfrak{J}_e + b_6\mathfrak{J}_f + \rho b_1\mathfrak{J}_g \text{ by (3.15c)}\\
&= b_4[\varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) - \Omega^2(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \varepsilon_d\mathbf{\Lambda} - \delta_a\mathbf{r}]\\
&\quad + b_5[2\varepsilon_h(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) - 3\varepsilon_g(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) + \varepsilon_d\dot{\mathbf{\Lambda}} - \varepsilon_d\Omega^2\mathbf{\Omega} + \mathfrak{y}_a\mathbf{r}]\\
&\quad + \rho\dot{\mathfrak{y}}[3\varepsilon_h(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + \varrho_t(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \varrho_u(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{y}_b\mathbf{\Lambda} - \mathfrak{y}_c\mathbf{\Omega} + \mathfrak{y}_d\mathbf{r} + \varepsilon_d\ddot{\mathbf{\Lambda}}]\\
&\quad - \ddot{\mathfrak{y}}[\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{y}_e\widehat{\mathbf{p}} - \varepsilon_e\mathfrak{x}_o\mathbf{\Omega} - \varepsilon_e\varphi_g\varphi_h\mathbf{\Lambda}]\\
&\quad + \dot{\mathfrak{y}}[\mathfrak{x}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + 2\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) + \varphi_i(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) - \mathfrak{x}_v(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{y}_f\widehat{\mathbf{p}} - \mathfrak{x}_p\varepsilon_e\mathbf{\Omega} - 2\mathfrak{x}_o\varepsilon_e\mathbf{\Lambda} - \varphi_g\varphi_h\varepsilon_e\dot{\mathbf{\Lambda}}]\\
&\quad + b_6[\mathfrak{y}_g(\mathbf{\Omega}\times\mathbf{r}) + \varepsilon_b^2(\mathbf{\Omega}\times\mathbf{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) - \varphi_a^2\dot{\mathbf{\Lambda}} + \mathfrak{y}_h\mathbf{r} + \Omega^2\varphi_a^2\mathbf{\Omega} + 3\varphi_c\mathbf{\Lambda}]\\
&\quad + \rho b_1[3\varepsilon_h\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{y}_i(\mathbf{\Omega}\times\mathbf{r}) - 3\varepsilon_h\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{y}_j\mathbf{\Lambda} + \mathfrak{y}_l\mathbf{r}\\
&\qquad + \mathfrak{y}_k\mathbf{\Omega} - \varphi_a^2\ddot{\mathbf{\Lambda}}] \text{ by (5.44)}
\end{aligned}
$$

$$
\begin{aligned}
&= (b_4\varepsilon_b + 2b_5\varepsilon_h + \rho\dot{\mathfrak{y}}\varrho_t - \ddot{\mathfrak{y}}\mathfrak{x}_r + \dot{\mathfrak{y}}\mathfrak{x}_s)(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + (-b_4\Omega^2 - 3b_5\varepsilon_g + \rho\dot{\mathfrak{y}}\varrho_u)(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\quad + (b_5\varepsilon_b + 3\rho\dot{\mathfrak{y}}\varepsilon_h - \ddot{\mathfrak{y}}\varphi_i + 2\dot{\mathfrak{y}}\mathfrak{x}_r)(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) + (\rho\dot{\mathfrak{y}}\varepsilon_b + \dot{\mathfrak{y}}\varphi_i)(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + (\ddot{\mathfrak{y}}\mathfrak{x}_u - \dot{\mathfrak{y}}\mathfrak{x}_v)(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\\
&\quad + (b_6\mathfrak{y}_g + \rho b_1\mathfrak{y}_i)(\mathbf{\Omega}\times\mathbf{r}) + (b_6\varepsilon_b^2 + 3\rho b_1\varepsilon_h\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \rho b_1\varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - \rho b_1\varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}})\\
&\quad - (b_6\varepsilon_b\Omega^2 + 3\rho b_1\varepsilon_h\Omega^2)(\mathbf{r}\times\mathbf{\Lambda}) + (b_4\varepsilon_d + \rho\dot{\mathfrak{y}}\mathfrak{y}_b + \ddot{\mathfrak{y}}\varepsilon_e\varphi_g\varphi_h - 2\dot{\mathfrak{y}}\mathfrak{x}_o\varepsilon_e + 3b_6\varphi_c + \rho b_1\mathfrak{y}_j)\mathbf{\Lambda}\\
&\quad + (-b_4\delta_a + b_5\mathfrak{y}_a + \rho\dot{\mathfrak{y}}\mathfrak{y}_d + b_6\mathfrak{y}_h + \rho b_1\mathfrak{y}_l)\mathbf{r} + (b_5\varepsilon_d - \dot{\mathfrak{y}}\varphi_g\varphi_h\varepsilon_e - b_6\varphi_a^2)\dot{\mathbf{\Lambda}} + (\dot{\mathfrak{y}}\mathfrak{y}_f - \ddot{\mathfrak{y}}\mathfrak{y}_e)\widehat{\mathbf{p}}\\
&\quad + (-b_5\varepsilon_d\Omega^2 - \rho\dot{\mathfrak{y}}\mathfrak{y}_c + \ddot{\mathfrak{y}}\varepsilon_e\mathfrak{x}_o - \dot{\mathfrak{y}}\mathfrak{x}_p\varepsilon_e + b_6\Omega^2\varphi_a^2 + \rho b_1\mathfrak{y}_k)\mathbf{\Omega} + (\rho\dot{\mathfrak{y}}\varepsilon_d - \rho b_1\varphi_a^2)\ddot{\mathbf{\Lambda}}
\end{aligned}
\tag{5.45a}
$$

$$\begin{aligned}
\mathfrak{J}_q &= -\ddot{\rho}\mathfrak{J}_h + b_1\mathfrak{J}_i + \rho^2\mathfrak{J}_j - b_2\mathfrak{J}_k + \rho\mathfrak{J}_l + \rho\mathfrak{J}_n + \mathfrak{J}_o \text{ by (3.15c)} \\
&= -\ddot{\rho}[\varphi_i\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) \\
&\qquad + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_o\mathfrak{y}_m\mathbf{\Omega} + \mathfrak{y}_n\mathbf{r} + \mathfrak{y}_o\widehat{\mathbf{p}} + \mathfrak{y}_p\mathbf{\Lambda}] \\
&\quad + b_1[2\mathfrak{x}_r\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\varepsilon_b(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_s\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - 2\mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) \\
&\qquad - \varphi_i\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{x}_v\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_p\mathfrak{y}_m\mathbf{\Omega} + \varphi_g\varphi_h\mathfrak{y}_m\dot{\mathbf{\Lambda}} + \mathfrak{y}_q\mathbf{r} + \mathfrak{y}_r\widehat{\mathbf{p}} + \mathfrak{y}_s\mathbf{\Lambda}] \\
&\quad + \rho^2[\mathfrak{y}_t(\mathbf{\Omega}\times\mathbf{\Lambda}) + 2\varepsilon_b\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{y}_u(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{y}_v(\mathbf{\Lambda}\times\mathbf{r}) - 3\varepsilon_b\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) \\
&\qquad + \varepsilon_b^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + \mathfrak{y}_w\mathbf{\Lambda} - \mathfrak{y}_x\dot{\mathbf{\Lambda}} - 3\varphi_c\ddot{\mathbf{\Lambda}} + \mathfrak{y}_y\mathbf{\Omega} + \mathfrak{y}_z\mathbf{r}] \\
&\quad - b_2[\varkappa_a(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_u\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda}) + 3\mathfrak{x}_u\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\qquad - \mathfrak{x}_u\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) + \varkappa_b\mathbf{r} + \varkappa_e\widehat{\mathbf{p}} + \varkappa_f\mathbf{\Omega} + \varphi_g\varphi_h\varkappa_d\mathbf{\Lambda} + \varkappa_c\dot{\mathbf{\Lambda}}] \\
&\quad + \rho[\varkappa_g(\mathbf{\Omega}\times\mathbf{\Lambda}) - 2\mathfrak{x}_v\varepsilon_h(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - 3\mathfrak{x}_s\varepsilon_g(\mathbf{r}\times\mathbf{\Omega}) - 6\mathfrak{x}_r\varepsilon_g(\mathbf{r}\times\mathbf{\Lambda}) + 3\mathfrak{x}_v\varepsilon_g(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_v\varepsilon_b(\mathbf{\Lambda}\times\widehat{\mathbf{p}}) \\
&\qquad + 2\varphi_i\varepsilon_h(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - 3\varphi_i\varepsilon_g(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \varphi_i\varepsilon_b(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + \varkappa_i\widehat{\mathbf{p}} + 2\mathfrak{x}_o\varkappa_h\mathbf{\Lambda} + \varkappa_j\mathbf{\Omega} + \varkappa_k\dot{\mathbf{\Lambda}} + \varkappa_l\mathbf{r}] \\
&\quad + \rho[\varkappa_m(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varrho_u\mathfrak{x}_r(\mathbf{\Omega}\times\mathbf{r}) + \varrho_u\varphi_i(\mathbf{\Lambda}\times\mathbf{r}) - \varrho_t\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Omega}) - \varrho_u\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{r}) - 3\varepsilon_h\mathfrak{x}_u(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) \\
&\qquad - \varepsilon_b\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \varepsilon_b\varphi_i(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) + \varkappa_o\widehat{\mathbf{p}} + \varkappa_p\mathbf{r} + \varkappa_q\mathbf{\Omega} + \varkappa_r\mathbf{\Lambda} + \varkappa_n\ddot{\mathbf{\Lambda}}] \\
&\quad + [\varkappa_s(\mathbf{\Omega}\times\mathbf{\Lambda}) + \varphi_i\mathfrak{x}_r(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \varkappa_t(\mathbf{\Omega}\times\widehat{\mathbf{p}}) + \varkappa_u(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) - \varphi_i\mathfrak{x}_u(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \varphi_i^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) \\
&\qquad + \varkappa_w\widehat{\mathbf{p}} + \varphi_g\varphi_h\varkappa_v\dot{\mathbf{\Lambda}} + \varkappa_x\mathbf{\Lambda} + \varkappa_y\mathbf{\Omega}]
\end{aligned}$$

$$\begin{aligned}
&= (-\ddot{\rho}\mathfrak{x}_r\Omega^2 + b_1\mathfrak{x}_s\Omega^2 + \rho^2\mathfrak{y}_u - 3b_2\mathfrak{x}_r\varepsilon_g + 3\rho\mathfrak{x}_s\varepsilon_g + \rho\varrho_u\mathfrak{x}_r)(\mathbf{\Omega}\times\mathbf{r}) \\
&\quad + (-\ddot{\rho}\varphi_i\varepsilon_b + 2b_1\mathfrak{x}_r\varepsilon_b + \rho^2\mathfrak{y}_t - b_2\varkappa_a + \rho\varkappa_g + \rho\varkappa_m + \varkappa_s)(\mathbf{\Omega}\times\mathbf{\Lambda}) \\
&\quad + (b_1\varphi_i\varepsilon_b + 2\rho^2\varepsilon_b\varepsilon_h + 2\rho\varphi_i\varepsilon_h + \rho\varepsilon_b\mathfrak{x}_r + \varphi_i\mathfrak{x}_r)(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) \\
&\quad + (\ddot{\rho}\mathfrak{x}_u\varepsilon_b - b_1\mathfrak{x}_v\varepsilon_b + 2b_2\mathfrak{x}_u\varepsilon_h - 2\rho\mathfrak{x}_v\varepsilon_h + \rho\varrho_t\mathfrak{x}_u + \varkappa_t)(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad + (-b_1\varphi_i\Omega^2 - 3\rho^2\varepsilon_b\varepsilon_g - 3\rho\varphi_i\varepsilon_g)(\mathbf{r}\times\dot{\mathbf{\Lambda}}) \\
&\quad + (\ddot{\rho}\varphi_i\Omega^2 - 2b_1\mathfrak{x}_r\Omega^2 - \rho^2\mathfrak{y}_v + 3b_2\varphi_i\varepsilon_g - 6\rho\mathfrak{x}_r\varepsilon_g - \rho\varrho_u\varphi_i)(\mathbf{r}\times\mathbf{\Lambda}) \\
&\quad + (-\ddot{\rho}\mathfrak{x}_u\Omega^2 + b_1\mathfrak{x}_v\Omega^2 - 3b_2\mathfrak{x}_u\varepsilon_g + 3\rho\mathfrak{x}_v\varepsilon_g + \rho\varrho_u\mathfrak{x}_u)(\mathbf{r}\times\widehat{\mathbf{p}}) - (\rho\varepsilon_b\mathfrak{x}_u + \varphi_i\mathfrak{x}_u)(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) \\
&\quad + (-b_2\mathfrak{x}_u\varepsilon_b + \rho\mathfrak{x}_v\varepsilon_b - 3\rho\varepsilon_h\mathfrak{x}_u + \varkappa_u)(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + (\rho^2\varepsilon_b^2 + \rho\varphi_i\varepsilon_b + \rho\varepsilon_b\varphi_i + \varphi_i^2)(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) \\
&\quad + (-\ddot{\rho}\mathfrak{y}_p + b_1\mathfrak{y}_s + \rho^2\mathfrak{y}_w - b_2\varphi_g\varphi_h\varkappa_d + 2\rho\mathfrak{x}_o\varkappa_h + \rho\varkappa_r + \varkappa_x)\mathbf{\Lambda} \\
&\quad + (-\ddot{\rho}\mathfrak{y}_n + b_1\mathfrak{y}_q + \rho^2\mathfrak{y}_z - b_2\varkappa_b + \rho\varkappa_l + \rho\varkappa_p)\mathbf{r} + (-\ddot{\rho}\mathfrak{y}_o + b_1\mathfrak{y}_r - b_2\varkappa_e + \rho\varkappa_i + \rho\varkappa_o + \varkappa_w)\widehat{\mathbf{p}} \\
&\quad + (-\ddot{\rho}\mathfrak{x}_o\mathfrak{y}_m + b_1\mathfrak{x}_p\mathfrak{y}_m + \rho^2\mathfrak{y}_y - b_2\varkappa_f + \rho\varkappa_j + \rho\varkappa_q + \varkappa_y)\mathbf{\Omega} \\
&\quad + (b_1\varphi_g\varphi_h\mathfrak{y}_m - \rho^2\mathfrak{y}_x - b_2\varkappa_c + \rho\varkappa_k + \varphi_g\varphi_h\varkappa_v)\dot{\mathbf{\Lambda}} + (-3\rho^2\varphi_c + \rho\varkappa_n)\ddot{\mathbf{\Lambda}}
\end{aligned} \tag{5.45b}$$

$$\begin{aligned}
&\mathfrak{J}_p + \mathfrak{J}_q \\
&= (b_4\varepsilon_b + 2b_5\varepsilon_h + \rho\dot{\mathfrak{Y}}\varrho_t - \ddot{\mathfrak{Y}}\mathfrak{x}_r + \dot{\mathfrak{Y}}\mathfrak{x}_s)(\boldsymbol{\kappa}\times\mathbf{\Omega}) + (-b_4\Omega^2 - 3b_5\varepsilon_g + \rho\dot{\mathfrak{Y}}\varrho_u)(\boldsymbol{\kappa}\times\mathbf{r}) \\
&\quad + (b_5\varepsilon_b + 3\rho\dot{\mathfrak{Y}}\varepsilon_h - \ddot{\mathfrak{Y}}\varphi_i + 2\dot{\mathfrak{Y}}\mathfrak{x}_r)(\boldsymbol{\kappa}\times\mathbf{\Lambda}) + (\rho\dot{\mathfrak{Y}}\varepsilon_b + \dot{\mathfrak{Y}}\varphi_i)(\boldsymbol{\kappa}\times\dot{\mathbf{\Lambda}}) + (\ddot{\mathfrak{Y}}\mathfrak{x}_u - \dot{\mathfrak{Y}}\mathfrak{x}_v)(\boldsymbol{\kappa}\times\widehat{\mathbf{p}}) \\
&\quad + (b_6\mathfrak{y}_g + \rho b_1\mathfrak{y}_i)(\mathbf{\Omega}\times\mathbf{r}) + (b_6\varepsilon_b^2 + 3\rho b_1\varepsilon_h\varepsilon_b)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \rho b_1\varepsilon_b^2(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) - \rho b_1\varepsilon_b\Omega^2(\mathbf{r}\times\dot{\mathbf{\Lambda}}) \\
&\quad - (b_6\varepsilon_b\Omega^2 + 3\rho b_1\varepsilon_h\Omega^2)(\mathbf{r}\times\mathbf{\Lambda}) + (b_4\varepsilon_d + \rho\dot{\mathfrak{Y}}\mathfrak{y}_b + \ddot{\mathfrak{Y}}\varepsilon_e\varphi_g\varphi_h - 2\dot{\mathfrak{Y}}\mathfrak{x}_o\varepsilon_e + 3b_6\varphi_c + \rho b_1\mathfrak{y}_j)\mathbf{\Lambda} \\
&\quad + (-b_4\delta_a + b_5\mathfrak{y}_a + \rho\dot{\mathfrak{Y}}\mathfrak{y}_d + b_6\mathfrak{y}_h + \rho b_1\mathfrak{y}_l)\mathbf{r} + (b_5\varepsilon_d - \dot{\mathfrak{Y}}\varphi_g\varphi_h\varepsilon_e - b_6\varphi_a^2)\dot{\mathbf{\Lambda}} + (\dot{\mathfrak{Y}}\mathfrak{y}_f - \ddot{\mathfrak{Y}}\mathfrak{y}_e)\widehat{\mathbf{p}} \\
&\quad + (-b_5\varepsilon_d\Omega^2 - \rho\dot{\mathfrak{Y}}\mathfrak{y}_c + \ddot{\mathfrak{Y}}\varepsilon_e\mathfrak{x}_o - \dot{\mathfrak{Y}}\mathfrak{x}_p\varepsilon_e + b_6\Omega^2\varphi_a^2 + \rho b_1\mathfrak{y}_k)\mathbf{\Omega} + (\rho\dot{\mathfrak{Y}}\varepsilon_d - \rho b_1\varphi_a^2)\ddot{\mathbf{\Lambda}} \\
&\quad + (-\ddot{\rho}\mathfrak{x}_r\Omega^2 + b_1\mathfrak{x}_s\Omega^2 + \rho^2\mathfrak{y}_u - 3b_2\mathfrak{x}_r\varepsilon_g + 3\rho\mathfrak{x}_s\varepsilon_g + \rho\varrho_u\mathfrak{x}_r)(\mathbf{\Omega}\times\mathbf{r}) \\
&\quad + (-\ddot{\rho}\varphi_i\varepsilon_b + 2b_1\mathfrak{x}_r\varepsilon_b + \rho^2\mathfrak{y}_t - b_2\varkappa_a + \rho\varkappa_g + \rho\varkappa_m + \varkappa_s)(\mathbf{\Omega}\times\mathbf{\Lambda}) + \cdots
\end{aligned}$$

$$\begin{aligned}
\cdots\quad & + (b_1\varphi_i\varepsilon_b + 2\rho^2\varepsilon_b\varepsilon_h + 2\rho\varphi_i\varepsilon_h + \rho\varepsilon_b\mathfrak{x}_r + \varphi_i\mathfrak{x}_r)(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) \\
& + (\ddot{\rho}\mathfrak{x}_u\varepsilon_b - b_1\mathfrak{x}_v\varepsilon_b + 2b_2\mathfrak{x}_u\varepsilon_h - 2\rho\mathfrak{x}_v\varepsilon_h + \rho\varrho_t\mathfrak{x}_u + \varkappa_t)(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
& + (-b_1\varphi_i\Omega^2 - 3\rho^2\varepsilon_b\varepsilon_g - 3\rho\varphi_i\varepsilon_g)(\mathbf{r}\times\dot{\mathbf{\Lambda}}) \\
& + (\ddot{\rho}\varphi_i\Omega^2 - 2b_1\mathfrak{x}_r\Omega^2 - \rho^2\mathfrak{y}_v + 3b_2\varphi_i\varepsilon_g - 6\rho\mathfrak{x}_r\varepsilon_g - \rho\varrho_u\varphi_i)(\mathbf{r}\times\mathbf{\Lambda}) \\
& + (-\ddot{\rho}\mathfrak{x}_u\Omega^2 + b_1\mathfrak{x}_v\Omega^2 - 3b_2\mathfrak{x}_u\varepsilon_g + 3\rho\mathfrak{x}_v\varepsilon_g + \rho\varrho_u\mathfrak{x}_u)(\mathbf{r}\times\widehat{\mathbf{p}}) - (\rho\varepsilon_b\mathfrak{x}_u + \varphi_i\mathfrak{x}_u)(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) \\
& + (-b_2\mathfrak{x}_u\varepsilon_b + \rho\mathfrak{x}_v\varepsilon_b - 3\rho\varepsilon_h\mathfrak{x}_u + \varkappa_u)(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + (\rho^2\varepsilon_b^2 + \rho\varphi_i\varepsilon_b + \rho\varepsilon_b\varphi_i + \varphi_i^2)(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) \\
& + (-\ddot{\rho}\mathfrak{y}_p + b_1\mathfrak{y}_s + \rho^2\mathfrak{y}_w - b_2\varphi_g\varphi_h\varkappa_d + 2\rho\mathfrak{x}_o\varkappa_h + \rho\varkappa_r + \varkappa_x)\mathbf{\Lambda} \\
& + (-\ddot{\rho}\mathfrak{y}_n + b_1\mathfrak{y}_q + \rho^2\mathfrak{y}_z - b_2\varkappa_b + \rho\varkappa_l + \rho\varkappa_p)\mathbf{r} + (-\ddot{\rho}\mathfrak{y}_o + b_1\mathfrak{y}_r - b_2\varkappa_e + \rho\varkappa_i + \rho\varkappa_o + \varkappa_w)\widehat{\mathbf{p}} \\
& + (-\ddot{\rho}\mathfrak{x}_o\mathfrak{y}_m + b_1\mathfrak{x}_p\mathfrak{y}_m + \rho^2\mathfrak{y}_y - b_2\varkappa_f + \rho\varkappa_j + \rho\varkappa_q + \varkappa_y)\mathbf{\Omega} \\
& + (b_1\varphi_g\varphi_h\mathfrak{y}_m - \rho^2\mathfrak{y}_x - b_2\varkappa_c + \rho\varkappa_k + \varphi_g\varphi_h\varkappa_v)\dot{\mathbf{\Lambda}} + (-3\rho^2\varphi_c + \rho\varkappa_n)\ddot{\mathbf{\Lambda}} \text{ by (5.45a) \& (5.45b)}
\end{aligned}$$

$$\begin{aligned}
&= (b_4\varepsilon_b + 2b_5\varepsilon_h + \rho\dot{\mathfrak{Y}}\varrho_t - \ddot{\mathfrak{Y}}\mathfrak{x}_r + \dot{\mathfrak{Y}}\mathfrak{x}_s)(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + (-b_4\Omega^2 - 3b_5\varepsilon_g + \rho\dot{\mathfrak{Y}}\varrho_u)(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) \\
&\quad + (b_5\varepsilon_b + 3\rho\dot{\mathfrak{Y}}\varepsilon_h - \ddot{\mathfrak{Y}}\varphi_i + 2\dot{\mathfrak{Y}}\mathfrak{x}_r)(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) + (\rho\dot{\mathfrak{Y}}\varepsilon_b + \dot{\mathfrak{Y}}\varphi_i)(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + (\ddot{\mathfrak{Y}}\mathfrak{x}_u - \dot{\mathfrak{Y}}\mathfrak{x}_v)(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) \\
&\quad + (b_6\mathfrak{y}_g + \rho b_1\mathfrak{y}_i - \ddot{\rho}\mathfrak{x}_r\Omega^2 + b_1\mathfrak{x}_s\Omega^2 + \rho^2\mathfrak{y}_u - 3b_2\mathfrak{x}_r\varepsilon_g + 3\rho\mathfrak{x}_s\varepsilon_g + \rho\varrho_u\mathfrak{x}_r)(\mathbf{\Omega}\times\mathbf{r}) \\
&\quad + (b_6\varepsilon_b^2 + 3\rho b_1\varepsilon_h\varepsilon_b - \ddot{\rho}\varphi_i\varepsilon_b + 2b_1\mathfrak{x}_r\varepsilon_b + \rho^2\mathfrak{y}_t - b_2\varkappa_a + \rho\varkappa_g + \rho\varkappa_m + \varkappa_s)(\mathbf{\Omega}\times\mathbf{\Lambda}) \\
&\quad + (\rho b_1\varepsilon_b^2 + b_1\varphi_i\varepsilon_b + 2\rho^2\varepsilon_b\varepsilon_h + 2\rho\varphi_i\varepsilon_h + \rho\varepsilon_b\mathfrak{x}_r + \varphi_i\mathfrak{x}_r)(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) \\
&\quad + (\ddot{\rho}\mathfrak{x}_u\varepsilon_b - b_1\mathfrak{x}_v\varepsilon_b + 2b_2\mathfrak{x}_u\varepsilon_h - 2\rho\mathfrak{x}_v\varepsilon_h + \rho\varrho_t\mathfrak{x}_u + \varkappa_t)(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad + (-\rho b_1\varepsilon_b\Omega^2 - b_1\varphi_i\Omega^2 - 3\rho^2\varepsilon_b\varepsilon_g - 3\rho\varphi_i\varepsilon_g)(\mathbf{r}\times\dot{\mathbf{\Lambda}}) \\
&\quad + (-b_6\varepsilon_b\Omega^2 - 3\rho b_1\varepsilon_h\Omega^2 + \ddot{\rho}\varphi_i\Omega^2 - 2b_1\mathfrak{x}_r\Omega^2 - \rho^2\mathfrak{y}_v + 3b_2\varphi_i\varepsilon_g - 6\rho\mathfrak{x}_r\varepsilon_g - \rho\varrho_u\varphi_i)(\mathbf{r}\times\mathbf{\Lambda}) \\
&\quad + (-\ddot{\rho}\mathfrak{x}_u\Omega^2 + b_1\mathfrak{x}_v\Omega^2 - 3b_2\mathfrak{x}_u\varepsilon_g + 3\rho\mathfrak{x}_v\varepsilon_g + \rho\varrho_u\mathfrak{x}_u)(\mathbf{r}\times\widehat{\mathbf{p}}) - (\rho\varepsilon_b\mathfrak{x}_u + \varphi_i\mathfrak{x}_u)(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) \\
&\quad + (-b_2\mathfrak{x}_u\varepsilon_b + \rho\mathfrak{x}_v\varepsilon_b - 3\rho\varepsilon_h\mathfrak{x}_u + \varkappa_u)(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + (\rho^2\varepsilon_b^2 + \rho\varphi_i\varepsilon_b + \rho\varepsilon_b\varphi_i + \varphi_i^2)(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}}) \\
&\quad + (b_4\varepsilon_d + \rho\dot{\mathfrak{Y}}\mathfrak{y}_b + \ddot{\mathfrak{Y}}\varepsilon_e\varphi_g\varphi_h - 2\dot{\mathfrak{Y}}\mathfrak{x}_o\varepsilon_e + 3b_6\varphi_c + \rho b_1\mathfrak{y}_j - \ddot{\rho}\mathfrak{y}_p + b_1\mathfrak{y}_s + \rho^2\mathfrak{y}_w \\
&\qquad - b_2\varphi_g\varphi_h\varkappa_d + 2\rho\mathfrak{x}_o\varkappa_h + \rho\varkappa_r + \varkappa_x)\mathbf{\Lambda} + \cdots
\end{aligned}$$

$$\begin{aligned}
\cdots\quad & + (-b_4\delta_a + b_5\mathfrak{y}_a + \rho\dot{\mathfrak{Y}}\mathfrak{y}_d + b_6\mathfrak{y}_h + \rho b_1\mathfrak{y}_l - \ddot{\rho}\mathfrak{y}_n + b_1\mathfrak{y}_q + \rho^2\mathfrak{y}_z - b_2\varkappa_b + \rho\varkappa_l + \rho\varkappa_p)\mathbf{r} \\
& + (\dot{\mathfrak{Y}}\mathfrak{y}_f - \ddot{\mathfrak{Y}}\mathfrak{y}_e - \ddot{\rho}\mathfrak{y}_o + b_1\mathfrak{y}_r - b_2\varkappa_e + \rho\varkappa_i + \rho\varkappa_o + \varkappa_w)\widehat{\mathbf{p}} \\
& + (-b_5\varepsilon_d\Omega^2 - \rho\dot{\mathfrak{Y}}\mathfrak{y}_c + \ddot{\mathfrak{Y}}\varepsilon_e\mathfrak{x}_o - \dot{\mathfrak{Y}}\mathfrak{x}_p\varepsilon_e + b_6\Omega^2\varphi_a^2 + \rho b_1\mathfrak{y}_k - \ddot{\rho}\mathfrak{x}_o\mathfrak{y}_m + b_1\mathfrak{x}_p\mathfrak{y}_m \\
& \quad + \rho^2\mathfrak{y}_y - b_2\varkappa_f + \rho\varkappa_j + \rho\varkappa_q + \varkappa_y)\mathbf{\Omega} \\
& + (b_5\varepsilon_d - \dot{\mathfrak{Y}}\varphi_g\varphi_h\varepsilon_e - b_6\varphi_a^2 + b_1\varphi_g\varphi_h\mathfrak{y}_m - \rho^2\mathfrak{y}_x - b_2\varkappa_c + \rho\varkappa_k + \varphi_g\varphi_h\varkappa_v)\dot{\mathbf{\Lambda}} \\
& + (\rho\dot{\mathfrak{Y}}\varepsilon_d - \rho b_1\varphi_a^2 - 3\rho^2\varphi_c + \rho\varkappa_n)\ddot{\mathbf{\Lambda}}
\end{aligned} \tag{5.45c}$$

$$\begin{aligned}
\boldsymbol{\mathfrak{J}}_r &= \dot{\mathfrak{Y}}\widehat{\boldsymbol{\kappa}} + b_1\mathbf{a} + \rho\dot{\mathbf{a}} + \dot{\mathbf{e}} \text{ by (3.15c)} \\
&= (c\mathfrak{x}_f - 2\mathfrak{x}_x)\widehat{\boldsymbol{\kappa}} + (\mathfrak{x}_i - 1)\mathbf{a} + \rho\dot{\mathbf{a}} + \dot{\mathbf{e}} \text{ by (5.42a) \& (5.43a)} \\
&= (c\mathfrak{x}_f - 2\mathfrak{x}_x)\widehat{\boldsymbol{\kappa}} + (\mathfrak{x}_i - 1)[(\mathbf{\Omega}\cdot\mathbf{r})\mathbf{\Omega} - \Omega^2\mathbf{r} + \mathbf{\Lambda}\times\mathbf{r}] + \rho[2\varepsilon_h\mathbf{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\mathbf{\Lambda}}\times\mathbf{r} - \Omega^2(\mathbf{\Omega}\times\mathbf{r}) \\
&\quad + \varepsilon_b\mathbf{\Lambda}] + \mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{x}_r\mathbf{\Omega} + \varphi_i\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}} \text{ by (1.4), (5.30a) \& (5.40a)} \\
&= (c\mathfrak{x}_f - 2\mathfrak{x}_x)\widehat{\boldsymbol{\kappa}} + \varepsilon_b(\mathfrak{x}_i - 1)\mathbf{\Omega} - \Omega^2(\mathfrak{x}_i - 1)\mathbf{r} + (\mathfrak{x}_i - 1)(\mathbf{\Lambda}\times\mathbf{r}) + 2\rho\varepsilon_h\mathbf{\Omega} - 3\rho\varepsilon_g\mathbf{r} + \rho(\dot{\mathbf{\Lambda}}\times\mathbf{r}) \\
&\quad - \rho\Omega^2(\mathbf{\Omega}\times\mathbf{r}) + \rho\varepsilon_b\mathbf{\Lambda} + \mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{x}_r\mathbf{\Omega} + \varphi_i\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}} \text{ by (5.1a)} \\
&= (\mathfrak{x}_i - 1)(\mathbf{\Lambda}\times\mathbf{r}) + \rho(\dot{\mathbf{\Lambda}}\times\mathbf{r}) - \rho\Omega^2(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + (c\mathfrak{x}_f - 2\mathfrak{x}_x)\widehat{\boldsymbol{\kappa}} \\
&\quad + [\varepsilon_b(\mathfrak{x}_i - 1) + 2\rho\varepsilon_h + \mathfrak{x}_r]\mathbf{\Omega} - [\Omega^2(\mathfrak{x}_i - 1) + 3\rho\varepsilon_g]\mathbf{r} + (\rho\varepsilon_b + \varphi_i)\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}}
\end{aligned}$$

$$\begin{aligned}
&= \mathfrak{v}_b\widehat{\boldsymbol{\kappa}} + \mathfrak{v}_c\mathbf{\Omega} - \mathfrak{v}_d\mathbf{r} + \mathfrak{v}_e\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}} + \mathfrak{v}_a(\mathbf{\Lambda}\times\mathbf{r}) + \rho(\dot{\mathbf{\Lambda}}\times\mathbf{r}) \\
&\quad - \rho\Omega^2(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) \text{ by (5.23r).}
\end{aligned} \tag{5.45d}$$

Equations (5.42a), (5.43a) and (5.23r) together give

$$\begin{gathered}\dot{\mathcal{Y}} = \mathfrak{v}_b, \quad \ddot{\mathcal{Y}} = \mathfrak{v}_f, \quad \dddot{\mathcal{Y}} = \mathfrak{v}_g \\ b_1 = \mathfrak{v}_a, \quad b_2 = \mathfrak{v}_h, \quad b_3 = \mathfrak{v}_i, \quad b_4 = \mathfrak{v}_j, \quad b_5 = \mathfrak{v}_k, \quad b_6 = \mathfrak{v}_l\end{gathered} \tag{5.46}$$

on account of which (5.45c) becomes

$$\begin{aligned}
&\mathfrak{J}_p + \mathfrak{J}_q \\
&= (\mathfrak{v}_j\varepsilon_b + 2\mathfrak{v}_k\varepsilon_h + \rho\mathfrak{v}_b\varrho_t - \mathfrak{v}_f\mathfrak{x}_r + \mathfrak{v}_b\mathfrak{x}_s)(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + (-\mathfrak{v}_j\Omega^2 - 3\mathfrak{v}_k\varepsilon_g + \rho\mathfrak{v}_b\varrho_u)(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + (\mathfrak{v}_k\varepsilon_b + 3\rho\mathfrak{v}_b\varepsilon_h - \mathfrak{v}_f\varphi_i + 2\mathfrak{v}_b\mathfrak{x}_r)(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + (\rho\mathfrak{v}_b\varepsilon_b + \mathfrak{v}_b\varphi_i)(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + (\mathfrak{v}_f\mathfrak{x}_u - \mathfrak{v}_b\mathfrak{x}_v)(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&\quad + (\mathfrak{v}_l\mathfrak{y}_g + \rho\mathfrak{v}_a\mathfrak{y}_i - \mathfrak{x}_j\mathfrak{x}_r\Omega^2 + \mathfrak{v}_a\mathfrak{x}_s\Omega^2 + \rho^2\mathfrak{y}_u - 3\mathfrak{v}_h\mathfrak{x}_r\varepsilon_g + 3\rho\mathfrak{x}_s\varepsilon_g + \rho\varrho_u\mathfrak{x}_r)(\boldsymbol{\Omega} \times \mathbf{r}) \\
&\quad + (\mathfrak{v}_l\varepsilon_b^2 + 3\rho\mathfrak{v}_a\varepsilon_h\varepsilon_b - \mathfrak{v}_f\varphi_i\varepsilon_b + 2\mathfrak{v}_a\mathfrak{x}_r\varepsilon_b + \rho^2\mathfrak{y}_t - \mathfrak{v}_h\varkappa_a + \rho\varkappa_g + \rho\varkappa_m + \varkappa_s)(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \\
&\quad + (\rho\mathfrak{v}_a\varepsilon_b^2 + \mathfrak{v}_a\varphi_i\varepsilon_b + 2\rho^2\varepsilon_b\varepsilon_h + 2\rho\varphi_i\varepsilon_h + \rho\varepsilon_b\mathfrak{x}_r + \varphi_i\mathfrak{x}_r)(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + (\mathfrak{v}_f\mathfrak{x}_u\varepsilon_b - \mathfrak{v}_a\mathfrak{x}_v\varepsilon_b + 2\mathfrak{v}_h\mathfrak{x}_u\varepsilon_h - 2\rho\mathfrak{x}_v\varepsilon_h + \rho\varrho_t\mathfrak{x}_u + \varkappa_t)(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad + (-\rho\mathfrak{v}_a\varepsilon_b\Omega^2 - \mathfrak{v}_a\varphi_i\Omega^2 - 3\rho^2\varepsilon_b\varepsilon_g - 3\rho\varphi_i\varepsilon_g)(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + (-\mathfrak{v}_l\varepsilon_b\Omega^2 - 3\rho\mathfrak{v}_a\varepsilon_h\Omega^2 + \mathfrak{v}_f\varphi_i\Omega^2 - 2\mathfrak{v}_a\mathfrak{x}_r\Omega^2 - \rho^2\mathfrak{y}_v + 3\mathfrak{v}_h\varphi_i\varepsilon_g - 6\rho\mathfrak{x}_r\varepsilon_g - \rho\varrho_u\varphi_i)(\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + (-\mathfrak{x}_j\mathfrak{x}_u\Omega^2 + \mathfrak{v}_a\mathfrak{x}_v\Omega^2 - 3\mathfrak{v}_h\mathfrak{x}_u\varepsilon_g + 3\rho\mathfrak{x}_v\varepsilon_g + \rho\varrho_u\mathfrak{x}_u)(\mathbf{r} \times \widehat{\mathbf{p}}) - (\rho\varepsilon_b\mathfrak{x}_u + \varphi_i\mathfrak{x}_u)(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + (-\mathfrak{v}_h\mathfrak{x}_u\varepsilon_b + \rho\mathfrak{x}_v\varepsilon_b - 3\rho\varepsilon_h\mathfrak{x}_u + \varkappa_u)(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + (\rho^2\varepsilon_b^2 + \rho\varphi_i\varepsilon_b + \rho\varepsilon_b\varphi_i + \varphi_i^2)(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + (\mathfrak{v}_j\varepsilon_d + \rho\mathfrak{v}_b\mathfrak{y}_b + \mathfrak{v}_f\varepsilon_e\varphi_g\varphi_h - 2\mathfrak{v}_b\mathfrak{x}_o\varepsilon_e + 3\mathfrak{v}_l\varphi_c + \rho\mathfrak{v}_a\mathfrak{y}_j - \mathfrak{v}_f\mathfrak{y}_p + \mathfrak{v}_a\mathfrak{y}_s + \rho^2\mathfrak{y}_w \\
&\qquad - \mathfrak{v}_h\varphi_g\varphi_h\varkappa_d + 2\rho\mathfrak{x}_o\varkappa_h + \rho\varkappa_r + \varkappa_x)\boldsymbol{\Lambda} \\
&\quad + (-\mathfrak{v}_j\delta_a + \mathfrak{v}_k\mathfrak{y}_a + \rho\mathfrak{v}_b\mathfrak{y}_d + \mathfrak{v}_l\mathfrak{y}_h + \rho\mathfrak{v}_a\mathfrak{y}_l - \mathfrak{v}_f\mathfrak{y}_n + \mathfrak{v}_a\mathfrak{y}_q + \rho^2\mathfrak{y}_z - \mathfrak{v}_h\varkappa_b + \rho\varkappa_l + \rho\varkappa_p)\mathbf{r} \\
&\quad + (\mathfrak{v}_b\mathfrak{y}_f - \mathfrak{v}_f\mathfrak{y}_e - \mathfrak{v}_f\mathfrak{y}_o + \mathfrak{v}_a\mathfrak{y}_r - \mathfrak{v}_h\varkappa_e + \rho\varkappa_i + \rho\varkappa_o + \varkappa_w)\widehat{\mathbf{p}} \\
&\quad + (-\mathfrak{v}_k\varepsilon_d\Omega^2 - \rho\mathfrak{v}_b\mathfrak{y}_c + \mathfrak{v}_f\varepsilon_e\mathfrak{x}_o - \mathfrak{v}_b\mathfrak{x}_p\varepsilon_e + \mathfrak{v}_l\Omega^2\varphi_a^2 + \rho\mathfrak{v}_a\mathfrak{y}_k - \mathfrak{v}_f\mathfrak{x}_o\mathfrak{y}_m + \mathfrak{v}_a\mathfrak{x}_p\mathfrak{y}_m \\
&\qquad + \rho^2\mathfrak{y}_y - \mathfrak{v}_h\varkappa_f + \rho\varkappa_j + \rho\varkappa_q + \varkappa_y)\boldsymbol{\Omega} \\
&\quad + (\mathfrak{v}_k\varepsilon_d - \mathfrak{v}_b\varphi_g\varphi_h\varepsilon_e - \mathfrak{v}_l\varphi_a^2 + \mathfrak{v}_a\varphi_g\varphi_h\mathfrak{y}_m - \rho^2\mathfrak{y}_x - \mathfrak{v}_h\varkappa_c + \rho\varkappa_k + \varphi_g\varphi_h\varkappa_v)\dot{\boldsymbol{\Lambda}} \\
&\quad + (\rho\mathfrak{v}_b\varepsilon_d - \rho\mathfrak{v}_a\varphi_a^2 - 3\rho^2\varphi_c + \rho\varkappa_n)\ddot{\boldsymbol{\Lambda}}
\end{aligned}$$

$$\begin{aligned}
&= [\mathfrak{v}_j\varepsilon_b + 2\mathfrak{v}_k\varepsilon_h\varrho_t - \mathfrak{v}_f\mathfrak{x}_r + \mathfrak{v}_b(\rho + \mathfrak{x}_s)](\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + (\rho\mathfrak{v}_b\varrho_u - \mathfrak{v}_j\Omega^2 - 3\mathfrak{v}_k\varepsilon_g)(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + [\mathfrak{v}_k\varepsilon_b - \mathfrak{v}_f\varphi_i + \mathfrak{v}_b(3\rho\varepsilon_h + 2\mathfrak{x}_r)](\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_b(\rho\varepsilon_b + \varphi_i)(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + (\mathfrak{v}_f\mathfrak{x}_u - \mathfrak{v}_b\mathfrak{x}_v)(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&\quad + [\mathfrak{v}_l\mathfrak{y}_g + \mathfrak{v}_a(\rho\mathfrak{y}_i + \mathfrak{x}_s\Omega^2) + \rho(\rho\mathfrak{y}_u + 3\mathfrak{x}_s\varepsilon_g) + \mathfrak{x}_r(\rho\varrho_u - \mathfrak{x}_j\Omega^2 - 3\mathfrak{v}_h\varepsilon_g)](\boldsymbol{\Omega} \times \mathbf{r}) \\
&\quad + [\varkappa_s - \mathfrak{v}_h\varkappa_a + \varepsilon_b(\mathfrak{v}_l\varepsilon_b - \mathfrak{x}_j\varphi_i + \mathfrak{v}_a(3\rho\varepsilon_h + 2\mathfrak{x}_r)) + \rho(\varkappa_g + \varkappa_m + \rho\mathfrak{y}_t)](\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \\
&\quad + [\rho\varepsilon_b(\mathfrak{x}_r + \mathfrak{v}_a\varepsilon_b + 2\rho\varepsilon_h) + \varphi_i(\mathfrak{x}_r + \mathfrak{v}_a\varepsilon_b + 2\rho\varepsilon_h)](\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + [\varkappa_t + \rho\varrho_t\mathfrak{x}_u + \varepsilon_b(\mathfrak{x}_j\mathfrak{x}_u - \mathfrak{v}_a\mathfrak{x}_v) + 2\varepsilon_h(\mathfrak{v}_h\mathfrak{x}_u - \rho\mathfrak{x}_v)](\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - [\mathfrak{v}_a\Omega^2(\rho\varepsilon_b + \varphi_i) + 3\rho\varepsilon_g(\rho\varepsilon_b + \varphi_i)](\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + [\varphi_i(3\mathfrak{v}_h\varepsilon_g + \mathfrak{x}_j\Omega^2) - \Omega^2(\mathfrak{v}_l\varepsilon_b + \mathfrak{v}_a(3\rho\varepsilon_h + 2\mathfrak{x}_r)) - \rho(\rho\mathfrak{y}_v + 6\mathfrak{x}_r\varepsilon_g + \varrho_u\varphi_i)](\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + [\rho\varrho_u\mathfrak{x}_u + \Omega^2(\mathfrak{v}_a\mathfrak{x}_v - \mathfrak{x}_j\mathfrak{x}_u) + 3\varepsilon_g(\rho\mathfrak{x}_v - \mathfrak{v}_h\mathfrak{x}_u)](\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u(\rho\varepsilon_b + \varphi_i)(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + [\varkappa_u - \mathfrak{v}_h\mathfrak{x}_u\varepsilon_b + \rho(\mathfrak{x}_v\varepsilon_b - 3\varepsilon_h\mathfrak{x}_u)](\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + [\rho\varepsilon_b(\rho\varepsilon_b + \varphi_i) + \varphi_i(\rho\varepsilon_b + \varphi_i)](\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad + [\varkappa_x - \mathfrak{x}_j\mathfrak{y}_p + 3\mathfrak{v}_l\varphi_c + \mathfrak{v}_j\varepsilon_d + \mathfrak{v}_b(\rho\mathfrak{y}_b - 2\mathfrak{x}_o\varepsilon_e) + \mathfrak{v}_a(\mathfrak{y}_s + \rho\mathfrak{y}_j) \\
&\qquad + \rho(\varkappa_r + \rho\mathfrak{y}_w + 2\mathfrak{x}_o\varkappa_h) + \varphi_g\varphi_h(\mathfrak{v}_f\varepsilon_e - \mathfrak{v}_h\varkappa_d)]\boldsymbol{\Lambda} + \cdots
\end{aligned}$$

$$
\begin{aligned}
\cdots \quad & + [\mathfrak{v}_k\mathfrak{y}_a - \mathfrak{v}_j\delta_a + \mathfrak{v}_l\mathfrak{y}_h - \mathfrak{x}_j\mathfrak{y}_n + \mathfrak{v}_a\mathfrak{y}_q - \mathfrak{v}_h\varkappa_b + \rho(\varkappa_l + \varkappa_p + \rho\mathfrak{y}_z + \mathfrak{v}_b\mathfrak{y}_d + \mathfrak{v}_a\mathfrak{y}_l)]\mathbf{r} \\
& + [\varkappa_w - \mathfrak{x}_j\mathfrak{y}_o + \mathfrak{v}_a\mathfrak{y}_r + \mathfrak{v}_b\mathfrak{y}_f - \mathfrak{v}_f\mathfrak{y}_e - \mathfrak{v}_h\varkappa_e + \rho(\varkappa_i + \varkappa_o)]\widehat{\mathbf{p}} \\
& + [\varkappa_y - \mathfrak{v}_h\varkappa_f + \Omega^2(\mathfrak{v}_l\varphi_a^2 - \mathfrak{v}_k\varepsilon_d) + \mathfrak{x}_o(\mathfrak{v}_f\varepsilon_e - \mathfrak{x}_j\mathfrak{y}_m) \\
& \quad + \rho(\mathfrak{v}_a\mathfrak{y}_k - \mathfrak{v}_b\mathfrak{y}_c + \varkappa_j + \varkappa_q + \rho\mathfrak{y}_y) + \mathfrak{x}_p(\mathfrak{v}_a\mathfrak{y}_m - \mathfrak{v}_b\varepsilon_e)]\boldsymbol{\Omega} \\
& + [\mathfrak{v}_k\varepsilon_d - \mathfrak{v}_l\varphi_a^2 - \mathfrak{v}_h\varkappa_c + \varphi_g\varphi_h(\varkappa_v - \mathfrak{v}_b\varepsilon_e + \mathfrak{v}_a\mathfrak{y}_m) + \rho(\varkappa_k - \rho\mathfrak{y}_x)]\dot{\boldsymbol{\Lambda}} \\
& + \rho(\mathfrak{v}_b\varepsilon_d - \mathfrak{v}_a\varphi_a^2 - 3\rho\varphi_c + \varkappa_n)\ddot{\boldsymbol{\Lambda}}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}}) \\
&\quad \text{by (5.23s), (5.23t) \& (5.24a).}
\end{aligned}
\tag{5.47}
$$

Consequently, we derive

$$
\begin{aligned}
& \boldsymbol{\Lambda} \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= \boldsymbol{\Lambda} \cdot [\mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x(\boldsymbol{\Lambda} \cdot \boldsymbol{\Lambda}) + \mathfrak{v}_y(\boldsymbol{\Lambda} \cdot \mathbf{r}) + \mathfrak{v}_z(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}}) + \mathfrak{o}_a(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega}) + \mathfrak{o}_b(\boldsymbol{\Lambda} \cdot \dot{\boldsymbol{\Lambda}}) + \mathfrak{o}_c(\boldsymbol{\Lambda} \cdot \ddot{\boldsymbol{\Lambda}}) + \mathfrak{v}_m[\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] \\
&\quad + \mathfrak{v}_n[\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o[\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_b\mathfrak{v}_e[\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_p[\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q[\boldsymbol{\Lambda} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{v}_r[\boldsymbol{\Lambda} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{v}_s[\boldsymbol{\Lambda} \cdot (\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_t[\boldsymbol{\Lambda} \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] - \mathfrak{v}_e\mathfrak{v}_d[\boldsymbol{\Lambda} \cdot (\mathbf{r} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_u[\boldsymbol{\Lambda} \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_v[\boldsymbol{\Lambda} \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] - \mathfrak{x}_u\mathfrak{v}_e[\boldsymbol{\Lambda} \cdot (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[\boldsymbol{\Lambda} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_e^2[\boldsymbol{\Lambda} \cdot (\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\Lambda^2 + \mathfrak{v}_y\varepsilon_h + \mathfrak{v}_z\varepsilon_i + \mathfrak{o}_a\varepsilon_g + \mathfrak{o}_b\varsigma_d + \mathfrak{o}_c\varsigma_i - \mathfrak{v}_m\mathfrak{d}_e - \mathfrak{v}_n\varepsilon_n - \mathfrak{v}_b\mathfrak{v}_e\mathfrak{d}_u + \mathfrak{v}_p\delta_b - \mathfrak{v}_q\varepsilon_m \\
&\quad - \mathfrak{v}_s\mathfrak{d}_v - \mathfrak{v}_t\mathfrak{d}_o - \mathfrak{v}_e\mathfrak{v}_d\mathfrak{d}_h + \mathfrak{v}_v\varepsilon_o + \mathfrak{x}_u\mathfrak{v}_e\mathfrak{d}_r \text{ by (5.1a), (5.13a), (5.23a), (5.23b) \& (5.23c)} \\
&= \mathfrak{o}_d \text{ by (5.24a)}
\end{aligned}
\tag{5.48a}
$$

$$
\begin{aligned}
& \mathbf{r} \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= \mathbf{r} \cdot [\mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x(\mathbf{r} \cdot \boldsymbol{\Lambda}) + \mathfrak{v}_y(\mathbf{r} \cdot \mathbf{r}) + \mathfrak{v}_z(\mathbf{r} \cdot \widehat{\mathbf{p}}) + \mathfrak{o}_a(\mathbf{r} \cdot \boldsymbol{\Omega}) + \mathfrak{o}_b(\mathbf{r} \cdot \dot{\boldsymbol{\Lambda}}) + \mathfrak{o}_c(\mathbf{r} \cdot \ddot{\boldsymbol{\Lambda}}) + \mathfrak{v}_m[\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] \\
&\quad + \mathfrak{v}_n[\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o[\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_b\mathfrak{v}_e[\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_p[\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q[\mathbf{r} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{v}_r[\mathbf{r} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{v}_s[\mathbf{r} \cdot (\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_t[\mathbf{r} \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] - \mathfrak{v}_e\mathfrak{v}_d[\mathbf{r} \cdot (\mathbf{r} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_u[\mathbf{r} \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_v[\mathbf{r} \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] - \mathfrak{x}_u\mathfrak{v}_e[\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_e^2[\mathbf{r} \cdot (\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\varepsilon_h + \mathfrak{v}_y r^2 + \mathfrak{v}_z\varepsilon_f + \mathfrak{o}_a\varepsilon_b + \mathfrak{o}_b\varsigma_n + \mathfrak{o}_c\varsigma_o + \mathfrak{v}_m\varepsilon_k + \mathfrak{v}_o\varepsilon_n + \mathfrak{v}_b\mathfrak{v}_e\mathfrak{d}_c - \mathfrak{v}_p\mathfrak{d}_t + \mathfrak{v}_r\varepsilon_m \\
&\quad - \mathfrak{v}_s\mathfrak{d}_l - \mathfrak{v}_t\varepsilon_l - \mathfrak{x}_u\mathfrak{v}_e\mathfrak{d}_n + \mathfrak{v}_w\varepsilon_o - \mathfrak{v}_e^2\mathfrak{d}_h \text{ by (5.1a), (5.23a), (5.23b) \& (5.23c)} \\
&= \mathfrak{o}_e \text{ by (5.24a)}
\end{aligned}
\tag{5.48b}
$$

$$
\begin{aligned}
&\widehat{\mathbf{p}}\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=\widehat{\mathbf{p}}\cdot[\mathfrak{v}_x\boldsymbol{\Lambda}+\mathfrak{v}_y\mathbf{r}+\mathfrak{v}_z\widehat{\mathbf{p}}+\mathfrak{o}_a\boldsymbol{\Omega}+\mathfrak{o}_b\dot{\boldsymbol{\Lambda}}+\mathfrak{o}_c\ddot{\boldsymbol{\Lambda}}+\mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})+\mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})+\mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})\\
&\quad+\mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{v}_q(\boldsymbol{\Omega}\times\mathbf{r})+\mathfrak{v}_r(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})+\mathfrak{v}_s(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_t(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})\\
&\quad-\mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_u(\mathbf{r}\times\boldsymbol{\Lambda})+\mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}})-\mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_w(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})+\mathfrak{v}_e^2(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})]\text{ by (5.47)}\\
&=\mathfrak{v}_x(\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda})+\mathfrak{v}_y(\widehat{\mathbf{p}}\cdot\mathbf{r})+\mathfrak{v}_z(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{p}})+\mathfrak{o}_a(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})+\mathfrak{o}_b(\widehat{\mathbf{p}}\cdot\dot{\boldsymbol{\Lambda}})+\mathfrak{o}_c(\widehat{\mathbf{p}}\cdot\ddot{\boldsymbol{\Lambda}})+\mathfrak{v}_m[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]\\
&\quad+\mathfrak{v}_n[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]+\mathfrak{v}_o[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})]+\mathfrak{v}_b\mathfrak{v}_e[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_p[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{v}_q[\widehat{\mathbf{p}}\cdot(\boldsymbol{\Omega}\times\mathbf{r})]\\
&\quad+\mathfrak{v}_r[\widehat{\mathbf{p}}\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})]+\mathfrak{v}_s[\widehat{\mathbf{p}}\cdot(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_t[\widehat{\mathbf{p}}\cdot(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})]-\mathfrak{v}_e\mathfrak{v}_d[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_u[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})]\\
&\quad+\mathfrak{v}_v[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]-\mathfrak{x}_u\mathfrak{v}_e[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_w[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]+\mathfrak{v}_e^2[\widehat{\mathbf{p}}\cdot(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})]\\
&=\mathfrak{v}_x\varepsilon_i+\mathfrak{v}_y\varepsilon_f+\mathfrak{v}_z+\mathfrak{o}_a\varepsilon_c+\mathfrak{o}_b\varsigma_a+\mathfrak{o}_c\varsigma_f-\mathfrak{v}_m\varepsilon_j+\mathfrak{v}_n\mathfrak{d}_t-\mathfrak{v}_o\delta_b-\mathfrak{v}_b\mathfrak{v}_e\mathfrak{d}_a+\mathfrak{v}_q\varepsilon_l\\
&\quad+\mathfrak{v}_r\mathfrak{d}_o+\mathfrak{v}_s\mathfrak{d}_p+\mathfrak{v}_e\mathfrak{v}_d\mathfrak{d}_n-\mathfrak{v}_u\varepsilon_o+\mathfrak{v}_e^2\mathfrak{d}_r\text{ by (5.1a), (5.13a), (5.23a), (5.23b) \& (5.23c)}\\
&=\mathfrak{o}_f\text{ by (5.24b)}
\end{aligned}
\tag{5.48c}
$$

$$
\begin{aligned}
&\boldsymbol{\Omega}\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=\boldsymbol{\Omega}\cdot[\mathfrak{v}_x\boldsymbol{\Lambda}+\mathfrak{v}_y\mathbf{r}+\mathfrak{v}_z\widehat{\mathbf{p}}+\mathfrak{o}_a\boldsymbol{\Omega}+\mathfrak{o}_b\dot{\boldsymbol{\Lambda}}+\mathfrak{o}_c\ddot{\boldsymbol{\Lambda}}+\mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})+\mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})+\mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})\\
&\quad+\mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{v}_q(\boldsymbol{\Omega}\times\mathbf{r})+\mathfrak{v}_r(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})+\mathfrak{v}_s(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_t(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})\\
&\quad-\mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_u(\mathbf{r}\times\boldsymbol{\Lambda})+\mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}})-\mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_w(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})+\mathfrak{v}_e^2(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})]\text{ by (5.47)}\\
&=\mathfrak{v}_x(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})+\mathfrak{v}_y(\boldsymbol{\Omega}\cdot\mathbf{r})+\mathfrak{v}_z(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})+\mathfrak{o}_a(\boldsymbol{\Omega}\cdot\boldsymbol{\Omega})+\mathfrak{o}_b(\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})+\mathfrak{o}_c(\boldsymbol{\Omega}\cdot\ddot{\boldsymbol{\Lambda}})+\mathfrak{v}_m[\boldsymbol{\Omega}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]\\
&\quad+\mathfrak{v}_n[\boldsymbol{\Omega}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]+\mathfrak{v}_o[\boldsymbol{\Omega}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})]+\mathfrak{v}_b\mathfrak{v}_e[\boldsymbol{\Omega}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_p[\boldsymbol{\Omega}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{v}_q[\boldsymbol{\Omega}\cdot(\boldsymbol{\Omega}\times\mathbf{r})]\\
&\quad+\mathfrak{v}_r[\boldsymbol{\Omega}\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})]+\mathfrak{v}_s[\boldsymbol{\Omega}\cdot(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_t[\boldsymbol{\Omega}\cdot(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})]-\mathfrak{v}_e\mathfrak{v}_d[\boldsymbol{\Omega}\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_u[\boldsymbol{\Omega}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})]\\
&\quad+\mathfrak{v}_v[\boldsymbol{\Omega}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]-\mathfrak{x}_u\mathfrak{v}_e[\boldsymbol{\Omega}\cdot(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_w[\boldsymbol{\Omega}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]+\mathfrak{v}_e^2[\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})]\\
&=\mathfrak{v}_x\varepsilon_g+\mathfrak{v}_y\varepsilon_b+\mathfrak{v}_z\varepsilon_c+\mathfrak{o}_a\Omega^2+\mathfrak{o}_b\varsigma_c+\mathfrak{o}_c\varsigma_h-\mathfrak{v}_n\varepsilon_k+\mathfrak{v}_o\mathfrak{d}_e-\mathfrak{v}_b\mathfrak{v}_e\mathfrak{d}_f+\mathfrak{v}_p\varepsilon_j-\mathfrak{v}_e\mathfrak{v}_d\mathfrak{d}_l\\
&\quad-\mathfrak{v}_u\varepsilon_m+\mathfrak{v}_v\varepsilon_l+\mathfrak{x}_u\mathfrak{v}_e\mathfrak{d}_p-\mathfrak{v}_w\mathfrak{d}_o+\mathfrak{v}_e^2\mathfrak{d}_v\text{ by (5.1a), (5.23a), (5.23b) \& (5.23c)}\\
&=\mathfrak{o}_g\text{ by (5.24b)}
\end{aligned}
\tag{5.48d}
$$

$$
\begin{aligned}
&\dot{\boldsymbol{\Lambda}}\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=\dot{\boldsymbol{\Lambda}}\cdot[\mathfrak{v}_x\boldsymbol{\Lambda}+\mathfrak{v}_y\mathbf{r}+\mathfrak{v}_z\widehat{\mathbf{p}}+\mathfrak{o}_a\boldsymbol{\Omega}+\mathfrak{o}_b\dot{\boldsymbol{\Lambda}}+\mathfrak{o}_c\ddot{\boldsymbol{\Lambda}}+\mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})+\mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})+\mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})\\
&\quad+\mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{v}_q(\boldsymbol{\Omega}\times\mathbf{r})+\mathfrak{v}_r(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})+\mathfrak{v}_s(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_t(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})\\
&\quad-\mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_u(\mathbf{r}\times\boldsymbol{\Lambda})+\mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}})-\mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})+\mathfrak{v}_w(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})+\mathfrak{v}_e^2(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})]\text{ by (5.47)}\\
&=\mathfrak{v}_x(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Lambda})+\mathfrak{v}_y(\dot{\boldsymbol{\Lambda}}\cdot\mathbf{r})+\mathfrak{v}_z(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})+\mathfrak{o}_a(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Omega})+\mathfrak{o}_b(\dot{\boldsymbol{\Lambda}}\cdot\dot{\boldsymbol{\Lambda}})+\mathfrak{o}_c(\dot{\boldsymbol{\Lambda}}\cdot\ddot{\boldsymbol{\Lambda}})+\mathfrak{v}_m[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]\\
&\quad+\mathfrak{v}_n[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]+\mathfrak{v}_o[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})]+\mathfrak{v}_b\mathfrak{v}_e[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_p[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{v}_q[\dot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Omega}\times\mathbf{r})]\\
&\quad+\mathfrak{v}_r[\dot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})]+\mathfrak{v}_s[\dot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_t[\dot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})]-\mathfrak{v}_e\mathfrak{v}_d[\dot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_u[\dot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})]\\
&\quad+\mathfrak{v}_v[\dot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]-\mathfrak{x}_u\mathfrak{v}_e[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})]+\mathfrak{v}_w[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]+\mathfrak{v}_e^2[\dot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})]\\
&=\mathfrak{v}_x\varsigma_d+\mathfrak{v}_y\varsigma_n+\mathfrak{v}_z\varsigma_a+\mathfrak{o}_a\varsigma_c+\mathfrak{o}_b\varsigma_e+\mathfrak{o}_c\varsigma_j+\mathfrak{v}_m\mathfrak{d}_f-\mathfrak{v}_n\mathfrak{d}_c+\mathfrak{v}_o\mathfrak{d}_u+\mathfrak{v}_p\mathfrak{d}_a+\mathfrak{v}_q\mathfrak{d}_l\\
&\quad+\mathfrak{v}_r\mathfrak{d}_v-\mathfrak{v}_t\mathfrak{d}_p-\mathfrak{v}_u\mathfrak{d}_h+\mathfrak{v}_v\mathfrak{d}_n+\mathfrak{v}_w\mathfrak{d}_r\text{ by (5.23a), (5.23b) \& (5.23c)}\\
&=\mathfrak{o}_h\text{ by (5.24b)}
\end{aligned}
\tag{5.48e}
$$

$$
\begin{aligned}
&\ddot{\mathbf{\Lambda}} \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= \ddot{\mathbf{\Lambda}} \cdot [\mathfrak{v}_x \mathbf{\Lambda} + \mathfrak{v}_y \mathbf{r} + \mathfrak{v}_z \widehat{\mathbf{p}} + \mathfrak{o}_a \mathbf{\Omega} + \mathfrak{o}_b \dot{\mathbf{\Lambda}} + \mathfrak{o}_c \ddot{\mathbf{\Lambda}} + \mathfrak{v}_m (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q (\mathbf{\Omega} \times \mathbf{r}) + \mathfrak{v}_r (\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{v}_s (\mathbf{\Omega} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_t (\mathbf{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e \mathfrak{v}_d (\mathbf{r} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_u (\mathbf{r} \times \mathbf{\Lambda}) + \mathfrak{v}_v (\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u \mathfrak{v}_e (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_w (\widehat{\mathbf{p}} \times \mathbf{\Lambda}) + \mathfrak{v}_e^2 (\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})] \text{ by (5.47)} \\
&= \mathfrak{v}_x (\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda}) + \mathfrak{v}_y (\ddot{\mathbf{\Lambda}} \cdot \mathbf{r}) + \mathfrak{v}_z (\ddot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}}) + \mathfrak{o}_a (\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Omega}) + \mathfrak{o}_b (\ddot{\mathbf{\Lambda}} \cdot \dot{\mathbf{\Lambda}}) + \mathfrak{o}_c (\ddot{\mathbf{\Lambda}} \cdot \ddot{\mathbf{\Lambda}}) + \mathfrak{v}_m [\ddot{\mathbf{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] \\
&\quad + \mathfrak{v}_n [\ddot{\mathbf{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o [\ddot{\mathbf{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda})] + \mathfrak{v}_b \mathfrak{v}_e [\ddot{\mathbf{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_p [\ddot{\mathbf{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q [\ddot{\mathbf{\Lambda}} \cdot (\mathbf{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{v}_r [\ddot{\mathbf{\Lambda}} \cdot (\mathbf{\Omega} \times \mathbf{\Lambda})] + \mathfrak{v}_s [\ddot{\mathbf{\Lambda}} \cdot (\mathbf{\Omega} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_t [\ddot{\mathbf{\Lambda}} \cdot (\mathbf{\Omega} \times \widehat{\mathbf{p}})] - \mathfrak{v}_e \mathfrak{v}_d [\ddot{\mathbf{\Lambda}} \cdot (\mathbf{r} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_u [\ddot{\mathbf{\Lambda}} \cdot (\mathbf{r} \times \mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_v [\ddot{\mathbf{\Lambda}} \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] - \mathfrak{x}_u \mathfrak{v}_e [\ddot{\mathbf{\Lambda}} \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_w [\ddot{\mathbf{\Lambda}} \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] + \mathfrak{v}_e^2 [\ddot{\mathbf{\Lambda}} \cdot (\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})] \\
&\qquad = \mathfrak{v}_x \varsigma_i + \mathfrak{v}_y \varsigma_o + \mathfrak{v}_z \varsigma_f + \mathfrak{o}_a \varsigma_h + \mathfrak{o}_b \varsigma_j + \mathfrak{o}_c \varsigma_k + \mathfrak{v}_m \mathfrak{d}_w - \mathfrak{v}_n \mathfrak{d}_d + \mathfrak{v}_o \mathfrak{d}_x + \mathfrak{v}_b \mathfrak{v}_e \mathfrak{d}_y + \mathfrak{v}_p \mathfrak{d}_b \\
&\qquad\quad + \mathfrak{v}_q \mathfrak{d}_m + \mathfrak{v}_r \mathfrak{d}_z + \mathfrak{v}_s \mathfrak{p}_a - \mathfrak{v}_t \mathfrak{d}_q + \mathfrak{v}_e \mathfrak{v}_d \mathfrak{d}_j - \mathfrak{v}_u \mathfrak{d}_i + \mathfrak{v}_v \mathfrak{p}_b - \mathfrak{x}_u \mathfrak{v}_e \mathfrak{p}_c + \mathfrak{v}_w \mathfrak{d}_s + \mathfrak{v}_e^2 \mathfrak{p}_d \\
&\qquad\quad \text{by (5.23a), (5.23b) \& (5.23c)} \\
&\qquad = \mathfrak{o}_i \text{ by (5.24b)}
\end{aligned}
\tag{5.48f}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot [\mathfrak{v}_x \mathbf{\Lambda} + \mathfrak{v}_y \mathbf{r} + \mathfrak{v}_z \widehat{\mathbf{p}} + \mathfrak{o}_a \mathbf{\Omega} + \mathfrak{o}_b \dot{\mathbf{\Lambda}} + \mathfrak{o}_c \ddot{\mathbf{\Lambda}} + \mathfrak{v}_m (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q (\mathbf{\Omega} \times \mathbf{r}) + \mathfrak{v}_r (\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{v}_s (\mathbf{\Omega} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_t (\mathbf{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e \mathfrak{v}_d (\mathbf{r} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_u (\mathbf{r} \times \mathbf{\Lambda}) + \mathfrak{v}_v (\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u \mathfrak{v}_e (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_w (\widehat{\mathbf{p}} \times \mathbf{\Lambda}) + \mathfrak{v}_e^2 (\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})] \text{ by (5.47)} \\
&= \mathfrak{v}_x [\mathbf{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] + \mathfrak{v}_y [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] + \mathfrak{v}_z [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] + \mathfrak{o}_a [\mathbf{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] + \mathfrak{o}_b [\dot{\mathbf{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] \\
&\quad + \mathfrak{o}_c [\ddot{\mathbf{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] + \mathfrak{v}_m [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] + \mathfrak{v}_n [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_p [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\mathbf{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{v}_r [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\mathbf{\Omega} \times \mathbf{\Lambda})] + \mathfrak{v}_s [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\mathbf{\Omega} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_t [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\mathbf{\Omega} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e \mathfrak{v}_d [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\mathbf{r} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_u [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\mathbf{r} \times \mathbf{\Lambda})] + \mathfrak{v}_v [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_w [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \cdot (\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})] \\
&= -\mathfrak{v}_x \mathfrak{d}_e + \mathfrak{v}_y \varepsilon_k - \mathfrak{v}_z \varepsilon_j + \mathfrak{o}_b \mathfrak{d}_f + \mathfrak{o}_c \mathfrak{d}_w \\
&\quad + \mathfrak{v}_m [\Omega^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Omega})^2] + \mathfrak{v}_n [(\mathbf{\Omega} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{\Omega} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o [(\mathbf{\Omega} \cdot \mathbf{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Lambda})(\mathbf{\Omega} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b \mathfrak{v}_e [(\mathbf{\Omega} \cdot \dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{\Lambda}})(\mathbf{\Omega} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p [(\mathbf{\Omega} \cdot \widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Omega})] + \mathfrak{v}_q [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Omega})(\mathbf{\Omega} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})\Omega^2] \\
&\quad + \mathfrak{v}_r [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Omega})(\mathbf{\Omega} \cdot \mathbf{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Lambda})\Omega^2] + \mathfrak{v}_s [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Omega})(\mathbf{\Omega} \cdot \dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{\Lambda}})\Omega^2] + \mathfrak{v}_t [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Omega})(\mathbf{\Omega} \cdot \widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})\Omega^2] \\
&\quad - \mathfrak{v}_e \mathfrak{v}_d [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{\Omega} \cdot \dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{\Lambda}})(\mathbf{r} \cdot \mathbf{\Omega})] + \mathfrak{v}_u [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{\Omega} \cdot \mathbf{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Lambda})(\mathbf{\Omega} \cdot \mathbf{r})] \\
&\quad + \mathfrak{v}_v [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{\Omega} \cdot \widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\mathbf{\Omega} \cdot \mathbf{r})] - \mathfrak{x}_u \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\mathbf{\Omega} \cdot \dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{\Lambda}})(\mathbf{\Omega} \cdot \widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_w [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\mathbf{\Omega} \cdot \mathbf{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Lambda})(\mathbf{\Omega} \cdot \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{\Lambda})(\mathbf{\Omega} \cdot \dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\mathbf{\Lambda}})(\mathbf{\Omega} \cdot \mathbf{\Lambda})] \\
&\quad \text{by (5.1a), (5.23b), (5.23c) \& (A.2)} \\
&\qquad = -\mathfrak{v}_x \mathfrak{d}_e + \mathfrak{v}_y \varepsilon_k - \mathfrak{v}_z \varepsilon_j + \mathfrak{o}_b \mathfrak{d}_f + \mathfrak{o}_c \mathfrak{d}_w + \mathfrak{v}_m (\Omega^2 - \varepsilon_a^2) + \mathfrak{v}_n (\varepsilon_b - \varepsilon_d \varepsilon_a) + \mathfrak{v}_o (\varepsilon_g - \delta_a \varepsilon_a) \\
&\qquad\quad + \mathfrak{v}_b \mathfrak{v}_e (\varsigma_c - \varsigma_b \varepsilon_a) + \mathfrak{v}_p (\varepsilon_c - \varepsilon_e \varepsilon_a) + \mathfrak{v}_q (\varepsilon_a \varepsilon_b - \varepsilon_d \Omega^2) + \mathfrak{v}_r (\varepsilon_a \varepsilon_g - \delta_a \Omega^2) \\
&\qquad\quad + \mathfrak{v}_s (\varepsilon_a \varsigma_c - \varsigma_b \Omega^2) + \mathfrak{v}_t (\varepsilon_a \varepsilon_c - \varepsilon_e \Omega^2) - \mathfrak{v}_e \mathfrak{v}_d (\varepsilon_d \varsigma_c - \varsigma_b \varepsilon_b) + \mathfrak{v}_u (\varepsilon_d \varepsilon_g - \delta_a \varepsilon_b) \\
&\qquad\quad + \mathfrak{v}_v (\varepsilon_d \varepsilon_c - \varepsilon_e \varepsilon_b) - \mathfrak{x}_u \mathfrak{v}_e (\varepsilon_e \varsigma_c - \varsigma_b \varepsilon_c) + \mathfrak{v}_w (\varepsilon_e \varepsilon_g - \delta_a \varepsilon_c) + \mathfrak{v}_e^2 (\delta_a \varsigma_c - \varsigma_b \varepsilon_g) \\
&\qquad\quad \text{by (5.1a), (5.13a) \& (5.23a)} \\
&\qquad = \mathfrak{o}_j \text{ by (5.24c)}
\end{aligned}
\tag{5.48g}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot [\mathfrak{v}_x \boldsymbol{\Lambda} + \mathfrak{v}_y \mathbf{r} + \mathfrak{v}_z \widehat{\mathbf{p}} + \mathfrak{o}_a \boldsymbol{\Omega} + \mathfrak{o}_b \dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c \ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q (\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s (\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t (\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e \mathfrak{v}_d (\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u (\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v (\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u \mathfrak{v}_e (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2 (\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x [\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_y [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_z [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{o}_a [\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{o}_b [\dot{\boldsymbol{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{o}_c [\ddot{\boldsymbol{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_m [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] + \mathfrak{v}_n [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_b \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_p [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{v}_r [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{v}_s [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_t [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e \mathfrak{v}_d [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\mathbf{r} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_u [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] + \mathfrak{v}_v [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x \varepsilon_n + \mathfrak{v}_z \mathfrak{d}_t - \mathfrak{o}_a \varepsilon_k - \mathfrak{o}_b \mathfrak{d}_c - \mathfrak{o}_c \mathfrak{d}_d + \mathfrak{v}_m [(\mathbf{r} \cdot \boldsymbol{\Omega}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})] + \mathfrak{v}_n [r^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})^2] \\
&\quad + \mathfrak{v}_o [(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_b \mathfrak{v}_e [(\mathbf{r} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})] \\
&\quad + \mathfrak{v}_p [(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})] + \mathfrak{v}_q [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) r^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad + \mathfrak{v}_r [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \boldsymbol{\Omega})] + \mathfrak{v}_s [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad + \mathfrak{v}_t [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Omega})] - \mathfrak{v}_e \mathfrak{v}_d [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{r} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}}) r^2] \\
&\quad + \mathfrak{v}_u [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda}) r^2] + \mathfrak{v}_v [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}}) r^2] \\
&\quad - \mathfrak{x}_u \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})(\mathbf{r} \cdot \widehat{\mathbf{p}})] + \mathfrak{v}_w [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})(\mathbf{r} \cdot \boldsymbol{\Lambda})] \text{ by (5.1a), (5.23b) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x \varepsilon_n + \mathfrak{v}_z \mathfrak{d}_t - \mathfrak{o}_a \varepsilon_k - \mathfrak{o}_b \mathfrak{d}_c - \mathfrak{o}_c \mathfrak{d}_d + \mathfrak{v}_m (\varepsilon_b - \varepsilon_a \varepsilon_d) + \mathfrak{v}_n (r^2 - \varepsilon_d^2) \\
&\quad + \mathfrak{v}_o (\varepsilon_h - \delta_a \varepsilon_d) + \mathfrak{v}_b \mathfrak{v}_e (\varsigma_n - \varsigma_b \varepsilon_d) + \mathfrak{v}_p (\varepsilon_f - \varepsilon_e \varepsilon_d) + \mathfrak{v}_q (\varepsilon_a r^2 - \varepsilon_d \varepsilon_b) \\
&\quad + \mathfrak{v}_r (\varepsilon_a \varepsilon_h - \delta_a \varepsilon_b) + \mathfrak{v}_s (\varepsilon_a \varsigma_n - \varsigma_b \varepsilon_b) + \mathfrak{v}_t (\varepsilon_a \varepsilon_f - \varepsilon_e \varepsilon_b) - \mathfrak{v}_e \mathfrak{v}_d (\varepsilon_d \varsigma_n - \varsigma_b r^2) \\
&\quad + \mathfrak{v}_u (\varepsilon_d \varepsilon_h - \delta_a r^2) + \mathfrak{v}_v (\varepsilon_d \varepsilon_f - \varepsilon_e r^2) - \mathfrak{x}_u \mathfrak{v}_e (\varepsilon_e \varsigma_n - \varsigma_b \varepsilon_f) + \mathfrak{v}_w (\varepsilon_e \varepsilon_h - \delta_a \varepsilon_f) \\
&\quad + \mathfrak{v}_e^2 (\delta_a \varsigma_n - \varsigma_b \varepsilon_h) \text{ by (5.1a), (5.13a) \& (5.23a)} \\
&= \mathfrak{o}_k \text{ by (5.24c)}
\end{aligned}
\tag{5.48h}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot [\mathfrak{v}_x \boldsymbol{\Lambda} + \mathfrak{v}_y \mathbf{r} + \mathfrak{v}_z \widehat{\mathbf{p}} + \mathfrak{o}_a \boldsymbol{\Omega} + \mathfrak{o}_b \dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c \ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q (\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s (\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t (\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e \mathfrak{v}_d (\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u (\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v (\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u \mathfrak{v}_e (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2 (\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x [\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_y [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_z [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{o}_a [\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{o}_b [\dot{\boldsymbol{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{o}_c [\ddot{\boldsymbol{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_m [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] + \mathfrak{v}_n [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_b \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_p [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{v}_r [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{v}_s [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_t [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e \mathfrak{v}_d [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\mathbf{r} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_u [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] + \mathfrak{v}_v [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_y\varepsilon_n - \mathfrak{v}_z\delta_b + \mathfrak{o}_a\mathfrak{d}_e + \mathfrak{o}_b\mathfrak{d}_u + \mathfrak{o}_c\mathfrak{d}_x + \mathfrak{v}_m[(\mathbf{\Lambda}\cdot\mathbf{\Omega}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\mathbf{\Lambda}\cdot\mathbf{r}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[\Lambda^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})^2] + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_p[(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_q[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\mathbf{r}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_r[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})\Lambda^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] + \mathfrak{v}_s[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_t[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] - \mathfrak{v}_e\mathfrak{v}_d[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\mathbf{r})] \\
&\quad + \mathfrak{v}_u[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})\Lambda^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\mathbf{r})] + \mathfrak{v}_v[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\mathbf{r})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_w[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})\Lambda^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_e^2[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})\Lambda^2] \text{ by (5.1a), (5.13a), (5.23b), (5.23c) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_y\varepsilon_n - \mathfrak{v}_z\delta_b + \mathfrak{o}_a\mathfrak{d}_e + \mathfrak{o}_b\mathfrak{d}_u + \mathfrak{o}_c\mathfrak{d}_x + \mathfrak{v}_m(\varepsilon_g - \varepsilon_a\delta_a) + \mathfrak{v}_n(\varepsilon_h - \varepsilon_d\delta_a) \\
&\quad + \mathfrak{v}_o(\Lambda^2 - \delta_a^2) + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_d - \varsigma_b\delta_a) + \mathfrak{v}_p(\varepsilon_i - \varepsilon_e\delta_a) + \mathfrak{v}_q(\varepsilon_a\varepsilon_h - \varepsilon_d\varepsilon_g) \\
&\quad + \mathfrak{v}_r(\varepsilon_a\Lambda^2 - \delta_a\varepsilon_g) + \mathfrak{v}_s(\varepsilon_a\varsigma_d - \varsigma_b\varepsilon_g) + \mathfrak{v}_t(\varepsilon_a\varepsilon_i - \varepsilon_e\varepsilon_g) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_d\varsigma_d - \varsigma_b\varepsilon_h) \\
&\quad + \mathfrak{v}_u(\varepsilon_d\Lambda^2 - \delta_a\varepsilon_h) + \mathfrak{v}_v(\varepsilon_d\varepsilon_i - \varepsilon_e\varepsilon_h) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_e\varsigma_d - \varsigma_b\varepsilon_i) + \mathfrak{v}_w(\varepsilon_e\Lambda^2 - \delta_a\varepsilon_i) \\
&\quad + \mathfrak{v}_e^2(\delta_a\varsigma_d - \varsigma_b\Lambda^2) \text{ by (5.1a), (5.13a) \& (5.23a)} \\
&= \mathfrak{o}_l \text{ by (5.24d)} \qquad \text{(5.48i)}
\end{aligned}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot[\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega}\times\mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_y[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{o}_a[\mathbf{\Omega}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] \\
&\quad + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_m[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})] + \mathfrak{v}_n[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_s[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{r}\times\mathbf{\Lambda})] + \mathfrak{v}_v[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\mathfrak{d}_u + \mathfrak{v}_y\mathfrak{d}_c - \mathfrak{v}_z\mathfrak{d}_a - \mathfrak{o}_a\mathfrak{d}_f + \mathfrak{o}_c\mathfrak{d}_y + \mathfrak{v}_m[(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_b\mathfrak{v}_e[(\dot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})^2] \\
&\quad + \mathfrak{v}_p[(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_q[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_r[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] + \mathfrak{v}_s[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})\dot{\mathbf{\Lambda}}^2 - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_t[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] - \mathfrak{v}_e\mathfrak{v}_d[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})\dot{\mathbf{\Lambda}}^2 - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r})] \\
&\quad + \mathfrak{v}_u[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r})] + \mathfrak{v}_v[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})\dot{\mathbf{\Lambda}}^2 - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_w[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_e^2[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda})\dot{\mathbf{\Lambda}}^2 - (\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})] \text{ by (5.23b), (5.23c) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\mathfrak{d}_u + \mathfrak{v}_y\mathfrak{d}_c - \mathfrak{v}_z\mathfrak{d}_a - \mathfrak{o}_a\mathfrak{d}_f + \mathfrak{o}_c\mathfrak{d}_y + \mathfrak{v}_m(\varsigma_c - \varepsilon_a\varsigma_b) + \mathfrak{v}_n(\varsigma_n - \varepsilon_d\varsigma_b) \\
&\quad + \mathfrak{v}_o(\varsigma_d - \delta_a\varsigma_b) + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_e - \varsigma_b^2) + \mathfrak{v}_p(\varsigma_a - \varepsilon_e\varsigma_b) + \mathfrak{v}_q(\varepsilon_a\varsigma_n - \varepsilon_d\varsigma_c) \\
&\quad + \mathfrak{v}_r(\varepsilon_a\varsigma_d - \delta_a\varsigma_c) + \mathfrak{v}_s(\varepsilon_a\varsigma_e - \varsigma_b\varsigma_c) + \mathfrak{v}_t(\varepsilon_a\varsigma_a - \varepsilon_e\varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_d\varsigma_e - \varsigma_b\varsigma_n) \\
&\quad + \mathfrak{v}_u(\varepsilon_d\varsigma_d - \delta_a\varsigma_n) + \mathfrak{v}_v(\varepsilon_d\varsigma_a - \varepsilon_e\varsigma_n) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_e\varsigma_e - \varsigma_b\varsigma_a) + \mathfrak{v}_w(\varepsilon_e\varsigma_d - \delta_a\varsigma_a) \\
&\quad + \mathfrak{v}_e^2(\delta_a\varsigma_e - \varsigma_b\varsigma_d) \text{ by (5.1a), (5.13a) \& (5.23a)} \\
&= \mathfrak{o}_m \text{ by (5.24d)} \qquad (5.48\text{j})
\end{aligned}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot [\mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_y[\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_z[\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{o}_a[\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{o}_b[\dot{\boldsymbol{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] \\
&\quad + \mathfrak{o}_c[\ddot{\boldsymbol{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_m[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] + \mathfrak{v}_n[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_p[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{v}_s[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_t[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\mathbf{r} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_u[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] + \mathfrak{v}_v[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_e^2[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\delta_b - \mathfrak{v}_y\mathfrak{d}_t + \mathfrak{o}_a\varepsilon_j + \mathfrak{o}_b\mathfrak{d}_a + \mathfrak{o}_c\mathfrak{d}_b + \mathfrak{v}_m[(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] + \mathfrak{v}_b\mathfrak{v}_e[(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_p[1 - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})^2] + \mathfrak{v}_q[(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] + \mathfrak{v}_r[(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] \\
&\quad + \mathfrak{v}_s[(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] + \mathfrak{v}_t[(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}} \cdot \mathbf{r})] + \mathfrak{v}_u[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \mathbf{r})] \\
&\quad + \mathfrak{v}_v[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \mathbf{r})] - \mathfrak{x}_u\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_e^2[(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda})] \text{ by (5.1a), (5.13a), (5.23b), (5.23c) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\delta_b - \mathfrak{v}_y\mathfrak{d}_t + \mathfrak{o}_a\varepsilon_j + \mathfrak{o}_b\mathfrak{d}_a + \mathfrak{o}_c\mathfrak{d}_b + \mathfrak{v}_m(\varepsilon_c - \varepsilon_a\varepsilon_e) + \mathfrak{v}_n(\varepsilon_f - \varepsilon_d\varepsilon_e) + \mathfrak{v}_o(\varepsilon_i - \delta_a\varepsilon_e) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_a - \varsigma_b\varepsilon_e) + \mathfrak{v}_p(1 - \varepsilon_e^2) + \mathfrak{v}_q(\varepsilon_a\varepsilon_f - \varepsilon_d\varepsilon_c) + \mathfrak{v}_r(\varepsilon_a\varepsilon_i - \delta_a\varepsilon_c) + \mathfrak{v}_s(\varepsilon_a\varsigma_a - \varsigma_b\varepsilon_c) \\
&\quad + \mathfrak{v}_t(\varepsilon_a - \varepsilon_e\varepsilon_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_d\varsigma_a - \varsigma_b\varepsilon_f) + \mathfrak{v}_u(\varepsilon_d\varepsilon_i - \delta_a\varepsilon_f) + \mathfrak{v}_v(\varepsilon_d - \varepsilon_e\varepsilon_f) \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_e\varsigma_a - \varsigma_b) + \mathfrak{v}_w(\varepsilon_e\varepsilon_i - \delta_a) + \mathfrak{v}_e^2(\delta_a\varsigma_a - \varsigma_b\varepsilon_i) \text{ by (5.1a), (5.13a) \& (5.23a)} \\
&= \mathfrak{o}_n \text{ by (5.24e)} \qquad (5.48\text{k})
\end{aligned}
$$

$$
\begin{aligned}
&(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\boldsymbol{\Omega} \times \mathbf{r}) \cdot [\mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda}\cdot(\mathbf{\Omega}\times\mathbf{r})] + \mathfrak{v}_y[\mathbf{r}\cdot(\mathbf{\Omega}\times\mathbf{r})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\mathbf{r})] + \mathfrak{o}_a[\mathbf{\Omega}\cdot(\mathbf{\Omega}\times\mathbf{r})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\mathbf{r})] + \mathfrak{v}_m[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})] + \mathfrak{v}_n[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_s[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{r}\times\mathbf{\Lambda})] + \mathfrak{v}_v[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\varepsilon_m + \mathfrak{v}_z\varepsilon_l + \mathfrak{o}_b\mathfrak{d}_l + \mathfrak{o}_c\mathfrak{d}_m + \mathfrak{v}_m[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\mathbf{\Omega}) - \Omega^2(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_n[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})r^2 - (\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_o[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})] \\
&\quad + \mathfrak{v}_p[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_q[\Omega^2 r^2 - (\mathbf{\Omega}\cdot\mathbf{r})^2] \\
&\quad + \mathfrak{v}_r[\Omega^2(\mathbf{r}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\mathbf{\Omega})] + \mathfrak{v}_s[\Omega^2(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\mathbf{r}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_t[\Omega^2(\mathbf{r}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Omega})] - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})r^2] \\
&\quad + \mathfrak{v}_u[(\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{\Lambda})r^2] + \mathfrak{v}_v[(\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{r}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Omega}\cdot\widehat{\mathbf{p}})r^2] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\mathbf{r}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_w[(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_e^2[(\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\mathbf{r}\cdot\mathbf{\Lambda})] \text{ by (5.1a), (5.23b) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\varepsilon_m + \mathfrak{v}_z\varepsilon_l + \mathfrak{o}_b\mathfrak{d}_l + \mathfrak{o}_c\mathfrak{d}_m + \mathfrak{v}_m(\varepsilon_a\varepsilon_b - \Omega^2\varepsilon_d) + \mathfrak{v}_n(\varepsilon_a r^2 - \varepsilon_b\varepsilon_d) \\
&\quad + \mathfrak{v}_o(\varepsilon_a\varepsilon_h - \varepsilon_g\varepsilon_d) + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_a\varsigma_n - \varsigma_c\varepsilon_d) + \mathfrak{v}_p(\varepsilon_a\varepsilon_f - \varepsilon_c\varepsilon_d) + \mathfrak{v}_q(\Omega^2 r^2 - \varepsilon_b^2) \\
&\quad + \mathfrak{v}_r(\Omega^2\varepsilon_h - \varepsilon_g\varepsilon_b) + \mathfrak{v}_s(\Omega^2\varsigma_n - \varsigma_c\varepsilon_b) + \mathfrak{v}_t(\Omega^2\varepsilon_f - \varepsilon_c\varepsilon_b) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_b\varsigma_n - \varsigma_c r^2) \\
&\quad + \mathfrak{v}_u(\varepsilon_b\varepsilon_h - \varepsilon_g r^2) + \mathfrak{v}_v(\varepsilon_b\varepsilon_f - \varepsilon_c r^2) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_c\varsigma_n - \varsigma_c\varepsilon_f) + \mathfrak{v}_w(\varepsilon_c\varepsilon_h - \varepsilon_g\varepsilon_f) \\
&\quad + \mathfrak{v}_e^2(\varepsilon_g\varsigma_n - \varsigma_c\varepsilon_h) \text{ by (5.1a) \& (5.23a)} \\
&= \mathfrak{o}_o \text{ by (5.24e)}
\end{aligned}
\tag{5.48l}
$$

$$
\begin{aligned}
&(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\mathbf{\Omega}\times\mathbf{\Lambda})\cdot[\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega}\times\mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda}\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_y[\mathbf{r}\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{o}_a[\mathbf{\Omega}\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_m[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})] + \mathfrak{v}_n[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_s[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\mathbf{r}\times\mathbf{\Lambda})] + \mathfrak{v}_v[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\mathbf{\Omega}\times\mathbf{\Lambda})\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_y\varepsilon_m + \mathfrak{v}_z\mathfrak{d}_o + \mathfrak{o}_b\mathfrak{d}_v + \mathfrak{o}_c\mathfrak{d}_z + \mathfrak{v}_m[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\mathbf{\Omega}) - \Omega^2(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\mathbf{r}) - (\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})\Lambda^2 - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[\Omega^2(\mathbf{\Lambda}\cdot\mathbf{r}) - (\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] + \mathfrak{v}_r[\Omega^2\Lambda^2 - (\mathbf{\Omega}\cdot\mathbf{\Lambda})^2] + \mathfrak{v}_s[\Omega^2(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_t[\Omega^2(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\mathbf{r})] \\
&\quad + \mathfrak{v}_u[(\mathbf{\Omega}\cdot\mathbf{r})\Lambda^2 - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\mathbf{r})] + \mathfrak{v}_v[(\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\mathbf{r})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_w[(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})\Lambda^2 - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_e^2[(\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})\Lambda^2] \text{ by (5.1a), (5.23b), (5.23c) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_y\varepsilon_m + \mathfrak{v}_z\mathfrak{d}_o + \mathfrak{o}_b\mathfrak{d}_v + \mathfrak{o}_c\mathfrak{d}_z + \mathfrak{v}_m(\varepsilon_a\varepsilon_g - \Omega^2\delta_a) + \mathfrak{v}_n(\varepsilon_a\varepsilon_h - \varepsilon_b\delta_a) \\
&\quad + \mathfrak{v}_o(\varepsilon_a\Lambda^2 - \varepsilon_g\delta_a) + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_a\varsigma_d - \varsigma_c\delta_a) + \mathfrak{v}_p(\varepsilon_a\varepsilon_i - \varepsilon_c\delta_a) + \mathfrak{v}_q(\Omega^2\varepsilon_h - \varepsilon_b\varepsilon_g) \\
&\quad + \mathfrak{v}_r(\Omega^2\Lambda^2 - \varepsilon_g^2) + \mathfrak{v}_s(\Omega^2\varsigma_d - \varsigma_c\varepsilon_g) + \mathfrak{v}_t(\Omega^2\varepsilon_i - \varepsilon_c\varepsilon_g) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_b\varsigma_d - \varsigma_c\varepsilon_h) \\
&\quad + \mathfrak{v}_u(\varepsilon_b\Lambda^2 - \varepsilon_g\varepsilon_h) + \mathfrak{v}_v(\varepsilon_b\varepsilon_i - \varepsilon_c\varepsilon_h) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_c\varsigma_d - \varsigma_c\varepsilon_i) + \mathfrak{v}_w(\varepsilon_c\Lambda^2 - \varepsilon_g\varepsilon_i) \\
&\quad + \mathfrak{v}_e^2(\varepsilon_g\varsigma_d - \varsigma_c\Lambda^2) \text{ by (5.1a), (5.13a) \& (5.23a)} \\
&= \mathfrak{o}_p \text{ by (5.24f)}
\end{aligned} \tag{5.48m}
$$

$$
\begin{aligned}
&(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot[\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega}\times\mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda}\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_y[\mathbf{r}\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{o}_a[\mathbf{\Omega}\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] \\
&\quad + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_m[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})] + \mathfrak{v}_n[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_s[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{r}\times\mathbf{\Lambda})] + \mathfrak{v}_v[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\mathfrak{d}_v - \mathfrak{v}_y\mathfrak{d}_l + \mathfrak{v}_z\mathfrak{d}_p + \mathfrak{o}_c\mathfrak{p}_a + \mathfrak{v}_m[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega}) - \Omega^2(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - (\mathbf{\Omega}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\mathbf{\Omega}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[\Omega^2(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - (\mathbf{\Omega}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] + \mathfrak{v}_r[\Omega^2(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] + \mathfrak{v}_s[\Omega^2\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})^2] \\
&\quad + \mathfrak{v}_t[\Omega^2(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{\Omega}\cdot\mathbf{r})\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r})] \\
&\quad + \mathfrak{v}_u[(\mathbf{\Omega}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r})] + \mathfrak{v}_v[(\mathbf{\Omega}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_w[(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_e^2[(\mathbf{\Omega}\cdot\mathbf{\Lambda})\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})] \text{ by (5.23b), (5.23c) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\mathfrak{d}_v - \mathfrak{v}_y\mathfrak{d}_l + \mathfrak{v}_z\mathfrak{d}_p + \mathfrak{o}_c\mathfrak{p}_a + \mathfrak{v}_m(\varepsilon_a\varsigma_c - \Omega^2\varsigma_b) + \mathfrak{v}_n(\varepsilon_a\varsigma_n - \varepsilon_b\varsigma_b) + \mathfrak{v}_o(\varepsilon_a\varsigma_d - \varepsilon_g\varsigma_b) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_a\varsigma_e - \varsigma_c\varsigma_b) + \mathfrak{v}_p(\varepsilon_a\varsigma_a - \varepsilon_c\varsigma_b) + \mathfrak{v}_q(\Omega^2\varsigma_n - \varepsilon_b\varsigma_c) + \mathfrak{v}_r(\Omega^2\varsigma_d - \varepsilon_g\varsigma_c) \\
&\quad + \mathfrak{v}_s(\Omega^2\varsigma_e - \varsigma_c^2) + \mathfrak{v}_t(\Omega^2\varsigma_a - \varepsilon_c\varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_b\varsigma_e - \varsigma_c\varsigma_n) + \mathfrak{v}_u(\varepsilon_b\varsigma_d - \varepsilon_g\varsigma_n) + \mathfrak{v}_v(\varepsilon_b\varsigma_a - \varepsilon_c\varsigma_n) \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_c\varsigma_e - \varsigma_c\varsigma_a) + \mathfrak{v}_w(\varepsilon_c\varsigma_d - \varepsilon_g\varsigma_a) + \mathfrak{v}_e^2(\varepsilon_g\varsigma_e - \varsigma_c\varsigma_d) \text{ by (5.1a) \& (5.23a)} \\
&= \mathfrak{o}_q \text{ by (5.24g)} \qquad (5.48\text{n})
\end{aligned}
$$

$$
\begin{aligned}
&(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= (\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot [\mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\boldsymbol{\Lambda} \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] + \mathfrak{v}_y[\mathbf{r} \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] + \mathfrak{v}_z[\widehat{\mathbf{p}} \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] + \mathfrak{o}_a[\boldsymbol{\Omega} \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] + \mathfrak{o}_b[\dot{\boldsymbol{\Lambda}} \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] \\
&\quad + \mathfrak{o}_c[\ddot{\boldsymbol{\Lambda}} \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] + \mathfrak{v}_m[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] + \mathfrak{v}_n[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_p[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{v}_s[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_t[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\mathbf{r} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_u[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] + \mathfrak{v}_v[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \mathfrak{v}_e^2[(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\mathfrak{d}_o - \mathfrak{v}_y\varepsilon_l - \mathfrak{o}_b\mathfrak{d}_p - \mathfrak{o}_c\mathfrak{d}_q + \mathfrak{v}_m[(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega}) - \Omega^2(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\boldsymbol{\Omega} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\boldsymbol{\Omega} \cdot \dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}}) - (\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[\Omega^2(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\boldsymbol{\Omega} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] + \mathfrak{v}_r[\Omega^2(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] \\
&\quad + \mathfrak{v}_s[\Omega^2(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\boldsymbol{\Omega} \cdot \dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] + \mathfrak{v}_t[\Omega^2 - (\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})^2] - \mathfrak{v}_e\mathfrak{v}_d[(\boldsymbol{\Omega} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\boldsymbol{\Omega} \cdot \dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}} \cdot \mathbf{r})] \\
&\quad + \mathfrak{v}_u[(\boldsymbol{\Omega} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \mathbf{r})] + \mathfrak{v}_v[(\boldsymbol{\Omega} \cdot \mathbf{r}) - (\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \mathbf{r})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\boldsymbol{\Omega} \cdot \dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_e^2[(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) - (\boldsymbol{\Omega} \cdot \dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda})] \text{ by (5.1a), (5.23b) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\mathfrak{d}_o - \mathfrak{v}_y\varepsilon_l - \mathfrak{o}_b\mathfrak{d}_p - \mathfrak{o}_c\mathfrak{d}_q + \mathfrak{v}_m(\varepsilon_a\varepsilon_c - \Omega^2\varepsilon_e) + \mathfrak{v}_n(\varepsilon_a\varepsilon_f - \varepsilon_b\varepsilon_e) + \mathfrak{v}_o(\varepsilon_a\varepsilon_i - \varepsilon_g\varepsilon_e) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_a\varsigma_a - \varsigma_c\varepsilon_e) + \mathfrak{v}_p(\varepsilon_a - \varepsilon_c\varepsilon_e) + \mathfrak{v}_q(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c) + \mathfrak{v}_r(\Omega^2\varepsilon_i - \varepsilon_g\varepsilon_c) \\
&\quad + \mathfrak{v}_s(\Omega^2\varsigma_a - \varsigma_c\varepsilon_c) + \mathfrak{v}_t(\Omega^2 - \varepsilon_c^2) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_b\varsigma_a - \varsigma_c\varepsilon_f) + \mathfrak{v}_u(\varepsilon_b\varepsilon_i - \varepsilon_g\varepsilon_f) + \mathfrak{v}_v(\varepsilon_b - \varepsilon_c\varepsilon_f) \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_c\varsigma_a - \varsigma_c) + \mathfrak{v}_w(\varepsilon_c\varepsilon_i - \varepsilon_g) + \mathfrak{v}_e^2(\varepsilon_g\varsigma_a - \varsigma_c\varepsilon_i) \text{ by (5.1a) \& (5.23a)} \\
&= \mathfrak{o}_r \text{ by (5.24h)} \qquad (5.48\text{o})
\end{aligned}
$$

$$
\begin{aligned}
&(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= (\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) \cdot [\mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\boldsymbol{\Lambda}\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_y[\mathbf{r}\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{o}_a[\boldsymbol{\Omega}\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{o}_b[\dot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})] \\
&\quad + \mathfrak{o}_c[\ddot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_m[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})] + \mathfrak{v}_n[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_p[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\boldsymbol{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})] + \mathfrak{v}_s[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_t[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_u[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\mathbf{r}\times\boldsymbol{\Lambda})] + \mathfrak{v}_v[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] + \mathfrak{v}_e^2[(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})\cdot(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\mathfrak{d}_h - \mathfrak{v}_z\mathfrak{d}_n + \mathfrak{o}_a\mathfrak{d}_l - \mathfrak{o}_c\mathfrak{d}_j + \mathfrak{v}_m[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Omega}) - (\mathbf{r}\cdot\boldsymbol{\Omega})(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\boldsymbol{\Lambda}}\cdot\mathbf{r}) - r^2(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Lambda}) - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})\dot{\boldsymbol{\Lambda}}^2 - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[(\mathbf{r}\cdot\boldsymbol{\Omega})(\dot{\boldsymbol{\Lambda}}\cdot\mathbf{r}) - r^2(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Omega})] + \mathfrak{v}_r[(\mathbf{r}\cdot\boldsymbol{\Omega})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Lambda}) - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Omega})] \\
&\quad + \mathfrak{v}_s[(\mathbf{r}\cdot\boldsymbol{\Omega})\dot{\boldsymbol{\Lambda}}^2 - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Omega})] + \mathfrak{v}_t[(\mathbf{r}\cdot\boldsymbol{\Omega})(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Omega})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[r^2\dot{\boldsymbol{\Lambda}}^2 - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})^2] + \mathfrak{v}_u[r^2(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Lambda}) - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\dot{\boldsymbol{\Lambda}}\cdot\mathbf{r})] \\
&\quad + \mathfrak{v}_v[r^2(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\mathbf{r})] - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{r}\cdot\widehat{\mathbf{p}})\dot{\boldsymbol{\Lambda}}^2 - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_w[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Lambda}) - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_e^2[(\mathbf{r}\cdot\boldsymbol{\Lambda})\dot{\boldsymbol{\Lambda}}^2 - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Lambda})] \\
&\quad \text{by (5.23b) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\mathfrak{d}_h - \mathfrak{v}_z\mathfrak{d}_n + \mathfrak{o}_a\mathfrak{d}_l - \mathfrak{o}_c\mathfrak{d}_j + \mathfrak{v}_m(\varepsilon_d\varsigma_c - \varepsilon_b\varsigma_b) + \mathfrak{v}_n(\varepsilon_d\varsigma_n - r^2\varsigma_b) + \mathfrak{v}_o(\varepsilon_d\varsigma_d - \varepsilon_h\varsigma_b) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_d\varsigma_e - \varsigma_n\varsigma_b) + \mathfrak{v}_p(\varepsilon_d\varsigma_a - \varepsilon_f\varsigma_b) + \mathfrak{v}_q(\varepsilon_b\varsigma_n - r^2\varsigma_c) + \mathfrak{v}_r(\varepsilon_b\varsigma_d - \varepsilon_h\varsigma_c) \\
&\quad + \mathfrak{v}_s(\varepsilon_b\varsigma_e - \varsigma_n\varsigma_c) + \mathfrak{v}_t(\varepsilon_b\varsigma_a - \varepsilon_f\varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(r^2\varsigma_e - \varsigma_n^2) + \mathfrak{v}_u(r^2\varsigma_d - \varepsilon_h\varsigma_n) \\
&\quad + \mathfrak{v}_v(r^2\varsigma_a - \varepsilon_f\varsigma_n) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_f\varsigma_e - \varsigma_n\varsigma_a) + \mathfrak{v}_w(\varepsilon_f\varsigma_d - \varepsilon_h\varsigma_a) + \mathfrak{v}_e^2(\varepsilon_h\varsigma_e - \varsigma_n\varsigma_d) \\
&\quad \text{by (5.1a) \& (5.23a)} \\
&= \mathfrak{o}_s \text{ by (5.24h)} \qquad (5.48\text{p})
\end{aligned}
$$

$$
\begin{aligned}
&(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\boldsymbol{\mathcal{J}}_p + \boldsymbol{\mathcal{J}}_q) \\
&= (\mathbf{r}\times\boldsymbol{\Lambda})\cdot[\mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega}\times\mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\boldsymbol{\Lambda}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})] + \mathfrak{v}_y[\mathbf{r}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})] + \mathfrak{o}_a[\boldsymbol{\Omega}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})] + \mathfrak{o}_b[\dot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})] \\
&\quad + \mathfrak{o}_c[\ddot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})] + \mathfrak{v}_m[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})] + \mathfrak{v}_n[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_p[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\boldsymbol{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})] + \mathfrak{v}_s[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_t[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_u[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\mathbf{r}\times\boldsymbol{\Lambda})] + \mathfrak{v}_v[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] + \mathfrak{v}_e^2[(\mathbf{r}\times\boldsymbol{\Lambda})\cdot(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_z\varepsilon_o - \mathfrak{o}_a\varepsilon_m - \mathfrak{o}_b\mathfrak{d}_h - \mathfrak{o}_c\mathfrak{d}_i + \mathfrak{v}_m[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega}) - (\mathbf{r}\cdot\boldsymbol{\Omega})(\boldsymbol{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\boldsymbol{\Lambda}\cdot\mathbf{r}) - r^2(\boldsymbol{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})\Lambda^2 - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\boldsymbol{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\boldsymbol{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[(\mathbf{r}\cdot\boldsymbol{\Omega})(\boldsymbol{\Lambda}\cdot\mathbf{r}) - r^2(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega})] + \mathfrak{v}_r[(\mathbf{r}\cdot\boldsymbol{\Omega})\Lambda^2 - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega})] \\
&\quad + \mathfrak{v}_s[(\mathbf{r}\cdot\boldsymbol{\Omega})(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega})] + \mathfrak{v}_t[(\mathbf{r}\cdot\boldsymbol{\Omega})(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[r^2(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\boldsymbol{\Lambda}\cdot\mathbf{r})] + \mathfrak{v}_u[r^2\Lambda^2 - (\mathbf{r}\cdot\boldsymbol{\Lambda})^2] + \mathfrak{v}_v[r^2(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\mathbf{r})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_w[(\mathbf{r}\cdot\widehat{\mathbf{p}})\Lambda^2 - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_e^2[(\mathbf{r}\cdot\boldsymbol{\Lambda})(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})\Lambda^2] \text{ by } (5.1\text{a}), (5.23\text{b}) \;\&\; (\text{A.2})
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_z\varepsilon_o - \mathfrak{o}_a\varepsilon_m - \mathfrak{o}_b\mathfrak{d}_h - \mathfrak{o}_c\mathfrak{d}_i + \mathfrak{v}_m(\varepsilon_d\varepsilon_g - \varepsilon_b\delta_a) + \mathfrak{v}_n(\varepsilon_d\varepsilon_h - r^2\delta_a) + \mathfrak{v}_o(\varepsilon_d\Lambda^2 - \varepsilon_h\delta_a) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_d\varsigma_d - \varsigma_n\delta_a) + \mathfrak{v}_p(\varepsilon_d\varepsilon_i - \varepsilon_f\delta_a) + \mathfrak{v}_q(\varepsilon_b\varepsilon_h - r^2\varepsilon_g) + \mathfrak{v}_r(\varepsilon_b\Lambda^2 - \varepsilon_h\varepsilon_g) \\
&\quad + \mathfrak{v}_s(\varepsilon_b\varsigma_d - \varsigma_n\varepsilon_g) + \mathfrak{v}_t(\varepsilon_b\varepsilon_i - \varepsilon_f\varepsilon_g) - \mathfrak{v}_e\mathfrak{v}_d(r^2\varsigma_d - \varsigma_n\varepsilon_h) + \mathfrak{v}_u(r^2\Lambda^2 - \varepsilon_h^2) \\
&\quad + \mathfrak{v}_v(r^2\varepsilon_i - \varepsilon_f\varepsilon_h) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_f\varsigma_d - \varsigma_n\varepsilon_i) + \mathfrak{v}_w(\varepsilon_f\Lambda^2 - \varepsilon_h\varepsilon_i) + \mathfrak{v}_e^2(\varepsilon_h\varsigma_d - \varsigma_n\Lambda^2) \\
&\quad \text{by } (5.1\text{a}), (5.13\text{a}) \;\&\; (5.23\text{a}) \\
&= \mathfrak{o}_t \text{ by } (5.24\text{i})
\end{aligned}
\tag{5.48q}
$$

$$
\begin{aligned}
&(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= (\mathbf{r}\times\widehat{\mathbf{p}})\cdot[\mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega}\times\mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})] \text{ by } (5.47)
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\boldsymbol{\Lambda}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] + \mathfrak{v}_y[\mathbf{r}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] + \mathfrak{o}_a[\boldsymbol{\Omega}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] + \mathfrak{o}_b[\dot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{o}_c[\ddot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] + \mathfrak{v}_m[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})] + \mathfrak{v}_n[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_p[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\boldsymbol{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})] + \mathfrak{v}_s[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\boldsymbol{\Omega}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_t[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_u[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\boldsymbol{\Lambda})] + \mathfrak{v}_v[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\mathbf{p}}\times\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] + \mathfrak{v}_e^2[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\boldsymbol{\Lambda}\times\dot{\boldsymbol{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\varepsilon_o + \mathfrak{o}_a\varepsilon_l + \mathfrak{o}_b\mathfrak{d}_n + \mathfrak{o}_c\mathfrak{p}_b + \mathfrak{v}_m[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega}) - (\mathbf{r}\cdot\boldsymbol{\Omega})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\mathbf{r}) - r^2(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda}) - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[(\mathbf{r}\cdot\boldsymbol{\Omega})(\widehat{\mathbf{p}}\cdot\mathbf{r}) - r^2(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})] + \mathfrak{v}_r[(\mathbf{r}\cdot\boldsymbol{\Omega})(\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda}) - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})] \\
&\quad + \mathfrak{v}_s[(\mathbf{r}\cdot\boldsymbol{\Omega})(\widehat{\mathbf{p}}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})] + \mathfrak{v}_t[(\mathbf{r}\cdot\boldsymbol{\Omega}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[r^2(\widehat{\mathbf{p}}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}}\cdot\mathbf{r})] + \mathfrak{v}_u[r^2(\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda}) - (\mathbf{r}\cdot\boldsymbol{\Lambda})(\widehat{\mathbf{p}}\cdot\mathbf{r})] + \mathfrak{v}_v[r^2 - (\mathbf{r}\cdot\widehat{\mathbf{p}})^2] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})] + \mathfrak{v}_w[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda}) - (\mathbf{r}\cdot\boldsymbol{\Lambda})] \\
&\quad + \mathfrak{v}_e^2[(\mathbf{r}\cdot\boldsymbol{\Lambda})(\widehat{\mathbf{p}}\cdot\dot{\boldsymbol{\Lambda}}) - (\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})(\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda})] \text{ by } (5.1\text{a}), (5.23\text{b}), (5.23\text{c}) \;\&\; (\text{A.2})
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\varepsilon_o + \mathfrak{o}_a\varepsilon_l + \mathfrak{o}_b\mathfrak{d}_n + \mathfrak{o}_c\mathfrak{p}_b + \mathfrak{v}_m(\varepsilon_d\varepsilon_c - \varepsilon_b\varepsilon_e) + \mathfrak{v}_n(\varepsilon_d\varepsilon_f - r^2\varepsilon_e) + \mathfrak{v}_o(\varepsilon_d\varepsilon_i - \varepsilon_h\varepsilon_e) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_d\varsigma_a - \varsigma_n\varepsilon_e) + \mathfrak{v}_p(\varepsilon_d - \varepsilon_f\varepsilon_e) + \mathfrak{v}_q(\varepsilon_b\varepsilon_f - r^2\varepsilon_c) + \mathfrak{v}_r(\varepsilon_b\varepsilon_i - \varepsilon_h\varepsilon_c) \\
&\quad + \mathfrak{v}_s(\varepsilon_b\varsigma_a - \varsigma_n\varepsilon_c) + \mathfrak{v}_t(\varepsilon_b - \varepsilon_f\varepsilon_c) - \mathfrak{v}_e\mathfrak{v}_d(r^2\varsigma_a - \varsigma_n\varepsilon_f) + \mathfrak{v}_u(r^2\varepsilon_i - \varepsilon_h\varepsilon_f) \\
&\quad + \mathfrak{v}_v(r^2 - \varepsilon_f^2) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_f\varsigma_a - \varsigma_n) + \mathfrak{v}_w(\varepsilon_f\varepsilon_i - \varepsilon_h) + \mathfrak{v}_e^2(\varepsilon_h\varsigma_a - \varsigma_n\varepsilon_i) \\
&\quad \text{by (5.1a) \& (5.23a)} \\
&= \mathfrak{o}_u \text{ by (5.24i)} \qquad (5.48\text{r})
\end{aligned}
$$

$$
\begin{aligned}
&(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot [\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda} \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_y[\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_z[\widehat{\mathbf{p}} \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{o}_a[\mathbf{\Omega} \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}} \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] \\
&\quad + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}} \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_m[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] + \mathfrak{v}_n[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\mathbf{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\mathbf{\Omega} \times \mathbf{\Lambda})] + \mathfrak{v}_s[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\mathbf{\Omega} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\mathbf{\Omega} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\mathbf{r} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\mathbf{r} \times \mathbf{\Lambda})] + \mathfrak{v}_v[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] + \mathfrak{v}_e^2[(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) \cdot (\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\mathfrak{d}_r + \mathfrak{v}_y\mathfrak{d}_n - \mathfrak{o}_a\mathfrak{d}_p + \mathfrak{o}_c\mathfrak{p}_c + \mathfrak{v}_m[(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Omega}) - (\widehat{\mathbf{p}} \cdot \mathbf{\Omega})(\dot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}} \cdot \mathbf{r}) - (\widehat{\mathbf{p}} \cdot \mathbf{r})(\dot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda}) - (\widehat{\mathbf{p}} \cdot \mathbf{\Lambda})(\dot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})\dot{\mathbf{\Lambda}}^2 - (\widehat{\mathbf{p}} \cdot \dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}}) - (\dot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[(\widehat{\mathbf{p}} \cdot \mathbf{\Omega})(\dot{\mathbf{\Lambda}} \cdot \mathbf{r}) - (\widehat{\mathbf{p}} \cdot \mathbf{r})(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})] + \mathfrak{v}_r[(\widehat{\mathbf{p}} \cdot \mathbf{\Omega})(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda}) - (\widehat{\mathbf{p}} \cdot \mathbf{\Lambda})(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})] \\
&\quad + \mathfrak{v}_s[(\widehat{\mathbf{p}} \cdot \mathbf{\Omega})\dot{\mathbf{\Lambda}}^2 - (\widehat{\mathbf{p}} \cdot \dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})] + \mathfrak{v}_t[(\widehat{\mathbf{p}} \cdot \mathbf{\Omega})(\dot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\widehat{\mathbf{p}} \cdot \mathbf{r})\dot{\mathbf{\Lambda}}^2 - (\widehat{\mathbf{p}} \cdot \dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}} \cdot \mathbf{r})] + \mathfrak{v}_u[(\widehat{\mathbf{p}} \cdot \mathbf{r})(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda}) - (\widehat{\mathbf{p}} \cdot \mathbf{\Lambda})(\dot{\mathbf{\Lambda}} \cdot \mathbf{r})] \\
&\quad + \mathfrak{v}_v[(\widehat{\mathbf{p}} \cdot \mathbf{r})(\dot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{r})] - \mathfrak{x}_u\mathfrak{v}_e[\dot{\mathbf{\Lambda}}^2 - (\widehat{\mathbf{p}} \cdot \dot{\mathbf{\Lambda}})^2] + \mathfrak{v}_w[(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda}) - (\widehat{\mathbf{p}} \cdot \mathbf{\Lambda})(\dot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_e^2[(\widehat{\mathbf{p}} \cdot \mathbf{\Lambda})\dot{\mathbf{\Lambda}}^2 - (\widehat{\mathbf{p}} \cdot \dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})] \text{ by (5.23b), (5.23c) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_x\mathfrak{d}_r + \mathfrak{v}_y\mathfrak{d}_n - \mathfrak{o}_a\mathfrak{d}_p + \mathfrak{o}_c\mathfrak{p}_c + \mathfrak{v}_m(\varepsilon_e\varsigma_c - \varepsilon_c\varsigma_b) + \mathfrak{v}_n(\varepsilon_e\varsigma_n - \varepsilon_f\varsigma_b) + \mathfrak{v}_o(\varepsilon_e\varsigma_d - \varepsilon_i\varsigma_b) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_e\varsigma_e - \varsigma_a\varsigma_b) + \mathfrak{v}_p(\varepsilon_e\varsigma_a - \varsigma_b) + \mathfrak{v}_q(\varepsilon_c\varsigma_n - \varepsilon_f\varsigma_c) + \mathfrak{v}_r(\varepsilon_c\varsigma_d - \varepsilon_i\varsigma_c) \\
&\quad + \mathfrak{v}_s(\varepsilon_c\varsigma_e - \varsigma_a\varsigma_c) + \mathfrak{v}_t(\varepsilon_c\varsigma_a - \varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_f\varsigma_e - \varsigma_a\varsigma_n) + \mathfrak{v}_u(\varepsilon_f\varsigma_d - \varepsilon_i\varsigma_n) \\
&\quad + \mathfrak{v}_v(\varepsilon_f\varsigma_a - \varsigma_n) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_e - \varsigma_a^2) + \mathfrak{v}_w(\varsigma_d - \varepsilon_i\varsigma_a) + \mathfrak{v}_e^2(\varepsilon_i\varsigma_e - \varsigma_a\varsigma_d) \\
&\quad \text{by (5.1a) \& (5.23a)} \\
&= \mathfrak{o}_v \text{ by (5.24j)} \qquad (5.48\text{s})
\end{aligned}
$$

$$
\begin{aligned}
&(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot [\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_y[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{o}_a[\mathbf{\Omega}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_m[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})] + \mathfrak{v}_n[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_s[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\mathbf{r}\times\mathbf{\Lambda})] + \mathfrak{v}_v[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_y\varepsilon_o - \mathfrak{o}_a\mathfrak{d}_o + \mathfrak{o}_b\mathfrak{d}_r + \mathfrak{o}_c\mathfrak{d}_s + \mathfrak{v}_m[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\mathbf{\Omega}) - (\widehat{\mathbf{p}}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\mathbf{r}) - (\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})\Lambda^2 - (\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\mathbf{r}) - (\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] + \mathfrak{v}_r[(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})\Lambda^2 - (\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_s[(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\mathbf{\Omega})] + \mathfrak{v}_t[(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Lambda}\cdot\mathbf{\Omega})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\mathbf{r})] + \mathfrak{v}_u[(\widehat{\mathbf{p}}\cdot\mathbf{r})\Lambda^2 - (\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\mathbf{r})] \\
&\quad + \mathfrak{v}_v[(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Lambda}\cdot\mathbf{r})] - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_w[\Lambda^2 - (\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})^2] \\
&\quad + \mathfrak{v}_e^2[(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}}) - (\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}})\Lambda^2] \text{ by (5.1a), (5.23b) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_y\varepsilon_o - \mathfrak{o}_a\mathfrak{d}_o + \mathfrak{o}_b\mathfrak{d}_r + \mathfrak{o}_c\mathfrak{d}_s + \mathfrak{v}_m(\varepsilon_e\varepsilon_g - \varepsilon_c\delta_a) + \mathfrak{v}_n(\varepsilon_e\varepsilon_h - \varepsilon_f\delta_a) + \mathfrak{v}_o(\varepsilon_e\Lambda^2 - \varepsilon_i\delta_a) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varepsilon_e\varsigma_d - \varsigma_a\delta_a) + \mathfrak{v}_p(\varepsilon_e\varepsilon_i - \delta_a) + \mathfrak{v}_q(\varepsilon_c\varepsilon_h - \varepsilon_f\varepsilon_g) + \mathfrak{v}_r(\varepsilon_c\Lambda^2 - \varepsilon_i\varepsilon_g) \\
&\quad + \mathfrak{v}_s(\varepsilon_c\varsigma_d - \varsigma_a\varepsilon_g) + \mathfrak{v}_t(\varepsilon_c\varepsilon_i - \varepsilon_g) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_f\varsigma_d - \varsigma_a\varepsilon_h) + \mathfrak{v}_u(\varepsilon_f\Lambda^2 - \varepsilon_i\varepsilon_h) \\
&\quad + \mathfrak{v}_v(\varepsilon_f\varepsilon_i - \varepsilon_h) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_d - \varsigma_a\varepsilon_i) + \mathfrak{v}_w(\Lambda^2 - \varepsilon_i^2) + \mathfrak{v}_e^2(\varepsilon_i\varsigma_d - \varsigma_a\Lambda^2) \\
&\quad \text{by (5.1a), (5.13a) \& (5.23a)} \\
&= \mathfrak{o}_w \text{ by (5.24j)} \qquad (5.48t)
\end{aligned}
$$

$$
\begin{aligned}
&(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot[\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega}\times\mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda}\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_y[\mathbf{r}\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] + \mathfrak{o}_a[\mathbf{\Omega}\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}}\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] \\
&\quad + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}}\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_m[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})] + \mathfrak{v}_n[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_s[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{r}\times\mathbf{\Lambda})] + \mathfrak{v}_v[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_y\mathfrak{d}_h + \mathfrak{v}_z\mathfrak{d}_r + \mathfrak{o}_a\mathfrak{d}_v + \mathfrak{o}_c\mathfrak{p}_d + \mathfrak{v}_m[(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega}) - (\mathbf{\Lambda}\cdot\mathbf{\Omega})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - (\mathbf{\Lambda}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - \Lambda^2(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[(\mathbf{\Lambda}\cdot\mathbf{\Omega})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r}) - (\mathbf{\Lambda}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] + \mathfrak{v}_r[(\mathbf{\Lambda}\cdot\mathbf{\Omega})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - \Lambda^2(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_s[(\mathbf{\Lambda}\cdot\mathbf{\Omega})\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] + \mathfrak{v}_t[(\mathbf{\Lambda}\cdot\mathbf{\Omega})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\mathbf{\Lambda}\cdot\mathbf{r})\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r})] + \mathfrak{v}_u[(\mathbf{\Lambda}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - \Lambda^2(\dot{\mathbf{\Lambda}}\cdot\mathbf{r})] \\
&\quad + \mathfrak{v}_v[(\mathbf{\Lambda}\cdot\mathbf{r})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}}) - (\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{r})] - \mathfrak{x}_u\mathfrak{v}_e[(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}})(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_w[(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda}) - \Lambda^2(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_e^2[\Lambda^2\dot{\mathbf{\Lambda}}^2 - (\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}})^2] \\
&\quad \text{by (5.23b), (5.23c) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_y\mathfrak{d}_h + \mathfrak{v}_z\mathfrak{d}_r + \mathfrak{o}_a\mathfrak{d}_v + \mathfrak{o}_c\mathfrak{p}_d + \mathfrak{v}_m(\delta_a\varsigma_c - \varepsilon_g\varsigma_b) + \mathfrak{v}_n(\delta_a\varsigma_n - \varepsilon_h\varsigma_b) + \mathfrak{v}_o(\delta_a\varsigma_d - \Lambda^2\varsigma_b) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\delta_a\varsigma_e - \varsigma_d\varsigma_b) + \mathfrak{v}_p(\delta_a\varsigma_a - \varepsilon_i\varsigma_b) + \mathfrak{v}_q(\varepsilon_g\varsigma_n - \varepsilon_h\varsigma_c) + \mathfrak{v}_r(\varepsilon_g\varsigma_d - \Lambda^2\varsigma_c) \\
&\quad + \mathfrak{v}_s(\varepsilon_g\varsigma_e - \varsigma_d\varsigma_c) + \mathfrak{v}_t(\varepsilon_g\varsigma_a - \varepsilon_i\varsigma_c) - \mathfrak{v}_e\mathfrak{v}_d(\varepsilon_h\varsigma_e - \varsigma_d\varsigma_n) + \mathfrak{v}_u(\varepsilon_h\varsigma_d - \Lambda^2\varsigma_n) \\
&\quad + \mathfrak{v}_v(\varepsilon_h\varsigma_a - \varepsilon_i\varsigma_n) - \mathfrak{x}_u\mathfrak{v}_e(\varepsilon_i\varsigma_e - \varsigma_d\varsigma_a) + \mathfrak{v}_w(\varepsilon_i\varsigma_d - \Lambda^2\varsigma_a) + \mathfrak{v}_e^2(\Lambda^2\varsigma_e - \varsigma_d^2) \\
&\quad \text{by (5.1a), (5.13a) \& (5.23a)} \\
&= \mathfrak{o}_x \text{ by (5.24k).} \qquad (5.48\text{u})
\end{aligned}
$$

Similarly, we derive

$$
\begin{aligned}
&\widehat{\boldsymbol{\kappa}}\cdot(\mathfrak{J}_p + \mathfrak{J}_q) \\
&= \widehat{\boldsymbol{\kappa}}\cdot[\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega}\times\mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda}) + \mathfrak{v}_y(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r}) + \mathfrak{v}_z(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}}) + \mathfrak{o}_a(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega}) + \mathfrak{o}_b(\widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{\Lambda}}) + \mathfrak{o}_c(\widehat{\boldsymbol{\kappa}}\cdot\ddot{\mathbf{\Lambda}}) + \mathfrak{v}_q[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_s[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] - \mathfrak{v}_e\mathfrak{v}_d[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_v[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] - \mathfrak{x}_u\mathfrak{v}_e[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x\delta_a + \mathfrak{v}_y\varepsilon_d + \mathfrak{v}_z\varepsilon_e + \mathfrak{o}_a\varepsilon_a + \mathfrak{o}_b\varsigma_b + \mathfrak{o}_c\varsigma_g + \mathfrak{v}_q\varepsilon_k - \mathfrak{v}_r\mathfrak{d}_e + \mathfrak{v}_s\mathfrak{d}_f - \mathfrak{v}_t\varepsilon_j + \mathfrak{v}_e\mathfrak{v}_d\mathfrak{d}_c \\
&\quad - \mathfrak{v}_u\varepsilon_n + \mathfrak{v}_v\mathfrak{d}_t - \mathfrak{x}_u\mathfrak{v}_e\mathfrak{d}_a + \mathfrak{v}_w\delta_b + \mathfrak{v}_e^2\mathfrak{d}_u \text{ by (5.1a), (5.13a), (5.23a), (5.23b) \& (5.23c)} \\
&= \mathfrak{o}_y \text{ by (5.24k)} \qquad (5.49\text{a})
\end{aligned}
$$

$$
\begin{aligned}
&(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot[\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega}\times\mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] \text{ by (5.47)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda}\cdot(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_y[\mathbf{r}\cdot(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})] + \mathfrak{o}_a[\mathbf{\Omega}\cdot(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}}\cdot(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}}\cdot(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_m[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})] + \mathfrak{v}_n[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_s[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\mathbf{\Lambda})] + \mathfrak{v}_v[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\ddot{\mathbf{\Lambda}}\times\widehat{\mathbf{p}})\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}
$$

$$\begin{aligned}
&= \mathfrak{v}_x\mathfrak{d}_s - \mathfrak{v}_y\mathfrak{p}_b + \mathfrak{o}_a\mathfrak{d}_q + \mathfrak{o}_b\mathfrak{p}_c + \mathfrak{v}_m[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\mathbf{\Omega}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\mathbf{r}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}}) - (\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})(\widehat{\mathbf{p}}\cdot\mathbf{r}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})] + \mathfrak{v}_r[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_s[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})(\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})] + \mathfrak{v}_t[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega}) - (\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})(\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})(\widehat{\mathbf{p}}\cdot\mathbf{r})] + \mathfrak{v}_u[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})(\widehat{\mathbf{p}}\cdot\mathbf{r})] \\
&\quad + \mathfrak{v}_v[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{r}) - (\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\mathbf{r})] - \mathfrak{x}_u\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})] \\
&\quad + \mathfrak{v}_w[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})(\widehat{\mathbf{p}}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})] \\
&\quad \text{by (5.23b), (5.23c) \& (A.2)}
\end{aligned}$$

$$\begin{aligned}
&= \mathfrak{v}_x\mathfrak{d}_s - \mathfrak{v}_y\mathfrak{p}_b + \mathfrak{o}_a\mathfrak{d}_q + \mathfrak{o}_b\mathfrak{p}_c + \mathfrak{v}_m(\varsigma_g\varepsilon_c - \varsigma_h\varepsilon_e) + \mathfrak{v}_n(\varsigma_g\varepsilon_f - \varsigma_o\varepsilon_e) + \mathfrak{v}_o(\varsigma_g\varepsilon_i - \varsigma_i\varepsilon_e) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_g\varsigma_a - \varsigma_j\varepsilon_e) + \mathfrak{v}_p(\varsigma_g - \varsigma_f\varepsilon_e) + \mathfrak{v}_q(\varsigma_h\varepsilon_f - \varsigma_o\varepsilon_c) + \mathfrak{v}_r(\varsigma_h\varepsilon_i - \varsigma_i\varepsilon_c) \\
&\quad + \mathfrak{v}_s(\varsigma_h\varsigma_a - \varsigma_j\varepsilon_c) + \mathfrak{v}_t(\varsigma_h - \varsigma_f\varepsilon_c) - \mathfrak{v}_e\mathfrak{v}_d(\varsigma_o\varsigma_a - \varsigma_j\varepsilon_f) + \mathfrak{v}_u(\varsigma_o\varepsilon_i - \varsigma_i\varepsilon_f) \\
&\quad + \mathfrak{v}_v(\varsigma_o - \varsigma_f\varepsilon_f) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_f\varsigma_a - \varsigma_j) + \mathfrak{v}_w(\varsigma_f\varepsilon_i - \varsigma_i) + \mathfrak{v}_e^2(\varsigma_i\varsigma_a - \varsigma_j\varepsilon_i) \\
&\quad \text{by (5.1a) \& (5.23a)} \\
&= \mathfrak{o}_z \text{ by (5.24l)}
\end{aligned} \tag{5.49b}$$

$$\begin{aligned}
&(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\mathfrak{J}_p + \mathfrak{J}_q) \\
&= (\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot[\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda}) \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega}\times\mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r}\times\widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})] \text{ by (5.47)}
\end{aligned}$$

$$\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda}\cdot(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] + \mathfrak{v}_y[\mathbf{r}\cdot(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] + \mathfrak{o}_a[\mathbf{\Omega}\cdot(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}}\cdot(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] \\
&\quad + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}}\cdot(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] + \mathfrak{v}_m[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})] + \mathfrak{v}_n[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + \mathfrak{v}_r[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})] + \mathfrak{v}_s[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\mathbf{r}\times\mathbf{\Lambda})] + \mathfrak{v}_v[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad - \mathfrak{x}_u\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\ddot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}$$

$$\begin{aligned}
&= -\mathfrak{v}_x\mathfrak{d}_i + \mathfrak{v}_z\mathfrak{p}_b - \mathfrak{o}_a\mathfrak{d}_m - \mathfrak{o}_b\mathfrak{d}_j + \mathfrak{v}_m[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\mathbf{\Omega}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_n[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})r^2 - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_b\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\widehat{\mathbf{p}}) - (\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{v}_q[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})r^2 - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{\Omega})] + \mathfrak{v}_r[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})(\mathbf{r}\cdot\mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\mathbf{\Omega})] \\
&\quad + \mathfrak{v}_s[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{r}\cdot\mathbf{\Omega})] + \mathfrak{v}_t[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})(\mathbf{r}\cdot\widehat{\mathbf{p}}) - (\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Omega})] \\
&\quad - \mathfrak{v}_e\mathfrak{v}_d[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})r^2] + \mathfrak{v}_u[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})r^2] \\
&\quad + \mathfrak{v}_v[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{r})(\mathbf{r}\cdot\widehat{\mathbf{p}}) - (\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})r^2] - \mathfrak{x}_u\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{r}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{v}_w[(\ddot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\widehat{\mathbf{p}})] + \mathfrak{v}_e^2[(\ddot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}}\cdot\dot{\mathbf{\Lambda}})(\mathbf{r}\cdot\mathbf{\Lambda})] \\
&\quad \text{by (5.23b), (5.23c) \& (A.2)}
\end{aligned}$$

$$= -\mathfrak{v}_x\mathfrak{d}_i + \mathfrak{v}_z\mathfrak{p}_b - \mathfrak{o}_a\mathfrak{d}_m - \mathfrak{o}_b\mathfrak{d}_j + \mathfrak{v}_m(\varsigma_g\varepsilon_b - \varsigma_h\varepsilon_d) + \mathfrak{v}_n(\varsigma_g r^2 - \varsigma_o\varepsilon_d) + \mathfrak{v}_o(\varsigma_g\varepsilon_h - \varsigma_i\varepsilon_d)$$
$$+ \mathfrak{v}_b\mathfrak{v}_e(\varsigma_g\varsigma_n - \varsigma_j\varepsilon_d) + \mathfrak{v}_p(\varsigma_g\varepsilon_f - \varsigma_f\varepsilon_d) + \mathfrak{v}_q(\varsigma_h r^2 - \varsigma_o\varepsilon_b) + \mathfrak{v}_r(\varsigma_h\varepsilon_h - \varsigma_i\varepsilon_b)$$
$$+ \mathfrak{v}_s(\varsigma_h\varsigma_n - \varsigma_j\varepsilon_b) + \mathfrak{v}_t(\varsigma_h\varepsilon_f - \varsigma_f\varepsilon_b) - \mathfrak{v}_e\mathfrak{v}_d(\varsigma_o\varsigma_n - \varsigma_j r^2) + \mathfrak{v}_u(\varsigma_o\varepsilon_h - \varsigma_i r^2)$$
$$+ \mathfrak{v}_v(\varsigma_o\varepsilon_f - \varsigma_f r^2) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_f\varsigma_n - \varsigma_j\varepsilon_f) + \mathfrak{v}_w(\varsigma_f\varepsilon_h - \varsigma_i\varepsilon_f) + \mathfrak{v}_e^2(\varsigma_i\varsigma_n - \varsigma_j\varepsilon_h)$$
by (5.1a) & (5.23a)
$$= \mathfrak{f}_a \text{ by (5.24m)} \tag{5.49c}$$

$$(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\boldsymbol{\mathcal{J}}_p + \boldsymbol{\mathcal{J}}_q)$$
$$= (\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot [\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda})$$
$$+ \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega} \times \widehat{\mathbf{p}})$$
$$- \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})] \text{ by (5.47)}$$

$$= \mathfrak{v}_x[\mathbf{\Lambda} \cdot (\ddot{\mathbf{\Lambda}} \times \mathbf{r})] + \mathfrak{v}_y[\mathbf{r} \cdot (\ddot{\mathbf{\Lambda}} \times \mathbf{r})] + \mathfrak{v}_z[\widehat{\mathbf{p}} \cdot (\ddot{\mathbf{\Lambda}} \times \mathbf{r})] + \mathfrak{o}_a[\mathbf{\Omega} \cdot (\ddot{\mathbf{\Lambda}} \times \mathbf{r})] + \mathfrak{o}_b[\dot{\mathbf{\Lambda}} \cdot (\ddot{\mathbf{\Lambda}} \times \mathbf{r})]$$
$$+ \mathfrak{o}_c[\ddot{\mathbf{\Lambda}} \cdot (\ddot{\mathbf{\Lambda}} \times \mathbf{r})] + \mathfrak{v}_m[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] + \mathfrak{v}_n[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{v}_o[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda})]$$
$$+ \mathfrak{v}_b\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_p[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_q[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{\Omega} \times \mathbf{r})]$$
$$+ \mathfrak{v}_r[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{\Omega} \times \mathbf{\Lambda})] + \mathfrak{v}_s[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{\Omega} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{\Omega} \times \widehat{\mathbf{p}})]$$
$$- \mathfrak{v}_e\mathfrak{v}_d[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{r} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_u[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{r} \times \mathbf{\Lambda})] + \mathfrak{v}_v[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{r} \times \widehat{\mathbf{p}})]$$
$$- \mathfrak{x}_u\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}})] + \mathfrak{v}_w[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] + \mathfrak{v}_e^2[(\ddot{\mathbf{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})]$$

$$= \mathfrak{v}_x\mathfrak{p}_e + \mathfrak{v}_z\mathfrak{p}_f + \mathfrak{o}_a\mathfrak{p}_g + \mathfrak{o}_b\mathfrak{p}_h + \mathfrak{o}_c\mathfrak{p}_i + \mathfrak{v}_m[(\ddot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \mathbf{\Omega}) - (\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})]$$
$$+ \mathfrak{v}_n[(\ddot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})r^2 - (\ddot{\mathbf{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_o[(\ddot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})]$$
$$+ \mathfrak{v}_b\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}} \cdot \dot{\mathbf{\Lambda}})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_p[(\ddot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\ddot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})]$$
$$+ \mathfrak{v}_q[(\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})r^2 - (\ddot{\mathbf{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \mathbf{\Omega})] + \mathfrak{v}_r[(\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})(\mathbf{r} \cdot \mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})(\mathbf{r} \cdot \mathbf{\Omega})]$$
$$+ \mathfrak{v}_s[(\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})(\mathbf{r} \cdot \dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}} \cdot \dot{\mathbf{\Lambda}})(\mathbf{r} \cdot \mathbf{\Omega})] + \mathfrak{v}_t[(\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\ddot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \mathbf{\Omega})]$$
$$- \mathfrak{v}_e\mathfrak{v}_d[(\ddot{\mathbf{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}} \cdot \dot{\mathbf{\Lambda}})r^2] + \mathfrak{v}_u[(\ddot{\mathbf{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})r^2]$$
$$+ \mathfrak{v}_v[(\ddot{\mathbf{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\ddot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})r^2] - \mathfrak{x}_u\mathfrak{v}_e[(\ddot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}} \cdot \dot{\mathbf{\Lambda}})(\mathbf{r} \cdot \widehat{\mathbf{p}})]$$
$$+ \mathfrak{v}_w[(\ddot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \mathbf{\Lambda}) - (\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})(\mathbf{r} \cdot \widehat{\mathbf{p}})] + \mathfrak{v}_e^2[(\ddot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})(\mathbf{r} \cdot \dot{\mathbf{\Lambda}}) - (\ddot{\mathbf{\Lambda}} \cdot \dot{\mathbf{\Lambda}})(\mathbf{r} \cdot \mathbf{\Lambda})]$$
by (5.23c) & (A.2)

$$= \mathfrak{v}_x\mathfrak{p}_e + \mathfrak{v}_z\mathfrak{p}_f + \mathfrak{o}_a\mathfrak{p}_g + \mathfrak{o}_b\mathfrak{p}_h + \mathfrak{o}_c\mathfrak{p}_i + \mathfrak{v}_m(\varsigma_p\varepsilon_b - \varsigma_m\varepsilon_d) + \mathfrak{v}_n(\varsigma_p r^2 - \varsigma_q\varepsilon_d)$$
$$+ \mathfrak{v}_o(\varsigma_p\varepsilon_h - \varsigma_r\varepsilon_d) + \mathfrak{v}_b\mathfrak{v}_e(\varsigma_p\varsigma_n - \varsigma_s\varepsilon_d) + \mathfrak{v}_p(\varsigma_p\varepsilon_f - \varsigma_l\varepsilon_d) + \mathfrak{v}_q(\varsigma_m r^2 - \varsigma_q\varepsilon_b)$$
$$+ \mathfrak{v}_r(\varsigma_m\varepsilon_h - \varsigma_r\varepsilon_b) + \mathfrak{v}_s(\varsigma_m\varsigma_n - \varsigma_s\varepsilon_b) + \mathfrak{v}_t(\varsigma_m\varepsilon_f - \varsigma_l\varepsilon_b) - \mathfrak{v}_e\mathfrak{v}_d(\varsigma_q\varsigma_n - \varsigma_s r^2)$$
$$+ \mathfrak{v}_u(\varsigma_q\varepsilon_h - \varsigma_r r^2) + \mathfrak{v}_v(\varsigma_q\varepsilon_f - \varsigma_l r^2) - \mathfrak{x}_u\mathfrak{v}_e(\varsigma_l\varsigma_n - \varsigma_s\varepsilon_f) + \mathfrak{v}_w(\varsigma_l\varepsilon_h - \varsigma_r\varepsilon_f)$$
$$+ \mathfrak{v}_e^2(\varsigma_r\varsigma_n - \varsigma_s\varepsilon_h) \text{ by (5.1a) \& (5.23a)}$$
$$= \mathfrak{f}_b \text{ by (5.24m).} \tag{5.49d}$$

As a consequence of the foregoing derivations, we get

$$|\boldsymbol{\mathcal{J}}_p + \boldsymbol{\mathcal{J}}_q|^2$$
$$= (\boldsymbol{\mathcal{J}}_p + \boldsymbol{\mathcal{J}}_q) \cdot [\mathfrak{v}_x\mathbf{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\mathbf{\Omega} + \mathfrak{o}_b\dot{\mathbf{\Lambda}} + \mathfrak{o}_c\ddot{\mathbf{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda})$$
$$+ \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\mathbf{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{v}_s(\mathbf{\Omega} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_t(\mathbf{\Omega} \times \widehat{\mathbf{p}})$$
$$- \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \mathbf{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\mathbf{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) + \mathfrak{v}_e^2(\mathbf{\Lambda} \times \dot{\mathbf{\Lambda}})] \text{ by (5.47)}$$

$$\begin{aligned}
&= \mathfrak{v}_x[\mathbf{\Lambda}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)] + \mathfrak{v}_y[\mathbf{r}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)] + \mathfrak{v}_z[\widehat{\mathbf{p}}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)] + \mathfrak{o}_a[\mathbf{\Omega}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)]\\
&\quad + \mathfrak{o}_b[\dot{\mathbf{\Lambda}}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)] + \mathfrak{o}_c[\ddot{\mathbf{\Lambda}}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)] + \mathfrak{v}_m[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})]\\
&\quad + \mathfrak{v}_n[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{v}_o[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})] + \mathfrak{v}_b\mathfrak{v}_e[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{\Lambda}})]\\
&\quad + \mathfrak{v}_p[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{v}_q[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{\Omega}\times\mathbf{r})] + \mathfrak{v}_r[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})]\\
&\quad + \mathfrak{v}_s[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{\Omega}\times\dot{\mathbf{\Lambda}})] + \mathfrak{v}_t[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{\Omega}\times\widehat{\mathbf{p}})] - \mathfrak{v}_e\mathfrak{v}_d[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{r}\times\dot{\mathbf{\Lambda}})]\\
&\quad + \mathfrak{v}_u[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{r}\times\mathbf{\Lambda})] + \mathfrak{v}_v[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] - \mathfrak{x}_u\mathfrak{v}_e[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})]\\
&\quad + \mathfrak{v}_w[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2[(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{\Lambda}\times\dot{\mathbf{\Lambda}})]
\end{aligned}$$

$$\begin{aligned}
&= \mathfrak{v}_x\mathfrak{o}_d + \mathfrak{v}_y\mathfrak{o}_e + \mathfrak{v}_z\mathfrak{o}_f + \mathfrak{o}_a\mathfrak{o}_g + \mathfrak{o}_b\mathfrak{o}_h + \mathfrak{o}_c\mathfrak{o}_i + \mathfrak{v}_m\mathfrak{o}_j + \mathfrak{v}_n\mathfrak{o}_k + \mathfrak{v}_o\mathfrak{o}_l + \mathfrak{v}_b\mathfrak{v}_e\mathfrak{o}_m + \mathfrak{v}_p\mathfrak{o}_n\\
&\quad + \mathfrak{v}_q\mathfrak{o}_o + \mathfrak{v}_r\mathfrak{o}_p + \mathfrak{v}_s\mathfrak{o}_q + \mathfrak{v}_t\mathfrak{o}_r - \mathfrak{v}_e\mathfrak{v}_d\mathfrak{o}_s + \mathfrak{v}_u\mathfrak{o}_t + \mathfrak{v}_v\mathfrak{o}_u - \mathfrak{x}_u\mathfrak{v}_e\mathfrak{o}_v + \mathfrak{v}_w\mathfrak{o}_w + \mathfrak{v}_e^2\mathfrak{o}_x\\
&\quad \text{by (5.49)}
\end{aligned}$$

$$\therefore |\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q| = \mathfrak{f}_c \text{ by (5.24n)} \tag{5.50a}$$

$$\begin{aligned}
|\boldsymbol{\mathfrak{J}}_r|^2 &= [\mathfrak{v}_b\widehat{\boldsymbol{\kappa}} + \mathfrak{v}_c\mathbf{\Omega} - \mathfrak{v}_d\mathbf{r} + \mathfrak{v}_e\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}} + \mathfrak{v}_a(\mathbf{\Lambda}\times\mathbf{r}) + \rho(\dot{\mathbf{\Lambda}}\times\mathbf{r}) - \rho\Omega^2(\mathbf{\Omega}\times\mathbf{r}) + \mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&\quad + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]^2 \text{ by (5.45d)}
\end{aligned}$$

$$\begin{aligned}
&= \mathfrak{v}_b^2 + 2\mathfrak{v}_c\mathfrak{v}_b(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Omega}) - 2\mathfrak{v}_d\mathfrak{v}_b(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r}) + 2\mathfrak{v}_e\mathfrak{v}_b(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{\Lambda}) - 2\mathfrak{x}_u\mathfrak{v}_b(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}}) + 2\mathfrak{v}_a\mathfrak{v}_b[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{\Lambda}\times\mathbf{r})]\\
&\quad + 2\rho\mathfrak{v}_b[\widehat{\boldsymbol{\kappa}}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})] - 2\rho\Omega^2\mathfrak{v}_b[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{\Omega}\times\mathbf{r})] + 2\mathfrak{x}_o\mathfrak{v}_b[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + 2\varphi_g\varphi_h\mathfrak{v}_b[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&\quad + \mathfrak{v}_c^2\Omega^2 - 2\mathfrak{v}_d\mathfrak{v}_c(\mathbf{\Omega}\cdot\mathbf{r}) + 2\mathfrak{v}_e\mathfrak{v}_c(\mathbf{\Omega}\cdot\mathbf{\Lambda}) - 2\mathfrak{x}_u\mathfrak{v}_c(\mathbf{\Omega}\cdot\widehat{\mathbf{p}}) + 2\mathfrak{v}_a\mathfrak{v}_c[\mathbf{\Omega}\cdot(\mathbf{\Lambda}\times\mathbf{r})]\\
&\quad + 2\rho\mathfrak{v}_c[\mathbf{\Omega}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})] + 2\varphi_g\varphi_h\mathfrak{v}_c[\mathbf{\Omega}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_d^2r^2 - 2\mathfrak{v}_e\mathfrak{v}_d(\mathbf{r}\cdot\mathbf{\Lambda}) + 2\mathfrak{x}_u\mathfrak{v}_d(\mathbf{r}\cdot\widehat{\mathbf{p}})\\
&\quad - 2\mathfrak{x}_o\mathfrak{v}_d[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})] - 2\mathfrak{v}_d\varphi_g\varphi_h[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{v}_e^2\Lambda^2 - 2\mathfrak{x}_u\mathfrak{v}_e(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}}) + 2\rho\mathfrak{v}_e[\mathbf{\Lambda}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})]\\
&\quad - 2\rho\Omega^2\mathfrak{v}_e[\mathbf{\Lambda}\cdot(\mathbf{\Omega}\times\mathbf{r})] + 2\mathfrak{x}_o\mathfrak{v}_e[\mathbf{\Lambda}\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + \mathfrak{x}_u^2 - 2\mathfrak{v}_a\mathfrak{x}_u[\widehat{\mathbf{p}}\cdot(\mathbf{\Lambda}\times\mathbf{r})] - 2\rho\mathfrak{x}_u[\widehat{\mathbf{p}}\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})]\\
&\quad + 2\rho\Omega^2\mathfrak{x}_u[\widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\mathbf{r})] + \mathfrak{v}_a^2[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\mathbf{\Lambda}\times\mathbf{r})] + 2\rho\mathfrak{v}_a[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})]\\
&\quad - 2\rho\Omega^2\mathfrak{v}_a[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{r})] + 2\mathfrak{x}_o\mathfrak{v}_a[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + 2\varphi_g\varphi_h\mathfrak{v}_a[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&\quad + \rho^2[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})] - 2\rho^2\Omega^2[(\mathbf{\Omega}\times\mathbf{r})\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})] + 2\mathfrak{x}_o\rho[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})]\\
&\quad + 2\varphi_g\varphi_h\rho[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \rho^2\Omega^4[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{r})] - 2\mathfrak{x}_o\rho\Omega^2[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})]\\
&\quad - 2\varphi_g\varphi_h\rho\Omega^2[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{x}_o^2[(\widehat{\mathbf{p}}\times\mathbf{\Omega})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + 2\varphi_g\varphi_h\mathfrak{x}_o[(\widehat{\mathbf{p}}\times\mathbf{\Omega})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&\quad + \varphi_g^2\varphi_h^2[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]
\end{aligned}$$

$$\begin{aligned}
&= \mathfrak{v}_b^2 + 2\mathfrak{v}_c\mathfrak{v}_b\varepsilon_a - 2\mathfrak{v}_d\mathfrak{v}_b\varepsilon_d + 2\mathfrak{v}_e\mathfrak{v}_b\delta_a - 2\mathfrak{x}_u\mathfrak{v}_b\varepsilon_e + 2\mathfrak{v}_a\mathfrak{v}_b\varepsilon_n + 2\rho\mathfrak{v}_b\mathfrak{d}_c - 2\rho\Omega^2\mathfrak{v}_b\varepsilon_k + 2\mathfrak{x}_o\mathfrak{v}_b\varepsilon_j\\
&\quad + 2\varphi_g\varphi_h\mathfrak{v}_b\delta_b + \mathfrak{v}_c^2\Omega^2 - 2\mathfrak{v}_d\mathfrak{v}_c\varepsilon_b + 2\mathfrak{v}_e\mathfrak{v}_c\varepsilon_g - 2\mathfrak{x}_u\mathfrak{v}_c\varepsilon_c + 2\mathfrak{v}_a\mathfrak{v}_c\varepsilon_m - 2\rho\mathfrak{v}_c\mathfrak{d}_l - 2\varphi_g\varphi_h\mathfrak{v}_c\mathfrak{d}_o\\
&\quad + \mathfrak{v}_d^2r^2 - 2\mathfrak{v}_e\mathfrak{v}_d\varepsilon_h + 2\mathfrak{x}_u\mathfrak{v}_d\varepsilon_f - 2\mathfrak{x}_o\mathfrak{v}_d\varepsilon_l - 2\mathfrak{v}_d\varphi_g\varphi_h\varepsilon_o + \mathfrak{v}_e^2\Lambda^2 - 2\mathfrak{x}_u\mathfrak{v}_e\varepsilon_i - 2\rho\mathfrak{v}_e\mathfrak{d}_h\\
&\quad + 2\rho\Omega^2\mathfrak{v}_e\varepsilon_m + 2\mathfrak{x}_o\mathfrak{v}_e\mathfrak{d}_o + \mathfrak{x}_u^2 - 2\mathfrak{v}_a\mathfrak{x}_u\varepsilon_o - 2\rho\mathfrak{x}_u\mathfrak{d}_n + 2\rho\Omega^2\mathfrak{x}_u\varepsilon_l + \mathfrak{v}_a^2[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\mathbf{\Lambda}\times\mathbf{r})]\\
&\quad + 2\rho\mathfrak{v}_a[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})] - 2\rho\Omega^2\mathfrak{v}_a[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{r})] + 2\mathfrak{x}_o\mathfrak{v}_a[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})]\\
&\quad + 2\varphi_g\varphi_h\mathfrak{v}_a[(\mathbf{\Lambda}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] - 2\rho^2\Omega^2[(\mathbf{\Omega}\times\mathbf{r})\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})] + \rho^2\Omega^4[(\mathbf{\Omega}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{r})]\\
&\quad - 2\mathfrak{x}_o\rho\Omega^2[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})] - 2\varphi_g\varphi_h\rho\Omega^2[(\mathbf{\Omega}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \rho^2[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})]\\
&\quad + 2\mathfrak{x}_o\rho[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + 2\varphi_g\varphi_h\rho[(\dot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \mathfrak{x}_o^2[(\widehat{\mathbf{p}}\times\mathbf{\Omega})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})]\\
&\quad + 2\varphi_g\varphi_h\mathfrak{x}_o[(\widehat{\mathbf{p}}\times\mathbf{\Omega})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] + \varphi_g^2\varphi_h^2[(\widehat{\mathbf{p}}\times\mathbf{\Lambda})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})] \text{ by (5.1a), (5.13a), (5.23a) \& (5.23b)}
\end{aligned}$$

$$
\begin{aligned}
&= \mathfrak{f}_d + \mathfrak{v}_a^2[\Lambda^2 r^2 - (\mathbf{\Lambda}\cdot\mathbf{r})^2] + 2\rho\mathfrak{v}_a[(\mathbf{\Lambda}\cdot\dot{\mathbf{\Lambda}})r^2 - (\mathbf{\Lambda}\cdot\mathbf{r})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}})] \\
&\quad - 2\rho\Omega^2\mathfrak{v}_a[(\mathbf{\Lambda}\cdot\mathbf{\Omega})r^2 - (\mathbf{\Lambda}\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{\Omega})] + 2\mathfrak{x}_o\mathfrak{v}_a[(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Omega}) - (\mathbf{\Lambda}\cdot\mathbf{\Omega})(\mathbf{r}\cdot\widehat{\mathbf{p}})] \\
&\quad + 2\varphi_g\varphi_h\mathfrak{v}_a[(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Lambda}) - \Lambda^2(\mathbf{r}\cdot\widehat{\mathbf{p}})] - 2\rho^2\Omega^2[(\mathbf{\Omega}\cdot\dot{\mathbf{\Lambda}})r^2 - (\mathbf{\Omega}\cdot\mathbf{r})(\mathbf{r}\cdot\dot{\mathbf{\Lambda}})] \\
&\quad + \rho^2\Omega^4[\Omega^2 r^2 - (\mathbf{\Omega}\cdot\mathbf{r})^2] - 2\mathfrak{x}_o\rho\Omega^2[(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Omega}) - \Omega^2(\mathbf{r}\cdot\widehat{\mathbf{p}})] \\
&\quad - 2\varphi_g\varphi_h\rho\Omega^2[(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Lambda}) - (\mathbf{\Omega}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\widehat{\mathbf{p}})] + \rho^2[\dot{\mathbf{\Lambda}}^2 r^2 - (\dot{\mathbf{\Lambda}}\cdot\mathbf{r})^2] \\
&\quad + 2\mathfrak{x}_o\rho[(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Omega}) - (\dot{\mathbf{\Lambda}}\cdot\mathbf{\Omega})(\mathbf{r}\cdot\widehat{\mathbf{p}})] + 2\varphi_g\varphi_h\rho[(\dot{\mathbf{\Lambda}}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{\Lambda}) - (\dot{\mathbf{\Lambda}}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\widehat{\mathbf{p}})] \\
&\quad + \mathfrak{x}_o^2[\Omega^2 - (\widehat{\mathbf{p}}\cdot\Omega)^2] + 2\varphi_g\varphi_h\mathfrak{x}_o[(\mathbf{\Omega}\cdot\mathbf{\Lambda}) - (\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})(\mathbf{\Omega}\cdot\widehat{\mathbf{p}})] + \varphi_g^2\varphi_h^2[\Lambda^2 - (\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})^2] \\
&\quad \text{by (5.24o) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{f}_d + \mathfrak{v}_a^2(\Lambda^2 r^2 - \varepsilon_h^2) + 2\rho\mathfrak{v}_a(\varsigma_d r^2 - \varepsilon_h\varsigma_n) - 2\rho\Omega^2\mathfrak{v}_a(\varepsilon_g r^2 - \varepsilon_h\varepsilon_b) + 2\mathfrak{x}_o\mathfrak{v}_a(\varepsilon_i\varepsilon_b - \varepsilon_g\varepsilon_f) \\
&\quad + 2\varphi_g\varphi_h\mathfrak{v}_a(\varepsilon_i\varepsilon_h - \Lambda^2\varepsilon_f) - 2\rho^2\Omega^2(\varsigma_c r^2 - \varepsilon_b\varsigma_n) + \rho^2\Omega^4(\Omega^2 r^2 - \varepsilon_b^2) - 2\mathfrak{x}_o\rho\Omega^2(\varepsilon_c\varepsilon_b - \Omega^2\varepsilon_f) \\
&\quad - 2\varphi_g\varphi_h\rho\Omega^2(\varepsilon_c\varepsilon_h - \varepsilon_g\varepsilon_f) + \rho^2(\varsigma_e r^2 - \varsigma_n^2) + 2\mathfrak{x}_o\rho(\varsigma_a\varepsilon_b - \varsigma_c\varepsilon_f) + 2\varphi_g\varphi_h\rho(\varsigma_a\varepsilon_h - \varsigma_d\varepsilon_f) \\
&\quad + \mathfrak{x}_o^2(\Omega^2 - \varepsilon_c^2) + 2\varphi_g\varphi_h\mathfrak{x}_o(\varepsilon_g - \varepsilon_i\varepsilon_c) + \varphi_g^2\varphi_h^2(\Lambda^2 - \varepsilon_i^2) \text{ by (5.1a) \& (5.23a)}
\end{aligned}
$$

$$\therefore |\boldsymbol{\mathfrak{J}}_r| = \mathfrak{f}_e \text{ by (5.24o)} \tag{5.50b}$$

Art 22h. *Development of equation* (3.15d).

The quantities defined by (3.15d) evaluate as

$$
\begin{aligned}
\aleph_1 &= \widehat{\boldsymbol{\kappa}}\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)} \\
&= \mathfrak{o}_y \text{ by (5.49a)}
\end{aligned} \tag{5.51a}
$$

$$
\begin{aligned}
\aleph_2 &= \mathbf{a}\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)} \\
&= (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot[\varepsilon_b\mathbf{\Omega} - \Omega^2\mathbf{r} + \mathbf{\Lambda}\times\mathbf{r}] \text{ by (1.4a) \& (5.1a)} \\
&= \varepsilon_b[\mathbf{\Omega}\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] - \Omega^2[\mathbf{r}\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + (\mathbf{\Lambda}\times\mathbf{r})\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= \varepsilon_b\mathfrak{o}_g - \Omega^2\mathfrak{o}_e - \mathfrak{o}_t \text{ by (5.48)}
\end{aligned} \tag{5.51b}
$$

$$
\begin{aligned}
\aleph_3 &= \dot{\mathbf{a}}\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)} \\
&= (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot[2\varepsilon_h\mathbf{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\mathbf{\Lambda}}\times\mathbf{r} - \Omega^2(\mathbf{\Omega}\times\mathbf{r}) + \varepsilon_b\mathbf{\Lambda}] \text{ by (5.30a)} \\
&= 2\varepsilon_h[\mathbf{\Omega}\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] - 3\varepsilon_g[\mathbf{r}\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + [(\dot{\mathbf{\Lambda}}\times\mathbf{r})\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] \\
&\qquad - \Omega^2[(\mathbf{\Omega}\times\mathbf{r})\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + \varepsilon_b[\mathbf{\Lambda}\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] \\
&= 2\varepsilon_h\mathfrak{o}_g - 3\varepsilon_g\mathfrak{o}_e - \mathfrak{o}_s - \Omega^2\mathfrak{o}_o + \varepsilon_b\mathfrak{o}_d \text{ by (5.48)}
\end{aligned} \tag{5.51c}
$$

$$
\begin{aligned}
\aleph_4 &= \ddot{\mathbf{a}}\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)} \\
&= (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot[3\varepsilon_h\mathbf{\Lambda} + \varepsilon_b\dot{\mathbf{\Lambda}} + \varrho_t\mathbf{\Omega} + \varrho_u\mathbf{r} + 2\varepsilon_b(\mathbf{\Lambda}\times\mathbf{\Omega}) - 3\Omega^2(\mathbf{\Lambda}\times\mathbf{r}) - 3\varepsilon_g(\mathbf{\Omega}\times\mathbf{r}) \\
&\qquad + (\ddot{\mathbf{\Lambda}}\times\mathbf{r})] \text{ by (5.30b)} \\
&= 3\varepsilon_h[(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot\mathbf{\Lambda}] + \varepsilon_b[(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot\dot{\mathbf{\Lambda}}] + \varrho_t[(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot\mathbf{\Omega}] + \varrho_u[(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot\mathbf{r}] \\
&\quad + 2\varepsilon_b[(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{\Lambda}\times\mathbf{\Omega})] - 3\Omega^2[(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{\Lambda}\times\mathbf{r})] - 3\varepsilon_g[(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot(\mathbf{\Omega}\times\mathbf{r})] \\
&\quad + [(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\cdot(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] \\
&= 3\varepsilon_h\mathfrak{o}_d + \varepsilon_b\mathfrak{o}_h + \varrho_t\mathfrak{o}_g + \varrho_u\mathfrak{o}_e - 2\varepsilon_b\mathfrak{o}_p + 3\Omega^2\mathfrak{o}_t - 3\varepsilon_g\mathfrak{o}_o + \mathfrak{f}_a \text{ by (5.48) \& (5.49c)}
\end{aligned} \tag{5.51d}
$$

$$
\begin{aligned}
\aleph_5 &= \ddddot{\mathbf{a}} \cdot (\boldsymbol{J}_p + \boldsymbol{J}_q) \text{ by (3.15d)} \\
&= (\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot [(\varrho_t + 3\varsigma_n)\boldsymbol{\Lambda} + 4\varepsilon_h\dot{\boldsymbol{\Lambda}} + \varepsilon_b\ddot{\boldsymbol{\Lambda}} + \varrho_v\boldsymbol{\Omega} + 5\varrho_w\mathbf{r} - 5\varepsilon_b(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) - 6\Omega^2(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \\
&\quad + 5\varepsilon_h(\boldsymbol{\Lambda} \times \boldsymbol{\Omega}) - 12\varepsilon_g(\boldsymbol{\Lambda} \times \mathbf{r}) + \varrho_u(\boldsymbol{\Omega} \times \mathbf{r}) + \dddot{\boldsymbol{\Lambda}} \times \mathbf{r}] \text{ by (5.30c)} \\
&= (\varrho_t + 3\varsigma_n)[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \boldsymbol{\Lambda}] + 4\varepsilon_h[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \dot{\boldsymbol{\Lambda}}] + \varepsilon_b[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \ddot{\boldsymbol{\Lambda}}] + \varrho_v[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \boldsymbol{\Omega}] \\
&\quad + 5\varrho_w[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \mathbf{r}] - 5\varepsilon_b[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}})] - 6\Omega^2[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&\quad + 5\varepsilon_h[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\boldsymbol{\Lambda} \times \boldsymbol{\Omega})] - 12\varepsilon_g[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\boldsymbol{\Lambda} \times \mathbf{r})] + \varrho_u[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + [(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\dddot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&= (\varrho_t + 3\varsigma_n)\mathfrak{o}_d + 4\varepsilon_h\mathfrak{o}_h + \varepsilon_b\mathfrak{o}_i + \varrho_v\mathfrak{o}_g + 5\varrho_w\mathfrak{o}_e - 5\varepsilon_b\mathfrak{o}_q + 6\Omega^2\mathfrak{o}_s - 5\varepsilon_h\mathfrak{o}_p \\
&\quad + 12\varepsilon_g\mathfrak{o}_t + \varrho_u\mathfrak{o}_o + \mathfrak{f}_b \text{ by (5.48) \& (5.49d)}
\end{aligned}
\tag{5.51e}
$$

$$
\begin{aligned}
\aleph_6 &= \ddddot{\mathbf{e}} \cdot (\boldsymbol{J}_p + \boldsymbol{J}_q) \text{ by (3.15d)} \\
&= (\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot [\mathfrak{x}_q(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + 3\mathfrak{x}_p(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + 3\mathfrak{x}_o(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \varphi_g\varphi_h(\widehat{\mathbf{p}} \times \ddot{\boldsymbol{\Lambda}}) \\
&\quad + \mathfrak{x}_t\boldsymbol{\Omega} + 3\mathfrak{x}_s\boldsymbol{\Lambda} + 3\mathfrak{x}_r\dot{\boldsymbol{\Lambda}} + \varphi_i\ddot{\boldsymbol{\Lambda}} - \mathfrak{x}_w\widehat{\mathbf{p}}] \text{ by (5.40c)} \\
&= \mathfrak{x}_q[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + 3\mathfrak{x}_p[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + 3\mathfrak{x}_o[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}})] \\
&\quad + \varphi_g\varphi_h[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot (\widehat{\mathbf{p}} \times \ddot{\boldsymbol{\Lambda}})] + \mathfrak{x}_t[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \boldsymbol{\Omega}] + 3\mathfrak{x}_s[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \boldsymbol{\Lambda}] \\
&\quad + 3\mathfrak{x}_r[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \dot{\boldsymbol{\Lambda}}] + \varphi_i[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \ddot{\boldsymbol{\Lambda}}] - \mathfrak{x}_w[(\boldsymbol{J}_p + \boldsymbol{J}_q) \cdot \widehat{\mathbf{p}}] \\
&= -\mathfrak{x}_q\mathfrak{o}_r + 3\mathfrak{x}_p\mathfrak{o}_w + 3\mathfrak{x}_o\mathfrak{o}_v - \varphi_g\varphi_h\mathfrak{o}_z + \mathfrak{x}_t\mathfrak{o}_g + 3\mathfrak{x}_s\mathfrak{o}_d + 3\mathfrak{x}_r\mathfrak{o}_h \\
&\quad + \varphi_i\mathfrak{o}_i - \mathfrak{x}_w\mathfrak{o}_f \text{ by (5.48) \& (5.49b)}
\end{aligned}
\tag{5.51f}
$$

$$
\begin{aligned}
&\ddddot{y}\aleph_1 + \ddddot{\rho}\aleph_2 + 3\dddot{\rho}\aleph_3 + b_3\aleph_4 + \rho\aleph_5 + \aleph_6 \\
&= \mathfrak{v}_g\aleph_1 + \mathfrak{x}_k\aleph_2 + 3\mathfrak{x}_j\aleph_3 + \mathfrak{v}_i\aleph_4 + \rho\aleph_5 + \aleph_6 \text{ by (5.46) \& (5.35)} \\
&= \mathfrak{v}_g\mathfrak{o}_y + \mathfrak{x}_k(\varepsilon_b\mathfrak{o}_g - \Omega^2\mathfrak{o}_e - \mathfrak{o}_t) + 3\mathfrak{x}_j(2\varepsilon_h\mathfrak{o}_g - 3\varepsilon_g\mathfrak{o}_e - \mathfrak{o}_s - \Omega^2\mathfrak{o}_o + \varepsilon_b\mathfrak{o}_d) \\
&\quad + \mathfrak{v}_i(3\varepsilon_h\mathfrak{o}_d + \varepsilon_b\mathfrak{o}_h + \varrho_t\mathfrak{o}_g + \varrho_u\mathfrak{o}_e - 2\varepsilon_b\mathfrak{o}_p + 3\Omega^2\mathfrak{o}_t - 3\varepsilon_g\mathfrak{o}_o + \mathfrak{f}_a) \\
&\quad - \mathfrak{x}_q\mathfrak{o}_r + 3\mathfrak{x}_p\mathfrak{o}_w + 3\mathfrak{x}_o\mathfrak{o}_v - \varphi_g\varphi_h\mathfrak{o}_z + \mathfrak{x}_t\mathfrak{o}_g + 3\mathfrak{x}_s\mathfrak{o}_d + 3\mathfrak{x}_r\mathfrak{o}_h + \varphi_i\mathfrak{o}_i - \mathfrak{x}_w\mathfrak{o}_f \\
&\quad + \rho[(\varrho_t + 3\varsigma_n)\mathfrak{o}_d + 4\varepsilon_h\mathfrak{o}_h + \varepsilon_b\mathfrak{o}_i + \varrho_v\mathfrak{o}_g + 5\varrho_w\mathfrak{o}_e - 5\varepsilon_b\mathfrak{o}_q + 6\Omega^2\mathfrak{o}_s - 5\varepsilon_h\mathfrak{o}_p \\
&\qquad + 12\varepsilon_g\mathfrak{o}_t + \varrho_u\mathfrak{o}_o + \mathfrak{f}_b] \text{ by (5.51)} \\
&= \mathfrak{f}_f \text{ by (5.24p).}
\end{aligned}
\tag{5.52}
$$

Art 22i. *Results of the computations.*

Substituting (5.50), (5.52), (5.45d) and (5.47) into (3.22) leads to

$$
\mathbb{K} = \frac{\mathfrak{f}_c}{(\mathfrak{f}_e)^3}, \quad \mathbb{T} = \frac{\mathfrak{f}_f}{(\mathfrak{f}_c)^2}
\tag{5.53a}
$$

$$
\begin{aligned}
\boldsymbol{\ell}_t = \frac{1}{\mathfrak{f}_e}\Big[&\mathfrak{v}_b\widehat{\boldsymbol{\kappa}} + \mathfrak{v}_c\boldsymbol{\Omega} - \mathfrak{v}_d\mathbf{r} + \mathfrak{v}_e\boldsymbol{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}} + \mathfrak{v}_a(\boldsymbol{\Lambda} \times \mathbf{r}) + \rho(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) - \rho\Omega^2(\boldsymbol{\Omega} \times \mathbf{r}) \\
&+ \mathfrak{x}_o(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})\Big]
\end{aligned}
\tag{5.53b}
$$

$$
\begin{aligned}
\boldsymbol{\ell}_b = \frac{1}{\mathfrak{f}_c}\Big[&\mathfrak{v}_x\boldsymbol{\Lambda} + \mathfrak{v}_y\mathbf{r} + \mathfrak{v}_z\widehat{\mathbf{p}} + \mathfrak{o}_a\boldsymbol{\Omega} + \mathfrak{o}_b\dot{\boldsymbol{\Lambda}} + \mathfrak{o}_c\ddot{\boldsymbol{\Lambda}} + \mathfrak{v}_m(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \mathfrak{v}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathfrak{v}_o(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&+ \mathfrak{v}_b\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_q(\boldsymbol{\Omega} \times \mathbf{r}) + \mathfrak{v}_r(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{v}_s(\boldsymbol{\Omega} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_t(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) \\
&- \mathfrak{v}_e\mathfrak{v}_d(\mathbf{r} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_u(\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{v}_v(\mathbf{r} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\mathfrak{v}_e(\widehat{\mathbf{p}} \times \dot{\boldsymbol{\Lambda}}) + \mathfrak{v}_w(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{v}_e^2(\boldsymbol{\Lambda} \times \dot{\boldsymbol{\Lambda}})\Big]
\end{aligned}
\tag{5.53c}
$$

as the complete set of equations describing the apparent path of the light source for a rotating observer.

Art 23. *Apparent geometry of obliquated rays.*

To evaluate (3.26) for a rotating observer, we introduce, in addition to (5.1), (5.13), (5.23) and (5.24), the quantities

$$
\begin{aligned}
&\mathfrak{K}_a = \mathfrak{Y}\mathfrak{v}_a - \rho\mathfrak{v}_b, \quad \mathfrak{K}_b = -\varphi_g\varphi_h\varepsilon_l - \mathfrak{v}_a(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b) + \varphi_g\varphi_h(\varphi_j - \varphi_i\varepsilon_c)\\
&\mathfrak{K}_c = \varphi_i\varepsilon_b - \varphi_j\varepsilon_f + \varphi_g\varphi_h\varepsilon_l, \quad \mathfrak{K}_d = \mathfrak{x}_u\varphi_i - \varphi_j(\mathfrak{v}_a\varepsilon_b + 2\rho\varepsilon_h + \mathfrak{x}_r)\\
&\mathfrak{K}_e = -\mathfrak{v}_b\varepsilon_a - \mathfrak{v}_a\varepsilon_m - \mathfrak{v}_a(\varphi_i\varepsilon_g - \varphi_j\varepsilon_i) + \rho\varphi_i(\Omega^4 - \varsigma_c) + \rho\varphi_j(\varsigma_a - \Omega^2\varepsilon_c) - \rho\varphi_g\varphi_h\mathfrak{d}_p\\
&\qquad - \rho(2\varepsilon_h\Omega^2 - 2\varepsilon_g\varepsilon_b - \mathfrak{d}_l) - (\mathfrak{x}_r\Omega^2 + \varphi_i\varepsilon_g - \mathfrak{x}_u\varepsilon_c)
\end{aligned}
\tag{5.54a}
$$

$$
\begin{aligned}
\mathfrak{K}_f &= -\mathfrak{v}_b\varphi_g\varphi_h\varepsilon_a - \mathfrak{v}_a\varphi_g\varphi_h\varepsilon_m + 2\rho\varphi_g\varphi_h(\varepsilon_g\varepsilon_b - \varepsilon_h\Omega^2) + \varphi_g\varphi_h\varepsilon_m\\
&\quad + \varphi_g\varphi_h(\mathfrak{x}_u\varepsilon_c - \mathfrak{x}_r\Omega^2 - \varphi_i\varepsilon_g + \varphi_g\varphi_h\mathfrak{d}_o) + \varphi_i(\mathfrak{x}_o\Omega^2 + \varphi_g\varphi_h\varepsilon_g) - \varphi_j(\mathfrak{x}_o\varepsilon_c + \varphi_g\varphi_h\varepsilon_i)\\
\mathfrak{K}_g &= \mathfrak{v}_b\varepsilon_d + \mathfrak{v}_b\varphi_g\varphi_h\varepsilon_e - \mathfrak{v}_a\varphi_a^2 - \mathfrak{v}_a\varphi_g\varphi_h(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)\\
&\quad + \rho\varphi_g\varphi_h(2\varepsilon_h\varepsilon_c - 3\varepsilon_g\varepsilon_f + \varepsilon_b\varepsilon_i - \Omega^2\varepsilon_l) + \rho\Omega^2(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b) - 3\rho\varphi_c\\
&\quad + \varphi_g\varphi_h(\mathfrak{x}_r\varepsilon_c + \varphi_i\varepsilon_i - \mathfrak{x}_u) + \mathfrak{x}_o(\varphi_j - \varphi_i\varepsilon_c) - (\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h - \mathfrak{x}_o\varepsilon_l)
\end{aligned}
\tag{5.54b}
$$

$$
\begin{aligned}
&\mathfrak{K}_h = \mathfrak{K}_f + \rho\mathfrak{y}_o + \mathfrak{Y}(\mathfrak{x}_o\varepsilon_a + \varphi_g\varphi_h\delta_a), \quad \mathfrak{K}_i = \mathfrak{K}_e + \rho(\mathfrak{y}_n + \rho\mathfrak{y}_h) - \rho\mathfrak{Y}(\varsigma_b - \Omega^2\varepsilon_a) - \mathfrak{K}_a\delta_a\\
&\mathfrak{K}_j = \mathfrak{K}_g + \mathfrak{x}_o(\rho\mathfrak{y}_m - \mathfrak{Y}\varepsilon_e) + \rho\Omega^2(\rho\varphi_a^2 - \mathfrak{Y}\varepsilon_d), \quad \mathfrak{K}_k = \mathfrak{K}_a\varepsilon_b - \mathfrak{v}_b\varphi_i + \mathfrak{Y}(\mathfrak{x}_r + 2\rho\varepsilon_h)\\
&\mathfrak{K}_l = \rho(\mathfrak{K}_c + \mathfrak{Y}\varepsilon_d - \rho\varphi_a^2), \quad \mathfrak{K}_m = \mathfrak{K}_b + \mathfrak{K}_a\varepsilon_d - \mathfrak{Y}\varphi_g\varphi_h\varepsilon_e + \rho(\mathfrak{y}_p + 3\rho\varphi_c)\\
&\mathfrak{K}_n = \mathfrak{K}_a\Omega^2 + 3\rho\mathfrak{Y}\varepsilon_g, \quad \mathfrak{K}_o = -\varphi_i\mathfrak{v}_d + \rho(\mathfrak{x}_r\Omega^2 + \rho\mathfrak{y}_g), \quad \mathfrak{K}_p = \mathfrak{v}_b\varphi_j - \mathfrak{Y}\mathfrak{x}_u\\
&\mathfrak{K}_q = \mathfrak{K}_d + \rho\mathfrak{x}_u\varepsilon_b, \quad \mathfrak{K}_r = \varphi_j\mathfrak{v}_d - \rho\mathfrak{x}_u\Omega^2
\end{aligned}
\tag{5.54c}
$$

$$
\begin{aligned}
\mathfrak{K}_r &= \mathfrak{K}_h + \mathfrak{K}_i\varepsilon_f + \mathfrak{K}_j\varepsilon_c + \mathfrak{K}_m\varepsilon_i + \mathfrak{K}_l\varsigma_a - \mathfrak{K}_k\varepsilon_j - \mathfrak{K}_n\mathfrak{d}_t - \mathfrak{Y}\mathfrak{v}_e\delta_b + \mathfrak{v}_e^2\mathfrak{d}_o + \mathfrak{K}_o\varepsilon_l + \rho\Omega^2\mathfrak{v}_e\varepsilon_o\\
\mathfrak{K}_s &= \mathfrak{K}_h\varepsilon_f + \mathfrak{K}_i r^2 + \mathfrak{K}_j\varepsilon_b + \mathfrak{K}_m\varepsilon_h + \mathfrak{K}_l\varsigma_n + \mathfrak{K}_k\varepsilon_k + \mathfrak{Y}\mathfrak{v}_e\varepsilon_n - \mathfrak{K}_p\mathfrak{d}_t + \mathfrak{v}_e^2\varepsilon_m + \mathfrak{K}_q\varepsilon_l\\
&\quad - \varphi_j\mathfrak{v}_e\varepsilon_o\\
\mathfrak{K}_t &= \mathfrak{K}_h\varepsilon_c + \mathfrak{K}_i\varepsilon_b + \mathfrak{K}_j\Omega^2 + \mathfrak{K}_m\varepsilon_g + \mathfrak{K}_l\varsigma_c + \mathfrak{K}_n\varepsilon_k + \mathfrak{Y}\mathfrak{v}_e\mathfrak{d}_e + \mathfrak{K}_p\varepsilon_j + \rho\Omega^2\mathfrak{v}_e\varepsilon_m\\
&\quad - \mathfrak{K}_r\varepsilon_l + \varphi_j\mathfrak{v}_e\mathfrak{d}_o\\
\mathfrak{K}_u &= \mathfrak{K}_h\varepsilon_i + \mathfrak{K}_i\varepsilon_h + \mathfrak{K}_j\varepsilon_g + \mathfrak{K}_m\Lambda^2 + \mathfrak{K}_l\varsigma_d - \mathfrak{K}_k\mathfrak{d}_e + \mathfrak{K}_n\varepsilon_n + \mathfrak{K}_p\delta_b + \mathfrak{K}_q\mathfrak{d}_o - \mathfrak{K}_o\varepsilon_m - \mathfrak{K}_r\varepsilon_o\\
\mathfrak{K}_v &= \mathfrak{K}_h\varsigma_a + \mathfrak{K}_i\varsigma_n + \mathfrak{K}_j\varsigma_c + \mathfrak{K}_m\varsigma_d + \mathfrak{K}_l\varsigma_e + \mathfrak{K}_k\mathfrak{d}_f + \mathfrak{K}_n\mathfrak{d}_c + \mathfrak{Y}\mathfrak{v}_e\mathfrak{d}_u + \mathfrak{K}_p\mathfrak{d}_a + \mathfrak{v}_e^2\mathfrak{d}_v + \mathfrak{K}_q\mathfrak{d}_p\\
&\quad + \mathfrak{K}_o\mathfrak{d}_l + \rho\Omega^2\mathfrak{v}_e\mathfrak{d}_h - \mathfrak{K}_r\mathfrak{d}_n - \varphi_j\mathfrak{v}_e\mathfrak{d}_r
\end{aligned}
\tag{5.54d}
$$

$$
\begin{aligned}
\mathfrak{K}_w &= -\mathfrak{K}_h\varepsilon_j + \mathfrak{K}_i\varepsilon_k - \mathfrak{K}_m\mathfrak{d}_e + \mathfrak{K}_l\mathfrak{d}_f + \mathfrak{K}_k(\Omega^2 - \varepsilon_a^2) - \mathfrak{K}_n(\varepsilon_b - \varepsilon_d\varepsilon_a) + \mathfrak{Y}\mathfrak{v}_e(\varepsilon_g - \delta_a\varepsilon_a)\\
&\quad + \mathfrak{K}_p(\varepsilon_c - \varepsilon_e\varepsilon_a) + \mathfrak{v}_e^2(\varepsilon_a\varepsilon_g - \delta_a\Omega^2) + \mathfrak{K}_q(\varepsilon_e\Omega^2 - \varepsilon_a\varepsilon_c) + \mathfrak{K}_o(\varepsilon_a\varepsilon_b - \varepsilon_d\Omega^2)\\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varepsilon_d\varepsilon_g - \delta_a\varepsilon_b) + \mathfrak{K}_r(\varepsilon_e\varepsilon_b - \varepsilon_d\varepsilon_c) - \varphi_j\mathfrak{v}_e(\varepsilon_e\varepsilon_g - \delta_a\varepsilon_c)\\
\mathfrak{K}_x &= \mathfrak{K}_h\mathfrak{d}_t - \mathfrak{K}_j\varepsilon_k - \mathfrak{K}_m\varepsilon_n - \mathfrak{K}_l\mathfrak{d}_c + \mathfrak{K}_k(\varepsilon_b - \varepsilon_a\varepsilon_d) - \mathfrak{K}_n(r^2 - \varepsilon_d^2) + \mathfrak{Y}\mathfrak{v}_e(\varepsilon_h - \delta_a\varepsilon_d)\\
&\quad + \mathfrak{K}_p(\varepsilon_f - \varepsilon_e\varepsilon_d) + \mathfrak{v}_e^2(\varepsilon_a\varepsilon_h - \delta_a\varepsilon_b) + \mathfrak{K}_q(\varepsilon_e\varepsilon_b - \varepsilon_a\varepsilon_f) + \mathfrak{K}_o(\varepsilon_a r^2 - \varepsilon_d\varepsilon_b)\\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varepsilon_d\varepsilon_h - \delta_a r^2) + \mathfrak{K}_r(\varepsilon_e r^2 - \varepsilon_d\varepsilon_f) - \varphi_j\mathfrak{v}_e(\varepsilon_e\varepsilon_h - \delta_a\varepsilon_f)
\end{aligned}
\tag{5.54e}
$$

$$\begin{aligned}
\mathfrak{K}_y &= -\mathfrak{K}_h\delta_b + \mathfrak{K}_i\varepsilon_n + \mathfrak{K}_j\mathfrak{d}_e + \mathfrak{K}_l\mathfrak{d}_u + \mathfrak{K}_k(\varepsilon_g - \varepsilon_a\delta_a) - \mathfrak{K}_n(\varepsilon_h - \varepsilon_d\delta_a) + \mathfrak{Y}\mathfrak{v}_e(\Lambda^2 - \delta_a^2) \\
&\quad + \mathfrak{K}_p(\varepsilon_i - \varepsilon_e\delta_a) + \mathfrak{v}_e^2(\varepsilon_a\Lambda^2 - \delta_a\varepsilon_g) + \mathfrak{K}_q(\varepsilon_e\varepsilon_g - \varepsilon_a\varepsilon_i) + \mathfrak{K}_o(\varepsilon_a\varepsilon_h - \varepsilon_d\varepsilon_g) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varepsilon_d\Lambda^2 - \delta_a\varepsilon_h) + \mathfrak{K}_r(\varepsilon_e\varepsilon_h - \varepsilon_d\varepsilon_i) - \varphi_j\mathfrak{v}_e(\varepsilon_e\Lambda^2 - \delta_a\varepsilon_i) \\
\mathfrak{K}_z &= -\mathfrak{K}_i\mathfrak{d}_t + \mathfrak{K}_j\varepsilon_j + \mathfrak{K}_m\delta_b + \mathfrak{K}_l\mathfrak{d}_a + \mathfrak{K}_k(\varepsilon_c - \varepsilon_a\varepsilon_e) - \mathfrak{K}_n(\varepsilon_f - \varepsilon_d\varepsilon_e) + \mathfrak{Y}\mathfrak{v}_e(\varepsilon_i - \delta_a\varepsilon_e) \\
&\quad + \mathfrak{K}_p(1 - \varepsilon_e^2) + \mathfrak{v}_e^2(\varepsilon_a\varepsilon_i - \delta_a\varepsilon_c) + \mathfrak{K}_q(\varepsilon_e\varepsilon_c - \varepsilon_a) + \mathfrak{K}_o(\varepsilon_a\varepsilon_f - \varepsilon_d\varepsilon_c) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varepsilon_d\varepsilon_i - \delta_a\varepsilon_f) + \mathfrak{K}_r(\varepsilon_e\varepsilon_f - \varepsilon_d) - \varphi_j\mathfrak{v}_e(\varepsilon_e\varepsilon_i - \delta_a)
\end{aligned} \tag{5.54f}$$

$$\begin{aligned}
\mathfrak{H}_a &= \mathfrak{K}_h\mathfrak{d}_o + \mathfrak{K}_i\varepsilon_m + \mathfrak{K}_l\mathfrak{d}_v + \mathfrak{K}_k(\varepsilon_a\varepsilon_g - \Omega^2\delta_a) - \mathfrak{K}_n(\varepsilon_a\varepsilon_h - \varepsilon_b\delta_a) + \mathfrak{Y}\mathfrak{v}_e(\varepsilon_a\Lambda^2 - \varepsilon_g\delta_a) \\
&\quad + \mathfrak{K}_p(\varepsilon_a\varepsilon_i - \varepsilon_c\delta_a) + \mathfrak{v}_e^2(\Omega^2\Lambda^2 - \varepsilon_g^2) + \mathfrak{K}_q(\varepsilon_c\varepsilon_g - \Omega^2\varepsilon_i) + \mathfrak{K}_o(\Omega^2\varepsilon_h - \varepsilon_b\varepsilon_g) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varepsilon_b\Lambda^2 - \varepsilon_g\varepsilon_h) + \mathfrak{K}_r(\varepsilon_c\varepsilon_h - \varepsilon_b\varepsilon_i) - \varphi_j\mathfrak{v}_e(\varepsilon_c\Lambda^2 - \varepsilon_g\varepsilon_i) \\
\mathfrak{H}_b &= \mathfrak{K}_i\varepsilon_l + \mathfrak{K}_m\mathfrak{d}_o + \mathfrak{K}_l\mathfrak{d}_p + \mathfrak{K}_k(\varepsilon_e\Omega^2 - \varepsilon_c\varepsilon_a) - \mathfrak{K}_n(\varepsilon_e\varepsilon_b - \varepsilon_f\varepsilon_a) + \mathfrak{Y}\mathfrak{v}_e(\varepsilon_e\varepsilon_g - \varepsilon_i\varepsilon_a) \\
&\quad + \mathfrak{K}_p(\varepsilon_e\varepsilon_c - \varepsilon_a) + \mathfrak{v}_e^2(\varepsilon_c\varepsilon_g - \varepsilon_i\Omega^2) + \mathfrak{K}_q(\Omega^2 - \varepsilon_c^2) + \mathfrak{K}_o(\varepsilon_c\varepsilon_b - \varepsilon_f\Omega^2) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varepsilon_f\varepsilon_g - \varepsilon_i\varepsilon_b) + \mathfrak{K}_r(\varepsilon_b - \varepsilon_f\varepsilon_c) - \varphi_j\mathfrak{v}_e(\varepsilon_g - \varepsilon_i\varepsilon)
\end{aligned} \tag{5.54g}$$

$$\begin{aligned}
\mathfrak{H}_c &= \mathfrak{K}_h\varepsilon_l - \mathfrak{K}_m\varepsilon_m + \mathfrak{K}_l\mathfrak{d}_l + \mathfrak{K}_k(\varepsilon_a\varepsilon_b - \Omega^2\varepsilon_d) - \mathfrak{K}_n(\varepsilon_a r^2 - \varepsilon_b\varepsilon_d) + \mathfrak{Y}\mathfrak{v}_e(\varepsilon_a\varepsilon_h - \varepsilon_g\varepsilon_d) \\
&\quad + \mathfrak{K}_p(\varepsilon_a\varepsilon_f - \varepsilon_c\varepsilon_d) + \mathfrak{v}_e^2(\Omega^2\varepsilon_h - \varepsilon_g\varepsilon_b) + \mathfrak{K}_q(\varepsilon_c\varepsilon_b - \Omega^2\varepsilon_f) + \mathfrak{K}_o(\Omega^2 r^2 - \varepsilon_b^2) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varepsilon_b\varepsilon_h - \varepsilon_g r^2) + \mathfrak{K}_r(\varepsilon_c r^2 - \varepsilon_b\varepsilon_f) - \varphi_j\mathfrak{v}_e(\varepsilon_c\varepsilon_h - \varepsilon_g\varepsilon_f) \\
\mathfrak{H}_d &= -\mathfrak{K}_h\varepsilon_o - \mathfrak{K}_j\varepsilon_m - \mathfrak{K}_l\mathfrak{d}_h + \mathfrak{K}_k(\varepsilon_d\varepsilon_g - \varepsilon_b\delta_a) - \mathfrak{K}_n(\varepsilon_d\varepsilon_h - r^2\delta_a) + \mathfrak{Y}\mathfrak{v}_e(\varepsilon_d\Lambda^2 - \varepsilon_h\delta_a) \\
&\quad + \mathfrak{K}_p(\varepsilon_d\varepsilon_i - \varepsilon_f\delta_a) + \mathfrak{v}_e^2(\varepsilon_b\Lambda^2 - \varepsilon_h\varepsilon_g) + \mathfrak{K}_q(\varepsilon_f\varepsilon_g - \varepsilon_b\varepsilon_i) + \mathfrak{K}_o(\varepsilon_b\varepsilon_h - r^2\varepsilon_g) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(r^2\Lambda^2 - \varepsilon_h^2) + \mathfrak{K}_r(\varepsilon_f\varepsilon_h - r^2\varepsilon_i) - \varphi_j\mathfrak{v}_e(\varepsilon_f\Lambda^2 - \varepsilon_h\varepsilon_i)
\end{aligned} \tag{5.54h}$$

$$\begin{aligned}
\mathfrak{H}_e &= -\mathfrak{K}_j\varepsilon_l - \mathfrak{K}_m\varepsilon_o - \mathfrak{K}_l\mathfrak{d}_n + \mathfrak{K}_k(\varepsilon_e\varepsilon_b - \varepsilon_c\varepsilon_d) - \mathfrak{K}_n(\varepsilon_e r^2 - \varepsilon_f\varepsilon_d) + \mathfrak{Y}\mathfrak{v}_e(\varepsilon_e\varepsilon_h - \varepsilon_i\varepsilon_d) \\
&\quad + \mathfrak{K}_p(\varepsilon_e\varepsilon_f - \varepsilon_d) + \mathfrak{v}_e^2(\varepsilon_c\varepsilon_h - \varepsilon_i\varepsilon_b) + \mathfrak{K}_q(\varepsilon_b - \varepsilon_c\varepsilon_f) + \mathfrak{K}_o(\varepsilon_c r^2 - \varepsilon_f\varepsilon_b) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varepsilon_f\varepsilon_h - \varepsilon_i r^2) + \mathfrak{K}_r(r^2 - \varepsilon_f^2) - \varphi_j\mathfrak{v}_e(\varepsilon_h - \varepsilon_i\varepsilon_f) \\
\mathfrak{H}_f &= \mathfrak{K}_i\varepsilon_o - \mathfrak{K}_j\mathfrak{d}_o + \mathfrak{K}_l\mathfrak{d}_r + \mathfrak{K}_k(\varepsilon_e\varepsilon_g - \varepsilon_c\delta_a) - \mathfrak{K}_n(\varepsilon_e\varepsilon_h - \varepsilon_f\delta_a) + \mathfrak{Y}\mathfrak{v}_e(\varepsilon_e\Lambda^2 - \varepsilon_i\delta_a) \\
&\quad + \mathfrak{K}_p(\varepsilon_e\varepsilon_i - \delta_a) + \mathfrak{v}_e^2(\varepsilon_c\Lambda^2 - \varepsilon_i\varepsilon_g) + \mathfrak{K}_q(\varepsilon_g - \varepsilon_c\varepsilon_i) + \mathfrak{K}_o(\varepsilon_c\varepsilon_h - \varepsilon_f\varepsilon_g) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varepsilon_f\Lambda^2 - \varepsilon_i\varepsilon_h) + \mathfrak{K}_r(\varepsilon_h - \varepsilon_f\varepsilon_i) - \varphi_j\mathfrak{v}_e(\Lambda^2 - \varepsilon_i^2)
\end{aligned} \tag{5.54i}$$

$$\begin{aligned}
\mathfrak{H}_g &= -\mathfrak{K}_i\mathfrak{d}_n + \mathfrak{K}_j\mathfrak{d}_p + \mathfrak{K}_m\mathfrak{d}_r + \mathfrak{K}_k(\varsigma_b\varepsilon_c - \varsigma_c\varepsilon_e) - \mathfrak{K}_n(\varsigma_b\varepsilon_f - \varsigma_n\varepsilon_e) + \mathfrak{Y}\mathfrak{v}_e(\varsigma_b\varepsilon_i - \varsigma_d\varepsilon_e) \\
&\quad + \mathfrak{K}_p(\varsigma_b - \varsigma_a\varepsilon_e) + \mathfrak{v}_e^2(\varsigma_c\varepsilon_i - \varsigma_d\varepsilon_c) + \mathfrak{K}_q(\varsigma_a\varepsilon_c - \varsigma_c) + \mathfrak{K}_o(\varsigma_c\varepsilon_f - \varsigma_n\varepsilon_c) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varsigma_n\varepsilon_i - \varsigma_d\varepsilon_f) + \mathfrak{K}_r(\varsigma_a\varepsilon_f - \varsigma_n) - \varphi_j\mathfrak{v}_e(\varsigma_a\varepsilon_i - \varsigma_d) \\
\mathfrak{H}_h &= \mathfrak{K}_h\mathfrak{d}_n - \mathfrak{K}_j\mathfrak{d}_l - \mathfrak{K}_m\mathfrak{d}_h + \mathfrak{K}_k(\varsigma_b\varepsilon_b - \varsigma_c\varepsilon_d) - \mathfrak{K}_n(\varsigma_b r^2 - \varsigma_n\varepsilon_d) + \mathfrak{Y}\mathfrak{v}_e(\varsigma_b\varepsilon_h - \varsigma_d\varepsilon_d) \\
&\quad + \mathfrak{K}_p(\varsigma_b\varepsilon_f - \varsigma_a\varepsilon_d) + \mathfrak{v}_e^2(\varsigma_c\varepsilon_h - \varsigma_d\varepsilon_b) + \mathfrak{K}_q(\varsigma_a\varepsilon_b - \varsigma_c\varepsilon_f) + \mathfrak{K}_o(\varsigma_c r^2 - \varsigma_n\varepsilon_b) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varsigma_n\varepsilon_h - \varsigma_d r^2) + \mathfrak{K}_r(\varsigma_a r^2 - \varsigma_n\varepsilon_f) - \varphi_j\mathfrak{v}_e(\varsigma_a\varepsilon_h - \varsigma_d\varepsilon_f)
\end{aligned} \tag{5.54j}$$

$$\begin{aligned}
\mathfrak{H}_i &= \mathfrak{K}_h\mathfrak{p}_b - \mathfrak{K}_j\mathfrak{d}_m - \mathfrak{K}_m\mathfrak{d}_i - \mathfrak{K}_l\mathfrak{d}_j + \mathfrak{K}_k(\varsigma_g\varepsilon_b - \varsigma_h\varepsilon_d) - \mathfrak{K}_n(\varsigma_g r^2 - \varsigma_o\varepsilon_d) \\
&\quad + \mathfrak{Y}\mathfrak{v}_e(\varsigma_g\varepsilon_h - \varsigma_i\varepsilon_d) + \mathfrak{K}_p(\varsigma_g\varepsilon_f - \varsigma_f\varepsilon_d) + \mathfrak{v}_e^2(\varsigma_h\varepsilon_h - \varsigma_i\varepsilon_b) + \mathfrak{K}_q(\varsigma_f\varepsilon_b - \varsigma_h\varepsilon_f) \\
&\quad + \mathfrak{K}_o(\varsigma_h r^2 - \varsigma_o\varepsilon_b) - \rho\Omega^2\mathfrak{v}_e(\varsigma_o\varepsilon_h - \varsigma_i r^2) + \mathfrak{K}_r(\varsigma_f r^2 - \varsigma_o\varepsilon_f) - \varphi_j\mathfrak{v}_e(\varsigma_f\varepsilon_h - \varsigma_i\varepsilon_f) \\
\mathfrak{H}_j &= \mathfrak{K}_h\varepsilon_e + \mathfrak{K}_i\varepsilon_d + \mathfrak{K}_j\varepsilon_a + \mathfrak{K}_m\delta_a + \mathfrak{K}_l\varsigma_b - \mathfrak{v}_e^2\mathfrak{d}_e + \mathfrak{K}_q\varepsilon_j + \mathfrak{K}_o\varepsilon_k \\
&\quad + \rho\Omega^2\mathfrak{v}_e\varepsilon_n - \mathfrak{K}_r\mathfrak{d}_t - \varphi_j\mathfrak{v}_e\delta_b
\end{aligned} \tag{5.54k}$$

$$
\begin{aligned}
\mathfrak{H}_k &= [\mathfrak{K}_h\mathfrak{K}_r + \mathfrak{K}_i\mathfrak{K}_s + \mathfrak{K}_j\mathfrak{K}_t + \mathfrak{K}_m\mathfrak{K}_u + \mathfrak{K}_l\mathfrak{K}_v + \mathfrak{K}_k\mathfrak{K}_w - \mathfrak{K}_n\mathfrak{K}_x + \mathfrak{Y}\mathfrak{v}_e\mathfrak{K}_y \\
&\quad + \mathfrak{K}_p\mathfrak{K}_z + \mathfrak{v}_e^2\mathfrak{H}_a + \mathfrak{K}_q\mathfrak{H}_b + \mathfrak{K}_o\mathfrak{H}_c - \rho\Omega^2\mathfrak{v}_e\mathfrak{H}_d + \mathfrak{K}_r\mathfrak{H}_e - \varphi_j\mathfrak{v}_e\mathfrak{H}_f]^{1/2} \\
\mathfrak{H}_l &= \mathfrak{v}_f\mathfrak{H}_j + \mathfrak{x}_j(\varepsilon_b\mathfrak{K}_t - \Omega^2\mathfrak{K}_s - \mathfrak{H}_d) + \mathfrak{v}_h(2\varepsilon_h\mathfrak{K}_t - 3\varepsilon_g\mathfrak{K}_s + \mathfrak{H}_h - \Omega^2\mathfrak{H}_c + \varepsilon_b\mathfrak{K}_u) \\
&\quad + \rho(3\varepsilon_h\mathfrak{K}_u + \varepsilon_b\mathfrak{K}_v + \varrho_t\mathfrak{K}_t + \varrho_u\mathfrak{K}_s - 2\varepsilon_b\mathfrak{H}_a + 3\Omega^2\mathfrak{H}_d - 3\varepsilon_g\mathfrak{H}_c + \mathfrak{H}_i) \\
&\quad + \mathfrak{x}_p\mathfrak{H}_b + 2\mathfrak{x}_o\mathfrak{H}_f - \varphi_g\varphi_h\mathfrak{H}_g + \mathfrak{x}_s\mathfrak{K}_t + 2\mathfrak{x}_r\mathfrak{K}_u + \varphi_i\mathfrak{K}_v - \mathfrak{x}_v\mathfrak{K}_r.
\end{aligned}
\tag{5.54l}
$$

Art 23a. *Development of equation* (3.24a).

With the above quantities in view, we derive

$$
\begin{aligned}
\mathcal{S}_a &= \widehat{\boldsymbol{\kappa}} \times \mathbf{u} \text{ by (3.24a)} \\
&= \widehat{\boldsymbol{\kappa}} \times (\boldsymbol{\Omega} \times \mathbf{r}) \text{ by (1.4c)} \\
&= \boldsymbol{\Omega}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) \text{ by (A.1)} \\
&= \varepsilon_d\boldsymbol{\Omega} - \varepsilon_a\mathbf{r} \text{ by (5.1a)}
\end{aligned}
\tag{5.55a}
$$

$$
\begin{aligned}
\mathcal{S}_b &= \widehat{\boldsymbol{\kappa}} \times \mathbf{a} \text{ by (3.24a)} \\
&= \widehat{\boldsymbol{\kappa}} \times (\varepsilon_b\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda} \times \mathbf{r}) \text{ by (1.4a) \& (5.1a)} \\
&= \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \Omega^2(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \widehat{\boldsymbol{\kappa}} \times (\boldsymbol{\Lambda} \times \mathbf{r}) \\
&= \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \Omega^2(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \boldsymbol{\Lambda}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda}) \text{ by (A.1)} \\
&= \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \Omega^2(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \varepsilon_d\boldsymbol{\Lambda} - \delta_a\mathbf{r} \text{ by (5.1a) \& (5.13a)}
\end{aligned}
\tag{5.55b}
$$

$$
\begin{aligned}
\mathcal{S}_c &= \widehat{\boldsymbol{\kappa}} \times \mathbf{e} \text{ by (3.24a)} \\
&= \widehat{\boldsymbol{\kappa}} \times [s_2(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}] \text{ by (1.4b)} \\
&= s_2[\widehat{\boldsymbol{\kappa}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + s_3(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - s_4(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&= s_2[\widehat{\mathbf{p}}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] + s_3(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - s_4(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \text{ by (A.1)} \\
&= s_2(\varepsilon_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Omega}) + s_3(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - s_4(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \text{ by (5.1a)} \\
&= \varphi_g\varphi_h(\varepsilon_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \varphi_j(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \text{ by (5.5)}
\end{aligned}
\tag{5.55c}
$$

$$
\begin{aligned}
\mathcal{S}_d &= \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}} \text{ by (3.24a)} \\
&= \widehat{\boldsymbol{\kappa}} \times [2\varepsilon_h\boldsymbol{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\boldsymbol{\Lambda}} \times \mathbf{r} - \Omega^2(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b\boldsymbol{\Lambda}] \text{ by (5.30a)} \\
&= 2\varepsilon_h(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - 3\varepsilon_g(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + [\widehat{\boldsymbol{\kappa}} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] - \Omega^2[\widehat{\boldsymbol{\kappa}} \times (\boldsymbol{\Omega} \times \mathbf{r})] + \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \\
&= 2\varepsilon_h(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - 3\varepsilon_g(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + [\dot{\boldsymbol{\Lambda}}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}})] \\
&\quad - \Omega^2[\boldsymbol{\Omega}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})] \text{ by (A.1)} \\
&= 2\varepsilon_h(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - 3\varepsilon_g(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \varepsilon_d\dot{\boldsymbol{\Lambda}} - \varsigma_b\mathbf{r} \\
&\quad - \Omega^2(\varepsilon_d\boldsymbol{\Omega} - \varepsilon_a\mathbf{r}) \text{ by (5.1a) \& (5.23a)}
\end{aligned}
\tag{5.55d}
$$

$$
\begin{aligned}
\mathcal{S}_e &= \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{e}} \text{ by (3.24a)} \\
&= \widehat{\boldsymbol{\kappa}} \times [\mathfrak{x}_o(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda}) + \mathfrak{x}_r\boldsymbol{\Omega} + \varphi_i\boldsymbol{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}}] \text{ by (5.40a)} \\
&= \mathfrak{x}_o[\widehat{\boldsymbol{\kappa}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \varphi_g\varphi_h[\widehat{\boldsymbol{\kappa}} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] + \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&= \mathfrak{x}_o[\widehat{\mathbf{p}}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] + \varphi_g\varphi_h[\widehat{\mathbf{p}}(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})] + \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) \\
&\quad + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \text{ by (A.1)} \\
&= \mathfrak{x}_o(\varepsilon_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Omega}) + \varphi_g\varphi_h(\delta_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Lambda}) + \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&\quad \text{by (5.1a) \& (5.13a)}
\end{aligned}
\tag{5.55e}
$$

$$
\begin{aligned}
\mathcal{S}_f &= \mathbf{a} \times \mathbf{u} \text{ by (3.24a)} \\
&= (\varepsilon_b \boldsymbol{\Omega} - \Omega^2 \mathbf{r} + \boldsymbol{\Lambda} \times \mathbf{r}) \times (\boldsymbol{\Omega} \times \mathbf{r}) \text{ by (1.4a), (1.4c) \& (5.1a)} \\
&= \varepsilon_b[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] - \Omega^2[\mathbf{r} \times (\boldsymbol{\Omega} \times \mathbf{r})] + (\boldsymbol{\Lambda} \times \mathbf{r}) \times (\boldsymbol{\Omega} \times \mathbf{r}) \\
&= \varepsilon_b[\boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \Omega^2 \mathbf{r}] - \Omega^2[r^2 \boldsymbol{\Omega} - \mathbf{r}(\mathbf{r} \cdot \boldsymbol{\Omega})] + [\boldsymbol{\Omega}(\mathbf{r} \cdot (\boldsymbol{\Lambda} \times \mathbf{r})) - \mathbf{r}((\boldsymbol{\Omega} \cdot (\boldsymbol{\Lambda} \times \mathbf{r})))] \\
&\quad \text{by (A.1) \& (A.5)} \\
&= \varepsilon_b(\varepsilon_b \boldsymbol{\Omega} - \Omega^2 \mathbf{r}) - \Omega^2(r^2 \boldsymbol{\Omega} - \varepsilon_b \mathbf{r}) - \varepsilon_m \mathbf{r} \text{ by (5.1a)} \\
&= (\varepsilon_b^2 - \Omega^2 r^2)\boldsymbol{\Omega} - \varepsilon_m \mathbf{r} = -\varphi_a^2 \boldsymbol{\Omega} - \varepsilon_m \mathbf{r} \text{ by (5.1b)}
\end{aligned} \tag{5.55f}
$$

$$
\begin{aligned}
\mathcal{S}_g &= \mathbf{a} \times \mathbf{e} \text{ by (3.24a)} \\
&= (\varepsilon_b \boldsymbol{\Omega} - \Omega^2 \mathbf{r} + \boldsymbol{\Lambda} \times \mathbf{r}) \times [s_2(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + s_3 \boldsymbol{\Omega} - s_4 \widehat{\mathbf{p}}] \text{ by (1.4a), (1.4b) \& (5.1a)} \\
&= s_2 \varepsilon_b[(\boldsymbol{\Omega} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + s_3 \varepsilon_b(\boldsymbol{\Omega} \times \boldsymbol{\Omega}) - s_4 \varepsilon_b(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) - s_2 \Omega^2[\mathbf{r} \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] \\
&\quad - s_3 \Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) + s_4 \Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) + s_2[(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] \\
&\quad - s_3[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \mathbf{r})] + s_4[\widehat{\mathbf{p}} \times (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&= s_4 \Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) - s_4 \varepsilon_b(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) - s_3 \Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) + s_2 \varepsilon_b[\Omega^2 \widehat{\mathbf{p}} - \boldsymbol{\Omega}(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})] \\
&\quad - s_2 \Omega^2[\widehat{\mathbf{p}}(\mathbf{r} \cdot \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\mathbf{r} \cdot \widehat{\mathbf{p}})] - s_3[\boldsymbol{\Lambda}(\boldsymbol{\Omega} \cdot \mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})] + s_4[\boldsymbol{\Lambda}(\widehat{\mathbf{p}} \cdot \mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda})] \\
&\quad + s_2[\widehat{\mathbf{p}}(\boldsymbol{\Omega} \cdot (\boldsymbol{\Lambda} \times \mathbf{r})) - \boldsymbol{\Omega}(\widehat{\mathbf{p}} \cdot (\boldsymbol{\Lambda} \times \mathbf{r}))] \text{ by (A.1) \& (A.5)}
\end{aligned}
$$

$$
\begin{aligned}
&= s_4 \Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) - s_4 \varepsilon_b(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) - s_3 \Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) + s_2 \varepsilon_b(\Omega^2 \widehat{\mathbf{p}} - \varepsilon_c \boldsymbol{\Omega}) \\
&\quad - s_2 \Omega^2(\varepsilon_b \widehat{\mathbf{p}} - \varepsilon_f \boldsymbol{\Omega}) - s_3(\varepsilon_b \boldsymbol{\Lambda} - \varepsilon_g \mathbf{r}) + s_4(\varepsilon_f \boldsymbol{\Lambda} - \varepsilon_i \mathbf{r}) + s_2(\varepsilon_m \widehat{\mathbf{p}} - \varepsilon_o \boldsymbol{\Omega}) \text{ by (5.1a)} \\
&= s_4 \Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) - s_4 \varepsilon_b(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) - s_3 \Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) + s_2 \varepsilon_b \Omega^2 \widehat{\mathbf{p}} - s_2 \varepsilon_b \varepsilon_c \boldsymbol{\Omega} \\
&\quad - s_2 \Omega^2 \varepsilon_b \widehat{\mathbf{p}} + s_2 \Omega^2 \varepsilon_f \boldsymbol{\Omega} - s_3 \varepsilon_b \boldsymbol{\Lambda} + s_3 \varepsilon_g \mathbf{r} + s_4 \varepsilon_f \boldsymbol{\Lambda} - s_4 \varepsilon_i \mathbf{r} + s_2 \varepsilon_m \widehat{\mathbf{p}} - s_2 \varepsilon_o \boldsymbol{\Omega} \\
&= s_4 \Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) - s_4 \varepsilon_b(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) - s_3 \Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) + s_2 \varepsilon_b \Omega^2 \widehat{\mathbf{p}} - s_2 \Omega^2 \varepsilon_b \widehat{\mathbf{p}} + s_2 \varepsilon_m \widehat{\mathbf{p}} \\
&\quad - s_2 \varepsilon_b \varepsilon_c \boldsymbol{\Omega} + s_2 \Omega^2 \varepsilon_f \boldsymbol{\Omega} - s_2 \varepsilon_o \boldsymbol{\Omega} - s_3 \varepsilon_b \boldsymbol{\Lambda} + s_4 \varepsilon_f \boldsymbol{\Lambda} + s_3 \varepsilon_g \mathbf{r} - s_4 \varepsilon_i \mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= s_4 \Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) - s_4 \varepsilon_b(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) - s_3 \Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) + s_2(\varepsilon_b \Omega^2 - \Omega^2 \varepsilon_b + \varepsilon_m)\widehat{\mathbf{p}} \\
&\quad + s_2(\Omega^2 \varepsilon_f - \varepsilon_b \varepsilon_c - \varepsilon_o)\boldsymbol{\Omega} + (s_4 \varepsilon_f - s_3 \varepsilon_b)\boldsymbol{\Lambda} + (s_3 \varepsilon_g - s_4 \varepsilon_i)\mathbf{r} \\
&= \varphi_j \Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) - \varphi_j \varepsilon_b(\boldsymbol{\Omega} \times \widehat{\mathbf{p}}) - \varphi_i \Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) + \varphi_g \varphi_h \varepsilon_m \widehat{\mathbf{p}} \\
&\quad + \varphi_g \varphi_h(\Omega^2 \varepsilon_f - \varepsilon_b \varepsilon_c - \varepsilon_o)\boldsymbol{\Omega} + (\varphi_j \varepsilon_f - \varphi_i \varepsilon_b)\boldsymbol{\Lambda} + (\varphi_i \varepsilon_g - \varphi_j \varepsilon_i)\mathbf{r} \text{ by (5.5)}
\end{aligned} \tag{5.55g}
$$

$$
\begin{aligned}
\mathcal{S}_h &= \mathbf{a} \times \dot{\mathbf{a}} \text{ by (3.24a)} \\
&= (\varepsilon_b \boldsymbol{\Omega} - \Omega^2 \mathbf{r} + \boldsymbol{\Lambda} \times \mathbf{r}) \times [2\varepsilon_h \boldsymbol{\Omega} - 3\varepsilon_g \mathbf{r} + \dot{\boldsymbol{\Lambda}} \times \mathbf{r} - \Omega^2(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b \boldsymbol{\Lambda}] \\
&\quad \text{by (1.4a), (5.1a) \& (5.30a)} \\
&= 2\varepsilon_h \varepsilon_b(\boldsymbol{\Omega} \times \boldsymbol{\Omega}) - 3\varepsilon_g \varepsilon_b(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b[\boldsymbol{\Omega} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] - \Omega^2 \varepsilon_b[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - 2\varepsilon_h \Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) + 3\varepsilon_g \Omega^2(\mathbf{r} \times \mathbf{r}) - \Omega^2[\mathbf{r} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \Omega^4[\mathbf{r} \times (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad - \varepsilon_b \Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) + 2\varepsilon_h[(\boldsymbol{\Lambda} \times \mathbf{r}) \times \boldsymbol{\Omega}] - 3\varepsilon_g[(\boldsymbol{\Lambda} \times \mathbf{r}) \times \mathbf{r}] + [(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&\quad - \Omega^2[(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\boldsymbol{\Omega} \times \mathbf{r})] + \varepsilon_b[(\boldsymbol{\Lambda} \times \mathbf{r}) \times \boldsymbol{\Lambda}] \\
&= -3\varepsilon_g \varepsilon_b(\boldsymbol{\Omega} \times \mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) - 2\varepsilon_h \Omega^2(\mathbf{r} \times \boldsymbol{\Omega}) - \varepsilon_b \Omega^2(\mathbf{r} \times \boldsymbol{\Lambda}) + \varepsilon_b[\boldsymbol{\Omega} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&\quad - \Omega^2 \varepsilon_b[\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})] - \Omega^2[\mathbf{r} \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \Omega^4[\mathbf{r} \times (\boldsymbol{\Omega} \times \mathbf{r})] - 2\varepsilon_h[\boldsymbol{\Omega} \times (\boldsymbol{\Lambda} \times \mathbf{r})] \\
&\quad + 3\varepsilon_g[\mathbf{r} \times (\boldsymbol{\Lambda} \times \mathbf{r})] - \varepsilon_b[\boldsymbol{\Lambda} \times (\boldsymbol{\Lambda} \times \mathbf{r})] + [(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] - \Omega^2[(\boldsymbol{\Lambda} \times \mathbf{r}) \times (\boldsymbol{\Omega} \times \mathbf{r})]
\end{aligned}
$$

$$
\begin{aligned}
&= -3\varepsilon_g\varepsilon_b(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \varepsilon_b[\dot{\boldsymbol{\Lambda}}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})]\\
&\quad - \Omega^2\varepsilon_b[\boldsymbol{\Omega}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}] - \Omega^2[r^2\dot{\boldsymbol{\Lambda}} - \mathbf{r}(\mathbf{r}\cdot\dot{\boldsymbol{\Lambda}})] + \Omega^4[r^2\boldsymbol{\Omega} - \mathbf{r}(\mathbf{r}\cdot\boldsymbol{\Omega})]\\
&\quad - 2\varepsilon_h[\boldsymbol{\Lambda}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})] + 3\varepsilon_g[r^2\boldsymbol{\Lambda} - \mathbf{r}(\mathbf{r}\cdot\boldsymbol{\Lambda})] - \varepsilon_b[\boldsymbol{\Lambda}(\boldsymbol{\Lambda}\cdot\mathbf{r}) - \Lambda^2\mathbf{r}]\\
&\quad + [\dot{\boldsymbol{\Lambda}}(\mathbf{r}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})) - \mathbf{r}(\dot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Lambda}\times\mathbf{r}))] - \Omega^2[\boldsymbol{\Omega}(\mathbf{r}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})) - \mathbf{r}(\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r}))]\\
&= -3\varepsilon_g\varepsilon_b(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \varepsilon_b(\varepsilon_b\dot{\boldsymbol{\Lambda}} - \varsigma_c\mathbf{r})\\
&\quad - \Omega^2\varepsilon_b(\varepsilon_b\boldsymbol{\Omega} - \Omega^2\mathbf{r}) - \Omega^2(r^2\dot{\boldsymbol{\Lambda}} - \varsigma_n\mathbf{r}) + \Omega^4(r^2\boldsymbol{\Omega} - \varepsilon_b\mathbf{r}) - 2\varepsilon_h(\varepsilon_b\boldsymbol{\Lambda} - \varepsilon_g\mathbf{r})\\
&\quad + 3\varepsilon_g(r^2\boldsymbol{\Lambda} - \varepsilon_h\mathbf{r}) - \varepsilon_b(\varepsilon_h\boldsymbol{\Lambda} - \Lambda^2\mathbf{r}) - \mathfrak{d}_h\mathbf{r} + \Omega^2\varepsilon_m\mathbf{r} \text{ by (5.1a), (5.23a) \& (5.23b)}
\end{aligned}
$$

$$
\begin{aligned}
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \varepsilon_b^2\dot{\boldsymbol{\Lambda}} - \varepsilon_b\varsigma_c\mathbf{r}\\
&\quad - \Omega^2\varepsilon_b^2\boldsymbol{\Omega} + \varepsilon_b\Omega^4\mathbf{r} - \Omega^2 r^2\dot{\boldsymbol{\Lambda}} + \Omega^2\varsigma_n\mathbf{r} + \Omega^4 r^2\boldsymbol{\Omega} - \Omega^4\varepsilon_b\mathbf{r} - 2\varepsilon_h\varepsilon_b\boldsymbol{\Lambda} + 2\varepsilon_h\varepsilon_g\mathbf{r}\\
&\quad + 3\varepsilon_g r^2\boldsymbol{\Lambda} - 3\varepsilon_g\varepsilon_h\mathbf{r} - \varepsilon_b\varepsilon_h\boldsymbol{\Lambda} + \varepsilon_b\Lambda^2\mathbf{r} + (\Omega^2\varepsilon_m - \mathfrak{d}_h)\mathbf{r}\\
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \varepsilon_b^2\dot{\boldsymbol{\Lambda}} - \Omega^2 r^2\dot{\boldsymbol{\Lambda}}\\
&\quad - \varepsilon_b\varsigma_c\mathbf{r} + \varepsilon_b\Omega^4\mathbf{r} + \Omega^2\varsigma_n\mathbf{r} - \Omega^4\varepsilon_b\mathbf{r} + 2\varepsilon_h\varepsilon_g\mathbf{r} - 3\varepsilon_g\varepsilon_h\mathbf{r} + \varepsilon_b\Lambda^2\mathbf{r} + (\Omega^2\varepsilon_m - \mathfrak{d}_h)\mathbf{r}\\
&\quad - \Omega^2\varepsilon_b^2\boldsymbol{\Omega} + \Omega^4 r^2\boldsymbol{\Omega} - 2\varepsilon_h\varepsilon_b\boldsymbol{\Lambda} + 3\varepsilon_g r^2\boldsymbol{\Lambda} - \varepsilon_b\varepsilon_h\boldsymbol{\Lambda}\\
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + (\varepsilon_b^2 - \Omega^2 r^2)\dot{\boldsymbol{\Lambda}}\\
&\quad + (-\varepsilon_b\varsigma_c + \Omega^2\varsigma_n - \varepsilon_g\varepsilon_h + \varepsilon_b\Lambda^2 + \Omega^2\varepsilon_m - \mathfrak{d}_h)\mathbf{r} - \Omega^2(\varepsilon_b^2 - \Omega^2 r^2)\boldsymbol{\Omega} + 3(\varepsilon_g r^2 - \varepsilon_h\varepsilon_b)\boldsymbol{\Lambda}\\
&= (2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) - \varphi_a^2\dot{\boldsymbol{\Lambda}} + \Omega^2\varphi_a^2\boldsymbol{\Omega} + 3\varphi_c\boldsymbol{\Lambda}\\
&\quad + [\varepsilon_b(\Lambda^2 - \varsigma_c) + \Omega^2(\varepsilon_m + \varsigma_n) - \varepsilon_g\varepsilon_h - \mathfrak{d}_h]\mathbf{r} \text{ by (5.1b)}
\end{aligned}
\tag{5.55h}
$$

$$
\begin{aligned}
\mathfrak{S}_i &= \mathbf{a}\times\dot{\mathbf{e}} \text{ by (3.24a)}\\
&= (\varepsilon_b\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda}\times\mathbf{r})\times[\mathfrak{x}_o(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda}) + \mathfrak{x}_r\boldsymbol{\Omega} + \varphi_i\boldsymbol{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}}]\\
&\qquad \text{by (1.4a), (5.1a) \& (5.40a)}\\
&= \mathfrak{x}_o\varepsilon_b[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \varphi_g\varphi_h\varepsilon_b[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] + \mathfrak{x}_r\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Omega}) + \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})\\
&\quad - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_o\Omega^2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] - \varphi_g\varphi_h\Omega^2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega})\\
&\quad - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) + \mathfrak{x}_o[(\boldsymbol{\Lambda}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \varphi_g\varphi_h[(\boldsymbol{\Lambda}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]\\
&\quad + \mathfrak{x}_r[(\boldsymbol{\Lambda}\times\mathbf{r})\times\boldsymbol{\Omega}] + \varphi_i[(\boldsymbol{\Lambda}\times\mathbf{r})\times\boldsymbol{\Lambda}] - \mathfrak{x}_u[(\boldsymbol{\Lambda}\times\mathbf{r})\times\widehat{\mathbf{p}}]\\
&= \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad + \mathfrak{x}_o\varepsilon_b[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \varphi_g\varphi_h\varepsilon_b[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] - \mathfrak{x}_o\Omega^2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})]\\
&\quad - \varphi_g\varphi_h\Omega^2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] - \mathfrak{x}_r[\boldsymbol{\Omega}\times(\boldsymbol{\Lambda}\times\mathbf{r})] - \varphi_i[\boldsymbol{\Lambda}\times(\boldsymbol{\Lambda}\times\mathbf{r})] + \mathfrak{x}_u[\widehat{\mathbf{p}}\times(\boldsymbol{\Lambda}\times\mathbf{r})]\\
&\quad + \mathfrak{x}_o[(\boldsymbol{\Lambda}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \varphi_g\varphi_h[(\boldsymbol{\Lambda}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]
\end{aligned}
$$

$$
\begin{aligned}
&= \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad + \mathfrak{x}_o\varepsilon_b[\Omega^2\widehat{\mathbf{p}} - \boldsymbol{\Omega}(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})] + \varphi_g\varphi_h\varepsilon_b[\widehat{\mathbf{p}}(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})] - \mathfrak{x}_o\Omega^2[\widehat{\mathbf{p}}(\mathbf{r}\cdot\boldsymbol{\Omega}) - \boldsymbol{\Omega}(\mathbf{r}\cdot\widehat{\mathbf{p}})]\\
&\quad - \varphi_g\varphi_h\Omega^2[\widehat{\mathbf{p}}(\mathbf{r}\cdot\boldsymbol{\Lambda}) - \boldsymbol{\Lambda}(\mathbf{r}\cdot\widehat{\mathbf{p}})] - \mathfrak{x}_r[\boldsymbol{\Lambda}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})] - \varphi_i[\boldsymbol{\Lambda}(\boldsymbol{\Lambda}\cdot\mathbf{r}) - \Lambda^2\mathbf{r}]\\
&\quad + \mathfrak{x}_u[\boldsymbol{\Lambda}(\widehat{\mathbf{p}}\cdot\mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda})] + \mathfrak{x}_o[\widehat{\mathbf{p}}(\boldsymbol{\Omega}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})) - \boldsymbol{\Omega}(\widehat{\mathbf{p}}\cdot(\boldsymbol{\Lambda}\times\mathbf{r}))]\\
&\quad + \varphi_g\varphi_h[\widehat{\mathbf{p}}(\boldsymbol{\Lambda}\cdot(\boldsymbol{\Lambda}\times\mathbf{r})) - \boldsymbol{\Lambda}(\widehat{\mathbf{p}}\cdot(\boldsymbol{\Lambda}\times\mathbf{r}))] \text{ by (A.1) \& (A.5)}\\
&= \varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad + \mathfrak{x}_o\varepsilon_b(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\boldsymbol{\Omega}) + \varphi_g\varphi_h\varepsilon_b(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_c\boldsymbol{\Lambda}) - \mathfrak{x}_o\Omega^2(\varepsilon_b\widehat{\mathbf{p}} - \varepsilon_f\boldsymbol{\Omega})\\
&\quad - \varphi_g\varphi_h\Omega^2(\varepsilon_h\widehat{\mathbf{p}} - \varepsilon_f\boldsymbol{\Lambda}) - \mathfrak{x}_r(\varepsilon_b\boldsymbol{\Lambda} - \varepsilon_g\mathbf{r}) - \varphi_i(\varepsilon_h\boldsymbol{\Lambda} - \Lambda^2\mathbf{r})\\
&\quad + \mathfrak{x}_u(\varepsilon_f\boldsymbol{\Lambda} - \varepsilon_i\mathbf{r}) + \mathfrak{x}_o(\varepsilon_m\widehat{\mathbf{p}} - \varepsilon_o\boldsymbol{\Omega}) - \varphi_g\varphi_h\varepsilon_o\boldsymbol{\Lambda} \text{ by (5.1a)}
\end{aligned}
$$

$$
\begin{aligned}
&= \varphi_i\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad + \mathfrak{x}_o\varepsilon_b\Omega^2\widehat{\mathbf{p}} - \mathfrak{x}_o\varepsilon_b\varepsilon_c\mathbf{\Omega} + \varphi_g\varphi_h\varepsilon_b\varepsilon_g\widehat{\mathbf{p}} - \varphi_g\varphi_h\varepsilon_b\varepsilon_c\mathbf{\Lambda} - \mathfrak{x}_o\Omega^2\varepsilon_b\widehat{\mathbf{p}} + \mathfrak{x}_o\Omega^2\varepsilon_f\mathbf{\Omega}\\
&\quad - \varphi_g\varphi_h\Omega^2\varepsilon_h\widehat{\mathbf{p}} + \varphi_g\varphi_h\Omega^2\varepsilon_f\mathbf{\Lambda} - \mathfrak{x}_r\varepsilon_b\mathbf{\Lambda} + \mathfrak{x}_r\varepsilon_g\mathbf{r} - \varphi_i\varepsilon_h\mathbf{\Lambda} + \varphi_i\Lambda^2\mathbf{r}\\
&\quad + \mathfrak{x}_u\varepsilon_f\mathbf{\Lambda} - \mathfrak{x}_u\varepsilon_i\mathbf{r} + \mathfrak{x}_o\varepsilon_m\widehat{\mathbf{p}} - \mathfrak{x}_o\varepsilon_o\mathbf{\Omega} - \varphi_g\varphi_h\varepsilon_o\mathbf{\Lambda}\\
&= \varphi_i\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad + \mathfrak{x}_o\varepsilon_b\Omega^2\widehat{\mathbf{p}} + \varphi_g\varphi_h\varepsilon_b\varepsilon_g\widehat{\mathbf{p}} - \mathfrak{x}_o\Omega^2\varepsilon_b\widehat{\mathbf{p}} - \varphi_g\varphi_h\Omega^2\varepsilon_h\widehat{\mathbf{p}} + \mathfrak{x}_o\varepsilon_m\widehat{\mathbf{p}} - \mathfrak{x}_o\varepsilon_b\varepsilon_c\mathbf{\Omega}\\
&\quad + \mathfrak{x}_o\Omega^2\varepsilon_f\mathbf{\Omega} - \mathfrak{x}_o\varepsilon_o\mathbf{\Omega} - \varphi_g\varphi_h\varepsilon_b\varepsilon_c\mathbf{\Lambda} + \varphi_g\varphi_h\Omega^2\varepsilon_f\mathbf{\Lambda} - \mathfrak{x}_r\varepsilon_b\mathbf{\Lambda} - \varphi_i\varepsilon_h\mathbf{\Lambda} + \mathfrak{x}_u\varepsilon_f\mathbf{\Lambda}\\
&\quad - \varphi_g\varphi_h\varepsilon_o\mathbf{\Lambda} + \mathfrak{x}_r\varepsilon_g\mathbf{r} + \varphi_i\Lambda^2\mathbf{r} - \mathfrak{x}_u\varepsilon_i\mathbf{r}\\
&= \varphi_i\varepsilon_b(\mathbf{\Omega}\times\mathbf{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\mathbf{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\mathbf{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\mathbf{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad + [\mathfrak{x}_o\varepsilon_m + \varphi_g\varphi_h(\varepsilon_b\varepsilon_g - \Omega^2\varepsilon_h)]\widehat{\mathbf{p}} + \mathfrak{x}_o(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)\mathbf{\Omega} + (\mathfrak{x}_r\varepsilon_g + \varphi_i\Lambda^2 - \mathfrak{x}_u\varepsilon_i)\mathbf{r} \qquad (5.55\text{i})\\
&\quad + [\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h + \varphi_g\varphi_h(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)]\mathbf{\Lambda}
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{S}_j &= \mathbf{u}\times\dot{\mathbf{a}} \text{ by (3.24a)}\\
&= (\mathbf{\Omega}\times\mathbf{r})\times[2\varepsilon_h\mathbf{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\mathbf{\Lambda}}\times\mathbf{r} - \Omega^2(\mathbf{\Omega}\times\mathbf{r}) + \varepsilon_b\mathbf{\Lambda}] \text{ by (1.4c) \& (5.30a)}\\
&= -2\varepsilon_h[\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})] + 3\varepsilon_g[\mathbf{r}\times(\mathbf{\Omega}\times\mathbf{r})] - \varepsilon_b[\mathbf{\Lambda}\times(\mathbf{\Omega}\times\mathbf{r})]\\
&\quad + [(\mathbf{\Omega}\times\mathbf{r})\times(\dot{\mathbf{\Lambda}}\times\mathbf{r})] - \Omega^2[(\mathbf{\Omega}\times\mathbf{r})\times(\mathbf{\Omega}\times\mathbf{r})]\\
&= -2\varepsilon_h[\mathbf{\Omega}(\mathbf{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}] + 3\varepsilon_g[r^2\mathbf{\Omega} - \mathbf{r}(\mathbf{r}\cdot\mathbf{\Omega})] - \varepsilon_b[\mathbf{\Omega}(\mathbf{\Lambda}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Lambda}\cdot\mathbf{\Omega})]\\
&\quad + [\dot{\mathbf{\Lambda}}(\mathbf{r}\cdot(\mathbf{\Omega}\times\mathbf{r})) - \mathbf{r}(\dot{\mathbf{\Lambda}}\cdot(\mathbf{\Omega}\times\mathbf{r}))] \text{ by (A.1) \& (A.5)}\\
&= -2\varepsilon_h(\varepsilon_b\mathbf{\Omega} - \Omega^2\mathbf{r}) + 3\varepsilon_g(r^2\mathbf{\Omega} - \varepsilon_b\mathbf{r}) - \varepsilon_b(\varepsilon_h\mathbf{\Omega} - \varepsilon_g\mathbf{r}) - \mathfrak{d}_l\mathbf{r} \text{ by (5.1a) \& (5.23b)}\\
&= -2\varepsilon_h\varepsilon_b\mathbf{\Omega} + 2\varepsilon_h\Omega^2\mathbf{r} + 3\varepsilon_g r^2\mathbf{\Omega} - 3\varepsilon_g\varepsilon_b\mathbf{r} - \varepsilon_b\varepsilon_h\mathbf{\Omega} + \varepsilon_b\varepsilon_g\mathbf{r} - \mathfrak{d}_l\mathbf{r}\\
&= -2\varepsilon_h\varepsilon_b\mathbf{\Omega} + 3\varepsilon_g r^2\mathbf{\Omega} - \varepsilon_b\varepsilon_h\mathbf{\Omega} + 2\varepsilon_h\Omega^2\mathbf{r} - 3\varepsilon_g\varepsilon_b\mathbf{r} + \varepsilon_b\varepsilon_g\mathbf{r} - \mathfrak{d}_l\mathbf{r}\\
&= 3(\varepsilon_g r^2 - \varepsilon_h\varepsilon_b)\mathbf{\Omega} + (2\varepsilon_h\Omega^2 - 2\varepsilon_g\varepsilon_b - \mathfrak{d}_l)\mathbf{r}\\
&= 3\varphi_c\mathbf{\Omega} + (2\varepsilon_h\Omega^2 - 2\varepsilon_g\varepsilon_b - \mathfrak{d}_l)\mathbf{r} \text{ by (5.1b)} \qquad (5.55\text{j})
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{S}_k &= \mathbf{u}\times\dot{\mathbf{e}} \text{ by (3.24a)}\\
&= (\mathbf{\Omega}\times\mathbf{r})\times[\mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Lambda}) + \mathfrak{x}_r\mathbf{\Omega} + \varphi_i\mathbf{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}}] \text{ by (5.40a)}\\
&= -\mathfrak{x}_r[\mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})] - \varphi_i[\mathbf{\Lambda}\times(\mathbf{\Omega}\times\mathbf{r})] + \mathfrak{x}_u[\widehat{\mathbf{p}}\times(\mathbf{\Omega}\times\mathbf{r})]\\
&\quad + \mathfrak{x}_o[(\mathbf{\Omega}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\mathbf{\Omega})] + \varphi_g\varphi_h[(\mathbf{\Omega}\times\mathbf{r})\times(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&= -\mathfrak{x}_r[\mathbf{\Omega}(\mathbf{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}] - \varphi_i[\mathbf{\Omega}(\mathbf{\Lambda}\cdot\mathbf{r}) - \mathbf{r}(\mathbf{\Lambda}\cdot\mathbf{\Omega})] + \mathfrak{x}_u[\mathbf{\Omega}(\widehat{\mathbf{p}}\cdot\mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})]\\
&\quad + \mathfrak{x}_o[\widehat{\mathbf{p}}(\mathbf{\Omega}\cdot(\mathbf{\Omega}\times\mathbf{r})) - \mathbf{\Omega}(\widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\mathbf{r}))] + \varphi_g\varphi_h[\widehat{\mathbf{p}}(\mathbf{\Lambda}\cdot(\mathbf{\Omega}\times\mathbf{r})) - \mathbf{\Lambda}(\widehat{\mathbf{p}}\cdot(\mathbf{\Omega}\times\mathbf{r}))]\\
&= -\mathfrak{x}_r(\varepsilon_b\mathbf{\Omega} - \Omega^2\mathbf{r}) - \varphi_i(\varepsilon_h\mathbf{\Omega} - \varepsilon_g\mathbf{r}) + \mathfrak{x}_u(\varepsilon_f\mathbf{\Omega} - \varepsilon_c\mathbf{r})\\
&\quad - \mathfrak{x}_o\varepsilon_l\mathbf{\Omega} + \varphi_g\varphi_h(-\varepsilon_m\widehat{\mathbf{p}} - \varepsilon_l\mathbf{\Lambda}) \text{ by (5.1a)}\\
&= -\mathfrak{x}_r\varepsilon_b\mathbf{\Omega} + \mathfrak{x}_r\Omega^2\mathbf{r} - \varphi_i\varepsilon_h\mathbf{\Omega} + \varphi_i\varepsilon_g\mathbf{r} + \mathfrak{x}_u\varepsilon_f\mathbf{\Omega} - \mathfrak{x}_u\varepsilon_c\mathbf{r} - \mathfrak{x}_o\varepsilon_l\mathbf{\Omega} - \varphi_g\varphi_h\varepsilon_m\widehat{\mathbf{p}} + \varphi_g\varphi_h\varepsilon_l\mathbf{\Lambda}\\
&= \varphi_g\varphi_h\varepsilon_l\mathbf{\Lambda} - \varphi_g\varphi_h\varepsilon_m\widehat{\mathbf{p}} + \mathfrak{x}_r\Omega^2\mathbf{r} + \varphi_i\varepsilon_g\mathbf{r} - \mathfrak{x}_u\varepsilon_c\mathbf{r} - \mathfrak{x}_r\varepsilon_b\mathbf{\Omega} - \varphi_i\varepsilon_h\mathbf{\Omega} + \mathfrak{x}_u\varepsilon_f\mathbf{\Omega} - \mathfrak{x}_o\varepsilon_l\mathbf{\Omega}\\
&= \varphi_g\varphi_h(\varepsilon_l\mathbf{\Lambda} - \varepsilon_m\widehat{\mathbf{p}}) + (\mathfrak{x}_r\Omega^2 + \varphi_i\varepsilon_g - \mathfrak{x}_u\varepsilon_c)\mathbf{r} + (\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h - \mathfrak{x}_o\varepsilon_l)\mathbf{\Omega} \qquad (5.55\text{k})
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{S}_l &= \mathbf{e}\times\dot{\mathbf{a}} \text{ by (3.24a)}\\
&= [s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}]\times[2\varepsilon_h\boldsymbol{\Omega} - 3\varepsilon_g\mathbf{r} + \dot{\boldsymbol{\Lambda}}\times\mathbf{r} - \Omega^2(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b\boldsymbol{\Lambda}]\\
&\quad \text{by (1.4b) \& (5.30a)}\\
&= -2\varepsilon_h s_2[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + 3\varepsilon_g s_2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + s_2[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\times(\dot{\boldsymbol{\Lambda}}\times\mathbf{r})]\\
&\quad - \Omega^2 s_2[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\times(\boldsymbol{\Omega}\times\mathbf{r})] - \varepsilon_b s_2[\boldsymbol{\Lambda}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + 2\varepsilon_h s_3(\boldsymbol{\Omega}\times\boldsymbol{\Omega}) - 3\varepsilon_g s_3(\boldsymbol{\Omega}\times\mathbf{r})\\
&\quad + s_3[\boldsymbol{\Omega}\times(\dot{\boldsymbol{\Lambda}}\times\mathbf{r})] - \Omega^2 s_3[\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})] + \varepsilon_b s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\\
&\quad + 3\varepsilon_g s_4(\widehat{\mathbf{p}}\times\mathbf{r}) - s_4[\widehat{\mathbf{p}}\times(\dot{\boldsymbol{\Lambda}}\times\mathbf{r})] + \Omega^2 s_4[\widehat{\mathbf{p}}\times(\boldsymbol{\Omega}\times\mathbf{r})] - \varepsilon_b s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&= -3\varepsilon_g s_3(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + 3\varepsilon_g s_4(\widehat{\mathbf{p}}\times\mathbf{r}) - \varepsilon_b s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad - 2\varepsilon_h s_2[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + 3\varepsilon_g s_2[\mathbf{r}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] - \varepsilon_b s_2[\boldsymbol{\Lambda}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + s_3[\boldsymbol{\Omega}\times(\dot{\boldsymbol{\Lambda}}\times\mathbf{r})]\\
&\quad - \Omega^2 s_3[\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r})] - s_4[\widehat{\mathbf{p}}\times(\dot{\boldsymbol{\Lambda}}\times\mathbf{r})] + \Omega^2 s_4[\widehat{\mathbf{p}}\times(\boldsymbol{\Omega}\times\mathbf{r})] + s_2[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\times(\dot{\boldsymbol{\Lambda}}\times\mathbf{r})]\\
&\quad - \Omega^2 s_2[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\times(\boldsymbol{\Omega}\times\mathbf{r})]
\end{aligned}
$$

$$
\begin{aligned}
&= -3\varepsilon_g s_3(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + 3\varepsilon_g s_4(\widehat{\mathbf{p}}\times\mathbf{r}) - \varepsilon_b s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad - 2\varepsilon_h s_2[\Omega^2\widehat{\mathbf{p}} - \boldsymbol{\Omega}(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})] + 3\varepsilon_g s_2[\widehat{\mathbf{p}}(\mathbf{r}\cdot\boldsymbol{\Omega}) - \boldsymbol{\Omega}(\mathbf{r}\cdot\widehat{\mathbf{p}})] - \varepsilon_b s_2[\widehat{\mathbf{p}}(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega}) - \boldsymbol{\Omega}(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})]\\
&\quad + s_3[\dot{\boldsymbol{\Lambda}}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \mathbf{r}(\boldsymbol{\Omega}\cdot\dot{\boldsymbol{\Lambda}})] - \Omega^2 s_3[\boldsymbol{\Omega}(\boldsymbol{\Omega}\cdot\mathbf{r}) - \Omega^2\mathbf{r}] - s_4[\dot{\boldsymbol{\Lambda}}(\widehat{\mathbf{p}}\cdot\mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}}\cdot\dot{\boldsymbol{\Lambda}})]\\
&\quad + \Omega^2 s_4[\boldsymbol{\Omega}(\widehat{\mathbf{p}}\cdot\mathbf{r}) - \mathbf{r}(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})] + s_2[\dot{\boldsymbol{\Lambda}}(\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})) - \mathbf{r}(\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}))]\\
&\quad - \Omega^2 s_2[\boldsymbol{\Omega}(\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})) - \mathbf{r}(\boldsymbol{\Omega}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}))] \text{ by (A.1) \& (A.5)}\\
&= -3\varepsilon_g s_3(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + 3\varepsilon_g s_4(\widehat{\mathbf{p}}\times\mathbf{r}) - \varepsilon_b s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad - 2\varepsilon_h s_2(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\boldsymbol{\Omega}) + 3\varepsilon_g s_2(\varepsilon_b\widehat{\mathbf{p}} - \varepsilon_f\boldsymbol{\Omega}) - \varepsilon_b s_2(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_i\boldsymbol{\Omega}) + s_3(\varepsilon_b\dot{\boldsymbol{\Lambda}} - \varsigma_c\mathbf{r})\\
&\quad - \Omega^2 s_3(\varepsilon_b\boldsymbol{\Omega} - \Omega^2\mathbf{r}) - s_4(\varepsilon_f\dot{\boldsymbol{\Lambda}} - \varsigma_a\mathbf{r}) + \Omega^2 s_4(\varepsilon_f\boldsymbol{\Omega} - \varepsilon_c\mathbf{r}) + s_2(\varepsilon_l\dot{\boldsymbol{\Lambda}} - \mathfrak{d}_p\mathbf{r})\\
&\quad - \Omega^2 s_2\varepsilon_l\boldsymbol{\Omega} \text{ by (5.1a), (5.23a) \& (5.23b)}
\end{aligned}
$$

$$
\begin{aligned}
&= -3\varepsilon_g s_3(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + 3\varepsilon_g s_4(\widehat{\mathbf{p}}\times\mathbf{r}) - \varepsilon_b s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad - 2\varepsilon_h s_2\Omega^2\widehat{\mathbf{p}} + 2\varepsilon_h s_2\varepsilon_c\boldsymbol{\Omega} + 3\varepsilon_g s_2\varepsilon_b\widehat{\mathbf{p}} - 3\varepsilon_g s_2\varepsilon_f\boldsymbol{\Omega} - \varepsilon_b s_2\varepsilon_g\widehat{\mathbf{p}} + \varepsilon_b s_2\varepsilon_i\boldsymbol{\Omega}\\
&\quad + s_3\varepsilon_b\dot{\boldsymbol{\Lambda}} - s_3\varsigma_c\mathbf{r} - \Omega^2 s_3\varepsilon_b\boldsymbol{\Omega} + \Omega^4 s_3\mathbf{r} - s_4\varepsilon_f\dot{\boldsymbol{\Lambda}} + s_4\varsigma_a\mathbf{r} + \Omega^2 s_4\varepsilon_f\boldsymbol{\Omega}\\
&\quad - \Omega^2 s_4\varepsilon_c\mathbf{r} + s_2\varepsilon_l\dot{\boldsymbol{\Lambda}} - s_2\mathfrak{d}_p\mathbf{r} - \Omega^2 s_2\varepsilon_l\boldsymbol{\Omega}\\
&= -3\varepsilon_g s_3(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + 3\varepsilon_g s_4(\widehat{\mathbf{p}}\times\mathbf{r}) - \varepsilon_b s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad - 2\varepsilon_h s_2\Omega^2\widehat{\mathbf{p}} + 3\varepsilon_g s_2\varepsilon_b\widehat{\mathbf{p}} - \varepsilon_b s_2\varepsilon_g\widehat{\mathbf{p}} + 2\varepsilon_h s_2\varepsilon_c\boldsymbol{\Omega} - 3\varepsilon_g s_2\varepsilon_f\boldsymbol{\Omega} + \varepsilon_b s_2\varepsilon_i\boldsymbol{\Omega}\\
&\quad - \Omega^2 s_3\varepsilon_b\boldsymbol{\Omega} + \Omega^2 s_4\varepsilon_f\boldsymbol{\Omega} - \Omega^2 s_2\varepsilon_l\boldsymbol{\Omega} - s_3\varsigma_c\mathbf{r} + \Omega^4 s_3\mathbf{r} + s_4\varsigma_a\mathbf{r} - s_2\mathfrak{d}_p\mathbf{r} - \Omega^2 s_4\varepsilon_c\mathbf{r}\\
&\quad + s_3\varepsilon_b\dot{\boldsymbol{\Lambda}} - s_4\varepsilon_f\dot{\boldsymbol{\Lambda}} + s_2\varepsilon_l\dot{\boldsymbol{\Lambda}}
\end{aligned}
$$

$$
\begin{aligned}
&= -3\varepsilon_g s_3(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + 3\varepsilon_g s_4(\widehat{\mathbf{p}}\times\mathbf{r}) - \varepsilon_b s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad + 2s_2(\varepsilon_g\varepsilon_b - \varepsilon_h\Omega^2)\widehat{\mathbf{p}} + [s_2(2\varepsilon_h\varepsilon_c - 3\varepsilon_g\varepsilon_f + \varepsilon_b\varepsilon_i - \Omega^2\varepsilon_l) + \Omega^2(s_4\varepsilon_f - s_3\varepsilon_b)]\boldsymbol{\Omega}\\
&\quad + [s_3(\Omega^4 - \varsigma_c) + s_4(\varsigma_a - \Omega^2\varepsilon_c) - s_2\mathfrak{d}_p]\mathbf{r} + (s_3\varepsilon_b - s_4\varepsilon_f + s_2\varepsilon_l)\dot{\boldsymbol{\Lambda}}\\
&= -3\varepsilon_g\varphi_i(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b\varphi_i(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - 2\varepsilon_h\varphi_j(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + 3\varepsilon_g\varphi_j(\widehat{\mathbf{p}}\times\mathbf{r}) - \varepsilon_b\varphi_j(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad + 2\varphi_g\varphi_h(\varepsilon_g\varepsilon_b - \varepsilon_h\Omega^2)\widehat{\mathbf{p}} + [\varphi_g\varphi_h(2\varepsilon_h\varepsilon_c - 3\varepsilon_g\varepsilon_f + \varepsilon_b\varepsilon_i - \Omega^2\varepsilon_l) + \Omega^2(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b)]\boldsymbol{\Omega} \quad (5.55l)\\
&\quad + [\varphi_i(\Omega^4 - \varsigma_c) + \varphi_j(\varsigma_a - \Omega^2\varepsilon_c) - \varphi_g\varphi_h\mathfrak{d}_p]\mathbf{r} + (\varphi_i\varepsilon_b - \varphi_j\varepsilon_f + \varphi_g\varphi_h\varepsilon_l)\dot{\boldsymbol{\Lambda}} \text{ by (5.5)}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}_m &= \mathbf{e}\times\dot{\mathbf{e}} \text{ by (3.24a)}\\
&= [s_2(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + s_3\boldsymbol{\Omega} - s_4\widehat{\mathbf{p}}]\times[\mathfrak{x}_o(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda}) + \mathfrak{x}_r\boldsymbol{\Omega} + \varphi_i\boldsymbol{\Lambda} - \mathfrak{x}_u\widehat{\mathbf{p}}]\\
&\quad \text{by (1.4b) \& (5.40a)}\\
&= \varphi_g\varphi_h s_2[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] - \mathfrak{x}_r s_2[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] - \varphi_i s_2[\boldsymbol{\Lambda}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \mathfrak{x}_u s_2[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})]\\
&\quad + \mathfrak{x}_o s_3[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \varphi_g\varphi_h s_3[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] + \varphi_i s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u s_3(\boldsymbol{\Omega}\times\widehat{\mathbf{p}})\\
&\quad - \mathfrak{x}_o s_4[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] - \varphi_g\varphi_h s_4[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] - \mathfrak{x}_r s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) - \varphi_i s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&= \varphi_i s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u s_3(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) - \varphi_i s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad - \mathfrak{x}_r s_2[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] - \varphi_i s_2[\boldsymbol{\Lambda}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \mathfrak{x}_u s_2[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})]\\
&\quad + \mathfrak{x}_o s_3[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})] + \varphi_g\varphi_h s_3[\boldsymbol{\Omega}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] - \mathfrak{x}_o s_4[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})]\\
&\quad - \varphi_g\varphi_h s_4[\widehat{\mathbf{p}}\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})] + \varphi_g\varphi_h s_2[(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\times(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]
\end{aligned}
$$

$$
\begin{aligned}
&= \varphi_i s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u s_3(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) - \varphi_i s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad - \mathfrak{x}_r s_2[\Omega^2\widehat{\mathbf{p}} - \boldsymbol{\Omega}(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})] - \varphi_i s_2[\widehat{\mathbf{p}}(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega}) - \boldsymbol{\Omega}(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})] + \mathfrak{x}_u s_2[\widehat{\mathbf{p}}(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega}) - \boldsymbol{\Omega}]\\
&\quad + \mathfrak{x}_o s_3[\Omega^2\widehat{\mathbf{p}} - \boldsymbol{\Omega}(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega})] + \varphi_g\varphi_h s_3[\widehat{\mathbf{p}}(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega}) - \boldsymbol{\Lambda}(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})] - \mathfrak{x}_o s_4[\widehat{\mathbf{p}}(\widehat{\mathbf{p}}\cdot\boldsymbol{\Omega}) - \boldsymbol{\Omega}]\\
&\quad - \varphi_g\varphi_h s_4[\widehat{\mathbf{p}}(\widehat{\mathbf{p}}\cdot\boldsymbol{\Lambda}) - \boldsymbol{\Lambda}] + \varphi_g\varphi_h s_2[\widehat{\mathbf{p}}(\boldsymbol{\Lambda}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})) - \boldsymbol{\Lambda}(\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}))] \text{ by (A.1) \& (A.5)}\\
&= \varphi_i s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u s_3(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) - \varphi_i s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad - \mathfrak{x}_r s_2(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\boldsymbol{\Omega}) - \varphi_i s_2(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_i\boldsymbol{\Omega}) + \mathfrak{x}_u s_2(\varepsilon_c\widehat{\mathbf{p}} - \boldsymbol{\Omega})\\
&\quad + \mathfrak{x}_o s_3(\Omega^2\widehat{\mathbf{p}} - \varepsilon_c\boldsymbol{\Omega}) + \varphi_g\varphi_h s_3(\varepsilon_g\widehat{\mathbf{p}} - \varepsilon_c\boldsymbol{\Lambda}) - \mathfrak{x}_o s_4(\varepsilon_c\widehat{\mathbf{p}} - \boldsymbol{\Omega})\\
&\quad - \varphi_g\varphi_h s_4(\varepsilon_i\widehat{\mathbf{p}} - \boldsymbol{\Lambda}) + \varphi_g\varphi_h s_2\mathfrak{d}_o\widehat{\mathbf{p}} \text{ by (5.1a) \& (5.23b)}
\end{aligned}
$$

$$
\begin{aligned}
&= \varphi_i s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u s_3(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) - \varphi_i s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad - \mathfrak{x}_r s_2\Omega^2\widehat{\mathbf{p}} + \mathfrak{x}_r s_2\varepsilon_c\boldsymbol{\Omega} - \varphi_i s_2\varepsilon_g\widehat{\mathbf{p}} + \varphi_i s_2\varepsilon_i\boldsymbol{\Omega} + \mathfrak{x}_u s_2\varepsilon_c\widehat{\mathbf{p}} - \mathfrak{x}_u s_2\boldsymbol{\Omega}\\
&\quad + \mathfrak{x}_o s_3\Omega^2\widehat{\mathbf{p}} - \mathfrak{x}_o s_3\varepsilon_c\boldsymbol{\Omega} + \varphi_g\varphi_h s_3\varepsilon_g\widehat{\mathbf{p}} - \varphi_g\varphi_h s_3\varepsilon_c\boldsymbol{\Lambda} - \mathfrak{x}_o s_4\varepsilon_c\widehat{\mathbf{p}} + \mathfrak{x}_o s_4\boldsymbol{\Omega}\\
&\quad - \varphi_g\varphi_h s_4\varepsilon_i\widehat{\mathbf{p}} + \varphi_g\varphi_h s_4\boldsymbol{\Lambda} + \varphi_g\varphi_h s_2\mathfrak{d}_o\widehat{\mathbf{p}}\\
&= \varphi_i s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u s_3(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) - \varphi_i s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda}) - \mathfrak{x}_r s_2\Omega^2\widehat{\mathbf{p}} - \varphi_i s_2\varepsilon_g\widehat{\mathbf{p}}\\
&\quad + \mathfrak{x}_u s_2\varepsilon_c\widehat{\mathbf{p}} + \mathfrak{x}_o s_3\Omega^2\widehat{\mathbf{p}} + \varphi_g\varphi_h s_3\varepsilon_g\widehat{\mathbf{p}} - \mathfrak{x}_o s_4\varepsilon_c\widehat{\mathbf{p}} - \varphi_g\varphi_h s_4\varepsilon_i\widehat{\mathbf{p}} + \varphi_g\varphi_h s_2\mathfrak{d}_o\widehat{\mathbf{p}}\\
&\quad + \mathfrak{x}_r s_2\varepsilon_c\boldsymbol{\Omega} + \varphi_i s_2\varepsilon_i\boldsymbol{\Omega} - \mathfrak{x}_u s_2\boldsymbol{\Omega} - \mathfrak{x}_o s_3\varepsilon_c\boldsymbol{\Omega} + \mathfrak{x}_o s_4\boldsymbol{\Omega} - \varphi_g\varphi_h s_3\varepsilon_c\boldsymbol{\Lambda} + \varphi_g\varphi_h s_4\boldsymbol{\Lambda}
\end{aligned}
$$

$$
\begin{aligned}
&= \varphi_i s_3(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u s_3(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) - \varphi_i s_4(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\\
&\quad + [s_2(\mathfrak{x}_u\varepsilon_c - \mathfrak{x}_r\Omega^2 - \varphi_i\varepsilon_g + \varphi_g\varphi_h\mathfrak{d}_o) + s_3(\mathfrak{x}_o\Omega^2 + \varphi_g\varphi_h\varepsilon_g) - s_4(\mathfrak{x}_o\varepsilon_c + \varphi_g\varphi_h\varepsilon_i)]\widehat{\mathbf{p}}\\
&\quad + [s_2(\mathfrak{x}_r\varepsilon_c + \varphi_i\varepsilon_i - \mathfrak{x}_u) + \mathfrak{x}_o(s_4 - s_3\varepsilon_c)]\boldsymbol{\Omega} + \varphi_g\varphi_h(s_4 - s_3\varepsilon_c)\boldsymbol{\Lambda}\\
&= \varphi_i^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varphi_i(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\varphi_j(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) - \varphi_i\varphi_j(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda}) + \varphi_g\varphi_h(\varphi_j - \varphi_i\varepsilon_c)\boldsymbol{\Lambda}\\
&\quad + [\varphi_g\varphi_h(\mathfrak{x}_u\varepsilon_c - \mathfrak{x}_r\Omega^2 - \varphi_i\varepsilon_g + \varphi_g\varphi_h\mathfrak{d}_o) + \varphi_i(\mathfrak{x}_o\Omega^2 + \varphi_g\varphi_h\varepsilon_g) - \varphi_j(\mathfrak{x}_o\varepsilon_c + \varphi_g\varphi_h\varepsilon_i)]\widehat{\mathbf{p}} \quad (5.55\text{m})\\
&\quad + [\varphi_g\varphi_h(\mathfrak{x}_r\varepsilon_c + \varphi_i\varepsilon_i - \mathfrak{x}_u) + \mathfrak{x}_o(\varphi_j - \varphi_i\varepsilon_c)]\boldsymbol{\Omega} \text{ by (5.5).}
\end{aligned}
$$

Art 23b. *Development of equation* (3.24b).

$$
\begin{aligned}
\mathcal{S}_t &= (\mathcal{Y}b_1 - \rho\dot{\mathcal{Y}})\mathcal{S}_b + \mathcal{Y}\mathcal{S}_n + \rho\mathcal{S}_o \text{ by (3.24b)}\\
&= (\mathcal{Y}\mathfrak{v}_a - \rho\mathfrak{v}_b)\mathcal{S}_b + \mathcal{Y}(\rho\mathcal{S}_d + \mathcal{S}_e) + \rho(\rho\mathcal{S}_h + \mathcal{S}_i) \text{ by (3.24b) \& (5.46)}\\
&= \mathfrak{K}_a\mathcal{S}_b + \mathcal{Y}\rho\mathcal{S}_d + \mathcal{Y}\mathcal{S}_e + \rho^2\mathcal{S}_h + \rho\mathcal{S}_i \text{ by (5.54a)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_a\{\varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) - \Omega^2(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \varepsilon_d\boldsymbol{\Lambda} - \delta_a\mathbf{r}\} \\
&\quad + \mathfrak{Y}\rho\{2\varepsilon_h(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) - 3\varepsilon_g(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) + \varepsilon_d\dot{\boldsymbol{\Lambda}} - \varsigma_b\mathbf{r} - \Omega^2(\varepsilon_d\boldsymbol{\Omega} - \varepsilon_a\mathbf{r})\} \\
&\quad + \mathfrak{Y}\{\mathfrak{x}_o(\varepsilon_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Omega}) + \varphi_g\varphi_h(\delta_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Lambda}) + \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\} \\
&\quad + \rho\{\varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\qquad + [\mathfrak{x}_o\varepsilon_m + \varphi_g\varphi_h(\varepsilon_b\varepsilon_g - \Omega^2\varepsilon_h)]\widehat{\mathbf{p}} + \mathfrak{x}_o(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)\boldsymbol{\Omega} + (\mathfrak{x}_r\varepsilon_g + \varphi_i\Lambda^2 - \mathfrak{x}_u\varepsilon_i)\mathbf{r} \\
&\qquad + [\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h + \varphi_g\varphi_h(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)]\boldsymbol{\Lambda}\} \\
&\quad + \rho^2\{(2\varepsilon_h\Omega^2 - 3\varepsilon_g\varepsilon_b)(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) - \varphi_a^2\dot{\boldsymbol{\Lambda}} + \Omega^2\varphi_a^2\boldsymbol{\Omega} + 3\varphi_c\boldsymbol{\Lambda} \\
&\qquad + [\varepsilon_b(\Lambda^2 - \varsigma_c) + \Omega^2(\varepsilon_m + \varsigma_n) - \varepsilon_g\varepsilon_h - \mathfrak{d}_h]\mathbf{r}\} \text{ by (5.55)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_a\{\varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) - \Omega^2(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \varepsilon_d\boldsymbol{\Lambda} - \delta_a\mathbf{r}\} \\
&\quad + \mathfrak{Y}\rho\{2\varepsilon_h(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) - 3\varepsilon_g(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) + \varepsilon_d\dot{\boldsymbol{\Lambda}} - \varsigma_b\mathbf{r} - \Omega^2(\varepsilon_d\boldsymbol{\Omega} - \varepsilon_a\mathbf{r})\} \\
&\quad + \mathfrak{Y}\{\mathfrak{x}_o(\varepsilon_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Omega}) + \varphi_g\varphi_h(\delta_a\widehat{\mathbf{p}} - \varepsilon_e\boldsymbol{\Lambda}) + \mathfrak{x}_r(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\} \\
&\quad + \rho\{\varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\qquad + \mathfrak{y}_o\widehat{\mathbf{p}} + \mathfrak{x}_o\mathfrak{y}_m\boldsymbol{\Omega} + \mathfrak{y}_n\mathbf{r} + \mathfrak{y}_p\boldsymbol{\Lambda}\} \\
&\quad + \rho^2\{\mathfrak{y}_g(\boldsymbol{\Omega}\times\mathbf{r}) + \varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) - \varphi_a^2\dot{\boldsymbol{\Lambda}} + \Omega^2\varphi_a^2\boldsymbol{\Omega} + 3\varphi_c\boldsymbol{\Lambda} + \mathfrak{y}_h\mathbf{r}\} \text{ by (5.23n)} \\
&= \mathfrak{K}_a\varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) - \mathfrak{K}_a\Omega^2(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{K}_a\varepsilon_d\boldsymbol{\Lambda} - \mathfrak{K}_a\delta_a\mathbf{r} + 2\rho\mathfrak{Y}\varepsilon_h(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) - 3\rho\mathfrak{Y}\varepsilon_g(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) \\
&\quad + \rho\mathfrak{Y}\varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) + \rho\mathfrak{Y}\varepsilon_d\dot{\boldsymbol{\Lambda}} - \rho\mathfrak{Y}\varsigma_b\mathbf{r} - \rho\mathfrak{Y}\Omega^2\varepsilon_d\boldsymbol{\Omega} + \rho\mathfrak{Y}\Omega^2\varepsilon_a\mathbf{r} + \mathfrak{Y}\mathfrak{x}_o\varepsilon_a\widehat{\mathbf{p}} - \mathfrak{Y}\mathfrak{x}_o\varepsilon_e\boldsymbol{\Omega} \\
&\quad + \mathfrak{Y}\varphi_g\varphi_h\delta_a\widehat{\mathbf{p}} - \mathfrak{Y}\varphi_g\varphi_h\varepsilon_e\boldsymbol{\Lambda} + \mathfrak{Y}\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) + \mathfrak{Y}\varphi_i(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) - \mathfrak{Y}\mathfrak{x}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) \\
&\quad + \rho\varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \rho\mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \rho\mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) - \rho\varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \rho\mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + \rho\mathfrak{y}_o\widehat{\mathbf{p}} + \rho\mathfrak{x}_o\mathfrak{y}_m\boldsymbol{\Omega} + \rho\mathfrak{y}_n\mathbf{r} + \rho\mathfrak{y}_p\boldsymbol{\Lambda} + \rho^2\mathfrak{y}_g(\boldsymbol{\Omega}\times\mathbf{r}) + \rho^2\varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \rho^2\varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) \\
&\quad - \rho^2\varphi_a^2\dot{\boldsymbol{\Lambda}} + \rho^2\Omega^2\varphi_a^2\boldsymbol{\Omega} + 3\rho^2\varphi_c\boldsymbol{\Lambda} + \rho^2\mathfrak{y}_h\mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= \rho\mathfrak{y}_o\widehat{\mathbf{p}} + \mathfrak{Y}\mathfrak{x}_o\varepsilon_a\widehat{\mathbf{p}} + \mathfrak{Y}\varphi_g\varphi_h\delta_a\widehat{\mathbf{p}} + \rho\mathfrak{y}_n\mathbf{r} - \mathfrak{K}_a\delta_a\mathbf{r} - \rho\mathfrak{Y}\varsigma_b\mathbf{r} + \rho\mathfrak{Y}\Omega^2\varepsilon_a\mathbf{r} + \rho^2\mathfrak{y}_h\mathbf{r} + \rho\mathfrak{x}_o\mathfrak{y}_m\boldsymbol{\Omega} \\
&\quad - \rho\mathfrak{Y}\Omega^2\varepsilon_d\boldsymbol{\Omega} - \mathfrak{Y}\mathfrak{x}_o\varepsilon_e\boldsymbol{\Omega} + \rho^2\Omega^2\varphi_a^2\boldsymbol{\Omega} + \rho\mathfrak{y}_p\boldsymbol{\Lambda} + \mathfrak{K}_a\varepsilon_d\boldsymbol{\Lambda} - \mathfrak{Y}\varphi_g\varphi_h\varepsilon_e\boldsymbol{\Lambda} + 3\rho^2\varphi_c\boldsymbol{\Lambda} \\
&\quad + \rho\mathfrak{Y}\varepsilon_d\dot{\boldsymbol{\Lambda}} - \rho^2\varphi_a^2\dot{\boldsymbol{\Lambda}} + \mathfrak{K}_a\varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) + 2\rho\mathfrak{Y}\varepsilon_h(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) + \mathfrak{Y}\mathfrak{x}_r(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) \\
&\quad - \mathfrak{K}_a\Omega^2(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - 3\rho\mathfrak{Y}\varepsilon_g(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \rho\mathfrak{Y}\varepsilon_b(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) + \mathfrak{Y}\varphi_i(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) \\
&\quad - \mathfrak{Y}\mathfrak{x}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \rho\varphi_i\varepsilon_b(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) + \rho^2\varepsilon_b^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \rho\mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) - \rho\mathfrak{x}_r\Omega^2(\mathbf{r}\times\boldsymbol{\Omega}) \\
&\quad + \rho^2\mathfrak{y}_g(\boldsymbol{\Omega}\times\mathbf{r}) - \rho\varphi_i\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) - \rho^2\varepsilon_b\Omega^2(\mathbf{r}\times\boldsymbol{\Lambda}) + \rho\mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}})
\end{aligned}
$$

$$
\begin{aligned}
&= [\rho\mathfrak{y}_o + \mathfrak{Y}(\mathfrak{x}_o\varepsilon_a + \varphi_g\varphi_h\delta_a)]\widehat{\mathbf{p}} + [\rho(\mathfrak{y}_n + \rho\mathfrak{y}_h) - \rho\mathfrak{Y}(\varsigma_b - \Omega^2\varepsilon_a) - \mathfrak{K}_a\delta_a]\mathbf{r} \\
&\quad + [\mathfrak{x}_o(\rho\mathfrak{y}_m - \mathfrak{Y}\varepsilon_e) + \rho\Omega^2(\rho\varphi_a^2 - \mathfrak{Y}\varepsilon_d)]\boldsymbol{\Omega} + [\mathfrak{K}_a\varepsilon_d - \mathfrak{Y}\varphi_g\varphi_h\varepsilon_e + \rho(\mathfrak{y}_p + 3\rho\varphi_c)]\boldsymbol{\Lambda} \\
&\quad + \rho(\mathfrak{Y}\varepsilon_d - \rho\varphi_a^2)\dot{\boldsymbol{\Lambda}} + [\mathfrak{K}_a\varepsilon_b + \mathfrak{Y}(\mathfrak{x}_r + 2\rho\varepsilon_h)](\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) - (\mathfrak{K}_a\Omega^2 + 3\rho\mathfrak{Y}\varepsilon_g)(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) \\
&\quad + \mathfrak{Y}(\rho\varepsilon_b + \varphi_i)(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda}) - \mathfrak{Y}\mathfrak{x}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \rho\varepsilon_b(\varphi_i + \rho\varepsilon_b)(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) - \rho\mathfrak{x}_u\varepsilon_b(\boldsymbol{\Omega}\times\widehat{\mathbf{p}}) \\
&\quad + \rho(\mathfrak{x}_r\Omega^2 + \rho\mathfrak{y}_g)(\boldsymbol{\Omega}\times\mathbf{r}) - \rho\Omega^2(\varphi_i + \rho\varepsilon_b)(\mathbf{r}\times\boldsymbol{\Lambda}) + \rho\mathfrak{x}_u\Omega^2(\mathbf{r}\times\widehat{\mathbf{p}})
\end{aligned} \tag{5.56a}
$$

$$
\begin{aligned}
\mathfrak{S}_u &= \dot{\mathfrak{y}}\mathfrak{S}_p + b_1\mathfrak{S}_q + \rho\mathfrak{S}_r + \mathfrak{S}_s \text{ by (3.24b)} \\
&= \mathfrak{v}_b(\mathfrak{S}_a - \mathfrak{S}_c) + \mathfrak{v}_a(\mathfrak{S}_f - \mathfrak{S}_g) + \rho(\mathfrak{S}_l - \mathfrak{S}_j) + (\mathfrak{S}_m - \mathfrak{S}_k) \text{ by (3.24b) \& (5.46)} \\
&= \mathfrak{v}_b\{\varepsilon_d\mathbf{\Omega} - \varepsilon_a\mathbf{r}\} - \mathfrak{v}_b\{\varphi_g\varphi_h(\varepsilon_a\widehat{\mathbf{p}} - \varepsilon_e\mathbf{\Omega}) + \varphi_i(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) - \varphi_j(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})\} + \mathfrak{v}_a\{-\varphi_a^2\mathbf{\Omega} - \varepsilon_m\mathbf{r}\} \\
&\quad - \mathfrak{v}_a\{\varphi_j\Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) - \varphi_j\varepsilon_b(\mathbf{\Omega} \times \widehat{\mathbf{p}}) - \varphi_i\Omega^2(\mathbf{r} \times \mathbf{\Omega}) + \varphi_g\varphi_h\varepsilon_m\widehat{\mathbf{p}} \\
&\qquad + \varphi_g\varphi_h(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)\mathbf{\Omega} + (\varphi_j\varepsilon_f - \varphi_i\varepsilon_b)\mathbf{\Lambda} + (\varphi_i\varepsilon_g - \varphi_j\varepsilon_i)\mathbf{r}\} \\
&\quad + \rho\{-3\varepsilon_g\varphi_i(\mathbf{\Omega} \times \mathbf{r}) + \varepsilon_b\varphi_i(\mathbf{\Omega} \times \mathbf{\Lambda}) - 2\varepsilon_h\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Omega}) + 3\varepsilon_g\varphi_j(\widehat{\mathbf{p}} \times \mathbf{r}) - \varepsilon_b\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \\
&\qquad + 2\varphi_g\varphi_h(\varepsilon_g\varepsilon_b - \varepsilon_h\Omega^2)\widehat{\mathbf{p}} + [\varphi_g\varphi_h(2\varepsilon_h\varepsilon_c - 3\varepsilon_g\varepsilon_f + \varepsilon_b\varepsilon_i - \Omega^2\varepsilon_l) + \Omega^2(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b)]\mathbf{\Omega} \\
&\qquad + [\varphi_i(\Omega^4 - \varsigma_c) + \varphi_j(\varsigma_a - \Omega^2\varepsilon_c) - \varphi_g\varphi_h\mathfrak{d}_p]\mathbf{r} + (\varphi_i\varepsilon_b - \varphi_j\varepsilon_f + \varphi_g\varphi_h\varepsilon_l)\dot{\mathbf{\Lambda}}\} \\
&\quad - \rho\{3\varphi_c\mathbf{\Omega} + (2\varepsilon_h\Omega^2 - 2\varepsilon_g\varepsilon_b - \mathfrak{d}_l)\mathbf{r}\} \\
&\quad + \{\varphi_i^2(\mathbf{\Omega} \times \mathbf{\Lambda}) - \mathfrak{x}_u\varphi_i(\mathbf{\Omega} \times \widehat{\mathbf{p}}) - \mathfrak{x}_r\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Omega}) - \varphi_i\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) + \varphi_g\varphi_h(\varphi_j - \varphi_i\varepsilon_c)\mathbf{\Lambda} \\
&\qquad + [\varphi_g\varphi_h(\mathfrak{x}_u\varepsilon_c - \mathfrak{x}_r\Omega^2 - \varphi_i\varepsilon_g + \varphi_g\varphi_h\mathfrak{d}_o) + \varphi_i(\mathfrak{x}_o\Omega^2 + \varphi_g\varphi_h\varepsilon_g) - \varphi_j(\mathfrak{x}_o\varepsilon_c + \varphi_g\varphi_h\varepsilon_i)]\widehat{\mathbf{p}} \\
&\qquad + [\varphi_g\varphi_h(\mathfrak{x}_r\varepsilon_c + \varphi_i\varepsilon_i - \mathfrak{x}_u) + \mathfrak{x}_o(\varphi_j - \varphi_i\varepsilon_c)]\mathbf{\Omega}\} \\
&\quad - \{\varphi_g\varphi_h(\varepsilon_l\mathbf{\Lambda} - \varepsilon_m\widehat{\mathbf{p}}) + (\mathfrak{x}_r\Omega^2 + \varphi_i\varepsilon_g - \mathfrak{x}_u\varepsilon_c)\mathbf{r} + (\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h - \mathfrak{x}_o\varepsilon_l)\mathbf{\Omega}\}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{v}_b\varepsilon_d\mathbf{\Omega} - \mathfrak{v}_b\varepsilon_a\mathbf{r} - \mathfrak{v}_b\varphi_g\varphi_h\varepsilon_a\widehat{\mathbf{p}} + \mathfrak{v}_b\varphi_g\varphi_h\varepsilon_e\mathbf{\Omega} - \mathfrak{v}_b\varphi_i(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_b\varphi_j(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \mathfrak{v}_a\varphi_a^2\mathbf{\Omega} \\
&\quad - \mathfrak{v}_a\varepsilon_m\mathbf{r} - \mathfrak{v}_a\varphi_j\Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) + \mathfrak{v}_a\varphi_j\varepsilon_b(\mathbf{\Omega} \times \widehat{\mathbf{p}}) + \mathfrak{v}_a\varphi_i\Omega^2(\mathbf{r} \times \mathbf{\Omega}) - \mathfrak{v}_a\varphi_g\varphi_h\varepsilon_m\widehat{\mathbf{p}} \\
&\quad - \mathfrak{v}_a\varphi_g\varphi_h(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)\mathbf{\Omega} - \mathfrak{v}_a(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b)\mathbf{\Lambda} - \mathfrak{v}_a(\varphi_i\varepsilon_g - \varphi_j\varepsilon_i)\mathbf{r} \\
&\quad - 3\rho\varepsilon_g\varphi_i(\mathbf{\Omega} \times \mathbf{r}) + \rho\varepsilon_b\varphi_i(\mathbf{\Omega} \times \mathbf{\Lambda}) - 2\rho\varepsilon_h\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Omega}) + 3\rho\varepsilon_g\varphi_j(\widehat{\mathbf{p}} \times \mathbf{r}) - \rho\varepsilon_b\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \\
&\quad + 2\rho\varphi_g\varphi_h(\varepsilon_g\varepsilon_b - \varepsilon_h\Omega^2)\widehat{\mathbf{p}} + \rho[\varphi_g\varphi_h(2\varepsilon_h\varepsilon_c - 3\varepsilon_g\varepsilon_f + \varepsilon_b\varepsilon_i - \Omega^2\varepsilon_l) + \Omega^2(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b)]\mathbf{\Omega} \\
&\quad + \rho[\varphi_i(\Omega^4 - \varsigma_c) + \varphi_j(\varsigma_a - \Omega^2\varepsilon_c) - \varphi_g\varphi_h\mathfrak{d}_p]\mathbf{r} + \rho(\varphi_i\varepsilon_b - \varphi_j\varepsilon_f + \varphi_g\varphi_h\varepsilon_l)\dot{\mathbf{\Lambda}} - 3\rho\varphi_c\mathbf{\Omega} \\
&\quad - \rho(2\varepsilon_h\Omega^2 - 2\varepsilon_g\varepsilon_b - \mathfrak{d}_l)\mathbf{r} + \varphi_i^2(\mathbf{\Omega} \times \mathbf{\Lambda}) - \mathfrak{x}_u\varphi_i(\mathbf{\Omega} \times \widehat{\mathbf{p}}) - \mathfrak{x}_r\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Omega}) - \varphi_i\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \\
&\quad + \varphi_g\varphi_h(\varphi_j - \varphi_i\varepsilon_c)\mathbf{\Lambda} + [\varphi_g\varphi_h(\mathfrak{x}_r\varepsilon_c + \varphi_i\varepsilon_i - \mathfrak{x}_u) + \mathfrak{x}_o(\varphi_j - \varphi_i\varepsilon_c)]\mathbf{\Omega} \\
&\quad - \varphi_g\varphi_h\varepsilon_l\mathbf{\Lambda} + \varphi_g\varphi_h\varepsilon_m\widehat{\mathbf{p}} - (\mathfrak{x}_r\Omega^2 + \varphi_i\varepsilon_g - \mathfrak{x}_u\varepsilon_c)\mathbf{r} - (\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h - \mathfrak{x}_o\varepsilon_l)\mathbf{\Omega} \\
&\quad + [\varphi_g\varphi_h(\mathfrak{x}_u\varepsilon_c - \mathfrak{x}_r\Omega^2 - \varphi_i\varepsilon_g + \varphi_g\varphi_h\mathfrak{d}_o) + \varphi_i(\mathfrak{x}_o\Omega^2 + \varphi_g\varphi_h\varepsilon_g) - \varphi_j(\mathfrak{x}_o\varepsilon_c + \varphi_g\varphi_h\varepsilon_i)]\widehat{\mathbf{p}}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{v}_b\varepsilon_a\mathbf{r} - \mathfrak{v}_a\varepsilon_m\mathbf{r} - \mathfrak{v}_a(\varphi_i\varepsilon_g - \varphi_j\varepsilon_i)\mathbf{r} + \rho[\varphi_i(\Omega^4 - \varsigma_c) + \varphi_j(\varsigma_a - \Omega^2\varepsilon_c) - \varphi_g\varphi_h\mathfrak{d}_p]\mathbf{r} \\
&\quad - \rho(2\varepsilon_h\Omega^2 - 2\varepsilon_g\varepsilon_b - \mathfrak{d}_l)\mathbf{r} - (\mathfrak{x}_r\Omega^2 + \varphi_i\varepsilon_g - \mathfrak{x}_u\varepsilon_c)\mathbf{r} \\
&\quad - \mathfrak{v}_b\varphi_g\varphi_h\varepsilon_a\widehat{\mathbf{p}} - \mathfrak{v}_a\varphi_g\varphi_h\varepsilon_m\widehat{\mathbf{p}} + 2\rho\varphi_g\varphi_h(\varepsilon_g\varepsilon_b - \varepsilon_h\Omega^2)\widehat{\mathbf{p}} + \varphi_g\varphi_h\varepsilon_m\widehat{\mathbf{p}} \\
&\quad + [\varphi_g\varphi_h(\mathfrak{x}_u\varepsilon_c - \mathfrak{x}_r\Omega^2 - \varphi_i\varepsilon_g + \varphi_g\varphi_h\mathfrak{d}_o) + \varphi_i(\mathfrak{x}_o\Omega^2 + \varphi_g\varphi_h\varepsilon_g) - \varphi_j(\mathfrak{x}_o\varepsilon_c + \varphi_g\varphi_h\varepsilon_i)]\widehat{\mathbf{p}} \\
&\quad + \mathfrak{v}_b\varepsilon_d\mathbf{\Omega} + \mathfrak{v}_b\varphi_g\varphi_h\varepsilon_e\mathbf{\Omega} - \mathfrak{v}_a\varphi_a^2\mathbf{\Omega} - \mathfrak{v}_a\varphi_g\varphi_h(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o)\mathbf{\Omega} \\
&\quad + \rho[\varphi_g\varphi_h(2\varepsilon_h\varepsilon_c - 3\varepsilon_g\varepsilon_f + \varepsilon_b\varepsilon_i - \Omega^2\varepsilon_l) + \Omega^2(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b)]\mathbf{\Omega} - 3\rho\varphi_c\mathbf{\Omega} \\
&\quad + [\varphi_g\varphi_h(\mathfrak{x}_r\varepsilon_c + \varphi_i\varepsilon_i - \mathfrak{x}_u) + \mathfrak{x}_o(\varphi_j - \varphi_i\varepsilon_c)]\mathbf{\Omega} - (\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h - \mathfrak{x}_o\varepsilon_l)\mathbf{\Omega} - \varphi_g\varphi_h\varepsilon_l\mathbf{\Lambda} \\
&\quad - \mathfrak{v}_a(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b)\mathbf{\Lambda} + \varphi_g\varphi_h(\varphi_j - \varphi_i\varepsilon_c)\mathbf{\Lambda} + \rho(\varphi_i\varepsilon_b - \varphi_j\varepsilon_f + \varphi_g\varphi_h\varepsilon_l)\dot{\mathbf{\Lambda}} - \mathfrak{v}_b\varphi_i(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \\
&\quad + \mathfrak{v}_b\varphi_j(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \mathfrak{v}_a\varphi_j\Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) + 3\rho\varepsilon_g\varphi_j(\widehat{\mathbf{p}} \times \mathbf{r}) + \mathfrak{v}_a\varphi_j\varepsilon_b(\mathbf{\Omega} \times \widehat{\mathbf{p}}) - \mathfrak{x}_u\varphi_i(\mathbf{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad - 2\rho\varepsilon_h\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Omega}) - \mathfrak{x}_r\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Omega}) + \mathfrak{v}_a\varphi_i\Omega^2(\mathbf{r} \times \mathbf{\Omega}) - 3\rho\varepsilon_g\varphi_i(\mathbf{\Omega} \times \mathbf{r}) \\
&\quad + \rho\varepsilon_b\varphi_i(\mathbf{\Omega} \times \mathbf{\Lambda}) + \varphi_i^2(\mathbf{\Omega} \times \mathbf{\Lambda}) - \rho\varepsilon_b\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) - \varphi_i\varphi_j(\widehat{\mathbf{p}} \times \mathbf{\Lambda})
\end{aligned}
$$

$$
\begin{aligned}
&= [-\mathfrak{v}_b\varepsilon_a - \mathfrak{v}_a\varepsilon_m - \mathfrak{v}_a(\varphi_i\varepsilon_g - \varphi_j\varepsilon_i) + \rho\varphi_i(\Omega^4 - \varsigma_c) + \rho\varphi_j(\varsigma_a - \Omega^2\varepsilon_c) - \rho\varphi_g\varphi_h\mathfrak{d}_p \\
&\qquad - \rho(2\varepsilon_h\Omega^2 - 2\varepsilon_g\varepsilon_b - \mathfrak{d}_l) - (\mathfrak{x}_r\Omega^2 + \varphi_i\varepsilon_g - \mathfrak{x}_u\varepsilon_c)]\mathbf{r} \\
&\quad + [-\mathfrak{v}_b\varphi_g\varphi_h\varepsilon_a - \mathfrak{v}_a\varphi_g\varphi_h\varepsilon_m + 2\rho\varphi_g\varphi_h(\varepsilon_g\varepsilon_b - \varepsilon_h\Omega^2) + \varphi_g\varphi_h\varepsilon_m \\
&\qquad + \varphi_g\varphi_h(\mathfrak{x}_u\varepsilon_c - \mathfrak{x}_r\Omega^2 - \varphi_i\varepsilon_g + \varphi_g\varphi_h\mathfrak{d}_o) + \varphi_i(\mathfrak{x}_o\Omega^2 + \varphi_g\varphi_h\varepsilon_g) - \varphi_j(\mathfrak{x}_o\varepsilon_c + \varphi_g\varphi_h\varepsilon_i)]\widehat{\mathbf{p}} \\
&\quad + [\mathfrak{v}_b\varepsilon_d + \mathfrak{v}_b\varphi_g\varphi_h\varepsilon_e - \mathfrak{v}_a\varphi_a^2 - \mathfrak{v}_a\varphi_g\varphi_h(\Omega^2\varepsilon_f - \varepsilon_b\varepsilon_c - \varepsilon_o) \\
&\qquad + \rho\varphi_g\varphi_h(2\varepsilon_h\varepsilon_c - 3\varepsilon_g\varepsilon_f + \varepsilon_b\varepsilon_i - \Omega^2\varepsilon_l) + \rho\Omega^2(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b) - 3\rho\varphi_c \\
&\qquad + \varphi_g\varphi_h(\mathfrak{x}_r\varepsilon_c + \varphi_i\varepsilon_i - \mathfrak{x}_u) + \mathfrak{x}_o(\varphi_j - \varphi_i\varepsilon_c) - (\mathfrak{x}_u\varepsilon_f - \mathfrak{x}_r\varepsilon_b - \varphi_i\varepsilon_h - \mathfrak{x}_o\varepsilon_l)]\mathbf{\Omega} \\
&\quad + [-\varphi_g\varphi_h\varepsilon_l - \mathfrak{v}_a(\varphi_j\varepsilon_f - \varphi_i\varepsilon_b) + \varphi_g\varphi_h(\varphi_j - \varphi_i\varepsilon_c)]\mathbf{\Lambda} + \rho(\varphi_i\varepsilon_b - \varphi_j\varepsilon_f + \varphi_g\varphi_h\varepsilon_l)\dot{\mathbf{\Lambda}} \\
&\quad - \mathfrak{v}_b\varphi_i(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_b\varphi_j(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \varphi_j(\mathfrak{v}_a\Omega^2 + 3\rho\varepsilon_g)(\widehat{\mathbf{p}} \times \mathbf{r}) \\
&\quad + [\mathfrak{x}_u\varphi_i - \varphi_j(\mathfrak{v}_a\varepsilon_b + 2\rho\varepsilon_h + \mathfrak{x}_r)](\widehat{\mathbf{p}} \times \mathbf{\Omega}) - \varphi_i(\mathfrak{v}_a\Omega^2 + 3\rho\varepsilon_g)(\mathbf{\Omega} \times \mathbf{r}) \\
&\quad + \varphi_i(\rho\varepsilon_b + \varphi_i)(\mathbf{\Omega} \times \mathbf{\Lambda}) - \varphi_j(\rho\varepsilon_b + \varphi_i)(\widehat{\mathbf{p}} \times \mathbf{\Lambda})
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_e\mathbf{r} + \mathfrak{K}_f\widehat{\mathbf{p}} + \mathfrak{K}_g\mathbf{\Omega} + \mathfrak{K}_b\mathbf{\Lambda} + \rho\mathfrak{K}_c\dot{\mathbf{\Lambda}} - \mathfrak{v}_b\varphi_i(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_b\varphi_j(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \varphi_j\mathfrak{v}_d(\widehat{\mathbf{p}} \times \mathbf{r}) \\
&\quad + \mathfrak{K}_d(\widehat{\mathbf{p}} \times \mathbf{\Omega}) - \varphi_i\mathfrak{v}_d(\mathbf{\Omega} \times \mathbf{r}) + \varphi_i\mathfrak{v}_e(\mathbf{\Omega} \times \mathbf{\Lambda}) - \varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \qquad (5.56\text{b})
\end{aligned}
$$

by (5.23r), (5.54a) & (5.54b).

Accordingly, we obtain

$$
\begin{aligned}
\mathfrak{S}_t + \mathfrak{S}_u &= [\rho\mathfrak{y}_o + \mathcal{Y}(\mathfrak{x}_o\varepsilon_a + \varphi_g\varphi_h\delta_a)]\widehat{\mathbf{p}} + [\rho(\mathfrak{y}_n + \rho\mathfrak{y}_h) - \rho\mathcal{Y}(\varsigma_b - \Omega^2\varepsilon_a) - \mathfrak{K}_a\delta_a]\mathbf{r} \\
&\quad + [\mathfrak{x}_o(\rho\mathfrak{y}_m - \mathcal{Y}\varepsilon_e) + \rho\Omega^2(\rho\varphi_a^2 - \mathcal{Y}\varepsilon_d)]\mathbf{\Omega} + [\mathfrak{K}_a\varepsilon_d - \mathcal{Y}\varphi_g\varphi_h\varepsilon_e + \rho(\mathfrak{y}_p + 3\rho\varphi_c)]\mathbf{\Lambda} \\
&\quad + \rho(\mathcal{Y}\varepsilon_d - \rho\varphi_a^2)\dot{\mathbf{\Lambda}} + [\mathfrak{K}_a\varepsilon_b + \mathcal{Y}(\mathfrak{x}_r + 2\rho\varepsilon_h)](\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) - (\mathfrak{K}_a\Omega^2 + 3\rho\mathcal{Y}\varepsilon_g)(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathcal{Y}(\rho\varepsilon_b + \varphi_i)(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) - \mathcal{Y}\mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \rho\varepsilon_b(\varphi_i + \rho\varepsilon_b)(\mathbf{\Omega} \times \mathbf{\Lambda}) - \rho\mathfrak{x}_u\varepsilon_b(\mathbf{\Omega} \times \widehat{\mathbf{p}}) \\
&\quad + \rho(\mathfrak{x}_r\Omega^2 + \rho\mathfrak{y}_g)(\mathbf{\Omega} \times \mathbf{r}) - \rho\Omega^2(\varphi_i + \rho\varepsilon_b)(\mathbf{r} \times \mathbf{\Lambda}) + \rho\mathfrak{x}_u\Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) \\
&\quad + \mathfrak{K}_e\mathbf{r} + \mathfrak{K}_f\widehat{\mathbf{p}} + \mathfrak{K}_g\mathbf{\Omega} + \mathfrak{K}_b\mathbf{\Lambda} + \rho\mathfrak{K}_c\dot{\mathbf{\Lambda}} - \mathfrak{v}_b\varphi_i(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + \mathfrak{v}_b\varphi_j(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \varphi_j\mathfrak{v}_d(\widehat{\mathbf{p}} \times \mathbf{r}) \\
&\quad + \mathfrak{K}_d(\widehat{\mathbf{p}} \times \mathbf{\Omega}) - \varphi_i\mathfrak{v}_d(\mathbf{\Omega} \times \mathbf{r}) + \varphi_i\mathfrak{v}_e(\mathbf{\Omega} \times \mathbf{\Lambda}) - \varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \text{ by (5.56)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_f\widehat{\mathbf{p}} + [\rho\mathfrak{y}_o + \mathcal{Y}(\mathfrak{x}_o\varepsilon_a + \varphi_g\varphi_h\delta_a)]\widehat{\mathbf{p}} + \mathfrak{K}_e\mathbf{r} + [\rho(\mathfrak{y}_n + \rho\mathfrak{y}_h) - \rho\mathcal{Y}(\varsigma_b - \Omega^2\varepsilon_a) - \mathfrak{K}_a\delta_a]\mathbf{r} \\
&\quad + \mathfrak{K}_g\mathbf{\Omega} + [\mathfrak{x}_o(\rho\mathfrak{y}_m - \mathcal{Y}\varepsilon_e) + \rho\Omega^2(\rho\varphi_a^2 - \mathcal{Y}\varepsilon_d)]\mathbf{\Omega} + \mathfrak{K}_b\mathbf{\Lambda} + [\mathfrak{K}_a\varepsilon_d - \mathcal{Y}\varphi_g\varphi_h\varepsilon_e + \rho(\mathfrak{y}_p + 3\rho\varphi_c)]\mathbf{\Lambda} \\
&\quad + \rho\mathfrak{K}_c\dot{\mathbf{\Lambda}} + \rho(\mathcal{Y}\varepsilon_d - \rho\varphi_a^2)\dot{\mathbf{\Lambda}} - \mathfrak{v}_b\varphi_i(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) + [\mathfrak{K}_a\varepsilon_b + \mathcal{Y}(\mathfrak{x}_r + 2\rho\varepsilon_h)](\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \\
&\quad - (\mathfrak{K}_a\Omega^2 + 3\rho\mathcal{Y}\varepsilon_g)(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathcal{Y}(\rho\varepsilon_b + \varphi_i)(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) + \mathfrak{v}_b\varphi_j(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \mathcal{Y}\mathfrak{x}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&\quad + \varphi_i\mathfrak{v}_e(\mathbf{\Omega} \times \mathbf{\Lambda}) + \rho\varepsilon_b(\varphi_i + \rho\varepsilon_b)(\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{K}_d(\widehat{\mathbf{p}} \times \mathbf{\Omega}) - \rho\mathfrak{x}_u\varepsilon_b(\mathbf{\Omega} \times \widehat{\mathbf{p}}) - \varphi_i\mathfrak{v}_d(\mathbf{\Omega} \times \mathbf{r}) \\
&\quad + \rho(\mathfrak{x}_r\Omega^2 + \rho\mathfrak{y}_g)(\mathbf{\Omega} \times \mathbf{r}) - \rho\Omega^2(\varphi_i + \rho\varepsilon_b)(\mathbf{r} \times \mathbf{\Lambda}) + \varphi_j\mathfrak{v}_d(\widehat{\mathbf{p}} \times \mathbf{r}) + \rho\mathfrak{x}_u\Omega^2(\mathbf{r} \times \widehat{\mathbf{p}}) \\
&\quad - \varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}} \times \mathbf{\Lambda})
\end{aligned}
$$

$$
\begin{aligned}
&= [\mathfrak{K}_f + \rho\mathfrak{y}_o + \mathcal{Y}(\mathfrak{x}_o\varepsilon_a + \varphi_g\varphi_h\delta_a)]\widehat{\mathbf{p}} + [\mathfrak{K}_e + \rho(\mathfrak{y}_n + \rho\mathfrak{y}_h) - \rho\mathcal{Y}(\varsigma_b - \Omega^2\varepsilon_a) - \mathfrak{K}_a\delta_a]\mathbf{r} \\
&\quad + [\mathfrak{K}_g + \mathfrak{x}_o(\rho\mathfrak{y}_m - \mathcal{Y}\varepsilon_e) + \rho\Omega^2(\rho\varphi_a^2 - \mathcal{Y}\varepsilon_d)]\mathbf{\Omega} + [\mathfrak{K}_b + \mathfrak{K}_a\varepsilon_d - \mathcal{Y}\varphi_g\varphi_h\varepsilon_e + \rho(\mathfrak{y}_p + 3\rho\varphi_c)]\mathbf{\Lambda} \\
&\quad + \rho(\mathfrak{K}_c + \mathcal{Y}\varepsilon_d - \rho\varphi_a^2)\dot{\mathbf{\Lambda}} + [\mathfrak{K}_a\varepsilon_b - \mathfrak{v}_b\varphi_i + \mathcal{Y}(\mathfrak{x}_r + 2\rho\varepsilon_h)](\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) \\
&\quad - (\mathfrak{K}_a\Omega^2 + 3\rho\mathcal{Y}\varepsilon_g)(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathcal{Y}(\rho\varepsilon_b + \varphi_i)(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) + (\mathfrak{v}_b\varphi_j - \mathcal{Y}\mathfrak{x}_u)(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&\quad + [\varphi_i\mathfrak{v}_e + \rho\varepsilon_b(\varphi_i + \rho\varepsilon_b)](\mathbf{\Omega} \times \mathbf{\Lambda}) + (\mathfrak{K}_d + \rho\mathfrak{x}_u\varepsilon_b)(\widehat{\mathbf{p}} \times \mathbf{\Omega}) + [-\varphi_i\mathfrak{v}_d + \rho(\mathfrak{x}_r\Omega^2 + \rho\mathfrak{y}_g)](\mathbf{\Omega} \times \mathbf{r}) \\
&\quad - \rho\Omega^2(\varphi_i + \rho\varepsilon_b)(\mathbf{r} \times \mathbf{\Lambda}) + (\varphi_j\mathfrak{v}_d - \rho\mathfrak{x}_u\Omega^2)(\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}} \times \mathbf{\Lambda})
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_h\widehat{\mathbf{p}} + \mathfrak{K}_i\mathbf{r} + \mathfrak{K}_j\mathbf{\Omega} + \mathfrak{K}_m\mathbf{\Lambda} + \mathfrak{K}_l\dot{\mathbf{\Lambda}} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) - \mathfrak{K}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) + \mathcal{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) + \mathfrak{K}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&\quad + \mathfrak{v}_e^2(\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{K}_q(\widehat{\mathbf{p}} \times \mathbf{\Omega}) + \mathfrak{K}_o(\mathbf{\Omega} \times \mathbf{r}) - \rho\Omega^2\mathfrak{v}_e(\mathbf{r} \times \mathbf{\Lambda}) + \mathfrak{K}_r(\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \qquad (5.57)
\end{aligned}
$$

by (5.23r) & (5.54c).

Art 23c. *Computation of the magnitude of* $\mathfrak{S}_t + \mathfrak{S}_u$.

To calculate the magnitude of the vector $\mathfrak{S}_t + \mathfrak{S}_u$, we first derive

$$
\begin{aligned}
&\widehat{\mathbf{p}} \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= \widehat{\mathbf{p}} \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o (\boldsymbol{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by } (5.57) \\
&= \mathfrak{K}_h (\widehat{\mathbf{p}} \cdot \widehat{\mathbf{p}}) + \mathfrak{K}_i (\widehat{\mathbf{p}} \cdot \mathbf{r}) + \mathfrak{K}_j (\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega}) + \mathfrak{K}_m (\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) + \mathfrak{K}_l (\widehat{\mathbf{p}} \cdot \dot{\boldsymbol{\Lambda}}) + \mathfrak{K}_k [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] \\
&\quad - \mathfrak{K}_n [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{Y} \mathfrak{v}_e [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [\widehat{\mathbf{p}} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{K}_q [\widehat{\mathbf{p}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] \\
&\quad + \mathfrak{K}_o [\widehat{\mathbf{p}} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [\widehat{\mathbf{p}} \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] + \mathfrak{K}_r [\widehat{\mathbf{p}} \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [\widehat{\mathbf{p}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
&= \mathfrak{K}_h + \mathfrak{K}_i \varepsilon_f + \mathfrak{K}_j \varepsilon_c + \mathfrak{K}_m \varepsilon_i + \mathfrak{K}_l \varsigma_a - \mathfrak{K}_k \varepsilon_j - \mathfrak{K}_n \mathfrak{d}_t - \mathfrak{Y} \mathfrak{v}_e \delta_b + \mathfrak{v}_e^2 \mathfrak{d}_o + \mathfrak{K}_o \varepsilon_l + \rho \Omega^2 \mathfrak{v}_e \varepsilon_o \\
&\qquad \text{by } (5.1\text{a}), (5.13\text{a}), (5.23\text{a}) \ \& \ (5.23\text{b}) \\
&= \mathfrak{K}_r \text{ by } (5.54\text{d})
\end{aligned} \tag{5.58a}
$$

$$
\begin{aligned}
&\mathbf{r} \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= \mathbf{r} \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o (\boldsymbol{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by } (5.57) \\
&= \mathfrak{K}_h (\mathbf{r} \cdot \widehat{\mathbf{p}}) + \mathfrak{K}_i (\mathbf{r} \cdot \mathbf{r}) + \mathfrak{K}_j (\mathbf{r} \cdot \boldsymbol{\Omega}) + \mathfrak{K}_m (\mathbf{r} \cdot \boldsymbol{\Lambda}) + \mathfrak{K}_l (\mathbf{r} \cdot \dot{\boldsymbol{\Lambda}}) + \mathfrak{K}_k [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] \\
&\quad - \mathfrak{K}_n [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{Y} \mathfrak{v}_e [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [\mathbf{r} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{K}_q [\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] \\
&\quad + \mathfrak{K}_o [\mathbf{r} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [\mathbf{r} \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] + \mathfrak{K}_r [\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
&= \mathfrak{K}_h \varepsilon_f + \mathfrak{K}_i r^2 + \mathfrak{K}_j \varepsilon_b + \mathfrak{K}_m \varepsilon_h + \mathfrak{K}_l \varsigma_n + \mathfrak{K}_k \varepsilon_k + \mathfrak{Y} \mathfrak{v}_e \varepsilon_n - \mathfrak{K}_p \mathfrak{d}_t + \mathfrak{v}_e^2 \varepsilon_m + \mathfrak{K}_q \varepsilon_l - \varphi_j \mathfrak{v}_e \varepsilon_o \\
&\qquad \text{by } (5.1\text{a}), (5.23\text{a}) \ \& \ (5.23\text{b}) \\
&= \mathfrak{K}_s \text{ by } (5.54\text{d})
\end{aligned} \tag{5.58b}
$$

$$
\begin{aligned}
&\boldsymbol{\Omega} \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= \boldsymbol{\Omega} \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o (\boldsymbol{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by } (5.57) \\
&= \mathfrak{K}_h (\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}}) + \mathfrak{K}_i (\boldsymbol{\Omega} \cdot \mathbf{r}) + \mathfrak{K}_j (\boldsymbol{\Omega} \cdot \boldsymbol{\Omega}) + \mathfrak{K}_m (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}) + \mathfrak{K}_l (\boldsymbol{\Omega} \cdot \dot{\boldsymbol{\Lambda}}) + \mathfrak{K}_k [\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] \\
&\quad - \mathfrak{K}_n [\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{Y} \mathfrak{v}_e [\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p [\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [\boldsymbol{\Omega} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{K}_q [\boldsymbol{\Omega} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] \\
&\quad + \mathfrak{K}_o [\boldsymbol{\Omega} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [\boldsymbol{\Omega} \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] + \mathfrak{K}_r [\boldsymbol{\Omega} \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [\boldsymbol{\Omega} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
&= \mathfrak{K}_h \varepsilon_c + \mathfrak{K}_i \varepsilon_b + \mathfrak{K}_j \Omega^2 + \mathfrak{K}_m \varepsilon_g + \mathfrak{K}_l \varsigma_c + \mathfrak{K}_n \varepsilon_k + \mathfrak{Y} \mathfrak{v}_e \mathfrak{d}_e + \mathfrak{K}_p \varepsilon_j + \rho \Omega^2 \mathfrak{v}_e \varepsilon_m - \mathfrak{K}_r \varepsilon_l \\
&\qquad + \varphi_j \mathfrak{v}_e \mathfrak{d}_o \text{ by } (5.1\text{a}), (5.23\text{a}) \ \& \ (5.23\text{b}) \\
&= \mathfrak{K}_t \text{ by } (5.54\text{d})
\end{aligned} \tag{5.58c}
$$

$$
\begin{aligned}
&\boldsymbol{\Lambda}\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&=\boldsymbol{\Lambda}\cdot[\mathfrak{K}_h\widehat{\mathbf{p}}+\mathfrak{K}_i\mathbf{r}+\mathfrak{K}_j\boldsymbol{\Omega}+\mathfrak{K}_m\boldsymbol{\Lambda}+\mathfrak{K}_l\dot{\boldsymbol{\Lambda}}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})-\mathfrak{K}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\quad+\mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})+\mathfrak{K}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{v}_e^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})+\mathfrak{K}_q(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})+\mathfrak{K}_o(\boldsymbol{\Omega}\times\mathbf{r})-\rho\Omega^2\mathfrak{v}_e(\mathbf{r}\times\boldsymbol{\Lambda})\\
&\quad+\mathfrak{K}_r(\widehat{\mathbf{p}}\times\mathbf{r})-\varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]\text{ by (5.57)}\\
&=\mathfrak{K}_h(\boldsymbol{\Lambda}\cdot\widehat{\mathbf{p}})+\mathfrak{K}_i(\boldsymbol{\Lambda}\cdot\mathbf{r})+\mathfrak{K}_j(\boldsymbol{\Lambda}\cdot\boldsymbol{\Omega})+\mathfrak{K}_m(\boldsymbol{\Lambda}\cdot\boldsymbol{\Lambda})+\mathfrak{K}_l(\boldsymbol{\Lambda}\cdot\dot{\boldsymbol{\Lambda}})+\mathfrak{K}_k[\boldsymbol{\Lambda}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]\\
&\quad-\mathfrak{K}_n[\boldsymbol{\Lambda}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]+\mathfrak{Y}\mathfrak{v}_e[\boldsymbol{\Lambda}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})]+\mathfrak{K}_p[\boldsymbol{\Lambda}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{v}_e^2[\boldsymbol{\Lambda}\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})]+\mathfrak{K}_q[\boldsymbol{\Lambda}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})]\\
&\quad+\mathfrak{K}_o[\boldsymbol{\Lambda}\cdot(\boldsymbol{\Omega}\times\mathbf{r})]-\rho\Omega^2\mathfrak{v}_e[\boldsymbol{\Lambda}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})]+\mathfrak{K}_r[\boldsymbol{\Lambda}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]-\varphi_j\mathfrak{v}_e[\boldsymbol{\Lambda}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]\\
&=\mathfrak{K}_h\varepsilon_i+\mathfrak{K}_i\varepsilon_h+\mathfrak{K}_j\varepsilon_g+\mathfrak{K}_m\Lambda^2+\mathfrak{K}_l\varsigma_d-\mathfrak{K}_k\mathfrak{d}_e+\mathfrak{K}_n\varepsilon_n+\mathfrak{K}_p\delta_b+\mathfrak{K}_q\mathfrak{d}_o-\mathfrak{K}_o\varepsilon_m-\mathfrak{K}_r\varepsilon_o\\
&\qquad\text{by (5.1a), (5.13a), (5.23a) \& (5.23b)}\\
&=\mathfrak{K}_u\text{ by (5.54d)}
\end{aligned}\tag{5.58d}
$$

$$
\begin{aligned}
&\dot{\boldsymbol{\Lambda}}\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&=\dot{\boldsymbol{\Lambda}}\cdot[\mathfrak{K}_h\widehat{\mathbf{p}}+\mathfrak{K}_i\mathbf{r}+\mathfrak{K}_j\boldsymbol{\Omega}+\mathfrak{K}_m\boldsymbol{\Lambda}+\mathfrak{K}_l\dot{\boldsymbol{\Lambda}}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})-\mathfrak{K}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\quad+\mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})+\mathfrak{K}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{v}_e^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})+\mathfrak{K}_q(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})+\mathfrak{K}_o(\boldsymbol{\Omega}\times\mathbf{r})-\rho\Omega^2\mathfrak{v}_e(\mathbf{r}\times\boldsymbol{\Lambda})\\
&\quad+\mathfrak{K}_r(\widehat{\mathbf{p}}\times\mathbf{r})-\varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]\text{ by (5.57)}\\
&=\mathfrak{K}_h(\dot{\boldsymbol{\Lambda}}\cdot\widehat{\mathbf{p}})+\mathfrak{K}_i(\dot{\boldsymbol{\Lambda}}\cdot\mathbf{r})+\mathfrak{K}_j(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Omega})+\mathfrak{K}_m(\dot{\boldsymbol{\Lambda}}\cdot\boldsymbol{\Lambda})+\mathfrak{K}_l(\dot{\boldsymbol{\Lambda}}\cdot\dot{\boldsymbol{\Lambda}})+\mathfrak{K}_k[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]\\
&\quad-\mathfrak{K}_n[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]+\mathfrak{Y}\mathfrak{v}_e[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})]+\mathfrak{K}_p[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{v}_e^2[\dot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})]+\mathfrak{K}_q[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})]\\
&\quad+\mathfrak{K}_o[\dot{\boldsymbol{\Lambda}}\cdot(\boldsymbol{\Omega}\times\mathbf{r})]-\rho\Omega^2\mathfrak{v}_e[\dot{\boldsymbol{\Lambda}}\cdot(\mathbf{r}\times\boldsymbol{\Lambda})]+\mathfrak{K}_r[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})-\varphi_j\mathfrak{v}_e[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]\\
&=\mathfrak{K}_h\varsigma_a+\mathfrak{K}_i\varsigma_n+\mathfrak{K}_j\varsigma_c+\mathfrak{K}_m\varsigma_d+\mathfrak{K}_l\varsigma_e+\mathfrak{K}_k\mathfrak{d}_f+\mathfrak{K}_n\mathfrak{d}_c+\mathfrak{Y}\mathfrak{v}_e\mathfrak{d}_u+\mathfrak{K}_p\mathfrak{d}_a+\mathfrak{v}_e^2\mathfrak{d}_v+\mathfrak{K}_q\mathfrak{d}_p\\
&\quad+\mathfrak{K}_o\mathfrak{d}_l+\rho\Omega^2\mathfrak{v}_e\mathfrak{d}_h-\mathfrak{K}_r\mathfrak{d}_n-\varphi_j\mathfrak{v}_e\mathfrak{d}_r\text{ by (5.23a), (5.23b) \& (5.23c)}\\
&=\mathfrak{K}_v\text{ by (5.54e)}
\end{aligned}\tag{5.58e}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&=(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot[\mathfrak{K}_h\widehat{\mathbf{p}}+\mathfrak{K}_i\mathbf{r}+\mathfrak{K}_j\boldsymbol{\Omega}+\mathfrak{K}_m\boldsymbol{\Lambda}+\mathfrak{K}_l\dot{\boldsymbol{\Lambda}}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})-\mathfrak{K}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\quad+\mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})+\mathfrak{K}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{v}_e^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})+\mathfrak{K}_q(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})+\mathfrak{K}_o(\boldsymbol{\Omega}\times\mathbf{r})-\rho\Omega^2\mathfrak{v}_e(\mathbf{r}\times\boldsymbol{\Lambda})\\
&\quad+\mathfrak{K}_r(\widehat{\mathbf{p}}\times\mathbf{r})-\varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]\text{ by (5.57)}\\
&=\mathfrak{K}_h[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]+\mathfrak{K}_i[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]+\mathfrak{K}_j[\boldsymbol{\Omega}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]+\mathfrak{K}_m[\boldsymbol{\Lambda}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]\\
&\quad+\mathfrak{K}_l[\dot{\boldsymbol{\Lambda}}\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]+\mathfrak{K}_k[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})]-\mathfrak{K}_n[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\quad+\mathfrak{Y}\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})]+\mathfrak{K}_p[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{v}_e^2[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\boldsymbol{\Omega}\times\boldsymbol{\Lambda})]\\
&\quad+\mathfrak{K}_q[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})]+\mathfrak{K}_o[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\boldsymbol{\Omega}\times\mathbf{r})]-\rho\Omega^2\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\mathbf{r}\times\boldsymbol{\Lambda})]\\
&\quad+\mathfrak{K}_r[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]-\varphi_j\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega})\cdot(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})]\\
&=-\mathfrak{K}_h\varepsilon_j+\mathfrak{K}_i\varepsilon_k-\mathfrak{K}_m\mathfrak{d}_e+\mathfrak{K}_l\mathfrak{d}_f+\mathfrak{K}_k[\Omega^2-(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Omega})^2]\\
&\quad-\mathfrak{K}_n[(\boldsymbol{\Omega}\cdot\mathbf{r})-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\boldsymbol{\Omega}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{Y}\mathfrak{v}_e[(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})-(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Lambda})(\boldsymbol{\Omega}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\mathfrak{K}_p[(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})-(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{v}_e^2[(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Omega})(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})-(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Lambda})\Omega^2]\\
&\quad+\mathfrak{K}_q[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})\Omega^2-(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Omega})(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})]+\mathfrak{K}_o[(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Omega})(\boldsymbol{\Omega}\cdot\mathbf{r})-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})\Omega^2]\\
&\quad-\rho\Omega^2\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})-(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Lambda})(\boldsymbol{\Omega}\cdot\mathbf{r})]+\mathfrak{K}_r[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\mathbf{r})-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})]\\
&\quad-\varphi_j\mathfrak{v}_e[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\boldsymbol{\Omega}\cdot\boldsymbol{\Lambda})-(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\Lambda})(\boldsymbol{\Omega}\cdot\widehat{\mathbf{p}})]\text{ by (5.1a), (5.23b) \& (A.2)}\\
&=-\mathfrak{K}_h\varepsilon_j+\mathfrak{K}_i\varepsilon_k-\mathfrak{K}_m\mathfrak{d}_e+\mathfrak{K}_l\mathfrak{d}_f+\mathfrak{K}_k(\Omega^2-\varepsilon_a^2)-\mathfrak{K}_n(\varepsilon_b-\varepsilon_d\varepsilon_a)+\mathfrak{Y}\mathfrak{v}_e(\varepsilon_g-\delta_a\varepsilon_a)\\
&\quad+\mathfrak{K}_p(\varepsilon_c-\varepsilon_e\varepsilon_a)+\mathfrak{v}_e^2(\varepsilon_a\varepsilon_g-\delta_a\Omega^2)+\mathfrak{K}_q(\varepsilon_e\Omega^2-\varepsilon_a\varepsilon_c)+\mathfrak{K}_o(\varepsilon_a\varepsilon_b-\varepsilon_d\Omega^2)\\
&\quad-\rho\Omega^2\mathfrak{v}_e(\varepsilon_d\varepsilon_g-\delta_a\varepsilon_b)+\mathfrak{K}_r(\varepsilon_e\varepsilon_b-\varepsilon_d\varepsilon_c)-\varphi_j\mathfrak{v}_e(\varepsilon_e\varepsilon_g-\delta_a\varepsilon_c)\text{ by (5.1a) \& (5.13a)}\\
&=\mathfrak{K}_w\text{ by (5.54e)}
\end{aligned}\tag{5.58f}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o (\boldsymbol{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by } (5.57) \\
&= \mathfrak{K}_h [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{K}_i [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{K}_j [\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{K}_m [\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{K}_l [\dot{\boldsymbol{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{K}_k [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] - \mathfrak{K}_n [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y} \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_q [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_r [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
\\
&= \mathfrak{K}_h \mathfrak{d}_t - \mathfrak{K}_j \varepsilon_k - \mathfrak{K}_m \varepsilon_n - \mathfrak{K}_l \mathfrak{d}_c + \mathfrak{K}_k [(\mathbf{r} \cdot \boldsymbol{\Omega}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad - \mathfrak{K}_n [r^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})^2] + \mathfrak{Y} \mathfrak{v}_e [(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_p [(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad + \mathfrak{K}_q [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Omega}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \widehat{\mathbf{p}})] + \mathfrak{K}_o [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) r^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda}) r^2] + \mathfrak{K}_r [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}}) r^2 - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \\
&\quad - \varphi_j \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \text{ by } (5.1\text{a}), (5.23\text{b}) \ \& \ (\text{A}.2) \\
&= \mathfrak{K}_h \mathfrak{d}_t - \mathfrak{K}_j \varepsilon_k - \mathfrak{K}_m \varepsilon_n - \mathfrak{K}_l \mathfrak{d}_c + \mathfrak{K}_k (\varepsilon_b - \varepsilon_a \varepsilon_d) - \mathfrak{K}_n (r^2 - \varepsilon_d^2) + \mathfrak{Y} \mathfrak{v}_e (\varepsilon_h - \delta_a \varepsilon_d) \\
&\quad + \mathfrak{K}_p (\varepsilon_f - \varepsilon_e \varepsilon_d) + \mathfrak{v}_e^2 (\varepsilon_a \varepsilon_h - \delta_a \varepsilon_b) + \mathfrak{K}_q (\varepsilon_e \varepsilon_b - \varepsilon_a \varepsilon_f) + \mathfrak{K}_o (\varepsilon_a r^2 - \varepsilon_d \varepsilon_b) \\
&\quad - \rho \Omega^2 \mathfrak{v}_e (\varepsilon_d \varepsilon_h - \delta_a r^2) + \mathfrak{K}_r (\varepsilon_e r^2 - \varepsilon_d \varepsilon_f) - \varphi_j \mathfrak{v}_e (\varepsilon_e \varepsilon_h - \delta_a \varepsilon_f) \text{ by } (5.1\text{a}) \ \& \ (5.13\text{a}) \\
&= \mathfrak{K}_x \text{ by } (5.54\text{e})
\end{aligned} \tag{5.58g}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o (\boldsymbol{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by } (5.57) \\
&= \mathfrak{K}_h [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_i [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_j [\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_m [\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_l [\dot{\boldsymbol{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_k [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] - \mathfrak{K}_n [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y} \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_q [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_r [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
\\
&= -\mathfrak{K}_h \delta_b + \mathfrak{K}_i \varepsilon_n + \mathfrak{K}_j \mathfrak{d}_e + \mathfrak{K}_l \mathfrak{d}_u + \mathfrak{K}_k [(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\boldsymbol{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad - \mathfrak{K}_n [(\boldsymbol{\Lambda} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\boldsymbol{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{Y} \mathfrak{v}_e [\Lambda^2 - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})^2] \\
&\quad + \mathfrak{K}_p [(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\boldsymbol{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) \Lambda^2 - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega})] \\
&\quad + \mathfrak{K}_q [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] + \mathfrak{K}_o [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\boldsymbol{\Lambda} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega})] \\
&\quad - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r}) \Lambda^2 - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\boldsymbol{\Lambda} \cdot \mathbf{r})] + \mathfrak{K}_r [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\boldsymbol{\Lambda} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] \\
&\quad - \varphi_j \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}}) \Lambda^2 - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] \text{ by } (5.1\text{a}), (5.13\text{a}), (5.23\text{b}), (5.23\text{c}) \ \& \ (\text{A}.2) \\
&= -\mathfrak{K}_h \delta_b + \mathfrak{K}_i \varepsilon_n + \mathfrak{K}_j \mathfrak{d}_e + \mathfrak{K}_l \mathfrak{d}_u + \mathfrak{K}_k (\varepsilon_g - \varepsilon_a \delta_a) - \mathfrak{K}_n (\varepsilon_h - \varepsilon_d \delta_a) + \mathfrak{Y} \mathfrak{v}_e (\Lambda^2 - \delta_a^2) \\
&\quad + \mathfrak{K}_p (\varepsilon_i - \varepsilon_e \delta_a) + \mathfrak{v}_e^2 (\varepsilon_a \Lambda^2 - \delta_a \varepsilon_g) + \mathfrak{K}_q (\varepsilon_e \varepsilon_g - \varepsilon_a \varepsilon_i) + \mathfrak{K}_o (\varepsilon_a \varepsilon_h - \varepsilon_d \varepsilon_g) \\
&\quad - \rho \Omega^2 \mathfrak{v}_e (\varepsilon_d \Lambda^2 - \delta_a \varepsilon_h) + \mathfrak{K}_r (\varepsilon_e \varepsilon_h - \varepsilon_d \varepsilon_i) - \varphi_j \mathfrak{v}_e (\varepsilon_e \Lambda^2 - \delta_a \varepsilon_i) \text{ by } (5.1\text{a}) \ \& \ (5.13\text{a}) \\
&= \mathfrak{K}_y \text{ by } (5.54\text{f})
\end{aligned} \tag{5.58h}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o (\boldsymbol{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by (5.57)} \\
&= \mathfrak{K}_h [\widehat{\mathbf{p}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_i [\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_j [\boldsymbol{\Omega} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_m [\boldsymbol{\Lambda} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] \\
&\quad + \mathfrak{K}_l [\dot{\boldsymbol{\Lambda}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_k [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] - \mathfrak{K}_n [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y} \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_q [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_r [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
&\\
&\quad = -\mathfrak{K}_i \mathfrak{d}_t + \mathfrak{K}_j \varepsilon_j + \mathfrak{K}_m \delta_b + \mathfrak{K}_l \mathfrak{d}_a + \mathfrak{K}_k [(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\qquad - \mathfrak{K}_n [(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{Y} \mathfrak{v}_e [(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{K}_p [1 - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})^2] \\
&\qquad + \mathfrak{v}_e^2 [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] + \mathfrak{K}_q [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})] \\
&\qquad + \mathfrak{K}_o [(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega})(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})] - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})(\widehat{\mathbf{p}} \cdot \mathbf{r})] \\
&\qquad + \mathfrak{K}_r [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})] - \varphi_j \mathfrak{v}_e [(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) - (\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda})] \\
&\qquad \text{by (5.1a), (5.13a), (5.23b) \& (A.2)} \\
&\quad = -\mathfrak{K}_i \mathfrak{d}_t + \mathfrak{K}_j \varepsilon_j + \mathfrak{K}_m \delta_b + \mathfrak{K}_l \mathfrak{d}_a + \mathfrak{K}_k (\varepsilon_c - \varepsilon_a \varepsilon_e) - \mathfrak{K}_n (\varepsilon_f - \varepsilon_d \varepsilon_e) + \mathfrak{Y} \mathfrak{v}_e (\varepsilon_i - \delta_a \varepsilon_e) \\
&\qquad + \mathfrak{K}_p (1 - \varepsilon_e^2) + \mathfrak{v}_e^2 (\varepsilon_a \varepsilon_i - \delta_a \varepsilon_c) + \mathfrak{K}_q (\varepsilon_e \varepsilon_c - \varepsilon_a) + \mathfrak{K}_o (\varepsilon_a \varepsilon_f - \varepsilon_d \varepsilon_c) \\
&\qquad - \rho \Omega^2 \mathfrak{v}_e (\varepsilon_d \varepsilon_i - \delta_a \varepsilon_f) + \mathfrak{K}_r (\varepsilon_e \varepsilon_f - \varepsilon_d) - \varphi_j \mathfrak{v}_e (\varepsilon_e \varepsilon_i - \delta_a) \text{ by (5.1a) \& (5.13a)} \\
&\quad = \mathfrak{K}_z \text{ by (5.54f)}
\end{aligned}
\tag{5.58i}
$$

$$
\begin{aligned}
&(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o (\boldsymbol{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by (5.57)} \\
&= \mathfrak{K}_h [\widehat{\mathbf{p}} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{K}_i [\mathbf{r} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{K}_j [\boldsymbol{\Omega} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{K}_m [\boldsymbol{\Lambda} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_l [\dot{\boldsymbol{\Lambda}} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] + \mathfrak{K}_k [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] - \mathfrak{K}_n [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y} \mathfrak{v}_e [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_q [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_r [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \\
&\\
&\quad = \mathfrak{K}_h \mathfrak{d}_o + \mathfrak{K}_i \varepsilon_m + \mathfrak{K}_l \mathfrak{d}_v + \mathfrak{K}_k [(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega}) - \Omega^2 (\boldsymbol{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\qquad - \mathfrak{K}_n [(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\boldsymbol{\Lambda} \cdot \mathbf{r}) - (\boldsymbol{\Omega} \cdot \mathbf{r})(\boldsymbol{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{Y} \mathfrak{v}_e [(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}}) \Lambda^2 - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\boldsymbol{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\qquad + \mathfrak{K}_p [(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}}) - (\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\boldsymbol{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_e^2 [\Omega^2 \Lambda^2 - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})^2] \\
&\qquad + \mathfrak{K}_q [(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega}) - \Omega^2 (\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] + \mathfrak{K}_o [\Omega^2 (\boldsymbol{\Lambda} \cdot \mathbf{r}) - (\boldsymbol{\Omega} \cdot \mathbf{r})(\boldsymbol{\Lambda} \cdot \boldsymbol{\Omega})] \\
&\qquad - \rho \Omega^2 \mathfrak{v}_e [(\boldsymbol{\Omega} \cdot \mathbf{r}) \Lambda^2 - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\boldsymbol{\Lambda} \cdot \mathbf{r})] + \mathfrak{K}_r [(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\boldsymbol{\Lambda} \cdot \mathbf{r}) - (\boldsymbol{\Omega} \cdot \mathbf{r})(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] \\
&\qquad - \varphi_j \mathfrak{v}_e [(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}}) \Lambda^2 - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\boldsymbol{\Lambda} \cdot \widehat{\mathbf{p}})] \text{ by (5.1a), (5.23b), (5.23c) \& (A.2)} \\
&\quad = \mathfrak{K}_h \mathfrak{d}_o + \mathfrak{K}_i \varepsilon_m + \mathfrak{K}_l \mathfrak{d}_v + \mathfrak{K}_k (\varepsilon_a \varepsilon_g - \Omega^2 \delta_a) - \mathfrak{K}_n (\varepsilon_a \varepsilon_h - \varepsilon_b \delta_a) + \mathfrak{Y} \mathfrak{v}_e (\varepsilon_a \Lambda^2 - \varepsilon_g \delta_a) \\
&\qquad + \mathfrak{K}_p (\varepsilon_a \varepsilon_i - \varepsilon_c \delta_a) + \mathfrak{v}_e^2 (\Omega^2 \Lambda^2 - \varepsilon_g^2) + \mathfrak{K}_q (\varepsilon_c \varepsilon_g - \Omega^2 \varepsilon_i) + \mathfrak{K}_o (\Omega^2 \varepsilon_h - \varepsilon_b \varepsilon_g) \\
&\qquad - \rho \Omega^2 \mathfrak{v}_e (\varepsilon_b \Lambda^2 - \varepsilon_g \varepsilon_h) + \mathfrak{K}_r (\varepsilon_c \varepsilon_h - \varepsilon_b \varepsilon_i) - \varphi_j \mathfrak{v}_e (\varepsilon_c \Lambda^2 - \varepsilon_g \varepsilon_i) \text{ by (5.1a) \& (5.13a)} \\
&\quad = \mathfrak{H}_a \text{ by (5.54g)}
\end{aligned}
\tag{5.58j}
$$

$$
\begin{aligned}
&(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o (\boldsymbol{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by (5.57)} \\
&= \mathfrak{K}_h [\widehat{\mathbf{p}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_i [\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_j [\boldsymbol{\Omega} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_m [\boldsymbol{\Lambda} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] \\
&\quad + \mathfrak{K}_l [\dot{\boldsymbol{\Lambda}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_k [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] - \mathfrak{K}_n [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y} \mathfrak{v}_e [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_q [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_r [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_i \varepsilon_l + \mathfrak{K}_m \mathfrak{d}_o + \mathfrak{K}_l \mathfrak{d}_p + \mathfrak{K}_k [(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}}) \Omega^2 - (\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad - \mathfrak{K}_n [(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\boldsymbol{\Omega} \cdot \mathbf{r}) - (\widehat{\mathbf{p}} \cdot \mathbf{r})(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{Y} \mathfrak{v}_e [(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}) - (\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda})(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_p [(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}}) - (\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_e^2 [(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}) - (\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda}) \Omega^2] + \mathfrak{K}_q [\Omega^2 - (\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})^2] \\
&\quad + \mathfrak{K}_o [(\widehat{\mathbf{p}} \cdot \boldsymbol{\Omega})(\boldsymbol{\Omega} \cdot \mathbf{r}) - (\widehat{\mathbf{p}} \cdot \mathbf{r}) \Omega^2] - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\mathbf{p}} \cdot \mathbf{r})(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}) - (\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda})(\boldsymbol{\Omega} \cdot \mathbf{r})] \\
&\quad + \mathfrak{K}_r [(\boldsymbol{\Omega} \cdot \mathbf{r}) - (\widehat{\mathbf{p}} \cdot \mathbf{r})(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})] - \varphi_j \mathfrak{v}_e [(\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}) - (\widehat{\mathbf{p}} \cdot \boldsymbol{\Lambda})(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})] \\
&\quad \text{by (5.1a), (5.23b) \& (A.2)} \\
&= \mathfrak{K}_i \varepsilon_l + \mathfrak{K}_m \mathfrak{d}_o + \mathfrak{K}_l \mathfrak{d}_p + \mathfrak{K}_k (\varepsilon_e \Omega^2 - \varepsilon_c \varepsilon_a) - \mathfrak{K}_n (\varepsilon_e \varepsilon_b - \varepsilon_f \varepsilon_a) + \mathfrak{Y} \mathfrak{v}_e (\varepsilon_e \varepsilon_g - \varepsilon_i \varepsilon_a) \\
&\quad + \mathfrak{K}_p (\varepsilon_e \varepsilon_c - \varepsilon_a) + \mathfrak{v}_e^2 (\varepsilon_c \varepsilon_g - \varepsilon_i \Omega^2) + \mathfrak{K}_q (\Omega^2 - \varepsilon_c^2) + \mathfrak{K}_o (\varepsilon_c \varepsilon_b - \varepsilon_f \Omega^2) \\
&\quad - \rho \Omega^2 \mathfrak{v}_e (\varepsilon_f \varepsilon_g - \varepsilon_i \varepsilon_b) + \mathfrak{K}_r (\varepsilon_b - \varepsilon_f \varepsilon_c) - \varphi_j \mathfrak{v}_e (\varepsilon_g - \varepsilon_i \varepsilon_c) \text{ by (5.1a)} \\
&= \mathfrak{H}_b \text{ by (5.54g)}
\end{aligned}
\tag{5.58k}
$$

$$
\begin{aligned}
&(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\boldsymbol{\Omega} \times \mathbf{r}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o (\boldsymbol{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by (5.57)} \\
&= \mathfrak{K}_h [\widehat{\mathbf{p}} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] + \mathfrak{K}_i [\mathbf{r} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] + \mathfrak{K}_j [\boldsymbol{\Omega} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] + \mathfrak{K}_m [\boldsymbol{\Lambda} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] \\
&\quad + \mathfrak{K}_l [\dot{\boldsymbol{\Lambda}} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] + \mathfrak{K}_k [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] - \mathfrak{K}_n [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y} \mathfrak{v}_e [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_q [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_r [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [(\boldsymbol{\Omega} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_h \varepsilon_l - \mathfrak{K}_m \varepsilon_m + \mathfrak{K}_l \mathfrak{d}_l + \mathfrak{K}_k [(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \boldsymbol{\Omega}) - \Omega^2 (\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad - \mathfrak{K}_n [(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}}) r^2 - (\boldsymbol{\Omega} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{Y} \mathfrak{v}_e [(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_p [(\boldsymbol{\Omega} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_e^2 [\Omega^2 (\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad + \mathfrak{K}_q [(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Omega}) - \Omega^2 (\mathbf{r} \cdot \widehat{\mathbf{p}})] + \mathfrak{K}_o [\Omega^2 r^2 - (\boldsymbol{\Omega} \cdot \mathbf{r})^2] \\
&\quad - \rho \Omega^2 \mathfrak{v}_e [(\boldsymbol{\Omega} \cdot \mathbf{r})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda}) r^2] + \mathfrak{K}_r [(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}}) r^2 - (\boldsymbol{\Omega} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \\
&\quad - \varphi_j \mathfrak{v}_e [(\boldsymbol{\Omega} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\boldsymbol{\Omega} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \text{ by (5.1a), (5.23b) \& (A.2)} \\
&= \mathfrak{K}_h \varepsilon_l - \mathfrak{K}_m \varepsilon_m + \mathfrak{K}_l \mathfrak{d}_l + \mathfrak{K}_k (\varepsilon_a \varepsilon_b - \Omega^2 \varepsilon_d) - \mathfrak{K}_n (\varepsilon_a r^2 - \varepsilon_b \varepsilon_d) + \mathfrak{Y} \mathfrak{v}_e (\varepsilon_a \varepsilon_h - \varepsilon_g \varepsilon_d) \\
&\quad + \mathfrak{K}_p (\varepsilon_a \varepsilon_f - \varepsilon_c \varepsilon_d) + \mathfrak{v}_e^2 (\Omega^2 \varepsilon_h - \varepsilon_g \varepsilon_b) + \mathfrak{K}_q (\varepsilon_c \varepsilon_b - \Omega^2 \varepsilon_f) + \mathfrak{K}_o (\Omega^2 r^2 - \varepsilon_b^2) \\
&\quad - \rho \Omega^2 \mathfrak{v}_e (\varepsilon_b \varepsilon_h - \varepsilon_g r^2) + \mathfrak{K}_r (\varepsilon_c r^2 - \varepsilon_b \varepsilon_f) - \varphi_j \mathfrak{v}_e (\varepsilon_c \varepsilon_h - \varepsilon_g \varepsilon_f) \text{ by (5.1a)} \\
&= \mathfrak{H}_c \text{ by (5.54h)}
\end{aligned}
\tag{5.58l}
$$

$$
\begin{aligned}
&(\mathbf{r}\times\mathbf{\Lambda})\cdot(\mathcal{S}_t+\mathcal{S}_u)\\
&=(\mathbf{r}\times\mathbf{\Lambda})\cdot[\mathfrak{K}_h\widehat{\mathbf{p}}+\mathfrak{K}_i\mathbf{r}+\mathfrak{K}_j\mathbf{\Omega}+\mathfrak{K}_m\mathbf{\Lambda}+\mathfrak{K}_l\dot{\mathbf{\Lambda}}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})-\mathfrak{K}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\quad+\mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})+\mathfrak{K}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{v}_e^2(\mathbf{\Omega}\times\mathbf{\Lambda})+\mathfrak{K}_q(\widehat{\mathbf{p}}\times\mathbf{\Omega})+\mathfrak{K}_o(\mathbf{\Omega}\times\mathbf{r})-\rho\Omega^2\mathfrak{v}_e(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad+\mathfrak{K}_r(\widehat{\mathbf{p}}\times\mathbf{r})-\varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\text{ by (5.57)}\\
&=\mathfrak{K}_h[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{\Lambda})]+\mathfrak{K}_i[\mathbf{r}\cdot(\mathbf{r}\times\mathbf{\Lambda})]+\mathfrak{K}_j[\mathbf{\Omega}\cdot(\mathbf{r}\times\mathbf{\Lambda})]+\mathfrak{K}_m[\mathbf{\Lambda}\cdot(\mathbf{r}\times\mathbf{\Lambda})]\\
&\quad+\mathfrak{K}_l[\dot{\mathbf{\Lambda}}\cdot(\mathbf{r}\times\mathbf{\Lambda})]+\mathfrak{K}_k[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})]-\mathfrak{K}_n[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\quad+\mathfrak{Y}\mathfrak{v}_e[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})]+\mathfrak{K}_p[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{v}_e^2[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})]\\
&\quad+\mathfrak{K}_q[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})]+\mathfrak{K}_o[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\mathbf{\Omega}\times\mathbf{r})]-\rho\Omega^2\mathfrak{v}_e[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\mathbf{r}\times\mathbf{\Lambda})]\\
&\quad+\mathfrak{K}_r[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]-\varphi_j\mathfrak{v}_e[(\mathbf{r}\times\mathbf{\Lambda})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&\\
&=-\mathfrak{K}_h\varepsilon_o-\mathfrak{K}_j\varepsilon_m-\mathfrak{K}_l\mathfrak{d}_h+\mathfrak{K}_k[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\mathbf{\Omega})-(\mathbf{r}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad-\mathfrak{K}_n[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\mathbf{r})-r^2(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{Y}\mathfrak{v}_e[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})\Lambda^2-(\mathbf{r}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\mathfrak{K}_p[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})-(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{v}_e^2[(\mathbf{r}\cdot\mathbf{\Omega})\Lambda^2-(\mathbf{r}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\mathbf{\Omega})]\\
&\quad+\mathfrak{K}_q[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\mathbf{\Omega})-(\mathbf{r}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})]+\mathfrak{K}_o[(\mathbf{r}\cdot\mathbf{\Omega})(\mathbf{\Lambda}\cdot\mathbf{r})-r^2(\mathbf{\Lambda}\cdot\mathbf{\Omega})]\\
&\quad-\rho\Omega^2\mathfrak{v}_e[r^2\Lambda^2-(\mathbf{r}\cdot\mathbf{\Lambda})^2]+\mathfrak{K}_r[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{\Lambda}\cdot\mathbf{r})-r^2(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})]\\
&\quad-\varphi_j\mathfrak{v}_e[(\mathbf{r}\cdot\widehat{\mathbf{p}})\Lambda^2-(\mathbf{r}\cdot\mathbf{\Lambda})(\mathbf{\Lambda}\cdot\widehat{\mathbf{p}})]\text{ by (5.1a), (5.23b) \& (A.2)}\\
&=-\mathfrak{K}_h\varepsilon_o-\mathfrak{K}_j\varepsilon_m-\mathfrak{K}_l\mathfrak{d}_h+\mathfrak{K}_k(\varepsilon_d\varepsilon_g-\varepsilon_b\delta_a)-\mathfrak{K}_n(\varepsilon_d\varepsilon_h-r^2\delta_a)+\mathfrak{Y}\mathfrak{v}_e(\varepsilon_d\Lambda^2-\varepsilon_h\delta_a)\\
&\quad+\mathfrak{K}_p(\varepsilon_d\varepsilon_i-\varepsilon_f\delta_a)+\mathfrak{v}_e^2(\varepsilon_b\Lambda^2-\varepsilon_h\varepsilon_g)+\mathfrak{K}_q(\varepsilon_f\varepsilon_g-\varepsilon_b\varepsilon_i)+\mathfrak{K}_o(\varepsilon_b\varepsilon_h-r^2\varepsilon_g)\\
&\quad-\rho\Omega^2\mathfrak{v}_e(r^2\Lambda^2-\varepsilon_h^2)+\mathfrak{K}_r(\varepsilon_f\varepsilon_h-r^2\varepsilon_i)-\varphi_j\mathfrak{v}_e(\varepsilon_f\Lambda^2-\varepsilon_h\varepsilon_i)\text{ by (5.1a) \& (5.13a)}\\
&=\mathfrak{H}_d\text{ by (5.54h)}
\end{aligned}
\tag{5.58m}
$$

$$
\begin{aligned}
&(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\mathcal{S}_t+\mathcal{S}_u)\\
&=(\widehat{\mathbf{p}}\times\mathbf{r})\cdot[\mathfrak{K}_h\widehat{\mathbf{p}}+\mathfrak{K}_i\mathbf{r}+\mathfrak{K}_j\mathbf{\Omega}+\mathfrak{K}_m\mathbf{\Lambda}+\mathfrak{K}_l\dot{\mathbf{\Lambda}}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})-\mathfrak{K}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\quad+\mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})+\mathfrak{K}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{v}_e^2(\mathbf{\Omega}\times\mathbf{\Lambda})+\mathfrak{K}_q(\widehat{\mathbf{p}}\times\mathbf{\Omega})+\mathfrak{K}_o(\mathbf{\Omega}\times\mathbf{r})-\rho\Omega^2\mathfrak{v}_e(\mathbf{r}\times\mathbf{\Lambda})\\
&\quad+\mathfrak{K}_r(\widehat{\mathbf{p}}\times\mathbf{r})-\varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\text{ by (5.57)}\\
&=\mathfrak{K}_h[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]+\mathfrak{K}_i[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]+\mathfrak{K}_j[\mathbf{\Omega}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]+\mathfrak{K}_m[\mathbf{\Lambda}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]\\
&\quad+\mathfrak{K}_l[\dot{\mathbf{\Lambda}}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]+\mathfrak{K}_k[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Omega})]-\mathfrak{K}_n[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\quad+\mathfrak{Y}\mathfrak{v}_e[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{\Lambda})]+\mathfrak{K}_p[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{v}_e^2[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{\Lambda})]\\
&\quad+\mathfrak{K}_q[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})]+\mathfrak{K}_o[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\mathbf{\Omega}\times\mathbf{r})]-\rho\Omega^2\mathfrak{v}_e[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\mathbf{r}\times\mathbf{\Lambda})]\\
&\quad+\mathfrak{K}_r[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]-\varphi_j\mathfrak{v}_e[(\widehat{\mathbf{p}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]\\
&\\
&=-\mathfrak{K}_j\varepsilon_l-\mathfrak{K}_m\varepsilon_o-\mathfrak{K}_l\mathfrak{d}_n+\mathfrak{K}_k[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\mathbf{\Omega})-(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad-\mathfrak{K}_n[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})r^2-(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{Y}\mathfrak{v}_e[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\mathbf{\Lambda})-(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\mathfrak{K}_p[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{r}\cdot\widehat{\mathbf{p}})-(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{v}_e^2[(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})(\mathbf{r}\cdot\mathbf{\Lambda})-(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\mathbf{\Omega})]\\
&\quad+\mathfrak{K}_q[(\mathbf{r}\cdot\mathbf{\Omega})-(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})(\mathbf{r}\cdot\widehat{\mathbf{p}})]+\mathfrak{K}_o[(\widehat{\mathbf{p}}\cdot\mathbf{\Omega})r^2-(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{\Omega})]\\
&\quad-\rho\Omega^2\mathfrak{v}_e[(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{\Lambda})-(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})r^2]+\mathfrak{K}_r[r^2-(\widehat{\mathbf{p}}\cdot\mathbf{r})^2]\\
&\quad-\varphi_j\mathfrak{v}_e[(\mathbf{r}\cdot\mathbf{\Lambda})-(\widehat{\mathbf{p}}\cdot\mathbf{\Lambda})(\mathbf{r}\cdot\widehat{\mathbf{p}})]\text{ by (5.1a), (5.23b) \& (A.2)}\\
&=-\mathfrak{K}_j\varepsilon_l-\mathfrak{K}_m\varepsilon_o-\mathfrak{K}_l\mathfrak{d}_n+\mathfrak{K}_k(\varepsilon_e\varepsilon_b-\varepsilon_c\varepsilon_d)-\mathfrak{K}_n(\varepsilon_e r^2-\varepsilon_f\varepsilon_d)+\mathfrak{Y}\mathfrak{v}_e(\varepsilon_e\varepsilon_h-\varepsilon_i\varepsilon_d)\\
&\quad+\mathfrak{K}_p(\varepsilon_e\varepsilon_f-\varepsilon_d)+\mathfrak{v}_e^2(\varepsilon_c\varepsilon_h-\varepsilon_i\varepsilon_b)+\mathfrak{K}_q(\varepsilon_b-\varepsilon_c\varepsilon_f)+\mathfrak{K}_o(\varepsilon_c r^2-\varepsilon_f\varepsilon_b)\\
&\quad-\rho\Omega^2\mathfrak{v}_e(\varepsilon_f\varepsilon_h-\varepsilon_i r^2)+\mathfrak{K}_r(r^2-\varepsilon_f^2)-\varphi_j\mathfrak{v}_e(\varepsilon_h-\varepsilon_i\varepsilon_f)\text{ by (5.1a)}\\
&=\mathfrak{H}_e\text{ by (5.54i)}
\end{aligned}
\tag{5.58n}
$$

$$
\begin{aligned}
&(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \mathbf{\Omega} + \mathfrak{K}_m \mathbf{\Lambda} + \mathfrak{K}_l \dot{\mathbf{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \mathbf{\Omega}) + \mathfrak{K}_o (\mathbf{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \mathbf{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] \text{ by (5.57)} \\
&= \mathfrak{K}_h [\widehat{\mathbf{p}} \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] + \mathfrak{K}_i [\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] + \mathfrak{K}_j [\mathbf{\Omega} \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] + \mathfrak{K}_m [\mathbf{\Lambda} \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] \\
&\quad + \mathfrak{K}_l [\dot{\mathbf{\Lambda}} \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] + \mathfrak{K}_k [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] - \mathfrak{K}_n [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y} \mathfrak{v}_e [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda})] + \mathfrak{K}_p [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\mathbf{\Omega} \times \mathbf{\Lambda})] \\
&\quad + \mathfrak{K}_q [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \mathbf{\Omega})] + \mathfrak{K}_o [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\mathbf{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\mathbf{r} \times \mathbf{\Lambda})] \\
&\quad + \mathfrak{K}_r [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [(\widehat{\mathbf{p}} \times \mathbf{\Lambda}) \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_i \varepsilon_o - \mathfrak{K}_j \mathfrak{d}_o + \mathfrak{K}_l \mathfrak{d}_r + \mathfrak{K}_k [(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda} \cdot \mathbf{\Omega}) - (\widehat{\mathbf{p}} \cdot \mathbf{\Omega})(\mathbf{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad - \mathfrak{K}_n [(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda} \cdot \mathbf{r}) - (\widehat{\mathbf{p}} \cdot \mathbf{r})(\mathbf{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{Y} \mathfrak{v}_e [(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}}) \Lambda^2 - (\widehat{\mathbf{p}} \cdot \mathbf{\Lambda})(\mathbf{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_p [(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{\Lambda} \cdot \widehat{\mathbf{p}}) - (\mathbf{\Lambda} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_e^2 [(\widehat{\mathbf{p}} \cdot \mathbf{\Omega}) \Lambda^2 - (\widehat{\mathbf{p}} \cdot \mathbf{\Lambda})(\mathbf{\Lambda} \cdot \mathbf{\Omega})] \\
&\quad + \mathfrak{K}_q [(\mathbf{\Lambda} \cdot \mathbf{\Omega}) - (\widehat{\mathbf{p}} \cdot \mathbf{\Omega})(\mathbf{\Lambda} \cdot \widehat{\mathbf{p}})] + \mathfrak{K}_o [(\widehat{\mathbf{p}} \cdot \mathbf{\Omega})(\mathbf{\Lambda} \cdot \mathbf{r}) - (\widehat{\mathbf{p}} \cdot \mathbf{r})(\mathbf{\Lambda} \cdot \mathbf{\Omega})] \\
&\quad - \rho \Omega^2 \mathfrak{v}_e [(\widehat{\mathbf{p}} \cdot \mathbf{r}) \Lambda^2 - (\widehat{\mathbf{p}} \cdot \mathbf{\Lambda})(\mathbf{\Lambda} \cdot \mathbf{r})] + \mathfrak{K}_r [(\mathbf{\Lambda} \cdot \mathbf{r}) - (\widehat{\mathbf{p}} \cdot \mathbf{r})(\mathbf{\Lambda} \cdot \widehat{\mathbf{p}})] \\
&\quad - \varphi_j \mathfrak{v}_e [\Lambda^2 - (\widehat{\mathbf{p}} \cdot \mathbf{\Lambda})^2] \text{ by (5.1a), (5.23b) \& (A.2)} \\
&= \mathfrak{K}_i \varepsilon_o - \mathfrak{K}_j \mathfrak{d}_o + \mathfrak{K}_l \mathfrak{d}_r + \mathfrak{K}_k (\varepsilon_e \varepsilon_g - \varepsilon_c \delta_a) - \mathfrak{K}_n (\varepsilon_e \varepsilon_h - \varepsilon_f \delta_a) + \mathfrak{Y} \mathfrak{v}_e (\varepsilon_e \Lambda^2 - \varepsilon_i \delta_a) \\
&\quad + \mathfrak{K}_p (\varepsilon_e \varepsilon_i - \delta_a) + \mathfrak{v}_e^2 (\varepsilon_c \Lambda^2 - \varepsilon_i \varepsilon_g) + \mathfrak{K}_q (\varepsilon_g - \varepsilon_c \varepsilon_i) + \mathfrak{K}_o (\varepsilon_c \varepsilon_h - \varepsilon_f \varepsilon_g) \\
&\quad - \rho \Omega^2 \mathfrak{v}_e (\varepsilon_f \Lambda^2 - \varepsilon_i \varepsilon_h) + \mathfrak{K}_r (\varepsilon_h - \varepsilon_f \varepsilon_i) - \varphi_j \mathfrak{v}_e (\Lambda^2 - \varepsilon_i^2) \text{ by (5.1a) \& (5.13a)} \\
&= \mathfrak{H}_f \text{ by (5.54i)}
\end{aligned}
\tag{5.58o}
$$

$$
\begin{aligned}
&(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \mathbf{\Omega} + \mathfrak{K}_m \mathbf{\Lambda} + \mathfrak{K}_l \dot{\mathbf{\Lambda}} + \mathfrak{K}_k (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega}) - \mathfrak{K}_n (\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y} \mathfrak{v}_e (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda}) + \mathfrak{K}_p (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2 (\mathbf{\Omega} \times \mathbf{\Lambda}) + \mathfrak{K}_q (\widehat{\mathbf{p}} \times \mathbf{\Omega}) + \mathfrak{K}_o (\mathbf{\Omega} \times \mathbf{r}) - \rho \Omega^2 \mathfrak{v}_e (\mathbf{r} \times \mathbf{\Lambda}) \\
&\quad + \mathfrak{K}_r (\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e (\widehat{\mathbf{p}} \times \mathbf{\Lambda})] \text{ by (5.57)} \\
&= \mathfrak{K}_h [\widehat{\mathbf{p}} \cdot (\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_i [\mathbf{r} \cdot (\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_j [\mathbf{\Omega} \cdot (\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_m [\mathbf{\Lambda} \cdot (\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}})] \\
&\quad + \mathfrak{K}_l [\dot{\mathbf{\Lambda}} \cdot (\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_k [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Omega})] - \mathfrak{K}_n [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y} \mathfrak{v}_e [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{\Lambda})] + \mathfrak{K}_p [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2 [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\mathbf{\Omega} \times \mathbf{\Lambda})] \\
&\quad + \mathfrak{K}_q [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \mathbf{\Omega})] + \mathfrak{K}_o [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\mathbf{\Omega} \times \mathbf{r})] - \rho \Omega^2 \mathfrak{v}_e [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\mathbf{r} \times \mathbf{\Lambda})] \\
&\quad + \mathfrak{K}_r [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e [(\dot{\mathbf{\Lambda}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \mathbf{\Lambda})]
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{K}_i \mathfrak{d}_n + \mathfrak{K}_j \mathfrak{d}_p + \mathfrak{K}_m \mathfrak{d}_r + \mathfrak{K}_k [(\dot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}} \cdot \mathbf{\Omega}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad - \mathfrak{K}_n [(\dot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{Y} \mathfrak{v}_e [(\dot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}} \cdot \mathbf{\Lambda}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_p [(\dot{\mathbf{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}}) - (\dot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_e^2 [(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})(\widehat{\mathbf{p}} \cdot \mathbf{\Lambda}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})(\widehat{\mathbf{p}} \cdot \mathbf{\Omega})] \\
&\quad + \mathfrak{K}_q [(\dot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \mathbf{\Omega}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})] + \mathfrak{K}_o [(\dot{\mathbf{\Lambda}} \cdot \mathbf{\Omega})(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \mathbf{\Omega})] \\
&\quad - \rho \Omega^2 \mathfrak{v}_e [(\dot{\mathbf{\Lambda}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \mathbf{\Lambda}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})(\widehat{\mathbf{p}} \cdot \mathbf{r})] + \mathfrak{K}_r [(\dot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{r})] \\
&\quad - \varphi_j \mathfrak{v}_e [(\dot{\mathbf{\Lambda}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \mathbf{\Lambda}) - (\dot{\mathbf{\Lambda}} \cdot \mathbf{\Lambda})] \text{ by (5.23b) \& (A.2)} \\
&= -\mathfrak{K}_i \mathfrak{d}_n + \mathfrak{K}_j \mathfrak{d}_p + \mathfrak{K}_m \mathfrak{d}_r + \mathfrak{K}_k (\varsigma_b \varepsilon_c - \varsigma_c \varepsilon_e) - \mathfrak{K}_n (\varsigma_b \varepsilon_f - \varsigma_n \varepsilon_e) + \mathfrak{Y} \mathfrak{v}_e (\varsigma_b \varepsilon_i - \varsigma_d \varepsilon_e) \\
&\quad + \mathfrak{K}_p (\varsigma_b - \varsigma_a \varepsilon_e) + \mathfrak{v}_e^2 (\varsigma_c \varepsilon_i - \varsigma_d \varepsilon_c) + \mathfrak{K}_q (\varsigma_a \varepsilon_c - \varsigma_c) + \mathfrak{K}_o (\varsigma_c \varepsilon_f - \varsigma_n \varepsilon_c) \\
&\quad - \rho \Omega^2 \mathfrak{v}_e (\varsigma_n \varepsilon_i - \varsigma_d \varepsilon_f) + \mathfrak{K}_r (\varsigma_a \varepsilon_f - \varsigma_n) - \varphi_j \mathfrak{v}_e (\varsigma_a \varepsilon_i - \varsigma_d) \text{ by (5.1a) \& (5.23a)} \\
&= \mathfrak{H}_g \text{ by (5.54j)}
\end{aligned}
\tag{5.58p}
$$

$$
\begin{aligned}
&(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\mathcal{S}_t + \mathcal{S}_u) \\
&= (\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o(\boldsymbol{\Omega} \times \mathbf{r}) - \rho\Omega^2\mathfrak{v}_e(\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r(\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by (5.57)} \\
&= \mathfrak{K}_h[\widehat{\mathbf{p}} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{K}_i[\mathbf{r} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{K}_j[\boldsymbol{\Omega} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{K}_m[\boldsymbol{\Lambda} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&\quad + \mathfrak{K}_l[\dot{\boldsymbol{\Lambda}} \cdot (\dot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{K}_k[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] - \mathfrak{K}_n[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y}\mathfrak{v}_e[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_q[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho\Omega^2\mathfrak{v}_e[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_r[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e[(\dot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_h\mathfrak{d}_n - \mathfrak{K}_j\mathfrak{d}_l - \mathfrak{K}_m\mathfrak{d}_h + \mathfrak{K}_k[(\dot{\boldsymbol{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \boldsymbol{\Omega}) - (\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad - \mathfrak{K}_n[(\dot{\boldsymbol{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})r^2 - (\dot{\boldsymbol{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{Y}\mathfrak{v}_e[(\dot{\boldsymbol{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_p[(\dot{\boldsymbol{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\dot{\boldsymbol{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_e^2[(\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad + \mathfrak{K}_q[(\dot{\boldsymbol{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Omega}) - (\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \widehat{\mathbf{p}})] + \mathfrak{K}_o[(\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})r^2 - (\dot{\boldsymbol{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad - \rho\Omega^2\mathfrak{v}_e[(\dot{\boldsymbol{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda})r^2] + \mathfrak{K}_r[(\dot{\boldsymbol{\Lambda}} \cdot \widehat{\mathbf{p}})r^2 - (\dot{\boldsymbol{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \\
&\quad - \varphi_j \mathfrak{v}_e[(\dot{\boldsymbol{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\dot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \text{ by (5.23b) \& (A.2)} \\
&= \mathfrak{K}_h\mathfrak{d}_n - \mathfrak{K}_j\mathfrak{d}_l - \mathfrak{K}_m\mathfrak{d}_h + \mathfrak{K}_k(\varsigma_b\varepsilon_b - \varsigma_c\varepsilon_d) - \mathfrak{K}_n(\varsigma_b r^2 - \varsigma_n\varepsilon_d) + \mathfrak{Y}\mathfrak{v}_e(\varsigma_b\varepsilon_h - \varsigma_d\varepsilon_d) \\
&\quad + \mathfrak{K}_p(\varsigma_b\varepsilon_f - \varsigma_a\varepsilon_d) + \mathfrak{v}_e^2(\varsigma_c\varepsilon_h - \varsigma_d\varepsilon_b) + \mathfrak{K}_q(\varsigma_a\varepsilon_b - \varsigma_c\varepsilon_f) + \mathfrak{K}_o(\varsigma_c r^2 - \varsigma_n\varepsilon_b) \\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varsigma_n\varepsilon_h - \varsigma_d r^2) + \mathfrak{K}_r(\varsigma_a r^2 - \varsigma_n\varepsilon_f) - \varphi_j \mathfrak{v}_e(\varsigma_a\varepsilon_h - \varsigma_d\varepsilon_f) \text{ by (5.1a) \& (5.23a)} \\
&= \mathfrak{H}_h \text{ by (5.54j)}
\end{aligned} \tag{5.58q}
$$

$$
\begin{aligned}
&(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\mathcal{S}_t + \mathcal{S}_u) \\
&= (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot [\mathfrak{K}_h \widehat{\mathbf{p}} + \mathfrak{K}_i \mathbf{r} + \mathfrak{K}_j \boldsymbol{\Omega} + \mathfrak{K}_m \boldsymbol{\Lambda} + \mathfrak{K}_l \dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \\
&\quad + \mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o(\boldsymbol{\Omega} \times \mathbf{r}) - \rho\Omega^2\mathfrak{v}_e(\mathbf{r} \times \boldsymbol{\Lambda}) \\
&\quad + \mathfrak{K}_r(\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j \mathfrak{v}_e(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by (5.57)} \\
&= \mathfrak{K}_h[\widehat{\mathbf{p}} \cdot (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{K}_i[\mathbf{r} \cdot (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{K}_j[\boldsymbol{\Omega} \cdot (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{K}_m[\boldsymbol{\Lambda} \cdot (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] \\
&\quad + \mathfrak{K}_l[\dot{\boldsymbol{\Lambda}} \cdot (\ddot{\boldsymbol{\Lambda}} \times \mathbf{r})] + \mathfrak{K}_k[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] - \mathfrak{K}_n[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad + \mathfrak{Y}\mathfrak{v}_e[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_q[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho\Omega^2\mathfrak{v}_e[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] \\
&\quad + \mathfrak{K}_r[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j \mathfrak{v}_e[(\ddot{\boldsymbol{\Lambda}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_h\mathfrak{p}_b - \mathfrak{K}_j\mathfrak{d}_m - \mathfrak{K}_m\mathfrak{d}_i - \mathfrak{K}_l\mathfrak{d}_j + \mathfrak{K}_k[(\ddot{\boldsymbol{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \boldsymbol{\Omega}) - (\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad - \mathfrak{K}_n[(\ddot{\boldsymbol{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})r^2 - (\ddot{\boldsymbol{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{Y}\mathfrak{v}_e[(\ddot{\boldsymbol{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_p[(\ddot{\boldsymbol{\Lambda}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\ddot{\boldsymbol{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{v}_e^2[(\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad + \mathfrak{K}_q[(\ddot{\boldsymbol{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Omega}) - (\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})(\mathbf{r} \cdot \widehat{\mathbf{p}})] + \mathfrak{K}_o[(\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Omega})r^2 - (\ddot{\boldsymbol{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \boldsymbol{\Omega})] \\
&\quad - \rho\Omega^2\mathfrak{v}_e[(\ddot{\boldsymbol{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda})r^2] + \mathfrak{K}_r[(\ddot{\boldsymbol{\Lambda}} \cdot \widehat{\mathbf{p}})r^2 - (\ddot{\boldsymbol{\Lambda}} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \\
&\quad - \varphi_j \mathfrak{v}_e[(\ddot{\boldsymbol{\Lambda}} \cdot \widehat{\mathbf{p}})(\mathbf{r} \cdot \boldsymbol{\Lambda}) - (\ddot{\boldsymbol{\Lambda}} \cdot \boldsymbol{\Lambda})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \text{ by (5.23b), (5.23c) \& (A.2)}
\end{aligned}
$$

$$\begin{aligned}
&= \mathfrak{K}_h\mathfrak{p}_b - \mathfrak{K}_j\mathfrak{d}_m - \mathfrak{K}_m\mathfrak{d}_i - \mathfrak{K}_l\mathfrak{d}_j + \mathfrak{K}_k(\varsigma_g\varepsilon_b - \varsigma_h\varepsilon_d) - \mathfrak{K}_n(\varsigma_g r^2 - \varsigma_o\varepsilon_d) + \mathfrak{Y}\mathfrak{v}_e(\varsigma_g\varepsilon_h - \varsigma_i\varepsilon_d)\\
&\quad + \mathfrak{K}_p(\varsigma_g\varepsilon_f - \varsigma_f\varepsilon_d) + \mathfrak{v}_e^2(\varsigma_h\varepsilon_h - \varsigma_i\varepsilon_b) + \mathfrak{K}_q(\varsigma_f\varepsilon_b - \varsigma_h\varepsilon_f) + \mathfrak{K}_o(\varsigma_h r^2 - \varsigma_o\varepsilon_b)\\
&\quad - \rho\Omega^2\mathfrak{v}_e(\varsigma_o\varepsilon_h - \varsigma_i r^2) + \mathfrak{K}_r(\varsigma_f r^2 - \varsigma_o\varepsilon_f) - \varphi_j\mathfrak{v}_e(\varsigma_f\varepsilon_h - \varsigma_i\varepsilon_f) \text{ by (5.1a) \& (5.23a)}\\
&= \mathfrak{H}_i \text{ by (5.54k)} \qquad (5.58\text{r})
\end{aligned}$$

$$\begin{aligned}
&\widehat{\boldsymbol{\kappa}} \cdot (\mathfrak{S}_t + \mathfrak{S}_u)\\
&= \widehat{\boldsymbol{\kappa}} \cdot [\mathfrak{K}_h\widehat{\mathbf{p}} + \mathfrak{K}_i\mathbf{r} + \mathfrak{K}_j\boldsymbol{\Omega} + \mathfrak{K}_m\boldsymbol{\Lambda} + \mathfrak{K}_l\dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r})\\
&\quad + \mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o(\boldsymbol{\Omega} \times \mathbf{r}) - \rho\Omega^2\mathfrak{v}_e(\mathbf{r} \times \boldsymbol{\Lambda})\\
&\quad + \mathfrak{K}_r(\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by (5.57)}\\
&= \mathfrak{K}_h(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}}) + \mathfrak{K}_i(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r}) + \mathfrak{K}_j(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Omega}) + \mathfrak{K}_m(\widehat{\boldsymbol{\kappa}} \cdot \boldsymbol{\Lambda}) + \mathfrak{K}_l(\widehat{\boldsymbol{\kappa}} \cdot \dot{\boldsymbol{\Lambda}}) + \mathfrak{K}_k[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})]\\
&\quad - \mathfrak{K}_n[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] + \mathfrak{Y}\mathfrak{v}_e[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2[\widehat{\boldsymbol{\kappa}} \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})]\\
&\quad + \mathfrak{K}_q[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o[\widehat{\boldsymbol{\kappa}} \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho\Omega^2\mathfrak{v}_e[\widehat{\boldsymbol{\kappa}} \cdot (\mathbf{r} \times \boldsymbol{\Lambda})] + \mathfrak{K}_r[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\mathbf{p}} \times \mathbf{r})]\\
&\quad - \varphi_j\mathfrak{v}_e[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})]\\
&= \mathfrak{K}_h\varepsilon_e + \mathfrak{K}_i\varepsilon_d + \mathfrak{K}_j\varepsilon_a + \mathfrak{K}_m\delta_a + \mathfrak{K}_l\varsigma_b - \mathfrak{v}_e^2\mathfrak{d}_e + \mathfrak{K}_q\varepsilon_j + \mathfrak{K}_o\varepsilon_k\\
&\quad + \rho\Omega^2\mathfrak{v}_e\varepsilon_n - \mathfrak{K}_r\mathfrak{d}_t - \varphi_j\mathfrak{v}_e\delta_b \text{ by (5.1a), (5.13a), (5.23a) \& (5.23b)}\\
&= \mathfrak{H}_j \text{ by (5.54k).} \qquad (5.58\text{s})
\end{aligned}$$

As a result of the foregoing derivations, we obtain

$$\begin{aligned}
|\mathfrak{S}_t + \mathfrak{S}_u|^2 &= (\mathfrak{S}_t + \mathfrak{S}_u) \cdot [\mathfrak{K}_h\widehat{\mathbf{p}} + \mathfrak{K}_i\mathbf{r} + \mathfrak{K}_j\boldsymbol{\Omega} + \mathfrak{K}_m\boldsymbol{\Lambda} + \mathfrak{K}_l\dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega}) - \mathfrak{K}_n(\widehat{\boldsymbol{\kappa}} \times \mathbf{r})\\
&\quad + \mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda}) + \mathfrak{K}_p(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{v}_e^2(\boldsymbol{\Omega} \times \boldsymbol{\Lambda}) + \mathfrak{K}_q(\widehat{\mathbf{p}} \times \boldsymbol{\Omega}) + \mathfrak{K}_o(\boldsymbol{\Omega} \times \mathbf{r})\\
&\quad - \rho\Omega^2\mathfrak{v}_e(\mathbf{r} \times \boldsymbol{\Lambda}) + \mathfrak{K}_r(\widehat{\mathbf{p}} \times \mathbf{r}) - \varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})] \text{ by (5.57)}\\
&= \mathfrak{K}_h[\widehat{\mathbf{p}} \cdot (\mathfrak{S}_t + \mathfrak{S}_u)] + \mathfrak{K}_i[\mathbf{r} \cdot (\mathfrak{S}_t + \mathfrak{S}_u)] + \mathfrak{K}_j[\boldsymbol{\Omega} \cdot (\mathfrak{S}_t + \mathfrak{S}_u)] + \mathfrak{K}_m[\boldsymbol{\Lambda} \cdot (\mathfrak{S}_t + \mathfrak{S}_u)]\\
&\quad + \mathfrak{K}_l[\dot{\boldsymbol{\Lambda}} \cdot (\mathfrak{S}_t + \mathfrak{S}_u)] + \mathfrak{K}_k[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Omega})] - \mathfrak{K}_n[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})]\\
&\quad + \mathfrak{Y}\mathfrak{v}_e[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\widehat{\boldsymbol{\kappa}} \times \boldsymbol{\Lambda})] + \mathfrak{K}_p[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{v}_e^2[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\boldsymbol{\Omega} \times \boldsymbol{\Lambda})]\\
&\quad + \mathfrak{K}_q[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Omega})] + \mathfrak{K}_o[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\boldsymbol{\Omega} \times \mathbf{r})] - \rho\Omega^2\mathfrak{v}_e[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\mathbf{r} \times \boldsymbol{\Lambda})]\\
&\quad + \mathfrak{K}_r[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] - \varphi_j\mathfrak{v}_e[(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\widehat{\mathbf{p}} \times \boldsymbol{\Lambda})]\\
&= \mathfrak{K}_h\mathfrak{K}_r + \mathfrak{K}_i\mathfrak{K}_s + \mathfrak{K}_j\mathfrak{K}_t + \mathfrak{K}_m\mathfrak{K}_u + \mathfrak{K}_l\mathfrak{K}_v + \mathfrak{K}_k\mathfrak{K}_w - \mathfrak{K}_n\mathfrak{K}_x + \mathfrak{Y}\mathfrak{v}_e\mathfrak{K}_y + \mathfrak{K}_p\mathfrak{K}_z\\
&\quad + \mathfrak{v}_e^2\mathfrak{H}_a + \mathfrak{K}_q\mathfrak{H}_b + \mathfrak{K}_o\mathfrak{H}_c - \rho\Omega^2\mathfrak{v}_e\mathfrak{H}_d + \mathfrak{K}_r\mathfrak{H}_e - \varphi_j\mathfrak{v}_e\mathfrak{H}_f \text{ by (5.58)}\\
\therefore |\mathfrak{S}_t + \mathfrak{S}_u| &= \mathfrak{H}_k \text{ by (5.54l).} \qquad (5.59)
\end{aligned}$$

Art 23d. *Development of equation* (3.24c).

Furthermore, we derive

$$\begin{aligned}
\Re_1 &= \widehat{\boldsymbol{\kappa}} \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \text{ by (3.24c)}\\
&= \mathfrak{H}_j \text{ by (5.58s)} \qquad (5.60\text{a})
\end{aligned}$$

$$\begin{aligned}
\Re_2 &= \mathbf{a} \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \text{ by (3.24c)}\\
&= (\varepsilon_b\boldsymbol{\Omega} - \Omega^2\mathbf{r} + \boldsymbol{\Lambda} \times \mathbf{r}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \text{ by (1.4a) \& (5.1a)}\\
&= \varepsilon_b[\boldsymbol{\Omega} \cdot (\mathfrak{S}_t + \mathfrak{S}_u)] - \Omega^2[\mathbf{r} \cdot (\mathfrak{S}_t + \mathfrak{S}_u)] + [(\mathfrak{S}_t + \mathfrak{S}_u) \cdot (\boldsymbol{\Lambda} \times \mathbf{r})]\\
&= \varepsilon_b\mathfrak{K}_t - \Omega^2\mathfrak{K}_s - \mathfrak{H}_d \text{ by (5.58)} \qquad (5.60\text{b})
\end{aligned}$$

$$\begin{aligned}
\Re_3 &= \dot{\mathbf{a}}\cdot(\mathcal{S}_t+\mathcal{S}_u) \text{ by (3.24c)}\\
&= (\mathcal{S}_t+\mathcal{S}_u)\cdot[2\varepsilon_h\mathbf{\Omega}-3\varepsilon_g\mathbf{r}+\dot{\mathbf{\Lambda}}\times\mathbf{r}-\Omega^2(\mathbf{\Omega}\times\mathbf{r})+\varepsilon_b\mathbf{\Lambda}] \text{ by (5.30a)}\\
&= 2\varepsilon_h[\mathbf{\Omega}\cdot(\mathcal{S}_t+\mathcal{S}_u)]-3\varepsilon_g[\mathbf{r}\cdot(\mathcal{S}_t+\mathcal{S}_u)]+[(\mathcal{S}_t+\mathcal{S}_u)\cdot(\dot{\mathbf{\Lambda}}\times\mathbf{r})]\\
&\quad -\Omega^2[(\mathcal{S}_t+\mathcal{S}_u)\cdot(\mathbf{\Omega}\times\mathbf{r})]+\varepsilon_b[\mathbf{\Lambda}\cdot(\mathcal{S}_t+\mathcal{S}_u)]\\
&= 2\varepsilon_h\mathfrak{K}_t-3\varepsilon_g\mathfrak{K}_s+\mathfrak{H}_h-\Omega^2\mathfrak{H}_c+\varepsilon_b\mathfrak{K}_u \text{ by (5.58)}
\end{aligned}\tag{5.60c}$$

$$\begin{aligned}
\Re_4 &= \ddot{\mathbf{a}}\cdot(\mathcal{S}_t+\mathcal{S}_u) \text{ by (3.24c)}\\
&= (\mathcal{S}_t+\mathcal{S}_u)\cdot[3\varepsilon_h\mathbf{\Lambda}+\varepsilon_b\dot{\mathbf{\Lambda}}+\varrho_t\mathbf{\Omega}+\varrho_u\mathbf{r}+2\varepsilon_b(\mathbf{\Lambda}\times\mathbf{\Omega})\\
&\quad -3\Omega^2(\mathbf{\Lambda}\times\mathbf{r})-3\varepsilon_g(\mathbf{\Omega}\times\mathbf{r})+(\ddot{\mathbf{\Lambda}}\times\mathbf{r})] \text{ by (5.30b)}\\
&= 3\varepsilon_h[\mathbf{\Lambda}\cdot(\mathcal{S}_t+\mathcal{S}_u)]+\varepsilon_b[\dot{\mathbf{\Lambda}}\cdot(\mathcal{S}_t+\mathcal{S}_u)]+\varrho_t[\mathbf{\Omega}\cdot(\mathcal{S}_t+\mathcal{S}_u)]+\varrho_u[\mathbf{r}\cdot(\mathcal{S}_t+\mathcal{S}_u)]\\
&\quad +2\varepsilon_b[(\mathcal{S}_t+\mathcal{S}_u)\cdot(\mathbf{\Lambda}\times\mathbf{\Omega})]-3\Omega^2[(\mathcal{S}_t+\mathcal{S}_u)\cdot(\mathbf{\Lambda}\times\mathbf{r})]\\
&\quad -3\varepsilon_g[(\mathcal{S}_t+\mathcal{S}_u)\cdot(\mathbf{\Omega}\times\mathbf{r})]+[(\mathcal{S}_t+\mathcal{S}_u)\cdot(\ddot{\mathbf{\Lambda}}\times\mathbf{r})]\\
&= 3\varepsilon_h\mathfrak{K}_u+\varepsilon_b\mathfrak{K}_v+\varrho_t\mathfrak{K}_t+\varrho_u\mathfrak{K}_s-2\varepsilon_b\mathfrak{H}_a+3\Omega^2\mathfrak{H}_d-3\varepsilon_g\mathfrak{H}_c+\mathfrak{H}_i \text{ by (5.58)}
\end{aligned}\tag{5.60d}$$

$$\begin{aligned}
\Re_5 &= \ddot{\mathbf{e}}\cdot(\mathcal{S}_t+\mathcal{S}_u) \text{ by (3.24c)}\\
&= (\mathcal{S}_t+\mathcal{S}_u)\cdot[\mathfrak{x}_p(\widehat{\mathbf{p}}\times\mathbf{\Omega})+2\mathfrak{x}_o(\widehat{\mathbf{p}}\times\mathbf{\Lambda})+\varphi_g\varphi_h(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})+\mathfrak{x}_s\mathbf{\Omega}+2\mathfrak{x}_r\mathbf{\Lambda}+\varphi_i\dot{\mathbf{\Lambda}}-\mathfrak{x}_v\widehat{\mathbf{p}}]\\
&= \mathfrak{x}_p[(\mathcal{S}_t+\mathcal{S}_u)\cdot(\widehat{\mathbf{p}}\times\mathbf{\Omega})]+2\mathfrak{x}_o[(\mathcal{S}_t+\mathcal{S}_u)\cdot(\widehat{\mathbf{p}}\times\mathbf{\Lambda})]+\varphi_g\varphi_h[(\mathcal{S}_t+\mathcal{S}_u)\cdot(\widehat{\mathbf{p}}\times\dot{\mathbf{\Lambda}})]\\
&\quad +\mathfrak{x}_s[\mathbf{\Omega}\cdot(\mathcal{S}_t+\mathcal{S}_u)]+2\mathfrak{x}_r[\mathbf{\Lambda}\cdot(\mathcal{S}_t+\mathcal{S}_u)]+\varphi_i[\dot{\mathbf{\Lambda}}\cdot(\mathcal{S}_t+\mathcal{S}_u)]-\mathfrak{x}_v[\widehat{\mathbf{p}}\cdot(\mathcal{S}_t+\mathcal{S}_u)]\\
&= \mathfrak{x}_p\mathfrak{H}_b+2\mathfrak{x}_o\mathfrak{H}_f-\varphi_g\varphi_h\mathfrak{H}_g+\mathfrak{x}_s\mathfrak{K}_t+2\mathfrak{x}_r\mathfrak{K}_u+\varphi_i\mathfrak{K}_v-\mathfrak{x}_v\mathfrak{K}_r \text{ by (5.58)}
\end{aligned}\tag{5.60e}$$

from which we obtain

$$\begin{aligned}
&\ddot{\mathcal{Y}}\Re_1+\ddot{\rho}\Re_2+b_2\Re_3+\rho\Re_4+\Re_5\\
&= \mathfrak{v}_f\Re_1+\mathfrak{x}_j\Re_2+\mathfrak{v}_h\Re_3+\rho\Re_4+\Re_5 \text{ by (5.35b) \& (5.46)}\\
&= \mathfrak{v}_f\mathfrak{H}_j+\mathfrak{x}_j(\varepsilon_b\mathfrak{K}_t-\Omega^2\mathfrak{K}_s-\mathfrak{H}_d)+\mathfrak{v}_h(2\varepsilon_h\mathfrak{K}_t-3\varepsilon_g\mathfrak{K}_s+\mathfrak{H}_h-\Omega^2\mathfrak{H}_c+\varepsilon_b\mathfrak{K}_u)\\
&\quad +\rho(3\varepsilon_h\mathfrak{K}_u+\varepsilon_b\mathfrak{K}_v+\varrho_t\mathfrak{K}_t+\varrho_u\mathfrak{K}_s-2\varepsilon_b\mathfrak{H}_a+3\Omega^2\mathfrak{H}_d-3\varepsilon_g\mathfrak{H}_c+\mathfrak{H}_i)\\
&\quad +\mathfrak{x}_p\mathfrak{H}_b+2\mathfrak{x}_o\mathfrak{H}_f-\varphi_g\varphi_h\mathfrak{H}_g+\mathfrak{x}_s\mathfrak{K}_t+2\mathfrak{x}_r\mathfrak{K}_u+\varphi_i\mathfrak{K}_v-\mathfrak{x}_v\mathfrak{K}_r \text{ by (5.60)}\\
&= \mathfrak{H}_l \text{ by (5.54l).}
\end{aligned}\tag{5.61}$$

We note also that

$$\begin{aligned}
\boldsymbol{\upsilon} &= \mathcal{Y}\widehat{\boldsymbol{\kappa}}+\rho\mathbf{a}-\mathbf{u}+\mathbf{e} \text{ by (3.14c)}\\
&= \mathcal{Y}\widehat{\boldsymbol{\kappa}}+\rho(\varepsilon_b\mathbf{\Omega}-\Omega^2\mathbf{r}+\mathbf{\Lambda}\times\mathbf{r})-(\mathbf{\Omega}\times\mathbf{r})+s_2(\widehat{\mathbf{p}}\times\mathbf{\Omega})+s_3\mathbf{\Omega}-s_4\widehat{\mathbf{p}}\\
&\qquad \text{by (1.4) \& (5.1a)}\\
&= \mathcal{Y}\widehat{\boldsymbol{\kappa}}+\rho(\varepsilon_b\mathbf{\Omega}-\Omega^2\mathbf{r}+\mathbf{\Lambda}\times\mathbf{r})-(\mathbf{\Omega}\times\mathbf{r})+\varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Omega})+\varphi_i\mathbf{\Omega}-\varphi_j\widehat{\mathbf{p}} \text{ by (5.28)}\\
&= \mathcal{Y}\widehat{\boldsymbol{\kappa}}+(\varphi_i+\rho\varepsilon_b)\mathbf{\Omega}-\varphi_j\widehat{\mathbf{p}}-\rho\Omega^2\mathbf{r}+\rho(\mathbf{\Lambda}\times\mathbf{r})-(\mathbf{\Omega}\times\mathbf{r})+\varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Omega})\\
&= \mathcal{Y}\widehat{\boldsymbol{\kappa}}+\mathfrak{v}_e\mathbf{\Omega}-\varphi_j\widehat{\mathbf{p}}-\rho\Omega^2\mathbf{r}+\rho(\mathbf{\Lambda}\times\mathbf{r})-(\mathbf{\Omega}\times\mathbf{r})+\varphi_g\varphi_h(\widehat{\mathbf{p}}\times\mathbf{\Omega}) \text{ by (5.23r).}
\end{aligned}\tag{5.62}$$

Art 23e. *Results of the computations.*

It follows by substituting (5.62), (5.61), (5.59) and (5.57) into (3.26) that

$$\overline{\mathbb{K}}=\frac{\mathfrak{H}_k}{c^3\mathcal{R}^3},\qquad \overline{\mathbb{T}}=\frac{\mathfrak{H}_l}{(\mathfrak{H}_k)^2}\tag{5.63a}$$

$$\begin{aligned}\overline{\boldsymbol{\ell}}_t &= \frac{1}{c\mathcal{R}}\Big[\mathfrak{Y}\widehat{\boldsymbol{\kappa}} - \varphi_j\widehat{\mathbf{p}} - \rho\Omega^2\mathbf{r} + \mathfrak{v}_e\boldsymbol{\Omega} - (\boldsymbol{\Omega}\times\mathbf{r}) + \rho(\boldsymbol{\Lambda}\times\mathbf{r}) + \varphi_g\varphi_h(\widehat{\mathbf{p}}\times\boldsymbol{\Omega})\Big]\\ \overline{\boldsymbol{\ell}}_b &= \frac{1}{\mathfrak{H}_k}\Big[\mathfrak{K}_h\widehat{\mathbf{p}} + \mathfrak{K}_i\mathbf{r} + \mathfrak{K}_j\boldsymbol{\Omega} + \mathfrak{K}_m\boldsymbol{\Lambda} + \mathfrak{K}_l\dot{\boldsymbol{\Lambda}} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Omega}) - \mathfrak{K}_n(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{Y}\mathfrak{v}_e(\widehat{\boldsymbol{\kappa}}\times\boldsymbol{\Lambda})\\ &\quad + \mathfrak{K}_p(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{v}_e^2(\boldsymbol{\Omega}\times\boldsymbol{\Lambda}) + \mathfrak{K}_q(\widehat{\mathbf{p}}\times\boldsymbol{\Omega}) + \mathfrak{K}_o(\boldsymbol{\Omega}\times\mathbf{r}) - \rho\Omega^2\mathfrak{v}_e(\mathbf{r}\times\boldsymbol{\Lambda})\\ &\quad + \mathfrak{K}_r(\widehat{\mathbf{p}}\times\mathbf{r}) - \varphi_j\mathfrak{v}_e(\widehat{\mathbf{p}}\times\boldsymbol{\Lambda})\Big]\end{aligned} \tag{5.63b}$$

is the complete set of equations describing the apparent geometry of obliquated rays for a rotating observer.

6 Gravitational obliquation

Art 24. *Apparent direction to a light source.*

To evaluate (3.6) for a gravitating observer, we introduce the quantities [9]

$$\begin{aligned}&\varepsilon_a = \widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}},\quad \varepsilon_b = \widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}},\quad \varepsilon_c = \widehat{\mathbf{r}}\cdot\widehat{\mathbf{p}},\quad \varepsilon_d = \widehat{\mathbf{r}}\cdot\mathbf{z},\quad \varepsilon_e = \widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\mathbf{h})\\ &\varepsilon_f = \widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h}) \neq 0,\quad \varepsilon_g = \widehat{\boldsymbol{\kappa}}\cdot(\mathbf{z}\times\mathbf{h}),\quad \varepsilon_h = \widehat{\mathbf{r}}\cdot(\mathbf{z}\times\mathbf{h}),\quad \varepsilon_i = \widehat{\mathbf{p}}\cdot(\mathbf{z}\times\mathbf{h})\end{aligned} \tag{6.1a}$$

$$\begin{gathered}\varphi_a = q/r^2,\quad \varphi_b = q/r,\quad \varphi_c = [(\varepsilon_b\varepsilon_e)/\varepsilon_f]^{1/2},\quad \varphi_d = \varphi_c^2 - 1 \neq 0,\quad \varphi_e = |\varphi_d|^{1/2}\\ \varphi_f = (2\rho\varphi_a\varepsilon_a + dc)/(2\varepsilon_f\varphi_d),\quad \varphi_g = -2(\rho\varphi_a\varepsilon_a + \varepsilon_f\varphi_f\varphi_c^2),\quad \varphi_h = (z^2 + 2q\varepsilon_d + q^2)^{1/2}/h\\ \varphi_i = (\varepsilon_g + \varphi_h\varepsilon_e)/(h^2\varphi_h),\quad \varphi_j = \varepsilon_h/(h^2\varphi_h),\quad \varphi_k = \varphi_f[r\varepsilon_b(q+\varepsilon_d) + h^{-2}\varepsilon_e(\varepsilon_i + \varphi_b\varepsilon_f)]\\ \varphi_l = (r^2h^2\varepsilon_b^2 + 2\varepsilon_b\varepsilon_e\varepsilon_f + \varepsilon_e^2)^{1/2},\quad \varphi_m = \varphi_h^2[\varphi_k + \varphi_h(\rho\varphi_a\varphi_j - \varphi_g\varphi_i)]\end{gathered} \tag{6.1b}$$

$$\begin{aligned}\varphi_n &= 2dc\varphi_h^2(2\varphi_f\varepsilon_e\varepsilon_b - \varphi_g) + \varphi_h^2\varphi_f(\varphi_f\varphi_l^2 - 2\rho\varphi_a\varepsilon_e\varepsilon_c - 4\varphi_g\varepsilon_b\varepsilon_e) + \varphi_h^2\varphi_g(\varphi_g - 2\rho\varphi_a\varepsilon_a)\\ \varphi_o &= 2dc\varphi_h\varphi_i(\varphi_k - \varphi_h\varphi_g\varphi_i) - \varphi_h^2\varphi_g\varphi_i(2\rho\varphi_a\varphi_j - \varphi_g\varphi_i) + \varphi_k^2 + 2\varphi_k\varphi_h(\rho\varphi_a\varphi_j - \varphi_g\varphi_i).\end{aligned} \tag{6.1c}$$

Art 24a. *Preliminary calculations.*

In view of the above quantities, we derive

$$\begin{aligned}\alpha &= \boldsymbol{\kappa}\cdot\mathbf{a} \text{ by (1.2b)}\\ &= \boldsymbol{\kappa}\cdot(-q\mathbf{r}/r^3) \text{ by (1.5a)}\\ &= -(q/r^3)(\boldsymbol{\kappa}\cdot\mathbf{r}) = -\kappa(q/r^2)\varepsilon_a \text{ by (6.1a)}\\ &= -\kappa\varphi_a\varepsilon_a \text{ by (6.1b)}\end{aligned} \tag{6.2a}$$

$$\begin{aligned}m &= \boldsymbol{\kappa}\cdot(\mathbf{r}\times\mathbf{h}) \text{ by (1.5c)}\\ &= \kappa[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\mathbf{h})]\\ &= \kappa\varepsilon_e \text{ by (6.1a)}\end{aligned} \tag{6.2b}$$

$$\begin{aligned}n &= \widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h}) \text{ by (1.5c)}\\ &= \varepsilon_f \text{ by (6.1a)}\end{aligned} \tag{6.2c}$$

[9] We note here that the angle between $\mathbf{z}$ and $\mathbf{r}$ is the true anomaly of the observer's orbit while the eccentricity (or Laplace-Runge-Lenz) vector $\mathbf{z}$ is perpendicular to the latus rectum of the orbit and the Hamilton vector $\mathbf{z}\times\mathbf{h}$ is parallel to the latus rectum.

$$
\begin{aligned}
\mho^2 &= (m/n)(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}}) \text{ by (1.5c)}\\
&= (m/n)(\kappa\varepsilon_b) \text{ by (6.1a)}\\
&= \kappa^2(\varepsilon_e/\varepsilon_f)\varepsilon_b \text{ by (6.2b) \& (6.2c)}\\
\therefore \mho &= \kappa\varphi_c \text{ by (6.1b)}
\end{aligned}
\tag{6.2d}
$$

$$
\begin{aligned}
\gamma^2 &= \left|1-\frac{\mho^2}{\kappa^2}\right| \text{ by (1.5a)}\\
&= \left|1-\varphi_c^2\right| \text{ by (6.2d)}\\
&= \left|\varphi_c^2-1\right| = |\varphi_d| \text{ by (6.1b)}\\
\therefore \gamma &= \varphi_e \text{ by (6.1b)}
\end{aligned}
\tag{6.2e}
$$

$$
\begin{aligned}
f_0 &= \frac{2\rho\alpha - d\omega_o}{n(\kappa^2-\mho^2)} \text{ by (1.5d)}\\
&= \frac{-2\rho\kappa\varphi_a\varepsilon_a - d\omega_o}{\varepsilon_f(\kappa^2-\kappa^2\varphi_c^2)} \text{ by (6.2a), (6.2c) \& (6.2d)}\\
&= \frac{-2\rho\kappa\varphi_a\varepsilon_a - d\omega_o}{\kappa^2\varepsilon_f(1-\varphi_c^2)} = \frac{2\rho\varphi_a\varepsilon_a + dc}{\kappa\varepsilon_f(\varphi_c^2-1)} \text{ by (1.2b)}\\
&= 2\varphi_f/\kappa \text{ by (6.1b)}
\end{aligned}
\tag{6.2f}
$$

$$
\begin{aligned}
f_1 &= \frac{(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})f_0}{2} \text{ by (1.5d)}\\
&= \frac{(\kappa\varepsilon_b)(2\varphi_f/\kappa)}{2} \text{ by (6.1a) \& (6.2f)}\\
&= \varepsilon_b\varphi_f
\end{aligned}
\tag{6.2g}
$$

$$
\begin{aligned}
f_2 &= \frac{mf_0}{2} \text{ by (1.5d)}\\
&= \frac{(\kappa\varepsilon_e)(2\varphi_f/\kappa)}{2} \text{ by (6.2b) \& (6.2f)}\\
&= \varepsilon_e\varphi_f.
\end{aligned}
\tag{6.2h}
$$

In addition, we have

$$
\begin{aligned}
\tau &= (2\rho\alpha - n\mho^2 f_0)\kappa^{-2} \text{ by (1.5b)}\\
&= (-2\rho\kappa\varphi_a\varepsilon_a - n\mho^2 f_0)\kappa^{-2} \text{ by (6.2a)}\\
&= [-2\rho\kappa\varphi_a\varepsilon_a - \varepsilon_f(\kappa^2\varphi_c^2)(2\varphi_f/\kappa)]\kappa^{-2} \text{ by (6.2c), (6.2d) \& (6.2f)}\\
&= -2(\rho\varphi_a\varepsilon_a + \varepsilon_f\varphi_f\varphi_c^2)/\kappa\\
&= \varphi_g/\kappa \text{ by (6.1b)}
\end{aligned}
\tag{6.3a}
$$

$$
\begin{aligned}
u^2 &= h^{-4}(\mathbf{z}\times\mathbf{h} + q\widehat{\mathbf{r}}\times\mathbf{h})^2 \text{ by (1.5c)}\\
&= h^{-4}[(\mathbf{z}\times\mathbf{h})\cdot(\mathbf{z}\times\mathbf{h}) + 2(\mathbf{z}\times\mathbf{h})\cdot(q\widehat{\mathbf{r}}\times\mathbf{h}) + (q\widehat{\mathbf{r}}\times\mathbf{h})\cdot(q\widehat{\mathbf{r}}\times\mathbf{h})]\\
&= h^{-4}[z^2h^2 - (\mathbf{z}\cdot\mathbf{h})^2 + 2(\mathbf{z}\cdot q\widehat{\mathbf{r}})h^2 - 2(\mathbf{z}\cdot\mathbf{h})(\mathbf{h}\cdot q\widehat{\mathbf{r}}) + q^2h^2 - (q\widehat{\mathbf{r}}\cdot\mathbf{h})^2] \text{ by (A.2)}\\
&= h^{-4}[z^2h^2 + 2qh^2(\mathbf{z}\cdot\widehat{\mathbf{r}}) + q^2h^2] \text{ by (1.5c)}\\
&= h^{-2}(z^2 + 2q\varepsilon_d + q^2) \text{ by (6.1a)}\\
\therefore u &= \varphi_h \text{ by (6.1b)}
\end{aligned}
\tag{6.3b}
$$

$$\begin{aligned}\cos\lambda &= (\boldsymbol{\kappa}/\kappa)\cdot(\mathbf{a}/a) \text{ by definition of } \lambda\\ &= \widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}} \text{ by (1.5a)}\\ &= \varepsilon_a \text{ by (6.1a)}\end{aligned} \tag{6.4a}$$

$$\begin{aligned}\cos\phi &= (\boldsymbol{\kappa}/\kappa)\cdot(\mathbf{u}/u) \text{ by definition of } \phi\\ &= (\widehat{\boldsymbol{\kappa}}/u)\cdot[h^{-2}(\mathbf{z}\times\mathbf{h}+q\widehat{\mathbf{r}}\times\mathbf{h})] \text{ by (1.5c)}\\ &= u^{-1}h^{-2}[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{z}\times\mathbf{h})+q\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{r}}\times\mathbf{h})]\\ &= \varphi_h^{-1}h^{-2}[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{z}\times\mathbf{h})+(q/r)\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\mathbf{h})] \text{ by (6.3b)}\\ &= \varphi_h^{-1}h^{-2}[\varepsilon_g+(q/r)\varepsilon_e] \text{ by (6.1a)}\\ &= \varphi_h^{-1}h^{-2}(\varepsilon_g+\varphi_b\varepsilon_e)=\varphi_i \text{ by (6.1b)}\end{aligned} \tag{6.4b}$$

$$\begin{aligned}\cos\theta &= (\mathbf{a}/a)\cdot(\mathbf{u}/u) \text{ by definition of } \theta\\ &= \widehat{\mathbf{r}}\cdot(\mathbf{u}/u) \text{ by (1.5a)}\\ &= (\widehat{\mathbf{r}}/u)\cdot[h^{-2}(\mathbf{z}\times\mathbf{h}+q\widehat{\mathbf{r}}\times\mathbf{h})] \text{ by (1.5c)}\\ &= \varphi_h^{-1}h^{-2}[\widehat{\mathbf{r}}\cdot(\mathbf{z}\times\mathbf{h})+q\widehat{\mathbf{r}}\cdot(\widehat{\mathbf{r}}\times\mathbf{h})] \text{ by (6.3b)}\\ &= \varphi_h^{-1}h^{-2}\varepsilon_h \text{ by (6.1a)}\\ &= \varphi_j \text{ by (6.1b).}\end{aligned} \tag{6.4c}$$

Art 24b. *Development of equation* (3.2a).

The various quantities defined in (3.2a) evaluate as

$$\begin{aligned}\mathcal{B} &= \mathbf{e}\cdot\mathbf{c} \text{ by (3.2a)}\\ &= \mathbf{c}\cdot[f_1(\mathbf{r}\times\mathbf{h})+f_2\widehat{\mathbf{p}}] \text{ by (1.5b)}\\ &= f_1c[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\mathbf{h})]+f_2c(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}}) \text{ by (1.1)}\\ &= f_1c\varepsilon_e+f_2c\varepsilon_b \text{ by (6.1a)}\\ &= \varepsilon_b\varphi_f c\varepsilon_e+\varepsilon_e\varphi_f c\varepsilon_b=2c\varphi_f\varepsilon_e\varepsilon_b \text{ by (6.2g) \& (6.2h)}\end{aligned} \tag{6.5a}$$

$$\begin{aligned}\mathcal{D} &= \mathbf{e}\cdot\mathbf{u} \text{ by (3.2a)}\\ &= h^{-2}[f_1(\mathbf{r}\times\mathbf{h})+f_2\widehat{\mathbf{p}}]\cdot(\mathbf{z}\times\mathbf{h}+q\widehat{\mathbf{r}}\times\mathbf{h}) \text{ by (1.5b) \& (1.5c)}\\ &= h^{-2}f_1[(\mathbf{r}\times\mathbf{h})\cdot(\mathbf{z}\times\mathbf{h})]+qh^{-2}f_1[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\mathbf{r}}\times\mathbf{h})]+h^{-2}f_2[\widehat{\mathbf{p}}\cdot(\mathbf{z}\times\mathbf{h})]\\ &\quad +qh^{-2}f_2[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{r}}\times\mathbf{h})]\\ &= h^{-2}f_1[(\mathbf{r}\cdot\mathbf{z})h^2-(\mathbf{r}\cdot\mathbf{h})(\mathbf{h}\cdot\mathbf{z})]+qh^{-2}f_1[(\mathbf{r}\cdot\widehat{\mathbf{r}})h^2-(\mathbf{r}\cdot\mathbf{h})(\mathbf{h}\cdot\widehat{\mathbf{r}})]\\ &\quad +h^{-2}f_2[\widehat{\mathbf{p}}\cdot(\mathbf{z}\times\mathbf{h})]+(q/r)h^{-2}f_2[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})] \text{ by (A.2)}\end{aligned}$$

$$\begin{aligned}&= rf_1(\widehat{\mathbf{r}}\cdot\mathbf{z})+qf_1r+h^{-2}f_2[\widehat{\mathbf{p}}\cdot(\mathbf{z}\times\mathbf{h})]+(q/r)h^{-2}f_2[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})] \text{ by (1.5c)}\\ &= rf_1\varepsilon_d+qf_1r+h^{-2}f_2\varepsilon_i+(q/r)h^{-2}f_2\varepsilon_f \text{ by (6.1a)}\\ &= rf_1(q+\varepsilon_d)+h^{-2}f_2[\varepsilon_i+(q/r)\varepsilon_f]\\ &= r\varepsilon_b\varphi_f(q+\varepsilon_d)+h^{-2}\varepsilon_e\varphi_f(\varepsilon_i+\varphi_b\varepsilon_f) \text{ by (6.1b), (6.2g) \& (6.2h)}\\ &= \varphi_k \text{ by (6.1b)}\end{aligned} \tag{6.5b}$$

$$\begin{aligned}\mathcal{A} &= \mathbf{e}\cdot\mathbf{a} \text{ by (3.2a)}\\ &= [f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}]\cdot(-\varphi_a\widehat{\mathbf{r}}) \text{ by (1.5a), (1.5b) \& (6.1b)}\\ &= -\varphi_a f_1[\widehat{\mathbf{r}}\cdot(\mathbf{r}\times\mathbf{h})] - \varphi_a f_2(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{r}})\\ &= -\varphi_a\varepsilon_e\varphi_f\varepsilon_c \text{ by (6.1a) \& (6.2h)}\end{aligned} \tag{6.5c}$$

$$\begin{aligned}\mathcal{H} &= \mathbf{e}\cdot\boldsymbol{\kappa} \text{ by (3.2a)}\\ &= [f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}]\cdot\boldsymbol{\kappa} \text{ by (1.5b)}\\ &= \kappa f_1[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\mathbf{h})] + \kappa f_2(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}}) = \kappa f_1\varepsilon_e + \kappa f_2\varepsilon_b \text{ by (6.1a)}\\ &= \kappa\varepsilon_b\varphi_f\varepsilon_e + \kappa\varepsilon_e\varphi_f\varepsilon_b = 2\kappa\varepsilon_b\varphi_f\varepsilon_e \text{ by (6.2g) \& (6.2h)}\end{aligned} \tag{6.5d}$$

$$\begin{aligned}\mathcal{E} &= \mathbf{e}\cdot\mathbf{e} \text{ by (3.2a)}\\ &= [f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}]\cdot[f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}] \text{ by (1.5b)}\\ &= f_1^2[(\mathbf{r}\times\mathbf{h})\cdot(\mathbf{r}\times\mathbf{h})] + 2f_1f_2[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})] + f_2^2(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{p}})\\ &= f_1^2[r^2h^2 - (\mathbf{r}\cdot\mathbf{h})^2] + 2f_1f_2\varepsilon_f + f_2^2 \text{ by (6.1a) \& (A.2)}\\ &= f_1^2r^2h^2 + 2f_1f_2\varepsilon_f + f_2^2 \text{ by (1.5c)}\\ &= (\varepsilon_b\varphi_f)^2r^2h^2 + 2(\varepsilon_b\varphi_f)(\varepsilon_e\varphi_f)\varepsilon_f + (\varepsilon_e\varphi_f)^2 \text{ by (6.2g) \& (6.2h)}\\ &= \varphi_f^2(r^2h^2\varepsilon_b^2 + 2\varepsilon_b\varepsilon_e\varepsilon_f + \varepsilon_e^2) = \varphi_f^2\varphi_l^2 \text{ by (6.1b).}\end{aligned} \tag{6.5e}$$

Art 24c. *Development of equations* (3.2b) *through* (3.2d).

Consequently, from the above derivations, we obtain

$$\begin{aligned}\mathcal{L}_0 &= \mathcal{D} - u\kappa\tau\cos\phi \text{ by (3.2b)}\\ &= \varphi_k - \varphi_h\varphi_g\varphi_i \text{ by (6.5b), (6.3b), (6.3a) \& (6.4b)}\end{aligned} \tag{6.6a}$$

$$\begin{aligned}\mathcal{L}_1 &= \rho a\cos\theta - \kappa\tau\cos\phi \text{ by (3.2b)}\\ &= \rho\varphi_a\varphi_j - \varphi_g\varphi_i \text{ by (1.5a), (6.1b), (6.4c), (6.3a) \& (6.4b)}\end{aligned} \tag{6.6b}$$

$$\begin{aligned}\mathcal{L}_2 &= \mathcal{L}_1 + \rho a\cos\theta \text{ by (3.2b)}\\ &= \rho\varphi_a\varphi_j - \varphi_g\varphi_i + \rho\varphi_a\varphi_j \text{ by (6.6b), (1.5a), (6.1b) \& (6.4c)}\\ &= 2\rho\varphi_a\varphi_j - \varphi_g\varphi_i\end{aligned} \tag{6.6c}$$

$$\begin{aligned}\mathcal{N}_1 &= 2du^2(\mathcal{B} - \tau\omega_o) \text{ by (3.2c)}\\ &= 2du^2(2c\varphi_f\varepsilon_e\varepsilon_b - c\kappa\tau) \text{ by (6.5a) \& (1.2b)}\\ &= 2d\varphi_h^2(2c\varphi_f\varepsilon_e\varepsilon_b - c\varphi_g) \text{ by (6.3b) \& (6.3a)}\\ &= 2dc\varphi_h^2(2\varphi_f\varepsilon_e\varepsilon_b - \varphi_g)\end{aligned} \tag{6.7a}$$

$$\begin{aligned}\mathcal{N}_2 &= 2duc\mathcal{L}_0\cos\phi \text{ by (3.2c)}\\ &= 2d(\varphi_h)c(\varphi_k - \varphi_h\varphi_g\varphi_i)(\varphi_i) \text{ by (6.3b), (6.6a) \& (6.4b)}\\ &= 2dc\varphi_h\varphi_i(\varphi_k - \varphi_h\varphi_g\varphi_i)\end{aligned} \tag{6.7b}$$

$$\begin{aligned}\mathcal{N}_3 &= u^2[\mathcal{E} + 2(\rho\mathcal{A} - \tau\mathcal{H})] \text{ by (3.2c)}\\ &= \varphi_h^2[\varphi_f^2\varphi_l^2 + 2\rho(-\varphi_a\varepsilon_e\varphi_f\varepsilon_c) - 2\tau(2\kappa\varepsilon_b\varphi_f\varepsilon_e)] \text{ by (6.3b), (6.5e), (6.5c) \& (6.5d)}\\ &= \varphi_h^2(\varphi_f^2\varphi_l^2 - 2\rho\varphi_a\varepsilon_e\varphi_f\varepsilon_c - 4\tau\kappa\varepsilon_b\varphi_f\varepsilon_e)\\ &= \varphi_h^2(\varphi_f^2\varphi_l^2 - 2\rho\varphi_a\varepsilon_e\varphi_f\varepsilon_c - 4\varphi_g\varepsilon_b\varphi_f\varepsilon_e) \text{ by (6.3a)}\\ &= \varphi_h^2\varphi_f(\varphi_f\varphi_l^2 - 2\rho\varphi_a\varepsilon_e\varepsilon_c - 4\varphi_g\varepsilon_b\varepsilon_e)\end{aligned} \tag{6.7c}$$

$$\begin{aligned}\mathcal{N}_4 &= u^2\kappa\tau(\kappa\tau - 2\rho a\cos\lambda) \text{ by (3.2d)}\\ &= \varphi_h^2\varphi_g(\varphi_g - 2\rho\varphi_a\varepsilon_a) \text{ by (6.3b), (6.3a), (1.5a), (6.1b) \& (6.4a)}\end{aligned} \tag{6.7d}$$

$$\begin{aligned}\mathcal{N}_5 &= u^2\kappa\tau\mathcal{L}_2\cos\phi \text{ by (3.2d)}\\ &= \varphi_h^2\varphi_g(2\rho\varphi_a\varphi_j - \varphi_g\varphi_i)\varphi_i \text{ by (6.3b), (6.3a), (6.6c) \& (6.4b)}\\ &= \varphi_h^2\varphi_g\varphi_i(2\rho\varphi_a\varphi_j - \varphi_g\varphi_i)\end{aligned} \tag{6.7e}$$

$$\begin{aligned}\mathcal{N}_6 &= \mathcal{D}(\mathcal{D} + 2u\mathcal{L}_1) \text{ by (3.2d)}\\ &= \varphi_k[\varphi_k + 2\varphi_h(\rho\varphi_a\varphi_j - \varphi_g\varphi_i)] \text{ by (6.5b), (6.3b) \& (6.6b)}\\ &= \varphi_k^2 + 2\varphi_k\varphi_h(\rho\varphi_a\varphi_j - \varphi_g\varphi_i)\end{aligned} \tag{6.7f}$$

$$\begin{aligned}2u^2(\mathcal{D} + u\mathcal{L}_1) &= \varphi_h^2[\varphi_k + \varphi_h(\rho\varphi_a\varphi_j - \varphi_g\varphi_i)] \text{ by (6.3b), (6.5b) \& (6.6b)}\\ &= \varphi_m \text{ by (6.1b)}\end{aligned} \tag{6.8a}$$

$$\begin{aligned}\mathcal{N}_1 + \mathcal{N}_3 + \mathcal{N}_4 &= 2dc\varphi_h^2(2\varphi_f\varepsilon_e\varepsilon_b - \varphi_g) + \varphi_h^2\varphi_f(\varphi_f\varphi_l^2 - 2\rho\varphi_a\varepsilon_e\varepsilon_c - 4\varphi_g\varepsilon_b\varepsilon_e)\\ &\quad + \varphi_h^2\varphi_g(\varphi_g - 2\rho\varphi_a\varepsilon_a) \text{ by (6.7)}\\ &= \varphi_n \text{ by (6.1c)}\end{aligned} \tag{6.8b}$$

$$\begin{aligned}\mathcal{N}_2 - \mathcal{N}_5 + \mathcal{N}_6 &= 2dc\varphi_h\varphi_i(\varphi_k - \varphi_h\varphi_g\varphi_i) - \varphi_h^2\varphi_g\varphi_i(2\rho\varphi_a\varphi_j - \varphi_g\varphi_i)\\ &\quad + \varphi_k^2 + 2\varphi_k\varphi_h(\rho\varphi_a\varphi_j - \varphi_g\varphi_i) \text{ by (6.7)}\\ &= \varphi_o \text{ by (6.1c).}\end{aligned} \tag{6.8c}$$

Art 24d. *Results of the computations.*

By substituting (6.8), (6.6a) and (6.4) into (3.3), we get

$$\begin{aligned}&\mathcal{L} = (\varphi_k - \varphi_h\varphi_g\varphi_i)/(\beta c^2), \quad \mathcal{P} = (\varphi_n - \varphi_m)/(\beta^2 c^4), \quad \mathcal{N} = (\varphi_n - \varphi_o)/(\beta^2 c^4)\\ &\mathcal{G} = \mathcal{L} - \beta + d\varphi_i + \rho\sigma\varphi_j, \quad \mathcal{R} = [\mathcal{P} + d^2 + \beta^2 + \rho^2\sigma^2 + 2d(\rho\sigma\varepsilon_a - \beta\varphi_i)]^{1/2}\\ &\mathcal{F} = [\mathcal{N} + d^2(1 - \varphi_i^2) + \rho^2\sigma^2(1 - \varphi_j^2) + 2d\rho\sigma(\varepsilon_a - \varphi_i\varphi_j)]^{1/2}\end{aligned} \tag{6.9a}$$

while from (3.6) and (6.4), we get

$$\tan\psi = \frac{[\mathcal{N} + d^2(1 - \varphi_i^2) + \rho^2\sigma^2(1 - \varphi_j^2) + 2d\rho\sigma(\varepsilon_a - \varphi_i\varphi_j)]^{1/2}}{\mathcal{L} - \beta + d\varphi_i + \rho\sigma\varphi_j} \tag{6.9b}$$

and by (1.2a), (1.2b), (1.5a), (1.6), (6.2a), (6.3) and (6.2e), we have

$$\begin{aligned}&\rho = \frac{\pi}{4d\omega_o}, \quad \pi = \frac{\vartheta}{\sqrt{1+\vartheta^2}}, \quad d = \gamma\left\{\frac{1+\sqrt{1+\vartheta^2}}{2}\right\}^{1/2}\\ &\vartheta = \alpha/(\gamma\omega_o)^2, \quad \alpha = -\kappa\varphi_a\varepsilon_a, \quad \gamma = \varphi_e, \quad \beta = \varphi_h/c, \quad \sigma = \varphi_a/c, \quad \mathcal{Y} = cd - \varphi_g.\end{aligned} \tag{6.9c}$$

Equations (6.1) and (6.9) give a complete prescription for calculating ψ for a gravitating observer.

Art 25. *Apparent drift of a light source.*

To evaluate (3.13) for a gravitating observer, it is convenient to introduce the following quantities in addition to those given by (6.1) and (6.9)

$$\delta_a = \widehat{\boldsymbol{\kappa}}\cdot\mathbf{h}, \quad \delta_b = \widehat{\boldsymbol{\kappa}}\cdot\mathbf{z}, \quad \delta_c = \widehat{\mathbf{p}}\cdot\mathbf{h}, \quad \delta_d = \widehat{\mathbf{p}}\cdot\mathbf{z}, \quad \delta_e = \widehat{\boldsymbol{\kappa}}\cdot(\mathbf{h}\times\widehat{\mathbf{p}}), \quad \delta_f = \widehat{\mathbf{r}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) \tag{6.10a}$$

$$\begin{aligned}
&\hbar_a = \delta_b + q\varepsilon_a, \quad \hbar_b = \varepsilon_d + q, \quad \hbar_c = \delta_d + q\varepsilon_c, \quad \hbar_d = h^{-2}(\mathcal{Y}\delta_a + \varepsilon_e\varphi_f\delta_c)\\
&\hbar_e = h^{-2}(\mathcal{Y}\hbar_a - \rho\varphi_a\hbar_b - r\varepsilon_b\varphi_f\varepsilon_h + \varepsilon_e\varphi_f\hbar_c), \quad \hbar_f = \varepsilon_e[\varphi_f\delta_e + (\rho\varphi_a/r)]\\
&\hbar_g = r\varepsilon_b\varphi_f h^2 - q, \quad \hbar_h = \hbar_f - \delta_b + \varepsilon_a\hbar_g, \quad \hbar_i = (1+\vartheta^2)^{1/2}, \quad \hbar_j = d/(\gamma^2\omega_o)\\
&\hbar_k = (\hbar_j - \rho\vartheta\hbar_i^2)/(4d^2\omega_o^2\hbar_i^3), \quad \hbar_l = (2\kappa\varphi_a\varepsilon_a\hbar_k - \rho)/(2r^2\kappa^2\varepsilon_f\varphi_d), \quad \hbar_m = \rho/r^2
\end{aligned} \tag{6.10b}$$

$$\begin{aligned}
&\hbar_n = \hbar_l\varepsilon_e\varepsilon_b + (\hbar_m/\kappa^2), \quad \hbar_o = 2(\hbar_m/\kappa) - 2\varphi_a\varepsilon_a(\hbar_k/r^2) - 2\kappa\varepsilon_f\hbar_l\varphi_c^2, \quad \hbar_p = (\beta c^2\mathcal{F})^{-1}\\
&\hbar_q = \mathcal{F}/h^2, \quad \hbar_r = \mathcal{G}\hbar_p(\hbar_e\hbar_a + \hbar_d\varphi_h^2\delta_a) - \hbar_q(\varepsilon_g + \varphi_b\varepsilon_e), \quad \hbar_s = -\varphi_a(\mathcal{G}\hbar_e\hbar_b\hbar_p - \varepsilon_h\hbar_q)\\
&\hbar_t = \kappa^2\hbar_q(\hbar_l\varepsilon_e\delta_c\delta_a - 2\hbar_n h^2), \quad \hbar_u = \kappa^2\hbar_q\hbar_l\delta_c\varepsilon_e\hbar_a, \quad \hbar_v = 2\kappa^2\mathcal{G}\hbar_p\hbar_e\hbar_n\\
&\hbar_w = \kappa^2\mathcal{G}\hbar_p\hbar_e\hbar_l\varepsilon_e\delta_c, \quad \hbar_x = \kappa^2\mathcal{G}\hbar_p\hbar_d\varphi_h^2\hbar_l\varepsilon_e\delta_c.
\end{aligned} \tag{6.10c}$$

Art 25a. *Preliminary calculations.*

We derive, in view of the above quantities,

$$\begin{aligned}
\mathbf{u}\times\widehat{\boldsymbol{\kappa}} &= -\widehat{\boldsymbol{\kappa}}\times\mathbf{u} = -h^{-2}\widehat{\boldsymbol{\kappa}}\times(\mathbf{z}\times\mathbf{h} + q\widehat{\mathbf{r}}\times\mathbf{h}) \text{ by (1.5c)}\\
&= -h^{-2}[\widehat{\boldsymbol{\kappa}}\times(\mathbf{z}\times\mathbf{h}) + q\widehat{\boldsymbol{\kappa}}\times(\widehat{\mathbf{r}}\times\mathbf{h})]\\
&= -h^{-2}[\mathbf{z}(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})] - qh^{-2}[\widehat{\mathbf{r}}(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}})] \text{ by (A.1)}\\
&= -h^{-2}(\delta_a\mathbf{z} - \delta_b\mathbf{h}) - qh^{-2}(\delta_a\widehat{\mathbf{r}} - \varepsilon_a\mathbf{h}) \text{ by (6.10a) \& (6.1a)}\\
&= -h^{-2}[\delta_a\mathbf{z} - \delta_b\mathbf{h} + q(\delta_a\widehat{\mathbf{r}} - \varepsilon_a\mathbf{h})]\\
&= h^{-2}[(\delta_b + q\varepsilon_a)\mathbf{h} - \delta_a\mathbf{z} - q\delta_a\widehat{\mathbf{r}}] = h^{-2}(\hbar_a\mathbf{h} - \delta_a\mathbf{z} - q\delta_a\widehat{\mathbf{r}}) \text{ by (6.10b)}
\end{aligned} \tag{6.11a}$$

$$\begin{aligned}
\mathbf{u}\times\mathbf{a} &= -\mathbf{a}\times\mathbf{u} = -h^{-2}(-\varphi_a\widehat{\mathbf{r}})\times(\mathbf{z}\times\mathbf{h} + q\widehat{\mathbf{r}}\times\mathbf{h}) \text{ by (1.5a), (6.1b) \& (1.5c)}\\
&= \varphi_a h^{-2}[\widehat{\mathbf{r}}\times(\mathbf{z}\times\mathbf{h}) + q\widehat{\mathbf{r}}\times(\widehat{\mathbf{r}}\times\mathbf{h})]\\
&= \varphi_a h^{-2}[\mathbf{z}(\widehat{\mathbf{r}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\mathbf{r}}\cdot\mathbf{z})] + q\varphi_a h^{-2}[\widehat{\mathbf{r}}(\widehat{\mathbf{r}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\mathbf{r}}\cdot\widehat{\mathbf{r}})] \text{ by (A.1)}\\
&= -\varphi_a h^{-2}\varepsilon_d\mathbf{h} - q\varphi_a h^{-2}\mathbf{h} \text{ by (1.5c) \& (6.1a)}\\
&= -\varphi_a h^{-2}(\varepsilon_d + q)\mathbf{h} = -h^{-2}\varphi_a\hbar_b\mathbf{h} \text{ by (6.10b)}
\end{aligned} \tag{6.11b}$$

$$\begin{aligned}
\mathbf{u}\times(\mathbf{r}\times\mathbf{h}) &= \mathbf{r}(\mathbf{u}\cdot\mathbf{h}) - \mathbf{h}(\mathbf{u}\cdot\mathbf{r}) = -\mathbf{h}(\mathbf{u}\cdot\mathbf{r}) \text{ by (A.1) \& (1.5c)}\\
&= -h^{-2}[\mathbf{r}\cdot(\mathbf{z}\times\mathbf{h} + q\widehat{\mathbf{r}}\times\mathbf{h})]\mathbf{h} \text{ by (1.5c)}\\
&= -rh^{-2}[\widehat{\mathbf{r}}\cdot(\mathbf{z}\times\mathbf{h})]\mathbf{h} = -h^{-2}r\varepsilon_h\mathbf{h} \text{ by (6.1a)}
\end{aligned} \tag{6.11c}$$

$$\begin{aligned}
\mathbf{u}\times\widehat{\mathbf{p}} &= -\widehat{\mathbf{p}}\times\mathbf{u} = -h^{-2}\widehat{\mathbf{p}}\times(\mathbf{z}\times\mathbf{h} + q\widehat{\mathbf{r}}\times\mathbf{h}) \text{ by (1.5c)}\\
&= -h^{-2}[\widehat{\mathbf{p}}\times(\mathbf{z}\times\mathbf{h})] - qh^{-2}[\widehat{\mathbf{p}}\times(\widehat{\mathbf{r}}\times\mathbf{h})]\\
&= -h^{-2}[\mathbf{z}(\widehat{\mathbf{p}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\mathbf{p}}\cdot\mathbf{z})] - qh^{-2}[\widehat{\mathbf{r}}(\widehat{\mathbf{p}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{r}})] \text{ by (A.1)}\\
&= -h^{-2}(\delta_c\mathbf{z} - \delta_d\mathbf{h}) - qh^{-2}(\delta_c\widehat{\mathbf{r}} - \varepsilon_c\mathbf{h}) \text{ by (6.1a) \& (6.10a)}\\
&= -h^{-2}[\delta_c\mathbf{z} - \delta_d\mathbf{h} + q(\delta_c\widehat{\mathbf{r}} - \varepsilon_c\mathbf{h})]\\
&= h^{-2}[(\delta_d + q\varepsilon_c)\mathbf{h} - \delta_c\mathbf{z} - q\delta_c\widehat{\mathbf{r}}] = h^{-2}(\hbar_c\mathbf{h} - \delta_c\mathbf{z} - q\delta_c\widehat{\mathbf{r}}) \text{ by (6.10b).}
\end{aligned} \tag{6.11d}$$

$$\begin{aligned}
\mathbf{u}\times\boldsymbol{\upsilon} &= \mathbf{u}\times(\mathcal{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e}) \text{ by (3.14c)}\\
&= \mathcal{Y}(\mathbf{u}\times\widehat{\boldsymbol{\kappa}}) + \rho(\mathbf{u}\times\mathbf{a}) + (\mathbf{u}\times\mathbf{e})\\
&= \mathcal{Y}h^{-2}(\hbar_a\mathbf{h} - \delta_a\mathbf{z} - q\delta_a\widehat{\mathbf{r}}) + \rho(-\varphi_a h^{-2}\hbar_b\mathbf{h}) + \mathbf{u}\times[f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}]\\
&\quad \text{by (6.11a), (6.11b) \& (1.5b)}
\end{aligned}$$

$$
\begin{aligned}
&= h^{-2}(\mathcal{Y}\hbar_a - \rho\varphi_a\hbar_b)\mathbf{h} - \mathcal{Y}h^{-2}(\delta_a\mathbf{z} + q\delta_a\widehat{\mathbf{r}}) + f_1[\mathbf{u}\times(\mathbf{r}\times\mathbf{h})] + f_2(\mathbf{u}\times\widehat{\mathbf{p}}) \\
&= h^{-2}(\mathcal{Y}\hbar_a - \rho\varphi_a\hbar_b)\mathbf{h} - \mathcal{Y}h^{-2}(\delta_a\mathbf{z} + q\delta_a\widehat{\mathbf{r}}) + f_1(-rh^{-2}\varepsilon_h\mathbf{h}) \\
&\quad + f_2h^{-2}(\hbar_c\mathbf{h} - \delta_c\mathbf{z} - q\delta_c\widehat{\mathbf{r}}) \text{ by (6.11c) \& (6.11d)} \\
&= h^{-2}(\mathcal{Y}\hbar_a - \rho\varphi_a\hbar_b - f_1r\varepsilon_h + f_2\hbar_c)\mathbf{h} - h^{-2}(\mathcal{Y}\delta_a + f_2\delta_c)(\mathbf{z} + q\widehat{\mathbf{r}}) \\
&= h^{-2}(\mathcal{Y}\hbar_a - \rho\varphi_a\hbar_b - r\varepsilon_b\varphi_f\varepsilon_h + \varepsilon_e\varphi_f\hbar_c)\mathbf{h} - h^{-2}(\mathcal{Y}\delta_a + \varepsilon_e\varphi_f\delta_c)(\mathbf{z} + q\widehat{\mathbf{r}}) \\
&\quad \text{by (6.2g) \& (6.2h)} \\
&= \hbar_e\mathbf{h} - \hbar_d(\mathbf{z} + q\widehat{\mathbf{r}}) \text{ by (6.10b)}
\end{aligned}
\tag{6.12a}
$$

$$
\begin{aligned}
\mathbf{h}\times\boldsymbol{\upsilon} &= \mathbf{h}\times(\mathcal{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e}) \text{ by (3.14c)} \\
&= \mathcal{Y}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \rho(\mathbf{h}\times\mathbf{a}) - (\mathbf{h}\times\mathbf{u}) + (\mathbf{h}\times\mathbf{e}) \\
&= \mathcal{Y}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \rho[\mathbf{h}\times(-\varphi_a\widehat{\mathbf{r}})] - h^{-2}[\mathbf{h}\times(\mathbf{z}\times\mathbf{h} + q\widehat{\mathbf{r}}\times\mathbf{h})] \\
&\quad + \mathbf{h}\times[f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}] \text{ by (1.5a), (6.1b), (1.5b) \& (1.5c)} \\
&= \mathcal{Y}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) - \rho\varphi_a(\mathbf{h}\times\widehat{\mathbf{r}}) - h^{-2}[\mathbf{h}\times(\mathbf{z}\times\mathbf{h})] - qh^{-2}[\mathbf{h}\times(\widehat{\mathbf{r}}\times\mathbf{h})] \\
&\quad + f_1[\mathbf{h}\times(\mathbf{r}\times\mathbf{h})] + f_2(\mathbf{h}\times\widehat{\mathbf{p}}) \\
&= \mathcal{Y}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) - \rho\varphi_a(\mathbf{h}\times\widehat{\mathbf{r}}) - h^{-2}[\mathbf{z}h^2 - \mathbf{h}(\mathbf{h}\cdot\mathbf{z})] - h^{-2}q[\widehat{\mathbf{r}}h^2 - \mathbf{h}(\mathbf{h}\cdot\widehat{\mathbf{r}})] \\
&\quad + f_1[\mathbf{r}h^2 - \mathbf{h}(\mathbf{h}\cdot\mathbf{r})] + f_2(\mathbf{h}\times\widehat{\mathbf{p}}) \text{ by (A.1)} \\
&= \mathcal{Y}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) - \rho\varphi_a(\mathbf{h}\times\widehat{\mathbf{r}}) - \mathbf{z} - q\widehat{\mathbf{r}} + f_1h^2\mathbf{r} + f_2(\mathbf{h}\times\widehat{\mathbf{p}}) \text{ by (1.5c)} \\
&= -\mathbf{z} + (r\varepsilon_b\varphi_fh^2 - q)\widehat{\mathbf{r}} + \mathcal{Y}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) - \rho\varphi_a(\mathbf{h}\times\widehat{\mathbf{r}}) + \varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) \\
&\quad \text{by (6.2g) \& (6.2h)} \\
&= -\mathbf{z} + \hbar_g\widehat{\mathbf{r}} + \mathcal{Y}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) - \rho\varphi_a(\mathbf{h}\times\widehat{\mathbf{r}}) + \varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) \text{ by (6.10b)}
\end{aligned}
\tag{6.12b}
$$

$$
\begin{aligned}
\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{h}\times\boldsymbol{\upsilon}) &= -\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z} + \hbar_g(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}}) + \mathcal{Y}[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{h}\times\widehat{\boldsymbol{\kappa}})] - \rho\varphi_a[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{h}\times\widehat{\mathbf{r}})] \\
&\quad + \varepsilon_e\varphi_f[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{h}\times\widehat{\mathbf{p}})] \text{ by (6.12b)} \\
&= -\delta_b + \hbar_g\varepsilon_a + (\rho\varphi_a/r)\varepsilon_e + \varepsilon_e\varphi_f\delta_e \text{ by (6.1a) \& (6.10a)} \\
&= -\delta_b + \hbar_g\varepsilon_a + \varepsilon_e[\varphi_f\delta_e + (\rho\varphi_a/r)] \\
&= \hbar_f - \delta_b + \varepsilon_a\hbar_g \text{ by (6.10b)} \\
&= \hbar_h \text{ by (6.10b)}
\end{aligned}
\tag{6.12c}
$$

$$
\begin{aligned}
q\widehat{\mathbf{r}} &= \mathbf{h}\times\mathbf{u} - \mathbf{z} \text{ by (1.5c)} \\
\therefore \mathbf{a} &= -(q/r^2)\widehat{\mathbf{r}} = r^{-2}(\mathbf{z} - \mathbf{h}\times\mathbf{u}) \text{ by (1.5a).}
\end{aligned}
\tag{6.13}
$$

Art 25b. *Development of equation* (3.8b).

The various quantities defined by (3.8b) evaluate as

$$
\begin{aligned}
\mathbf{g}_1 &= (\boldsymbol{\kappa}\cdot\nabla_u)\,\mathbf{a} \text{ by (3.8b)} \\
&= (\boldsymbol{\kappa}\cdot\nabla_u)\,[r^{-2}(\mathbf{z} - \mathbf{h}\times\mathbf{u})] \text{ by (6.13)} \\
&= r^{-2}(\boldsymbol{\kappa}\cdot\nabla_u)\,(\mathbf{z} - \mathbf{h}\times\mathbf{u}) = -r^{-2}(\boldsymbol{\kappa}\cdot\nabla_u)\,(\mathbf{h}\times\mathbf{u}) \text{ by (A.22)} \\
&= -r^{-2}\{\mathbf{h}\times[(\boldsymbol{\kappa}\cdot\nabla_u)\,\mathbf{u}] - \mathbf{u}\times[(\boldsymbol{\kappa}\cdot\nabla_u)\,\mathbf{h}]\} \text{ by (A.23)} \\
&= -r^{-2}\{\mathbf{h}\times[(\boldsymbol{\kappa}\cdot\nabla_u)\,\mathbf{u}]\} = r^{-2}(\boldsymbol{\kappa}\times\mathbf{h}) \text{ by (A.21)}
\end{aligned}
\tag{6.14a}
$$

$$
\begin{aligned}
\mathbf{g}_2 &= \nabla_u \times \mathbf{a} \text{ by (3.8b)} \\
&= \nabla_u \times [r^{-2}(\mathbf{z} + \mathbf{u} \times \mathbf{h})] \text{ by (6.13)} \\
&= r^{-2}[\nabla_u \times (\mathbf{z} + \mathbf{u} \times \mathbf{h})] = r^{-2}[\nabla_u \times (\mathbf{u} \times \mathbf{h})] \text{ by (A.22)} \\
&= r^{-2}[\mathbf{u}(\nabla_u \cdot \mathbf{h}) - (\mathbf{u} \cdot \nabla_u)\,\mathbf{h} + (\mathbf{h} \cdot \nabla_u)\,\mathbf{u} - \mathbf{h}(\nabla_u \cdot \mathbf{u})] \text{ by (A.13)} \\
&= r^{-2}[(\mathbf{h} \cdot \nabla_u)\,\mathbf{u} - \mathbf{h}(\nabla_u \cdot \mathbf{u})] = r^{-2}(\mathbf{h} - 3\mathbf{h}) \text{ by (A.21) \& (A.6)} \\
&= -2r^{-2}\mathbf{h} \qquad (6.14b)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{g}_3 &= (\mathbf{u} \cdot \nabla_u)\,\mathbf{a} \text{ by (3.8b)} \\
&= (\mathbf{u} \cdot \nabla_u)\,[r^{-2}(\mathbf{z} + \mathbf{u} \times \mathbf{h})] \text{ by (6.13)} \\
&= r^{-2}[(\mathbf{u} \cdot \nabla_u)\,(\mathbf{z} + \mathbf{u} \times \mathbf{h})] = r^{-2}[(\mathbf{u} \cdot \nabla_u)\,(\mathbf{u} \times \mathbf{h})] \text{ by (A.22)} \\
&= r^{-2}\{\mathbf{u} \times [(\mathbf{u} \cdot \nabla_u)\,\mathbf{h}] - \mathbf{h} \times [(\mathbf{u} \cdot \nabla_u)\,\mathbf{u}]\} \text{ by (A.23)} \\
&= -r^{-2}\{\mathbf{h} \times [(\mathbf{u} \cdot \nabla_u)\,\mathbf{u}]\} = -r^{-2}(\mathbf{h} \times \mathbf{u}) \text{ by (A.21)} \\
&= -r^{-2}(\mathbf{z} + q\widehat{\mathbf{r}}) \text{ by (1.5c)} \qquad (6.14c)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{g}_4 &= (\boldsymbol{\upsilon} \cdot \nabla_u)\,\mathbf{a} \text{ by (3.8b)} \\
&= (\boldsymbol{\upsilon} \cdot \nabla_u)\,[r^{-2}(\mathbf{z} + \mathbf{u} \times \mathbf{h})] \text{ by (6.13)} \\
&= r^{-2}[(\boldsymbol{\upsilon} \cdot \nabla_u)\,(\mathbf{z} + \mathbf{u} \times \mathbf{h})] = r^{-2}[(\boldsymbol{\upsilon} \cdot \nabla_u)\,(\mathbf{u} \times \mathbf{h})] \text{ by (A.22)} \\
&= r^{-2}\{\mathbf{u} \times [(\boldsymbol{\upsilon} \cdot \nabla_u)\,\mathbf{h}] - \mathbf{h} \times [(\boldsymbol{\upsilon} \cdot \nabla_u)\,\mathbf{u}]\} \text{ by (A.23)} \\
&= -r^{-2}\{\mathbf{h} \times [(\boldsymbol{\upsilon} \cdot \nabla_u)\,\mathbf{u}]\} = -r^{-2}(\mathbf{h} \times \boldsymbol{\upsilon}) \text{ by (A.21)} \qquad (6.14d)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{g}_5 &= \nabla_u \gamma \text{ by (3.8b)} \\
&= \nabla_u \left| 1 - \frac{\mho^2}{\kappa^2} \right|^{1/2} \text{ by (1.5a)} \\
&= 0 \text{ by (1.5c)} \qquad (6.14e)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{g}_6 &= \mathbf{g}_1 + \boldsymbol{\kappa} \times \mathbf{g}_2 \text{ by (3.8b)} \\
&= r^{-2}(\boldsymbol{\kappa} \times \mathbf{h}) - 2r^{-2}(\boldsymbol{\kappa} \times \mathbf{h}) \text{ by (6.14a) \& (6.14b)} \\
&= r^{-2}(\mathbf{h} \times \boldsymbol{\kappa}) \qquad (6.14f) \\
\mathbf{g}_7 &= \nu_2 \mathbf{g}_6 - \nu_3 \mathbf{g}_5 \text{ by (3.8b)} \\
&= \nu_2 r^{-2}(\mathbf{h} \times \boldsymbol{\kappa}) \text{ by (6.14f) \& (6.14e)} \qquad (6.14g)
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{g}_8 &= \nu_1 \mathbf{g}_6 - \nu_6 \mathbf{g}_5 \text{ by (3.8b)} \\
&= \nu_1 r^{-2}(\mathbf{h} \times \boldsymbol{\kappa}) \text{ by (6.14f) \& (6.14e)} \qquad (6.14h) \\
\mathbf{g}_9 &= \nu_7 \mathbf{g}_6 - \nu_8 \mathbf{g}_5 \text{ by (3.8b)} \\
&= \nu_7 r^{-2}(\mathbf{h} \times \boldsymbol{\kappa}) \text{ by (6.14f) \& (6.14e)} \qquad (6.14i)
\end{aligned}
$$

$$
\begin{aligned}
\nu_7 &= \nu_5 \left[\frac{\nu_2}{\nu_4^2} - \frac{\vartheta \nu_1}{d} \right] = \frac{(1+\vartheta^2)^{-1/2}}{4d\omega_o} \left[\frac{1}{(1+\vartheta^2)\gamma^2\omega_o^2} - \frac{\vartheta\rho}{d\omega_o} \right] \text{ by (3.8a)} \\
&= \frac{(1+\vartheta^2)^{-3/2}}{4d\omega_o} \left[\frac{1}{\gamma^2\omega_o^2} - \frac{\rho\vartheta(1+\vartheta^2)}{d\omega_o} \right] = \frac{1}{4d^2\omega_o^2\hbar_i^3} \left[\frac{d}{\gamma^2\omega_o} - \rho\vartheta\hbar_i^2 \right] \text{ by (6.10b)} \\
&= (\hbar_j - \rho\vartheta\hbar_i^2)/(4d^2\omega_o^2\hbar_i^3) = \hbar_k \text{ by (6.10b)} \qquad (6.15)
\end{aligned}
$$

$$
\begin{aligned}
\nabla_u f_0 &= \nabla_u \left[\frac{2\rho\alpha - d\omega_o}{n(\kappa^2 - \mho^2)}\right] \text{ by (1.5d)} \\
&= [n(\kappa^2 - \mho^2)]^{-1}\nabla_u(2\rho\alpha - d\omega_o) \text{ by (A.16) \& (1.5c)} \\
&= [n(\kappa^2 - \mho^2)]^{-1}(2\rho\nabla_u\alpha + 2\alpha\nabla_u\rho - \omega_o\nabla_u d) \text{ by (A.16)} \\
&= [n(\kappa^2 - \mho^2)]^{-1}(2\rho\mathbf{g}_6 + 2\alpha\mathbf{g}_9 - \omega_o\mathbf{g}_8) \text{ by (3.9)} \\
&= r^{-2}[n(\kappa^2 - \mho^2)]^{-1}(2\rho + 2\alpha\nu_7 - \omega_o\nu_1)(\mathbf{h}\times\boldsymbol{\kappa}) \text{ by (6.14)} \\
&= r^{-2}[n(\kappa^2 - \kappa^2\varphi_c^2)]^{-1}(\rho + 2\alpha\nu_7)(\mathbf{h}\times\boldsymbol{\kappa}) \text{ by (3.8a) \& (6.2d)} \\
&= [r^2\kappa^2\varepsilon_f(1-\varphi_c^2)]^{-1}(\rho - 2\kappa\varphi_a\varepsilon_a\nu_7)(\mathbf{h}\times\boldsymbol{\kappa}) \text{ by (6.2a) \& (6.2c)} \\
&= [(\rho - 2\kappa\varphi_a\varepsilon_a\hbar_k)/(-r^2\kappa^2\varepsilon_f\varphi_d)](\mathbf{h}\times\boldsymbol{\kappa}) \text{ by (6.1b) \& (6.15)} \\
&= 2\hbar_l(\mathbf{h}\times\boldsymbol{\kappa}) \text{ by (6.10b)} \qquad (6.16a)
\end{aligned}
$$

$$
\begin{aligned}
\nabla_u f_1 &= \nabla_u[(1/2)(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})f_0] \text{ by (1.5d)} \\
&= (1/2)(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})\nabla_u f_0 \text{ by (A.15)} \\
&= (1/2)(\kappa\varepsilon_b)[2\hbar_l(\mathbf{h}\times\boldsymbol{\kappa})] \text{ by (6.1a) \& (6.16a)} \\
&= \kappa\varepsilon_b\hbar_l(\mathbf{h}\times\boldsymbol{\kappa}) \qquad (6.16b) \\
\nabla_u f_2 &= \nabla_u[(1/2)mf_0] \text{ by (1.5d)} \\
&= (1/2)m\nabla_u f_0 \text{ by (A.15)} \\
&= (1/2)(\kappa\varepsilon_e)[2\hbar_l(\mathbf{h}\times\boldsymbol{\kappa})] \text{ by (6.2b) \& (6.16a)} \\
&= \kappa\varepsilon_e\hbar_l(\mathbf{h}\times\boldsymbol{\kappa}). \qquad (6.16c)
\end{aligned}
$$

Art 25c. *Development of equation* (3.8c).

The quantities defined by (3.8c) become

$$
\begin{aligned}
\jmath_1 &= \nabla_u \times \mathbf{e} \text{ by (3.8c)} \\
&= \nabla \times [f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}] \text{ by (1.5b)} \\
&= f_1[\nabla_u \times (\mathbf{r}\times\mathbf{h})] - (\mathbf{r}\times\mathbf{h})\times(\nabla_u f_1) + f_2(\nabla_u\times\widehat{\mathbf{p}}) - \widehat{\mathbf{p}}\times(\nabla_u f_2) \text{ by (A.12)} \\
&= -(\mathbf{r}\times\mathbf{h})\times(\nabla_u f_1) - \widehat{\mathbf{p}}\times(\nabla_u f_2) \\
&= -(\mathbf{r}\times\mathbf{h})\times[\kappa\varepsilon_b\hbar_l(\mathbf{h}\times\boldsymbol{\kappa})] - \widehat{\mathbf{p}}\times[\kappa\varepsilon_e\hbar_l(\mathbf{h}\times\boldsymbol{\kappa})] \text{ by (6.16b) \& (6.16c)} \\
&= -\kappa\varepsilon_b\hbar_l[\mathbf{h}(\boldsymbol{\kappa}\cdot(\mathbf{r}\times\mathbf{h})) - \boldsymbol{\kappa}(\mathbf{h}\cdot(\mathbf{r}\times\mathbf{h}))] - \kappa\varepsilon_e\hbar_l[\mathbf{h}(\widehat{\mathbf{p}}\cdot\boldsymbol{\kappa}) - \boldsymbol{\kappa}(\widehat{\mathbf{p}}\cdot\mathbf{h})] \text{ by (A.1) \& (A.5)} \\
&= -\kappa^2\varepsilon_b\hbar_l[\mathbf{h}(\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\mathbf{h}))] - \kappa^2\varepsilon_e\hbar_l[\mathbf{h}(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}}) - \widehat{\boldsymbol{\kappa}}(\widehat{\mathbf{p}}\cdot\mathbf{h})] \\
&= -\kappa^2\varepsilon_b\hbar_l\varepsilon_e\mathbf{h} - \kappa^2\varepsilon_e\hbar_l(\varepsilon_b\mathbf{h} - \delta_c\widehat{\boldsymbol{\kappa}}) \text{ by (6.1a) \& (6.10a)} \\
&= -2\kappa^2\hbar_l\varepsilon_e\varepsilon_b\mathbf{h} + \kappa^2\hbar_l\varepsilon_e\delta_c\widehat{\boldsymbol{\kappa}} \qquad (6.17a)
\end{aligned}
$$

$$
\begin{aligned}
\jmath_2 &= (\mathbf{u}\cdot\nabla_u)\,\mathbf{e} \text{ by (3.8c)} \\
&= (\mathbf{u}\cdot\nabla_u)\,[f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}] \text{ by (1.5b)} \\
&= (\mathbf{r}\times\mathbf{h})[\mathbf{u}\cdot(\nabla_u f_1)] + f_1[(\mathbf{u}\cdot\nabla_u)\,(\mathbf{r}\times\mathbf{h})] + \widehat{\mathbf{p}}[\mathbf{u}\cdot(\nabla_u f_2)] + f_2[(\mathbf{u}\cdot\nabla_u)\,\widehat{\mathbf{p}}] \text{ by (A.22)} \\
&= (\mathbf{r}\times\mathbf{h})[\mathbf{u}\cdot(\nabla_u f_1)] + \widehat{\mathbf{p}}[\mathbf{u}\cdot(\nabla_u f_2)] \\
&= \kappa\varepsilon_b\hbar_l(\mathbf{r}\times\mathbf{h})[\mathbf{u}\cdot(\mathbf{h}\times\boldsymbol{\kappa})] + \kappa\varepsilon_e\hbar_l\widehat{\mathbf{p}}[\mathbf{u}\cdot(\mathbf{h}\times\boldsymbol{\kappa})] \text{ by (6.16)} \\
&= \kappa\hbar_l[\boldsymbol{\kappa}\cdot(\mathbf{u}\times\mathbf{h})][\varepsilon_b(\mathbf{r}\times\mathbf{h}) + \varepsilon_e\widehat{\mathbf{p}}] \text{ by (A.4)} \\
&= \kappa\hbar_l[\boldsymbol{\kappa}\cdot(-\mathbf{z} - q\widehat{\mathbf{r}})][\varepsilon_b(\mathbf{r}\times\mathbf{h}) + \varepsilon_e\widehat{\mathbf{p}}] \text{ by (1.5c)} \\
&= -\kappa^2\hbar_l(\delta_b + q\varepsilon_a)[\varepsilon_b(\mathbf{r}\times\mathbf{h}) + \varepsilon_e\widehat{\mathbf{p}}] \text{ by (6.1a) \& (6.10a)} \\
&= -\kappa^2\hbar_l\hbar_a[\varepsilon_e\widehat{\mathbf{p}} + \varepsilon_b(\mathbf{r}\times\mathbf{h})] \text{ by (6.10b)} \qquad (6.17b)
\end{aligned}
$$

$$\begin{aligned}
\boldsymbol{\jmath}_3 &= (\boldsymbol{\upsilon}\cdot\nabla_u)\,\mathbf{e} \text{ by (3.8c)}\\
&= (\boldsymbol{\upsilon}\cdot\nabla_u)\,[f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}] \text{ by (1.5b)}\\
&= (\mathbf{r}\times\mathbf{h})[\boldsymbol{\upsilon}\cdot(\nabla_u f_1)] + f_1[(\boldsymbol{\upsilon}\cdot\nabla_u)\,(\mathbf{r}\times\mathbf{h})] + \widehat{\mathbf{p}}[\boldsymbol{\upsilon}\cdot(\nabla_u f_2)]\\
&\quad + f_2[(\boldsymbol{\upsilon}\cdot\nabla_u)\,\widehat{\mathbf{p}}] \text{ by (A.22)}\\
&= (\mathbf{r}\times\mathbf{h})[\boldsymbol{\upsilon}\cdot(\nabla_u f_1)] + \widehat{\mathbf{p}}[\boldsymbol{\upsilon}\cdot(\nabla_u f_2)]\\
&= \kappa\varepsilon_b\hbar_l(\mathbf{r}\times\mathbf{h})[\boldsymbol{\upsilon}\cdot(\mathbf{h}\times\boldsymbol{\kappa})] + \kappa\varepsilon_e\hbar_l\widehat{\mathbf{p}}[\boldsymbol{\upsilon}\cdot(\mathbf{h}\times\boldsymbol{\kappa})] \text{ by (6.16)}\\
&= -\kappa\hbar_l[\varepsilon_e\widehat{\mathbf{p}} + \varepsilon_b(\mathbf{r}\times\mathbf{h})][\boldsymbol{\kappa}\cdot(\mathbf{h}\times\boldsymbol{\upsilon})] \text{ by (A.4)}\\
&= -\kappa^2\hbar_l\hbar_h[\varepsilon_e\widehat{\mathbf{p}} + \varepsilon_b(\mathbf{r}\times\mathbf{h})] \text{ by (6.12c)}
\end{aligned}\tag{6.17c}$$

$$\begin{aligned}
\boldsymbol{\jmath}_4 &= \boldsymbol{\jmath}_1 + \rho\mathbf{g}_2 \text{ by (3.8c)}\\
&= -2\kappa^2\hbar_l\varepsilon_e\varepsilon_b\mathbf{h} + \kappa^2\hbar_l\varepsilon_e\delta_c\widehat{\boldsymbol{\kappa}} + \rho(-2r^{-2}\mathbf{h}) \text{ by (6.14b) \& (6.17a)}\\
&= -2(\kappa^2\hbar_l\varepsilon_e\varepsilon_b + \hbar_m)\mathbf{h} + \kappa^2\hbar_l\varepsilon_e\delta_c\widehat{\boldsymbol{\kappa}} \text{ by (6.10b)}\\
&= -2\kappa^2\hbar_n\mathbf{h} + \kappa^2\hbar_l\varepsilon_e\delta_c\widehat{\boldsymbol{\kappa}} \text{ by (6.10c)}
\end{aligned}\tag{6.17d}$$

$$\begin{aligned}
\boldsymbol{\jmath}_5 &= \boldsymbol{\jmath}_2 + \rho\mathbf{g}_3 \text{ by (3.8c)}\\
&= -\kappa^2\hbar_l\hbar_a[\varepsilon_e\widehat{\mathbf{p}} + \varepsilon_b(\mathbf{r}\times\mathbf{h})] + \rho[-r^{-2}(\mathbf{z} + q\widehat{\mathbf{r}})] \text{ by (6.17b) \& (6.14c)}\\
&= -\kappa^2\hbar_l\hbar_a\varepsilon_e\widehat{\mathbf{p}} - \hbar_m\mathbf{z} - q\hbar_m\widehat{\mathbf{r}} - \kappa^2\hbar_l\hbar_a\varepsilon_b(\mathbf{r}\times\mathbf{h})
\end{aligned}\tag{6.17e}$$

$$\begin{aligned}
\boldsymbol{\jmath}_6 &= \boldsymbol{\jmath}_3 + \rho\mathbf{g}_4 \text{ by (3.8c)}\\
&= -\kappa^2\hbar_l\hbar_h[\varepsilon_e\widehat{\mathbf{p}} + \varepsilon_b(\mathbf{r}\times\mathbf{h})] + \rho[-r^{-2}(\mathbf{h}\times\boldsymbol{\upsilon})] \text{ by (6.17c) \& (6.14d)}\\
&= -\kappa^2\hbar_l\hbar_h\varepsilon_e\widehat{\mathbf{p}} - \kappa^2\hbar_l\hbar_h\varepsilon_b(\mathbf{r}\times\mathbf{h}) - \hbar_m(\mathbf{h}\times\boldsymbol{\upsilon}) \text{ by (6.10c)}
\end{aligned}\tag{6.17f}$$

$$\begin{aligned}
\boldsymbol{\jmath}_7 &= \nabla_u\tau \text{ by (3.8c)}\\
&= \nabla_u[\kappa^{-2}(2\rho\alpha - n\mho^2 f_0)] \text{ by (1.5b)}\\
&= \kappa^{-2}(2\rho\nabla_u\alpha + 2\alpha\nabla_u\rho - n\mho^2\nabla_u f_0) \text{ by (A.16)}\\
&= \kappa^{-2}[2\rho\mathbf{g}_6 + 2\alpha\mathbf{g}_9 - 2n\mho^2\hbar_l(\mathbf{h}\times\boldsymbol{\kappa})] \text{ by (3.9) \& (6.16a)}\\
&= \kappa^{-2}[2\rho r^{-2}(\mathbf{h}\times\boldsymbol{\kappa}) + 2\alpha\nu_7 r^{-2}(\mathbf{h}\times\boldsymbol{\kappa}) - 2n\mho^2\hbar_l(\mathbf{h}\times\boldsymbol{\kappa})] \text{ by (6.14f) \& (6.14i)}\\
&= \kappa^{-2}(2\hbar_m - 2\kappa\varphi_a\varepsilon_a\nu_7 r^{-2} - 2\varepsilon_f\mho^2\hbar_l)(\mathbf{h}\times\boldsymbol{\kappa}) \text{ by (6.10b), (6.2a) \& (6.2c)}\\
&= [2(\hbar_m/\kappa) - 2\varphi_a\varepsilon_a(\hbar_k/r^2) - 2\kappa\varepsilon_f\hbar_l\varphi_c^2](\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) \text{ by (6.15) \& (6.2d)}\\
&= \hbar_o(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) \text{ by (6.10c)}
\end{aligned}\tag{6.17g}$$

$$\begin{aligned}
\boldsymbol{\jmath}_8 &= (\mathbf{u}\times\boldsymbol{\upsilon})/|\mathbf{u}\times\boldsymbol{\upsilon}| \text{ by (3.8c)}\\
&= (\beta c^2\mathcal{F})^{-1}(\mathbf{u}\times\boldsymbol{\upsilon}) \text{ by (3.5c)}\\
&= (\beta c^2\mathcal{F})^{-1}(\hbar_e\mathbf{h} - \hbar_d\mathbf{z} - q\hbar_d\widehat{\mathbf{r}}) \text{ by (6.12a)}\\
&= \hbar_p(\hbar_e\mathbf{h} - \hbar_d\mathbf{z} - q\hbar_d\widehat{\mathbf{r}}) \text{ by (6.10c)}
\end{aligned}\tag{6.17h}$$

$$\begin{aligned}
\boldsymbol{\jmath}_9 &= \boldsymbol{\jmath}_8\times\mathbf{u} \text{ by (3.8c)}\\
&= [\hbar_p(\hbar_e\mathbf{h} - \hbar_d\mathbf{z} - q\hbar_d\widehat{\mathbf{r}})]\times\mathbf{u} \text{ by (6.17h)}\\
&= \hbar_p[\hbar_e(\mathbf{h}\times\mathbf{u}) - \hbar_d(\mathbf{z}\times\mathbf{u}) - q\hbar_d(\widehat{\mathbf{r}}\times\mathbf{u})]\\
&= \hbar_p[\hbar_e(\mathbf{z} + q\widehat{\mathbf{r}}) - \hbar_d(\mathbf{z}\times\mathbf{u}) + q\hbar_d(\mathbf{h}/r)] \text{ by (1.5c)}\\
&= \hbar_p[\hbar_e(\mathbf{z} + q\widehat{\mathbf{r}}) + \varphi_b\hbar_d\mathbf{h} + \hbar_d(\mathbf{u}\times\mathbf{z})] \text{ by (6.1b)}
\end{aligned}$$

$$
\begin{aligned}
&= \hbar_p[\hbar_e(\mathbf{z}+q\widehat{\mathbf{r}}) + \varphi_b\hbar_d\mathbf{h} + \hbar_d(\mathbf{u}\times\{\mathbf{h}\times\mathbf{u} - q\widehat{\mathbf{r}}\})] \text{ by (1.5c)}\\
&= \hbar_p[\hbar_e(\mathbf{z}+q\widehat{\mathbf{r}}) + \varphi_b\hbar_d\mathbf{h} + \hbar_d\mathbf{u}\times(\mathbf{h}\times\mathbf{u}) - q\hbar_d(\mathbf{u}\times\widehat{\mathbf{r}})]\\
&= \hbar_p[\hbar_e(\mathbf{z}+q\widehat{\mathbf{r}}) + \varphi_b\hbar_d\mathbf{h} + \hbar_d\mathbf{u}\times(\mathbf{h}\times\mathbf{u}) - (q/r)\hbar_d\mathbf{h}] \text{ by (1.5c)}\\
&= \hbar_p[\hbar_e(\mathbf{z}+q\widehat{\mathbf{r}}) + \varphi_b\hbar_d\mathbf{h} + \hbar_d\mathbf{u}\times(\mathbf{h}\times\mathbf{u}) - \varphi_b\hbar_d\mathbf{h}] \text{ by (6.1b)}\\
&= \hbar_p[\hbar_e(\mathbf{z}+q\widehat{\mathbf{r}}) + \hbar_d(\mathbf{h}u^2 - \mathbf{u}(\mathbf{u}\cdot\mathbf{h}))] \text{ by (A.1)}\\
&= \hbar_p\hbar_e\mathbf{z} + q\hbar_p\hbar_e\widehat{\mathbf{r}} + \hbar_p\hbar_d\varphi_h^2\mathbf{h} \text{ by (1.5c) \& (6.3b)}
\end{aligned} \tag{6.17i}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_0 &= \mathcal{G}\boldsymbol{\jmath}_9 - \mathcal{F}\mathbf{u} \text{ by (3.8c)}\\
&= \mathcal{G}[\hbar_p\hbar_e\mathbf{z} + q\hbar_p\hbar_e\widehat{\mathbf{r}} + \hbar_p\hbar_d\varphi_h^2\mathbf{h}] - \mathcal{F}h^{-2}[(\mathbf{z}\times\mathbf{h}) + q(\widehat{\mathbf{r}}\times\mathbf{h})] \text{ by (6.17i) \& (1.5c)}\\
&= \mathcal{G}\hbar_p\hbar_e\mathbf{z} + q\mathcal{G}\hbar_p\hbar_e\widehat{\mathbf{r}} + \mathcal{G}\hbar_p\hbar_d\varphi_h^2\mathbf{h} - \hbar_q(\mathbf{z}\times\mathbf{h}) - q\hbar_q(\widehat{\mathbf{r}}\times\mathbf{h}) \text{ by (6.10c).}
\end{aligned} \tag{6.17j}
$$

From the foregoing derivations, we obtain

$$
\begin{aligned}
\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\jmath}_0 &= \widehat{\boldsymbol{\kappa}}\cdot[\mathcal{G}\hbar_p\hbar_e\mathbf{z} + q\mathcal{G}\hbar_p\hbar_e\widehat{\mathbf{r}} + \mathcal{G}\hbar_p\hbar_d\varphi_h^2\mathbf{h} - \hbar_q(\mathbf{z}\times\mathbf{h}) - q\hbar_q(\widehat{\mathbf{r}}\times\mathbf{h})] \text{ by (6.17j)}\\
&= \mathcal{G}\hbar_p\hbar_e(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z}) + q\mathcal{G}\hbar_p\hbar_e(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}}) + \mathcal{G}\hbar_p\hbar_d\varphi_h^2(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h}) - \hbar_q[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{z}\times\mathbf{h})] - q\hbar_q[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{r}}\times\mathbf{h})]\\
&= \mathcal{G}\hbar_p\hbar_e\delta_b + q\mathcal{G}\hbar_p\hbar_e\varepsilon_a + \mathcal{G}\hbar_p\hbar_d\varphi_h^2\delta_a - \hbar_q\varepsilon_g - (q/r)\hbar_q\varepsilon_e \text{ by (6.10a) \& (6.1a)}\\
&= \mathcal{G}\hbar_p[\hbar_e(\delta_b + q\varepsilon_a) + \hbar_d\varphi_h^2\delta_a] - \hbar_q(\varepsilon_g + \varphi_b\varepsilon_e) \text{ by (6.1b)}\\
&= \mathcal{G}\hbar_p(\hbar_e\hbar_a + \hbar_d\varphi_h^2\delta_a) - \hbar_q(\varepsilon_g + \varphi_b\varepsilon_e) \text{ by (6.10b)}\\
&= \hbar_r \text{ by (6.10c)}
\end{aligned} \tag{6.18a}
$$

$$
\begin{aligned}
\mathbf{c}\cdot\boldsymbol{\jmath}_0 &= c(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\jmath}_0) = c\hbar_r \text{ by (6.18a)}\\
\boldsymbol{\kappa}\cdot\boldsymbol{\jmath}_0 &= \kappa(\widehat{\boldsymbol{\kappa}}\cdot\boldsymbol{\jmath}_0) = \kappa\hbar_r \text{ by (6.18a)}
\end{aligned} \tag{6.18b}
$$

$$
\begin{aligned}
\mathbf{a}\cdot\boldsymbol{\jmath}_0 &= (-\varphi_a\widehat{\mathbf{r}})\cdot\boldsymbol{\jmath}_0 \text{ by (1.5a) \& (6.1b)}\\
&= -\varphi_a\widehat{\mathbf{r}}\cdot[\mathcal{G}\hbar_p\hbar_e\mathbf{z} + q\mathcal{G}\hbar_p\hbar_e\widehat{\mathbf{r}} + \mathcal{G}\hbar_p\hbar_d\varphi_h^2\mathbf{h} - \hbar_q(\mathbf{z}\times\mathbf{h}) - q\hbar_q(\widehat{\mathbf{r}}\times\mathbf{h})] \text{ by (6.17j)}\\
&= -\varphi_a[\mathcal{G}\hbar_p\hbar_e(\widehat{\mathbf{r}}\cdot\mathbf{z}) + q\mathcal{G}\hbar_p\hbar_e(\widehat{\mathbf{r}}\cdot\widehat{\mathbf{r}}) + \mathcal{G}\hbar_p\hbar_d\varphi_h^2(\widehat{\mathbf{r}}\cdot\mathbf{h}) - \hbar_q(\widehat{\mathbf{r}}\cdot(\mathbf{z}\times\mathbf{h})) - q\hbar_q(\widehat{\mathbf{r}}\cdot(\widehat{\mathbf{r}}\times\mathbf{h}))]\\
&= -\varphi_a(\mathcal{G}\hbar_p\hbar_e\varepsilon_d + q\mathcal{G}\hbar_p\hbar_e - \hbar_q\varepsilon_h) \text{ by (6.1a) \& (1.5c)}\\
&= -\mathcal{G}\hbar_p\hbar_e\varphi_a(\varepsilon_d + q) + \varphi_a\hbar_q\varepsilon_h = -\mathcal{G}\hbar_p\hbar_e\varphi_a\hbar_b + \varphi_a\hbar_q\varepsilon_h \text{ by (6.10b)}\\
&= -\varphi_a(\mathcal{G}\hbar_e\hbar_b\hbar_p - \varepsilon_h\hbar_q) = \hbar_s \text{ by (6.10c)}
\end{aligned} \tag{6.18c}
$$

$$
\begin{aligned}
\boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 = -\boldsymbol{\jmath}_4\times\boldsymbol{\jmath}_0 &= -(-2\kappa^2\hbar_n\mathbf{h} + \kappa^2\hbar_l\varepsilon_e\delta_c\widehat{\boldsymbol{\kappa}})\times[\mathcal{G}\hbar_p\hbar_e\mathbf{z} + q\mathcal{G}\hbar_p\hbar_e\widehat{\mathbf{r}} + \mathcal{G}\hbar_p\hbar_d\varphi_h^2\mathbf{h}\\
&\quad - \hbar_q(\mathbf{z}\times\mathbf{h}) - q\hbar_q(\widehat{\mathbf{r}}\times\mathbf{h})] \text{ by (6.17d) \& (6.17j)}\\
&= 2\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\mathbf{z}) + 2q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\widehat{\mathbf{r}}) + 2\mathcal{G}\hbar_p\hbar_d\varphi_h^2\kappa^2\hbar_n(\mathbf{h}\times\mathbf{h})\\
&\quad - 2\hbar_q\kappa^2\hbar_n[\mathbf{h}\times(\mathbf{z}\times\mathbf{h})] - 2q\hbar_q\kappa^2\hbar_n[\mathbf{h}\times(\widehat{\mathbf{r}}\times\mathbf{h})] - \mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\\
&\quad - q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_d\varphi_h^2\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) + \hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c[\widehat{\boldsymbol{\kappa}}\times(\mathbf{z}\times\mathbf{h})]\\
&\quad + q\hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c[\widehat{\boldsymbol{\kappa}}\times(\widehat{\mathbf{r}}\times\mathbf{h})]
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\mathbf{z}) + 2q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\\
&\quad - q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_d\varphi_h^2\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) - 2\hbar_q\kappa^2\hbar_n[\mathbf{h}\times(\mathbf{z}\times\mathbf{h})]\\
&\quad - 2q\hbar_q\kappa^2\hbar_n[\mathbf{h}\times(\widehat{\mathbf{r}}\times\mathbf{h})] + \hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c[\widehat{\boldsymbol{\kappa}}\times(\mathbf{z}\times\mathbf{h})] + q\hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c[\widehat{\boldsymbol{\kappa}}\times(\widehat{\mathbf{r}}\times\mathbf{h})]\\
&= 2\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\mathbf{z}) + 2q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) - q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\\
&\quad - \mathcal{G}\hbar_p\hbar_d\varphi_h^2\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) - 2\hbar_q\kappa^2\hbar_n[\mathbf{z}h^2 - \mathbf{h}(\mathbf{h}\cdot\mathbf{z})] - 2q\hbar_q\kappa^2\hbar_n[\widehat{\mathbf{r}}h^2 - \mathbf{h}(\mathbf{h}\cdot\widehat{\mathbf{r}})]\\
&\quad + \hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c[\mathbf{z}(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})] + q\hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c[\widehat{\mathbf{r}}(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}})] \text{ by (A.1)}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\mathbf{z}) + 2q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) \\
&\quad - q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_d\varphi_h^2\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) - 2\hbar_q\kappa^2\hbar_n h^2\mathbf{z} - 2q\hbar_q\kappa^2\hbar_n h^2\widehat{\mathbf{r}} \\
&\quad + \hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h})\mathbf{z} - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})\mathbf{h}] + q\hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h})\widehat{\mathbf{r}} - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}})\mathbf{h}] \text{ by (1.5c)} \\
&= 2\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\mathbf{z}) + 2q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) \\
&\quad - q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_d\varphi_h^2\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) - 2\hbar_q\kappa^2\hbar_n h^2\mathbf{z} - 2q\hbar_q\kappa^2\hbar_n h^2\widehat{\mathbf{r}} \\
&\quad + \hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c(\delta_a\mathbf{z} - \delta_b\mathbf{h}) + q\hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c(\delta_a\widehat{\mathbf{r}} - \varepsilon_a\mathbf{h}) \text{ by (6.1a) \& (6.10a)}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\mathbf{z}) + 2q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_n(\mathbf{h}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) \\
&\quad - q\mathcal{G}\hbar_p\hbar_e\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \mathcal{G}\hbar_p\hbar_d\varphi_h^2\kappa^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) - 2\hbar_q\kappa^2\hbar_n h^2\mathbf{z} + \hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c\delta_a\mathbf{z} \\
&\quad - 2q\hbar_q\kappa^2\hbar_n h^2\widehat{\mathbf{r}} + q\hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c\delta_a\widehat{\mathbf{r}} - \hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c\delta_b\mathbf{h} - q\hbar_q\kappa^2\hbar_l\varepsilon_e\delta_c\varepsilon_a\mathbf{h} \\
&= 2\kappa^2\mathcal{G}\hbar_p\hbar_e\hbar_n(\mathbf{h}\times\mathbf{z}) + 2\kappa^2 q\mathcal{G}\hbar_p\hbar_e\hbar_n(\mathbf{h}\times\widehat{\mathbf{r}}) - \kappa^2\mathcal{G}\hbar_p\hbar_e\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) \\
&\quad - \kappa^2 q\mathcal{G}\hbar_p\hbar_e\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \kappa^2\mathcal{G}\hbar_p\hbar_d\varphi_h^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) + \kappa^2\hbar_q(\hbar_l\varepsilon_e\delta_c\delta_a - 2\hbar_n h^2)\mathbf{z} \\
&\quad + \kappa^2 q\hbar_q(\hbar_l\varepsilon_e\delta_c\delta_a - 2\hbar_n h^2)\widehat{\mathbf{r}} - \kappa^2\hbar_q\hbar_l\delta_c\varepsilon_e(\delta_b + q\varepsilon_a)\mathbf{h}
\end{aligned}
$$

$$
\begin{aligned}
&= \hbar_t\mathbf{z} + q\hbar_t\widehat{\mathbf{r}} - \kappa^2\hbar_q\hbar_l\delta_c\varepsilon_e\hbar_a\mathbf{h} + 2\kappa^2\mathcal{G}\hbar_p\hbar_e\hbar_n(\mathbf{h}\times\mathbf{z}) + 2\kappa^2 q\mathcal{G}\hbar_p\hbar_e\hbar_n(\mathbf{h}\times\widehat{\mathbf{r}}) \\
&\quad - \kappa^2\mathcal{G}\hbar_p\hbar_e\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) - \kappa^2 q\mathcal{G}\hbar_p\hbar_e\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \kappa^2\mathcal{G}\hbar_p\hbar_d\varphi_h^2\hbar_l\varepsilon_e\delta_c(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) \\
&\quad \text{by (6.10b) \& (6.10c)} \\
&= \hbar_t\mathbf{z} + q\hbar_t\widehat{\mathbf{r}} - \hbar_u\mathbf{h} + \hbar_v(\mathbf{h}\times\mathbf{z}) + q\hbar_v(\mathbf{h}\times\widehat{\mathbf{r}}) - \hbar_w(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) - q\hbar_w(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) \\
&\quad - \hbar_x(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) \text{ by (6.10c)}
\end{aligned}
\tag{6.18d}
$$

$$
\begin{aligned}
&\mathcal{X}_1\mathbf{u} - \mathcal{X}_2\boldsymbol{\upsilon} + \mathcal{X}_3\boldsymbol{\jmath}_6 - \mathcal{X}_4\boldsymbol{\jmath}_5 + (\mathbf{c}\cdot\boldsymbol{\jmath}_0)\mathbf{g}_8 - (\boldsymbol{\kappa}\cdot\boldsymbol{\jmath}_0)\boldsymbol{\jmath}_7 + (\mathbf{a}\cdot\boldsymbol{\jmath}_0)\mathbf{g}_9 + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \\
&= \mathcal{X}_1\mathbf{u} - \mathcal{X}_2\boldsymbol{\upsilon} + \mathcal{X}_3\boldsymbol{\jmath}_6 - \mathcal{X}_4\boldsymbol{\jmath}_5 + c\hbar_r\mathbf{g}_8 - \kappa\hbar_r\boldsymbol{\jmath}_7 + \hbar_s\mathbf{g}_9 + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \text{ by (6.18)} \\
&= \mathcal{X}_1\mathbf{u} - \mathcal{X}_2(\mathcal{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e}) + \mathcal{X}_3\boldsymbol{\jmath}_6 - \mathcal{X}_4\boldsymbol{\jmath}_5 + c\hbar_r[\nu_1 r^{-2}(\mathbf{h}\times\boldsymbol{\kappa})] - \kappa\hbar_r\boldsymbol{\jmath}_7 \\
&\quad + \hbar_s[\nu_7 r^{-2}(\mathbf{h}\times\boldsymbol{\kappa})] + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \text{ by (3.14c), (6.14h) \& (6.14i)} \\
&= (\mathcal{X}_1+\mathcal{X}_2)\mathbf{u} - \rho\mathcal{X}_2\mathbf{a} - \mathcal{X}_2\mathbf{e} + \mathcal{X}_3\boldsymbol{\jmath}_6 - \mathcal{X}_4\boldsymbol{\jmath}_5 - \kappa\hbar_r\boldsymbol{\jmath}_7 - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + c\kappa\hbar_r\nu_1 r^{-2}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) \\
&\quad + \kappa\hbar_s\nu_7 r^{-2}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \\
&= (\mathcal{X}_1+\mathcal{X}_2)\mathbf{u} - \rho\mathcal{X}_2\mathbf{a} - \mathcal{X}_2\mathbf{e} + \mathcal{X}_3\boldsymbol{\jmath}_6 - \mathcal{X}_4\boldsymbol{\jmath}_5 - \kappa\hbar_r\boldsymbol{\jmath}_7 - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} \\
&\quad + r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k)(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \text{ by (1.2b), (3.8a) \& (6.15)}
\end{aligned}
$$

$$
\begin{aligned}
&= (\mathcal{X}_1+\mathcal{X}_2)\mathbf{u} - \rho\mathcal{X}_2\mathbf{a} - \mathcal{X}_2\mathbf{e} + \mathcal{X}_3[-\kappa^2\hbar_l\hbar_h\varepsilon_e\widehat{\mathbf{p}} - \kappa^2\hbar_l\hbar_h\varepsilon_b(\mathbf{r}\times\mathbf{h}) - \hbar_m(\mathbf{h}\times\boldsymbol{\upsilon})] \\
&\quad - \mathcal{X}_4[-\kappa^2\hbar_l\hbar_a\varepsilon_e\widehat{\mathbf{p}} - \hbar_m\mathbf{z} - q\hbar_m\widehat{\mathbf{r}} - \kappa^2\hbar_l\hbar_a\varepsilon_b(\mathbf{r}\times\mathbf{h})] - \kappa\hbar_r[\hbar_o(\mathbf{h}\times\widehat{\boldsymbol{\kappa}})] \\
&\quad - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k)(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \text{ by (6.17f), (6.17e) \& (6.17g)} \\
&= (\mathcal{X}_1+\mathcal{X}_2)\mathbf{u} - \rho\mathcal{X}_2\mathbf{a} - \mathcal{X}_2\mathbf{e} - \mathcal{X}_3\kappa^2\hbar_l\hbar_h\varepsilon_e\widehat{\mathbf{p}} - \mathcal{X}_3\kappa^2\hbar_l\hbar_h\varepsilon_b(\mathbf{r}\times\mathbf{h}) \\
&\quad - \mathcal{X}_3\hbar_m[-\mathbf{z} + \hbar_g\widehat{\mathbf{r}} + \mathcal{Y}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) - \rho\varphi_a(\mathbf{h}\times\widehat{\mathbf{r}}) + \varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}})] \\
&\quad + \mathcal{X}_4\kappa^2\hbar_l\hbar_a\varepsilon_e\widehat{\mathbf{p}} + \mathcal{X}_4\hbar_m\mathbf{z} + \mathcal{X}_4 q\hbar_m\widehat{\mathbf{r}} + \mathcal{X}_4\kappa^2\hbar_l\hbar_a\varepsilon_b(\mathbf{r}\times\mathbf{h}) - \kappa\hbar_r\hbar_o(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) \\
&\quad - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k)(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \text{ by (6.12b)}
\end{aligned}
$$

$$
\begin{aligned}
&= (\mathcal{X}_1 + \mathcal{X}_2)\mathbf{u} - \rho\mathcal{X}_2\mathbf{a} - \mathcal{X}_2\mathbf{e} - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + \mathcal{X}_4\kappa^2\hbar_l\hbar_a\varepsilon_e\widehat{\mathbf{p}} - \mathcal{X}_3\kappa^2\hbar_l\hbar_h\varepsilon_e\widehat{\mathbf{p}} \\
&\quad + \mathcal{X}_4\hbar_m\mathbf{z} + \mathcal{X}_3\hbar_m\mathbf{z} - \mathcal{X}_3\hbar_m\hbar_g\widehat{\mathbf{r}} + \mathcal{X}_4 q\hbar_m\widehat{\mathbf{r}} - \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) \\
&\quad - \mathcal{X}_3\kappa^2\hbar_l\hbar_h\varepsilon_b(\mathbf{r}\times\mathbf{h}) + \mathcal{X}_4\kappa^2\hbar_l\hbar_a\varepsilon_b(\mathbf{r}\times\mathbf{h}) + \mathcal{X}_3\hbar_m\rho\varphi_a(\mathbf{h}\times\widehat{\mathbf{r}}) \\
&\quad - \mathcal{X}_3\hbar_m\mathcal{Y}(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) - \kappa\hbar_r\hbar_o(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k)(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \\
&= (\mathcal{X}_1 + \mathcal{X}_2)\mathbf{u} - \rho\mathcal{X}_2\mathbf{a} - \mathcal{X}_2\mathbf{e} - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + \kappa^2\varepsilon_e\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h)\widehat{\mathbf{p}} \\
&\quad + \hbar_m(\mathcal{X}_4 + \mathcal{X}_3)\mathbf{z} + \hbar_m(q\mathcal{X}_4 - \mathcal{X}_3\hbar_g)\widehat{\mathbf{r}} - \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) \\
&\quad + [r\kappa^2\varepsilon_b\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - \rho\mathcal{X}_3\hbar_m\varphi_a](\widehat{\mathbf{r}}\times\mathbf{h}) \\
&\quad + [r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k) - \mathcal{Y}\mathcal{X}_3\hbar_m - \kappa\hbar_r\hbar_o](\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4
\end{aligned}
$$

$$
\begin{aligned}
&= h^{-2}(\mathcal{X}_1 + \mathcal{X}_2)(\mathbf{z}\times\mathbf{h} + q\widehat{\mathbf{r}}\times\mathbf{h}) - \rho\mathcal{X}_2(-\varphi_a\widehat{\mathbf{r}}) - \mathcal{X}_2[f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}] \\
&\quad - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + \kappa^2\varepsilon_e\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h)\widehat{\mathbf{p}} + \hbar_m(\mathcal{X}_4 + \mathcal{X}_3)\mathbf{z} + \hbar_m(q\mathcal{X}_4 - \mathcal{X}_3\hbar_g)\widehat{\mathbf{r}} \\
&\quad - \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) + [r\kappa^2\varepsilon_b\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - \rho\mathcal{X}_3\hbar_m\varphi_a](\widehat{\mathbf{r}}\times\mathbf{h}) \\
&\quad + [r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k) - \mathcal{Y}\mathcal{X}_3\hbar_m - \kappa\hbar_r\hbar_o](\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \text{ by (1.5) \& (6.1b)} \\
&= h^{-2}(\mathcal{X}_1 + \mathcal{X}_2)(\mathbf{z}\times\mathbf{h}) + qh^{-2}(\mathcal{X}_1 + \mathcal{X}_2)(\widehat{\mathbf{r}}\times\mathbf{h}) + \rho\mathcal{X}_2\varphi_a\widehat{\mathbf{r}} - \mathcal{X}_2\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{h}) \\
&\quad - \mathcal{X}_2\varepsilon_e\varphi_f\widehat{\mathbf{p}} - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + \kappa^2\varepsilon_e\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h)\widehat{\mathbf{p}} + \hbar_m(\mathcal{X}_4 + \mathcal{X}_3)\mathbf{z} + \hbar_m(q\mathcal{X}_4 - \mathcal{X}_3\hbar_g)\widehat{\mathbf{r}} \\
&\quad - \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) + [r\kappa^2\varepsilon_b\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - \rho\mathcal{X}_3\hbar_m\varphi_a](\widehat{\mathbf{r}}\times\mathbf{h}) \\
&\quad + [r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k) - \mathcal{Y}\mathcal{X}_3\hbar_m - \kappa\hbar_r\hbar_o](\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \text{ by (6.2g) \& (6.2h)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} - \mathcal{X}_2\varepsilon_e\varphi_f\widehat{\mathbf{p}} + \kappa^2\varepsilon_e\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h)\widehat{\mathbf{p}} + \hbar_m(\mathcal{X}_4 + \mathcal{X}_3)\mathbf{z} \\
&\quad + \rho\mathcal{X}_2\varphi_a\widehat{\mathbf{r}} + \hbar_m(q\mathcal{X}_4 - \mathcal{X}_3\hbar_g)\widehat{\mathbf{r}} + h^{-2}(\mathcal{X}_1 + \mathcal{X}_2)(\mathbf{z}\times\mathbf{h}) - \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) \\
&\quad - r\mathcal{X}_2\varepsilon_b\varphi_f(\widehat{\mathbf{r}}\times\mathbf{h}) + qh^{-2}(\mathcal{X}_1 + \mathcal{X}_2)(\widehat{\mathbf{r}}\times\mathbf{h}) + [r\kappa^2\varepsilon_b\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - \rho\mathcal{X}_3\hbar_m\varphi_a](\widehat{\mathbf{r}}\times\mathbf{h}) \\
&\quad + [r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k) - \mathcal{Y}\mathcal{X}_3\hbar_m - \kappa\hbar_r\hbar_o](\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4 \\
&= \hbar_m(\mathcal{X}_4 + \mathcal{X}_3)\mathbf{z} - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + [\kappa^2\varepsilon_e\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - \mathcal{X}_2\varepsilon_e\varphi_f]\widehat{\mathbf{p}} \\
&\quad + [\rho\mathcal{X}_2\varphi_a + \hbar_m(q\mathcal{X}_4 - \mathcal{X}_3\hbar_g)]\widehat{\mathbf{r}} + h^{-2}(\mathcal{X}_1 + \mathcal{X}_2)(\mathbf{z}\times\mathbf{h}) - \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) \\
&\quad + [qh^{-2}(\mathcal{X}_1 + \mathcal{X}_2) + r\kappa^2\varepsilon_b\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - r\mathcal{X}_2\varepsilon_b\varphi_f - \rho\mathcal{X}_3\hbar_m\varphi_a](\widehat{\mathbf{r}}\times\mathbf{h}) \\
&\quad + [r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k) - \mathcal{Y}\mathcal{X}_3\hbar_m - \kappa\hbar_r\hbar_o](\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \boldsymbol{\jmath}_0\times\boldsymbol{\jmath}_4
\end{aligned}
$$

$$
\begin{aligned}
&= \hbar_m(\mathcal{X}_4 + \mathcal{X}_3)\mathbf{z} - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + [\kappa^2\varepsilon_e\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - \mathcal{X}_2\varepsilon_e\varphi_f]\widehat{\mathbf{p}} \\
&\quad + [\rho\mathcal{X}_2\varphi_a + \hbar_m(q\mathcal{X}_4 - \mathcal{X}_3\hbar_g)]\widehat{\mathbf{r}} + h^{-2}(\mathcal{X}_1 + \mathcal{X}_2)(\mathbf{z}\times\mathbf{h}) - \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) \\
&\quad + [qh^{-2}(\mathcal{X}_1 + \mathcal{X}_2) + r\kappa^2\varepsilon_b\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - r\mathcal{X}_2\varepsilon_b\varphi_f - \rho\mathcal{X}_3\hbar_m\varphi_a](\widehat{\mathbf{r}}\times\mathbf{h}) \\
&\quad + [r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k) - \mathcal{Y}\mathcal{X}_3\hbar_m - \kappa\hbar_r\hbar_o](\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \hbar_t\mathbf{z} + q\hbar_t\widehat{\mathbf{r}} - \hbar_u\mathbf{h} + \hbar_v(\mathbf{h}\times\mathbf{z}) \\
&\quad + q\hbar_v(\mathbf{h}\times\widehat{\mathbf{r}}) - \hbar_w(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) - q\hbar_w(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \hbar_x(\widehat{\boldsymbol{\kappa}}\times\mathbf{h}) \text{ by (6.18d)} \\
&= \hbar_t\mathbf{z} + \hbar_m(\mathcal{X}_4 + \mathcal{X}_3)\mathbf{z} - \hbar_u\mathbf{h} - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + [\kappa^2\varepsilon_e\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - \mathcal{X}_2\varepsilon_e\varphi_f]\widehat{\mathbf{p}} \\
&\quad + q\hbar_t\widehat{\mathbf{r}} + [\rho\mathcal{X}_2\varphi_a + \hbar_m(q\mathcal{X}_4 - \mathcal{X}_3\hbar_g)]\widehat{\mathbf{r}} - \hbar_w(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) \\
&\quad - q\hbar_w(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \hbar_v(\mathbf{z}\times\mathbf{h}) + h^{-2}(\mathcal{X}_1 + \mathcal{X}_2)(\mathbf{z}\times\mathbf{h}) - \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) \\
&\quad + [qh^{-2}(\mathcal{X}_1 + \mathcal{X}_2) + r\kappa^2\varepsilon_b\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h) - r\mathcal{X}_2\varepsilon_b\varphi_f - \rho\mathcal{X}_3\hbar_m\varphi_a](\widehat{\mathbf{r}}\times\mathbf{h}) - q\hbar_v(\widehat{\mathbf{r}}\times\mathbf{h}) \\
&\quad + \hbar_x(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + [r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k) - \mathcal{Y}\mathcal{X}_3\hbar_m - \kappa\hbar_r\hbar_o](\mathbf{h}\times\widehat{\boldsymbol{\kappa}})
\end{aligned}
$$

$$
\begin{aligned}
&= -\hbar_u\mathbf{h} - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + [\hbar_t + \hbar_m(\mathcal{X}_4 + \mathcal{X}_3)]\mathbf{z} + [-\mathcal{X}_2\varepsilon_e\varphi_f + \kappa^2\varepsilon_e\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h)]\widehat{\mathbf{p}} \\
&\quad + [q\hbar_t + \rho\mathcal{X}_2\varphi_a + \hbar_m(q\mathcal{X}_4 - \mathcal{X}_3\hbar_g)]\widehat{\mathbf{r}} - \hbar_w(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) - q\hbar_w(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f(\mathbf{h}\times\widehat{\mathbf{p}}) \\
&\quad + [-r\mathcal{X}_2\varepsilon_b\varphi_f - \rho\mathcal{X}_3\hbar_m\varphi_a - q\hbar_v + qh^{-2}(\mathcal{X}_1 + \mathcal{X}_2) + r\kappa^2\varepsilon_b\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h)](\widehat{\mathbf{r}}\times\mathbf{h}) \\
&\quad + [\hbar_x - \mathcal{Y}\mathcal{X}_3\hbar_m - \kappa\hbar_r\hbar_o + r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k)](\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + [-\hbar_v + h^{-2}(\mathcal{X}_1 + \mathcal{X}_2)](\mathbf{z}\times\mathbf{h}).
\end{aligned}
\tag{6.19}
$$

Art 25d. *Results of the computations.*

It follows from (3.13) and (6.19) that

$$\begin{aligned}\mathfrak{X} = \frac{1}{\beta c^2\mathcal{R}^2}\Big[&-\hbar_u\mathbf{h} - \mathcal{Y}\mathcal{X}_2\widehat{\boldsymbol{\kappa}} + \imath_0\mathbf{z} + \imath_1\widehat{\mathbf{p}} + \imath_2\widehat{\mathbf{r}} - \hbar_w(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) - q\hbar_w(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\\ &- \imath_3(\mathbf{h}\times\widehat{\mathbf{p}}) + \imath_4(\widehat{\mathbf{r}}\times\mathbf{h}) + \imath_5(\mathbf{h}\times\widehat{\boldsymbol{\kappa}}) + \imath_6(\mathbf{z}\times\mathbf{h})\Big]\end{aligned} \tag{6.20a}$$

where

$$\begin{aligned}&\imath_0 = \hbar_t + \hbar_m(\mathcal{X}_4 + \mathcal{X}_3), \quad \imath_1 = -\mathcal{X}_2\varepsilon_e\varphi_f + \kappa^2\varepsilon_e\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h)\\ &\imath_2 = q\hbar_t + \rho\mathcal{X}_2\varphi_a + \hbar_m(q\mathcal{X}_4 - \mathcal{X}_3\hbar_g), \quad \imath_3 = \mathcal{X}_3\hbar_m\varepsilon_e\varphi_f\\ &\imath_4 = -r\mathcal{X}_2\varepsilon_b\varphi_f - \rho\mathcal{X}_3\hbar_m\varphi_a - q\hbar_v + qh^{-2}(\mathcal{X}_1 + \mathcal{X}_2) + r\kappa^2\varepsilon_b\hbar_l(\mathcal{X}_4\hbar_a - \mathcal{X}_3\hbar_h)\\ &\imath_5 = \hbar_x - \mathcal{Y}\mathcal{X}_3\hbar_m - \kappa\hbar_r\hbar_o + r^{-2}(\rho\hbar_r + \kappa\hbar_s\hbar_k), \quad \imath_6 = -\hbar_v + h^{-2}(\mathcal{X}_1 + \mathcal{X}_2)\end{aligned} \tag{6.20b}$$

and with $\mathcal{R}, \mathcal{G}, \mathcal{F}$ given by (6.9a),

$$\mathcal{X}_1 = \frac{\mathcal{R}^2(\beta+\mathcal{G})}{\beta\mathcal{F}}, \quad \mathcal{X}_2 = \frac{\mathcal{R}^2+\beta\mathcal{G}}{\mathcal{F}}, \quad \mathcal{X}_3 = \frac{\beta\mathcal{G}}{\mathcal{F}}, \quad \mathcal{X}_4 = \frac{\mathcal{R}^2}{\mathcal{F}}. \tag{6.20c}$$

Furthermore, we have from (3.7c), (1.5a) and (6.20a) that

$$\begin{aligned}\Theta &= \dot{\psi}/a = (\mathfrak{X}\cdot\mathbf{a})/a = -(\widehat{\mathbf{r}}\cdot\mathfrak{X})\\ &= \frac{1}{\beta c^2\mathcal{R}^2}\Big[\hbar_u(\widehat{\mathbf{r}}\cdot\mathbf{h}) + \mathcal{Y}\mathcal{X}_2(\widehat{\mathbf{r}}\cdot\widehat{\boldsymbol{\kappa}}) - \imath_0(\widehat{\mathbf{r}}\cdot\mathbf{z}) - \imath_1(\widehat{\mathbf{r}}\cdot\widehat{\mathbf{p}}) - \imath_2(\widehat{\mathbf{r}}\cdot\widehat{\mathbf{r}})\\ &\quad + \hbar_w[\widehat{\mathbf{r}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})] + q\hbar_w[\widehat{\mathbf{r}}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})] + \imath_3[\widehat{\mathbf{r}}\cdot(\mathbf{h}\times\widehat{\mathbf{p}})] - \imath_4[\widehat{\mathbf{r}}\cdot(\widehat{\mathbf{r}}\times\mathbf{h})]\\ &\quad - \imath_5[\widehat{\mathbf{r}}\cdot(\mathbf{h}\times\widehat{\boldsymbol{\kappa}})] - \imath_6[\widehat{\mathbf{r}}\cdot(\mathbf{z}\times\mathbf{h})]\Big]\\ &= \frac{1}{\beta c^2\mathcal{R}^2}\Big[\mathcal{Y}\mathcal{X}_2\varepsilon_a - \imath_0\varepsilon_d - \imath_1\varepsilon_c - \imath_2 + \hbar_w\delta_f + \imath_3(\varepsilon_f/r) - \imath_5(\varepsilon_e/r) - \imath_6\varepsilon_h\Big]\\ &\quad \text{by (6.1a) \& (6.10a).}\end{aligned} \tag{6.20d}$$

Equations (6.10) and (6.20) completely determine the slope and the variation of obliquation for a gravitating observer.

Art 26. *Apparent path of a light source.*

Equation (3.22) can be evaluated for a gravitating observer as follows. We introduce the following quantities in addition to those defined by (6.1), (6.9) and (6.10),

$$\varsigma_a = \mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}), \quad \varsigma_b = \mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{z}), \quad \varsigma_c = \widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\mathbf{z}) \tag{6.21a}$$

$$\begin{aligned}&\varrho_a = q/r^3, \quad \varrho_b = \varepsilon_h/(rh^2), \quad \varrho_c = \varrho_a(\hbar_a + 3\varrho_b\varepsilon_e), \quad \varrho_d = \varrho_a(\hbar_c + 3\varrho_b\varepsilon_f)\\ &\varrho_e = \varepsilon_b(\varepsilon_e\hbar_c - \varepsilon_f\hbar_a)/(2\varphi_c\varepsilon_f^2), \quad \varrho_f = \varrho_e(\varrho_e\varepsilon_f - 2\hbar_c\varphi_c)/(\varphi_c\varepsilon_f), \quad \varrho_g = \pm(\varphi_c\varrho_e)/\varphi_e\\ &\varrho_h = [\varrho_f\hbar_c\varphi_c + \varrho_e(3\varrho_f\varepsilon_f - \varrho_e\hbar_c - 2\varrho_a\varepsilon_f\varphi_c)]/(\varphi_c\varepsilon_f), \quad \varrho_i = (1/\varphi_e)[\pm(\varrho_e^2 - \varrho_f\varphi_c) - \varrho_g^2]\\ &\varrho_j = (1/\varphi_e)[\pm(\varrho_h\varphi_c - 3\varrho_e\varrho_f) - 3\varrho_g\varrho_i], \quad \varrho_k = \varphi_h/r, \quad \varrho_l = 2\varrho_a + 15\varrho_b^2 - 3\varrho_k^2\\ &\varrho_m = \varrho_a(9\varrho_k - 8\varrho_a - 45\varrho_b^2), \quad \varrho_n = 15\varrho_a\varrho_b(2\varrho_a - 3\varrho_k + 7\varrho_b^2), \quad \varrho_o = h^{-2}(\varepsilon_g + \varepsilon_e\varphi_b)\\ &\varrho_p = \varrho_a(3r\varrho_b\varepsilon_a - \varrho_o), \quad \varrho_q = \varrho_a(6\varrho_b\varrho_o - r\varrho_l\varepsilon_a), \quad \varrho_r = \varrho_m\varrho_o + r\varrho_n\varepsilon_a\end{aligned} \tag{6.21b}$$

$$\begin{aligned}&\varrho_s = \frac{1}{\varphi_e^2\omega_o^2}\left[\varrho_p + \frac{2\varphi_a\varepsilon_a\varrho_g}{\varphi_e}\right], \quad \varrho_t = \frac{1}{\varphi_e^2\omega_o^2}\left[\varrho_q - \frac{4\varrho_p\varrho_g}{\varphi_e} + \frac{2\varphi_a\varepsilon_a\varrho_i}{\varphi_e} - \frac{6\varphi_a\varepsilon_a\varrho_g^2}{\varphi_e^2}\right]\\ &\varrho_u = \frac{1}{\varphi_e^2\omega_o^2}\left[\varrho_r - \frac{6\varrho_q\varrho_g}{\varphi_e} - \frac{6\varrho_p\varrho_i}{\varphi_e} + \frac{2\varphi_a\varepsilon_a\varrho_j}{\varphi_e} + \frac{18\varrho_p\varrho_g^2}{\varphi_e^2} - \frac{18\varphi_a\varepsilon_a\varrho_g\varrho_i}{\varphi_e^2} + \frac{24\varphi_a\varepsilon_a\varrho_g^3}{\varphi_e^3}\right]\\ &\varrho_v = \varrho_s/\hbar_i^3, \quad \varrho_w = (\varrho_t\hbar_i^2 - 3\vartheta\varrho_s^2)/\hbar_i^5, \quad \varrho_x = [\varrho_u\hbar_i^4 - 9\kappa\vartheta\varrho_s\varrho_t\hbar_i^2 - 3\kappa^2\varrho_s^3(1-4\vartheta^2)]/\hbar_i^7\end{aligned} \tag{6.21c}$$

$$
\begin{aligned}
&\mathfrak{x}_a = d\left[\frac{\varrho_g}{\varphi_e} + \frac{\kappa\pi\varphi_e^2\varrho_s}{4d^2}\right], \quad \mathfrak{x}_b = \frac{\varrho_g}{\varphi_e} - \frac{\kappa\pi\varphi_e^2\varrho_s}{4d^2}, \quad \mathfrak{x}_c = \frac{\kappa}{4d}\left[\kappa\varrho_v\varphi_e\varrho_s + 2\pi\varrho_g\varrho_s + \pi\varphi_e\varrho_t\right] \\
&\mathfrak{x}_d = \frac{\varrho_i}{\varphi_e} - \frac{\varrho_g^2}{\varphi_e^2}, \quad \mathfrak{x}_e = d\mathfrak{x}_d + \mathfrak{x}_a\mathfrak{x}_b + \varphi_e\mathfrak{x}_c, \quad \mathfrak{x}_f = d\left[\frac{\varrho_j}{\varphi_e} - \frac{3\varrho_g\varrho_i}{\varphi_e^2} + \frac{2\varrho_g^3}{\varphi_e^3}\right] \\
&\mathfrak{x}_g = \mathfrak{x}_f + \mathfrak{x}_e\mathfrak{x}_b + \varrho_g\mathfrak{x}_c + 2\mathfrak{x}_a\left[\mathfrak{x}_d - \frac{\kappa^2\varrho_v\varphi_e^2\varrho_s}{4d^2} - \frac{\kappa\pi\varphi_e\varrho_g\varrho_s}{2d^2} - \frac{\kappa\pi\varphi_e^2\varrho_t}{4d^2} + \frac{\kappa\pi\varphi_e^2\mathfrak{x}_a\varrho_s}{4d^3}\right] \\
&\qquad + \frac{\kappa\varphi_e}{4d}\left[\kappa\left\{\varrho_w\varphi_e\varrho_s + 3\varrho_v\varrho_g\varrho_s + 2\varrho_v\varphi_e\varrho_t\right\} + \pi\left\{2\varrho_i\varrho_s + 3\varrho_g\varrho_t + \varphi_e\varrho_u\right\}\right]
\end{aligned} \tag{6.21d}
$$

$$
\begin{aligned}
&\mathfrak{x}_h = \frac{\kappa d\varrho_v - \pi\mathfrak{x}_a}{4\omega_o d^2}, \quad \mathfrak{x}_i = -\frac{\mathfrak{x}_a(d\kappa\varrho_v - \pi\mathfrak{x}_a)}{2\omega_o d^3} + \frac{d\kappa\varrho_w - \pi\mathfrak{x}_e}{4\omega_o d^2} \\
&\mathfrak{x}_j = \frac{(3\mathfrak{x}_a^2 - d\mathfrak{x}_e)(d\kappa\varrho_v - \pi\mathfrak{x}_a)}{2\omega_o d^4} - \frac{\mathfrak{x}_a(d\kappa\varrho_w - \pi\mathfrak{x}_e)}{\omega_o d^3} + \frac{\kappa(\mathfrak{x}_a\varrho_w + d\varrho_x - \varrho_v\mathfrak{x}_e) - \pi\mathfrak{x}_g}{4\omega_o d^2} \\
&\mathfrak{x}_k = \frac{\rho\varrho_p - \mathfrak{x}_h\varphi_a\varepsilon_a - \frac{1}{2}\mathfrak{x}_a c - \varphi_f\varphi_d\hbar_c + 2\varepsilon_f\varphi_f\varphi_c\varrho_e}{\kappa\varepsilon_f\varphi_d} \\
&\mathfrak{x}_l = \frac{\varepsilon_a\varphi_a\mathfrak{x}_i - \rho\varrho_q - 2\mathfrak{x}_h\varrho_p + \frac{1}{2}\mathfrak{x}_e c + 4\kappa\mathfrak{x}_k\varrho_e\varepsilon_f\varphi_c + 4\varphi_f\varphi_c\hbar_c\varrho_e - 2\varrho_e^2\varphi_f\varepsilon_f + 2\varrho_f\varphi_f\varepsilon_f\varphi_c}{\kappa\varepsilon_f\varphi_d} \\
&\qquad - \frac{2\kappa\mathfrak{x}_k\hbar_c - \varepsilon_f\varrho_a\varphi_f}{\kappa\varepsilon_f}
\end{aligned} \tag{6.21e}
$$

$$
\begin{aligned}
\mathfrak{x}_m = &-\frac{\rho\varrho_r - \mathfrak{x}_j\varphi_a\varepsilon_a + 3\mathfrak{x}_i\varrho_p + 3\mathfrak{x}_h\varrho_q - \frac{1}{2}\mathfrak{x}_g c}{\kappa\varepsilon_f\varphi_d} + \frac{\hbar_c\varphi_d - 2\varepsilon_f\varphi_c\varrho_e}{\varepsilon_f\varphi_d}\left\{\mathfrak{x}_l + \frac{2\kappa\mathfrak{x}_k\hbar_c - \varrho_a\varepsilon_f\varphi_f}{\kappa\varepsilon_f}\right\} \\
&- \frac{\varepsilon_f[\kappa(-2\mathfrak{x}_l\hbar_c + 2\mathfrak{x}_k\varrho_a\varepsilon_f + \mathfrak{x}_k\varrho_a\varepsilon_f) + \varrho_d\varphi_f] + \hbar_c(2\kappa\mathfrak{x}_k\hbar_c - \varrho_a\varepsilon_f\varphi_f)}{\kappa\varepsilon_f^2} \\
&- \frac{8\kappa\varrho_e\mathfrak{x}_k\hbar_c\varphi_c - 4\varrho_e\varrho_a\varepsilon_f\varphi_f\varphi_c - 4\hbar_c\varrho_e^2\varphi_f + 6\hbar_c\varrho_f\varphi_f\varphi_c - 2\varrho_e^2\hbar_c\varphi_f}{\kappa\varepsilon_f\varphi_d} \\
&- \frac{4\kappa\varrho_e\mathfrak{x}_l\varepsilon_f\varphi_c - 6\kappa\varrho_e^2\mathfrak{x}_k\varepsilon_f - 6\varrho_e\varrho_f\varphi_f\varepsilon_f + 6\kappa\varrho_f\mathfrak{x}_k\varepsilon_f\varphi_c + 2\varrho_h\varphi_f\varepsilon_f\varphi_c}{\kappa\varepsilon_f\varphi_d}
\end{aligned} \tag{6.21f}
$$

$$
\begin{aligned}
&\mathfrak{x}_n = -\hbar_a\varphi_f - \kappa\varepsilon_e\mathfrak{x}_k, \quad \mathfrak{x}_o = -\varrho_a\varepsilon_e\varphi_f + \kappa(2\hbar_a\mathfrak{x}_k + \varepsilon_e\mathfrak{x}_l), \quad \mathfrak{x}_p = \varepsilon_b(\kappa\mathfrak{x}_l - \varphi_f\varrho_a) \\
&\mathfrak{x}_q = \varrho_c\varphi_f + \kappa(3\varrho_a\varepsilon_e\mathfrak{x}_k - 3\hbar_a\mathfrak{x}_l + \varepsilon_e\mathfrak{x}_m), \quad \mathfrak{x}_r = \varepsilon_b(\kappa\mathfrak{x}_m + 3\kappa\mathfrak{x}_k\varrho_a + 3\varrho_a\varrho_b\varphi_f) \\
&\mathfrak{x}_s = \varepsilon_b(3\kappa\mathfrak{x}_l - \varrho_a\varphi_f), \quad \mathfrak{x}_t = \rho\varrho_p - \mathfrak{x}_h\varphi_a\varepsilon_a + \hbar_c\varphi_c^2\varphi_f - 2\varrho_e\varepsilon_f\varphi_c\varphi_f + \kappa\mathfrak{x}_k\varepsilon_f\varphi_c^2 \\
&\mathfrak{x}_u = -\varepsilon_a\varphi_a\mathfrak{x}_i + 2\mathfrak{x}_h\varrho_p + \rho\varrho_q + \varrho_a\varepsilon_f\varphi_c^2\varphi_f + 4\hbar_c\varrho_e\varphi_c\varphi_f - 2\kappa\hbar_c\mathfrak{x}_k\varphi_c^2 - 2\varrho_e^2\varepsilon_f\varphi_f \\
&\qquad + 2\varrho_f\varepsilon_f\varphi_c\varphi_f + 4\kappa\varrho_e\mathfrak{x}_k\varepsilon_f\varphi_c - \kappa\varepsilon_f\mathfrak{x}_l\varphi_c^2 \\
&\mathfrak{x}_v = -\varphi_a\varepsilon_a\mathfrak{x}_j + 3\varrho_p\mathfrak{x}_i + 3\varrho_q\mathfrak{x}_h + \varrho_r\rho - \varrho_d\varphi_c^2\varphi_f - \kappa\varepsilon_f\mathfrak{x}_k\varrho_a\varphi_c^2 + 6\varrho_e\varrho_a\varepsilon_f\varphi_c\varphi_f \\
&\qquad + 6\varrho_e^2\hbar_c\varphi_f - 6\varrho_f\hbar_c\varphi_c\varphi_f - 12\kappa\varrho_e\mathfrak{x}_k\hbar_c\varphi_c - 2\kappa\mathfrak{x}_k\varrho_a\varepsilon_f\varphi_c^2 + 3\kappa\hbar_c\mathfrak{x}_l\varphi_c^2 + 4\varrho_e\varrho_f\varepsilon_f\varphi_f \\
&\qquad + 6\kappa\varrho_e^2\mathfrak{x}_k\varepsilon_f + 2\varrho_e\varrho_f\varepsilon_f\varphi_f - 2\varrho_h\varepsilon_f\varphi_c\varphi_f - 6\kappa\varrho_f\mathfrak{x}_k\varepsilon_f\varphi_c - 6\kappa\varrho_e\mathfrak{x}_l\varepsilon_f\varphi_c - \kappa\varepsilon_f\mathfrak{x}_m\varphi_c^2
\end{aligned} \tag{6.21g}
$$

$$
\begin{aligned}
&\mathfrak{y}_a = c\mathfrak{x}_a - 2\mathfrak{x}_t, \quad \mathfrak{y}_b = c\mathfrak{x}_e - 2\mathfrak{x}_u, \quad \mathfrak{y}_c = c\mathfrak{x}_g - 2\mathfrak{x}_v, \quad \mathfrak{y}_d = \mathfrak{x}_h - 1, \quad \mathfrak{y}_e = 2\mathfrak{x}_h - 1 \\
&\mathfrak{y}_f = 3\mathfrak{x}_h - 1, \quad \mathfrak{y}_g = \mathfrak{x}_i\mathfrak{y}_a - \mathfrak{y}_b\mathfrak{y}_d, \quad \mathfrak{y}_h = \mathfrak{y}_a\mathfrak{y}_e - \rho\mathfrak{y}_b, \quad \mathfrak{y}_i = \mathfrak{y}_d\mathfrak{y}_e - \rho\mathfrak{x}_i \\
&\mathfrak{y}_j = \varrho_a^2(\varrho_l - 18\varrho_b^2), \quad \mathfrak{y}_k = \varphi_b\varepsilon_b(\varrho_a\varphi_f + 2\kappa\varrho_b\mathfrak{x}_k), \quad \mathfrak{y}_l = 2\varphi_b(\kappa\varrho_a\varepsilon_b\mathfrak{x}_k + \varrho_b\mathfrak{x}_p) \\
&\mathfrak{y}_m = \varepsilon_b\varphi_b[6\varrho_a\varrho_b\varphi_f + \kappa\mathfrak{x}_k(\varrho_l - 6\varrho_b^2)], \quad \mathfrak{y}_n = 2\kappa\mathfrak{x}_k\mathfrak{x}_n + \mathfrak{x}_o\varphi_f, \quad \mathfrak{y}_o = \mathfrak{x}_p\mathfrak{x}_n + \kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_o \\
&\mathfrak{y}_p = 2\kappa^2\varepsilon_b\mathfrak{x}_k^2 - \mathfrak{x}_p\varphi_f, \quad \mathfrak{y}_q = r(\varepsilon_c\mathfrak{y}_o + \varepsilon_b\mathfrak{y}_p\hbar_b), \quad \mathfrak{y}_r = \varphi_f\mathfrak{y}_b + 2\kappa\mathfrak{x}_k\mathfrak{y}_a \\
&\mathfrak{y}_s = \mathfrak{y}_h - 6\rho\varrho_b\mathfrak{y}_a, \quad \mathfrak{y}_t = \mathfrak{y}_i - 6\rho\varrho_b\mathfrak{y}_d, \quad \mathfrak{y}_u = \mathfrak{x}_o\mathfrak{y}_a - \mathfrak{x}_n\mathfrak{y}_b, \quad \mathfrak{y}_v = \kappa\varepsilon_b\mathfrak{x}_k\mathfrak{y}_b + \mathfrak{x}_p\mathfrak{y}_a \\
&\mathfrak{y}_w = 3\varrho_b\mathfrak{y}_h - \rho\varrho_l\mathfrak{y}_a - \mathfrak{y}_g, \quad \mathfrak{y}_x = \varphi_a\mathfrak{y}_w + q\varepsilon_b\mathfrak{y}_r, \quad \mathfrak{y}_y = r\varepsilon_a\mathfrak{y}_v + \varrho_a^2\mathfrak{y}_t, \quad \mathfrak{y}_z = z^2 + q\varepsilon_d
\end{aligned} \tag{6.21h}
$$

$$\begin{aligned}
\varkappa_a &= \varrho_a(\mathfrak{x}_n\mathfrak{x}_i - \mathfrak{x}_o\mathfrak{y}_d) - 3\varrho_a\varrho_b(\mathfrak{x}_n\mathfrak{y}_e - \rho\mathfrak{x}_o) + \rho\varrho_a\varrho_l\mathfrak{x}_n - \varepsilon_b\varphi_b\mathfrak{y}_n \\
\varkappa_b &= \varrho_a\varepsilon_b[\varphi_f(3\varrho_b\mathfrak{y}_e - \mathfrak{x}_i - \rho\varrho_l) + 2\kappa\mathfrak{x}_k(3\rho\varrho_b - \mathfrak{y}_d)] \\
\varkappa_c &= \varrho_a(\mathfrak{x}_n\mathfrak{y}_e - \rho\mathfrak{x}_o - 6\rho\varrho_b\mathfrak{x}_n), \quad \varkappa_d = \varrho_a\varepsilon_b(-\varphi_f\mathfrak{y}_e - 2\rho\kappa\mathfrak{x}_k + 6\rho\varrho_b\varphi_f) \\
\varkappa_e &= \varphi_b(\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_i + \mathfrak{x}_p\mathfrak{y}_d) + \rho(\rho\mathfrak{y}_j - \mathfrak{y}_l + \mathfrak{y}_m) - \mathfrak{y}_k\mathfrak{y}_e - \mathfrak{y}_q \\
\varkappa_f &= \varrho_a\mathfrak{y}_d - 3\rho\varrho_a\varrho_b + \varphi_b\varepsilon_b\varphi_f, \quad \varkappa_g = \delta_a\mathfrak{y}_v + \delta_c\mathfrak{y}_o, \quad \varkappa_h = \varkappa_e - \mathfrak{y}_y
\end{aligned} \tag{6.21i}$$

$$\begin{aligned}
\varkappa_i &= [\mathfrak{y}_a^2 + 2\mathfrak{x}_n\mathfrak{y}_a\varepsilon_b - 2\varepsilon_b\varphi_f\mathfrak{y}_a\delta_b - 2\rho\varrho_a\mathfrak{y}_a\varrho_o - 2r\varkappa_f\mathfrak{y}_a\varepsilon_a - 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{y}_a\varepsilon_e + \mathfrak{x}_n^2 \\
&\quad - 2\varepsilon_b\varphi_f\mathfrak{x}_n\delta_d - 2\varrho_a\mathfrak{x}_n(\rho/h^2)(\varepsilon_i + \varepsilon_f\varphi_b) - 2r\varkappa_f\mathfrak{x}_n\varepsilon_c - 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_n\varepsilon_f + z^2\varepsilon_b^2\varphi_f^2 \\
&\quad - 2\rho r\varrho_b\varphi_b\varrho_a\varepsilon_b\varphi_f + 2r\varkappa_f\varepsilon_b\varphi_f\varepsilon_d - 2r\kappa\varepsilon_b^2\mathfrak{x}_k\varphi_f\varepsilon_h + (\rho\varrho_a\varphi_h)^2 + 2\rho r^2\varkappa_f\varrho_a\varrho_b \\
&\quad + 2\kappa r\hbar_b\varepsilon_b\mathfrak{x}_k\rho\varrho_a + (r\varkappa_f)^2 + (\kappa r h\varepsilon_b\mathfrak{x}_k)^2]^{1/2} \\
\varkappa_j &= \varkappa_g r^2 + \mathfrak{y}_u\varsigma_a + r\varepsilon_b\mathfrak{y}_r\delta_f + \varepsilon_b\mathfrak{y}_n\varsigma_b - \varrho_a\mathfrak{y}_s\delta_a - \varkappa_c\delta_c \\
\varkappa_k &= h^2\varkappa_h + \varepsilon_e(\mathfrak{y}_x/r) - \mathfrak{y}_u\delta_e + \varepsilon_b\mathfrak{y}_r\varepsilon_g + \varepsilon_b\mathfrak{y}_n\varepsilon_i - \varkappa_a\varepsilon_f + r\varkappa_b\varepsilon_h + \varrho_a\mathfrak{y}_s\hbar_a + \varkappa_c\hbar_c + \varkappa_d\mathfrak{y}_z \\
\varkappa_l &= (\varkappa_h/r)\varepsilon_e + \mathfrak{y}_x(1-\varepsilon_a^2) + \mathfrak{y}_u(\varepsilon_c - \varepsilon_b\varepsilon_a) + \varepsilon_b\mathfrak{y}_r(\varepsilon_d - \delta_b\varepsilon_a) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b\varepsilon_d - \delta_b\varepsilon_c) \\
&\quad + r\varkappa_a(\varepsilon_a\varepsilon_c - \varepsilon_b) + r\varkappa_b(\varepsilon_a\varepsilon_d - \delta_b) - \varrho_a\mathfrak{y}_s(r\varrho_b - \varrho_o\varepsilon_a) - \varkappa_c(r\varepsilon_b\varrho_b - \varrho_o\varepsilon_c) \\
&\quad - \varkappa_d(r\delta_b\varrho_b - \varrho_o\varepsilon_d)
\end{aligned} \tag{6.21j}$$

$$\begin{aligned}
\varkappa_m &= \varkappa_g\varsigma_a - \varkappa_h\delta_e + \mathfrak{y}_x(\varepsilon_c - \varepsilon_a\varepsilon_b) + \mathfrak{y}_u(1-\varepsilon_b^2) + \varepsilon_b\mathfrak{y}_r(\delta_d - \delta_b\varepsilon_b) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b\delta_d - \delta_b) \\
&\quad + r\varkappa_a(\varepsilon_a - \varepsilon_b\varepsilon_c) + r\varkappa_b(\varepsilon_a\delta_d - \delta_b\varepsilon_c) + \varrho_a\mathfrak{y}_s\varrho_o\varepsilon_b + \varkappa_c\varrho_o + \varkappa_d\varrho_o\delta_d \\
&\quad - h^{-2}(\varrho_a\mathfrak{y}_s + \varkappa_c\varepsilon_b + \varkappa_d\delta_b)(\varepsilon_i + \varepsilon_f\varphi_b) \\
\varkappa_n &= r\varkappa_g\delta_f + \varkappa_h\varepsilon_g + \mathfrak{y}_x(\varepsilon_d - \varepsilon_a\delta_b) + \mathfrak{y}_u(\delta_d - \varepsilon_b\delta_b) + \varepsilon_b\mathfrak{y}_r(z^2 - \delta_b^2) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b z^2 - \delta_b\delta_d) \\
&\quad + r\varkappa_a(\varepsilon_a\delta_d - \varepsilon_b\varepsilon_d) + r\varkappa_b(\varepsilon_a z^2 - \delta_b\varepsilon_d) + \varrho_a\mathfrak{y}_s(r\varrho_b\varphi_b + \varrho_o\delta_b) + \varkappa_c(r\varepsilon_b\varrho_b\varphi_b + \varrho_o\delta_d) \\
&\quad + \varkappa_d(r\varrho_b\varphi_b\delta_b + \varrho_o z^2) \\
\varkappa_o &= \varkappa_g\varsigma_b + \varkappa_h\varepsilon_i + \mathfrak{y}_x(\varepsilon_b\varepsilon_d - \varepsilon_c\delta_b) + \mathfrak{y}_u(\varepsilon_b\delta_d - \delta_b) + \varepsilon_b\mathfrak{y}_r(\varepsilon_b z^2 - \delta_d\delta_b) + \varepsilon_b\mathfrak{y}_n(z^2 - \delta_d^2) \\
&\quad + r\varkappa_a(\varepsilon_c\delta_d - \varepsilon_d) + r\varkappa_b(\varepsilon_c z^2 - \delta_d\varepsilon_d) + r\mathfrak{y}_s\varepsilon_b\varrho_a\varrho_b\varphi_b + r\varkappa_c\varrho_b\varphi_b + r\varkappa_d\delta_d\varrho_b\varphi_b \\
&\quad + h^{-2}(\varrho_a\mathfrak{y}_s\delta_b + \varkappa_c\delta_d + \varkappa_d z^2)(\varepsilon_i + \varepsilon_f\varphi_b)
\end{aligned} \tag{6.21k}$$

$$\begin{aligned}
\varkappa_p &= -\varkappa_h\varepsilon_f + r\mathfrak{y}_x(\varepsilon_a\varepsilon_c - \varepsilon_b) + r\mathfrak{y}_u(\varepsilon_a - \varepsilon_c\varepsilon_b) + r\varepsilon_b\mathfrak{y}_r(\varepsilon_a\delta_d - \varepsilon_d\varepsilon_b) + r\varepsilon_b\mathfrak{y}_n(\varepsilon_c\delta_d - \varepsilon_d) \\
&\quad + r^2\varkappa_a(1-\varepsilon_c^2) + r^2\varkappa_b(\delta_d - \varepsilon_d\varepsilon_c) + r^2(\varepsilon_b\mathfrak{y}_s\varrho_a\varrho_b + \varkappa_c\varrho_b + \varkappa_d\delta_d\varrho_b) \\
&\quad - (r/h^2)(\varrho_a\mathfrak{y}_s\varepsilon_a + \varkappa_c\varepsilon_c + \varkappa_d\varepsilon_d)(\varepsilon_i + \varepsilon_f\varphi_b) \\
\varkappa_q &= r\varkappa_h\varepsilon_h + r\mathfrak{y}_x(\varepsilon_a\varepsilon_d - \delta_b) + r\mathfrak{y}_u(\varepsilon_a\delta_d - \varepsilon_c\delta_b) + r\varepsilon_b\mathfrak{y}_r(\varepsilon_a z^2 - \varepsilon_d\delta_b) \\
&\quad + r\varepsilon_b\mathfrak{y}_n(\varepsilon_c z^2 - \varepsilon_d\delta_d) + r^2\varkappa_a(\delta_d - \varepsilon_c\varepsilon_d) + r^2\varkappa_b(z^2 - \varepsilon_d^2) + r^2\varrho_a\mathfrak{y}_s\varrho_b(\varepsilon_a\varphi_b + \delta_b) \\
&\quad + r^2\varkappa_c\varrho_b(\varepsilon_c\varphi_b + \delta_d) + r^2\varkappa_d\varrho_b(\varepsilon_d\varphi_b + z^2) \\
\varkappa_r &= \varkappa_g\delta_a - \varkappa_h\hbar_a + \mathfrak{y}_x(r\varrho_b - \varepsilon_a\varrho_o) - \mathfrak{y}_u\varepsilon_b\varrho_o - \varepsilon_b\mathfrak{y}_r(r\varrho_b\varphi_b + \delta_b\varrho_o) \\
&\quad - \varepsilon_b^2\mathfrak{y}_n r\varrho_b\varphi_b - r^2\varkappa_a\varepsilon_b\varrho_b - r^2\varkappa_b\varrho_b(\varepsilon_a\varphi_b + \delta_b) - \varrho_a\mathfrak{y}_s(\varphi_h^2 - \varrho_o^2) - \varkappa_c\varepsilon_b\varphi_h^2 \\
&\quad - \varkappa_d(\delta_b\varphi_h^2 + r\varrho_o\varrho_b\varphi_b) + h^{-2}(\mathfrak{y}_u - \varepsilon_b\mathfrak{y}_n\delta_b + \varkappa_a r\varepsilon_a + \varkappa_c\varrho_o)(\varepsilon_i + \varepsilon_f\varphi_b)
\end{aligned} \tag{6.21l}$$

$$\begin{aligned}
\varkappa_s &= \varkappa_g\delta_c - \varkappa_h\hbar_c + \mathfrak{y}_x(r\varepsilon_b\varrho_b - \varepsilon_c\varrho_o) - \mathfrak{y}_u\varrho_o - \varepsilon_b\mathfrak{y}_r(r\varepsilon_b\varrho_b\varphi_b + \delta_d\varrho_o) - r\varepsilon_b\mathfrak{y}_n\varrho_b\varphi_b \\
&\quad - r^2\varrho_b[\varkappa_a + \varkappa_b(\varepsilon_c\varphi_b + \delta_d)] - \varphi_h^2(\varrho_a\mathfrak{y}_s\varepsilon_b + \varkappa_c + \varkappa_d\delta_d) + h^{-4}\varkappa_c(\varepsilon_i + \varepsilon_f\varphi_b)^2 \\
&\quad + h^{-2}(\mathfrak{y}_u\varepsilon_b - \varepsilon_b\mathfrak{y}_n\delta_d + r\varkappa_a\varepsilon_c + \varrho_o\varrho_a\mathfrak{y}_s - r\varkappa_d\varrho_b\varphi_b)(\varepsilon_i + \varepsilon_f\varphi_b) \\
\varkappa_t &= -\varkappa_h\mathfrak{y}_z + \mathfrak{y}_x(r\delta_b\varrho_b - \varepsilon_d\varrho_o) - \mathfrak{y}_u\delta_d\varrho_o - \varepsilon_b\mathfrak{y}_r(r\delta_b\varrho_b\varphi_b + z^2\varrho_o) - r\varepsilon_b\mathfrak{y}_n\delta_d\varrho_b\varphi_b \\
&\quad - r^2\varkappa_a\delta_d\varrho_b - r^2\varkappa_b\varrho_b(\varepsilon_d\varphi_b + z^2) - \varrho_a\mathfrak{y}_s(\delta_b\varphi_h^2 + r\varrho_b\varphi_b\varrho_o) - \varkappa_c\delta_d\varphi_h^2 \\
&\quad - \varkappa_d(z^2\varphi_h^2 - r^2\varrho_b^2\varphi_b^2) + h^{-2}(\mathfrak{y}_u\delta_b - z^2\varepsilon_b\mathfrak{y}_n + r\varkappa_a\varepsilon_d - r\varkappa_c\varrho_b\varphi_b)(\varepsilon_i + \varepsilon_f\varphi_b)
\end{aligned} \tag{6.21m}$$

$$\begin{aligned}
\varkappa_u &= [\varkappa_g\varkappa_j + \varkappa_h\varkappa_k + \mathfrak{y}_x\varkappa_l + \mathfrak{y}_u\varkappa_m + \varepsilon_b\mathfrak{y}_r\varkappa_n + \varepsilon_b\mathfrak{y}_n\varkappa_o + \varkappa_a\varkappa_p + \varkappa_b\varkappa_q - \varrho_a\mathfrak{y}_s\varkappa_r \\
&\quad - \varkappa_c\varkappa_s - \varkappa_d\varkappa_t]^{1/2} \\
\varkappa_v &= r\varkappa_g\varepsilon_a + \varkappa_h\delta_a + \varepsilon_b\mathfrak{y}_n\varsigma_c - \varkappa_a\varsigma_a - r\varkappa_b\delta_f - (\varkappa_c/h^2)(\delta_c\hbar_a - \hbar_c\delta_a) + (\varkappa_d/h^2)\mathfrak{y}_z\delta_a \\
\varkappa_w &= r\varkappa_g\varepsilon_c + \varkappa_h\delta_c - (\mathfrak{y}_x/r)\varsigma_a - \varepsilon_b\mathfrak{y}_r\varsigma_c - \varkappa_b\varsigma_b + \varrho_a(\mathfrak{y}_s/h^2)(\delta_c\hbar_a - \hbar_c\delta_a) + (\varkappa_d/h^2)\mathfrak{y}_z\delta_c \\
\varkappa_x &= r\varkappa_g\varepsilon_d - \mathfrak{y}_x\delta_f + \mathfrak{y}_u\varsigma_c + \varkappa_a\varsigma_b - (\mathfrak{y}_z/h^2)(\mathfrak{y}_s\varrho_a\delta_a + \varkappa_c\delta_c) \\
\varkappa_y &= -r\mathfrak{y}_x\delta_a + r\mathfrak{y}_u(\varepsilon_a\delta_c - \varepsilon_c\delta_a) - r\varepsilon_b\varepsilon_d(\mathfrak{y}_r\delta_a + \mathfrak{y}_n\delta_c) + r^2(\varkappa_a\delta_c + \varrho_a\mathfrak{y}_s\varrho_b\delta_a + \varkappa_c\varrho_b\delta_c) \\
\varkappa_z &= -\mathfrak{y}_x\varepsilon_d\delta_a + \mathfrak{y}_u(\delta_b\delta_c - \delta_d\delta_a) - z^2\varepsilon_b(\mathfrak{y}_r\delta_a + \mathfrak{y}_n\delta_c) + r(\varkappa_g\varepsilon_h + \varkappa_a\varepsilon_d\delta_c) \\
&\quad - r\varrho_b\varphi_b(\varrho_a\mathfrak{y}_s\delta_a + \varkappa_c\delta_c)
\end{aligned} \tag{6.21n}$$

$$\begin{aligned}
&\mathfrak{v}_a = h^{-2}(\varkappa_z + \varphi_b\varkappa_y), \quad \mathfrak{v}_b = \mathfrak{x}_q\varkappa_w - \mathfrak{x}_s\varkappa_x - \varphi_b\mathfrak{x}_s\varkappa_j + \mathfrak{x}_r\varkappa_y \\
&\mathfrak{v}_c = -\varkappa_j\mathfrak{x}_j + 3\mathfrak{x}_i(3\varrho_b\varkappa_j - \mathfrak{v}_a) + \mathfrak{y}_f(6\varrho_b\mathfrak{v}_a - \varrho_l\varkappa_j), \quad \mathfrak{v}_d = \varrho_a\varphi_b(\rho/h^2) + \kappa\varepsilon_b\mathfrak{x}_k \\
&\mathfrak{v}_e = h^{-2}(\varrho_a\mathfrak{y}_s\delta_a + \varkappa_c\delta_c), \quad \mathfrak{v}_f = \varkappa_h + h^{-2}(\varrho_a\mathfrak{y}_s\hbar_a + \varkappa_c\hbar_c + \varkappa_d\mathfrak{y}_z), \quad \mathfrak{v}_g = r\varkappa_g - q\mathfrak{v}_e.
\end{aligned} \tag{6.21o}$$

Art 26a. *Development of* (1.5).

Bearing the foregoing quantities in mind, we derive

$$\begin{aligned}
\dot m &= [\boldsymbol\kappa\cdot(\mathbf r\times\mathbf h)]' \text{ by (1.5c)} \\
&= \boldsymbol\kappa\cdot(\mathbf u\times\mathbf h) = -\boldsymbol\kappa\cdot(\mathbf h\times\mathbf u) \\
&= -\boldsymbol\kappa\cdot\mathbf z - q(\boldsymbol\kappa\cdot\widehat{\mathbf r}) \text{ by (1.5c)} \\
&= -\kappa(\delta_b + q\varepsilon_a) \text{ by (6.1a) \& (6.10a)} \\
&= -\kappa\hbar_a \text{ by (6.10b)}
\end{aligned} \tag{6.22a}$$

$$\begin{aligned}
\ddot m &= [-\boldsymbol\kappa\cdot\mathbf z - q(\boldsymbol\kappa\cdot\widehat{\mathbf r})]' \text{ by (6.22a)} \\
&= -q(\boldsymbol\kappa\cdot\dot{\widehat{\mathbf r}}) = -qr^{-3}[\boldsymbol\kappa\cdot\{\mathbf r\times(\mathbf u\times\mathbf r)\}] \\
&= -qr^{-3}[\boldsymbol\kappa\cdot(\mathbf r\times\mathbf h)] \text{ by (1.5c)} \\
&= -\kappa\varrho_a\varepsilon_e \text{ by (6.21b) \& (6.1a)}
\end{aligned} \tag{6.22b}$$

$$\begin{aligned}
\dddot m &= [-qr^{-3}\{\boldsymbol\kappa\cdot(\mathbf r\times\mathbf h)\}]' \text{ by (6.22b)} \\
&= -q[r^{-3}]'[\boldsymbol\kappa\cdot(\mathbf r\times\mathbf h)] - qr^{-3}[\boldsymbol\kappa\cdot(\mathbf r\times\mathbf h)]' \\
&= 3qr^{-4}(\widehat{\mathbf r}\cdot\mathbf u)[\boldsymbol\kappa\cdot(\mathbf r\times\mathbf h)] - qr^{-3}[\boldsymbol\kappa\cdot(\mathbf u\times\mathbf h)] \\
&= 3qr^{-4}h^{-2}[\widehat{\mathbf r}\cdot(\mathbf z\times\mathbf h)][\boldsymbol\kappa\cdot(\mathbf r\times\mathbf h)] - qr^{-3}[\boldsymbol\kappa\cdot(-\mathbf z - q\widehat{\mathbf r})] \text{ by (1.5c)} \\
&= 3qr^{-4}h^{-2}\varepsilon_h\varepsilon_e\kappa - qr^{-3}(-\kappa\delta_b - q\kappa\varepsilon_a) \text{ by (6.1a) \& (6.10a)} \\
&= 3qr^{-4}h^{-2}\varepsilon_h\varepsilon_e\kappa + \kappa qr^{-3}\hbar_a \text{ by (6.10b)} \\
&= \kappa qr^{-3}(\hbar_a + 3r^{-1}h^{-2}\varepsilon_h\varepsilon_e) = \kappa\varrho_a(\hbar_a + 3\varrho_b\varepsilon_e) = \kappa\varrho_c \text{ by (6.21b)}
\end{aligned} \tag{6.22c}$$

$$\begin{aligned}
\dot n &= [\widehat{\mathbf p}\cdot(\mathbf r\times\mathbf h)]' \text{ by (1.5c)} \\
&= \widehat{\mathbf p}\cdot(\mathbf u\times\mathbf h) = -\widehat{\mathbf p}\cdot(\mathbf h\times\mathbf u) \\
&= -\widehat{\mathbf p}\cdot\mathbf z - q(\widehat{\mathbf p}\cdot\widehat{\mathbf r}) \text{ by (1.5c)} \\
&= -(\delta_d + q\varepsilon_c) \text{ by (6.1a) \& (6.10a)} \\
&= -\hbar_c \text{ by (6.10b)}
\end{aligned} \tag{6.23a}$$

$$\begin{aligned}\ddot{n} &= [-\widehat{\mathbf{p}}\cdot\mathbf{z} - q(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{r}})]' \text{ by (6.23a)}\\ &= -q(\widehat{\mathbf{p}}\cdot\dot{\widehat{\mathbf{r}}}) = -qr^{-3}[\widehat{\mathbf{p}}\cdot\{\mathbf{r}\times(\mathbf{u}\times\mathbf{r})\}]\\ &= -qr^{-3}[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})] \text{ by (1.5c)}\\ &= -\varrho_a\varepsilon_f \text{ by (6.21b) \& (6.1a)}\end{aligned} \tag{6.23b}$$

$$\begin{aligned}\dddot{n} &= [-qr^{-3}\{\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})\}]' \text{ by (6.23b)}\\ &= -q[r^{-3}]'[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})] - qr^{-3}[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})]'\\ &= 3qr^{-4}(\widehat{\mathbf{r}}\cdot\mathbf{u})[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})] - qr^{-3}[\widehat{\mathbf{p}}\cdot(\mathbf{u}\times\mathbf{h})]\\ &= 3qr^{-4}h^{-2}[\widehat{\mathbf{r}}\cdot(\mathbf{z}\times\mathbf{h})][\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})] - qr^{-3}[\widehat{\mathbf{p}}\cdot(-\mathbf{z}-q\widehat{\mathbf{r}})] \text{ by (1.5c)}\\ &= 3qr^{-4}h^{-2}\varepsilon_h\varepsilon_f - qr^{-3}(-\delta_d - q\varepsilon_c) \text{ by (6.1a) \& (6.10a)}\\ &= 3qr^{-4}h^{-2}\varepsilon_h\varepsilon_f + qr^{-3}\hbar_c \text{ by (6.10b)}\\ &= qr^{-3}(\hbar_c + 3r^{-1}h^{-2}\varepsilon_h\varepsilon_f) = \varrho_a(\hbar_c + 3\varrho_b\varepsilon_f) = \varrho_d \text{ by (6.21b)}\end{aligned} \tag{6.23c}$$

$$\begin{aligned}n\ddot{m} - m\ddot{n} &= n(-\kappa\varrho_a\varepsilon_e) - m(-\varrho_a\varepsilon_f) \text{ by (6.22b) \& (6.23b)}\\ &= -\kappa\varrho_a\varepsilon_e n + \varrho_a\varepsilon_f m\\ &= -\kappa\varrho_a\varepsilon_e(\varepsilon_f) + \varrho_a\varepsilon_f(\kappa\varepsilon_e) \text{ by (6.2b) \& (6.2c)}\\ &= -\kappa\varrho_a\varepsilon_e\varepsilon_f + \kappa\varrho_a\varepsilon_f\varepsilon_e = 0\end{aligned} \tag{6.23d}$$

$$\begin{aligned}[\mho^2]' &= [(m/n)(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})]' \text{ by (1.5c)}\\ &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(m/n)]' = (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(n\dot{m} - m\dot{n})/n^2]\\ \therefore \dot{\mho} &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(n\dot{m} - m\dot{n})/(2\mho n^2)]\\ &= \kappa\varepsilon_b(n\dot{m} - m\dot{n})/(2\mho n^2) \text{ by (6.1a)}\\ &= \kappa\varepsilon_b[n(-\kappa\hbar_a) - m(-\hbar_c)]/(2\mho n^2) \text{ by (6.22a) \& (6.23a)}\\ &= \kappa\varepsilon_b[\varepsilon_f(-\kappa\hbar_a) - \kappa\varepsilon_e(-\hbar_c)]/(2\kappa\varphi_c\varepsilon_f^2) \text{ by (6.2b), (6.2c) \& (6.2d)}\\ &= \kappa\varepsilon_b(-\kappa\varepsilon_f\hbar_a + \kappa\varepsilon_e\hbar_c)/(2\kappa\varphi_c\varepsilon_f^2)\\ &= \kappa\varepsilon_b(\varepsilon_e\hbar_c - \varepsilon_f\hbar_a)/(2\varphi_c\varepsilon_f^2) = \kappa\varrho_e \text{ by (6.21b)}\end{aligned} \tag{6.24a}$$

$$\begin{aligned}\ddot{\mho} &= [(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})\{(n\dot{m} - m\dot{n})/(2\mho n^2)\}]' = (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})[(n\dot{m} - m\dot{n})/(2\mho n^2)]' \text{ by (6.24a)}\\ &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})\left[\frac{(2\mho n^2)(n\dot{m} - m\dot{n})' - (n\dot{m} - m\dot{n})(2\mho n^2)'}{(2\mho n^2)^2}\right]\\ &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})\left[\frac{(2\mho n^2)(n\dot{m} - m\dot{n})'}{(2\mho n^2)^2}\right] - (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})\left[\frac{(n\dot{m} - m\dot{n})(2\mho n^2)'}{(2\mho n^2)^2}\right]\\ &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})\left[\frac{(n\dot{m} - m\dot{n})'}{2\mho n^2}\right] - \dot{\mho}\left[\frac{(2\mho n^2)'}{2\mho n^2}\right] \text{ by (6.24a)}\\ &= (\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})\left[\frac{\dot{n}\dot{m} + n\ddot{m} - \dot{m}\dot{n} - m\ddot{n}}{2\mho n^2}\right] - \dot{\mho}\left[\frac{2\dot{\mho}n^2 + 4\mho n\dot{n}}{2\mho n^2}\right]\end{aligned}$$

$$
\begin{aligned}
&= \frac{(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})(n\ddot{m} - m\ddot{n}) - 2\dot{\mho}(\dot{\mho}n^2 + 2\mho n\dot{n})}{2\mho n^2} = -\frac{\dot{\mho}(\dot{\mho}n + 2\mho\dot{n})}{\mho n} \text{ by (6.23d)} \\
&= -\frac{(\kappa\varrho_e)[(\kappa\varrho_e)n + 2\mho(-\hbar_c)]}{\mho n} \text{ by (6.23a) \& (6.24a)} \\
&= -\frac{\kappa\varrho_e[\kappa\varrho_e(\varepsilon_f) - 2\hbar_c(\kappa\varphi_c)]}{(\kappa\varphi_c)(\varepsilon_f)} \text{ by (6.2c) \& (6.2d)} \\
&= -\kappa\varrho_e(\varrho_e\varepsilon_f - 2\hbar_c\varphi_c)/(\varphi_c\varepsilon_f) = -\kappa\varrho_f \text{ by (6.21b)}
\end{aligned} \tag{6.24b}
$$

$$
\begin{aligned}
\dddot{\mho} &= \left[-\frac{\dot{\mho}(\dot{\mho}n + 2\mho\dot{n})}{\mho n}\right]' \text{ by (6.24b)} \\
&= \frac{-(\mho n)[\dot{\mho}(\dot{\mho}n - 2\mho\dot{n})]' + [\dot{\mho}(\dot{\mho}n + 2\mho\dot{n})](\mho n)'}{(\mho n)^2} \\
&= \frac{-[\dot{\mho}(\dot{\mho}n - 2\mho\dot{n})]' - \ddot{\mho}(\mho n)'}{\mho n} \text{ by (6.24b)} \\
&= \frac{-\ddot{\mho}(\dot{\mho}n - 2\mho\dot{n}) - \dot{\mho}(\ddot{\mho}n + \dot{\mho}\dot{n} - 2\dot{\mho}\dot{n} - 2\mho\ddot{n}) - \ddot{\mho}(\dot{\mho}n + \mho\dot{n})}{\mho n} \\
&= \frac{-\ddot{\mho}\dot{\mho}n + 2\ddot{\mho}\mho\dot{n} - \dot{\mho}\ddot{\mho}n + \dot{\mho}^2\dot{n} + 2\mho\dot{\mho}\ddot{n} - \ddot{\mho}\dot{\mho}n - \ddot{\mho}\mho\dot{n}}{\mho n} \\
&= \frac{\ddot{\mho}\mho\dot{n} - 3\dot{\mho}\ddot{\mho}n + \dot{\mho}^2\dot{n} + 2\mho\dot{\mho}\ddot{n}}{\mho n}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(-\kappa\varrho_f)\mho(-\hbar_c) - 3(\kappa\varrho_e)(-\kappa\varrho_f)n + (\kappa\varrho_e)^2(-\hbar_c) + 2\mho(\kappa\varrho_e)(-\varrho_a\varepsilon_f)}{\mho n} \\
&\quad \text{by (6.23), (6.24a) \& (6.24b)} \\
&= \frac{\kappa\varrho_f\hbar_c\mho + 3\kappa^2\varrho_e\varrho_f n - \kappa^2\varrho_e^2\hbar_c - 2\kappa\varrho_e\varrho_a\varepsilon_f\mho}{\mho n} \\
&= \frac{\kappa\varrho_f\hbar_c(\kappa\varphi_c) + 3\kappa^2\varrho_e\varrho_f(\varepsilon_f) - \kappa^2\varrho_e^2\hbar_c - 2\kappa\varrho_e\varrho_a\varepsilon_f(\kappa\varphi_c)}{(\kappa\varphi_c)(\varepsilon_f)} \text{ by (6.2c) \& (6.2d)} \\
&= \frac{\kappa^2\varrho_f\hbar_c\varphi_c + 3\kappa^2\varrho_e\varrho_f\varepsilon_f - \kappa^2\varrho_e^2\hbar_c - 2\kappa^2\varrho_e\varrho_a\varepsilon_f\varphi_c}{\kappa\varphi_c\varepsilon_f} \\
&= \kappa[\varrho_f\hbar_c\varphi_c + \varrho_e(3\varrho_f\varepsilon_f - \varrho_e\hbar_c - 2\varrho_a\varepsilon_f\varphi_c)]/(\varphi_c\varepsilon_f) = \kappa\varrho_h \text{ by (6.21b).}
\end{aligned} \tag{6.24c}
$$

Art 26b. *Derivatives of* **a** *and* γ.

Thus, from the foregoing derivations, we obtain[10]

$$
\begin{aligned}
\dot{\gamma} &= \left[\left|1 - \frac{\mho^2}{\kappa^2}\right|^{1/2}\right]' \text{ by (1.5a)} \\
&= \pm\frac{1}{2}\left|1 - \frac{\mho^2}{\kappa^2}\right|^{-1/2}\left[\frac{\mho^2}{\kappa^2}\right]' = \pm\frac{1}{2\gamma}\left[\frac{2\mho\dot{\mho}}{\kappa^2}\right] \text{ by (1.5a)} \\
&= \pm\frac{\mho\dot{\mho}}{\gamma\kappa^2} = \pm\frac{(\kappa\varphi_c)(\kappa\varrho_e)}{\varphi_e\kappa^2} \text{ by (6.24a), (6.2d) \& (6.2e)} \\
&= \pm(\varphi_c\varrho_e)/\varphi_e = \varrho_g \text{ by (6.21b)}
\end{aligned} \tag{6.25a}
$$

[10]In the expressions for the derivatives of γ, we are to take the positive sign when $\mho > \kappa$ and the negative sign when $\mho < \kappa$. The case $\mho = \kappa$ is forbidden since γ is nonzero.

$$\begin{aligned}
\ddot{\gamma} &= \pm\left[\frac{\mho\dot{\mho}}{\gamma\kappa^2}\right]' \text{ by (6.25a)} \\
&= \pm\frac{1}{\kappa^2}\left[\frac{\gamma(\mho\dot{\mho})' - (\mho\dot{\mho})\dot{\gamma}}{\gamma^2}\right] = \pm\left[\frac{\gamma(\mho\dot{\mho})'}{\gamma^2\kappa^2} - \frac{(\mho\dot{\mho})\dot{\gamma}}{\gamma^2\kappa^2}\right] \\
&= \pm\left[\frac{\dot{\mho}^2 + \mho\ddot{\mho}}{\gamma\kappa^2}\right] - \left[\frac{\dot{\gamma}^2}{\gamma}\right] \text{ by (6.25a)} \\
&= \pm\left[\frac{(\kappa\varrho_e)^2 + \mho(-\kappa\varrho_f)}{\gamma\kappa^2}\right] - \left[\frac{(\varrho_g)^2}{\gamma}\right] \text{ by (6.24) \& (6.25a)} \\
&= \pm\left[\frac{\kappa^2\varrho_e^2 - \kappa\varrho_f\mho}{\gamma\kappa^2}\right] - \left[\frac{\varrho_g^2}{\gamma}\right] = \pm\left[\frac{\kappa^2\varrho_e^2 - \kappa\varrho_f(\kappa\varphi_c)}{(\varphi_e)\kappa^2}\right] - \left[\frac{\varrho_g^2}{(\varphi_e)}\right] \text{ by (6.2d) \& (6.2e)} \\
&= (1/\varphi_e)[\pm(\varrho_e^2 - \varrho_f\varphi_c) - \varrho_g^2] = \varrho_i \text{ by (6.21b)}
\end{aligned} \tag{6.25b}$$

$$\begin{aligned}
\dddot{\gamma} &= \pm\left[\frac{\dot{\mho}^2 + \mho\ddot{\mho}}{\gamma\kappa^2}\right]' - \left[\frac{\dot{\gamma}^2}{\gamma}\right]' \text{ by (6.25b)} \\
&= \pm\frac{1}{\kappa^2}\left[\frac{\gamma(\dot{\mho}^2 + \mho\ddot{\mho})' - (\dot{\mho}^2 + \mho\ddot{\mho})\dot{\gamma}}{\gamma^2}\right] - \left[\frac{\gamma(\dot{\gamma}^2)' - \dot{\gamma}^3}{\gamma^2}\right] \\
&= \pm\left[\frac{\gamma(\dot{\mho}^2 + \mho\ddot{\mho})'}{\gamma^2\kappa^2}\right] - \left[\pm\frac{(\dot{\mho}^2 + \mho\ddot{\mho})\dot{\gamma}}{\gamma^2\kappa^2}\right] - \left[\frac{\gamma(\dot{\gamma}^2)'}{\gamma^2}\right] + \left[\frac{\dot{\gamma}^3}{\gamma^2}\right] \\
&= \pm\left[\frac{\gamma(\dot{\mho}^2 + \mho\ddot{\mho})'}{\gamma^2\kappa^2}\right] - \left[\frac{\gamma(\dot{\gamma}^2)'}{\gamma^2}\right] - \frac{\dot{\gamma}}{\gamma}\left[\pm\frac{(\dot{\mho}^2 + \mho\ddot{\mho})}{\gamma\kappa^2} - \frac{\dot{\gamma}^2}{\gamma}\right]
\end{aligned}$$

$$\begin{aligned}
&= \pm\left[\frac{2\dot{\mho}\ddot{\mho} + \dot{\mho}\ddot{\mho} + \mho\dddot{\mho}}{\gamma\kappa^2}\right] - \left[\frac{2\dot{\gamma}\ddot{\gamma}}{\gamma}\right] - \frac{\dot{\gamma}\ddot{\gamma}}{\gamma} \text{ by (6.25b)} \\
&= \pm\left[\frac{3\dot{\mho}\ddot{\mho} + \mho\dddot{\mho}}{\gamma\kappa^2}\right] - \frac{3\dot{\gamma}\ddot{\gamma}}{\gamma} \\
&= \pm\left[\frac{3(\kappa\varrho_e)(-\kappa\varrho_f) + \mho(\kappa\varrho_h)}{\gamma\kappa^2}\right] - \frac{3(\varrho_g)(\varrho_i)}{\gamma} \text{ by (6.24), (6.25a) \& (6.25b)} \\
&= \pm\left[\frac{-3\kappa^2\varrho_e\varrho_f + \kappa\varrho_h\mho}{\gamma\kappa^2}\right] - \frac{3\varrho_g\varrho_i}{\gamma} \\
&= \pm\left[\frac{-3\kappa^2\varrho_e\varrho_f + \kappa\varrho_h(\kappa\varphi_c)}{(\varphi_e)\kappa^2}\right] - \frac{3\varrho_g\varrho_i}{(\varphi_e)} \text{ by (6.2d) \& (6.2e)} \\
&= (1/\varphi_e)[\pm(\varrho_h\varphi_c - 3\varrho_e\varrho_f) - 3\varrho_g\varrho_i] = \varrho_j \text{ by (6.21b).}
\end{aligned} \tag{6.25c}$$

We have also that

$$\begin{aligned}
\dot{\mathbf{a}} &= (-q\mathbf{r}/r^3)' \text{ by (1.5a)} \\
&= -q\left[\frac{r^3\mathbf{u} - 3r^2\dot{r}\mathbf{r}}{r^6}\right] = -\frac{q\mathbf{u}}{r^3} + \frac{3q\dot{r}\mathbf{r}}{r^4} = -\frac{q\mathbf{u}}{r^3} + \frac{3q(\widehat{\mathbf{r}}\cdot\mathbf{u})\widehat{\mathbf{r}}}{r^3} \\
&= -\varrho_a\mathbf{u} + 3\varrho_a(\widehat{\mathbf{r}}\cdot\mathbf{u})\widehat{\mathbf{r}} \text{ by (6.21b)} \\
&= -\varrho_a\mathbf{u} + 3\varrho_a[h^{-2}\widehat{\mathbf{r}}\cdot(\mathbf{z}\times\mathbf{h})]\widehat{\mathbf{r}} \text{ by (1.5c)} \\
&= -\varrho_a\mathbf{u} + 3\varrho_a[h^{-2}\varepsilon_h]\widehat{\mathbf{r}} \text{ by (6.1a)} \\
&= -\varrho_a\mathbf{u} + 3r\varrho_a\varrho_b\widehat{\mathbf{r}} \text{ by (6.21b)} \\
&= -\varrho_a\mathbf{u} + 3\varrho_a\varrho_b\mathbf{r}
\end{aligned} \tag{6.26a}$$

$$\begin{aligned}
\ddot{\mathbf{a}} &= \left[-\frac{q\mathbf{u}}{r^3}\right]' + \left[\frac{3q(\mathbf{r}\cdot\mathbf{u})\mathbf{r}}{r^5}\right]' \text{ by (6.26a)}\\
&= -q\left[\frac{r^3\mathbf{a} - 3r^2\dot{r}\mathbf{u}}{r^6}\right] + 3q\left[\frac{r^5[(\mathbf{r}\cdot\mathbf{u})\mathbf{r}]'}{r^{10}} - \frac{5r^4\dot{r}[(\mathbf{r}\cdot\mathbf{u})\mathbf{r}]}{r^{10}}\right]\\
&= -q\left[\frac{\mathbf{a}}{r^3} - \frac{3\dot{r}\mathbf{u}}{r^4} - \frac{3(\mathbf{r}\cdot\mathbf{u})'\mathbf{r}}{r^5} - \frac{3(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^5} + \frac{15\dot{r}(\mathbf{r}\cdot\mathbf{u})\mathbf{r}}{r^6}\right]\\
&= -q\left[\frac{\mathbf{a}}{r^3} - \frac{3\mathbf{r}(\mathbf{r}\cdot\mathbf{u})'}{r^5} - \frac{3(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^5} - \frac{3\dot{r}\mathbf{u}}{r^4} + \frac{15\dot{r}(\mathbf{r}\cdot\mathbf{u})\mathbf{r}}{r^6}\right]\\
&= -\frac{q\mathbf{a}}{r^3} + \frac{3q\mathbf{r}(\mathbf{r}\cdot\mathbf{u})'}{r^5} + \frac{3q(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^5} + \frac{3q\dot{r}\mathbf{u}}{r^4} - \frac{15q\dot{r}(\mathbf{r}\cdot\mathbf{u})\mathbf{r}}{r^6}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{q\mathbf{a}}{r^3} + \frac{3q(\mathbf{u}\cdot\mathbf{u})\mathbf{r}}{r^5} + \frac{3q(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^5} + \frac{3q(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^5} + \frac{3q\dot{r}\mathbf{u}}{r^4} - \frac{15q\dot{r}(\mathbf{r}\cdot\mathbf{u})\mathbf{r}}{r^6}\\
&= -\frac{q\mathbf{a}}{r^3} + \frac{3q(\mathbf{u}\cdot\mathbf{u})\mathbf{r}}{r^5} + \frac{3q(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^5} + \frac{3q(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^5} + \frac{3q(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^5} - \frac{15q(\mathbf{r}\cdot\mathbf{u})^2\mathbf{r}}{r^7}\\
&= \frac{q^2\mathbf{r}}{r^6} + \frac{3q(\mathbf{u}\cdot\mathbf{u})\mathbf{r}}{r^5} + \frac{3q(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^5} + \frac{6q(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^5} - \frac{15q(\mathbf{r}\cdot\mathbf{u})^2\mathbf{r}}{r^7} \text{ by (1.5a)}\\
&= \frac{q^2\mathbf{r}}{r^6} + \frac{3q\varphi_h^2\mathbf{r}}{r^5} - \frac{3q^2\mathbf{r}}{r^6} + \frac{6q[h^{-2}\mathbf{r}\cdot(\mathbf{z}\times\mathbf{h})]\mathbf{u}}{r^5} - \frac{15q[h^{-2}\mathbf{r}\cdot(\mathbf{z}\times\mathbf{h})]^2\mathbf{r}}{r^7}\\
&\quad \text{by (1.5a), (6.3b) \& (1.5c)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{3q\varphi_h^2\mathbf{r}}{r^5} - \frac{2q^2\mathbf{r}}{r^6} + \frac{6q\varepsilon_h\mathbf{u}}{h^2r^4} - \frac{15q\varepsilon_h^2\mathbf{r}}{h^4r^5} \text{ by (6.1a)}\\
&= \frac{3q\varphi_h^2\mathbf{r}}{r^5} - \frac{2q^2\mathbf{r}}{r^6} + \frac{6q\varrho_b\mathbf{u}}{r^3} - \frac{15q\varrho_b^2\mathbf{r}}{r^3} \text{ by (6.21b)}\\
&= 3\varrho_a(\varphi_h/r)^2\mathbf{r} - 2\varrho_a^2\mathbf{r} + 6\varrho_a\varrho_b\mathbf{u} - 15\varrho_a\varrho_b^2\mathbf{r} \text{ by (6.21b)}\\
&= 6\varrho_a\varrho_b\mathbf{u} - \varrho_a[2\varrho_a + 15\varrho_b^2 - 3(\varphi_h/r)^2]\mathbf{r}\\
&= 6\varrho_a\varrho_b\mathbf{u} - \varrho_a(2\varrho_a + 15\varrho_b^2 - 3\varrho_k^2)\mathbf{r} = 6\varrho_a\varrho_b\mathbf{u} - \varrho_a\varrho_l\mathbf{r} \text{ by (6.21b)} \qquad (6.26b)
\end{aligned}$$

$$\begin{aligned}
\dddot{\mathbf{a}} &= \left[\frac{q^2\mathbf{r}}{r^6} + \frac{3q(\mathbf{u}\cdot\mathbf{u})\mathbf{r}}{r^5} + \frac{3q(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^5} + \frac{6q(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^5} - \frac{15q(\mathbf{r}\cdot\mathbf{u})^2\mathbf{r}}{r^7}\right]' \text{ by (6.26b)}\\
&= q^2\left[\frac{r^6\mathbf{u} - 6r^5\dot{r}\mathbf{r}}{r^{12}}\right] + 3q\left[\frac{r^5[2(\mathbf{a}\cdot\mathbf{u})\mathbf{r} + (\mathbf{u}\cdot\mathbf{u})\mathbf{u}] - 5r^4\dot{r}[(\mathbf{u}\cdot\mathbf{u})\mathbf{r}]}{r^{10}}\right]\\
&\quad + 3q\left[\frac{r^5[(\mathbf{r}\cdot\mathbf{a})\mathbf{u} + (\mathbf{u}\cdot\mathbf{a} + \mathbf{r}\cdot\dot{\mathbf{a}})\mathbf{r}] - 5r^4\dot{r}[(\mathbf{r}\cdot\mathbf{a})\mathbf{r}]}{r^{10}}\right]\\
&\quad + 6q\left[\frac{r^5[(\mathbf{r}\cdot\mathbf{u})\mathbf{a} + (\mathbf{u}\cdot\mathbf{u} + \mathbf{r}\cdot\mathbf{a})\mathbf{u}] - 5r^4\dot{r}[(\mathbf{r}\cdot\mathbf{u})\mathbf{u}]}{r^{10}}\right]\\
&\quad - 15q\left[\frac{r^7[(\mathbf{r}\cdot\mathbf{u})^2\mathbf{u} + 2(\mathbf{r}\cdot\mathbf{u})(\mathbf{u}\cdot\mathbf{u} + \mathbf{r}\cdot\mathbf{a})\mathbf{r}] - 7r^6\dot{r}[(\mathbf{r}\cdot\mathbf{u})^2\mathbf{r}]}{r^{14}}\right]
\end{aligned}$$

$$\begin{aligned}
&= q^2\left[\frac{r^6\mathbf{u}}{r^{12}}\right] - q^2\left[\frac{6r^5\dot{r}\mathbf{r}}{r^{12}}\right] + 3q\left[\frac{2r^5(\mathbf{a}\cdot\mathbf{u})\mathbf{r}}{r^{10}}\right] + 3q\left[\frac{r^5(\mathbf{u}\cdot\mathbf{u})\mathbf{u}}{r^{10}}\right] + 3q\left[\frac{-5r^4\dot{r}(\mathbf{u}\cdot\mathbf{u})\mathbf{r}}{r^{10}}\right]\\
&\quad + 3q\left[\frac{r^5(\mathbf{r}\cdot\mathbf{a})\mathbf{u}}{r^{10}}\right] + 3q\left[\frac{r^5(\mathbf{u}\cdot\mathbf{a})\mathbf{r}}{r^{10}}\right] + 3q\left[\frac{r^5(\mathbf{r}\cdot\dot{\mathbf{a}})\mathbf{r}}{r^{10}}\right] + 3q\left[\frac{-5r^4\dot{r}(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^{10}}\right]\\
&\quad + 6q\left[\frac{r^5(\mathbf{r}\cdot\mathbf{u})\mathbf{a}}{r^{10}}\right] + 6q\left[\frac{r^5(\mathbf{u}\cdot\mathbf{u})\mathbf{u}}{r^{10}}\right] + 6q\left[\frac{r^5(\mathbf{r}\cdot\mathbf{a})\mathbf{u}}{r^{10}}\right] + 6q\left[\frac{-5r^4\dot{r}(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^{10}}\right]\\
&\quad - 15q\left[\frac{r^7(\mathbf{r}\cdot\mathbf{u})^2\mathbf{u}}{r^{14}}\right] - 15q\left[\frac{2r^7(\mathbf{r}\cdot\mathbf{u})(\mathbf{u}\cdot\mathbf{u})\mathbf{r}}{r^{14}}\right] - 15q\left[\frac{2r^7(\mathbf{r}\cdot\mathbf{u})(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^{14}}\right] - 15q\left[\frac{-7r^6\dot{r}(\mathbf{r}\cdot\mathbf{u})^2\mathbf{r}}{r^{14}}\right]
\end{aligned}$$

$$\begin{aligned}
&= \frac{q^2\mathbf{u}}{r^6} - \frac{6q^2\dot{r}\mathbf{r}}{r^7} + \frac{6q(\mathbf{a}\cdot\mathbf{u})\mathbf{r}}{r^5} + \frac{3qu^2\mathbf{u}}{r^5} - \frac{15qu^2\dot{r}\mathbf{r}}{r^6} + \frac{3q(\mathbf{r}\cdot\mathbf{a})\mathbf{u}}{r^5} + \frac{3q(\mathbf{u}\cdot\mathbf{a})\mathbf{r}}{r^5} \\
&\quad + \frac{3q(\mathbf{r}\cdot\dot{\mathbf{a}})\mathbf{r}}{r^5} - \frac{15q\dot{r}(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^6} + \frac{6q(\mathbf{r}\cdot\mathbf{u})\mathbf{a}}{r^5} + \frac{6qu^2\mathbf{u}}{r^5} + \frac{6q(\mathbf{r}\cdot\mathbf{a})\mathbf{u}}{r^5} - \frac{30q\dot{r}(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^6} \\
&\quad - \frac{15q(\mathbf{r}\cdot\mathbf{u})^2\mathbf{u}}{r^7} - \frac{30qu^2(\mathbf{r}\cdot\mathbf{u})\mathbf{r}}{r^7} - \frac{30q(\mathbf{r}\cdot\mathbf{u})(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^7} + \frac{105q\dot{r}(\mathbf{r}\cdot\mathbf{u})^2\mathbf{r}}{r^8} \\
&= \frac{q^2\mathbf{u}}{r^6} - \frac{6q^2(\widehat{\mathbf{r}}\cdot\mathbf{u})\mathbf{r}}{r^7} + \frac{6q(\mathbf{a}\cdot\mathbf{u})\mathbf{r}}{r^5} + \frac{3qu^2\mathbf{u}}{r^5} - \frac{15qu^2(\widehat{\mathbf{r}}\cdot\mathbf{u})\mathbf{r}}{r^6} + \frac{3q(\mathbf{r}\cdot\mathbf{a})\mathbf{u}}{r^5} + \frac{3q(\mathbf{u}\cdot\mathbf{a})\mathbf{r}}{r^5} \\
&\quad + \frac{3q(\mathbf{r}\cdot\dot{\mathbf{a}})\mathbf{r}}{r^5} - \frac{15q(\widehat{\mathbf{r}}\cdot\mathbf{u})(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^6} + \frac{6q(\mathbf{r}\cdot\mathbf{u})\mathbf{a}}{r^5} + \frac{6qu^2\mathbf{u}}{r^5} + \frac{6q(\mathbf{r}\cdot\mathbf{a})\mathbf{u}}{r^5} - \frac{30q(\widehat{\mathbf{r}}\cdot\mathbf{u})(\mathbf{r}\cdot\mathbf{u})\mathbf{u}}{r^6} \\
&\quad - \frac{15q(\mathbf{r}\cdot\mathbf{u})^2\mathbf{u}}{r^7} - \frac{30qu^2(\mathbf{r}\cdot\mathbf{u})\mathbf{r}}{r^7} - \frac{30q(\mathbf{r}\cdot\mathbf{u})(\mathbf{r}\cdot\mathbf{a})\mathbf{r}}{r^7} + \frac{105q(\widehat{\mathbf{r}}\cdot\mathbf{u})(\mathbf{r}\cdot\mathbf{u})^2\mathbf{r}}{r^8}
\end{aligned}$$

$$\begin{aligned}
&= \frac{q^2\mathbf{u}}{r^6} - \frac{6q^2(\widehat{\mathbf{r}}\cdot\mathbf{u})\widehat{\mathbf{r}}}{r^6} + \frac{6q(\mathbf{a}\cdot\mathbf{u})\widehat{\mathbf{r}}}{r^4} + \frac{3qu^2\mathbf{u}}{r^5} - \frac{15qu^2(\widehat{\mathbf{r}}\cdot\mathbf{u})\widehat{\mathbf{r}}}{r^5} + \frac{3q(\widehat{\mathbf{r}}\cdot\mathbf{a})\mathbf{u}}{r^4} + \frac{3q(\mathbf{u}\cdot\mathbf{a})\widehat{\mathbf{r}}}{r^4} \\
&\quad + \frac{3q(\widehat{\mathbf{r}}\cdot\dot{\mathbf{a}})\widehat{\mathbf{r}}}{r^3} - \frac{15q(\widehat{\mathbf{r}}\cdot\mathbf{u})(\widehat{\mathbf{r}}\cdot\mathbf{a})\widehat{\mathbf{r}}}{r^4} + \frac{6q(\widehat{\mathbf{r}}\cdot\mathbf{u})\mathbf{a}}{r^4} + \frac{6qu^2\mathbf{u}}{r^5} + \frac{6q(\widehat{\mathbf{r}}\cdot\mathbf{a})\mathbf{u}}{r^4} - \frac{30q(\widehat{\mathbf{r}}\cdot\mathbf{u})^2\mathbf{u}}{r^5} \\
&\quad - \frac{15q(\widehat{\mathbf{r}}\cdot\mathbf{u})^2\mathbf{u}}{r^5} - \frac{30qu^2(\widehat{\mathbf{r}}\cdot\mathbf{u})\widehat{\mathbf{r}}}{r^5} - \frac{30q(\widehat{\mathbf{r}}\cdot\mathbf{u})(\widehat{\mathbf{r}}\cdot\mathbf{a})\widehat{\mathbf{r}}}{r^4} + \frac{105q(\widehat{\mathbf{r}}\cdot\mathbf{u})^3\widehat{\mathbf{r}}}{r^5} \\
&= \varrho_a^2\mathbf{u} - 6\varrho_a^2(\varepsilon_h/h^2)\widehat{\mathbf{r}} + 6(\varrho_a/r)(-\varphi_a\widehat{\mathbf{r}}\cdot\mathbf{u})\widehat{\mathbf{r}} + 3\varrho_a(u^2/r^2)\mathbf{u} - 15\varrho_a(u^2/r^2)(\varepsilon_h/h^2)\widehat{\mathbf{r}} \\
&\quad + 3(\varrho_a/r)(-\varphi_a)\mathbf{u} + 3(\varrho_a/r)(-\varphi_a\widehat{\mathbf{r}}\cdot\mathbf{u})\widehat{\mathbf{r}} + 3\varrho_a[\widehat{\mathbf{r}}\cdot(-\varrho_a\mathbf{u} + 3\varrho_a\varrho_b\mathbf{r})]\widehat{\mathbf{r}} \\
&\quad - 15\varrho_a[\varepsilon_h/(rh^2)](-\varphi_a)\widehat{\mathbf{r}} + 6(\varrho_a/r)(\varepsilon_h/h^2)(-\varphi_a\widehat{\mathbf{r}}) + 6\varrho_a(u^2/r^2)\mathbf{u} + 6(\varrho_a/r)(-\varphi_a)\mathbf{u} \\
&\quad - 30\varrho_a[\varepsilon_h/(rh^2)]^2\mathbf{u} - 15\varrho_a[\varepsilon_h/(rh^2)]^2\mathbf{u} - 30\varrho_a r(u^2/r^2)[\varepsilon_h/(rh^2)]\widehat{\mathbf{r}} - 30\varrho_a[\varepsilon_h/(rh^2)](-\varphi_a)\widehat{\mathbf{r}} \\
&\quad + 105r\varrho_a[\varepsilon_h/(rh^2)]^3\widehat{\mathbf{r}} \text{ by (1.5a), (1.5c), (6.1a), (6.1b), (6.21b) \& (6.26a)}
\end{aligned}$$

$$\begin{aligned}
&= \varrho_a^2\mathbf{u} - 6r\varrho_b\varrho_a^2\widehat{\mathbf{r}} - 6\varphi_a\varrho_a[\varepsilon_h/(rh^2)]\widehat{\mathbf{r}} + 9\varrho_a(u^2/r^2)\mathbf{u} - 15r\varrho_a\varrho_b(u^2/r^2)\widehat{\mathbf{r}} - 3\varphi_a(\varrho_a/r)\mathbf{u} \\
&\quad - 3\varphi_a\varrho_a[\varepsilon_h/(rh^2)]\widehat{\mathbf{r}} + 3\varrho_a[-\varrho_a(\varepsilon_h/h^2) + 3r\varrho_a\varrho_b]\widehat{\mathbf{r}} + 15\varphi_a\varrho_a\varrho_b\widehat{\mathbf{r}} - 6\varphi_a\varrho_a\varrho_b\widehat{\mathbf{r}} \\
&\quad - 6\varphi_a(\varrho_a/r)\mathbf{u} - 30\varrho_a\varrho_b^2\mathbf{u} - 15\varrho_a\varrho_b^2\mathbf{u} - 30r\varrho_a\varrho_b(u^2/r^2)\widehat{\mathbf{r}} + 30\varphi_a\varrho_a\varrho_b\widehat{\mathbf{r}} + 105r\varrho_a\varrho_b^3\widehat{\mathbf{r}} \\
&\quad \text{by (1.5c), (6.1a), (6.1b) \& (6.21b)} \\
&= \varrho_a^2\mathbf{u} - 6r\varrho_b\varrho_a^2\widehat{\mathbf{r}} - 6\varphi_a\varrho_a\varrho_b\widehat{\mathbf{r}} + 9\varrho_a(\varphi_h^2/r^2)\mathbf{u} - 15r\varrho_a\varrho_b(\varphi_h^2/r^2)\widehat{\mathbf{r}} - 3\varrho_a^2\mathbf{u} \\
&\quad - 3\varphi_a\varrho_a\varrho_b\widehat{\mathbf{r}} + 3\varrho_a[-r\varrho_a\varrho_b + 3r\varrho_a\varrho_b]\widehat{\mathbf{r}} + 15\varphi_a\varrho_a\varrho_b\widehat{\mathbf{r}} - 6\varphi_a\varrho_a\varrho_b\widehat{\mathbf{r}} \\
&\quad - 6\varrho_a^2\mathbf{u} - 30\varrho_a\varrho_b^2\mathbf{u} - 15\varrho_a\varrho_b^2\mathbf{u} - 30r\varrho_a\varrho_b(\varphi_h^2/r^2)\widehat{\mathbf{r}} + 30\varphi_a\varrho_a\varrho_b\widehat{\mathbf{r}} + 105r\varrho_a\varrho_b^3\widehat{\mathbf{r}} \\
&\quad \text{by (6.21b), (6.3b) \& (6.1b)}
\end{aligned}$$

$$\begin{aligned}
&= (\varrho_a^2 + 9\varrho_a\varrho_k - 3\varrho_a^2 - 6\varrho_a^2 - 30\varrho_a\varrho_b^2 - 15\varrho_a\varrho_b^2)\mathbf{u} \\
&\quad + (-6r\varrho_b\varrho_a^2 - 6\varphi_a\varrho_a\varrho_b - 15r\varrho_a\varrho_b\varrho_k - 3\varphi_a\varrho_a\varrho_b + 6r\varrho_a^2\varrho_b + 15\varphi_a\varrho_a\varrho_b - 6\varphi_a\varrho_a\varrho_b \\
&\qquad - 30r\varrho_a\varrho_b\varrho_k + 30\varphi_a\varrho_a\varrho_b + 105r\varrho_a\varrho_b^3)\widehat{\mathbf{r}} \text{ by (6.21b)} \\
&= \varrho_a(9\varrho_k - 8\varrho_a - 45\varrho_b^2)\mathbf{u} + (-6r\varrho_b\varrho_a^2 - 6\varphi_a\varrho_a\varrho_b - 3\varphi_a\varrho_a\varrho_b + 15\varphi_a\varrho_a\varrho_b - 6\varphi_a\varrho_a\varrho_b \\
&\qquad + 30\varphi_a\varrho_a\varrho_b - 15r\varrho_a\varrho_b\varrho_k + 6r\varrho_a^2\varrho_b - 30r\varrho_a\varrho_b\varrho_k + 105r\varrho_a\varrho_b^3)\widehat{\mathbf{r}} \\
&= \varrho_a(9\varrho_k - 8\varrho_a - 45\varrho_b^2)\mathbf{u} + \varrho_a\varrho_b(30\varphi_a - 45r\varrho_k + 105r\varrho_b^2)\widehat{\mathbf{r}} \\
&= \varrho_a(9\varrho_k - 8\varrho_a - 45\varrho_b^2)\mathbf{u} + \varrho_a\varrho_b(30r\varrho_a - 45r\varrho_k + 105r\varrho_b^2)\widehat{\mathbf{r}} \text{ by (6.1b) \& (6.21b)} \\
&= \varrho_a(9\varrho_k - 8\varrho_a - 45\varrho_b^2)\mathbf{u} + 15\varrho_a\varrho_b(2\varrho_a - 3\varrho_k + 7\varrho_b^2)\mathbf{r} = \varrho_m\mathbf{u} + \varrho_n\mathbf{r} \text{ by (6.21b).} \qquad (6.26c)
\end{aligned}$$

Art 26c. *Development of equations* (3.17) *through* (3.21).

We derive also

$$\begin{aligned}\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u} &= h^{-2}[\widehat{\boldsymbol{\kappa}} \cdot (\mathbf{z} \times \mathbf{h} + q\widehat{\mathbf{r}} \times \mathbf{h})] \text{ by (1.5c)} \\ &= h^{-2}[\widehat{\boldsymbol{\kappa}} \cdot (\mathbf{z} \times \mathbf{h})] + qh^{-2}[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\mathbf{r}} \times \mathbf{h})] \\ &= h^{-2}\varepsilon_g + h^{-2}\varepsilon_e(q/r) \text{ by (6.1a)} \\ &= h^{-2}(\varepsilon_g + \varepsilon_e\varphi_b) \text{ by (6.1b)} \\ &= \varrho_o \text{ by (6.21b)} \end{aligned} \tag{6.27a}$$

$$\begin{aligned}\widehat{\mathbf{r}} \cdot \mathbf{u} &= h^{-2}[\widehat{\mathbf{r}} \cdot (\mathbf{z} \times \mathbf{h} + q\widehat{\mathbf{r}} \times \mathbf{h})] \text{ by (1.5c)} \\ &= h^{-2}[\widehat{\mathbf{r}} \cdot (\mathbf{z} \times \mathbf{h})] \\ &= \varepsilon_h/h^2 \text{ by (6.1a)} \\ &= r\varrho_b \text{ by (6.21b)} \end{aligned} \tag{6.27b}$$

$$\begin{aligned}\widehat{\mathbf{p}} \cdot \mathbf{u} &= h^{-2}[\widehat{\mathbf{p}} \cdot (\mathbf{z} \times \mathbf{h} + q\widehat{\mathbf{r}} \times \mathbf{h})] \text{ by (1.5c)} \\ &= h^{-2}[\widehat{\mathbf{p}} \cdot (\mathbf{z} \times \mathbf{h})] + qh^{-2}[\widehat{\mathbf{p}} \cdot (\widehat{\mathbf{r}} \times \mathbf{h})] \\ &= h^{-2}\varepsilon_i + h^{-2}\varepsilon_f(q/r) \text{ by (6.1a)} \\ &= h^{-2}(\varepsilon_i + \varepsilon_f\varphi_b) \text{ by (6.1b)} \end{aligned} \tag{6.27c}$$

$$\begin{aligned}\mathbf{z} \cdot \mathbf{u} &= h^{-2}[\mathbf{z} \cdot (\mathbf{z} \times \mathbf{h} + q\widehat{\mathbf{r}} \times \mathbf{h})] \text{ by (1.5c)} \\ &= h^{-2}[\mathbf{z} \cdot (\mathbf{z} \times \mathbf{h})] + qh^{-2}[\mathbf{z} \cdot (\widehat{\mathbf{r}} \times \mathbf{h})] \\ &= -h^{-2}\varepsilon_h(q/r) = -h^{-2}\varepsilon_h\varphi_b \text{ by (6.1a) \& (6.1b)} \\ &= -r\varrho_b\varphi_b \text{ by (6.21b)} \end{aligned} \tag{6.27d}$$

$$\begin{aligned}(\mathbf{r} \times \mathbf{h}) \cdot \mathbf{u} &= h^{-2}[(\mathbf{r} \times \mathbf{h}) \cdot (\mathbf{z} \times \mathbf{h} + q\widehat{\mathbf{r}} \times \mathbf{h})] \text{ by (1.5c)} \\ &= h^{-2}[(\mathbf{r} \times \mathbf{h}) \cdot (\mathbf{z} \times \mathbf{h})] + qrh^{-2}[(\widehat{\mathbf{r}} \times \mathbf{h}) \cdot (\widehat{\mathbf{r}} \times \mathbf{h})] \\ &= h^{-2}[(\mathbf{r} \cdot \mathbf{z})(\mathbf{h} \cdot \mathbf{h}) - (\mathbf{r} \cdot \mathbf{h})(\mathbf{h} \cdot \mathbf{z})] + qrh^{-2}[h^2 - (\widehat{\mathbf{r}} \cdot \mathbf{h})^2] \text{ by (A.2)} \\ &= h^{-2}(r\varepsilon_d h^2) + qrh^{-2}(h^2) \text{ by (6.1a) \& (1.5c)} \\ &= r(\varepsilon_d + q) = r\hbar_b \text{ by (6.10b)} \end{aligned} \tag{6.27e}$$

$$\begin{aligned}(\mathbf{z} \times \mathbf{h}) \cdot \mathbf{u} &= h^{-2}[(\mathbf{z} \times \mathbf{h}) \cdot (\mathbf{z} \times \mathbf{h} + q\widehat{\mathbf{r}} \times \mathbf{h})] \text{ by (1.5c)} \\ &= h^{-2}[(\mathbf{z} \times \mathbf{h}) \cdot (\mathbf{z} \times \mathbf{h})] + qh^{-2}[(\mathbf{z} \times \mathbf{h}) \cdot (\widehat{\mathbf{r}} \times \mathbf{h})] \\ &= h^{-2}[z^2h^2 - (\mathbf{z} \cdot \mathbf{h})^2] + qh^{-2}[(\mathbf{z} \cdot \widehat{\mathbf{r}})h^2 - (\mathbf{z} \cdot \mathbf{h})(\widehat{\mathbf{r}} \cdot \mathbf{h})] \text{ by (A.2)} \\ &= h^{-2}(z^2h^2) + qh^{-2}(\varepsilon_d h^2) \text{ by (6.1a) \& (1.5c)} \\ &= z^2 + q\varepsilon_d = \mathfrak{y}_z \text{ by (6.21h).} \end{aligned} \tag{6.27f}$$

Equations (3.17) through (3.21) therefore evaluate as

$$\begin{aligned}\dot{\alpha} &= \boldsymbol{\kappa} \cdot \dot{\mathbf{a}} \text{ by (3.17)} \\ &= \boldsymbol{\kappa} \cdot (-\varrho_a\mathbf{u} + 3\varrho_a\varrho_b\mathbf{r}) \text{ by (6.26a)} \\ &= -\kappa\varrho_a(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u}) + 3\kappa\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r}) \\ &= -\kappa\varrho_a\varrho_o + 3\kappa r\varrho_a\varrho_b\varepsilon_a \text{ by (6.27a) \& (6.1a)} \\ &= \kappa\varrho_a(3r\varrho_b\varepsilon_a - \varrho_o) = \kappa\varrho_p \text{ by (6.21b)} \end{aligned} \tag{6.28a}$$

$$\begin{aligned}
\ddot{\alpha} &= \boldsymbol{\kappa} \cdot \ddot{\mathbf{a}} \text{ by (3.17)} \\
&= \boldsymbol{\kappa} \cdot (6\varrho_a\varrho_b\mathbf{u} - \varrho_a\varrho_l\mathbf{r}) \text{ by (6.26b)} \\
&= 6\kappa\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u}) - \kappa r\varrho_a\varrho_l(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{r}}) \\
&= 6\kappa\varrho_a\varrho_b\varrho_o - \kappa r\varrho_a\varrho_l\varepsilon_a \text{ by (6.27a) \& (6.1a)} \\
&= \kappa\varrho_a(6\varrho_b\varrho_o - r\varrho_l\varepsilon_a) = \kappa\varrho_q \text{ by (6.21b)}
\end{aligned} \tag{6.28b}$$

$$\begin{aligned}
\dddot{\alpha} &= \boldsymbol{\kappa} \cdot \dddot{\mathbf{a}} \text{ by (3.17)} \\
&= \boldsymbol{\kappa} \cdot (\varrho_m\mathbf{u} + \varrho_n\mathbf{r}) \text{ by (6.26c)} \\
&= \kappa\varrho_m(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{u}) + \kappa r\varrho_n(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{r}}) \\
&= \kappa\varrho_m\varrho_o + \kappa r\varrho_n\varepsilon_a \text{ by (6.27a) \& (6.1a)} \\
&= \kappa(\varrho_m\varrho_o + r\varrho_n\varepsilon_a) = \kappa\varrho_r \text{ by (6.21b)}
\end{aligned} \tag{6.28c}$$

$$\begin{aligned}
\dot{\vartheta} &= \frac{1}{\gamma^2\omega_o^2}\left[\dot{\alpha} - \frac{2\alpha\dot{\gamma}}{\gamma}\right] \text{ by (3.18a)} \\
&= \frac{1}{\varphi_e^2\omega_o^2}\left[\dot{\alpha} - \frac{2(-\kappa\varphi_a\varepsilon_a)\dot{\gamma}}{\varphi_e}\right] \text{ by (6.2a) \& (6.2e)} \\
&= \frac{1}{\varphi_e^2\omega_o^2}\left[(\kappa\varrho_p) + \frac{2\kappa\varphi_a\varepsilon_a(\varrho_g)}{\varphi_e}\right] \text{ by (6.25a) \& (6.28a)} \\
&= \frac{\kappa}{\varphi_e^2\omega_o^2}\left[\varrho_p + \frac{2\varphi_a\varepsilon_a\varrho_g}{\varphi_e}\right] = \kappa\varrho_s \text{ by (6.21c)}
\end{aligned} \tag{6.29a}$$

$$\begin{aligned}
\ddot{\vartheta} &= \frac{1}{\gamma^2\omega_o^2}\left[\ddot{\alpha} - \frac{4\dot{\alpha}\dot{\gamma}}{\gamma} - \frac{2\alpha\ddot{\gamma}}{\gamma} + \frac{6\alpha\dot{\gamma}^2}{\gamma^2}\right] \text{ by (3.18b)} \\
&= \frac{1}{\varphi_e^2\omega_o^2}\left[\ddot{\alpha} - \frac{4\dot{\alpha}\dot{\gamma}}{\varphi_e} - \frac{2(-\kappa\varphi_a\varepsilon_a)\ddot{\gamma}}{\varphi_e} + \frac{6(-\kappa\varphi_a\varepsilon_a)\dot{\gamma}^2}{\varphi_e^2}\right] \text{ by (6.2a) \& (6.2e)} \\
&= \frac{1}{\varphi_e^2\omega_o^2}\left[(\kappa\varrho_q) - \frac{4(\kappa\varrho_p)(\varrho_g)}{\varphi_e} + \frac{2\kappa\varphi_a\varepsilon_a(\varrho_i)}{\varphi_e} - \frac{6\kappa\varphi_a\varepsilon_a(\varrho_g)^2}{\varphi_e^2}\right] \text{ by (6.25) \& (6.28)} \\
&= \frac{\kappa}{\varphi_e^2\omega_o^2}\left[\varrho_q - \frac{4\varrho_p\varrho_g}{\varphi_e} + \frac{2\varphi_a\varepsilon_a\varrho_i}{\varphi_e} - \frac{6\varphi_a\varepsilon_a\varrho_g^2}{\varphi_e^2}\right] = \kappa\varrho_t \text{ by (6.21c)}
\end{aligned} \tag{6.29b}$$

$$\begin{aligned}
\dddot{\vartheta} &= \frac{1}{\gamma^2\omega_o^2}\left[\dddot{\alpha} - \frac{6\ddot{\alpha}\dot{\gamma}}{\gamma} - \frac{6\dot{\alpha}\ddot{\gamma}}{\gamma} + \frac{18\dot{\alpha}\dot{\gamma}^2}{\gamma^2} - \frac{2\alpha\dddot{\gamma}}{\gamma} + \frac{18\alpha\dot{\gamma}\ddot{\gamma}}{\gamma^2} - \frac{24\alpha\dot{\gamma}^3}{\gamma^3}\right] \text{ by (3.18c)} \\
&= \frac{1}{\varphi_e^2\omega_o^2}\left[\dddot{\alpha} - \frac{6\ddot{\alpha}\dot{\gamma}}{\varphi_e} - \frac{6\dot{\alpha}\ddot{\gamma}}{\varphi_e} + \frac{18\dot{\alpha}\dot{\gamma}^2}{\varphi_e^2} - \frac{2(-\kappa\varphi_a\varepsilon_a)\dddot{\gamma}}{\varphi_e} + \frac{18(-\kappa\varphi_a\varepsilon_a)\dot{\gamma}\ddot{\gamma}}{\varphi_e^2} - \frac{24(-\kappa\varphi_a\varepsilon_a)\dot{\gamma}^3}{\varphi_e^3}\right] \\
&\quad \text{by (6.2a) \& (6.2e)} \\
&= \frac{1}{\varphi_e^2\omega_o^2}\left[(\kappa\varrho_r) - \frac{6(\kappa\varrho_q)(\varrho_g)}{\varphi_e} - \frac{6(\kappa\varrho_p)(\varrho_i)}{\varphi_e} + \frac{18(\kappa\varrho_p)(\varrho_g)^2}{\varphi_e^2} + \frac{2\kappa\varphi_a\varepsilon_a(\varrho_j)}{\varphi_e} - \frac{18\kappa\varphi_a\varepsilon_a(\varrho_g)(\varrho_i)}{\varphi_e^2}\right. \\
&\quad \left. + \frac{24\kappa\varphi_a\varepsilon_a(\varrho_g)^3}{\varphi_e^3}\right] \text{ by (6.25) \& (6.28)} \\
&= \frac{\kappa}{\varphi_e^2\omega_o^2}\left[\varrho_r - \frac{6\varrho_q\varrho_g}{\varphi_e} - \frac{6\varrho_p\varrho_i}{\varphi_e} + \frac{2\varphi_a\varepsilon_a\varrho_j}{\varphi_e} + \frac{18\varrho_p\varrho_g^2}{\varphi_e^2} - \frac{18\varphi_a\varepsilon_a\varrho_g\varrho_i}{\varphi_e^2} + \frac{24\varphi_a\varepsilon_a\varrho_g^3}{\varphi_e^3}\right] \\
&= \kappa\varrho_u \text{ by (6.21c)}
\end{aligned} \tag{6.29c}$$

$$\begin{aligned}\dot\pi &= \dot\vartheta(1+\vartheta^2)^{-3/2} \text{ by (3.19a)}\\ &= \dot\vartheta/\hbar_i^3 \text{ by (6.10b)}\\ &= (\kappa\varrho_s)/\hbar_i^3 \text{ by (6.29a)}\\ &= \kappa\varrho_v \text{ by (6.21c)}\end{aligned} \tag{6.30a}$$

$$\begin{aligned}\ddot\pi &= (1+\vartheta^2)^{-5/2}[\ddot\vartheta(1+\vartheta^2) - 3\vartheta\dot\vartheta^2] \text{ by (3.19b)}\\ &= (1/\hbar_i^5)(\ddot\vartheta\hbar_i^2 - 3\vartheta\dot\vartheta^2) \text{ by (6.10b)}\\ &= (1/\hbar_i^5)[(\kappa\varrho_t)\hbar_i^2 - 3\vartheta(\kappa\varrho_s)^2] \text{ by (6.29)}\\ &= \kappa(\varrho_t\hbar_i^2 - 3\vartheta\varrho_s^2)/\hbar_i^5 = \kappa\varrho_w \text{ by (6.21c)}\end{aligned} \tag{6.30b}$$

$$\begin{aligned}\dddot\pi &= (1+\vartheta^2)^{-7/2}[\dddot\vartheta(1+\vartheta^2)^2 - 9\vartheta\dot\vartheta\ddot\vartheta(1+\vartheta^2) - 3\dot\vartheta^3(1-4\vartheta^2)] \text{ by (3.19c)}\\ &= [\dddot\vartheta\hbar_i^4 - 9\vartheta\dot\vartheta\ddot\vartheta\hbar_i^2 - 3\dot\vartheta^3(1-4\vartheta^2)]/\hbar_i^7 \text{ by (6.10b)}\\ &= [(\kappa\varrho_u)\hbar_i^4 - 9\vartheta(\kappa\varrho_s)(\kappa\varrho_t)\hbar_i^2 - 3(\kappa\varrho_s)^3(1-4\vartheta^2)]/\hbar_i^7 \text{ by (6.29)}\\ &= \kappa[\varrho_u\hbar_i^4 - 9\kappa\vartheta\varrho_s\varrho_t\hbar_i^2 - 3\kappa^2\varrho_s^3(1-4\vartheta^2)]/\hbar_i^7 = \kappa\varrho_x \text{ by (6.21c)}\end{aligned} \tag{6.30c}$$

$$\begin{aligned}\dot d &= \frac{\dot\gamma d}{\gamma} + \frac{\pi\gamma^2\dot\vartheta}{4d} \text{ by (3.20a)}\\ &= \frac{\dot\gamma d}{\varphi_e} + \frac{\pi(\varphi_e)^2\dot\vartheta}{4d} \text{ by (6.2e)}\\ &= \frac{\varrho_g d}{\varphi_e} + \frac{\pi\varphi_e^2\kappa\varrho_s}{4d} \text{ by (6.25a) \& (6.29a)}\\ &= d\left[\frac{\varrho_g}{\varphi_e} + \frac{\kappa\pi\varphi_e^2\varrho_s}{4d^2}\right] = \mathfrak{x}_a \text{ by (6.21d)}\end{aligned} \tag{6.31a}$$

$$\begin{aligned}\ddot d &= d\left[\frac{\ddot\gamma}{\gamma} - \frac{\dot\gamma^2}{\gamma^2}\right] + \dot d\left[\frac{\dot\gamma}{\gamma} - \frac{\pi\gamma^2\dot\vartheta}{4d^2}\right] + \frac{\gamma}{4d}\left[\dot\pi\gamma\dot\vartheta + 2\pi\dot\gamma\dot\vartheta + \pi\gamma\ddot\vartheta\right] \text{ by (3.20b)}\\ &= d\left[\frac{\ddot\gamma}{\varphi_e} - \frac{\dot\gamma^2}{\varphi_e^2}\right] + \dot d\left[\frac{\dot\gamma}{\varphi_e} - \frac{\pi(\varphi_e)^2\dot\vartheta}{4d^2}\right] + \frac{\varphi_e}{4d}\left[\dot\pi(\varphi_e)\dot\vartheta + 2\pi\dot\gamma\dot\vartheta + \pi(\varphi_e)\ddot\vartheta\right] \text{ by (6.2e)}\\ &= d\left[\frac{\varrho_i}{\varphi_e} - \frac{\varrho_g^2}{\varphi_e^2}\right] + \mathfrak{x}_a\left[\frac{\varrho_g}{\varphi_e} - \frac{\pi\varphi_e^2(\kappa\varrho_s)}{4d^2}\right] + \frac{\varphi_e}{4d}\left[(\kappa\varrho_v)\varphi_e(\kappa\varrho_s) + 2\pi\varrho_g(\kappa\varrho_s) + \pi\varphi_e(\kappa\varrho_t)\right]\\ &\qquad \text{by (6.25), (6.29), (6.30a), \& (6.31a)}\\ &= d\left[\frac{\varrho_i}{\varphi_e} - \frac{\varrho_g^2}{\varphi_e^2}\right] + \mathfrak{x}_a\left[\frac{\varrho_g}{\varphi_e} - \frac{\kappa\pi\varphi_e^2\varrho_s}{4d^2}\right] + \frac{\kappa\varphi_e}{4d}\left[\kappa\varrho_v\varphi_e\varrho_s + 2\pi\varrho_g\varrho_s + \pi\varphi_e\varrho_t\right]\\ &= d\mathfrak{x}_d + \mathfrak{x}_a\mathfrak{x}_b + \varphi_e\mathfrak{x}_c = \mathfrak{x}_e \text{ by (6.21d)}\end{aligned} \tag{6.31b}$$

$$\begin{aligned}\dddot d &= d\left[\frac{\dddot\gamma}{\gamma} - \frac{3\dot\gamma\ddot\gamma}{\gamma^2} + \frac{2\dot\gamma^3}{\gamma^3}\right] + 2\dot d\left[\frac{\ddot\gamma}{\gamma} - \frac{\dot\gamma^2}{\gamma^2} - \frac{\dot\pi\gamma^2\dot\vartheta}{4d^2} - \frac{\pi\gamma\dot\gamma\dot\vartheta}{2d^2} - \frac{\pi\gamma^2\ddot\vartheta}{4d^2} + \frac{\pi\gamma^2\dot d\dot\vartheta}{4d^3}\right] + \ddot d\left[\frac{\dot\gamma}{\gamma} - \frac{\pi\gamma^2\dot\vartheta}{4d^2}\right]\\ &\quad + \frac{\dot\gamma}{4d}\left[\dot\pi\gamma\dot\vartheta + 2\pi\dot\gamma\dot\vartheta + \pi\gamma\ddot\vartheta\right] + \frac{\gamma}{4d}\left[\ddot\pi\gamma\dot\vartheta + 3\dot\pi\dot\gamma\dot\vartheta + 2\dot\pi\gamma\ddot\vartheta + 2\pi\ddot\gamma\dot\vartheta + 3\pi\dot\gamma\ddot\vartheta + \pi\gamma\dddot\vartheta\right]\\ &= d\left[\frac{\dddot\gamma}{\gamma} - \frac{3\dot\gamma\ddot\gamma}{\gamma^2} + \frac{2\dot\gamma^3}{\gamma^3}\right] + 2\dot d\left[\frac{\ddot\gamma}{\gamma} - \frac{\dot\gamma^2}{\gamma^2} - \frac{\dot\pi\gamma^2\dot\vartheta}{4d^2} - \frac{\pi\gamma\dot\gamma\dot\vartheta}{2d^2} - \frac{\pi\gamma^2\ddot\vartheta}{4d^2} + \frac{\pi\gamma^2\dot d\dot\vartheta}{4d^3}\right] + \ddot d\mathfrak{x}_b\\ &\quad + \dot\gamma\mathfrak{x}_c + \frac{\gamma}{4d}\left[\ddot\pi\gamma\dot\vartheta + 3\dot\pi\dot\gamma\dot\vartheta + 2\dot\pi\gamma\ddot\vartheta + 2\pi\ddot\gamma\dot\vartheta + 3\pi\dot\gamma\ddot\vartheta + \pi\gamma\dddot\vartheta\right] \text{ by (6.31b) \& (6.21d)}\end{aligned}$$

$$
\begin{aligned}
&= d\left[\frac{\varrho_j}{\varphi_e} - \frac{3\varrho_g\varrho_i}{\varphi_e^2} + \frac{2\varrho_g^3}{\varphi_e^3}\right] + 2\dot{d}\left[\frac{\varrho_i}{\varphi_e} - \frac{\varrho_g^2}{\varphi_e^2} - \frac{\dot{\pi}\varphi_e^2\dot{\vartheta}}{4d^2} - \frac{\pi\varphi_e\varrho_g\dot{\vartheta}}{2d^2} - \frac{\pi\varphi_e^2\ddot{\vartheta}}{4d^2} + \frac{\pi\varphi_e^2\dot{d}\dot{\vartheta}}{4d^3}\right] + \ddot{d}\mathfrak{x}_b \\
&\quad + \varrho_g\mathfrak{x}_c + \frac{\varphi_e}{4d}\left[\ddot{\pi}\varphi_e\dot{\vartheta} + 3\dot{\pi}\varrho_g\dot{\vartheta} + 2\dot{\pi}\varphi_e\ddot{\vartheta} + 2\pi\varrho_i\dot{\vartheta} + 3\pi\varrho_g\ddot{\vartheta} + \pi\varphi_e\dddot{\vartheta}\right] \text{ by (6.2e) \& (6.25)} \\
&= \mathfrak{x}_f + 2\mathfrak{x}_a\left[\mathfrak{x}_d - \frac{(\kappa\varrho_v)\varphi_e^2(\kappa\varrho_s)}{4d^2} - \frac{\pi\varphi_e\varrho_g(\kappa\varrho_s)}{2d^2} - \frac{\pi\varphi_e^2(\kappa\varrho_t)}{4d^2} + \frac{\pi\varphi_e^2\mathfrak{x}_a(\kappa\varrho_s)}{4d^3}\right] + \mathfrak{x}_e\mathfrak{x}_b + \varrho_g\mathfrak{x}_c \\
&\quad + \frac{\varphi_e}{4d}\left[(\kappa\varrho_w)\varphi_e(\kappa\varrho_s) + 3(\kappa\varrho_v)\varrho_g(\kappa\varrho_s) + 2(\kappa\varrho_v)\varphi_e(\kappa\varrho_t) + 2\pi\varrho_i(\kappa\varrho_s) + 3\pi\varrho_g(\kappa\varrho_t) + \pi\varphi_e(\kappa\varrho_u)\right] \\
&\quad \text{by (6.21d), (6.29), (6.30), (6.31a) \& (6.31b)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{x}_f + \mathfrak{x}_e\mathfrak{x}_b + \varrho_g\mathfrak{x}_c + 2\mathfrak{x}_a\left[\mathfrak{x}_d - \frac{\kappa^2\varrho_v\varphi_e^2\varrho_s}{4d^2} - \frac{\kappa\pi\varphi_e\varrho_g\varrho_s}{2d^2} - \frac{\kappa\pi\varphi_e^2\varrho_t}{4d^2} + \frac{\kappa\pi\varphi_e^2\mathfrak{x}_a\varrho_s}{4d^3}\right] \\
&\quad + \frac{\kappa\varphi_e}{4d}\left[\kappa\left\{\varrho_w\varphi_e\varrho_s + 3\varrho_v\varrho_g\varrho_s + 2\varrho_v\varphi_e\varrho_t\right\} + \pi\left\{2\varrho_i\varrho_s + 3\varrho_g\varrho_t + \varphi_e\varrho_u\right\}\right] \\
&= \mathfrak{x}_g \text{ by (6.21d)}
\end{aligned}
\tag{6.31c}
$$

$$
\begin{aligned}
\dot{\rho} &= \frac{d\dot{\pi} - \pi\dot{d}}{4\omega_o d^2} \text{ by (3.21a)} \\
&= \frac{d(\kappa\varrho_v) - \pi(\mathfrak{x}_a)}{4\omega_o d^2} \text{ by (6.30a) \& (6.31a)} \\
&= \frac{d\kappa\varrho_v - \pi\mathfrak{x}_a}{4\omega_o d^2} = \mathfrak{x}_h \text{ by (6.21e)}
\end{aligned}
\tag{6.32a}
$$

$$
\begin{aligned}
\ddot{\rho} &= -\frac{\dot{d}(d\dot{\pi} - \pi\dot{d})}{2\omega_o d^3} + \frac{d\ddot{\pi} - \pi\ddot{d}}{4\omega_o d^2} \text{ by (3.21b)} \\
&= -\frac{\mathfrak{x}_a[d(\kappa\varrho_v) - \pi\mathfrak{x}_a]}{2\omega_o d^3} + \frac{d(\kappa\varrho_w) - \pi\mathfrak{x}_e}{4\omega_o d^2} \text{ by (6.30) \& (6.31)} \\
&= -\frac{\mathfrak{x}_a(d\kappa\varrho_v - \pi\mathfrak{x}_a)}{2\omega_o d^3} + \frac{d\kappa\varrho_w - \pi\mathfrak{x}_e}{4\omega_o d^2} = \mathfrak{x}_i \text{ by (6.21e)}
\end{aligned}
\tag{6.32b}
$$

$$
\begin{aligned}
\dddot{\rho} &= \frac{(3\dot{d}^2 - d\ddot{d})(d\dot{\pi} - \pi\dot{d})}{2\omega_o d^4} - \frac{\dot{d}(d\ddot{\pi} - \pi\ddot{d})}{\omega_o d^3} + \frac{\dot{d}\ddot{\pi} + d\dddot{\pi} - \dot{\pi}\ddot{d} - \pi\dddot{d}}{4\omega_o d^2} \text{ by (3.21c)} \\
&= \frac{(3\mathfrak{x}_a^2 - d\mathfrak{x}_e)[d(\kappa\varrho_v) - \pi\mathfrak{x}_a]}{2\omega_o d^4} - \frac{\mathfrak{x}_a[d(\kappa\varrho_w) - \pi\mathfrak{x}_e]}{\omega_o d^3} + \frac{\mathfrak{x}_a(\kappa\varrho_w) + d(\kappa\varrho_x) - (\kappa\varrho_v)\mathfrak{x}_e - \pi\mathfrak{x}_g}{4\omega_o d^2} \\
&\quad \text{by (6.30) \& (6.31)} \\
&= \frac{(3\mathfrak{x}_a^2 - d\mathfrak{x}_e)(d\kappa\varrho_v - \pi\mathfrak{x}_a)}{2\omega_o d^4} - \frac{\mathfrak{x}_a(d\kappa\varrho_w - \pi\mathfrak{x}_e)}{\omega_o d^3} + \frac{\kappa(\mathfrak{x}_a\varrho_w + d\varrho_x - \varrho_v\mathfrak{x}_e) - \pi\mathfrak{x}_g}{4\omega_o d^2} \\
&= \mathfrak{x}_j \text{ by (6.21e).}
\end{aligned}
\tag{6.32c}
$$

Art 26d. *Derivatives of τ and* $\mathbf{e}$.

To calculate the derivatives of τ and $\mathbf{e}$ we first derive

$$\begin{aligned}
\dot{f}_0 &= \left[\frac{2\rho\alpha - d\omega_o}{n(\kappa^2-\mho^2)}\right]' \text{ by (1.5d)} \\
&= \frac{[n(\kappa^2-\mho^2)](2\rho\alpha - d\omega_o)' - (2\rho\alpha - d\omega_o)[n(\kappa^2-\mho^2)]'}{[n(\kappa^2-\mho^2)]^2} \\
&= \frac{(2\rho\alpha - d\omega_o)'}{n(\kappa^2-\mho^2)} - \frac{f_0[n(\kappa^2-\mho^2)]'}{n(\kappa^2-\mho^2)} \text{ by (1.5d)} \\
&= \frac{2\rho\dot{\alpha} + 2\dot{\rho}\alpha - \dot{d}\omega_o}{n(\kappa^2-\mho^2)} - \frac{f_0[\dot{n}(\kappa^2-\mho^2) + n(-2\mho\dot{\mho})]}{n(\kappa^2-\mho^2)} \\
&= \frac{2\rho\dot{\alpha} + 2\dot{\rho}\alpha - \dot{d}\omega_o - f_0\dot{n}(\kappa^2-\mho^2) + 2f_0 n\mho\dot{\mho}}{n(\kappa^2-\mho^2)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{2\rho(\kappa\varrho_p) + 2\mathfrak{x}_h\alpha - \mathfrak{x}_a\omega_o - f_0(-\hbar_c)(\kappa^2-\mho^2) + 2f_0 n\mho(\kappa\varrho_e)}{n(\kappa^2-\mho^2)} \\
&\quad \text{by (6.28a), (6.32a), (6.31a), (6.23a) \& (6.24a)} \\
&= \frac{2\kappa\rho\varrho_p + 2\mathfrak{x}_h(-\kappa\varphi_a\varepsilon_a) - \mathfrak{x}_a\omega_o + (2\varphi_f/\kappa)\hbar_c(\kappa^2-\kappa^2\varphi_c^2) + 2\kappa(2\varphi_f/\kappa)\varepsilon_f(\kappa\varphi_c)\varrho_e}{\varepsilon_f(\kappa^2-\kappa^2\varphi_c^2)} \text{ by (6.2)} \\
&= \frac{2\kappa\rho\varrho_p - 2\kappa\mathfrak{x}_h\varphi_a\varepsilon_a - \mathfrak{x}_a\omega_o + 2\kappa\varphi_f\hbar_c(1-\varphi_c^2) + 4\kappa\varphi_f\varepsilon_f\varphi_c\varrho_e}{\kappa^2\varepsilon_f(1-\varphi_c^2)} \\
&= \frac{2\rho\varrho_p - 2\mathfrak{x}_h\varphi_a\varepsilon_a - \mathfrak{x}_a c - 2\varphi_f\varphi_d\hbar_c + 4\varepsilon_f\varphi_f\varphi_c\varrho_e}{-\kappa\varepsilon_f\varphi_d} \text{ by (1.2b) \& (6.1b)} \\
&= -2\mathfrak{x}_k \text{ by (6.21e)} \qquad (6.33\text{a})
\end{aligned}$$

$$\begin{aligned}
\ddot{f}_0 &= \left[\frac{2\rho\dot{\alpha} + 2\dot{\rho}\alpha - \dot{d}\omega_o - f_0\dot{n}(\kappa^2-\mho^2) + 2f_0 n\mho\dot{\mho}}{n(\kappa^2-\mho^2)}\right]' \text{ by (6.33a)} \\
&= \frac{[n(\kappa^2-\mho^2)][2\rho\dot{\alpha} + 2\dot{\rho}\alpha - \dot{d}\omega_o - f_0\dot{n}(\kappa^2-\mho^2) + 2f_0 n\mho\dot{\mho}]'}{[n(\kappa^2-\mho^2)]^2} \\
&\quad - \frac{[2\rho\dot{\alpha} + 2\dot{\rho}\alpha - \dot{d}\omega_o - f_0\dot{n}(\kappa^2-\mho^2) + 2f_0 n\mho\dot{\mho}][n(\kappa^2-\mho^2)]'}{[n(\kappa^2-\mho^2)]^2} \\
&= \frac{[2\rho\dot{\alpha} + 2\dot{\rho}\alpha - \dot{d}\omega_o - f_0\dot{n}(\kappa^2-\mho^2) + 2f_0 n\mho\dot{\mho}]'}{n(\kappa^2-\mho^2)} - \frac{\dot{f}_0[n(\kappa^2-\mho^2)]'}{n(\kappa^2-\mho^2)} \text{ by (6.33a)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(2\rho\dot{\alpha} + 2\dot{\rho}\alpha)'}{n(\kappa^2-\mho^2)} - \frac{(\dot{d}\omega_o)'}{n(\kappa^2-\mho^2)} - \frac{[f_0\dot{n}(\kappa^2-\mho^2)]'}{n(\kappa^2-\mho^2)} + \frac{2(f_0 n\mho\dot{\mho})'}{n(\kappa^2-\mho^2)} - \frac{\dot{f}_0[n(\kappa^2-\mho^2)]'}{n(\kappa^2-\mho^2)} \\
&= \frac{2\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} + 2\ddot{\rho}\alpha + 2\dot{\rho}\dot{\alpha}}{n(\kappa^2-\mho^2)} - \frac{\ddot{d}\omega_o}{n(\kappa^2-\mho^2)} - \frac{\dot{f}_0\dot{n}(\kappa^2-\mho^2) + f_0\ddot{n}(\kappa^2-\mho^2) + f_0\dot{n}(-2\mho\dot{\mho})}{n(\kappa^2-\mho^2)} \\
&\quad + \frac{2\dot{f}_0 n\mho\dot{\mho} + 2f_0\dot{n}\mho\dot{\mho} + 2f_0 n\dot{\mho}^2 + 2f_0 n\mho\ddot{\mho}}{n(\kappa^2-\mho^2)} - \frac{\dot{f}_0[\dot{n}(\kappa^2-\mho^2) + n(-2\mho\dot{\mho})]}{n(\kappa^2-\mho^2)} \\
&= \frac{2\rho\ddot{\alpha} + 2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} - \ddot{d}\omega_o}{n(\kappa^2-\mho^2)} - \frac{\dot{f}_0\dot{n}(\kappa^2-\mho^2) + f_0\ddot{n}(\kappa^2-\mho^2) - 2f_0\dot{n}\mho\dot{\mho}}{n(\kappa^2-\mho^2)} \\
&\quad + \frac{2\dot{f}_0 n\mho\dot{\mho} + 2f_0\dot{n}\mho\dot{\mho} + 2f_0 n\dot{\mho}^2 + 2f_0 n\mho\ddot{\mho}}{n(\kappa^2-\mho^2)} - \frac{\dot{f}_0\dot{n}(\kappa^2-\mho^2) - 2\dot{f}_0 n\mho\dot{\mho}}{n(\kappa^2-\mho^2)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2\rho\ddot{\alpha} + 2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} - \ddot{d}\omega_o}{n(\kappa^2 - \mho^2)} - \frac{2\dot{f}_0\dot{n} + f_0\ddot{n}}{n} + \frac{4\dot{f}_0 n\mho\dot{\mho} + 4f_0\dot{n}\mho\dot{\mho} + 2f_0 n\dot{\mho}^2 + 2f_0 n\mho\ddot{\mho}}{n(\kappa^2 - \mho^2)} \\
&= \frac{2\rho(\kappa\varrho_q) + 2\mathfrak{x}_i\alpha + 4\mathfrak{x}_h(\kappa\varrho_p) - \mathfrak{x}_e\omega_o}{n(\kappa^2 - \mho^2)} - \frac{2(-2\mathfrak{x}_k)(-\hbar_c) + f_0(-\varrho_a\varepsilon_f)}{n} \\
&\quad + \frac{4(-2\mathfrak{x}_k)n\mho(\kappa\varrho_e) + 4f_0(-\hbar_c)\mho(\kappa\varrho_e) + 2f_0 n(\kappa\varrho_e)^2 + 2f_0 n\mho(-\kappa\varrho_f)}{n(\kappa^2 - \mho^2)} \\
&\quad \text{by } (6.28), (6.32), (6.31), (6.33\text{a}), (6.23) \ \&\ (6.24) \\
&= \frac{2\kappa\rho\varrho_q + 2\mathfrak{x}_i\alpha + 4\kappa\mathfrak{x}_h\varrho_p - \mathfrak{x}_e\omega_o}{n(\kappa^2 - \mho^2)} - \frac{4\mathfrak{x}_k\hbar_c - \varepsilon_f\varrho_a f_0}{n} \\
&\quad + \frac{-8\kappa\mathfrak{x}_k\varrho_e n\mho - 4\kappa\hbar_c\varrho_e f_0\mho + 2\kappa^2\varrho_e^2 f_0 n - 2\kappa\varrho_f f_0 n\mho}{n(\kappa^2 - \mho^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\kappa\rho\varrho_q + 2\mathfrak{x}_i(-\kappa\varphi_a\varepsilon_a) + 4\kappa\mathfrak{x}_h\varrho_p - \mathfrak{x}_e\omega_o}{\varepsilon_f(\kappa^2 - \kappa^2\varphi_c^2)} - \frac{4\mathfrak{x}_k\hbar_c - \varepsilon_f\varrho_a(2\varphi_f/\kappa)}{\varepsilon_f} \\
&\quad + \frac{-8\kappa\mathfrak{x}_k\varrho_e\varepsilon_f(\kappa\varphi_c) - 4\kappa\hbar_c\varrho_e(2\varphi_f/\kappa)(\kappa\varphi_c) + 2\kappa^2\varrho_e^2(2\varphi_f/\kappa)\varepsilon_f - 2\kappa\varrho_f(2\varphi_f/\kappa)\varepsilon_f(\kappa\varphi_c)}{\varepsilon_f(\kappa^2 - \kappa^2\varphi_c^2)} \\
&\quad \text{by } (6.2) \\
&= \frac{2\rho\varrho_q - 2\varepsilon_a\varphi_a\mathfrak{x}_i + 4\mathfrak{x}_h\varrho_p - \mathfrak{x}_e c}{-\kappa\varepsilon_f\varphi_d} - \frac{4\kappa\mathfrak{x}_k\hbar_c - 2\varepsilon_f\varrho_a\varphi_f}{\kappa\varepsilon_f} \\
&\quad + \frac{-8\kappa\mathfrak{x}_k\varrho_e\varepsilon_f\varphi_c - 8\varphi_f\varphi_c\hbar_c\varrho_e + 4\varrho_e^2\varphi_f\varepsilon_f - 4\varrho_f\varphi_f\varepsilon_f\varphi_c}{-\kappa\varepsilon_f\varphi_d} \text{ by (1.2b) \& (6.1b)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\varepsilon_a\varphi_a\mathfrak{x}_i - 2\rho\varrho_q - 4\mathfrak{x}_h\varrho_p + \mathfrak{x}_e c + 8\kappa\mathfrak{x}_k\varrho_e\varepsilon_f\varphi_c + 8\varphi_f\varphi_c\hbar_c\varrho_e - 4\varrho_e^2\varphi_f\varepsilon_f + 4\varrho_f\varphi_f\varepsilon_f\varphi_c}{\kappa\varepsilon_f\varphi_d} \\
&\quad - \frac{4\kappa\mathfrak{x}_k\hbar_c - 2\varepsilon_f\varrho_a\varphi_f}{\kappa\varepsilon_f} = 2\mathfrak{x}_l \text{ by (6.21e)}
\end{aligned}
\tag{6.33b}
$$

$$
\begin{aligned}
\dddot{f}_0 &= \left[\frac{2\rho\ddot{\alpha} + 2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} - \ddot{d}\omega_o}{n(\kappa^2 - \mho^2)}\right]' - \left[\frac{2\dot{f}_0\dot{n} + f_0\ddot{n}}{n}\right]' + \left[\frac{4\dot{f}_0 n\mho\dot{\mho} + 4f_0\dot{n}\mho\dot{\mho} + 2f_0 n\dot{\mho}^2 + 2f_0 n\mho\ddot{\mho}}{n(\kappa^2 - \mho^2)}\right]' \\
&\quad \text{by } (6.33\text{b}) \\
&= \frac{[n(\kappa^2 - \mho^2)][2\rho\ddot{\alpha} + 2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} - \ddot{d}\omega_o]' - [2\rho\ddot{\alpha} + 2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} - \ddot{d}\omega_o][n(\kappa^2 - \mho^2)]'}{[n(\kappa^2 - \mho^2)]^2} \\
&\quad - \frac{n[2\dot{f}_0\dot{n} + f_0\ddot{n}]' - [2\dot{f}_0\dot{n} + f_0\ddot{n}]\dot{n}}{n^2} + \frac{[n(\kappa^2 - \mho^2)][4\dot{f}_0 n\mho\dot{\mho} + 4f_0\dot{n}\mho\dot{\mho} + 2f_0 n\dot{\mho}^2 + 2f_0 n\mho\ddot{\mho}]'}{[n(\kappa^2 - \mho^2)]^2} \\
&\quad - \frac{[4\dot{f}_0 n\mho\dot{\mho} + 4f_0\dot{n}\mho\dot{\mho} + 2f_0 n\dot{\mho}^2 + 2f_0 n\mho\ddot{\mho}][n(\kappa^2 - \mho^2)]'}{[n(\kappa^2 - \mho^2)]^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left[2\rho\ddot{\alpha} + 2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} - \ddot{d}\omega_o\right]'}{n(\kappa^2 - \mho^2)} - \frac{[2\rho\ddot{\alpha} + 2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} - \ddot{d}\omega_o][n(\kappa^2 - \mho^2)]'}{[n(\kappa^2 - \mho^2)]^2} \\
&\quad - \frac{n\left[2\dot{f}_0\dot{n} + f_0\ddot{n}\right]' - [2\dot{f}_0\dot{n} + f_0\ddot{n}]\dot{n}}{n^2} + \frac{\left[4\dot{f}_0 n\mho\dot{\mho} + 4f_0\dot{n}\mho\dot{\mho} + 2f_0 n\dot{\mho}^2 + 2f_0 n\mho\ddot{\mho}\right]'}{n(\kappa^2 - \mho^2)} \\
&\quad - \frac{[4\dot{f}_0 n\mho\dot{\mho} + 4f_0\dot{n}\mho\dot{\mho} + 2f_0 n\dot{\mho}^2 + 2f_0 n\mho\ddot{\mho}][n(\kappa^2 - \mho^2)]'}{[n(\kappa^2 - \mho^2)]^2} \\
&= \frac{\left[2\rho\ddot{\alpha} + 2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} - \ddot{d}\omega_o\right]'}{n(\kappa^2 - \mho^2)} - \frac{[n(\kappa^2 - \mho^2)]'}{n(\kappa^2 - \mho^2)}\left[\ddot{f}_0 + \frac{2\dot{f}_0\dot{n} + f_0\ddot{n}}{n}\right] \\
&\quad - \frac{n\left[2\dot{f}_0\dot{n} + f_0\ddot{n}\right]' - [2\dot{f}_0\dot{n} + f_0\ddot{n}]\dot{n}}{n^2} + \frac{\left[4\dot{f}_0 n\mho\dot{\mho} + 4f_0\dot{n}\mho\dot{\mho} + 2f_0 n\dot{\mho}^2 + 2f_0 n\mho\ddot{\mho}\right]'}{n(\kappa^2 - \mho^2)}
\end{aligned}
$$

by (6.33b)

$$
\begin{aligned}
&= \frac{2\dot{\rho}\ddot{\alpha} + 2\rho\dddot{\alpha} + 2\dddot{\rho}\alpha + 2\ddot{\rho}\dot{\alpha} + 4\ddot{\rho}\dot{\alpha} + 4\dot{\rho}\ddot{\alpha} - \dddot{d}\omega_o}{n(\kappa^2 - \mho^2)} - \frac{\dot{n}(\kappa^2 - \mho^2) + n(-2\mho\dot{\mho})}{n(\kappa^2 - \mho^2)}\left[\ddot{f}_0 + \frac{2\dot{f}_0\dot{n} + f_0\ddot{n}}{n}\right] \\
&\quad - \frac{n[2\ddot{f}_0\dot{n} + 2\dot{f}_0\ddot{n} + \dot{f}_0\ddot{n} + f_0\dddot{n}]}{n^2} + \frac{[2\dot{f}_0\dot{n} + f_0\ddot{n}]\dot{n}}{n^2} + \frac{4\ddot{f}_0 n\mho\dot{\mho} + 4\dot{f}_0\dot{n}\mho\dot{\mho} + 4\dot{f}_0 n\dot{\mho}^2 + 4\dot{f}_0 n\mho\ddot{\mho}}{n(\kappa^2 - \mho^2)} \\
&\quad \frac{+4\dot{f}_0\dot{n}\mho\dot{\mho} + 4f_0\ddot{n}\mho\dot{\mho} + 4f_0\dot{n}\dot{\mho}^2 + 4f_0\dot{n}\mho\ddot{\mho} + 2\dot{f}_0 n\dot{\mho}^2 + 2f_0\dot{n}\dot{\mho}^2 + 4f_0 n\dot{\mho}\ddot{\mho}}{n(\kappa^2 - \mho^2)} \\
&\quad + \frac{2\dot{f}_0 n\mho\ddot{\mho} + 2f_0\dot{n}\mho\ddot{\mho} + 2f_0 n\dot{\mho}\ddot{\mho} + 2f_0 n\mho\dddot{\mho}}{n(\kappa^2 - \mho^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\rho\dddot{\alpha} + 2\dddot{\rho}\alpha + 6\ddot{\rho}\dot{\alpha} + 6\dot{\rho}\ddot{\alpha} - \dddot{d}\omega_o}{n(\kappa^2 - \mho^2)} - \frac{\dot{n}(\kappa^2 - \mho^2) - 2n\mho\dot{\mho}}{n(\kappa^2 - \mho^2)}\left[\ddot{f}_0 + \frac{2\dot{f}_0\dot{n} + f_0\ddot{n}}{n}\right] \\
&\quad - \frac{n[2\ddot{f}_0\dot{n} + 2\dot{f}_0\ddot{n} + \dot{f}_0\ddot{n} + f_0\dddot{n}] - \dot{n}[2\dot{f}_0\dot{n} + f_0\ddot{n}]}{n^2} \\
&\quad + \frac{8\dot{f}_0\dot{n}\mho\dot{\mho} + 4f_0\ddot{n}\mho\dot{\mho} + 4f_0\dot{n}\dot{\mho}^2 + 6f_0\dot{n}\mho\ddot{\mho} + 2f_0\dot{n}\dot{\mho}^2}{n(\kappa^2 - \mho^2)} \\
&\quad + \frac{4\ddot{f}_0 n\mho\dot{\mho} + 6\dot{f}_0 n\dot{\mho}^2 + 6f_0 n\dot{\mho}\ddot{\mho} + 6\dot{f}_0 n\mho\ddot{\mho} + 2f_0 n\mho\dddot{\mho}}{n(\kappa^2 - \mho^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\rho(\kappa\varrho_r) + 2\mathfrak{x}_j\alpha + 6\mathfrak{x}_i(\kappa\varrho_p) + 6\mathfrak{x}_h(\kappa\varrho_q) - \mathfrak{x}_g\omega_o}{n(\kappa^2 - \mho^2)} \\
&\quad - \frac{(-\hbar_c)(\kappa^2 - \mho^2) - 2n\mho(\kappa\varrho_e)}{n(\kappa^2 - \mho^2)}\left[(2\mathfrak{x}_l) + \frac{2(-2\mathfrak{x}_k)(-\hbar_c) + f_0(-\varrho_a\varepsilon_f)}{n}\right] \\
&\quad - \frac{n[2(2\mathfrak{x}_l)(-\hbar_c) + 2(-2\mathfrak{x}_k)(-\varrho_a\varepsilon_f) + (-2\mathfrak{x}_k)(-\varrho_a\varepsilon_f) + f_0(\varrho_d)] - (-\hbar_c)[2(-2\mathfrak{x}_k)(-\hbar_c) + f_0(-\varrho_a\varepsilon_f)]}{n^2} \\
&\quad + \frac{8(-2\mathfrak{x}_k)(-\hbar_c)\mho(\kappa\varrho_e) + 4f_0(-\varrho_a\varepsilon_f)\mho(\kappa\varrho_e) + 4f_0(-\hbar_c)(\kappa\varrho_e)^2 + 6f_0(-\hbar_c)\mho(-\kappa\varrho_f) + 2f_0(-\hbar_c)(\kappa\varrho_e)^2}{n(\kappa^2 - \mho^2)} \\
&\quad + \frac{4(2\mathfrak{x}_l)n\mho(\kappa\varrho_e) + 6(-2\mathfrak{x}_k)n(\kappa\varrho_e)^2 + 6f_0 n(\kappa\varrho_e)(-\kappa\varrho_f) + 6(-2\mathfrak{x}_k)n\mho(-\kappa\varrho_f) + 2f_0 n\mho(\kappa\varrho_h)}{n(\kappa^2 - \mho^2)}
\end{aligned}
$$

by (6.28), (6.32), (6.31), (6.23), (6.24), (6.33a) & (6.33b)

$$
\begin{aligned}
&= \frac{2\kappa\rho\varrho_r + 2\mathfrak{x}_j\alpha + 6\kappa\mathfrak{x}_i\varrho_p + 6\kappa\mathfrak{x}_h\varrho_q - \mathfrak{x}_g\omega_o}{n(\kappa^2 - \mho^2)} - \frac{-\hbar_c(\kappa^2 - \mho^2) - 2\kappa\varrho_e n\mho}{n(\kappa^2 - \mho^2)}\left[2\mathfrak{x}_l + \frac{4\mathfrak{x}_k\hbar_c - \varrho_a\varepsilon_f f_0}{n}\right] \\
&\quad - \frac{n[-4\mathfrak{x}_l\hbar_c + 4\mathfrak{x}_k\varrho_a\varepsilon_f + 2\mathfrak{x}_k\varrho_a\varepsilon_f + \varrho_d f_0] + \hbar_c[4\mathfrak{x}_k\hbar_c - \varrho_a\varepsilon_f f_0]}{n^2} \\
&\quad + \frac{16\kappa\varrho_e\mathfrak{x}_k\hbar_c\mho - 4\kappa\varrho_e\varrho_a\varepsilon_f f_0\mho - 4\kappa^2\hbar_c\varrho_e^2 f_0 + 6\kappa\hbar_c\varrho_f f_0\mho - 2\kappa^2\varrho_e^2\hbar_c f_0}{n(\kappa^2 - \mho^2)} \\
&\quad + \frac{8\kappa\varrho_e\mathfrak{x}_l n\mho - 12\kappa^2\varrho_e^2\mathfrak{x}_k n - 6\kappa^2\varrho_e\varrho_f f_0 n + 12\kappa\varrho_f\mathfrak{x}_k n\mho + 2\kappa\varrho_h f_0 n\mho}{n(\kappa^2 - \mho^2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\kappa\rho\varrho_r + 2\mathfrak{x}_j(-\kappa\varphi_a\varepsilon_a) + 6\kappa\mathfrak{x}_i\varrho_p + 6\kappa\mathfrak{x}_h\varrho_q - \mathfrak{x}_g\omega_o}{\varepsilon_f(\kappa^2 - \kappa^2\varphi_c^2)} \\
&\quad - \frac{-\hbar_c(\kappa^2 - \kappa^2\varphi_c^2) - 2\kappa\varrho_e\varepsilon_f(\kappa\varphi_c)}{\varepsilon_f(\kappa^2 - \kappa^2\varphi_c^2)}\left[2\mathfrak{x}_l + \frac{4\mathfrak{x}_k\hbar_c - \varrho_a\varepsilon_f(2\varphi_f/\kappa)}{\varepsilon_f}\right] \\
&\quad - \frac{\varepsilon_f[-4\mathfrak{x}_l\hbar_c + 4\mathfrak{x}_k\varrho_a\varepsilon_f + 2\mathfrak{x}_k\varrho_a\varepsilon_f + \varrho_d(2\varphi_f/\kappa)] + \hbar_c[4\mathfrak{x}_k\hbar_c - \varrho_a\varepsilon_f(2\varphi_f/\kappa)]}{\varepsilon_f^2} \\
&\quad + \frac{16\kappa\varrho_e\mathfrak{x}_k\hbar_c(\kappa\varphi_c) - 4\kappa\varrho_e\varrho_a\varepsilon_f(2\varphi_f/\kappa)(\kappa\varphi_c) - 4\kappa^2\hbar_c\varrho_e^2(2\varphi_f/\kappa) + 6\kappa\hbar_c\varrho_f(2\varphi_f/\kappa)(\kappa\varphi_c)}{\varepsilon_f(\kappa^2 - \kappa^2\varphi_c^2)} \\
&\quad + \frac{-2\kappa^2\varrho_e^2\hbar_c(2\varphi_f/\kappa) + 8\kappa\varrho_e\mathfrak{x}_l\varepsilon_f(\kappa\varphi_c) - 12\kappa^2\varrho_e^2\mathfrak{x}_k\varepsilon_f - 6\kappa^2\varrho_e\varrho_f(2\varphi_f/\kappa)\varepsilon_f}{\varepsilon_f(\kappa^2 - \kappa^2\varphi_c^2)} \\
&\quad + \frac{12\kappa\varrho_f\mathfrak{x}_k\varepsilon_f(\kappa\varphi_c) + 2\kappa\varrho_h(2\varphi_f/\kappa)\varepsilon_f(\kappa\varphi_c)}{\varepsilon_f(\kappa^2 - \kappa^2\varphi_c^2)} \text{ by (6.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\rho\varrho_r - 2\mathfrak{x}_j\varphi_a\varepsilon_a + 6\mathfrak{x}_i\varrho_p + 6\mathfrak{x}_h\varrho_q - \mathfrak{x}_g c}{-\kappa\varepsilon_f\varphi_d} - \frac{\hbar_c\varphi_d - 2\varepsilon_f\varphi_c\varrho_e}{-\varepsilon_f\varphi_d}\left\{2\mathfrak{x}_l + \frac{4\kappa\mathfrak{x}_k\hbar_c - 2\varrho_a\varepsilon_f\varphi_f}{\kappa\varepsilon_f}\right\} \\
&\quad - \frac{\varepsilon_f[\kappa(-4\mathfrak{x}_l\hbar_c + 4\mathfrak{x}_k\varrho_a\varepsilon_f + 2\mathfrak{x}_k\varrho_a\varepsilon_f) + 2\varrho_d\varphi_f] + \hbar_c(4\kappa\mathfrak{x}_k\hbar_c - 2\varrho_a\varepsilon_f\varphi_f)}{\kappa\varepsilon_f^2} \\
&\quad + \frac{16\kappa\varrho_e\mathfrak{x}_k\hbar_c\varphi_c - 8\varrho_e\varrho_a\varepsilon_f\varphi_f\varphi_c - 8\hbar_c\varrho_e^2\varphi_f + 12\hbar_c\varrho_f\varphi_f\varphi_c - 4\varrho_e^2\hbar_c\varphi_f}{-\kappa\varepsilon_f\varphi_d} \\
&\quad + \frac{8\kappa\varrho_e\mathfrak{x}_l\varepsilon_f\varphi_c - 12\kappa\varrho_e^2\mathfrak{x}_k\varepsilon_f - 12\varrho_e\varrho_f\varphi_f\varepsilon_f + 12\kappa\varrho_f\mathfrak{x}_k\varepsilon_f\varphi_c + 4\varrho_h\varphi_f\varepsilon_f\varphi_c}{-\kappa\varepsilon_f\varphi_d} \text{ by (1.2b) \& (6.1b)}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\left[-\frac{\rho\varrho_r - \mathfrak{x}_j\varphi_a\varepsilon_a + 3\mathfrak{x}_i\varrho_p + 3\mathfrak{x}_h\varrho_q - \frac{1}{2}\mathfrak{x}_g c}{\kappa\varepsilon_f\varphi_d} + \frac{\hbar_c\varphi_d - 2\varepsilon_f\varphi_c\varrho_e}{\varepsilon_f\varphi_d}\left\{\mathfrak{x}_l + \frac{2\kappa\mathfrak{x}_k\hbar_c - \varrho_a\varepsilon_f\varphi_f}{\kappa\varepsilon_f}\right\}\right. \\
&\quad - \frac{\varepsilon_f[\kappa(-2\mathfrak{x}_l\hbar_c + 2\mathfrak{x}_k\varrho_a\varepsilon_f + \mathfrak{x}_k\varrho_a\varepsilon_f) + \varrho_d\varphi_f] + \hbar_c(2\kappa\mathfrak{x}_k\hbar_c - \varrho_a\varepsilon_f\varphi_f)}{\kappa\varepsilon_f^2} \\
&\quad - \frac{8\kappa\varrho_e\mathfrak{x}_k\hbar_c\varphi_c - 4\varrho_e\varrho_a\varepsilon_f\varphi_f\varphi_c - 4\hbar_c\varrho_e^2\varphi_f + 6\hbar_c\varrho_f\varphi_f\varphi_c - 2\varrho_e^2\hbar_c\varphi_f}{\kappa\varepsilon_f\varphi_d} \\
&\quad \left. - \frac{4\kappa\varrho_e\mathfrak{x}_l\varepsilon_f\varphi_c - 6\kappa\varrho_e^2\mathfrak{x}_k\varepsilon_f - 6\varrho_e\varrho_f\varphi_f\varepsilon_f + 6\kappa\varrho_f\mathfrak{x}_k\varepsilon_f\varphi_c + 2\varrho_h\varphi_f\varepsilon_f\varphi_c}{\kappa\varepsilon_f\varphi_d}\right] \\
&= 2\mathfrak{x}_m \text{ by (6.21f).} \qquad (6.33c)
\end{aligned}
$$

In consequence of the foregoing derivations, we obtain

$$
\begin{aligned}
\dot{f}_1 &= (1/2)[(\boldsymbol{\kappa}\cdot\widehat{\mathbf{p}})f_0]' \text{ by (1.5d)} \\
&= (\kappa/2)(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})\dot{f}_0 \\
&= (\kappa/2)(\varepsilon_b)(-2\mathfrak{x}_k) \text{ by (6.1a) \& (6.33a)} \\
&= -\kappa\varepsilon_b\mathfrak{x}_k \qquad (6.34a)
\end{aligned}
$$

$$\begin{aligned}\ddot{f}_1 &= (\kappa/2)(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})\ddot{f}_0 \text{ by (6.34a)}\\ &= (\kappa/2)\varepsilon_b(2\mathfrak{x}_l) \text{ by (6.1a) \& (6.33b)}\\ &= \kappa\varepsilon_b\mathfrak{x}_l \end{aligned}\tag{6.34b}$$

$$\begin{aligned}\dddot{f}_1 &= (\kappa/2)(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})\dddot{f}_0 \text{ by (6.34b)}\\ &= (\kappa/2)\varepsilon_b(2\mathfrak{x}_m) \text{ by (6.1a) \& (6.33c)}\\ &= \kappa\varepsilon_b\mathfrak{x}_m \end{aligned}\tag{6.34c}$$

$$\begin{aligned}\dot{f}_2 &= (1/2)(mf_0)' \text{ by (1.5d)}\\ &= (1/2)(\dot{m}f_0 + m\dot{f}_0)\\ &= (1/2)[(-\kappa\hbar_a)(2\varphi_f/\kappa) + (\kappa\varepsilon_e)(-2\mathfrak{x}_k)] \text{ by (6.22a), (6.2f), (6.2b) \& (6.33a)}\\ &= -\hbar_a\varphi_f - \kappa\varepsilon_e\mathfrak{x}_k = \mathfrak{x}_n \text{ by (6.21g)}\end{aligned}\tag{6.35a}$$

$$\begin{aligned}\ddot{f}_2 &= (1/2)(\dot{m}f_0 + m\dot{f}_0)' \text{ by (6.35a)}\\ &= (1/2)(\ddot{m}f_0 + \dot{m}\dot{f}_0 + \dot{m}\dot{f}_0 + m\ddot{f}_0)\\ &= (1/2)(\ddot{m}f_0 + 2\dot{m}\dot{f}_0 + m\ddot{f}_0)\\ &= (1/2)[(-\kappa\varrho_a\varepsilon_e)(2\varphi_f/\kappa) + 2(-\kappa\hbar_a)(-2\mathfrak{x}_k) + (\kappa\varepsilon_e)(2\mathfrak{x}_l)] \text{ by (6.22), (6.2) \& (6.33)}\\ &= -\varrho_a\varepsilon_e\varphi_f + \kappa(2\hbar_a\mathfrak{x}_k + \varepsilon_e\mathfrak{x}_l) = \mathfrak{x}_o \text{ by (6.21g)}\end{aligned}\tag{6.35b}$$

$$\begin{aligned}\dddot{f}_2 &= (1/2)(\ddot{m}f_0 + 2\dot{m}\dot{f}_0 + m\ddot{f}_0)' \text{ by (6.35b)}\\ &= (1/2)(\dddot{m}f_0 + \ddot{m}\dot{f}_0 + 2\ddot{m}\dot{f}_0 + 2\dot{m}\ddot{f}_0 + \dot{m}\ddot{f}_0 + m\dddot{f}_0)\\ &= (1/2)(\dddot{m}f_0 + 3\ddot{m}\dot{f}_0 + 3\dot{m}\ddot{f}_0 + m\dddot{f}_0)\\ &= (1/2)[(\kappa\varrho_c)(2\varphi_f/\kappa) + 3(-\kappa\varrho_a\varepsilon_e)(-2\mathfrak{x}_k) + 3(-\kappa\hbar_a)(2\mathfrak{x}_l) + (\kappa\varepsilon_e)(2\mathfrak{x}_m)]\\ &\quad \text{by (6.22), (6.2) \& (6.33)}\\ &= \varrho_c\varphi_f + \kappa(3\varrho_a\varepsilon_e\mathfrak{x}_k - 3\hbar_a\mathfrak{x}_l + \varepsilon_e\mathfrak{x}_m) = \mathfrak{x}_q \text{ by (6.21g).}\end{aligned}\tag{6.35c}$$

The derivatives of $\mathbf{e}$ are computed as

$$\begin{aligned}\dot{\mathbf{e}} &= [f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}]' \text{ by (1.5b)}\\ &= \dot{f}_1(\mathbf{r}\times\mathbf{h}) + f_1(\mathbf{u}\times\mathbf{h}) + \dot{f}_2\widehat{\mathbf{p}}\\ &= \dot{f}_1(\mathbf{r}\times\mathbf{h}) - f_1(\mathbf{z} + q\widehat{\mathbf{r}}) + \dot{f}_2\widehat{\mathbf{p}} \text{ by (1.5c)}\\ &= (-\kappa\varepsilon_b\mathfrak{x}_k)(\mathbf{r}\times\mathbf{h}) - (\varepsilon_b\varphi_f)(\mathbf{z} + q\widehat{\mathbf{r}}) + (\mathfrak{x}_n)\widehat{\mathbf{p}} \text{ by (6.34a), (6.2g) \& (6.35a)}\\ &= -\kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h}) - \varepsilon_b\varphi_f(\mathbf{z} + q\widehat{\mathbf{r}}) + \mathfrak{x}_n\widehat{\mathbf{p}}\\ &= \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - q\varepsilon_b\varphi_f\widehat{\mathbf{r}} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})\end{aligned}\tag{6.36a}$$

$$\begin{aligned}\ddot{\mathbf{e}} &= [\dot{f}_1(\mathbf{r}\times\mathbf{h}) + f_1(\mathbf{u}\times\mathbf{h}) + \dot{f}_2\widehat{\mathbf{p}}]' \text{ by (6.36a)}\\ &= \ddot{f}_1(\mathbf{r}\times\mathbf{h}) + \dot{f}_1(\mathbf{u}\times\mathbf{h}) + \dot{f}_1(\mathbf{u}\times\mathbf{h}) + f_1(\mathbf{a}\times\mathbf{h}) + \ddot{f}_2\widehat{\mathbf{p}}\\ &= \ddot{f}_1(\mathbf{r}\times\mathbf{h}) + 2\dot{f}_1(\mathbf{u}\times\mathbf{h}) + f_1(\mathbf{a}\times\mathbf{h}) + \ddot{f}_2\widehat{\mathbf{p}}\\ &= (\kappa\varepsilon_b\mathfrak{x}_l)(\mathbf{r}\times\mathbf{h}) + 2(-\kappa\varepsilon_b\mathfrak{x}_k)(\mathbf{u}\times\mathbf{h}) + (\varepsilon_b\varphi_f)(\mathbf{a}\times\mathbf{h}) + \mathfrak{x}_o\widehat{\mathbf{p}} \text{ by (6.34), (6.2g) \& (6.35b)}\\ &= \kappa\varepsilon_b\mathfrak{x}_l(\mathbf{r}\times\mathbf{h}) - 2\kappa\varepsilon_b\mathfrak{x}_k(-\mathbf{z} - q\widehat{\mathbf{r}}) + \varepsilon_b\varphi_f[(-q\mathbf{r}/r^3)\times\mathbf{h}] + \mathfrak{x}_o\widehat{\mathbf{p}} \text{ by (1.5a) \& (1.5c)}\\ &= \kappa\varepsilon_b\mathfrak{x}_l(\mathbf{r}\times\mathbf{h}) + 2\kappa\varepsilon_b\mathfrak{x}_k(\mathbf{z} + q\widehat{\mathbf{r}}) - \varepsilon_b\varphi_f\varrho_a(\mathbf{r}\times\mathbf{h}) + \mathfrak{x}_o\widehat{\mathbf{p}} \text{ by (6.21b)}\\ &= \mathfrak{x}_o\widehat{\mathbf{p}} + 2\kappa\varepsilon_b\mathfrak{x}_k\mathbf{z} + 2\kappa q\varepsilon_b\mathfrak{x}_k\widehat{\mathbf{r}} + \varepsilon_b(\kappa\mathfrak{x}_l - \varphi_f\varrho_a)(\mathbf{r}\times\mathbf{h})\\ &= \mathfrak{x}_o\widehat{\mathbf{p}} + 2\kappa\varepsilon_b\mathfrak{x}_k\mathbf{z} + 2\kappa q\varepsilon_b\mathfrak{x}_k\widehat{\mathbf{r}} + \mathfrak{x}_p(\mathbf{r}\times\mathbf{h}) \text{ by (6.21g)}\end{aligned}\tag{6.36b}$$

$$
\begin{aligned}
\dddot{\mathbf{e}} &= [\ddot{f}_1(\mathbf{r}\times\mathbf{h}) + 2\dot{f}_1(\mathbf{u}\times\mathbf{h}) + f_1(\mathbf{a}\times\mathbf{h}) + \ddot{f}_2\widehat{\mathbf{p}}]' \text{ by (6.36b)}\\
&= \dddot{f}_1(\mathbf{r}\times\mathbf{h}) + \ddot{f}_1(\mathbf{u}\times\mathbf{h}) + 2\ddot{f}_1(\mathbf{u}\times\mathbf{h}) + 2\dot{f}_1(\mathbf{a}\times\mathbf{h}) + \dot{f}_1(\mathbf{a}\times\mathbf{h}) + f_1(\dot{\mathbf{a}}\times\mathbf{h}) + \dddot{f}_2\widehat{\mathbf{p}}\\
&= \dddot{f}_1(\mathbf{r}\times\mathbf{h}) + 3\ddot{f}_1(\mathbf{u}\times\mathbf{h}) + 3\dot{f}_1(\mathbf{a}\times\mathbf{h}) + f_1(\dot{\mathbf{a}}\times\mathbf{h}) + \dddot{f}_2\widehat{\mathbf{p}}\\
&= \dddot{f}_1(\mathbf{r}\times\mathbf{h}) + 3\ddot{f}_1(\mathbf{u}\times\mathbf{h}) + 3\dot{f}_1[(-\varrho_a\mathbf{r})\times\mathbf{h}] + f_1[(-\varrho_a\mathbf{u} + 3\varrho_a\varrho_b\mathbf{r})\times\mathbf{h}] + \dddot{f}_2\widehat{\mathbf{p}}\\
&\quad \text{by (1.5a), (6.21b) \& (6.26a)}
\end{aligned}
$$

$$
\begin{aligned}
&= \dddot{f}_1(\mathbf{r}\times\mathbf{h}) + 3\ddot{f}_1(\mathbf{u}\times\mathbf{h}) - 3\dot{f}_1\varrho_a(\mathbf{r}\times\mathbf{h}) + f_1[-\varrho_a(\mathbf{u}\times\mathbf{h}) + 3\varrho_a\varrho_b(\mathbf{r}\times\mathbf{h})] + \dddot{f}_2\widehat{\mathbf{p}}\\
&= (\dddot{f}_1 - 3\dot{f}_1\varrho_a + 3\varrho_a\varrho_b f_1)(\mathbf{r}\times\mathbf{h}) + (3\ddot{f}_1 - \varrho_a f_1)(\mathbf{u}\times\mathbf{h}) + \dddot{f}_2\widehat{\mathbf{p}}\\
&= [(\kappa\varepsilon_b\mathfrak{x}_m) - 3(-\kappa\varepsilon_b\mathfrak{x}_k)\varrho_a + 3\varrho_a\varrho_b(\varepsilon_b\varphi_f)](\mathbf{r}\times\mathbf{h}) + [3(\kappa\varepsilon_b\mathfrak{x}_l) - \varrho_a(\varepsilon_b\varphi_f)](\mathbf{u}\times\mathbf{h}) + \mathfrak{x}_q\widehat{\mathbf{p}}\\
&\quad \text{by (6.34), (6.2g) \& (6.35c)}\\
&= \varepsilon_b(\kappa\mathfrak{x}_m + 3\kappa\mathfrak{x}_k\varrho_a + 3\varrho_a\varrho_b\varphi_f)(\mathbf{r}\times\mathbf{h}) + \varepsilon_b(3\kappa\mathfrak{x}_l - \varrho_a\varphi_f)(\mathbf{u}\times\mathbf{h}) + \mathfrak{x}_q\widehat{\mathbf{p}}\\
&= \mathfrak{x}_r(\mathbf{r}\times\mathbf{h}) + \mathfrak{x}_s(-\mathbf{z} - q\widehat{\mathbf{r}}) + \mathfrak{x}_q\widehat{\mathbf{p}} \text{ by (6.21g) \& (1.5c)}\\
&= \mathfrak{x}_q\widehat{\mathbf{p}} - \mathfrak{x}_s\mathbf{z} - q\mathfrak{x}_s\widehat{\mathbf{r}} + \mathfrak{x}_r(\mathbf{r}\times\mathbf{h}) \qquad (6.36c)
\end{aligned}
$$

$$
\begin{aligned}
\dot{\tau} &= [(2\rho\alpha - n\mho^2 f_0)\kappa^{-2}]' \text{ by (1.5b)}\\
&= \kappa^{-2}(2\dot{\rho}\alpha + 2\rho\dot{\alpha} - \dot{n}\mho^2 f_0 - 2n\mho\dot{\mho}f_0 - n\mho^2\dot{f}_0)\\
&= \kappa^{-2}[2\mathfrak{x}_h\alpha + 2\rho(\kappa\varrho_p) - (-\hbar_c)\mho^2 f_0 - 2n\mho(\kappa\varrho_e)f_0 - n\mho^2(-2\mathfrak{x}_k)]\\
&\quad \text{by (6.32a), (6.28a), (6.23a), (6.24a) \& (6.33a)}\\
&= \kappa^{-2}(2\mathfrak{x}_h\alpha + 2\kappa\rho\varrho_p + \hbar_c\mho^2 f_0 - 2\kappa\varrho_e n\mho f_0 + 2\mathfrak{x}_k n\mho^2)\\
&= \kappa^{-2}[2\mathfrak{x}_h(-\kappa\varphi_a\varepsilon_a) + 2\kappa\rho\varrho_p + \hbar_c(\kappa\varphi_c)^2(2\varphi_f/\kappa) - 2\kappa\varrho_e\varepsilon_f(\kappa\varphi_c)(2\varphi_f/\kappa)\\
&\qquad + 2\mathfrak{x}_k\varepsilon_f(\kappa\varphi_c)^2] \text{ by (6.2)}\\
&= \kappa^{-2}(-2\kappa\mathfrak{x}_h\varphi_a\varepsilon_a + 2\kappa\rho\varrho_p + 2\kappa\hbar_c\varphi_c^2\varphi_f - 4\kappa\varrho_e\varepsilon_f\varphi_c\varphi_f + 2\kappa^2\mathfrak{x}_k\varepsilon_f\varphi_c^2)\\
&= 2\kappa^{-1}(\rho\varrho_p - \mathfrak{x}_h\varphi_a\varepsilon_a + \hbar_c\varphi_c^2\varphi_f - 2\varrho_e\varepsilon_f\varphi_c\varphi_f + \kappa\mathfrak{x}_k\varepsilon_f\varphi_c^2) = 2\mathfrak{x}_t/\kappa \text{ by (6.21g)} \qquad (6.37a)
\end{aligned}
$$

$$
\begin{aligned}
\ddot{\tau} &= \kappa^{-2}(2\dot{\rho}\alpha + 2\rho\dot{\alpha} - \dot{n}\mho^2 f_0 - 2n\mho\dot{\mho}f_0 - n\mho^2\dot{f}_0)' \text{ by (6.37a)}\\
&= \kappa^{-2}[2\ddot{\rho}\alpha + 2\dot{\rho}\dot{\alpha} + 2\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} - \ddot{n}\mho^2 f_0 - 2\dot{n}\mho\dot{\mho}f_0 - \dot{n}\mho^2\dot{f}_0 - 2\dot{n}\mho\dot{\mho}f_0 - 2n\dot{\mho}^2 f_0\\
&\qquad - 2n\mho\ddot{\mho}f_0 - 2n\mho\dot{\mho}\dot{f}_0 - \dot{n}\mho^2\dot{f}_0 - 2n\mho\dot{\mho}\dot{f}_0 - n\mho^2\ddot{f}_0]\\
&= \kappa^{-2}(2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} - \ddot{n}\mho^2 f_0 - 4\dot{n}\mho\dot{\mho}f_0 - 2\dot{n}\mho^2\dot{f}_0 - 2n\dot{\mho}^2 f_0\\
&\qquad - 2n\mho\ddot{\mho}f_0 - 4n\mho\dot{\mho}\dot{f}_0 - n\mho^2\ddot{f}_0)
\end{aligned}
$$

$$
\begin{aligned}
&= \kappa^{-2}[2\mathfrak{x}_i\alpha + 4\mathfrak{x}_h(\kappa\varrho_p) + 2\rho(\kappa\varrho_q) - (-\varrho_a\varepsilon_f)\mho^2 f_0 - 4(-\hbar_c)\mho(\kappa\varrho_e)f_0 - 2(-\hbar_c)\mho^2(-2\mathfrak{x}_k)\\
&\quad - 2n(\kappa\varrho_e)^2 f_0 - 2n\mho(-\kappa\varrho_f)f_0 - 4n\mho(\kappa\varrho_e)(-2\mathfrak{x}_k) - n\mho^2(2\mathfrak{x}_l)]\\
&\quad \text{by (6.32), (6.28), (6.23), (6.24) \& (6.33)}\\
&= \kappa^{-2}(2\mathfrak{x}_i\alpha + 4\kappa\mathfrak{x}_h\varrho_p + 2\kappa\rho\varrho_q + \varrho_a\varepsilon_f\mho^2 f_0 + 4\kappa\hbar_c\varrho_e\mho f_0 - 4\hbar_c\mathfrak{x}_k\mho^2\\
&\quad - 2\kappa^2\varrho_e^2 n f_0 + 2\kappa\varrho_f n\mho f_0 + 8\kappa\varrho_e\mathfrak{x}_k n\mho - 2\mathfrak{x}_l n\mho^2)\\
&= \kappa^{-2}[2\mathfrak{x}_i(-\kappa\varphi_a\varepsilon_a) + 4\kappa\mathfrak{x}_h\varrho_p + 2\kappa\rho\varrho_q + \varrho_a\varepsilon_f(\kappa\varphi_c)^2(2\varphi_f/\kappa) + 4\kappa\hbar_c\varrho_e(\kappa\varphi_c)(2\varphi_f/\kappa)\\
&\quad - 4\hbar_c\mathfrak{x}_k(\kappa\varphi_c)^2 - 2\kappa^2\varrho_e^2\varepsilon_f(2\varphi_f/\kappa) + 2\kappa\varrho_f\varepsilon_f(\kappa\varphi_c)(2\varphi_f/\kappa) + 8\kappa\varrho_e\mathfrak{x}_k\varepsilon_f(\kappa\varphi_c)\\
&\quad - 2\mathfrak{x}_l\varepsilon_f(\kappa\varphi_c)^2] \text{ by (6.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= 2\kappa^{-1}(-\varepsilon_a\varphi_a\mathfrak{x}_i + 2\mathfrak{x}_h\varrho_p + \rho\varrho_q + \varrho_a\varepsilon_f\varphi_c^2\varphi_f + 4\hbar_c\varrho_e\varphi_c\varphi_f - 2\kappa\hbar_c\mathfrak{x}_k\varphi_c^2 \\
&\quad - 2\varrho_e^2\varepsilon_f\varphi_f + 2\varrho_f\varepsilon_f\varphi_c\varphi_f + 4\kappa\varrho_e\mathfrak{x}_k\varepsilon_f\varphi_c - \kappa\varepsilon_f\mathfrak{x}_l\varphi_c^2) \\
&= 2\mathfrak{x}_u/\kappa \text{ by (6.21g)}
\end{aligned}
\tag{6.37b}
$$

$$
\begin{aligned}
\dddot{\tau} &= \kappa^{-2}[(2\ddot{\rho}\alpha + 4\dot{\rho}\dot{\alpha} + 2\rho\ddot{\alpha} - \ddot{n}\mho^2 f_0 - 4\dot{n}\mho\dot{\mho}f_0 - 2\dot{n}\mho^2\dot{f}_0 - 2n\dot{\mho}^2 f_0)' \\
&\quad + (-2n\mho\ddot{\mho}f_0 - 4n\mho\dot{\mho}\dot{f}_0 - n\mho^2\ddot{f}_0)' \text{ by (6.37b)} \\
&= \kappa^{-2}[2\dddot{\rho}\alpha + 2\ddot{\rho}\dot{\alpha} + 4\ddot{\rho}\dot{\alpha} + 4\dot{\rho}\ddot{\alpha} + 2\dot{\rho}\ddot{\alpha} + 2\rho\dddot{\alpha} - \dddot{n}\mho^2 f_0 - 2\ddot{n}\mho\dot{\mho}f_0 - \ddot{n}\mho^2\dot{f}_0 \\
&\quad - 4\ddot{n}\mho\dot{\mho}f_0 - 4\dot{n}\dot{\mho}^2 f_0 - 4\dot{n}\mho\ddot{\mho}f_0 - 4\dot{n}\mho\dot{\mho}\dot{f}_0 - 2\ddot{n}\mho^2\dot{f}_0 - 4\dot{n}\mho\dot{\mho}\dot{f}_0 - 2\dot{n}\mho^2\ddot{f}_0 \\
&\quad - 2\dot{n}\dot{\mho}^2 f_0 - 4n\dot{\mho}\ddot{\mho}f_0 - 2n\dot{\mho}^2\dot{f}_0 - 2\dot{n}\mho\ddot{\mho}f_0 - 2n\dot{\mho}\ddot{\mho}f_0 - 2n\mho\dddot{\mho}f_0 - 2n\mho\ddot{\mho}\dot{f}_0 \\
&\quad - 4\dot{n}\mho\dot{\mho}\dot{f}_0 - 4n\dot{\mho}^2\dot{f}_0 - 4n\mho\ddot{\mho}\dot{f}_0 - 4n\mho\dot{\mho}\ddot{f}_0 - \dot{n}\mho^2\ddot{f}_0 - 2n\mho\dot{\mho}\ddot{f}_0 - n\mho^2\dddot{f}_0] \\
&= \kappa^{-2}[2\dddot{\rho}\alpha + 6\ddot{\rho}\dot{\alpha} + 6\dot{\rho}\ddot{\alpha} + 2\rho\dddot{\alpha} - \dddot{n}\mho^2 f_0 - \ddot{n}\mho^2\dot{f}_0 - 6\ddot{n}\mho\dot{\mho}f_0 - 6\dot{n}\dot{\mho}^2 f_0 \\
&\quad - 6\dot{n}\mho\ddot{\mho}f_0 - 12\dot{n}\mho\dot{\mho}\dot{f}_0 - 2\ddot{n}\mho^2\dot{f}_0 - 3\dot{n}\mho^2\ddot{f}_0 - 4n\dot{\mho}\ddot{\mho}f_0 - 6n\dot{\mho}^2\dot{f}_0 \\
&\quad - 2n\dot{\mho}\ddot{\mho}f_0 - 2n\mho\dddot{\mho}f_0 - 6n\mho\ddot{\mho}\dot{f}_0 - 6n\mho\dot{\mho}\ddot{f}_0 - n\mho^2\dddot{f}_0]
\end{aligned}
$$

$$
\begin{aligned}
&= \kappa^{-2}[2\mathfrak{x}_j\alpha + 6\mathfrak{x}_i(\kappa\varrho_p) + 6\mathfrak{x}_h(\kappa\varrho_q) + 2\rho(\kappa\varrho_r) - \varrho_d\mho^2 f_0 - (-\varrho_a\varepsilon_f)\mho^2(-2\mathfrak{x}_k) \\
&\quad - 6(-\varrho_a\varepsilon_f)\mho(\kappa\varrho_e)f_0 - 6(-\hbar_c)(\kappa\varrho_e)^2 f_0 - 6(-\hbar_c)\mho(-\kappa\varrho_f)f_0 - 12(-\hbar_c)\mho(\kappa\varrho_e)(-2\mathfrak{x}_k) \\
&\quad - 2(-\varrho_a\varepsilon_f)\mho^2(-2\mathfrak{x}_k) - 3(-\hbar_c)\mho^2(2\mathfrak{x}_l) - 4n(\kappa\varrho_e)(-\kappa\varrho_f)f_0 \\
&\quad - 6n(\kappa\varrho_e)^2(-2\mathfrak{x}_k) - 2n(\kappa\varrho_e)(-\kappa\varrho_f)f_0 - 2n\mho(\kappa\varrho_h)f_0 - 6n\mho(-\kappa\varrho_f)(-2\mathfrak{x}_k) \\
&\quad - 6n\mho(\kappa\varrho_e)(2\mathfrak{x}_l) - n\mho^2(2\mathfrak{x}_m)] \text{ by (6.32), (6.28), (6.23), (6.24) \& (6.33)} \\
&= \kappa^{-2}[2\mathfrak{x}_j\alpha + 6\kappa\varrho_p\mathfrak{x}_i + 6\kappa\varrho_q\mathfrak{x}_h + 2\kappa\varrho_r\rho - \varrho_d\mho^2 f_0 - 2\mathfrak{x}_k\varrho_a\varepsilon_f\mho^2 + 6\kappa\varrho_e\varrho_a\varepsilon_f\mho f_0 \\
&\quad + 6\kappa^2\varrho_e^2\hbar_c f_0 - 6\kappa\varrho_f\hbar_c\mho f_0 - 24\kappa\varrho_e\mathfrak{x}_k\hbar_c\mho - 4\mathfrak{x}_k\varrho_a\varepsilon_f\mho^2 + 6\mathfrak{x}_l\hbar_c\mho^2 \\
&\quad + 4\kappa^2\varrho_e\varrho_f n f_0 + 12\kappa^2\varrho_e^2\mathfrak{x}_k n + 2\kappa^2\varrho_e\varrho_f n f_0 - 2\kappa\varrho_h n\mho f_0 - 12\kappa\varrho_f\mathfrak{x}_k n\mho \\
&\quad - 12\kappa\varrho_e\mathfrak{x}_l n\mho - 2\mathfrak{x}_m n\mho^2]
\end{aligned}
$$

$$
\begin{aligned}
&= \kappa^{-2}[2\mathfrak{x}_j(-\kappa\varphi_a\varepsilon_a) + 6\kappa\varrho_p\mathfrak{x}_i + 6\kappa\varrho_q\mathfrak{x}_h + 2\kappa\varrho_r\rho - \varrho_d(\kappa\varphi_c)^2(2\varphi_f/\kappa) - 2\mathfrak{x}_k\varrho_a\varepsilon_f(\kappa\varphi_c)^2 \\
&\quad + 6\kappa\varrho_e\varrho_a\varepsilon_f(\kappa\varphi_c)(2\varphi_f/\kappa) + 6\kappa^2\varrho_e^2\hbar_c(2\varphi_f/\kappa) - 6\kappa\varrho_f\hbar_c(\kappa\varphi_c)(2\varphi_f/\kappa) - 24\kappa\varrho_e\mathfrak{x}_k\hbar_c(\kappa\varphi_c) \\
&\quad - 4\mathfrak{x}_k\varrho_a\varepsilon_f(\kappa\varphi_c)^2 + 6\mathfrak{x}_l\hbar_c(\kappa\varphi_c)^2 + 4\kappa^2\varrho_e\varrho_f\varepsilon_f(2\varphi_f/\kappa) + 12\kappa^2\varrho_e^2\mathfrak{x}_k\varepsilon_f \\
&\quad + 2\kappa^2\varrho_e\varrho_f\varepsilon_f(2\varphi_f/\kappa) - 2\kappa\varrho_h\varepsilon_f(\kappa\varphi_c)(2\varphi_f/\kappa) - 12\kappa\varrho_f\mathfrak{x}_k\varepsilon_f(\kappa\varphi_c) \\
&\quad - 12\kappa\varrho_e\mathfrak{x}_l\varepsilon_f(\kappa\varphi_c) - 2\mathfrak{x}_m\varepsilon_f(\kappa\varphi_c)^2] \text{ by (6.2)} \\
&= \kappa^{-2}[-2\kappa\varphi_a\varepsilon_a\mathfrak{x}_j + 6\kappa\varrho_p\mathfrak{x}_i + 6\kappa\varrho_q\mathfrak{x}_h + 2\kappa\varrho_r\rho - 2\varrho_d\kappa\varphi_c^2\varphi_f - 2\kappa^2\varepsilon_f\mathfrak{x}_k\varrho_a\varphi_c^2 \\
&\quad + 12\kappa\varrho_e\varrho_a\varepsilon_f\varphi_c\varphi_f + 12\kappa\varrho_e^2\hbar_c\varphi_f - 12\kappa\varrho_f\hbar_c\varphi_c\varphi_f - 24\kappa^2\varrho_e\mathfrak{x}_k\hbar_c\varphi_c \\
&\quad - 4\kappa^2\mathfrak{x}_k\varrho_a\varepsilon_f\varphi_c^2 + 6\kappa^2\hbar_c\mathfrak{x}_l\varphi_c^2 + 8\kappa\varrho_e\varrho_f\varepsilon_f\varphi_f + 12\kappa^2\varrho_e^2\mathfrak{x}_k\varepsilon_f \\
&\quad + 4\kappa\varrho_e\varrho_f\varepsilon_f\varphi_f - 4\kappa\varrho_h\varepsilon_f\varphi_c\varphi_f - 12\kappa^2\varrho_f\mathfrak{x}_k\varepsilon_f\varphi_c - 12\kappa^2\varrho_e\mathfrak{x}_l\varepsilon_f\varphi_c - 2\kappa^2\varepsilon_f\mathfrak{x}_m\varphi_c^2]
\end{aligned}
$$

$$
\begin{aligned}
&= 2\kappa^{-1}[-\varphi_a\varepsilon_a\mathfrak{x}_j + 3\varrho_p\mathfrak{x}_i + 3\varrho_q\mathfrak{x}_h + \varrho_r\rho - \varrho_d\varphi_c^2\varphi_f - \kappa\varepsilon_f\mathfrak{x}_k\varrho_a\varphi_c^2 \\
&\quad + 6\varrho_e\varrho_a\varepsilon_f\varphi_c\varphi_f + 6\varrho_e^2\hbar_c\varphi_f - 6\varrho_f\hbar_c\varphi_c\varphi_f - 12\kappa\varrho_e\mathfrak{x}_k\hbar_c\varphi_c \\
&\quad - 2\kappa\mathfrak{x}_k\varrho_a\varepsilon_f\varphi_c^2 + 3\kappa\hbar_c\mathfrak{x}_l\varphi_c^2 + 4\varrho_e\varrho_f\varepsilon_f\varphi_f + 6\kappa\varrho_e^2\mathfrak{x}_k\varepsilon_f \\
&\quad + 2\varrho_e\varrho_f\varepsilon_f\varphi_f - 2\varrho_h\varepsilon_f\varphi_c\varphi_f - 6\kappa\varrho_f\mathfrak{x}_k\varepsilon_f\varphi_c - 6\kappa\varrho_e\mathfrak{x}_l\varepsilon_f\varphi_c - \kappa\varepsilon_f\mathfrak{x}_m\varphi_c^2] \\
&= 2\mathfrak{x}_v/\kappa \text{ by (6.21g).}
\end{aligned}
\tag{6.37c}
$$

Art 26e. *Development of equation* (3.15a).

We are now ready to evaluate the quantities defined by (3.15). We start with

$$\begin{aligned}\dot{\mathfrak{y}} &= c\dot{d} - \kappa\dot{\tau} \text{ by (3.15a)}\\ &= c\mathfrak{x}_a - \kappa(2\mathfrak{x}_t/\kappa) \text{ by (6.31a) \& (6.37a)}\\ &= c\mathfrak{x}_a - 2\mathfrak{x}_t = \mathfrak{y}_a \text{ by (6.21h)}\end{aligned} \tag{6.38a}$$

$$\begin{aligned}\ddot{\mathfrak{y}} &= c\ddot{d} - \kappa\ddot{\tau} \text{ by (3.15a)}\\ &= c\mathfrak{x}_e - \kappa(2\mathfrak{x}_u/\kappa) \text{ by (6.31b) \& (6.37b)}\\ &= c\mathfrak{x}_e - 2\mathfrak{x}_u = \mathfrak{y}_b \text{ by (6.21h)}\end{aligned} \tag{6.38b}$$

$$\begin{aligned}\dddot{\mathfrak{y}} &= c\dddot{d} - \kappa\dddot{\tau} \text{ by (3.15a)}\\ &= c\mathfrak{x}_g - \kappa(2\mathfrak{x}_v/\kappa) \text{ by (6.31c) \& (6.37c)}\\ &= c\mathfrak{x}_g - 2\mathfrak{x}_v = \mathfrak{y}_c \text{ by (6.21h)}\end{aligned} \tag{6.38c}$$

$$\begin{aligned}b_1 &= \dot{\rho} - 1 \text{ by (3.15a)}\\ &= \mathfrak{x}_h - 1 \text{ by (6.32a)}\\ &= \mathfrak{y}_d \text{ by (6.21h)}\end{aligned} \tag{6.39a}$$

$$\begin{aligned}b_2 &= 2\dot{\rho} - 1 \text{ by (3.15a)}\\ &= 2\mathfrak{x}_h - 1 \text{ by (6.32a)}\\ &= \mathfrak{y}_e \text{ by (6.21h)}\end{aligned} \tag{6.39b}$$

$$\begin{aligned}b_3 &= 3\dot{\rho} - 1 \text{ by (3.15a)}\\ &= 3\mathfrak{x}_h - 1 \text{ by (6.32a)}\\ &= \mathfrak{y}_f \text{ by (6.21h)}\end{aligned} \tag{6.39c}$$

$$\begin{aligned}b_4 &= \ddot{\rho}\dot{\mathfrak{y}} - \ddot{\mathfrak{y}}b_1 \text{ by (3.15a)}\\ &= \mathfrak{x}_i\mathfrak{y}_a - \mathfrak{y}_b\mathfrak{y}_d \text{ by (6.32b), (6.38) \& (6.39a)}\\ &= \mathfrak{y}_g \text{ by (6.21h)}\end{aligned} \tag{6.39d}$$

$$\begin{aligned}b_5 &= \dot{\mathfrak{y}}b_2 - \rho\ddot{\mathfrak{y}} \text{ by (3.15a)}\\ &= \mathfrak{y}_a\mathfrak{y}_e - \rho\mathfrak{y}_b \text{ by (6.38) \& (6.39b)}\\ &= \mathfrak{y}_h \text{ by (6.21h)}\end{aligned} \tag{6.39e}$$

$$\begin{aligned}b_6 &= b_1b_2 - \rho\ddot{\rho} \text{ by (3.15a)}\\ &= \mathfrak{y}_d\mathfrak{y}_e - \rho\mathfrak{x}_i \text{ by (6.32b), (6.39a) \& (6.39b)}\\ &= \mathfrak{y}_i \text{ by (6.21h).}\end{aligned} \tag{6.39f}$$

Art 26f. *Development of equation* (3.15b).

We have furthermore that

$$\begin{aligned}\mathfrak{J}_a &= \widehat{\boldsymbol{\kappa}} \times \mathbf{a} \text{ by (3.15b)}\\ &= \widehat{\boldsymbol{\kappa}} \times (-\varrho_a\mathbf{r}) \text{ by (1.5a) \& (6.21b)}\\ &= -\varrho_a(\widehat{\boldsymbol{\kappa}} \times \mathbf{r})\end{aligned} \tag{6.40a}$$

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_b &= \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}} \text{ by (3.15b)}\\ &= \widehat{\boldsymbol{\kappa}} \times (-\varrho_a \mathbf{u} + 3\varrho_a\varrho_b \mathbf{r}) \text{ by (6.26a)}\\ &= -\varrho_a(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) + 3\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}} \times \mathbf{r})\end{aligned} \tag{6.40b}$$

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_c &= \widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{a}} \text{ by (3.15b)}\\ &= \widehat{\boldsymbol{\kappa}} \times (6\varrho_a\varrho_b \mathbf{u} - \varrho_a\varrho_l \mathbf{r}) \text{ by (6.26b)}\\ &= 6\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) - \varrho_a\varrho_l(\widehat{\boldsymbol{\kappa}} \times \mathbf{r})\end{aligned} \tag{6.40c}$$

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_d &= \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{e}} \text{ by (3.15b)}\\ &= \widehat{\boldsymbol{\kappa}} \times [\mathfrak{x}_n \widehat{\mathbf{p}} - \varepsilon_b\varphi_f \mathbf{z} - q\varepsilon_b\varphi_f \widehat{\mathbf{r}} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r} \times \mathbf{h})] \text{ by (6.36a)}\\ &= \mathfrak{x}_n(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) - q\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) - \kappa\varepsilon_b\mathfrak{x}_k[\widehat{\boldsymbol{\kappa}} \times (\mathbf{r} \times \mathbf{h})]\\ &= \mathfrak{x}_n(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) - q\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) - \kappa\varepsilon_b\mathfrak{x}_k[\mathbf{r}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{h}) - \mathbf{h}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})] \text{ by (A.1)}\\ &= \mathfrak{x}_n(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) - q\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) - \kappa\varepsilon_b\mathfrak{x}_k(\delta_a \mathbf{r} - r\varepsilon_a \mathbf{h}) \text{ by (6.10a) \& (6.1a)}\\ &= \kappa r\varepsilon_a\varepsilon_b\mathfrak{x}_k \mathbf{h} - \kappa\varepsilon_b\mathfrak{x}_k\delta_a \mathbf{r} + \mathfrak{x}_n(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) - q\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}})\end{aligned} \tag{6.40d}$$

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_e &= \widehat{\boldsymbol{\kappa}} \times \ddot{\mathbf{e}} \text{ by (3.15b)}\\ &= \widehat{\boldsymbol{\kappa}} \times [\mathfrak{x}_o \widehat{\mathbf{p}} + 2\kappa\varepsilon_b\mathfrak{x}_k \mathbf{z} + 2\kappa q\varepsilon_b\mathfrak{x}_k \widehat{\mathbf{r}} + \mathfrak{x}_p(\mathbf{r} \times \mathbf{h})] \text{ by (6.36b)}\\ &= \mathfrak{x}_o(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + 2\kappa\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + 2\kappa q\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) + \mathfrak{x}_p[\widehat{\boldsymbol{\kappa}} \times (\mathbf{r} \times \mathbf{h})]\\ &= \mathfrak{x}_o(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + 2\kappa\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + 2\kappa q\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) + \mathfrak{x}_p[\mathbf{r}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{h}) - \mathbf{h}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})] \text{ by (A.1)}\\ &= \mathfrak{x}_o(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + 2\kappa\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + 2\kappa q\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) + \mathfrak{x}_p(\delta_a \mathbf{r} - r\varepsilon_a \mathbf{h}) \text{ by (6.10a) \& (6.1a)}\\ &= \mathfrak{x}_p\delta_a \mathbf{r} - r\mathfrak{x}_p\varepsilon_a \mathbf{h} + \mathfrak{x}_o(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + 2\kappa\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + 2\kappa q\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}})\end{aligned} \tag{6.40e}$$

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_f &= \mathbf{a} \times \dot{\mathbf{a}} \text{ by (3.15b)}\\ &= (-\varrho_a \mathbf{r}) \times (-\varrho_a \mathbf{u} + 3\varrho_a\varrho_b \mathbf{r}) \text{ by (1.5a), (6.21b) \& (6.26a)}\\ &= -\varrho_a^2(\mathbf{u} \times \mathbf{r}) = -\varrho_a^2 \mathbf{h} \text{ by (1.5c)}\end{aligned} \tag{6.40f}$$

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_g &= \mathbf{a} \times \ddot{\mathbf{a}} \text{ by (3.15b)}\\ &= (-\varrho_a \mathbf{r}) \times (6\varrho_a\varrho_b \mathbf{u} - \varrho_a\varrho_l \mathbf{r}) \text{ by (1.5a), (6.21b) \& (6.26b)}\\ &= 6\varrho_a^2\varrho_b(\mathbf{u} \times \mathbf{r}) = 6\varrho_a^2\varrho_b \mathbf{h} \text{ by (1.5c)}\end{aligned} \tag{6.40g}$$

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_h &= \mathbf{a} \times \dot{\mathbf{e}} \text{ by (3.15b)}\\ &= (-\varrho_a \mathbf{r}) \times [\mathfrak{x}_n \widehat{\mathbf{p}} - \varepsilon_b\varphi_f \mathbf{z} - q\varepsilon_b\varphi_f \widehat{\mathbf{r}} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r} \times \mathbf{h})] \text{ by (1.5a), (6.21b) \& (6.36a)}\\ &= -\varrho_a\mathfrak{x}_n(\mathbf{r} \times \widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) + \kappa\varrho_a\varepsilon_b\mathfrak{x}_k[\mathbf{r} \times (\mathbf{r} \times \mathbf{h})]\\ &= -\varrho_a\mathfrak{x}_n(\mathbf{r} \times \widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) + \kappa\varrho_a\varepsilon_b\mathfrak{x}_k[(\mathbf{r} \cdot \mathbf{h})\mathbf{r} - r^2\mathbf{h}] \text{ by (A.1)}\\ &= -\varrho_a\mathfrak{x}_n(\mathbf{r} \times \widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) - \kappa\varrho_a\varepsilon_b r^2\mathfrak{x}_k \mathbf{h} \text{ by (1.5c)}\\ &= -\varrho_a\mathfrak{x}_n(\mathbf{r} \times \widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) - \kappa\varphi_b\varepsilon_b\mathfrak{x}_k \mathbf{h} \text{ by (6.1b) \& (6.21b)}\end{aligned} \tag{6.40h}$$

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_i &= \mathbf{a} \times \ddot{\mathbf{e}} \text{ by (3.15b)}\\ &= (-\varrho_a \mathbf{r}) \times [\mathfrak{x}_o \widehat{\mathbf{p}} + 2\kappa\varepsilon_b\mathfrak{x}_k \mathbf{z} + 2\kappa q\varepsilon_b\mathfrak{x}_k \widehat{\mathbf{r}} + \mathfrak{x}_p(\mathbf{r} \times \mathbf{h})] \text{ by (1.5a), (6.21b) \& (6.36b)}\\ &= -\varrho_a\mathfrak{x}_o(\mathbf{r} \times \widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{r} \times \mathbf{z}) - \varrho_a\mathfrak{x}_p[\mathbf{r} \times (\mathbf{r} \times \mathbf{h})]\\ &= -\varrho_a\mathfrak{x}_o(\mathbf{r} \times \widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{r} \times \mathbf{z}) - \varrho_a\mathfrak{x}_p[(\mathbf{r} \cdot \mathbf{h})\mathbf{r} - r^2\mathbf{h}] \text{ by (A.1)}\\ &= -\varrho_a\mathfrak{x}_o(\mathbf{r} \times \widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{r} \times \mathbf{z}) + \varrho_a r^2\mathfrak{x}_p \mathbf{h} \text{ by (1.5c)}\\ &= -\varrho_a\mathfrak{x}_o(\mathbf{r} \times \widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{r} \times \mathbf{z}) + \varphi_b\mathfrak{x}_p \mathbf{h} \text{ by (6.1b) \& (6.21b)}\end{aligned} \tag{6.40i}$$

$$\begin{aligned}
\mathfrak{J}_j &= \dot{\mathbf{a}} \times \ddot{\mathbf{a}} \text{ by (3.15b)} \\
&= (-\varrho_a \mathbf{u} + 3\varrho_a\varrho_b \mathbf{r}) \times (6\varrho_a\varrho_b \mathbf{u} - \varrho_a\varrho_l \mathbf{r}) \text{ by (6.26)} \\
&= -6\varrho_a^2\varrho_b(\mathbf{u}\times\mathbf{u}) + \varrho_a^2\varrho_l(\mathbf{u}\times\mathbf{r}) + 18\varrho_a^2\varrho_b^2(\mathbf{r}\times\mathbf{u}) - 3\varrho_a^2\varrho_b\varrho_l(\mathbf{r}\times\mathbf{r}) \\
&= (\varrho_a^2\varrho_l - 18\varrho_a^2\varrho_b^2)(\mathbf{u}\times\mathbf{r}) = \varrho_a^2(\varrho_l - 18\varrho_b^2)\mathbf{h} \text{ by (1.5c)} \\
&= \mathfrak{y}_j \mathbf{h} \text{ by (6.21h)}
\end{aligned} \tag{6.40j}$$

$$\begin{aligned}
\mathfrak{J}_k &= \dot{\mathbf{a}} \times \dot{\mathbf{e}} \text{ by (3.15b)} \\
&= (-\varrho_a \mathbf{u} + 3\varrho_a\varrho_b \mathbf{r}) \times [\mathfrak{x}_n \widehat{\mathbf{p}} - \varepsilon_b\varphi_f \mathbf{z} - q\varepsilon_b\varphi_f \widehat{\mathbf{r}} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})] \text{ by (6.26a) \& (6.36a)} \\
&= -\varrho_a\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + q\varrho_a\varepsilon_b\varphi_f(\mathbf{u}\times\widehat{\mathbf{r}}) + \kappa\varrho_a\varepsilon_b\mathfrak{x}_k[\mathbf{u}\times(\mathbf{r}\times\mathbf{h})] \\
&\quad + 3\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - 3\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) - 3q\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{r}\times\widehat{\mathbf{r}}) - 3\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k[\mathbf{r}\times(\mathbf{r}\times\mathbf{h})] \\
&= -\varrho_a\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + q\varrho_a\varepsilon_b\varphi_f(\mathbf{u}\times\widehat{\mathbf{r}}) + \kappa\varrho_a\varepsilon_b\mathfrak{x}_k[(\mathbf{u}\cdot\mathbf{h})\mathbf{r} - (\mathbf{u}\cdot\mathbf{r})\mathbf{h}] \\
&\quad + 3\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - 3\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) - 3\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k[(\mathbf{r}\cdot\mathbf{h})\mathbf{r} - r^2\mathbf{h}] \text{ by (A.1)}
\end{aligned}$$

$$\begin{aligned}
&= -\varrho_a\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + (q/r)\varrho_a\varepsilon_b\varphi_f\mathbf{h} + \kappa\varrho_a\varepsilon_b\mathfrak{x}_k(-r^2\varrho_b\mathbf{h}) \\
&\quad + 3\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - 3\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) - 3\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(-r^2\mathbf{h}) \text{ by (1.5c) \& (6.27b)} \\
&= -\varrho_a\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + \varphi_b\varrho_a\varepsilon_b\varphi_f\mathbf{h} - \kappa\varphi_b\varrho_b\varepsilon_b\mathfrak{x}_k\mathbf{h} \\
&\quad + 3\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - 3\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) + 3\kappa\varphi_b\varrho_b\varepsilon_b\mathfrak{x}_k\mathbf{h} \text{ by (6.1b) \& (6.21b)} \\
&= -\varrho_a\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + 3\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - 3\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) \\
&\quad + \varphi_b\varepsilon_b(\varrho_a\varphi_f + 2\kappa\varrho_b\mathfrak{x}_k)\mathbf{h} \\
&= -\varrho_a\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + 3\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad - 3\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) + \mathfrak{y}_k\mathbf{h} \text{ by (6.21h)}
\end{aligned} \tag{6.40k}$$

$$\begin{aligned}
\mathfrak{J}_l &= \dot{\mathbf{a}} \times \ddot{\mathbf{e}} \text{ by (3.15b)} \\
&= (-\varrho_a \mathbf{u} + 3\varrho_a\varrho_b \mathbf{r}) \times [\mathfrak{x}_o\widehat{\mathbf{p}} + 2\kappa\varepsilon_b\mathfrak{x}_k\mathbf{z} + 2\kappa q\varepsilon_b\mathfrak{x}_k\widehat{\mathbf{r}} + \mathfrak{x}_p(\mathbf{r}\times\mathbf{h})] \text{ by (6.26a) \& (6.36b)} \\
&= -\varrho_a\mathfrak{x}_o(\mathbf{u}\times\widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\mathbf{z}) - 2\kappa q\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\widehat{\mathbf{r}}) - \varrho_a\mathfrak{x}_p[\mathbf{u}\times(\mathbf{r}\times\mathbf{h})] \\
&\quad + 3\varrho_a\varrho_b\mathfrak{x}_o(\mathbf{r}\times\widehat{\mathbf{p}}) + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{z}) + 6\kappa q\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\widehat{\mathbf{r}}) + 3\varrho_a\varrho_b\mathfrak{x}_p[\mathbf{r}\times(\mathbf{r}\times\mathbf{h})] \\
&= -\varrho_a\mathfrak{x}_o(\mathbf{u}\times\widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\mathbf{z}) - 2\kappa q\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\widehat{\mathbf{r}}) - \varrho_a\mathfrak{x}_p[(\mathbf{u}\cdot\mathbf{h})\mathbf{r} - (\mathbf{u}\cdot\mathbf{r})\mathbf{h}] \\
&\quad + 3\varrho_a\varrho_b\mathfrak{x}_o(\mathbf{r}\times\widehat{\mathbf{p}}) + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{z}) + 3\varrho_a\varrho_b\mathfrak{x}_p[(\mathbf{r}\cdot\mathbf{h})\mathbf{r} - r^2\mathbf{h}] \text{ by (A.1)}
\end{aligned}$$

$$\begin{aligned}
&= -\varrho_a\mathfrak{x}_o(\mathbf{u}\times\widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\mathbf{z}) - 2\kappa(q/r)\varrho_a\varepsilon_b\mathfrak{x}_k\mathbf{h} - \varrho_a\mathfrak{x}_p(-r^2\varrho_b\mathbf{h}) \\
&\quad + 3\varrho_a\varrho_b\mathfrak{x}_o(\mathbf{r}\times\widehat{\mathbf{p}}) + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{z}) + 3\varrho_a\varrho_b\mathfrak{x}_p(-r^2\mathbf{h}) \text{ by (1.5c) \& (6.27b)} \\
&= -\varrho_a\mathfrak{x}_o(\mathbf{u}\times\widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\mathbf{z}) - 2\kappa\varphi_b\varrho_a\varepsilon_b\mathfrak{x}_k\mathbf{h} + \varphi_b\mathfrak{x}_p\varrho_b\mathbf{h} \\
&\quad + 3\varrho_a\varrho_b\mathfrak{x}_o(\mathbf{r}\times\widehat{\mathbf{p}}) + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{z}) - 3\varphi_b\varrho_b\mathfrak{x}_p\mathbf{h} \text{ by (6.1b) \& (6.21b)} \\
&= -\varrho_a\mathfrak{x}_o(\mathbf{u}\times\widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\mathbf{z}) + 3\varrho_a\varrho_b\mathfrak{x}_o(\mathbf{r}\times\widehat{\mathbf{p}}) + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{z}) \\
&\quad - 2\varphi_b(\kappa\varrho_a\varepsilon_b\mathfrak{x}_k + \varrho_b\mathfrak{x}_p)\mathbf{h} \\
&= -\varrho_a\mathfrak{x}_o(\mathbf{u}\times\widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\mathbf{z}) + 3\varrho_a\varrho_b\mathfrak{x}_o(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{z}) - \mathfrak{y}_l\mathbf{h} \text{ by (6.21h)}
\end{aligned} \tag{6.40l}$$

$$\begin{aligned}
\mathfrak{J}_n &= \dot{\mathbf{e}} \times \ddot{\mathbf{a}} \text{ by (3.15b)} \\
&= [\mathfrak{x}_n \widehat{\mathbf{p}} - \varepsilon_b\varphi_f \mathbf{z} - q\varepsilon_b\varphi_f \widehat{\mathbf{r}} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})] \times (6\varrho_a\varrho_b\mathbf{u} - \varrho_a\varrho_l\mathbf{r}) \text{ by (6.36a) \& (6.26b)} \\
&= -6\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + 6\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + 6q\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\widehat{\mathbf{r}}) + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k[\mathbf{u}\times(\mathbf{r}\times\mathbf{h})] \\
&\quad + \varrho_a\varrho_l\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - \varepsilon_b\varrho_a\varrho_l\varphi_f(\mathbf{r}\times\mathbf{z}) - q\varepsilon_b\varrho_a\varrho_l\varphi_f(\mathbf{r}\times\widehat{\mathbf{r}}) - \kappa\varepsilon_b\varrho_a\varrho_l\mathfrak{x}_k[\mathbf{r}\times(\mathbf{r}\times\mathbf{h})] \\
&= -6\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + 6\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + 6q\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\widehat{\mathbf{r}}) + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k[(\mathbf{u}\cdot\mathbf{h})\mathbf{r} - (\mathbf{u}\cdot\mathbf{r})\mathbf{h}] \\
&\quad + \varrho_a\varrho_l\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - \varepsilon_b\varrho_a\varrho_l\varphi_f(\mathbf{r}\times\mathbf{z}) - \kappa\varepsilon_b\varrho_a\varrho_l\mathfrak{x}_k[(\mathbf{r}\cdot\mathbf{h})\mathbf{r} - r^2\mathbf{h}] \text{ by (A.1)}
\end{aligned}$$

$$\begin{aligned}
&= -6\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + 6\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + 6(q/r)\varrho_a\varrho_b\varepsilon_b\varphi_f\mathbf{h} + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(-r^2\varrho_b\mathbf{h})\\
&\quad + \varrho_a\varrho_l\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - \varepsilon_b\varrho_a\varrho_l\varphi_f(\mathbf{r}\times\mathbf{z}) - \kappa\varepsilon_b\varrho_a\varrho_l\mathfrak{x}_k(-r^2\mathbf{h}) \text{ by (1.5c) \& (6.27b)}\\
&= -6\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + 6\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + 6\varphi_b\varrho_a\varrho_b\varepsilon_b\varphi_f\mathbf{h} - 6\kappa\varphi_b\varrho_b^2\varepsilon_b\mathfrak{x}_k\mathbf{h}\\
&\quad + \varrho_a\varrho_l\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - \varepsilon_b\varrho_a\varrho_l\varphi_f(\mathbf{r}\times\mathbf{z}) + \kappa\varepsilon_b\varphi_b\varrho_l\mathfrak{x}_k\mathbf{h} \text{ by (6.1b) \& (6.21b)}\\
&= -6\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + 6\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + \varrho_a\varrho_l\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - \varepsilon_b\varrho_a\varrho_l\varphi_f(\mathbf{r}\times\mathbf{z})\\
&\quad + \varepsilon_b\varphi_b[6\varrho_a\varrho_b\varphi_f + \kappa\mathfrak{x}_k(\varrho_l - 6\varrho_b^2)]\mathbf{h}\\
&= -6\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + 6\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + \varrho_a\varrho_l\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad - \varepsilon_b\varrho_a\varrho_l\varphi_f(\mathbf{r}\times\mathbf{z}) + \mathfrak{y}_m\mathbf{h} \text{ by (6.21h)}
\end{aligned} \tag{6.40m}$$

$$\begin{aligned}
\mathfrak{J}_o &= \dot{\mathbf{e}}\times\ddot{\mathbf{e}} \text{ by (3.15b)}\\
&= [\mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - q\varepsilon_b\varphi_f\widehat{\mathbf{r}} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})]\times[\mathfrak{x}_o\widehat{\mathbf{p}} + 2\kappa\varepsilon_b\mathfrak{x}_k\mathbf{z} + 2\kappa q\varepsilon_b\mathfrak{x}_k\widehat{\mathbf{r}} + \mathfrak{x}_p(\mathbf{r}\times\mathbf{h})] \text{ by (6.36)}\\
&= 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + 2\kappa q\varepsilon_b\mathfrak{x}_k\mathfrak{x}_n(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) + \mathfrak{x}_p\mathfrak{x}_n[\widehat{\mathbf{p}}\times(\mathbf{r}\times\mathbf{h})] - \mathfrak{x}_o\varepsilon_b\varphi_f(\mathbf{z}\times\widehat{\mathbf{p}})\\
&\quad - 2\kappa q\varepsilon_b^2\mathfrak{x}_k\varphi_f(\mathbf{z}\times\widehat{\mathbf{r}}) - \mathfrak{x}_p\varepsilon_b\varphi_f[\mathbf{z}\times(\mathbf{r}\times\mathbf{h})] - \mathfrak{x}_o q\varepsilon_b\varphi_f(\widehat{\mathbf{r}}\times\widehat{\mathbf{p}})\\
&\quad - 2\kappa\varepsilon_b^2\mathfrak{x}_k q\varphi_f(\widehat{\mathbf{r}}\times\mathbf{z}) - \mathfrak{x}_p q\varepsilon_b\varphi_f[\widehat{\mathbf{r}}\times(\mathbf{r}\times\mathbf{h})] + \kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_o[\widehat{\mathbf{p}}\times(\mathbf{r}\times\mathbf{h})]\\
&\quad + 2\kappa^2\varepsilon_b^2\mathfrak{x}_k^2[\mathbf{z}\times(\mathbf{r}\times\mathbf{h})] + 2q\kappa^2\varepsilon_b^2\mathfrak{x}_k^2[\widehat{\mathbf{r}}\times(\mathbf{r}\times\mathbf{h})]\\
&= \varepsilon_b(2\kappa\mathfrak{x}_k\mathfrak{x}_n + \mathfrak{x}_o\varphi_f)(\widehat{\mathbf{p}}\times\mathbf{z}) + q\varepsilon_b(2\kappa\mathfrak{x}_k\mathfrak{x}_n + \mathfrak{x}_o\varphi_f)(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) + (\mathfrak{x}_p\mathfrak{x}_n + \kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_o)[\widehat{\mathbf{p}}\times(\mathbf{r}\times\mathbf{h})]\\
&\quad + \varepsilon_b(2\kappa^2\varepsilon_b\mathfrak{x}_k^2 - \mathfrak{x}_p\varphi_f)[\mathbf{z}\times(\mathbf{r}\times\mathbf{h})] + q\varepsilon_b(2\kappa^2\varepsilon_b\mathfrak{x}_k^2 - \mathfrak{x}_p\varphi_f)[\widehat{\mathbf{r}}\times(\mathbf{r}\times\mathbf{h})]
\end{aligned}$$

$$\begin{aligned}
&= \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + q\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_o[(\widehat{\mathbf{p}}\cdot\mathbf{h})\mathbf{r} - (\widehat{\mathbf{p}}\cdot\mathbf{r})\mathbf{h}]\\
&\quad + \varepsilon_b\mathfrak{y}_p[(\mathbf{z}\cdot\mathbf{h})\mathbf{r} - (\mathbf{z}\cdot\mathbf{r})\mathbf{h}] + q\varepsilon_b\mathfrak{y}_p[(\widehat{\mathbf{r}}\cdot\mathbf{h})\mathbf{r} - r\mathbf{h}] \text{ by (6.21h) \& (A.1)}\\
&= \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + q\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_o(\delta_c\mathbf{r} - r\varepsilon_c\mathbf{h}) + \varepsilon_b\mathfrak{y}_p(-r\varepsilon_d\mathbf{h})\\
&\quad + q\varepsilon_b\mathfrak{y}_p(-r\mathbf{h}) \text{ by (1.5c), (6.10a) \& (6.1a)}\\
&= \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + q\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_o\delta_c\mathbf{r} - r[\varepsilon_c\mathfrak{y}_o + \varepsilon_b\mathfrak{y}_p(\varepsilon_d + q)]\mathbf{h}\\
&= \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + q\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_o\delta_c\mathbf{r} - r(\varepsilon_c\mathfrak{y}_o + \varepsilon_b\mathfrak{y}_p\hbar_b)\mathbf{h} \text{ by (6.10b)}\\
&= \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + q\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_o\delta_c\mathbf{r} - \mathfrak{y}_q\mathbf{h} \text{ by (6.21h).}
\end{aligned} \tag{6.40n}$$

Art 26g. *Development of equation* (3.15c).

From the foregoing derivations, we obtain

$$\begin{aligned}
\mathfrak{J}_p &= b_4\mathfrak{J}_a + b_5\mathfrak{J}_b + \rho\dot{\mathfrak{Y}}\mathfrak{J}_c - \ddot{\mathfrak{Y}}\mathfrak{J}_d + \dot{\mathfrak{Y}}\mathfrak{J}_e + b_6\mathfrak{J}_f + \rho b_1\mathfrak{J}_g \text{ by (3.15c)}\\
&= \mathfrak{y}_g\mathfrak{J}_a + \mathfrak{y}_h\mathfrak{J}_b + \rho\mathfrak{y}_a\mathfrak{J}_c - \mathfrak{y}_b\mathfrak{J}_d + \mathfrak{y}_a\mathfrak{J}_e + \mathfrak{y}_i\mathfrak{J}_f + \rho\mathfrak{y}_d\mathfrak{J}_g \text{ by (6.38) \& (6.39)}\\
&= \mathfrak{y}_g[-\varrho_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \mathfrak{y}_h[-\varrho_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + 3\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \rho\mathfrak{y}_a[6\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \varrho_a\varrho_l(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\quad - \mathfrak{y}_b[\kappa r\varepsilon_a\varepsilon_b\mathfrak{x}_k\mathbf{h} - \kappa\varepsilon_b\mathfrak{x}_k\delta_a\mathbf{r} + \mathfrak{x}_n(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) - \varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) - q\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]\\
&\quad + \mathfrak{y}_a[\mathfrak{x}_p\delta_a\mathbf{r} - r\mathfrak{x}_p\varepsilon_a\mathbf{h} + \mathfrak{x}_o(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + 2\kappa\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + 2\kappa q\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]\\
&\quad + \mathfrak{y}_i[-\varrho_a^2\mathbf{h}] + \rho\mathfrak{y}_d[6\varrho_a^2\varrho_b\mathbf{h}] \text{ by (6.40)}\\
&= -\mathfrak{y}_g\varrho_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \mathfrak{y}_h\varrho_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + 3\mathfrak{y}_h\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + 6\rho\mathfrak{y}_a\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \rho\mathfrak{y}_a\varrho_a\varrho_l(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\\
&\quad - \mathfrak{y}_b\kappa r\varepsilon_a\varepsilon_b\mathfrak{x}_k\mathbf{h} + \mathfrak{y}_b\kappa\varepsilon_b\mathfrak{x}_k\delta_a\mathbf{r} - \mathfrak{y}_b\mathfrak{x}_n(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{y}_b\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \mathfrak{y}_b q\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\\
&\quad + \mathfrak{y}_a\mathfrak{x}_p\delta_a\mathbf{r} - \mathfrak{y}_a r\mathfrak{x}_p\varepsilon_a\mathbf{h} + \mathfrak{y}_a\mathfrak{x}_o(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + 2\mathfrak{y}_a\kappa\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + 2\mathfrak{y}_a\kappa q\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\\
&\quad - \mathfrak{y}_i\varrho_a^2\mathbf{h} + 6\rho\mathfrak{y}_d\varrho_a^2\varrho_b\mathbf{h}
\end{aligned}$$

$$
\begin{aligned}
&= -\mathfrak{y}_g\varrho_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + 3\mathfrak{y}_h\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \rho\mathfrak{y}_a\varrho_a\varrho_l(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \mathfrak{y}_b q\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + 2\mathfrak{y}_a\kappa q\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\\
&\quad - \mathfrak{y}_h\varrho_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + 6\rho\mathfrak{y}_a\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \mathfrak{y}_b\mathfrak{x}_n(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{y}_a\mathfrak{x}_o(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{y}_b\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\\
&\quad + 2\mathfrak{y}_a\kappa\varepsilon_b\mathfrak{x}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \mathfrak{y}_b\kappa\varepsilon_b\mathfrak{x}_k\delta_a\mathbf{r} + \mathfrak{y}_a\mathfrak{x}_p\delta_a\mathbf{r} - \mathfrak{y}_b\kappa r\varepsilon_a\varepsilon_b\mathfrak{x}_k\mathbf{h} - \mathfrak{y}_a r\mathfrak{x}_p\varepsilon_a\mathbf{h}\\
&\quad - \mathfrak{y}_i\varrho_a^2\mathbf{h} + 6\rho\mathfrak{y}_d\varrho_a^2\varrho_b\mathbf{h}\\
&= (-r\mathfrak{y}_g\varrho_a + 3r\mathfrak{y}_h\varrho_a\varrho_b - r\rho\mathfrak{y}_a\varrho_a\varrho_l + \mathfrak{y}_b q\varepsilon_b\varphi_f + 2\mathfrak{y}_a\kappa q\varepsilon_b\mathfrak{x}_k)(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\\
&\quad + (-\mathfrak{y}_h\varrho_a + 6\rho\mathfrak{y}_a\varrho_a\varrho_b)(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + (-\mathfrak{y}_b\mathfrak{x}_n + \mathfrak{y}_a\mathfrak{x}_o)(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + (\mathfrak{y}_b\varepsilon_b\varphi_f + 2\mathfrak{y}_a\kappa\varepsilon_b\mathfrak{x}_k)(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\\
&\quad + (\mathfrak{y}_b\kappa\varepsilon_b\mathfrak{x}_k\delta_a + \mathfrak{y}_a\mathfrak{x}_p\delta_a)\mathbf{r} + (-\mathfrak{y}_b\kappa r\varepsilon_a\varepsilon_b\mathfrak{x}_k - \mathfrak{y}_a r\mathfrak{x}_p\varepsilon_a - \mathfrak{y}_i\varrho_a^2 + 6\rho\mathfrak{y}_d\varrho_a^2\varrho_b)\mathbf{h}\\
&= (-\varphi_a\mathfrak{y}_g + 3\varphi_a\varrho_b\mathfrak{y}_h - \rho\varphi_a\varrho_l\mathfrak{y}_a + q\varepsilon_b\varphi_f\mathfrak{y}_b + 2\kappa q\varepsilon_b\mathfrak{x}_k\mathfrak{y}_a)(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\\
&\quad + \varrho_a(6\rho\varrho_b\mathfrak{y}_a - \mathfrak{y}_h)(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + (\mathfrak{x}_o\mathfrak{y}_a - \mathfrak{x}_n\mathfrak{y}_b)(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b(\varphi_f\mathfrak{y}_b + 2\kappa\mathfrak{x}_k\mathfrak{y}_a)(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\\
&\quad + \delta_a(\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{y}_b + \mathfrak{x}_p\mathfrak{y}_a)\mathbf{r} + [-r\varepsilon_a(\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{y}_b + \mathfrak{x}_p\mathfrak{y}_a) - \varrho_a^2(\mathfrak{y}_i - 6\rho\varrho_b\mathfrak{y}_d)]\mathbf{h} \text{ by (6.1b) \& (6.21b)}
\end{aligned}
$$

$$
\begin{aligned}
&= [\varphi_a(3\varrho_b\mathfrak{y}_h - \rho\varrho_l\mathfrak{y}_a - \mathfrak{y}_g) + q\varepsilon_b(\varphi_f\mathfrak{y}_b + 2\kappa\mathfrak{x}_k\mathfrak{y}_a)](\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\\
&\quad - \varrho_a(\mathfrak{y}_h - 6\rho\varrho_b\mathfrak{y}_a)(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + (\mathfrak{x}_o\mathfrak{y}_a - \mathfrak{x}_n\mathfrak{y}_b)(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b(\varphi_f\mathfrak{y}_b + 2\kappa\mathfrak{x}_k\mathfrak{y}_a)(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\\
&\quad + \delta_a(\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{y}_b + \mathfrak{x}_p\mathfrak{y}_a)\mathbf{r} - [r\varepsilon_a(\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{y}_b + \mathfrak{x}_p\mathfrak{y}_a) + \varrho_a^2(\mathfrak{y}_i - 6\rho\varrho_b\mathfrak{y}_d)]\mathbf{h}\\
&= (\varphi_a\mathfrak{y}_w + q\varepsilon_b\mathfrak{y}_r)(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\\
&\quad + \delta_a\mathfrak{y}_v\mathbf{r} - (r\varepsilon_a\mathfrak{y}_v + \varrho_a^2\mathfrak{y}_t)\mathbf{h} \text{ by (6.21h)}\\
&= \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \delta_a\mathfrak{y}_v\mathbf{r} - \mathfrak{y}_y\mathbf{h} \text{ by (6.21h)}
\end{aligned}
\tag{6.41a}
$$

$$
\begin{aligned}
\boldsymbol{\mathfrak{J}}_q &= -\ddot{\rho}\boldsymbol{\mathfrak{J}}_h + b_1\boldsymbol{\mathfrak{J}}_i + \rho^2\boldsymbol{\mathfrak{J}}_j - b_2\boldsymbol{\mathfrak{J}}_k + \rho\boldsymbol{\mathfrak{J}}_l + \rho\boldsymbol{\mathfrak{J}}_n + \boldsymbol{\mathfrak{J}}_o \text{ by (3.15c)}\\
&= -\mathfrak{x}_i\boldsymbol{\mathfrak{J}}_h + \mathfrak{y}_d\boldsymbol{\mathfrak{J}}_i + \rho^2\boldsymbol{\mathfrak{J}}_j - \mathfrak{y}_e\boldsymbol{\mathfrak{J}}_k + \rho\boldsymbol{\mathfrak{J}}_l + \rho\boldsymbol{\mathfrak{J}}_n + \boldsymbol{\mathfrak{J}}_o \text{ by (6.32b) \& (6.39)}\\
&= -\mathfrak{x}_i[-\varrho_a\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) - \kappa\varphi_b\varepsilon_b\mathfrak{x}_k\mathbf{h}] + \mathfrak{y}_d[-\varrho_a\mathfrak{x}_o(\mathbf{r}\times\widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{z}) + \varphi_b\mathfrak{x}_p\mathbf{h}]\\
&\quad + \rho^2[\mathfrak{y}_j\mathbf{h}] - \mathfrak{y}_e[-\varrho_a\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + 3\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - 3\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) + \mathfrak{y}_k\mathbf{h}]\\
&\quad + \rho[-\varrho_a\mathfrak{x}_o(\mathbf{u}\times\widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\mathbf{z}) + 3\varrho_a\varrho_b\mathfrak{x}_o(\mathbf{r}\times\widehat{\mathbf{p}}) + 6\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{z}) - \mathfrak{y}_l\mathbf{h}]\\
&\quad + \rho[-6\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + 6\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + \varrho_a\varrho_l\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - \varepsilon_b\varrho_a\varrho_l\varphi_f(\mathbf{r}\times\mathbf{z}) + \mathfrak{y}_m\mathbf{h}]\\
&\quad + [\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + q\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_o\delta_c\mathbf{r} - \mathfrak{y}_q\mathbf{h}] \text{ by (6.40)}
\end{aligned}
$$

$$
\begin{aligned}
&= \varrho_a\mathfrak{x}_n\mathfrak{x}_i(\mathbf{r}\times\widehat{\mathbf{p}}) - \varrho_a\varepsilon_b\varphi_f\mathfrak{x}_i(\mathbf{r}\times\mathbf{z}) + \kappa\varphi_b\varepsilon_b\mathfrak{x}_k\mathfrak{x}_i\mathbf{h} - \varrho_a\mathfrak{x}_o\mathfrak{y}_d(\mathbf{r}\times\widehat{\mathbf{p}}) - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k\mathfrak{y}_d(\mathbf{r}\times\mathbf{z}) + \varphi_b\mathfrak{x}_p\mathfrak{y}_d\mathbf{h}\\
&\quad + \rho^2\mathfrak{y}_j\mathbf{h} + \varrho_a\mathfrak{x}_n\mathfrak{y}_e(\mathbf{u}\times\widehat{\mathbf{p}}) - \varrho_a\varepsilon_b\varphi_f\mathfrak{y}_e(\mathbf{u}\times\mathbf{z}) - 3\varrho_a\varrho_b\mathfrak{x}_n\mathfrak{y}_e(\mathbf{r}\times\widehat{\mathbf{p}}) + 3\varrho_a\varrho_b\varepsilon_b\varphi_f\mathfrak{y}_e(\mathbf{r}\times\mathbf{z}) - \mathfrak{y}_k\mathfrak{y}_e\mathbf{h}\\
&\quad - \rho\varrho_a\mathfrak{x}_o(\mathbf{u}\times\widehat{\mathbf{p}}) - 2\rho\kappa\varrho_a\varepsilon_b\mathfrak{x}_k(\mathbf{u}\times\mathbf{z}) + 3\rho\varrho_a\varrho_b\mathfrak{x}_o(\mathbf{r}\times\widehat{\mathbf{p}}) + 6\rho\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{z}) - \rho\mathfrak{y}_l\mathbf{h}\\
&\quad - 6\rho\varrho_a\varrho_b\mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}}) + 6\rho\varrho_a\varrho_b\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z}) + \rho\varrho_a\varrho_l\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) - \rho\varepsilon_b\varrho_a\varrho_l\varphi_f(\mathbf{r}\times\mathbf{z}) + \rho\mathfrak{y}_m\mathbf{h}\\
&\quad + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + q\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_o\delta_c\mathbf{r} - \mathfrak{y}_q\mathbf{h}\\
&= (\varrho_a\mathfrak{x}_n\mathfrak{x}_i - \varrho_a\mathfrak{x}_o\mathfrak{y}_d - 3\varrho_a\varrho_b\mathfrak{x}_n\mathfrak{y}_e + 3\rho\varrho_a\varrho_b\mathfrak{x}_o + \rho\varrho_a\varrho_l\mathfrak{x}_n)(\mathbf{r}\times\widehat{\mathbf{p}}) - (q/r)\varepsilon_b\mathfrak{y}_n(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad + (-\varrho_a\varepsilon_b\varphi_f\mathfrak{x}_i - 2\kappa\varrho_a\varepsilon_b\mathfrak{x}_k\mathfrak{y}_d + 3\varrho_a\varrho_b\varepsilon_b\varphi_f\mathfrak{y}_e + 6\rho\kappa\varrho_a\varrho_b\varepsilon_b\mathfrak{x}_k - \rho\varepsilon_b\varrho_a\varrho_l\varphi_f)(\mathbf{r}\times\mathbf{z})\\
&\quad + (\varrho_a\mathfrak{x}_n\mathfrak{y}_e - \rho\varrho_a\mathfrak{x}_o - 6\rho\varrho_a\varrho_b\mathfrak{x}_n)(\mathbf{u}\times\widehat{\mathbf{p}}) + (-\varrho_a\varepsilon_b\varphi_f\mathfrak{y}_e - 2\rho\kappa\varrho_a\varepsilon_b\mathfrak{x}_k + 6\rho\varrho_a\varrho_b\varepsilon_b\varphi_f)(\mathbf{u}\times\mathbf{z})\\
&\quad + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + \kappa\varphi_b\varepsilon_b\mathfrak{x}_k\mathfrak{x}_i\mathbf{h} + \varphi_b\mathfrak{x}_p\mathfrak{y}_d\mathbf{h} + \rho^2\mathfrak{y}_j\mathbf{h} - \mathfrak{y}_k\mathfrak{y}_e\mathbf{h} - \rho\mathfrak{y}_l\mathbf{h} + \rho\mathfrak{y}_m\mathbf{h} - \mathfrak{y}_q\mathbf{h} + \mathfrak{y}_o\delta_c\mathbf{r}
\end{aligned}
$$

$$
\begin{aligned}
&= [\varrho_a(\mathfrak{x}_n\mathfrak{x}_i - \mathfrak{x}_o\mathfrak{y}_d) - 3\varrho_a\varrho_b(\mathfrak{x}_n\mathfrak{y}_e - \rho\mathfrak{x}_o) + \rho\varrho_a\varrho_l\mathfrak{x}_n - \varepsilon_b\varphi_b\mathfrak{y}_n](\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad + \varrho_a\varepsilon_b[\varphi_f(3\varrho_b\mathfrak{y}_e - \mathfrak{x}_i - \rho\varrho_l) + 2\kappa\mathfrak{x}_k(3\rho\varrho_b - \mathfrak{y}_d)](\mathbf{r}\times\mathbf{z})\\
&\quad + \varrho_a(\mathfrak{x}_n\mathfrak{y}_e - \rho\mathfrak{x}_o - 6\rho\varrho_b\mathfrak{x}_n)(\mathbf{u}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b(-\varphi_f\mathfrak{y}_e - 2\rho\kappa\mathfrak{x}_k + 6\rho\varrho_b\varphi_f)(\mathbf{u}\times\mathbf{z})\\
&\quad + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + [\varphi_b(\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_i + \mathfrak{x}_p\mathfrak{y}_d) + \rho(\rho\mathfrak{y}_j - \mathfrak{y}_l + \mathfrak{y}_m) - \mathfrak{y}_k\mathfrak{y}_e - \mathfrak{y}_q]\mathbf{h}\\
&\quad + \mathfrak{y}_o\delta_c\mathbf{r} \text{ by (6.1b)}\\
&= \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) + \varkappa_b(\mathbf{r}\times\mathbf{z}) + \varkappa_c(\mathbf{u}\times\widehat{\mathbf{p}}) + \varkappa_d(\mathbf{u}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})\\
&\quad + \varkappa_e\mathbf{h} + \mathfrak{y}_o\delta_c\mathbf{r} \text{ by (6.21i)}
\end{aligned}
\tag{6.41b}
$$

$$
\begin{aligned}
\mathfrak{J}_r &= \dot{\mathfrak{Y}}\widehat{\boldsymbol{\kappa}} + b_1\mathbf{a} + \rho\dot{\mathbf{a}} + \dot{\mathbf{e}} \text{ by (3.15c)}\\
&= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{y}_d\mathbf{a} + \rho\dot{\mathbf{a}} + \dot{\mathbf{e}} \text{ by (6.38a) \& (6.39a)}\\
&= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{y}_d(-\varrho_a\mathbf{r}) + \rho(-\varrho_a\mathbf{u} + 3\varrho_a\varrho_b\mathbf{r}) + [\mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - q\varepsilon_b\varphi_f\widehat{\mathbf{r}} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})]\\
&\quad \text{by (1.5a), (6.21b), (6.26a) \& (6.36a)}\\
&= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} - \varrho_a\mathfrak{y}_d\mathbf{r} - \rho\varrho_a\mathbf{u} + 3\rho\varrho_a\varrho_b\mathbf{r} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - q\varepsilon_b\varphi_f\widehat{\mathbf{r}} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})\\
&= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \rho\varrho_a\mathbf{u} + [-\varrho_a\mathfrak{y}_d + 3\rho\varrho_a\varrho_b - (q/r)\varepsilon_b\varphi_f]\mathbf{r} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})\\
&= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \rho\varrho_a\mathbf{u} - (\varrho_a\mathfrak{y}_d - 3\rho\varrho_a\varrho_b + \varphi_b\varepsilon_b\varphi_f)\mathbf{r} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h}) \text{ by (6.1b)}\\
&= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \rho\varrho_a\mathbf{u} - \varkappa_f\mathbf{r} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h}) \text{ by (6.21i)}
\end{aligned}
\tag{6.41c}
$$

$$
\begin{aligned}
\mathfrak{J}_p + \mathfrak{J}_q &= \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \delta_a\mathfrak{y}_v\mathbf{r} - \mathfrak{y}_y\mathbf{h}\\
&\quad + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) + \varkappa_b(\mathbf{r}\times\mathbf{z}) + \varkappa_c(\mathbf{u}\times\widehat{\mathbf{p}}) + \varkappa_d(\mathbf{u}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})\\
&\quad + \varkappa_e\mathbf{h} + \mathfrak{y}_o\delta_c\mathbf{r} \text{ by (6.41a) \& (6.41b)}\\
&= (\delta_a\mathfrak{y}_v + \delta_c\mathfrak{y}_o)\mathbf{r} + (\varkappa_e - \mathfrak{y}_y)\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\\
&\quad + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) + \varkappa_b(\mathbf{r}\times\mathbf{z}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u}) - \varkappa_d(\mathbf{z}\times\mathbf{u})\\
&= \varkappa_g\mathbf{r} + \varkappa_h\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad + \varkappa_b(\mathbf{r}\times\mathbf{z}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u}) - \varkappa_d(\mathbf{z}\times\mathbf{u}) \text{ by (6.21i).}
\end{aligned}
\tag{6.41d}
$$

Art 26h. *Magnitude of the vector $\mathfrak{J}_r$.*

We derive

$$
\begin{aligned}
|\mathfrak{J}_r|^2 = (\mathfrak{J}_r\cdot\mathfrak{J}_r) &= [\mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \rho\varrho_a\mathbf{u} - \varkappa_f\mathbf{r} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})]\cdot[\mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}}\\
&\quad - \varepsilon_b\varphi_f\mathbf{z} - \rho\varrho_a\mathbf{u} - \varkappa_f\mathbf{r} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})] \text{ by (6.41c)}\\
&= \mathfrak{y}_a^2(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\boldsymbol{\kappa}}) + 2\mathfrak{x}_n\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}}) - 2\varepsilon_b\varphi_f\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z}) - 2\rho\varrho_a\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})\\
&\quad - 2\varkappa_f\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r}) - 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{y}_a[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\mathbf{h})] + \mathfrak{x}_n^2(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{p}}) - 2\varepsilon_b\varphi_f\mathfrak{x}_n(\widehat{\mathbf{p}}\cdot\mathbf{z})\\
&\quad - 2\rho\varrho_a\mathfrak{x}_n(\widehat{\mathbf{p}}\cdot\mathbf{u}) - 2\varkappa_f\mathfrak{x}_n(\widehat{\mathbf{p}}\cdot\mathbf{r}) - 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_n[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{h})] + \varepsilon_b^2\varphi_f^2(\mathbf{z}\cdot\mathbf{z})\\
&\quad + 2\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{z}\cdot\mathbf{u}) + 2\varkappa_f\varepsilon_b\varphi_f(\mathbf{z}\cdot\mathbf{r}) + 2\kappa\varepsilon_b^2\mathfrak{x}_k\varphi_f[\mathbf{z}\cdot(\mathbf{r}\times\mathbf{h})] + \rho^2\varrho_a^2(\mathbf{u}\cdot\mathbf{u})\\
&\quad + 2\varkappa_f\rho\varrho_a(\mathbf{u}\cdot\mathbf{r}) + 2\kappa\varepsilon_b\mathfrak{x}_k\rho\varrho_a[\mathbf{u}\cdot(\mathbf{r}\times\mathbf{h})] + \varkappa_f^2(\mathbf{r}\cdot\mathbf{r}) + 2\kappa\varepsilon_b\mathfrak{x}_k\varkappa_f[\mathbf{r}\cdot(\mathbf{r}\times\mathbf{h})]\\
&\quad + \kappa^2\varepsilon_b^2\mathfrak{x}_k^2[(\mathbf{r}\times\mathbf{h})\cdot(\mathbf{r}\times\mathbf{h})]
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{y}_a^2 + 2\mathfrak{x}_n\mathfrak{y}_a\varepsilon_b - 2\varepsilon_b\varphi_f\mathfrak{y}_a\delta_b - 2\rho\varrho_a\mathfrak{y}_a\varrho_o - 2r\varkappa_f\mathfrak{y}_a\varepsilon_a - 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{y}_a\varepsilon_e + \mathfrak{x}_n^2\\
&\quad - 2\varepsilon_b\varphi_f\mathfrak{x}_n\delta_d - 2\rho\varrho_a\mathfrak{x}_n[h^{-2}(\varepsilon_i + \varepsilon_f\varphi_b)] - 2r\varkappa_f\mathfrak{x}_n\varepsilon_c - 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_n\varepsilon_f + z^2\varepsilon_b^2\varphi_f^2\\
&\quad + 2\rho\varrho_a\varepsilon_b\varphi_f(-r\varrho_b\varphi_b) + 2r\varkappa_f\varepsilon_b\varphi_f\varepsilon_d - 2r\kappa\varepsilon_b^2\mathfrak{x}_k\varphi_f\varepsilon_h + \rho^2\varrho_a^2\varphi_h^2\\
&\quad + 2r\varkappa_f\rho\varrho_a(r\varrho_b) + 2\kappa\varepsilon_b\mathfrak{x}_k\rho\varrho_a(r\hbar_b) + r^2\varkappa_f^2 + \kappa^2\varepsilon_b^2\mathfrak{x}_k^2[r^2h^2 - (\mathbf{r}\cdot\mathbf{h})^2]\\
&\quad \text{by (6.1a), (6.3b), (6.10a), (6.27) \& (A.2)}\\
&= \mathfrak{y}_a^2 + 2\mathfrak{x}_n\mathfrak{y}_a\varepsilon_b - 2\varepsilon_b\varphi_f\mathfrak{y}_a\delta_b - 2\rho\varrho_a\mathfrak{y}_a\varrho_o - 2r\varkappa_f\mathfrak{y}_a\varepsilon_a - 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{y}_a\varepsilon_e + \mathfrak{x}_n^2\\
&\quad - 2\varepsilon_b\varphi_f\mathfrak{x}_n\delta_d - 2\varrho_a\mathfrak{x}_n(\rho/h^2)(\varepsilon_i + \varepsilon_f\varphi_b) - 2r\varkappa_f\mathfrak{x}_n\varepsilon_c - 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{x}_n\varepsilon_f + z^2\varepsilon_b^2\varphi_f^2\\
&\quad - 2\rho r\varrho_b\varphi_b\varrho_a\varepsilon_b\varphi_f + 2r\varkappa_f\varepsilon_b\varphi_f\varepsilon_d - 2r\kappa\varepsilon_b^2\mathfrak{x}_k\varphi_f\varepsilon_h + \rho^2\varrho_a^2\varphi_h^2\\
&\quad + 2\rho r^2\varkappa_f\varrho_a\varrho_b + 2\kappa r\hbar_b\varepsilon_b\mathfrak{x}_k\rho\varrho_a + r^2\varkappa_f^2 + \kappa^2r^2h^2\varepsilon_b^2\mathfrak{x}_k^2 \text{ by (1.5c)}\\
&\therefore |\mathfrak{J}_r| = \varkappa_i \text{ by (6.21j).}
\end{aligned}
\tag{6.42}
$$

Art 26i. *Magnitude of the vector* $\mathfrak{J}_p + \mathfrak{J}_q$.

We derive also

$$\begin{aligned}
&\mathbf{r}\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=\mathbf{r}\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})] \text{ by (6.41d)}\\
&=\varkappa_g(\mathbf{r}\cdot\mathbf{r})+\varkappa_h(\mathbf{r}\cdot\mathbf{h})+\mathfrak{y}_x[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\varepsilon_b\mathfrak{y}_r[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&\quad+\varkappa_a[\mathbf{r}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]+\varkappa_b[\mathbf{r}\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]-\varkappa_d[\mathbf{r}\cdot(\mathbf{z}\times\mathbf{u})]\\
&=\varkappa_g r^2+\mathfrak{y}_u\varsigma_a+r\varepsilon_b\mathfrak{y}_r\delta_f+\varepsilon_b\mathfrak{y}_n\varsigma_b-\varrho_a\mathfrak{y}_s[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{u}\times\mathbf{r})]-\varkappa_c[\widehat{\mathbf{p}}\cdot(\mathbf{u}\times\mathbf{r})]-\varkappa_d[\mathbf{z}\cdot(\mathbf{u}\times\mathbf{r})]\\
&\quad\text{by (1.5c), (6.21a), (6.10a) \& (A.4)}\\
&=\varkappa_g r^2+\mathfrak{y}_u\varsigma_a+r\varepsilon_b\mathfrak{y}_r\delta_f+\varepsilon_b\mathfrak{y}_n\varsigma_b-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h})-\varkappa_c(\widehat{\mathbf{p}}\cdot\mathbf{h})-\varkappa_d(\mathbf{z}\cdot\mathbf{h}) \text{ by (1.5c)}\\
&=\varkappa_g r^2+\mathfrak{y}_u\varsigma_a+r\varepsilon_b\mathfrak{y}_r\delta_f+\varepsilon_b\mathfrak{y}_n\varsigma_b-\varrho_a\mathfrak{y}_s\delta_a-\varkappa_c\delta_c \text{ by (6.10a)}\\
&=\varkappa_j \text{ by (6.21j)}
\end{aligned}\tag{6.43a}$$

$$\begin{aligned}
&\mathbf{h}\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=\mathbf{h}\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})] \text{ by (6.41d)}\\
&=\varkappa_g(\mathbf{h}\cdot\mathbf{r})+\varkappa_h(\mathbf{h}\cdot\mathbf{h})+\mathfrak{y}_x[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\varepsilon_b\mathfrak{y}_r[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[\mathbf{h}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&\quad+\varkappa_a[\mathbf{h}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]+\varkappa_b[\mathbf{h}\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[\mathbf{h}\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]-\varkappa_d[\mathbf{h}\cdot(\mathbf{z}\times\mathbf{u})]\\
&=h^2\varkappa_h+(\mathfrak{y}_x/r)\varepsilon_e-\mathfrak{y}_u\delta_e+\varepsilon_b\mathfrak{y}_r\varepsilon_g+\varepsilon_b\mathfrak{y}_n\varepsilon_i-\varkappa_a\varepsilon_f+r\varkappa_b\varepsilon_h-\varrho_a\mathfrak{y}_s[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{u}\times\mathbf{h})]\\
&\quad-\varkappa_c[\widehat{\mathbf{p}}\cdot(\mathbf{u}\times\mathbf{h})]-\varkappa_d[\mathbf{z}\cdot(\mathbf{u}\times\mathbf{h})] \text{ by (6.1a), (6.10a) \& (A.4)}
\end{aligned}$$

$$\begin{aligned}
&=h^2\varkappa_h+(\mathfrak{y}_x/r)\varepsilon_e-\mathfrak{y}_u\delta_e+\varepsilon_b\mathfrak{y}_r\varepsilon_g+\varepsilon_b\mathfrak{y}_n\varepsilon_i-\varkappa_a\varepsilon_f+r\varkappa_b\varepsilon_h-\varrho_a\mathfrak{y}_s[\widehat{\boldsymbol{\kappa}}\cdot(-\mathbf{z}-q\widehat{\mathbf{r}})]\\
&\quad-\varkappa_c[\widehat{\mathbf{p}}\cdot(-\mathbf{z}-q\widehat{\mathbf{r}})]-\varkappa_d[\mathbf{z}\cdot(-\mathbf{z}-q\widehat{\mathbf{r}})] \text{ by (1.5c)}\\
&=h^2\varkappa_h+(\mathfrak{y}_x/r)\varepsilon_e-\mathfrak{y}_u\delta_e+\varepsilon_b\mathfrak{y}_r\varepsilon_g+\varepsilon_b\mathfrak{y}_n\varepsilon_i-\varkappa_a\varepsilon_f+r\varkappa_b\varepsilon_h-\varrho_a\mathfrak{y}_s[-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})-q(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}})]\\
&\quad-\varkappa_c[-(\widehat{\mathbf{p}}\cdot\mathbf{z})-q(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{r}})]-\varkappa_d[-(\mathbf{z}\cdot\mathbf{z})-q(\mathbf{z}\cdot\widehat{\mathbf{r}})]\\
&=h^2\varkappa_h+\varepsilon_e(\mathfrak{y}_x/r)-\mathfrak{y}_u\delta_e+\varepsilon_b\mathfrak{y}_r\varepsilon_g+\varepsilon_b\mathfrak{y}_n\varepsilon_i-\varkappa_a\varepsilon_f+r\varkappa_b\varepsilon_h-\varrho_a\mathfrak{y}_s(-\delta_b-q\varepsilon_a)\\
&\quad-\varkappa_c(-\delta_d-q\varepsilon_c)-\varkappa_d(-z^2-q\varepsilon_d) \text{ by (6.1a) \& (6.10a)}\\
&=h^2\varkappa_h+\varepsilon_e(\mathfrak{y}_x/r)-\mathfrak{y}_u\delta_e+\varepsilon_b\mathfrak{y}_r\varepsilon_g+\varepsilon_b\mathfrak{y}_n\varepsilon_i-\varkappa_a\varepsilon_f+r\varkappa_b\varepsilon_h+\varrho_a\mathfrak{y}_s\hbar_a+\varkappa_c\hbar_c+\varkappa_d\mathfrak{y}_z\\
&\quad\text{by (6.10b) \& (6.21h)}\\
&=\varkappa_k \text{ by (6.21j)}
\end{aligned}\tag{6.43b}$$

$$\begin{aligned}
&(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})] \text{ by (6.41d)}\\
&=\varkappa_g[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\varkappa_h[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_x[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\varepsilon_b\mathfrak{y}_r[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\varkappa_a[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]\\
&\quad+\varkappa_b[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]\\
&\quad-\varkappa_d[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}$$

$$\begin{aligned}
&= (\varkappa_h/r)\varepsilon_e + \mathfrak{y}_x[1-(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}})^2] + \mathfrak{y}_u[(\widehat{\mathbf{r}}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{r}}\cdot\widehat{\boldsymbol{\kappa}})] + \varepsilon_b\mathfrak{y}_r[(\widehat{\mathbf{r}}\cdot\mathbf{z}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\widehat{\mathbf{r}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \varepsilon_b\mathfrak{y}_n[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{r}}\cdot\mathbf{z}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\widehat{\mathbf{r}}\cdot\widehat{\mathbf{p}})] + \varkappa_a[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\widehat{\mathbf{r}}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{r}}\cdot\mathbf{r})] \\
&\quad + \varkappa_b[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\widehat{\mathbf{r}}\cdot\mathbf{z}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\widehat{\mathbf{r}}\cdot\mathbf{r})] - \varrho_a\mathfrak{y}_s[(\widehat{\mathbf{r}}\cdot\mathbf{u}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\widehat{\mathbf{r}}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad - \varkappa_c[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{r}}\cdot\mathbf{u}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\widehat{\mathbf{r}}\cdot\widehat{\mathbf{p}})] - \varkappa_d[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\widehat{\mathbf{r}}\cdot\mathbf{u}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\widehat{\mathbf{r}}\cdot\mathbf{z})] \\
&\quad \text{by (6.1a) \& (A.2)} \\
&= (\varkappa_h/r)\varepsilon_e + \mathfrak{y}_x(1-\varepsilon_a^2) + \mathfrak{y}_u(\varepsilon_c - \varepsilon_b\varepsilon_a) + \varepsilon_b\mathfrak{y}_r(\varepsilon_d - \delta_b\varepsilon_a) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b\varepsilon_d - \delta_b\varepsilon_c) \\
&\quad + \varkappa_a(r\varepsilon_a\varepsilon_c - r\varepsilon_b) + \varkappa_b(r\varepsilon_a\varepsilon_d - r\delta_b) - \varrho_a\mathfrak{y}_s(r\varrho_b - \varrho_o\varepsilon_a) - \varkappa_c(r\varepsilon_b\varrho_b - \varrho_o\varepsilon_c) \\
&\quad - \varkappa_d(r\delta_b\varrho_b - \varrho_o\varepsilon_d) \text{ by (6.1a), (6.10a) \& (6.27)} \\
&= \varkappa_l \text{ by (6.21j)}
\end{aligned} \tag{6.43c}$$

$$\begin{aligned}
&(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= (\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot[\varkappa_g\mathbf{r} + \varkappa_h\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + \varkappa_b(\mathbf{r}\times\mathbf{z}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u}) - \varkappa_d(\mathbf{z}\times\mathbf{u})] \text{ by (6.41d)} \\
&= \varkappa_g[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \varkappa_h[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] + \mathfrak{y}_x[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})] + \mathfrak{y}_u[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] \\
&\quad + \varepsilon_b\mathfrak{y}_r[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})] + \varepsilon_b\mathfrak{y}_n[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})] + \varkappa_a[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad + \varkappa_b[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\mathbf{z})] - \varrho_a\mathfrak{y}_s[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})] - \varkappa_c[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})] \\
&\quad - \varkappa_d[(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}$$

$$\begin{aligned}
&= \varkappa_g\varsigma_a - \varkappa_h\delta_e + \mathfrak{y}_x[(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{r}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{y}_u[1-(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})^2] \\
&\quad + \varepsilon_b\mathfrak{y}_r[(\widehat{\mathbf{p}}\cdot\mathbf{z}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] + \varepsilon_b\mathfrak{y}_n[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\mathbf{z}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})] + \varkappa_a[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\mathbf{r})] \\
&\quad + \varkappa_b[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\widehat{\mathbf{p}}\cdot\mathbf{z}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\widehat{\mathbf{p}}\cdot\mathbf{r})] - \varrho_a\mathfrak{y}_s[(\widehat{\mathbf{p}}\cdot\mathbf{u}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})] - \varkappa_c[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\mathbf{u}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})] \\
&\quad - \varkappa_d[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\widehat{\mathbf{p}}\cdot\mathbf{u}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\widehat{\mathbf{p}}\cdot\mathbf{z})] \text{ by (6.21a), (6.10a) \& (A.2)} \\
&= \varkappa_g\varsigma_a - \varkappa_h\delta_e + \mathfrak{y}_x(\varepsilon_c - \varepsilon_a\varepsilon_b) + \mathfrak{y}_u(1-\varepsilon_b^2) + \varepsilon_b\mathfrak{y}_r(\delta_d - \delta_b\varepsilon_b) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b\delta_d - \delta_b) + \varkappa_a(r\varepsilon_a - r\varepsilon_b\varepsilon_c) \\
&\quad + \varkappa_b(r\varepsilon_a\delta_d - r\delta_b\varepsilon_c) - \varrho_a\mathfrak{y}_s[(\widehat{\mathbf{p}}\cdot\mathbf{u}) - \varrho_o\varepsilon_b] - \varkappa_c[\varepsilon_b(\widehat{\mathbf{p}}\cdot\mathbf{u}) - \varrho_o] - \varkappa_d[\delta_b(\widehat{\mathbf{p}}\cdot\mathbf{u}) - \varrho_o\delta_d] \\
&\quad \text{by (6.1a), (6.10a) \& (6.27)}
\end{aligned}$$

$$\begin{aligned}
&= \varkappa_g\varsigma_a - \varkappa_h\delta_e + \mathfrak{y}_x(\varepsilon_c - \varepsilon_a\varepsilon_b) + \mathfrak{y}_u(1-\varepsilon_b^2) + \varepsilon_b\mathfrak{y}_r(\delta_d - \delta_b\varepsilon_b) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b\delta_d - \delta_b) \\
&\quad + \varkappa_a(r\varepsilon_a - r\varepsilon_b\varepsilon_c) + \varkappa_b(r\varepsilon_a\delta_d - r\delta_b\varepsilon_c) - \varrho_a\mathfrak{y}_s(\widehat{\mathbf{p}}\cdot\mathbf{u}) + \varrho_a\mathfrak{y}_s\varrho_o\varepsilon_b \\
&\quad - \varkappa_c\varepsilon_b(\widehat{\mathbf{p}}\cdot\mathbf{u}) + \varkappa_c\varrho_o - \varkappa_d\delta_b(\widehat{\mathbf{p}}\cdot\mathbf{u}) + \varkappa_d\varrho_o\delta_d \\
&= \varkappa_g\varsigma_a - \varkappa_h\delta_e + \mathfrak{y}_x(\varepsilon_c - \varepsilon_a\varepsilon_b) + \mathfrak{y}_u(1-\varepsilon_b^2) + \varepsilon_b\mathfrak{y}_r(\delta_d - \delta_b\varepsilon_b) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b\delta_d - \delta_b) \\
&\quad + r\varkappa_a(\varepsilon_a - \varepsilon_b\varepsilon_c) + r\varkappa_b(\varepsilon_a\delta_d - \delta_b\varepsilon_c) + \varrho_a\mathfrak{y}_s\varrho_o\varepsilon_b + \varkappa_c\varrho_o + \varkappa_d\varrho_o\delta_d \\
&\quad - (\varrho_a\mathfrak{y}_s + \varkappa_c\varepsilon_b + \varkappa_d\delta_b)(\widehat{\mathbf{p}}\cdot\mathbf{u}) \\
&= \varkappa_g\varsigma_a - \varkappa_h\delta_e + \mathfrak{y}_x(\varepsilon_c - \varepsilon_a\varepsilon_b) + \mathfrak{y}_u(1-\varepsilon_b^2) + \varepsilon_b\mathfrak{y}_r(\delta_d - \delta_b\varepsilon_b) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b\delta_d - \delta_b) \\
&\quad + r\varkappa_a(\varepsilon_a - \varepsilon_b\varepsilon_c) + r\varkappa_b(\varepsilon_a\delta_d - \delta_b\varepsilon_c) + \varrho_a\mathfrak{y}_s\varrho_o\varepsilon_b + \varkappa_c\varrho_o + \varkappa_d\varrho_o\delta_d \\
&\quad - h^{-2}(\varrho_a\mathfrak{y}_s + \varkappa_c\varepsilon_b + \varkappa_d\delta_b)(\varepsilon_i + \varepsilon_f\varphi_b) \text{ by (6.27c)} \\
&= \varkappa_m \text{ by (6.21k)}
\end{aligned} \tag{6.43d}$$

$$\begin{aligned}
&(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&= (\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot[\varkappa_g\mathbf{r} + \varkappa_h\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\quad + \varkappa_b(\mathbf{r}\times\mathbf{z}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u}) - \varkappa_d(\mathbf{z}\times\mathbf{u})] \text{ by (6.41d)} \\
&= \varkappa_g[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})] + \varkappa_h[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})] + \mathfrak{y}_x[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})] + \mathfrak{y}_u[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] \\
&\quad + \varepsilon_b\mathfrak{y}_r[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})] + \varepsilon_b\mathfrak{y}_n[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})] + \varkappa_a[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\quad + \varkappa_b[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\mathbf{r}\times\mathbf{z})] - \varrho_a\mathfrak{y}_s[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})] - \varkappa_c[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})] \\
&\quad - \varkappa_d[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}$$

$$
\begin{aligned}
&= r\varkappa_g\delta_f + \varkappa_h\varepsilon_g + \mathfrak{y}_x[(\mathbf{z}\cdot\widehat{\mathbf{r}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{y}_u[(\mathbf{z}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \varepsilon_b\mathfrak{y}_r[z^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})^2] + \varepsilon_b\mathfrak{y}_n[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})z^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{z}\cdot\widehat{\mathbf{p}})] + \varkappa_a[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{z}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\mathbf{r})] \\
&\quad + \varkappa_b[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})z^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{z}\cdot\mathbf{r})] - \varrho_a\mathfrak{y}_s[(\mathbf{z}\cdot\mathbf{u}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] - \varkappa_c[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\mathbf{u}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\mathbf{z}\cdot\widehat{\mathbf{p}})] \\
&\quad - \varkappa_d[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{z}\cdot\mathbf{u}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})z^2] \text{ by (6.1a), (6.10a) \& (A.2)} \\
&= r\varkappa_g\delta_f + \varkappa_h\varepsilon_g + \mathfrak{y}_x(\varepsilon_d - \varepsilon_a\delta_b) + \mathfrak{y}_u(\delta_d - \varepsilon_b\delta_b) + \varepsilon_b\mathfrak{y}_r(z^2 - \delta_b^2) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b z^2 - \delta_b\delta_d) \\
&\quad + \varkappa_a(r\varepsilon_a\delta_d - r\varepsilon_b\varepsilon_d) + \varkappa_b(r\varepsilon_a z^2 - r\delta_b\varepsilon_d) - \varrho_a\mathfrak{y}_s(-r\varrho_b\varphi_b - \varrho_o\delta_b) - \varkappa_c[\varepsilon_b(-r\varrho_b\varphi_b) - \varrho_o\delta_d] \\
&\quad - \varkappa_d[\delta_b(-r\varrho_b\varphi_b) - \varrho_o z^2] \text{ by (6.1a), (6.10a) \& (6.27)} \\
&= r\varkappa_g\delta_f + \varkappa_h\varepsilon_g + \mathfrak{y}_x(\varepsilon_d - \varepsilon_a\delta_b) + \mathfrak{y}_u(\delta_d - \varepsilon_b\delta_b) + \varepsilon_b\mathfrak{y}_r(z^2 - \delta_b^2) + \varepsilon_b\mathfrak{y}_n(\varepsilon_b z^2 - \delta_b\delta_d) \\
&\quad + r\varkappa_a(\varepsilon_a\delta_d - \varepsilon_b\varepsilon_d) + r\varkappa_b(\varepsilon_a z^2 - \delta_b\varepsilon_d) + \varrho_a\mathfrak{y}_s(r\varrho_b\varphi_b + \varrho_o\delta_b) + \varkappa_c(r\varepsilon_b\varrho_b\varphi_b + \varrho_o\delta_d) \qquad (6.43e) \\
&\quad + \varkappa_d(r\varrho_b\varphi_b\delta_b + \varrho_o z^2) = \varkappa_n \text{ by (6.21k)}
\end{aligned}
$$

$$
\begin{aligned}
&(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \\
&\; = (\widehat{\mathbf{p}}\times\mathbf{z})\cdot[\varkappa_g\mathbf{r} + \varkappa_h\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) \\
&\qquad + \varkappa_b(\mathbf{r}\times\mathbf{z}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u}) - \varkappa_d(\mathbf{z}\times\mathbf{u})] \text{ by (6.41d)} \\
&\; = \varkappa_g[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})] + \varkappa_h[\mathbf{h}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})] + \mathfrak{y}_x[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})] + \mathfrak{y}_u[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})] \\
&\qquad + \varepsilon_b\mathfrak{y}_r[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})] + \varepsilon_b\mathfrak{y}_n[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})] + \varkappa_a[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})] \\
&\qquad + \varkappa_b[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\mathbf{r}\times\mathbf{z})] - \varrho_a\mathfrak{y}_s[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})] - \varkappa_c[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})] \\
&\qquad - \varkappa_d[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_g\varsigma_b + \varkappa_h\varepsilon_i + \mathfrak{y}_x[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\widehat{\mathbf{r}}) - (\widehat{\mathbf{p}}\cdot\widehat{\mathbf{r}})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{y}_u[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\widehat{\mathbf{p}}) - (\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \varepsilon_b\mathfrak{y}_r[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})z^2 - (\widehat{\mathbf{p}}\cdot\mathbf{z})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] + \varepsilon_b\mathfrak{y}_n[z^2 - (\widehat{\mathbf{p}}\cdot\mathbf{z})^2] + \varkappa_a[(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{z}\cdot\widehat{\mathbf{p}}) - (\mathbf{z}\cdot\mathbf{r})] \\
&\quad + \varkappa_b[(\widehat{\mathbf{p}}\cdot\mathbf{r})z^2 - (\widehat{\mathbf{p}}\cdot\mathbf{z})(\mathbf{z}\cdot\mathbf{r})] - \varrho_a\mathfrak{y}_s[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\mathbf{u}) - (\widehat{\mathbf{p}}\cdot\mathbf{u})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad - \varkappa_c[(\mathbf{z}\cdot\mathbf{u}) - (\widehat{\mathbf{p}}\cdot\mathbf{u})(\mathbf{z}\cdot\widehat{\mathbf{p}})] - \varkappa_d[(\widehat{\mathbf{p}}\cdot\mathbf{z})(\mathbf{z}\cdot\mathbf{u}) - (\widehat{\mathbf{p}}\cdot\mathbf{u})z^2] \text{ by (6.21a), (6.1a) \& (A.2)} \\
&= \varkappa_g\varsigma_b + \varkappa_h\varepsilon_i + \mathfrak{y}_x(\varepsilon_b\varepsilon_d - \varepsilon_c\delta_b) + \mathfrak{y}_u(\varepsilon_b\delta_d - \delta_b) \\
&\quad + \varepsilon_b\mathfrak{y}_r(\varepsilon_b z^2 - \delta_d\delta_b) + \varepsilon_b\mathfrak{y}_n(z^2 - \delta_d^2) + \varkappa_a(r\varepsilon_c\delta_d - r\varepsilon_d) \\
&\quad + \varkappa_b(r\varepsilon_c z^2 - r\delta_d\varepsilon_d) - \varrho_a\mathfrak{y}_s[\varepsilon_b(-r\varrho_b\varphi_b) - (\widehat{\mathbf{p}}\cdot\mathbf{u})\delta_b] - \varkappa_c[(-r\varrho_b\varphi_b) - (\widehat{\mathbf{p}}\cdot\mathbf{u})\delta_d] \\
&\quad - \varkappa_d[\delta_d(-r\varrho_b\varphi_b) - (\widehat{\mathbf{p}}\cdot\mathbf{u})z^2] \text{ by (6.1a), (6.10a) \& (6.27)}
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_g\varsigma_b + \varkappa_h\varepsilon_i + \mathfrak{y}_x(\varepsilon_b\varepsilon_d - \varepsilon_c\delta_b) + \mathfrak{y}_u(\varepsilon_b\delta_d - \delta_b) + \varepsilon_b\mathfrak{y}_r(\varepsilon_b z^2 - \delta_d\delta_b) + \varepsilon_b\mathfrak{y}_n(z^2 - \delta_d^2) \\
&\quad + r\varkappa_a(\varepsilon_c\delta_d - \varepsilon_d) + r\varkappa_b(\varepsilon_c z^2 - \delta_d\varepsilon_d) + \varrho_a\mathfrak{y}_s\varepsilon_b(r\varrho_b\varphi_b) + \varrho_a\mathfrak{y}_s\delta_b(\widehat{\mathbf{p}}\cdot\mathbf{u}) + \varkappa_c r\varrho_b\varphi_b \\
&\quad + \varkappa_c\delta_d(\widehat{\mathbf{p}}\cdot\mathbf{u}) + \varkappa_d\delta_d(r\varrho_b\varphi_b) + \varkappa_d z^2(\widehat{\mathbf{p}}\cdot\mathbf{u}) \\
&= \varkappa_g\varsigma_b + \varkappa_h\varepsilon_i + \mathfrak{y}_x(\varepsilon_b\varepsilon_d - \varepsilon_c\delta_b) + \mathfrak{y}_u(\varepsilon_b\delta_d - \delta_b) + \varepsilon_b\mathfrak{y}_r(\varepsilon_b z^2 - \delta_d\delta_b) + \varepsilon_b\mathfrak{y}_n(z^2 - \delta_d^2) \\
&\quad + r\varkappa_a(\varepsilon_c\delta_d - \varepsilon_d) + r\varkappa_b(\varepsilon_c z^2 - \delta_d\varepsilon_d) + r\mathfrak{y}_s\varepsilon_b\varrho_a\varrho_b\varphi_b + r\varkappa_c\varrho_b\varphi_b + r\varkappa_d\delta_d\varrho_b\varphi_b \\
&\quad + (\varrho_a\mathfrak{y}_s\delta_b + \varkappa_c\delta_d + \varkappa_d z^2)(\widehat{\mathbf{p}}\cdot\mathbf{u}) \\
&= \varkappa_g\varsigma_b + \varkappa_h\varepsilon_i + \mathfrak{y}_x(\varepsilon_b\varepsilon_d - \varepsilon_c\delta_b) + \mathfrak{y}_u(\varepsilon_b\delta_d - \delta_b) + \varepsilon_b\mathfrak{y}_r(\varepsilon_b z^2 - \delta_d\delta_b) + \varepsilon_b\mathfrak{y}_n(z^2 - \delta_d^2) \\
&\quad + r\varkappa_a(\varepsilon_c\delta_d - \varepsilon_d) + r\varkappa_b(\varepsilon_c z^2 - \delta_d\varepsilon_d) + r\mathfrak{y}_s\varepsilon_b\varrho_a\varrho_b\varphi_b + r\varkappa_c\varrho_b\varphi_b + r\varkappa_d\delta_d\varrho_b\varphi_b \\
&\quad + h^{-2}(\varrho_a\mathfrak{y}_s\delta_b + \varkappa_c\delta_d + \varkappa_d z^2)(\varepsilon_i + \varepsilon_f\varphi_b) \text{ by (6.27c)} \\
&= \varkappa_o \text{ by (6.21k)} \qquad (6.43f)
\end{aligned}
$$

$$
\begin{aligned}
&(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=(\mathbf{r}\times\widehat{\mathbf{p}})\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})]\text{ by (6.41d)}\\
&=\varkappa_g[\mathbf{r}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]+\varkappa_h[\mathbf{h}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]+\mathfrak{y}_x[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\varepsilon_b\mathfrak{y}_r[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\varkappa_a[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]\\
&\quad+\varkappa_b[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]\\
&\quad-\varkappa_d[(\mathbf{r}\times\widehat{\mathbf{p}})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}
$$

$$
\begin{aligned}
&=-\varkappa_h\varepsilon_f+\mathfrak{y}_x[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\widehat{\mathbf{r}})-(\mathbf{r}\cdot\widehat{\mathbf{r}})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{y}_u[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})-(\mathbf{r}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\varepsilon_b\mathfrak{y}_r[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\mathbf{z})-(\mathbf{r}\cdot\mathbf{z})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})]+\varepsilon_b\mathfrak{y}_n[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\mathbf{z})-(\mathbf{r}\cdot\mathbf{z})]+\varkappa_a[r^2-(\mathbf{r}\cdot\widehat{\mathbf{p}})^2]\\
&\quad+\varkappa_b[r^2(\widehat{\mathbf{p}}\cdot\mathbf{z})-(\mathbf{r}\cdot\mathbf{z})(\widehat{\mathbf{p}}\cdot\mathbf{r})]-\varrho_a\mathfrak{y}_s[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\widehat{\mathbf{p}}\cdot\mathbf{u})-(\mathbf{r}\cdot\mathbf{u})(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad-\varkappa_c[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\widehat{\mathbf{p}}\cdot\mathbf{u})-(\mathbf{r}\cdot\mathbf{u})]-\varkappa_d[(\mathbf{r}\cdot\mathbf{z})(\widehat{\mathbf{p}}\cdot\mathbf{u})-(\mathbf{r}\cdot\mathbf{u})(\widehat{\mathbf{p}}\cdot\mathbf{z})]\text{ by (6.1a) \& (A.2)}\\
&=-\varkappa_h\varepsilon_f+\mathfrak{y}_x(r\varepsilon_a\varepsilon_c-r\varepsilon_b)+\mathfrak{y}_u(r\varepsilon_a-r\varepsilon_c\varepsilon_b)+\varepsilon_b\mathfrak{y}_r(r\varepsilon_a\delta_d-r\varepsilon_d\varepsilon_b)+\varepsilon_b\mathfrak{y}_n(r\varepsilon_c\delta_d-r\varepsilon_d)\\
&\quad+\varkappa_a(r^2-r^2\varepsilon_c^2)+\varkappa_b(r^2\delta_d-r^2\varepsilon_d\varepsilon_c)-\varrho_a\mathfrak{y}_s[r\varepsilon_a(\widehat{\mathbf{p}}\cdot\mathbf{u})-r(r\varrho_b)\varepsilon_b]-\varkappa_c[r\varepsilon_c(\widehat{\mathbf{p}}\cdot\mathbf{u})-r(r\varrho_b)]\\
&\quad-\varkappa_d[r\varepsilon_d(\widehat{\mathbf{p}}\cdot\mathbf{u})-r(r\varrho_b)\delta_d]\text{ by (6.1a), (6.10a) \& (6.27)}\\
&=-\varkappa_h\varepsilon_f+r\mathfrak{y}_x(\varepsilon_a\varepsilon_c-\varepsilon_b)+r\mathfrak{y}_u(\varepsilon_a-\varepsilon_c\varepsilon_b)+r\varepsilon_b\mathfrak{y}_r(\varepsilon_a\delta_d-\varepsilon_d\varepsilon_b)+r\varepsilon_b\mathfrak{y}_n(\varepsilon_c\delta_d-\varepsilon_d)\\
&\quad+r^2\varkappa_a(1-\varepsilon_c^2)+r^2\varkappa_b(\delta_d-\varepsilon_d\varepsilon_c)-\varrho_a\mathfrak{y}_s r\varepsilon_a(\widehat{\mathbf{p}}\cdot\mathbf{u})+r^2\varepsilon_b\mathfrak{y}_s\varrho_a\varrho_b-\varkappa_c r\varepsilon_c(\widehat{\mathbf{p}}\cdot\mathbf{u})+r^2\varkappa_c\varrho_b\\
&\quad-\varkappa_d r\varepsilon_d(\widehat{\mathbf{p}}\cdot\mathbf{u})+r^2\varkappa_d\delta_d\varrho_b
\end{aligned}
$$

$$
\begin{aligned}
&=-\varkappa_h\varepsilon_f+r\mathfrak{y}_x(\varepsilon_a\varepsilon_c-\varepsilon_b)+r\mathfrak{y}_u(\varepsilon_a-\varepsilon_c\varepsilon_b)+r\varepsilon_b\mathfrak{y}_r(\varepsilon_a\delta_d-\varepsilon_d\varepsilon_b)+r\varepsilon_b\mathfrak{y}_n(\varepsilon_c\delta_d-\varepsilon_d)\\
&\quad+r^2\varkappa_a(1-\varepsilon_c^2)+r^2\varkappa_b(\delta_d-\varepsilon_d\varepsilon_c)+r^2\varepsilon_b\mathfrak{y}_s\varrho_a\varrho_b+r^2\varkappa_c\varrho_b+r^2\varkappa_d\delta_d\varrho_b\\
&\quad-r(\varrho_a\mathfrak{y}_s\varepsilon_a+\varkappa_c\varepsilon_c+\varkappa_d\varepsilon_d)(\widehat{\mathbf{p}}\cdot\mathbf{u})\\
&=-\varkappa_h\varepsilon_f+r\mathfrak{y}_x(\varepsilon_a\varepsilon_c-\varepsilon_b)+r\mathfrak{y}_u(\varepsilon_a-\varepsilon_c\varepsilon_b)+r\varepsilon_b\mathfrak{y}_r(\varepsilon_a\delta_d-\varepsilon_d\varepsilon_b)+r\varepsilon_b\mathfrak{y}_n(\varepsilon_c\delta_d-\varepsilon_d)\\
&\quad+r^2\varkappa_a(1-\varepsilon_c^2)+r^2\varkappa_b(\delta_d-\varepsilon_d\varepsilon_c)+r^2\varepsilon_b\mathfrak{y}_s\varrho_a\varrho_b+r^2\varkappa_c\varrho_b+r^2\varkappa_d\delta_d\varrho_b\\
&\quad-(r/h^2)(\varrho_a\mathfrak{y}_s\varepsilon_a+\varkappa_c\varepsilon_c+\varkappa_d\varepsilon_d)(\varepsilon_i+\varepsilon_f\varphi_b)\text{ by (6.27c)}\\
&=\varkappa_p\text{ by (6.21l)}
\end{aligned}
\tag{6.43g}
$$

$$
\begin{aligned}
&(\mathbf{r}\times\mathbf{z})\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=(\mathbf{r}\times\mathbf{z})\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})]\text{ by (6.41d)}\\
&=\varkappa_g[\mathbf{r}\cdot(\mathbf{r}\times\mathbf{z})]+\varkappa_h[\mathbf{h}\cdot(\mathbf{r}\times\mathbf{z})]+\mathfrak{y}_x[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\varepsilon_b\mathfrak{y}_r[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\varkappa_a[(\mathbf{r}\times\mathbf{z})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]\\
&\quad+\varkappa_b[(\mathbf{r}\times\mathbf{z})\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]\\
&\quad-\varkappa_d[(\mathbf{r}\times\mathbf{z})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}
$$

$$
\begin{aligned}
&= r\varkappa_h\varepsilon_h + \mathfrak{y}_x[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\widehat{\mathbf{r}}) - (\mathbf{r}\cdot\widehat{\mathbf{r}})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{y}_u[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\widehat{\mathbf{p}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad + \varepsilon_b\mathfrak{y}_r[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})z^2 - (\mathbf{r}\cdot\mathbf{z})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] + \varepsilon_b\mathfrak{y}_n[(\mathbf{r}\cdot\widehat{\mathbf{p}})z^2 - (\mathbf{r}\cdot\mathbf{z})(\mathbf{z}\cdot\widehat{\mathbf{p}})] + \varkappa_a[r^2(\mathbf{z}\cdot\widehat{\mathbf{p}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\mathbf{r})]\\
&\quad + \varkappa_b[r^2z^2 - (\mathbf{r}\cdot\mathbf{z})^2] - \varrho_a\mathfrak{y}_s[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\mathbf{u}) - (\mathbf{r}\cdot\mathbf{u})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})] - \varkappa_c[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\mathbf{u}) - (\mathbf{r}\cdot\mathbf{u})(\mathbf{z}\cdot\widehat{\mathbf{p}})]\\
&\quad - \varkappa_d[(\mathbf{r}\cdot\mathbf{z})(\mathbf{z}\cdot\mathbf{u}) - (\mathbf{r}\cdot\mathbf{u})z^2] \text{ by (6.1a) \& (A.2)}\\
&= r\varkappa_h\varepsilon_h + \mathfrak{y}_x(r\varepsilon_a\varepsilon_d - r\delta_b) + \mathfrak{y}_u(r\varepsilon_a\delta_d - r\varepsilon_c\delta_b) + \varepsilon_b\mathfrak{y}_r(r\varepsilon_a z^2 - r\varepsilon_d\delta_b) + \varepsilon_b\mathfrak{y}_n(r\varepsilon_c z^2 - r\varepsilon_d\delta_d)\\
&\quad + \varkappa_a(r^2\delta_d - r^2\varepsilon_c\varepsilon_d) + \varkappa_b(r^2z^2 - r^2\varepsilon_d^2) - \varrho_a\mathfrak{y}_s[r\varepsilon_a(-r\varrho_b\varphi_b) - r(r\varrho_b)\delta_b]\\
&\quad - \varkappa_c[r\varepsilon_c(-r\varrho_b\varphi_b) - r(r\varrho_b)\delta_d] - \varkappa_d[r\varepsilon_d(-r\varrho_b\varphi_b) - r(r\varrho_b)z^2] \text{ by (6.1a), (6.10a) \& (6.27)}\\
&= r\varkappa_h\varepsilon_h + r\mathfrak{y}_x(\varepsilon_a\varepsilon_d - \delta_b) + r\mathfrak{y}_u(\varepsilon_a\delta_d - \varepsilon_c\delta_b) + r\varepsilon_b\mathfrak{y}_r(\varepsilon_a z^2 - \varepsilon_d\delta_b) + r\varepsilon_b\mathfrak{y}_n(\varepsilon_c z^2 - \varepsilon_d\delta_d)\\
&\quad + r^2\varkappa_a(\delta_d - \varepsilon_c\varepsilon_d) + r^2\varkappa_b(z^2 - \varepsilon_d^2) + r^2\varrho_a\mathfrak{y}_s\varrho_b(\varepsilon_a\varphi_b + \delta_b) + r^2\varkappa_c\varrho_b(\varepsilon_c\varphi_b + \delta_d)\\
&\quad + r^2\varkappa_d\varrho_b(\varepsilon_d\varphi_b + z^2)\\
&= \varkappa_q \text{ by (6.21l)}
\end{aligned}
\tag{6.43h}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)\\
&\quad = (\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot[\varkappa_g\mathbf{r} + \varkappa_h\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\qquad + \varkappa_b(\mathbf{r}\times\mathbf{z}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u}) - \varkappa_d(\mathbf{z}\times\mathbf{u})] \text{ by (6.41d)}\\
&\quad = \varkappa_g[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})] + \varkappa_h[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})] + \mathfrak{y}_x[(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})] + \mathfrak{y}_u[(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\qquad + \varepsilon_b\mathfrak{y}_r[(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})] + \varepsilon_b\mathfrak{y}_n[(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})] + \varkappa_a[(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]\\
&\qquad + \varkappa_b[(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\mathbf{r}\times\mathbf{z})] - \varrho_a\mathfrak{y}_s[(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})] - \varkappa_c[(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]\\
&\qquad - \varkappa_d[(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_g[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{u}\times\mathbf{r})] + \varkappa_h[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{u}\times\mathbf{h})] + \mathfrak{y}_x[(\mathbf{u}\cdot\widehat{\mathbf{r}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{r}})(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{y}_u[(\mathbf{u}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad + \varepsilon_b\mathfrak{y}_r[(\mathbf{u}\cdot\mathbf{z}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})] + \varepsilon_b\mathfrak{y}_n[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{u}\cdot\mathbf{z}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{u}\cdot\widehat{\mathbf{p}})]\\
&\quad + \varkappa_a[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{u}\cdot\widehat{\mathbf{p}}) - (\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{u}\cdot\mathbf{r})] + \varkappa_b[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{u}\cdot\mathbf{z}) - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{u}\cdot\mathbf{r})] - \varrho_a\mathfrak{y}_s[u^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})^2]\\
&\quad - \varkappa_c[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})u^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\mathbf{u}\cdot\widehat{\mathbf{p}})] - \varkappa_d[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})u^2 - (\widehat{\boldsymbol{\kappa}}\cdot\mathbf{u})(\mathbf{u}\cdot\mathbf{z})] \text{ by (A.2) \& (A.4)}\\
&= \varkappa_g(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h}) + \varkappa_h[\widehat{\boldsymbol{\kappa}}\cdot(-\mathbf{z} - q\widehat{\mathbf{r}})] + \mathfrak{y}_x(r\varrho_b - \varepsilon_a\varrho_o) + \mathfrak{y}_u[(\mathbf{u}\cdot\widehat{\mathbf{p}}) - \varepsilon_b\varrho_o]\\
&\quad + \varepsilon_b\mathfrak{y}_r(-r\varrho_b\varphi_b - \delta_b\varrho_o) + \varepsilon_b\mathfrak{y}_n[\varepsilon_b(-r\varrho_b\varphi_b) - \delta_b(\mathbf{u}\cdot\widehat{\mathbf{p}})]\\
&\quad + \varkappa_a[r\varepsilon_a(\mathbf{u}\cdot\widehat{\mathbf{p}}) - r\varepsilon_b(r\varrho_b)] + \varkappa_b[r\varepsilon_a(-r\varrho_b\varphi_b) - r\delta_b(r\varrho_b)] - \varrho_a\mathfrak{y}_s(\varphi_h^2 - \varrho_o^2)\\
&\quad - \varkappa_c[\varepsilon_b\varphi_h^2 - \varrho_o(\mathbf{u}\cdot\widehat{\mathbf{p}})] - \varkappa_d[\delta_b\varphi_h^2 - \varrho_o(-r\varrho_b\varphi_b)] \text{ by (1.5c), (6.27) \& (6.3b)}
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_g\delta_a - \varkappa_h(\delta_b + q\varepsilon_a) + \mathfrak{y}_x(r\varrho_b - \varepsilon_a\varrho_o) + \mathfrak{y}_u(\mathbf{u}\cdot\widehat{\mathbf{p}}) - \mathfrak{y}_u\varepsilon_b\varrho_o - \varepsilon_b\mathfrak{y}_r(r\varrho_b\varphi_b + \delta_b\varrho_o)\\
&\quad - \varepsilon_b\mathfrak{y}_n r\varepsilon_b\varrho_b\varphi_b - \varepsilon_b\mathfrak{y}_n\delta_b(\mathbf{u}\cdot\widehat{\mathbf{p}}) + \varkappa_a r\varepsilon_a(\mathbf{u}\cdot\widehat{\mathbf{p}}) - r^2\varkappa_a\varepsilon_b\varrho_b - r^2\varkappa_b\varrho_b(\varepsilon_a\varphi_b + \delta_b)\\
&\quad - \varrho_a\mathfrak{y}_s(\varphi_h^2 - \varrho_o^2) - \varkappa_c\varepsilon_b\varphi_h^2 + \varkappa_c\varrho_o(\mathbf{u}\cdot\widehat{\mathbf{p}}) - \varkappa_d(\delta_b\varphi_h^2 + r\varrho_o\varrho_b\varphi_b) \text{ by (6.1a) \& (6.10a)}\\
&= \varkappa_g\delta_a - \varkappa_h(\delta_b + q\varepsilon_a) + \mathfrak{y}_x(r\varrho_b - \varepsilon_a\varrho_o) - \mathfrak{y}_u\varepsilon_b\varrho_o - \varepsilon_b\mathfrak{y}_r(r\varrho_b\varphi_b + \delta_b\varrho_o)\\
&\quad - \varepsilon_b\mathfrak{y}_n r\varepsilon_b\varrho_b\varphi_b - r^2\varkappa_a\varepsilon_b\varrho_b - r^2\varkappa_b\varrho_b(\varepsilon_a\varphi_b + \delta_b) - \varrho_a\mathfrak{y}_s(\varphi_h^2 - \varrho_o^2) - \varkappa_c\varepsilon_b\varphi_h^2\\
&\quad - \varkappa_d(\delta_b\varphi_h^2 + r\varrho_o\varrho_b\varphi_b) + \mathfrak{y}_u(\mathbf{u}\cdot\widehat{\mathbf{p}}) - \varepsilon_b\mathfrak{y}_n\delta_b(\mathbf{u}\cdot\widehat{\mathbf{p}}) + \varkappa_a r\varepsilon_a(\mathbf{u}\cdot\widehat{\mathbf{p}}) + \varkappa_c\varrho_o(\mathbf{u}\cdot\widehat{\mathbf{p}})
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_g\delta_a - \varkappa_h\hbar_a + \mathfrak{y}_x(r\varrho_b - \varepsilon_a\varrho_o) - \mathfrak{y}_u\varepsilon_b\varrho_o - \varepsilon_b\mathfrak{y}_r(r\varrho_b\varphi_b + \delta_b\varrho_o)\\
&\quad - \varepsilon_b\mathfrak{y}_n r\varepsilon_b\varrho_b\varphi_b - r^2\varkappa_a\varepsilon_b\varrho_b - r^2\varkappa_b\varrho_b(\varepsilon_a\varphi_b + \delta_b) - \varrho_a\mathfrak{y}_s(\varphi_h^2 - \varrho_o^2) - \varkappa_c\varepsilon_b\varphi_h^2\\
&\quad - \varkappa_d(\delta_b\varphi_h^2 + r\varrho_o\varrho_b\varphi_b) + (\mathfrak{y}_u - \varepsilon_b\mathfrak{y}_n\delta_b + \varkappa_a r\varepsilon_a + \varkappa_c\varrho_o)(\mathbf{u}\cdot\widehat{\mathbf{p}}) \text{ by (6.10b)}\\
&= \varkappa_g\delta_a - \varkappa_h\hbar_a + \mathfrak{y}_x(r\varrho_b - \varepsilon_a\varrho_o) - \mathfrak{y}_u\varepsilon_b\varrho_o - \varepsilon_b\mathfrak{y}_r(r\varrho_b\varphi_b + \delta_b\varrho_o)\\
&\quad - \varepsilon_b^2\mathfrak{y}_n r\varrho_b\varphi_b - r^2\varkappa_a\varepsilon_b\varrho_b - r^2\varkappa_b\varrho_b(\varepsilon_a\varphi_b + \delta_b) - \varrho_a\mathfrak{y}_s(\varphi_h^2 - \varrho_o^2) - \varkappa_c\varepsilon_b\varphi_h^2\\
&\quad - \varkappa_d(\delta_b\varphi_h^2 + r\varrho_o\varrho_b\varphi_b) + h^{-2}(\mathfrak{y}_u - \varepsilon_b\mathfrak{y}_n\delta_b + \varkappa_a r\varepsilon_a + \varkappa_c\varrho_o)(\varepsilon_i + \varepsilon_f\varphi_b) \text{ by (6.27c)}\\
&= \varkappa_r \text{ by (6.21l)}
\end{aligned}
\tag{6.43i}
$$

$$
\begin{aligned}
&(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\mathfrak{J}_p + \mathfrak{J}_q)\\
&\quad = (\widehat{\mathbf{p}}\times\mathbf{u})\cdot[\varkappa_g\mathbf{r} + \varkappa_h\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z}) + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\qquad + \varkappa_b(\mathbf{r}\times\mathbf{z}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u}) - \varkappa_d(\mathbf{z}\times\mathbf{u})] \text{ by (6.41d)}\\
&\quad = \varkappa_g[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{u})] + \varkappa_h[\mathbf{h}\cdot(\widehat{\mathbf{p}}\times\mathbf{u})] + \mathfrak{y}_x[(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})] + \mathfrak{y}_u[(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\qquad + \varepsilon_b\mathfrak{y}_r[(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})] + \varepsilon_b\mathfrak{y}_n[(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})] + \varkappa_a[(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]\\
&\qquad + \varkappa_b[(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\mathbf{r}\times\mathbf{z})] - \varrho_a\mathfrak{y}_s[(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})] - \varkappa_c[(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]\\
&\qquad - \varkappa_d[(\widehat{\mathbf{p}}\times\mathbf{u})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_g[\widehat{\mathbf{p}}\cdot(\mathbf{u}\times\mathbf{r})] + \varkappa_h[\widehat{\mathbf{p}}\cdot(\mathbf{u}\times\mathbf{h})] + \mathfrak{y}_x[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{u}\cdot\widehat{\mathbf{r}}) - (\widehat{\mathbf{p}}\cdot\widehat{\mathbf{r}})(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{y}_u[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{u}\cdot\widehat{\mathbf{p}}) - (\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad + \varepsilon_b\mathfrak{y}_r[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{u}\cdot\mathbf{z}) - (\widehat{\mathbf{p}}\cdot\mathbf{z})(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})] + \varepsilon_b\mathfrak{y}_n[(\mathbf{u}\cdot\mathbf{z}) - (\widehat{\mathbf{p}}\cdot\mathbf{z})(\mathbf{u}\cdot\widehat{\mathbf{p}})] + \varkappa_a[(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{u}\cdot\widehat{\mathbf{p}}) - (\mathbf{u}\cdot\mathbf{r})]\\
&\quad + \varkappa_b[(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{u}\cdot\mathbf{z}) - (\widehat{\mathbf{p}}\cdot\mathbf{z})(\mathbf{u}\cdot\mathbf{r})] - \varrho_a\mathfrak{y}_s[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})u^2 - (\widehat{\mathbf{p}}\cdot\mathbf{u})(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})] - \varkappa_c[u^2 - (\widehat{\mathbf{p}}\cdot\mathbf{u})^2]\\
&\quad - \varkappa_d[(\widehat{\mathbf{p}}\cdot\mathbf{z})u^2 - (\widehat{\mathbf{p}}\cdot\mathbf{u})(\mathbf{u}\cdot\mathbf{z})] \text{ by (A.4) \& (A.2)}\\
&= \varkappa_g(\widehat{\mathbf{p}}\cdot\mathbf{h}) + \varkappa_h[\widehat{\mathbf{p}}\cdot(-\mathbf{z} - q\widehat{\mathbf{r}})] + \mathfrak{y}_x[\varepsilon_b(r\varrho_b) - \varepsilon_c\varrho_o] + \mathfrak{y}_u[\varepsilon_b(\mathbf{u}\cdot\widehat{\mathbf{p}}) - \varrho_o]\\
&\quad + \varepsilon_b\mathfrak{y}_r[\varepsilon_b(-r\varrho_b\varphi_b) - \delta_d\varrho_o] + \varepsilon_b\mathfrak{y}_n[(-r\varrho_b\varphi_b) - \delta_d(\mathbf{u}\cdot\widehat{\mathbf{p}})] + \varkappa_a[r\varepsilon_c(\mathbf{u}\cdot\widehat{\mathbf{p}}) - r(r\varrho_b)]\\
&\quad + \varkappa_b[r\varepsilon_c(-r\varrho_b\varphi_b) - r\delta_d(r\varrho_b)] - \varrho_a\mathfrak{y}_s[\varepsilon_b\varphi_h^2 - (\widehat{\mathbf{p}}\cdot\mathbf{u})\varrho_o] - \varkappa_c[\varphi_h^2 - (\widehat{\mathbf{p}}\cdot\mathbf{u})^2]\\
&\quad - \varkappa_d[\delta_d\varphi_h^2 - (\widehat{\mathbf{p}}\cdot\mathbf{u})(-r\varrho_b\varphi_b)] \text{ by (1.5c), (6.1a), (6.3b) \& (6.10a)}\\
&= \varkappa_g\delta_c + \varkappa_h(-\delta_d - q\varepsilon_c) + \mathfrak{y}_x(r\varepsilon_b\varrho_b - \varepsilon_c\varrho_o) + \mathfrak{y}_u\varepsilon_b(\mathbf{u}\cdot\widehat{\mathbf{p}}) - \mathfrak{y}_u\varrho_o\\
&\quad - \varepsilon_b\mathfrak{y}_r(r\varepsilon_b\varrho_b\varphi_b + \delta_d\varrho_o) - \varepsilon_b\mathfrak{y}_n r\varrho_b\varphi_b - \varepsilon_b\mathfrak{y}_n\delta_d(\mathbf{u}\cdot\widehat{\mathbf{p}}) + r\varkappa_a\varepsilon_c(\mathbf{u}\cdot\widehat{\mathbf{p}}) - r^2\varkappa_a\varrho_b\\
&\quad - r^2\varkappa_b\varrho_b(\varepsilon_c\varphi_b + \delta_d) - \varrho_a\mathfrak{y}_s\varepsilon_b\varphi_h^2 + \varrho_o\varrho_a\mathfrak{y}_s(\widehat{\mathbf{p}}\cdot\mathbf{u}) - \varkappa_c\varphi_h^2 + \varkappa_c(\widehat{\mathbf{p}}\cdot\mathbf{u})^2\\
&\quad - \varkappa_d\delta_d\varphi_h^2 - r\varkappa_d\varrho_b\varphi_b(\widehat{\mathbf{p}}\cdot\mathbf{u}) \text{ by (6.10a)}
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_g\delta_c - \varkappa_h(\delta_d + q\varepsilon_c) + \mathfrak{y}_x(r\varepsilon_b\varrho_b - \varepsilon_c\varrho_o) - \mathfrak{y}_u\varrho_o - \varepsilon_b\mathfrak{y}_r(r\varepsilon_b\varrho_b\varphi_b + \delta_d\varrho_o) - \varepsilon_b\mathfrak{y}_n r\varrho_b\varphi_b\\
&\quad - r^2\varkappa_a\varrho_b - r^2\varkappa_b\varrho_b(\varepsilon_c\varphi_b + \delta_d) - \varrho_a\mathfrak{y}_s\varepsilon_b\varphi_h^2 - \varkappa_c\varphi_h^2 - \varkappa_d\delta_d\varphi_h^2 + \mathfrak{y}_u\varepsilon_b(\mathbf{u}\cdot\widehat{\mathbf{p}}) - \varepsilon_b\mathfrak{y}_n\delta_d(\mathbf{u}\cdot\widehat{\mathbf{p}})\\
&\quad + r\varkappa_a\varepsilon_c(\mathbf{u}\cdot\widehat{\mathbf{p}}) + \varrho_o\varrho_a\mathfrak{y}_s(\widehat{\mathbf{p}}\cdot\mathbf{u}) + \varkappa_c(\widehat{\mathbf{p}}\cdot\mathbf{u})^2 - r\varkappa_d\varrho_b\varphi_b(\widehat{\mathbf{p}}\cdot\mathbf{u})\\
&= \varkappa_g\delta_c - \varkappa_h(\delta_d + q\varepsilon_c) + \mathfrak{y}_x(r\varepsilon_b\varrho_b - \varepsilon_c\varrho_o) - \mathfrak{y}_u\varrho_o - \varepsilon_b\mathfrak{y}_r(r\varepsilon_b\varrho_b\varphi_b + \delta_d\varrho_o)\\
&\quad - \varepsilon_b\mathfrak{y}_n r\varrho_b\varphi_b - r^2\varkappa_a\varrho_b - r^2\varkappa_b\varrho_b(\varepsilon_c\varphi_b + \delta_d) - \varrho_a\mathfrak{y}_s\varepsilon_b\varphi_h^2 - \varkappa_c\varphi_h^2 - \varkappa_d\delta_d\varphi_h^2\\
&\quad + \varkappa_c(\widehat{\mathbf{p}}\cdot\mathbf{u})^2 + (\mathfrak{y}_u\varepsilon_b - \varepsilon_b\mathfrak{y}_n\delta_d + r\varkappa_a\varepsilon_c + \varrho_o\varrho_a\mathfrak{y}_s - r\varkappa_d\varrho_b\varphi_b)(\widehat{\mathbf{p}}\cdot\mathbf{u})\\
&= \varkappa_g\delta_c - \varkappa_h\hbar_c + \mathfrak{y}_x(r\varepsilon_b\varrho_b - \varepsilon_c\varrho_o) - \mathfrak{y}_u\varrho_o - \varepsilon_b\mathfrak{y}_r(r\varepsilon_b\varrho_b\varphi_b + \delta_d\varrho_o) - r\varepsilon_b\mathfrak{y}_n\varrho_b\varphi_b\\
&\quad - r^2\varrho_b[\varkappa_a + \varkappa_b(\varepsilon_c\varphi_b + \delta_d)] - \varphi_h^2(\varrho_a\mathfrak{y}_s\varepsilon_b + \varkappa_c + \varkappa_d\delta_d) + h^{-4}\varkappa_c(\varepsilon_i + \varepsilon_f\varphi_b)^2\\
&\quad + h^{-2}(\mathfrak{y}_u\varepsilon_b - \varepsilon_b\mathfrak{y}_n\delta_d + r\varkappa_a\varepsilon_c + \varrho_o\varrho_a\mathfrak{y}_s - r\varkappa_d\varrho_b\varphi_b)(\varepsilon_i + \varepsilon_f\varphi_b) \text{ by (6.27c) \& (6.10b)}\\
&= \varkappa_s \text{ by (6.21m)}
\end{aligned}
\tag{6.43j}
$$

$$
\begin{aligned}
&(\mathbf{z}\times\mathbf{u})\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\\
&= (\mathbf{z}\times\mathbf{u})\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})] \text{ by (6.41d)}\\
&= \varkappa_g[\mathbf{r}\cdot(\mathbf{z}\times\mathbf{u})]+\varkappa_h[\mathbf{h}\cdot(\mathbf{z}\times\mathbf{u})]+\mathfrak{y}_x[(\mathbf{z}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[(\mathbf{z}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\varepsilon_b\mathfrak{y}_r[(\mathbf{z}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[(\mathbf{z}\times\mathbf{u})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\varkappa_a[(\mathbf{z}\times\mathbf{u})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]\\
&\quad+\varkappa_b[(\mathbf{z}\times\mathbf{u})\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[(\mathbf{z}\times\mathbf{u})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[(\mathbf{z}\times\mathbf{u})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]\\
&\quad-\varkappa_d[(\mathbf{z}\times\mathbf{u})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_g[\mathbf{z}\cdot(\mathbf{u}\times\mathbf{r})]+\varkappa_h[\mathbf{z}\cdot(\mathbf{u}\times\mathbf{h})]+\mathfrak{y}_x[(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{u}\cdot\widehat{\mathbf{r}})-(\mathbf{z}\cdot\widehat{\mathbf{r}})(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\mathfrak{y}_u[(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{u}\cdot\widehat{\mathbf{p}})-(\mathbf{z}\cdot\widehat{\mathbf{p}})(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})]+\varepsilon_b\mathfrak{y}_r[(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{u}\cdot\mathbf{z})-z^2(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\varepsilon_b\mathfrak{y}_n[(\mathbf{z}\cdot\widehat{\mathbf{p}})(\mathbf{u}\cdot\mathbf{z})-z^2(\mathbf{u}\cdot\widehat{\mathbf{p}})]+\varkappa_a[(\mathbf{z}\cdot\mathbf{r})(\mathbf{u}\cdot\widehat{\mathbf{p}})-(\mathbf{z}\cdot\widehat{\mathbf{p}})(\mathbf{u}\cdot\mathbf{r})]\\
&\quad+\varkappa_b[(\mathbf{z}\cdot\mathbf{r})(\mathbf{u}\cdot\mathbf{z})-z^2(\mathbf{u}\cdot\mathbf{r})]-\varrho_a\mathfrak{y}_s[(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})u^2-(\mathbf{z}\cdot\mathbf{u})(\mathbf{u}\cdot\widehat{\boldsymbol{\kappa}})]-\varkappa_c[(\mathbf{z}\cdot\widehat{\mathbf{p}})u^2-(\mathbf{z}\cdot\mathbf{u})(\mathbf{u}\cdot\widehat{\mathbf{p}})]\\
&\quad-\varkappa_d[z^2u^2-(\mathbf{z}\cdot\mathbf{u})^2] \text{ by (A.2) \& (A.4)}\\
&= \varkappa_g(\mathbf{z}\cdot\mathbf{h})+\varkappa_h[\mathbf{z}\cdot(-\mathbf{z}-q\widehat{\mathbf{r}})]+\mathfrak{y}_x[\delta_b(r\varrho_b)-\varepsilon_d\varrho_o]+\mathfrak{y}_u[\delta_b(\mathbf{u}\cdot\widehat{\mathbf{p}})-\delta_d\varrho_o]\\
&\quad+\varepsilon_b\mathfrak{y}_r[\delta_b(-r\varrho_b\varphi_b)-z^2\varrho_o]+\varepsilon_b\mathfrak{y}_n[\delta_d(-r\varrho_b\varphi_b)-z^2(\mathbf{u}\cdot\widehat{\mathbf{p}})]+\varkappa_a[r\varepsilon_d(\mathbf{u}\cdot\widehat{\mathbf{p}})-r\delta_d(r\varrho_b)]\\
&\quad+\varkappa_b[r\varepsilon_d(-r\varrho_b\varphi_b)-rz^2(r\varrho_b)]-\varrho_a\mathfrak{y}_s[\delta_b\varphi_h^2-(-r\varrho_b\varphi_b)\varrho_o]-\varkappa_c[\delta_d\varphi_h^2-(-r\varrho_b\varphi_b)(\mathbf{u}\cdot\widehat{\mathbf{p}})]\\
&\quad-\varkappa_d[z^2\varphi_h^2-(-r\varrho_b\varphi_b)^2] \text{ by (1.5c), (6.10a), (6.27) \& (6.3b)}
\end{aligned}
$$

$$
\begin{aligned}
&= \varkappa_h(-z^2-q\varepsilon_d)+\mathfrak{y}_x(r\delta_b\varrho_b-\varepsilon_d\varrho_o)+\mathfrak{y}_u\delta_b(\mathbf{u}\cdot\widehat{\mathbf{p}})-\mathfrak{y}_u\delta_d\varrho_o\\
&\quad-\varepsilon_b\mathfrak{y}_r(r\delta_b\varrho_b\varphi_b+z^2\varrho_o)-r\varepsilon_b\mathfrak{y}_n\delta_d\varrho_b\varphi_b-z^2\varepsilon_b\mathfrak{y}_n(\mathbf{u}\cdot\widehat{\mathbf{p}})+r\varkappa_a\varepsilon_d(\mathbf{u}\cdot\widehat{\mathbf{p}})-r^2\varkappa_a\delta_d\varrho_b\\
&\quad-r^2\varkappa_b\varrho_b(\varepsilon_d\varphi_b+z^2)-\varrho_a\mathfrak{y}_s(\delta_b\varphi_h^2+r\varrho_b\varphi_b\varrho_o)-\varkappa_c\delta_d\varphi_h^2-r\varkappa_c\varrho_b\varphi_b(\mathbf{u}\cdot\widehat{\mathbf{p}})\\
&\quad-\varkappa_d(z^2\varphi_h^2-r^2\varrho_b^2\varphi_b^2) \text{ by (1.5c), (6.1a) \& (6.10a)}\\
&= -\varkappa_h(z^2+q\varepsilon_d)+\mathfrak{y}_x(r\delta_b\varrho_b-\varepsilon_d\varrho_o)-\mathfrak{y}_u\delta_d\varrho_o-\varepsilon_b\mathfrak{y}_r(r\delta_b\varrho_b\varphi_b+z^2\varrho_o)-r\varepsilon_b\mathfrak{y}_n\delta_d\varrho_b\varphi_b-r^2\varkappa_a\delta_d\varrho_b\\
&\quad-r^2\varkappa_b\varrho_b(\varepsilon_d\varphi_b+z^2)-\varrho_a\mathfrak{y}_s(\delta_b\varphi_h^2+r\varrho_b\varphi_b\varrho_o)-\varkappa_c\delta_d\varphi_h^2-\varkappa_d(z^2\varphi_h^2-r^2\varrho_b^2\varphi_b^2)\\
&\quad+\mathfrak{y}_u\delta_b(\mathbf{u}\cdot\widehat{\mathbf{p}})-z^2\varepsilon_b\mathfrak{y}_n(\mathbf{u}\cdot\widehat{\mathbf{p}})+r\varkappa_a\varepsilon_d(\mathbf{u}\cdot\widehat{\mathbf{p}})-r\varkappa_c\varrho_b\varphi_b(\mathbf{u}\cdot\widehat{\mathbf{p}})
\end{aligned}
$$

$$
\begin{aligned}
&= -\varkappa_h\mathfrak{y}_z+\mathfrak{y}_x(r\delta_b\varrho_b-\varepsilon_d\varrho_o)-\mathfrak{y}_u\delta_d\varrho_o-\varepsilon_b\mathfrak{y}_r(r\delta_b\varrho_b\varphi_b+z^2\varrho_o)-r\varepsilon_b\mathfrak{y}_n\delta_d\varrho_b\varphi_b-r^2\varkappa_a\delta_d\varrho_b\\
&\quad-r^2\varkappa_b\varrho_b(\varepsilon_d\varphi_b+z^2)-\varrho_a\mathfrak{y}_s(\delta_b\varphi_h^2+r\varrho_b\varphi_b\varrho_o)-\varkappa_c\delta_d\varphi_h^2-\varkappa_d(z^2\varphi_h^2-r^2\varrho_b^2\varphi_b^2)\\
&\quad+(\mathfrak{y}_u\delta_b-z^2\varepsilon_b\mathfrak{y}_n+r\varkappa_a\varepsilon_d-r\varkappa_c\varrho_b\varphi_b)(\mathbf{u}\cdot\widehat{\mathbf{p}}) \text{ by (6.21h)}\\
&= -\varkappa_h\mathfrak{y}_z+\mathfrak{y}_x(r\delta_b\varrho_b-\varepsilon_d\varrho_o)-\mathfrak{y}_u\delta_d\varrho_o-\varepsilon_b\mathfrak{y}_r(r\delta_b\varrho_b\varphi_b+z^2\varrho_o)-r\varepsilon_b\mathfrak{y}_n\delta_d\varrho_b\varphi_b-r^2\varkappa_a\delta_d\varrho_b\\
&\quad-r^2\varkappa_b\varrho_b(\varepsilon_d\varphi_b+z^2)-\varrho_a\mathfrak{y}_s(\delta_b\varphi_h^2+r\varrho_b\varphi_b\varrho_o)-\varkappa_c\delta_d\varphi_h^2-\varkappa_d(z^2\varphi_h^2-r^2\varrho_b^2\varphi_b^2)\\
&\quad+h^{-2}(\mathfrak{y}_u\delta_b-z^2\varepsilon_b\mathfrak{y}_n+r\varkappa_a\varepsilon_d-r\varkappa_c\varrho_b\varphi_b)(\varepsilon_i+\varepsilon_f\varphi_b) \text{ by (6.27c)}\\
&= \varkappa_t \text{ by (6.21m)} \qquad (6.43k)
\end{aligned}
$$

$$
\begin{aligned}
&|\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q|^2 = (\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \\
&= (\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot [\varkappa_g \mathbf{r} + \varkappa_h \mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \varepsilon_b \mathfrak{y}_r(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \varepsilon_b \mathfrak{y}_n(\widehat{\mathbf{p}} \times \mathbf{z}) \\
&\quad + \varkappa_a(\mathbf{r} \times \widehat{\mathbf{p}}) + \varkappa_b(\mathbf{r} \times \mathbf{z}) - \varrho_a \mathfrak{y}_s(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}} \times \mathbf{u}) - \varkappa_d(\mathbf{z} \times \mathbf{u})] \text{ by (6.41d)} \\
&= \varkappa_g[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot \mathbf{r}] + \varkappa_h[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot \mathbf{h}] + \mathfrak{y}_x[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}})] + \mathfrak{y}_u[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] \\
&\quad + \varepsilon_b \mathfrak{y}_r[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{z})] + \varepsilon_b \mathfrak{y}_n[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\widehat{\mathbf{p}} \times \mathbf{z})] + \varkappa_a[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] \\
&\quad + \varkappa_b[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\mathbf{r} \times \mathbf{z})] - \varrho_a \mathfrak{y}_s[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{u})] - \varkappa_c[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\widehat{\mathbf{p}} \times \mathbf{u})] \\
&\quad - \varkappa_d[(\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \cdot (\mathbf{z} \times \mathbf{u})] \\
&= \varkappa_g \varkappa_j + \varkappa_h \varkappa_k + \mathfrak{y}_x \varkappa_l + \mathfrak{y}_u \varkappa_m + \varepsilon_b \mathfrak{y}_r \varkappa_n + \varepsilon_b \mathfrak{y}_n \varkappa_o + \varkappa_a \varkappa_p + \varkappa_b \varkappa_q - \varrho_a \mathfrak{y}_s \varkappa_r - \varkappa_c \varkappa_s \\
&\quad - \varkappa_d \varkappa_t \text{ by (6.43)} \\
&\therefore |\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q| = \varkappa_u \text{ by (6.21n).}
\end{aligned} \tag{6.44}
$$

Art 26j. *Development of equation* (3.15d).

To evaluate the quantities defined by eqnrefkpath2d, we first derive

$$
\begin{aligned}
\mathbf{z} \times \mathbf{u} &= h^{-2}[\mathbf{z} \times (\mathbf{z} \times \mathbf{h} + q\widehat{\mathbf{r}} \times \mathbf{h})] \text{ by (1.5c)} \\
&= h^{-2}[\mathbf{z} \times (\mathbf{z} \times \mathbf{h}) + q\mathbf{z} \times (\widehat{\mathbf{r}} \times \mathbf{h})] \\
&= h^{-2}[\mathbf{z}(\mathbf{z} \cdot \mathbf{h}) - z^2 \mathbf{h} + q\widehat{\mathbf{r}}(\mathbf{z} \cdot \mathbf{h}) - q\mathbf{h}(\mathbf{z} \cdot \widehat{\mathbf{r}})] \text{ by (A.1)} \\
&= h^{-2}(-z^2 \mathbf{h} - q\varepsilon_d \mathbf{h}) \text{ by (1.5c) \& (6.1a)} \\
&= -h^{-2}(z^2 + q\varepsilon_d)\mathbf{h} = -h^{-2}\mathfrak{y}_z \mathbf{h} \text{ by (6.21h)}
\end{aligned} \tag{6.45a}
$$

$$
\begin{aligned}
&\widehat{\boldsymbol{\kappa}} \cdot (\boldsymbol{\mathfrak{I}}_p + \boldsymbol{\mathfrak{I}}_q) \\
&= \widehat{\boldsymbol{\kappa}} \cdot [\varkappa_g \mathbf{r} + \varkappa_h \mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \varepsilon_b \mathfrak{y}_r(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \varepsilon_b \mathfrak{y}_n(\widehat{\mathbf{p}} \times \mathbf{z}) + \varkappa_a(\mathbf{r} \times \widehat{\mathbf{p}}) \\
&\quad + \varkappa_b(\mathbf{r} \times \mathbf{z}) - \varrho_a \mathfrak{y}_s(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}} \times \mathbf{u}) - \varkappa_d(\mathbf{z} \times \mathbf{u})] \text{ by (6.41d)} \\
&= \varkappa_g(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r}) + \varkappa_h(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{h}) + \mathfrak{y}_x[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}})] + \mathfrak{y}_u[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \varepsilon_b \mathfrak{y}_r[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{z})] + \varepsilon_b \mathfrak{y}_n[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\mathbf{p}} \times \mathbf{z})] \\
&\quad + \varkappa_a[\widehat{\boldsymbol{\kappa}} \cdot (\mathbf{r} \times \widehat{\mathbf{p}})] + \varkappa_b[\widehat{\boldsymbol{\kappa}} \cdot (\mathbf{r} \times \mathbf{z})] - \varrho_a \mathfrak{y}_s[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{u})] - \varkappa_c[\widehat{\boldsymbol{\kappa}} \cdot (\widehat{\mathbf{p}} \times \mathbf{u})] - \varkappa_d[\widehat{\boldsymbol{\kappa}} \cdot (\mathbf{z} \times \mathbf{u})] \\
&= r\varkappa_g \varepsilon_a + \varkappa_h \delta_a + \varepsilon_b \mathfrak{y}_n \varsigma_c - \varkappa_a \varsigma_a - r\varkappa_b \delta_f - h^{-2}\varkappa_c[\widehat{\boldsymbol{\kappa}} \cdot (\delta_c \mathbf{z} + q\delta_c \widehat{\mathbf{r}} - \hbar_c \mathbf{h})] \\
&\quad - h^{-2}\varkappa_d[\widehat{\boldsymbol{\kappa}} \cdot (-\mathfrak{y}_z \mathbf{h})] \text{ by (6.1a), (6.10a), (6.21a), (6.11d) \& (6.45a)} \\
&= r\varkappa_g \varepsilon_a + \varkappa_h \delta_a + \varepsilon_b \mathfrak{y}_n \varsigma_c - \varkappa_a \varsigma_a - r\varkappa_b \delta_f - h^{-2}\varkappa_c[\delta_c(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{z}) + q\delta_c(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{r}}) - \hbar_c(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{h})] \\
&\quad + h^{-2}\varkappa_d \mathfrak{y}_z(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{h}) \\
&= r\varkappa_g \varepsilon_a + \varkappa_h \delta_a + \varepsilon_b \mathfrak{y}_n \varsigma_c - \varkappa_a \varsigma_a - r\varkappa_b \delta_f - h^{-2}\varkappa_c(\delta_c \delta_b + q\delta_c \varepsilon_a - \hbar_c \delta_a) \\
&\quad + h^{-2}\varkappa_d \mathfrak{y}_z \delta_a \text{ by (6.10a) \& (6.1a)} \\
&= r\varkappa_g \varepsilon_a + \varkappa_h \delta_a + \varepsilon_b \mathfrak{y}_n \varsigma_c - \varkappa_a \varsigma_a - r\varkappa_b \delta_f - h^{-2}\varkappa_c[\delta_c(\delta_b + q\varepsilon_a) - \hbar_c \delta_a] + h^{-2}\varkappa_d \mathfrak{y}_z \delta_a \\
&= r\varkappa_g \varepsilon_a + \varkappa_h \delta_a + \varepsilon_b \mathfrak{y}_n \varsigma_c - \varkappa_a \varsigma_a - r\varkappa_b \delta_f - (\varkappa_c/h^2)(\delta_c \hbar_a - \hbar_c \delta_a) + (\varkappa_d/h^2)\mathfrak{y}_z \delta_a \text{ by (6.10b)} \\
&= \varkappa_v \text{ by (6.21n)}
\end{aligned} \tag{6.46a}
$$

$$
\begin{aligned}
&\widehat{\mathbf{p}}\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=\widehat{\mathbf{p}}\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})]\text{ by (6.41d)}\\
&=\varkappa_g(\widehat{\mathbf{p}}\cdot\mathbf{r})+\varkappa_h(\widehat{\mathbf{p}}\cdot\mathbf{h})+\mathfrak{y}_x[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\varepsilon_b\mathfrak{y}_r[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&\quad+\varkappa_a[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]+\varkappa_b[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]-\varkappa_d[\widehat{\mathbf{p}}\cdot(\mathbf{z}\times\mathbf{u})]\\
&=r\varkappa_g\varepsilon_c+\varkappa_h\delta_c-(\mathfrak{y}_x/r)\varsigma_a-\varepsilon_b\mathfrak{y}_r\varsigma_c-\varkappa_b\varsigma_b+\varrho_a\mathfrak{y}_s[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]\\
&\quad-\varkappa_d[\widehat{\mathbf{p}}\cdot(\mathbf{z}\times\mathbf{u})]\text{ by (6.1a), (6.10a), (6.21a) \& (A.4)}\\
&=r\varkappa_g\varepsilon_c+\varkappa_h\delta_c-(\mathfrak{y}_x/r)\varsigma_a-\varepsilon_b\mathfrak{y}_r\varsigma_c-\varkappa_b\varsigma_b+\varrho_a(\mathfrak{y}_s/h^2)[\widehat{\boldsymbol{\kappa}}\cdot(\delta_c\mathbf{z}+q\delta_c\widehat{\mathbf{r}}-\hbar_c\mathbf{h})]\\
&\quad-(\varkappa_d/h^2)[\widehat{\mathbf{p}}\cdot(-\mathfrak{y}_z\mathbf{h})]\text{ by (6.11d) \& (6.45a)}\\
&=r\varkappa_g\varepsilon_c+\varkappa_h\delta_c-(\mathfrak{y}_x/r)\varsigma_a-\varepsilon_b\mathfrak{y}_r\varsigma_c-\varkappa_b\varsigma_b+\varrho_a(\mathfrak{y}_s/h^2)(\delta_c\delta_b+q\delta_c\varepsilon_a-\hbar_c\delta_a)+(\varkappa_d/h^2)\mathfrak{y}_z\delta_c\\
&\quad\text{by (6.1a) \& (6.10a)}\\
&=r\varkappa_g\varepsilon_c+\varkappa_h\delta_c-(\mathfrak{y}_x/r)\varsigma_a-\varepsilon_b\mathfrak{y}_r\varsigma_c-\varkappa_b\varsigma_b+\varrho_a(\mathfrak{y}_s/h^2)[\delta_c(\delta_b+q\varepsilon_a)-\hbar_c\delta_a]+(\varkappa_d/h^2)\mathfrak{y}_z\delta_c\\
&=r\varkappa_g\varepsilon_c+\varkappa_h\delta_c-(\mathfrak{y}_x/r)\varsigma_a-\varepsilon_b\mathfrak{y}_r\varsigma_c-\varkappa_b\varsigma_b+\varrho_a(\mathfrak{y}_s/h^2)(\delta_c\hbar_a-\hbar_c\delta_a)+(\varkappa_d/h^2)\mathfrak{y}_z\delta_c\text{ by (6.10b)}\\
&=\varkappa_w\text{ by (6.21n)}
\end{aligned}
\tag{6.46b}
$$

$$
\begin{aligned}
&\mathbf{z}\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=\mathbf{z}\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})]\text{ by (6.41d)}\\
&=\varkappa_g(\mathbf{z}\cdot\mathbf{r})+\varkappa_h(\mathbf{z}\cdot\mathbf{h})+\mathfrak{y}_x[\mathbf{z}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[\mathbf{z}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\varepsilon_b\mathfrak{y}_r[\mathbf{z}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[\mathbf{z}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&\quad+\varkappa_a[\mathbf{z}\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]+\varkappa_b[\mathbf{z}\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[\mathbf{z}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[\mathbf{z}\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]-\varkappa_d[\mathbf{z}\cdot(\mathbf{z}\times\mathbf{u})]\\
&=r\varkappa_g\varepsilon_d-\mathfrak{y}_x\delta_f+\mathfrak{y}_u\varsigma_c+\varkappa_a\varsigma_b+\varrho_a\mathfrak{y}_s[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{z}\times\mathbf{u})]\\
&\quad+\varkappa_c[\widehat{\mathbf{p}}\cdot(\mathbf{z}\times\mathbf{u})]\text{ by (1.5c), (6.1a), (6.10a), (6.21a) \& (A.4)}\\
&=r\varkappa_g\varepsilon_d-\mathfrak{y}_x\delta_f+\mathfrak{y}_u\varsigma_c+\varkappa_a\varsigma_b+\varrho_a(\mathfrak{y}_s/h^2)[\widehat{\boldsymbol{\kappa}}\cdot(-\mathfrak{y}_z\mathbf{h})]+(\varkappa_c/h^2)[\widehat{\mathbf{p}}\cdot(-\mathfrak{y}_z\mathbf{h})]\text{ by (6.45a)}\\
&=r\varkappa_g\varepsilon_d-\mathfrak{y}_x\delta_f+\mathfrak{y}_u\varsigma_c+\varkappa_a\varsigma_b-\varrho_a(\mathfrak{y}_s/h^2)(\mathfrak{y}_z\delta_a)-(\varkappa_c/h^2)(\mathfrak{y}_z\delta_c)\text{ by (6.10a)}\\
&=r\varkappa_g\varepsilon_d-\mathfrak{y}_x\delta_f+\mathfrak{y}_u\varsigma_c+\varkappa_a\varsigma_b-(\mathfrak{y}_z/h^2)(\mathfrak{y}_s\varrho_a\delta_a+\varkappa_c\delta_c)\\
&=\varkappa_x\text{ by (6.21n)}
\end{aligned}
\tag{6.46c}
$$

$$
\begin{aligned}
&(\mathbf{r}\times\mathbf{h})\cdot(\mathfrak{J}_p+\mathfrak{J}_q)\\
&=(\mathbf{r}\times\mathbf{h})\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\quad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})]\text{ by (6.41d)}\\
&=\varkappa_g[\mathbf{r}\cdot(\mathbf{r}\times\mathbf{h})]+\varkappa_h[\mathbf{h}\cdot(\mathbf{r}\times\mathbf{h})]+\mathfrak{y}_x[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\varepsilon_b\mathfrak{y}_r[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\varkappa_a[(\mathbf{r}\times\mathbf{h})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]\\
&\quad+\varkappa_b[(\mathbf{r}\times\mathbf{h})\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]-\varkappa_d[(\mathbf{r}\times\mathbf{h})\cdot(\mathbf{z}\times\mathbf{u})]\\
&\qquad=\mathfrak{y}_x[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\widehat{\mathbf{r}})-(\mathbf{r}\cdot\widehat{\mathbf{r}})(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{y}_u[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\widehat{\mathbf{p}})-(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\qquad\quad+\varepsilon_b\mathfrak{y}_r[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\mathbf{z})-(\mathbf{r}\cdot\mathbf{z})(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})]+\varepsilon_b\mathfrak{y}_n[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\mathbf{z})-(\mathbf{r}\cdot\mathbf{z})(\mathbf{h}\cdot\widehat{\mathbf{p}})]\\
&\qquad\quad+\varkappa_a[r^2(\mathbf{h}\cdot\widehat{\mathbf{p}})-(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\mathbf{r})]+\varkappa_b[r^2(\mathbf{h}\cdot\mathbf{z})-(\mathbf{r}\cdot\mathbf{z})(\mathbf{h}\cdot\mathbf{r})]\\
&\qquad\quad-\varrho_a\mathfrak{y}_s[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\mathbf{u})-(\mathbf{r}\cdot\mathbf{u})(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})]-\varkappa_c[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\mathbf{u})-(\mathbf{r}\cdot\mathbf{u})(\mathbf{h}\cdot\widehat{\mathbf{p}})]\\
&\qquad\quad-\varkappa_d[(\mathbf{r}\cdot\mathbf{z})(\mathbf{h}\cdot\mathbf{u})-(\mathbf{r}\cdot\mathbf{u})(\mathbf{h}\cdot\mathbf{z})]\text{ by (A.2)}\\
&\qquad=\mathfrak{y}_x(-r\delta_a)+\mathfrak{y}_u(r\varepsilon_a\delta_c-r\varepsilon_c\delta_a)+\varepsilon_b\mathfrak{y}_r(-r\varepsilon_d\delta_a)+\varepsilon_b\mathfrak{y}_n(-r\varepsilon_d\delta_c)+\varkappa_a(r^2\delta_c)\\
&\qquad\quad-\varrho_a\mathfrak{y}_s[-r(r\varrho_b)\delta_a]-\varkappa_c[-r(r\varrho_b)\delta_c]\text{ by (1.5c), (6.10a) \& (6.27b)}\\
&\qquad=-r\mathfrak{y}_x\delta_a+r\mathfrak{y}_u(\varepsilon_a\delta_c-\varepsilon_c\delta_a)-r\varepsilon_b\varepsilon_d(\mathfrak{y}_r\delta_a+\mathfrak{y}_n\delta_c)+r^2(\varkappa_a\delta_c+\varrho_a\mathfrak{y}_s\varrho_b\delta_a+\varkappa_c\varrho_b\delta_c)\\
&\qquad=\varkappa_y\text{ by (6.21n)}
\end{aligned}
\tag{6.46d}
$$

$$
\begin{aligned}
&(\mathbf{z}\times\mathbf{h})\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\\
&\quad=(\mathbf{z}\times\mathbf{h})\cdot[\varkappa_g\mathbf{r}+\varkappa_h\mathbf{h}+\mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})+\mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})+\varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}})\\
&\qquad+\varkappa_b(\mathbf{r}\times\mathbf{z})-\varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})-\varkappa_c(\widehat{\mathbf{p}}\times\mathbf{u})-\varkappa_d(\mathbf{z}\times\mathbf{u})]\text{ by (6.41d)}\\
&\quad=\varkappa_g[\mathbf{r}\cdot(\mathbf{z}\times\mathbf{h})]+\varkappa_h[\mathbf{h}\cdot(\mathbf{z}\times\mathbf{h})]+\mathfrak{y}_x[(\mathbf{z}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})]+\mathfrak{y}_u[(\mathbf{z}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\qquad+\varepsilon_b\mathfrak{y}_r[(\mathbf{z}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\varepsilon_b\mathfrak{y}_n[(\mathbf{z}\times\mathbf{h})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\varkappa_a[(\mathbf{z}\times\mathbf{h})\cdot(\mathbf{r}\times\widehat{\mathbf{p}})]\\
&\qquad+\varkappa_b[(\mathbf{z}\times\mathbf{h})\cdot(\mathbf{r}\times\mathbf{z})]-\varrho_a\mathfrak{y}_s[(\mathbf{z}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{u})]-\varkappa_c[(\mathbf{z}\times\mathbf{h})\cdot(\widehat{\mathbf{p}}\times\mathbf{u})]\\
&\qquad-\varkappa_d[(\mathbf{z}\times\mathbf{h})\cdot(\mathbf{z}\times\mathbf{u})]
\end{aligned}
$$

$$
\begin{aligned}
&=r\varkappa_g\varepsilon_h+\mathfrak{y}_x[(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\widehat{\mathbf{r}})-(\mathbf{z}\cdot\widehat{\mathbf{r}})(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{y}_u[(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\widehat{\mathbf{p}})-(\mathbf{z}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\varepsilon_b\mathfrak{y}_r[(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\mathbf{z})-z^2(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})]+\varepsilon_b\mathfrak{y}_n[(\mathbf{z}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\mathbf{z})-z^2(\mathbf{h}\cdot\widehat{\mathbf{p}})]\\
&\quad+\varkappa_a[(\mathbf{z}\cdot\mathbf{r})(\mathbf{h}\cdot\widehat{\mathbf{p}})-(\mathbf{z}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\mathbf{r})]+\varkappa_b[(\mathbf{z}\cdot\mathbf{r})(\mathbf{h}\cdot\mathbf{z})-z^2(\mathbf{h}\cdot\mathbf{r})]\\
&\quad-\varrho_a\mathfrak{y}_s[(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\mathbf{u})-(\mathbf{z}\cdot\mathbf{u})(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})]-\varkappa_c[(\mathbf{z}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\mathbf{u})-(\mathbf{z}\cdot\mathbf{u})(\mathbf{h}\cdot\widehat{\mathbf{p}})]\\
&\quad-\varkappa_d[z^2(\mathbf{h}\cdot\mathbf{u})-(\mathbf{z}\cdot\mathbf{u})(\mathbf{h}\cdot\mathbf{z})]\text{ by (6.1a) \& (A.2)}\\
&=r\varkappa_g\varepsilon_h+\mathfrak{y}_x(-\varepsilon_d\delta_a)+\mathfrak{y}_u(\delta_b\delta_c-\delta_d\delta_a)+\varepsilon_b\mathfrak{y}_r(-z^2\delta_a)+\varepsilon_b\mathfrak{y}_n(-z^2\delta_c)+\varkappa_a(r\varepsilon_d\delta_c)\\
&\quad-\varrho_a\mathfrak{y}_s[-(-r\varrho_b\varphi_b)\delta_a]-\varkappa_c[-(-r\varrho_b\varphi_b)\delta_c]\text{ by (1.5c), (6.1a), (6.10a) \& (6.27d)}\\
&=r\varkappa_g\varepsilon_h-\mathfrak{y}_x\varepsilon_d\delta_a+\mathfrak{y}_u(\delta_b\delta_c-\delta_d\delta_a)-z^2\varepsilon_b(\mathfrak{y}_r\delta_a+\mathfrak{y}_n\delta_c)+r(\varkappa_a\varepsilon_d\delta_c-\varrho_a\mathfrak{y}_s\varrho_b\varphi_b\delta_a-\varkappa_c\varrho_b\varphi_b\delta_c)\\
&=-\mathfrak{y}_x\varepsilon_d\delta_a+\mathfrak{y}_u(\delta_b\delta_c-\delta_d\delta_a)-z^2\varepsilon_b(\mathfrak{y}_r\delta_a+\mathfrak{y}_n\delta_c)+r(\varkappa_g\varepsilon_h+\varkappa_a\varepsilon_d\delta_c)-r\varrho_b\varphi_b(\varrho_a\mathfrak{y}_s\delta_a+\varkappa_c\delta_c)\\
&=\varkappa_z\text{ by (6.21n)}
\end{aligned}\tag{6.46e}
$$

$$
\begin{aligned}
\mathbf{u}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)&=h^{-2}[(\mathbf{z}\times\mathbf{h}+q\widehat{\mathbf{r}}\times\mathbf{h})\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)]\text{ by (1.5c)}\\
&=h^{-2}[(\mathbf{z}\times\mathbf{h})\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)+(q/r)(\mathbf{r}\times\mathbf{h})\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)]\\
&=h^{-2}(\varkappa_z+\varphi_b\varkappa_y)\text{ by (6.1b), (6.46d) \& (6.46e)}\\
&=\mathfrak{v}_a\text{ by (6.21o).}
\end{aligned}\tag{6.46f}
$$

Consequently, we obtain

$$
\begin{aligned}
\aleph_1&=\widehat{\boldsymbol{\kappa}}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\text{ by (3.15d)}\\
&=\varkappa_v\text{ by (6.46a)}
\end{aligned}\tag{6.47a}
$$

$$
\begin{aligned}
\aleph_2&=\mathbf{a}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\text{ by (3.15d)}\\
&=(-\varrho_a\mathbf{r})\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\text{ by (1.5a) \& (6.21b)}\\
&=-\varrho_a\varkappa_j\text{ by (6.43a)}
\end{aligned}\tag{6.47b}
$$

$$
\begin{aligned}
\aleph_3&=\dot{\mathbf{a}}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\text{ by (3.15d)}\\
&=(-\varrho_a\mathbf{u}+3\varrho_a\varrho_b\mathbf{r})\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\text{ by (6.26a)}\\
&=-\varrho_a[\mathbf{u}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)]+3\varrho_a\varrho_b[\mathbf{r}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)]\\
&=-\varrho_a\mathfrak{v}_a+3\varrho_a\varrho_b\varkappa_j\text{ by (6.46f) \& (6.43a)}\\
&=\varrho_a(3\varrho_b\varkappa_j-\mathfrak{v}_a)
\end{aligned}\tag{6.47c}
$$

$$
\begin{aligned}
\aleph_4&=\ddot{\mathbf{a}}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\text{ by (3.15d)}\\
&=(6\varrho_a\varrho_b\mathbf{u}-\varrho_a\varrho_l\mathbf{r})\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)\text{ by (6.26b)}\\
&=6\varrho_a\varrho_b[\mathbf{u}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)]-\varrho_a\varrho_l[\mathbf{r}\cdot(\boldsymbol{\mathfrak{J}}_p+\boldsymbol{\mathfrak{J}}_q)]\\
&=6\varrho_a\varrho_b\mathfrak{v}_a-\varrho_a\varrho_l\varkappa_j\text{ by (6.46f) \& (6.43a)}\\
&=\varrho_a(6\varrho_b\mathfrak{v}_a-\varrho_l\varkappa_j)
\end{aligned}\tag{6.47d}
$$

$$\begin{aligned}\aleph_5 &= \ddot{\mathbf{a}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)}\\ &= (\varrho_m \mathbf{u} + \varrho_n \mathbf{r}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (6.26c)}\\ &= \varrho_m[\mathbf{u} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + \varrho_n[\mathbf{r} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)]\\ &= \varrho_m \mathfrak{v}_a + \varrho_n \varkappa_j \text{ by (6.46f) \& (6.43a)}\end{aligned} \tag{6.47e}$$

$$\begin{aligned}\aleph_6 &= \ddot{\mathbf{e}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (3.15d)}\\ &= [\mathfrak{x}_q \widehat{\mathbf{p}} - \mathfrak{x}_s \mathbf{z} - q\mathfrak{x}_s \widehat{\mathbf{r}} + \mathfrak{x}_r(\mathbf{r} \times \mathbf{h})] \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q) \text{ by (6.36c)}\\ &= \mathfrak{x}_q[\widehat{\mathbf{p}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] - \mathfrak{x}_s[\mathbf{z} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] - q\mathfrak{x}_s[\widehat{\mathbf{r}} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + \mathfrak{x}_r[(\mathbf{r} \times \mathbf{h}) \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)]\\ &= \mathfrak{x}_q \varkappa_w - \mathfrak{x}_s \varkappa_x - (q/r)\mathfrak{x}_s[\mathbf{r} \cdot (\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q)] + \mathfrak{x}_r \varkappa_y \text{ by (6.46)}\\ &= \mathfrak{x}_q \varkappa_w - \mathfrak{x}_s \varkappa_x - \varphi_b \mathfrak{x}_s \varkappa_j + \mathfrak{x}_r \varkappa_y \text{ by (6.43a) \& (6.1b)}\\ &= \mathfrak{v}_b \text{ by (6.21o)}\end{aligned} \tag{6.47f}$$

from which we derive

$$\begin{aligned}&\ddot{\mathfrak{Y}}\aleph_1 + \dddot{\rho}\aleph_2 + 3\ddot{\rho}\aleph_3 + b_3\aleph_4 + \rho\aleph_5 + \aleph_6\\ &= \ddot{\mathfrak{Y}}(\varkappa_v) + \dddot{\rho}(-\varrho_a\varkappa_j) + 3\ddot{\rho}[\varrho_a(3\varrho_b\varkappa_j - \mathfrak{v}_a)] + b_3[\varrho_a(6\varrho_b\mathfrak{v}_a - \varrho_l\varkappa_j)] + \rho(\varrho_m\mathfrak{v}_a + \varrho_n\varkappa_j) + \mathfrak{v}_b\\ &\quad \text{by (6.47)}\\ &= \varkappa_v(\ddot{\mathfrak{Y}}) - \varrho_a\varkappa_j(\dddot{\rho}) + 3\varrho_a(3\varrho_b\varkappa_j - \mathfrak{v}_a)(\ddot{\rho}) + \varrho_a(6\varrho_b\mathfrak{v}_a - \varrho_l\varkappa_j)(b_3) + \rho(\varrho_m\mathfrak{v}_a + \varrho_n\varkappa_j) + \mathfrak{v}_b\\ &= \varkappa_v(\mathfrak{y}_c) - \varrho_a\varkappa_j(\mathfrak{x}_j) + 3\varrho_a(3\varrho_b\varkappa_j - \mathfrak{v}_a)(\mathfrak{x}_i) + \varrho_a(6\varrho_b\mathfrak{v}_a - \varrho_l\varkappa_j)(\mathfrak{y}_f) + \rho(\varrho_m\mathfrak{v}_a + \varrho_n\varkappa_j) + \mathfrak{v}_b\\ &\quad \text{by (6.38), (6.32) \& (6.39c)}\\ &= \varkappa_v\mathfrak{y}_c + \varrho_a[-\varkappa_j\mathfrak{x}_j + 3\mathfrak{x}_i(3\varrho_b\varkappa_j - \mathfrak{v}_a) + \mathfrak{y}_f(6\varrho_b\mathfrak{v}_a - \varrho_l\varkappa_j)] + \rho(\varrho_m\mathfrak{v}_a + \varrho_n\varkappa_j) + \mathfrak{v}_b\\ &= \mathfrak{v}_b + \varkappa_v\mathfrak{y}_c + \varrho_a\mathfrak{v}_c + \rho(\varrho_m\mathfrak{v}_a + \varrho_n\varkappa_j) \text{ by (6.21o).}\end{aligned} \tag{6.48}$$

Moreover, we derive

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_r &= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \rho\varrho_a\mathbf{u} - \varkappa_f\mathbf{r} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r} \times \mathbf{h}) \text{ by (6.41c)}\\ &= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \varkappa_f\mathbf{r} - h^{-2}\rho\varrho_a(\mathbf{z} \times \mathbf{h} + q\widehat{\mathbf{r}} \times \mathbf{h}) - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r} \times \mathbf{h}) \text{ by (1.5c)}\\ &= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \varkappa_f\mathbf{r} - h^{-2}\rho\varrho_a(\mathbf{z} \times \mathbf{h}) - h^{-2}\rho\varrho_a(q/r)(\mathbf{r} \times \mathbf{h}) - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r} \times \mathbf{h})\\ &= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \varkappa_f\mathbf{r} - h^{-2}\rho\varrho_a(\mathbf{z} \times \mathbf{h}) - (h^{-2}\rho\varrho_a\varphi_b + \kappa\varepsilon_b\mathfrak{x}_k)(\mathbf{r} \times \mathbf{h}) \text{ by (6.1b)}\\ &= \mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \varkappa_f\mathbf{r} - h^{-2}\rho\varrho_a(\mathbf{z} \times \mathbf{h}) - \mathfrak{v}_d(\mathbf{r} \times \mathbf{h}) \text{ by (6.21o)}\end{aligned} \tag{6.49a}$$

$$\begin{aligned}\boldsymbol{\mathfrak{J}}_p + \boldsymbol{\mathfrak{J}}_q &= \varkappa_g\mathbf{r} + \varkappa_h\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}} \times \mathbf{z}) + \varkappa_a(\mathbf{r} \times \widehat{\mathbf{p}})\\ &\quad + \varkappa_b(\mathbf{r} \times \mathbf{z}) - \varrho_a\mathfrak{y}_s(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) - \varkappa_c(\widehat{\mathbf{p}} \times \mathbf{u}) - \varkappa_d(\mathbf{z} \times \mathbf{u}) \text{ by (6.41d)}\\ &= \varkappa_g\mathbf{r} + \varkappa_h\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}} \times \mathbf{z}) + \varkappa_a(\mathbf{r} \times \widehat{\mathbf{p}})\\ &\quad + \varkappa_b(\mathbf{r} \times \mathbf{z}) - \varrho_a\mathfrak{y}_s[h^{-2}(\delta_a\mathbf{z} + q\delta_a\widehat{\mathbf{r}} - \hbar_a\mathbf{h})] - \varkappa_c[h^{-2}(\delta_c\mathbf{z} + q\delta_c\widehat{\mathbf{r}} - \hbar_c\mathbf{h})]\\ &\quad - \varkappa_d[-h^{-2}\mathfrak{y}_z\mathbf{h}] \text{ by (6.11) \& (6.45a)}\\ &= \varkappa_g\mathbf{r} + \varkappa_h\mathbf{h} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}} \times \mathbf{z}) + \varkappa_a(\mathbf{r} \times \widehat{\mathbf{p}})\\ &\quad + \varkappa_b(\mathbf{r} \times \mathbf{z}) - h^{-2}\varrho_a\mathfrak{y}_s\delta_a\mathbf{z} - qh^{-2}\varrho_a\mathfrak{y}_s\delta_a\widehat{\mathbf{r}} + h^{-2}\varrho_a\mathfrak{y}_s\hbar_a\mathbf{h} - h^{-2}\varkappa_c\delta_c\mathbf{z} - qh^{-2}\varkappa_c\delta_c\widehat{\mathbf{r}}\\ &\quad + h^{-2}\varkappa_c\hbar_c\mathbf{h} + h^{-2}\varkappa_d\mathfrak{y}_z\mathbf{h}\end{aligned}$$

$$
\begin{aligned}
&= r\varkappa_g\widehat{\mathbf{r}} - qh^{-2}\varrho_a\mathfrak{y}_s\delta_a\widehat{\mathbf{r}} - qh^{-2}\varkappa_c\delta_c\widehat{\mathbf{r}} + \varkappa_h\mathbf{h} + h^{-2}\varrho_a\mathfrak{y}_s\hbar_a\mathbf{h} + h^{-2}\varkappa_c\hbar_c\mathbf{h} + h^{-2}\varkappa_d\mathfrak{y}_z\mathbf{h}\\
&\quad - h^{-2}\varrho_a\mathfrak{y}_s\delta_a\mathbf{z} - h^{-2}\varkappa_c\delta_c\mathbf{z} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})\\
&\quad + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) + \varkappa_b(\mathbf{r}\times\mathbf{z})\\
&= [r\varkappa_g - qh^{-2}(\varrho_a\mathfrak{y}_s\delta_a + \varkappa_c\delta_c)]\widehat{\mathbf{r}} + [\varkappa_h + h^{-2}(\varrho_a\mathfrak{y}_s\hbar_a + \varkappa_c\hbar_c + \varkappa_d\mathfrak{y}_z)]\mathbf{h}\\
&\quad - h^{-2}(\varrho_a\mathfrak{y}_s\delta_a + \varkappa_c\delta_c)\mathbf{z} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})\\
&\quad + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) + \varkappa_b(\mathbf{r}\times\mathbf{z})\\
&= \mathfrak{v}_g\widehat{\mathbf{r}} + \mathfrak{v}_f\mathbf{h} - \mathfrak{v}_e\mathbf{z} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})\\
&\quad + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) + \varkappa_b(\mathbf{r}\times\mathbf{z}).
\end{aligned}
\tag{6.49b}
$$

Art 26k. *Results of the computations.*

Substituting (6.49), (6.48), (6.44) and (6.42) into (3.22) gives

$$
\mathbb{K} = \frac{\varkappa_u}{(\varkappa_i)^3}, \quad \mathbb{T} = \frac{\mathfrak{v}_b + \varkappa_v\mathfrak{y}_c + \varrho_a\mathfrak{v}_c + \rho(\varrho_m\mathfrak{v}_a + \varrho_n\varkappa_j)}{(\varkappa_u)^2}
\tag{6.50a}
$$

$$
\begin{aligned}
\boldsymbol{\ell}_t &= \frac{1}{\varkappa_i}\Big[\mathfrak{y}_a\widehat{\boldsymbol{\kappa}} + \mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - \varkappa_f\mathbf{r} - \varrho_a(\rho/h^2)(\mathbf{z}\times\mathbf{h}) - \mathfrak{v}_d(\mathbf{r}\times\mathbf{h})\Big]\\
\boldsymbol{\ell}_b &= \frac{1}{\varkappa_u}\Big[\mathfrak{v}_g\widehat{\mathbf{r}} + \mathfrak{v}_f\mathbf{h} - \mathfrak{v}_e\mathbf{z} + \mathfrak{y}_x(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) + \mathfrak{y}_u(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \varepsilon_b\mathfrak{y}_r(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \varepsilon_b\mathfrak{y}_n(\widehat{\mathbf{p}}\times\mathbf{z})\\
&\qquad + \varkappa_a(\mathbf{r}\times\widehat{\mathbf{p}}) + \varkappa_b(\mathbf{r}\times\mathbf{z})\Big]
\end{aligned}
\tag{6.50b}
$$

as the complete set of equations describing the apparent path of the light source for a gravitating observer.

Art 27. *Apparent geometry of obliquated rays.*

To evaluate (3.26) for a gravitating observer, we introduce, in addition to (6.1), (6.10) and (6.21), the quantities

$$
\begin{aligned}
&\mathfrak{K}_a = h^{-2}[\hbar_c\mathfrak{x}_n + r\kappa\varepsilon_b\varepsilon_h\mathfrak{x}_k - \varepsilon_b\varphi_f(\mathfrak{y}_z + h^2\varphi_b)], \quad \mathfrak{K}_b = h^{-2}\delta_c\mathfrak{x}_n\\
&\mathfrak{K}_c = \mathfrak{x}_n + \kappa\varepsilon_e\mathfrak{x}_k, \quad \mathfrak{K}_d = r\varepsilon_b\varphi_f(\varepsilon_c\mathfrak{K}_c - \varepsilon_b\varphi_f\hbar_b), \quad \mathfrak{K}_e = \varrho_a/h^2, \quad \mathfrak{K}_f = \varepsilon_e\varphi_f\delta_c\mathfrak{K}_e\\
&\mathfrak{K}_g = \varphi_f(2\varphi_b\varrho_b\varepsilon_b + \mathfrak{K}_e\varepsilon_e\hbar_c), \quad \mathfrak{K}_h = \mathcal{Y}\mathfrak{y}_d - \rho\mathfrak{y}_a, \quad \mathfrak{K}_i = \mathcal{Y}\delta_a(\rho\mathfrak{K}_e\varphi_b + \kappa\varepsilon_b\mathfrak{x}_k)\\
&\mathfrak{K}_j = \rho(\mathcal{Y}\mathfrak{K}_e\hbar_a - \rho\varrho_a^2) + \kappa\varepsilon_b\mathfrak{x}_k(r\mathcal{Y}\varepsilon_a - \rho\varphi_b), \quad \mathfrak{K}_k = \varrho_a(3\rho\mathcal{Y}\varrho_b - \mathfrak{K}_h) - \mathcal{Y}\varepsilon_b\varphi_f\varphi_b\\
&\mathfrak{K}_l = \mathfrak{K}_b - \rho\mathfrak{K}_f + \mathfrak{y}_a(\delta_a/h^2), \quad \mathfrak{K}_m = \mathfrak{y}_a\delta_a(\varphi_b/h^2) - \varepsilon_b\varphi_f(\delta_a\mathfrak{y}_a + \delta_c\mathfrak{K}_c) + \varphi_b(\mathfrak{K}_b - \rho\mathfrak{K}_f)\\
&\mathfrak{K}_n = \mathfrak{K}_d - \mathfrak{K}_a + h^{-2}(\varphi_a\hbar_b\mathfrak{y}_d - \mathfrak{y}_a\hbar_a) + r\varepsilon_b\varphi_f(\varepsilon_a\mathfrak{y}_a - r\varrho_a\mathfrak{y}_d) + \rho(\mathfrak{K}_g - 3\varrho_a\varrho_b)\\
&\mathfrak{K}_o = \varepsilon_e\varphi_f[\varrho_a(3\rho\varrho_b - \mathfrak{y}_d) - \varepsilon_b\varphi_b\varphi_f], \quad \mathfrak{K}_p = \mathfrak{K}_j + \mathfrak{K}_n, \quad \mathfrak{K}_q = \mathfrak{K}_l - \rho\mathcal{Y}\mathfrak{K}_e\delta_a\\
&\mathfrak{K}_r = \mathfrak{K}_m - \mathfrak{K}_i, \quad \mathfrak{K}_s = \mathcal{Y}\mathfrak{x}_n - \varepsilon_e\varphi_f\mathfrak{y}_a, \quad \mathfrak{K}_t = \mathfrak{K}_o + \rho\varrho_a\mathfrak{x}_n
\end{aligned}
\tag{6.51a}
$$

$$
\begin{aligned}
\mathfrak{K}_u &= \mathfrak{K}_p h^2 + \mathfrak{K}_k\varepsilon_e - \mathcal{Y}\varepsilon_b\varphi_f\varepsilon_g - \mathfrak{K}_s\delta_e + \mathfrak{K}_t\varepsilon_f + r\rho\varrho_a\varepsilon_b\varphi_f\varepsilon_h - \varepsilon_b\varepsilon_e\varphi_f^2\varepsilon_i\\
\mathfrak{K}_v &= \mathfrak{K}_q z^2 + r(\mathfrak{K}_r\varepsilon_d - \mathfrak{K}_k\delta_f) + \mathfrak{K}_s\varsigma_c - \mathfrak{K}_t\varsigma_b\\
\mathfrak{K}_w &= r(\mathfrak{K}_q\varepsilon_d + r\mathfrak{K}_r - \mathcal{Y}\varepsilon_b\varphi_f\delta_f) + \mathfrak{K}_s\varsigma_a - \varepsilon_b\varepsilon_e\varsigma_b\varphi_f^2\\
\mathfrak{K}_x &= \mathfrak{K}_p\varepsilon_e - r\mathfrak{K}_q\delta_f + r^2\mathfrak{K}_k(1-\varepsilon_a^2) - r\mathcal{Y}\varepsilon_b\varphi_f(\varepsilon_d - \delta_b\varepsilon_a) + r\mathfrak{K}_s(\varepsilon_c - \varepsilon_b\varepsilon_a)\\
&\quad + r^2\mathfrak{K}_t(\varepsilon_b - \varepsilon_a\varepsilon_c) + r^2\rho\varrho_a\varepsilon_b\varphi_f(\varepsilon_a\varepsilon_d - \delta_b) - r\varepsilon_b\varepsilon_e\varphi_f^2(\varepsilon_b\varepsilon_d - \delta_b\varepsilon_c)\\
\mathfrak{K}_y &= \mathfrak{K}_p\varepsilon_g + r\mathfrak{K}_r\delta_f + r\mathfrak{K}_k(\varepsilon_d - \varepsilon_a\delta_b) - \mathcal{Y}\varepsilon_b\varphi_f(z^2 - \delta_b^2) + \mathfrak{K}_s(\delta_d - \varepsilon_b\delta_b)\\
&\quad + r\mathfrak{K}_t(\varepsilon_b\varepsilon_d - \varepsilon_a\delta_d) + r\rho\varrho_a\varepsilon_b\varphi_f(z^2\varepsilon_a - \delta_b\varepsilon_d) - \varepsilon_b\varepsilon_e\varphi_f^2(z^2\varepsilon_b - \delta_b\delta_d)\\
\mathfrak{K}_z &= -\mathfrak{K}_p\delta_e + \mathfrak{K}_q\varsigma_c + \mathfrak{K}_r\varsigma_a + r\mathfrak{K}_k(\varepsilon_c - \varepsilon_a\varepsilon_b) - \mathcal{Y}\varepsilon_b\varphi_f(\delta_d - \delta_b\varepsilon_b) + \mathfrak{K}_s(1 - \varepsilon_b^2)\\
&\quad + r\mathfrak{K}_t(\varepsilon_b\varepsilon_c - \varepsilon_a) + r\rho\varrho_a\varepsilon_b\varphi_f(\varepsilon_a\delta_d - \delta_b\varepsilon_c) - \varepsilon_b\varepsilon_e\varphi_f^2(\varepsilon_b\delta_d - \delta_b)
\end{aligned}
\tag{6.51b}
$$

$$
\begin{aligned}
\mathfrak{H}_a &= \mathfrak{K}_p\varepsilon_f - \mathfrak{K}_q\varsigma_b + r^2\mathfrak{K}_k(\varepsilon_b - \varepsilon_c\varepsilon_a) - r\mathcal{Y}\varepsilon_b\varphi_f(\varepsilon_b\varepsilon_d - \delta_d\varepsilon_a) + r\mathfrak{K}_s(\varepsilon_b\varepsilon_c - \varepsilon_a) \\
&\quad + r^2\mathfrak{K}_t(1 - \varepsilon_c^2) - r\varepsilon_b\varepsilon_e\varphi_f^2(\varepsilon_d - \delta_d\varepsilon_c) + r^2\rho\varrho_a\varepsilon_b\varphi_f(\varepsilon_c\varepsilon_d - \delta_d) \\
\mathfrak{H}_b &= \mathfrak{K}_p\varepsilon_i + \mathfrak{K}_r\varsigma_b + r\mathfrak{K}_k(\varepsilon_b\varepsilon_d - \varepsilon_c\delta_b) - \mathcal{Y}\varepsilon_b\varphi_f(z^2\varepsilon_b - \delta_d\delta_b) + \mathfrak{K}_s(\varepsilon_b\delta_d - \delta_b) \\
&\quad + r\mathfrak{K}_t(\varepsilon_d - \varepsilon_c\delta_d) - \varepsilon_b\varepsilon_e\varphi_f^2(z^2 - \delta_d^2) + r\rho\varrho_a\varepsilon_b\varphi_f(z^2\varepsilon_c - \delta_d\varepsilon_d) \\
\mathfrak{H}_c &= r\mathfrak{K}_p\varepsilon_h + r^2\mathfrak{K}_k(\varepsilon_a\varepsilon_d - \delta_b) - r\mathcal{Y}\varepsilon_b\varphi_f(z^2\varepsilon_a - \varepsilon_d\delta_b) + r\mathfrak{K}_s(\varepsilon_a\delta_d - \varepsilon_c\delta_b) \\
&\quad + r^2\mathfrak{K}_t(\varepsilon_c\varepsilon_d - \delta_d) - r\varepsilon_b\varepsilon_e\varphi_f^2(z^2\varepsilon_c - \varepsilon_d\delta_d) + r^2\rho\varrho_a\varepsilon_b\varphi_f(z^2 - \varepsilon_d^2) \\
\mathfrak{H}_d &= (\mathfrak{K}_p\mathfrak{K}_u + \mathfrak{K}_q\mathfrak{K}_v + \mathfrak{K}_r\mathfrak{K}_w + \mathfrak{K}_k\mathfrak{K}_x - \mathcal{Y}\varepsilon_b\varphi_f\mathfrak{K}_y + \mathfrak{K}_s\mathfrak{K}_z + \mathfrak{K}_t\mathfrak{H}_a \\
&\quad - \varepsilon_b\varepsilon_e\varphi_f^2\mathfrak{H}_b + \rho\varrho_a\varepsilon_b\varphi_f\mathfrak{H}_c)^{1/2}
\end{aligned}
\tag{6.51c}
$$

$$
\begin{aligned}
\mathfrak{H}_e &= \mathfrak{K}_p\delta_a + \mathfrak{K}_q\delta_b + r\mathfrak{K}_r\varepsilon_a + \mathfrak{K}_t\varsigma_a - r\rho\varrho_a\varepsilon_b\varphi_f\delta_f - \varepsilon_b\varepsilon_e\varphi_f^2\varsigma_c \\
\mathfrak{H}_f &= -r\mathfrak{K}_q\varrho_b\varphi_b + r^2\mathfrak{K}_r\varrho_b - \mathfrak{K}_k\delta_a + \varepsilon_b\varphi_f\mathfrak{y}_z\delta_a(\mathcal{Y}/h^2) - (\mathfrak{K}_s/h^2)(\hbar_c\delta_a - \delta_c\hbar_a) \\
&\quad - \mathfrak{K}_t\delta_c + \varepsilon_b\varepsilon_e\varphi_f^2\mathfrak{y}_z(\delta_c/h^2) \\
\mathfrak{H}_g &= \mathfrak{K}_p\delta_c + \mathfrak{K}_q\delta_d + r\mathfrak{K}_r\varepsilon_c - \mathfrak{K}_k\varsigma_a + \mathcal{Y}\varepsilon_b\varphi_f\varsigma_c - \rho\varrho_a\varepsilon_b\varphi_f\varsigma_b \\
\mathfrak{H}_h &= r[-\mathfrak{K}_q\varepsilon_h - r(\mathfrak{K}_k\delta_a + \mathfrak{K}_t\delta_c) + \mathfrak{K}_s(\varepsilon_a\delta_c - \varepsilon_c\delta_a) + \varepsilon_b\varepsilon_d\varphi_f(\mathcal{Y}\delta_a + \varepsilon_e\varphi_f\delta_c)] \\
\mathfrak{H}_i &= \mathfrak{y}_b\mathfrak{H}_e + \mathfrak{x}_o\mathfrak{H}_g + \mathfrak{x}_p\mathfrak{H}_h - \varrho_a[\mathfrak{x}_i\mathfrak{K}_w + \mathfrak{y}_e(\mathfrak{H}_f - 3\varrho_b\mathfrak{K}_w) + \rho(\varrho_l\mathfrak{K}_w - 6\varrho_b\mathfrak{H}_f)] \\
&\quad + 2\kappa\varepsilon_b\mathfrak{x}_k(\mathfrak{K}_v + \varphi_b\mathfrak{K}_w) \\
\mathfrak{H}_j &= \varepsilon_b\varphi_f - (\varphi_b/h^2)
\end{aligned}
\tag{6.51d}
$$

Art 27a. *Development of equation* (3.24a).

In view of the above quantities, we derive

$$
\begin{aligned}
\mathcal{S}_a &= \widehat{\boldsymbol{\kappa}} \times \mathbf{u} \text{ by (3.24a)} \\
&= h^{-2}(\delta_a\mathbf{z} + q\delta_a\widehat{\mathbf{r}} - \hbar_a\mathbf{h}) \text{ by (6.11a)}
\end{aligned}
\tag{6.52a}
$$

$$
\begin{aligned}
\mathcal{S}_b &= \widehat{\boldsymbol{\kappa}} \times \mathbf{a} \text{ by (3.24a)} \\
&= -\varrho_a(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \text{ by (6.40a)}
\end{aligned}
\tag{6.52b}
$$

$$
\begin{aligned}
\mathcal{S}_c &= \widehat{\boldsymbol{\kappa}} \times \mathbf{e} \text{ by (3.24a)} \\
&= \widehat{\boldsymbol{\kappa}} \times [f_1(\mathbf{r} \times \mathbf{h}) + f_2\widehat{\mathbf{p}}] \text{ by (1.5b)} \\
&= f_1[\widehat{\boldsymbol{\kappa}} \times (\mathbf{r} \times \mathbf{h})] + f_2(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \\
&= f_1[\mathbf{r}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{h}) - \mathbf{h}(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})] + f_2(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \text{ by (A.1)} \\
&= f_1(\delta_a\mathbf{r} - r\varepsilon_a\mathbf{h}) + f_2(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \text{ by (6.1a) \& (6.10a)} \\
&= \varepsilon_b\varphi_f(\delta_a\mathbf{r} - r\varepsilon_a\mathbf{h}) + \varepsilon_e\varphi_f(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \text{ by (6.2g) \& (6.2h)}
\end{aligned}
\tag{6.52c}
$$

$$
\begin{aligned}
\mathcal{S}_d &= \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{a}} \text{ by (3.24a)} \\
&= -\varrho_a(\widehat{\boldsymbol{\kappa}} \times \mathbf{u}) + 3\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \text{ by (6.40b)} \\
&= (\varrho_a/h^2)(\hbar_a\mathbf{h} - \delta_a\mathbf{z} - q\delta_a\widehat{\mathbf{r}}) + 3\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) \text{ by (6.11a)}
\end{aligned}
\tag{6.52d}
$$

$$
\begin{aligned}
\mathcal{S}_e &= \widehat{\boldsymbol{\kappa}} \times \dot{\mathbf{e}} \text{ by (3.24a)} \\
&= \kappa r\varepsilon_a\varepsilon_b\mathfrak{x}_k\mathbf{h} - \kappa\varepsilon_b\mathfrak{x}_k\delta_a\mathbf{r} + \mathfrak{x}_n(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) - q\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{r}}) \text{ by (6.40d)}
\end{aligned}
\tag{6.52e}
$$

$$\begin{aligned}\mathcal{S}_f &= \mathbf{a}\times\mathbf{u} \text{ by (3.24a)}\\ &= h^{-2}\varphi_a\hbar_b\mathbf{h} \text{ by (6.11b)} \qquad (6.52f)\\ \mathcal{S}_g &= \mathbf{a}\times\mathbf{e} \text{ by (3.24a)}\\ &= (-\varrho_a\mathbf{r})\times[f_1(\mathbf{r}\times\mathbf{h})+f_2\widehat{\mathbf{p}}] \text{ by (1.5) \& (6.21b)}\\ &= -\varrho_a f_1[\mathbf{r}\times(\mathbf{r}\times\mathbf{h})]-\varrho_a f_2(\mathbf{r}\times\widehat{\mathbf{p}})\\ &= -\varrho_a f_1[\mathbf{r}(\mathbf{r}\cdot\mathbf{h})-r^2\mathbf{h}]-\varrho_a f_2(\mathbf{r}\times\widehat{\mathbf{p}}) \text{ by (A.1)}\\ &= r^2\varrho_a f_1\mathbf{h}-\varrho_a f_2(\mathbf{r}\times\widehat{\mathbf{p}}) \text{ by (1.5c)}\\ &= r^2\varrho_a\varepsilon_b\varphi_f\mathbf{h}-\varrho_a\varepsilon_e\varphi_f(\mathbf{r}\times\widehat{\mathbf{p}}) \text{ by (6.2g) \& (6.2h)} \qquad (6.52g)\end{aligned}$$

$$\begin{aligned}\mathcal{S}_h &= \mathbf{a}\times\dot{\mathbf{a}} \text{ by (3.24a)}\\ &= -\varrho_a^2\mathbf{h} \text{ by (6.40f)} \qquad (6.52h)\\ \mathcal{S}_i &= \mathbf{a}\times\dot{\mathbf{e}} \text{ by (3.24a)}\\ &= -\varrho_a\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}})+\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})-\kappa\varphi_b\varepsilon_b\mathfrak{x}_k\mathbf{h} \text{ by (6.40h)} \qquad (6.52i)\\ \mathcal{S}_j &= \mathbf{u}\times\dot{\mathbf{a}} \text{ by (3.24a)}\\ &= \mathbf{u}\times(-\varrho_a\mathbf{u}+3\varrho_a\varrho_b\mathbf{r}) \text{ by (6.26a)}\\ &= 3\varrho_a\varrho_b(\mathbf{u}\times\mathbf{r}) = 3\varrho_a\varrho_b\mathbf{h} \text{ by (1.5c)} \qquad (6.52j)\end{aligned}$$

$$\begin{aligned}\mathcal{S}_k &= \mathbf{u}\times\dot{\mathbf{e}} \text{ by (3.24a)}\\ &= \mathbf{u}\times[\mathfrak{x}_n\widehat{\mathbf{p}}-\varepsilon_b\varphi_f\mathbf{z}-q\varepsilon_b\varphi_f\widehat{\mathbf{r}}-\kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})] \text{ by (6.36a)}\\ &= \mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}})-\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z})-q\varepsilon_b\varphi_f(\mathbf{u}\times\widehat{\mathbf{r}})-\kappa\varepsilon_b\mathfrak{x}_k[\mathbf{u}\times(\mathbf{r}\times\mathbf{h})]\\ &= \mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}})-\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z})-\varepsilon_b\varphi_f(q/r)(\mathbf{u}\times\mathbf{r})-\kappa\varepsilon_b\mathfrak{x}_k(-h^{-2}r\varepsilon_h\mathbf{h}) \text{ by (6.11c)}\\ &= \mathfrak{x}_n(\mathbf{u}\times\widehat{\mathbf{p}})-\varepsilon_b\varphi_f(\mathbf{u}\times\mathbf{z})-\varepsilon_b\varphi_f\varphi_b\mathbf{h}+h^{-2}r\kappa\varepsilon_b\varepsilon_h\mathfrak{x}_k\mathbf{h} \text{ by (1.5c) \& (6.1b)}\\ &= \mathfrak{x}_n[h^{-2}(\hbar_c\mathbf{h}-\delta_c\mathbf{z}-q\delta_c\widehat{\mathbf{r}})]+\varepsilon_b\varphi_f[-h^{-2}\mathfrak{y}_z\mathbf{h}]-\varepsilon_b\varphi_f\varphi_b\mathbf{h}\\ &\quad +h^{-2}r\kappa\varepsilon_b\varepsilon_h\mathfrak{x}_k\mathbf{h} \text{ by (6.11d) \& (6.45a)}\\ &= h^{-2}\hbar_c\mathfrak{x}_n\mathbf{h}-h^{-2}\delta_c\mathfrak{x}_n\mathbf{z}-h^{-2}q\delta_c\mathfrak{x}_n\widehat{\mathbf{r}}-h^{-2}\varepsilon_b\varphi_f\mathfrak{y}_z\mathbf{h}-\varepsilon_b\varphi_f\varphi_b\mathbf{h}+h^{-2}r\kappa\varepsilon_b\varepsilon_h\mathfrak{x}_k\mathbf{h}\\ &= h^{-2}[\hbar_c\mathfrak{x}_n+r\kappa\varepsilon_b\varepsilon_h\mathfrak{x}_k-\varepsilon_b\varphi_f(\mathfrak{y}_z+h^2\varphi_b)]\mathbf{h}-h^{-2}\delta_c\mathfrak{x}_n(\mathbf{z}+q\widehat{\mathbf{r}})\\ &= \mathfrak{K}_a\mathbf{h}-\mathfrak{K}_b(\mathbf{z}+q\widehat{\mathbf{r}}) \text{ by (6.51a)} \qquad (6.52k)\end{aligned}$$

$$\begin{aligned}\mathcal{S}_l &= \mathbf{e}\times\dot{\mathbf{a}} \text{ by (3.24a)}\\ &= [f_1(\mathbf{r}\times\mathbf{h})+f_2\widehat{\mathbf{p}}]\times(-\varrho_a\mathbf{u}+3\varrho_a\varrho_b\mathbf{r}) \text{ by (1.5b) \& (6.26a)}\\ &= \varrho_a f_1[\mathbf{u}\times(\mathbf{r}\times\mathbf{h})]-3\varrho_a\varrho_b f_1[\mathbf{r}\times(\mathbf{r}\times\mathbf{h})]-\varrho_a f_2(\widehat{\mathbf{p}}\times\mathbf{u})+3\varrho_a\varrho_b f_2(\widehat{\mathbf{p}}\times\mathbf{r})\\ &= \varrho_a f_1[\mathbf{r}(\mathbf{u}\cdot\mathbf{h})-\mathbf{h}(\mathbf{u}\cdot\mathbf{r})]-3\varrho_a\varrho_b f_1[\mathbf{r}(\mathbf{r}\cdot\mathbf{h})-r^2\mathbf{h}]+\varrho_a f_2[h^{-2}(\hbar_c\mathbf{h}-\delta_c\mathbf{z}-q\delta_c\widehat{\mathbf{r}})]\\ &\quad +3\varrho_a\varrho_b f_2(\widehat{\mathbf{p}}\times\mathbf{r}) \text{ by (A.1) \& (6.11d)}\\ &= \varrho_a f_1[-r(r\varrho_b)\mathbf{h}]-3\varrho_a\varrho_b f_1(-r^2\mathbf{h})+f_2(\varrho_a/h^2)(\hbar_c\mathbf{h}-\delta_c\mathbf{z}-q\delta_c\widehat{\mathbf{r}})\\ &\quad +3\varrho_a\varrho_b f_2(\widehat{\mathbf{p}}\times\mathbf{r}) \text{ by (1.5c) \& (6.27b)}\end{aligned}$$

$$\begin{aligned}&= -r^2\varrho_a f_1\varrho_b\mathbf{h}+3r^2\varrho_a\varrho_b f_1\mathbf{h}+f_2(\varrho_a/h^2)\hbar_c\mathbf{h}-f_2\delta_c(\varrho_a/h^2)(\mathbf{z}+q\widehat{\mathbf{r}})+3\varrho_a\varrho_b f_2(\widehat{\mathbf{p}}\times\mathbf{r})\\ &= [2\varphi_b\varrho_b f_1+(\varrho_a/h^2)\hbar_c f_2]\mathbf{h}-f_2\delta_c(\varrho_a/h^2)(\mathbf{z}+q\widehat{\mathbf{r}})+3\varrho_a\varrho_b f_2(\widehat{\mathbf{p}}\times\mathbf{r})\\ &\quad \text{by (6.1b) \& (6.21b)}\\ &= \varphi_f[2\varphi_b\varrho_b\varepsilon_b+(\varrho_a/h^2)\varepsilon_e\hbar_c]\mathbf{h}-\varepsilon_e\varphi_f\delta_c(\varrho_a/h^2)(\mathbf{z}+q\widehat{\mathbf{r}})+3\varrho_a\varrho_b\varepsilon_e\varphi_f(\widehat{\mathbf{p}}\times\mathbf{r})\\ &\quad \text{by (6.2g) \& (6.2h)}\\ &= \mathfrak{K}_g\mathbf{h}-\mathfrak{K}_f(\mathbf{z}+q\widehat{\mathbf{r}})+3\varrho_a\varrho_b\varepsilon_e\varphi_f(\widehat{\mathbf{p}}\times\mathbf{r}) \text{ by (6.51a)} \qquad (6.52l)\end{aligned}$$

$$
\begin{aligned}
\mathfrak{S}_m &= \mathbf{e}\times\dot{\mathbf{e}} \text{ by } (3.24\text{a}) \\
&= [f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}}]\times[\mathfrak{x}_n\widehat{\mathbf{p}} - \varepsilon_b\varphi_f\mathbf{z} - q\varepsilon_b\varphi_f\widehat{\mathbf{r}} - \kappa\varepsilon_b\mathfrak{x}_k(\mathbf{r}\times\mathbf{h})] \text{ by } (1.5\text{b}) \ \& \ (6.36\text{a}) \\
&= -\mathfrak{x}_n f_1[\widehat{\mathbf{p}}\times(\mathbf{r}\times\mathbf{h})] + \varepsilon_b\varphi_f f_1[\mathbf{z}\times(\mathbf{r}\times\mathbf{h})] + q\varepsilon_b\varphi_f f_1[\widehat{\mathbf{r}}\times(\mathbf{r}\times\mathbf{h})] \\
&\quad - \varepsilon_b\varphi_f f_2(\widehat{\mathbf{p}}\times\mathbf{z}) - q\varepsilon_b\varphi_f f_2(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) - \kappa\varepsilon_b\mathfrak{x}_k f_2[\widehat{\mathbf{p}}\times(\mathbf{r}\times\mathbf{h})] \\
&= -\mathfrak{x}_n f_1[\mathbf{r}(\widehat{\mathbf{p}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\mathbf{p}}\cdot\mathbf{r})] + \varepsilon_b\varphi_f f_1[\mathbf{r}(\mathbf{z}\cdot\mathbf{h}) - \mathbf{h}(\mathbf{z}\cdot\mathbf{r})] + q\varepsilon_b\varphi_f f_1[\mathbf{r}(\widehat{\mathbf{r}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\mathbf{r}}\cdot\mathbf{r})] \\
&\quad - \varepsilon_b\varphi_f f_2(\widehat{\mathbf{p}}\times\mathbf{z}) - q\varepsilon_b\varphi_f f_2(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) - \kappa\varepsilon_b\mathfrak{x}_k f_2[\mathbf{r}(\widehat{\mathbf{p}}\cdot\mathbf{h}) - \mathbf{h}(\widehat{\mathbf{p}}\cdot\mathbf{r})] \text{ by } (\text{A}.1) \\
&= -\mathfrak{x}_n f_1(\delta_c\mathbf{r} - r\varepsilon_c\mathbf{h}) + \varepsilon_b\varphi_f f_1(-r\varepsilon_d\mathbf{h}) + q\varepsilon_b\varphi_f f_1(-r\mathbf{h}) - \varepsilon_b\varphi_f f_2(\widehat{\mathbf{p}}\times\mathbf{z}) \\
&\quad - q\varepsilon_b\varphi_f f_2(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) - \kappa\varepsilon_b\mathfrak{x}_k f_2(\delta_c\mathbf{r} - r\varepsilon_c\mathbf{h}) \text{ by } (1.5\text{c}), (6.1\text{a}) \ \& \ (6.10\text{a})
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{x}_n\varepsilon_b\varphi_f(\delta_c\mathbf{r} - r\varepsilon_c\mathbf{h}) + \varepsilon_b^2\varphi_f^2(-r\varepsilon_d\mathbf{h}) + q\varepsilon_b^2\varphi_f^2(-r\mathbf{h}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z}) \\
&\quad - q\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) - \kappa\varepsilon_b\mathfrak{x}_k\varepsilon_e\varphi_f(\delta_c\mathbf{r} - r\varepsilon_c\mathbf{h}) \text{ by } (6.2\text{g}) \ \& \ (6.2\text{h}) \\
&= -\mathfrak{x}_n\varepsilon_b\varphi_f\delta_c\mathbf{r} + r\mathfrak{x}_n\varepsilon_b\varphi_f\varepsilon_c\mathbf{h} - r\varepsilon_b^2\varphi_f^2\varepsilon_d\mathbf{h} - rq\varepsilon_b^2\varphi_f^2\mathbf{h} - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z}) \\
&\quad - q\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) - \kappa\varepsilon_b\mathfrak{x}_k\varepsilon_e\varphi_f\delta_c\mathbf{r} + r\kappa\varepsilon_b\mathfrak{x}_k\varepsilon_e\varphi_f\varepsilon_c\mathbf{h} \\
&= -\varepsilon_b\varphi_f\delta_c(\mathfrak{x}_n + \kappa\varepsilon_e\mathfrak{x}_k)\mathbf{r} + r\varepsilon_b\varphi_f[\varepsilon_c(\mathfrak{x}_n + \kappa\varepsilon_e\mathfrak{x}_k) - \varepsilon_b\varphi_f(\varepsilon_d + q)]\mathbf{h} \\
&\quad - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z}) - q\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) \\
&= -\varepsilon_b\varphi_f\delta_c\mathfrak{K}_c\mathbf{r} + \mathfrak{K}_d\mathbf{h} - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z}) - q\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\widehat{\mathbf{r}}) \text{ by } (6.51\text{a}).
\end{aligned} \qquad (6.52\text{m})
$$

Art 27b. *Development of equation* (3.24b).

Using the foregoing equations, we derive

$$
\begin{aligned}
\mathfrak{S}_t &= (\mathfrak{Y}b_1 - \rho\dot{\mathfrak{Y}})\mathfrak{S}_b + \mathfrak{Y}\mathfrak{S}_n + \rho\mathfrak{S}_o \text{ by } (3.24\text{b}) \\
&= (\mathfrak{Y}\mathfrak{y}_d - \rho\mathfrak{y}_a)\mathfrak{S}_b + \mathfrak{Y}(\rho\mathfrak{S}_d + \mathfrak{S}_e) + \rho(\rho\mathfrak{S}_h + \mathfrak{S}_i) \text{ by } (6.39\text{a}), (6.38\text{a}) \ \& \ (3.24\text{b}) \\
&= \mathfrak{K}_h\mathfrak{S}_b + \rho\mathfrak{Y}\mathfrak{S}_d + \mathfrak{Y}\mathfrak{S}_e + \rho^2\mathfrak{S}_h + \rho\mathfrak{S}_i \text{ by } (6.51\text{a}) \\
&= \mathfrak{K}_h[-\varrho_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] + \rho\mathfrak{Y}[\mathfrak{K}_e(\hbar_a\mathbf{h} - \delta_a\mathbf{z} - q\delta_a\widehat{\mathbf{r}}) + 3\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})] \\
&\quad + \mathfrak{Y}[\kappa r\varepsilon_a\varepsilon_b\mathfrak{x}_k\mathbf{h} - \kappa\varepsilon_b\mathfrak{x}_k\delta_a\mathbf{r} + \mathfrak{x}_n(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) - \varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) - q\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}})] + \rho^2[-\varrho_a^2\mathbf{h}] \\
&\quad + \rho[-\varrho_a\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) + \varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) - \kappa\varphi_b\varepsilon_b\mathfrak{x}_k\mathbf{h}] \text{ by } (6.51\text{a}) \ \& \ (6.52) \\
&= -\mathfrak{K}_h\varrho_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + \rho\mathfrak{Y}\mathfrak{K}_e\hbar_a\mathbf{h} - \rho\mathfrak{Y}\mathfrak{K}_e\delta_a\mathbf{z} - q\rho\mathfrak{Y}\mathfrak{K}_e\delta_a\widehat{\mathbf{r}} + 3\rho\mathfrak{Y}\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) \\
&\quad + \kappa r\mathfrak{Y}\varepsilon_a\varepsilon_b\mathfrak{x}_k\mathbf{h} - \kappa\mathfrak{Y}\varepsilon_b\mathfrak{x}_k\delta_a\mathbf{r} + \mathfrak{Y}\mathfrak{x}_n(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) - \mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) - q\mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{r}}) - \rho^2\varrho_a^2\mathbf{h} \\
&\quad - \rho\varrho_a\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) - \kappa\rho\varphi_b\varepsilon_b\mathfrak{x}_k\mathbf{h}
\end{aligned}
$$

$$
\begin{aligned}
&= \rho\mathfrak{Y}\mathfrak{K}_e\hbar_a\mathbf{h} + \kappa r\mathfrak{Y}\varepsilon_a\varepsilon_b\mathfrak{x}_k\mathbf{h} - \rho^2\varrho_a^2\mathbf{h} - \kappa\rho\varphi_b\varepsilon_b\mathfrak{x}_k\mathbf{h} - \rho\mathfrak{Y}\mathfrak{K}_e\delta_a\mathbf{z} - \rho\mathfrak{Y}\mathfrak{K}_e\delta_a(q/r)\mathbf{r} - \kappa\mathfrak{Y}\varepsilon_b\mathfrak{x}_k\delta_a\mathbf{r} \\
&\quad - \mathfrak{K}_h\varrho_a(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) + 3\rho\mathfrak{Y}\varrho_a\varrho_b(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \mathfrak{Y}\varepsilon_b\varphi_f(q/r)(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \mathfrak{Y}\mathfrak{x}_n(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) \\
&\quad - \rho\varrho_a\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) \\
&= [\rho(\mathfrak{Y}\mathfrak{K}_e\hbar_a - \rho\varrho_a^2) + \kappa\varepsilon_b\mathfrak{x}_k(r\mathfrak{Y}\varepsilon_a - \rho\varphi_b)]\mathbf{h} - \rho\mathfrak{Y}\mathfrak{K}_e\delta_a\mathbf{z} - \mathfrak{Y}\delta_a(\rho\mathfrak{K}_e\varphi_b + \kappa\varepsilon_b\mathfrak{x}_k)\mathbf{r} \\
&\quad + [\varrho_a(3\rho\mathfrak{Y}\varrho_b - \mathfrak{K}_h) - \mathfrak{Y}\varepsilon_b\varphi_f\varphi_b](\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \mathfrak{Y}\mathfrak{x}_n(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) \\
&\quad - \rho\varrho_a\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) \text{ by } (6.1\text{b}) \\
&= \mathfrak{K}_j\mathbf{h} - \rho\mathfrak{Y}\mathfrak{K}_e\delta_a\mathbf{z} - \mathfrak{K}_i\mathbf{r} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \mathfrak{Y}\mathfrak{x}_n(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) \\
&\quad - \rho\varrho_a\mathfrak{x}_n(\mathbf{r}\times\widehat{\mathbf{p}}) + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) \text{ by } (6.51\text{a})
\end{aligned} \qquad (6.53\text{a})
$$

$$
\begin{aligned}
\mathfrak{S}_u &= \dot{\mathfrak{Y}}\mathfrak{S}_p + b_1\mathfrak{S}_q + \rho\mathfrak{S}_r + \mathfrak{S}_s \text{ by (3.24b)}\\
&= \mathfrak{y}_a(\mathfrak{S}_a - \mathfrak{S}_c) + \mathfrak{y}_d(\mathfrak{S}_f - \mathfrak{S}_g) + \rho(\mathfrak{S}_l - \mathfrak{S}_j) + \mathfrak{S}_m - \mathfrak{S}_k \text{ by (6.38a), (6.39a) \& (3.24b)}\\
&= \mathfrak{y}_a\mathfrak{S}_a - \mathfrak{y}_a\mathfrak{S}_c + \mathfrak{y}_d\mathfrak{S}_f - \mathfrak{y}_d\mathfrak{S}_g + \rho\mathfrak{S}_l - \rho\mathfrak{S}_j + \mathfrak{S}_m - \mathfrak{S}_k\\
&= \mathfrak{y}_a[h^{-2}(\delta_a\mathbf{z} + q\delta_a\widehat{\mathbf{r}} - \hbar_a\mathbf{h})] - \mathfrak{y}_a[\varepsilon_b\varphi_f(\delta_a\mathbf{r} - r\varepsilon_a\mathbf{h}) + \varepsilon_e\varphi_f(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{y}_d[h^{-2}\varphi_a\hbar_b\mathbf{h}]\\
&\quad - \mathfrak{y}_d[r^2\varrho_a\varepsilon_b\varphi_f\mathbf{h} - \varrho_a\varepsilon_e\varphi_f(\mathbf{r} \times \widehat{\mathbf{p}})] + \rho[\mathfrak{K}_g\mathbf{h} - \mathfrak{K}_f(\mathbf{z} + q\widehat{\mathbf{r}}) + 3\varrho_a\varrho_b\varepsilon_e\varphi_f(\widehat{\mathbf{p}} \times \mathbf{r})] - \rho[3\varrho_a\varrho_b\mathbf{h}]\\
&\quad - \varepsilon_b\varphi_f\delta_c\mathfrak{K}_c\mathbf{r} + \mathfrak{K}_d\mathbf{h} - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z}) - q\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \widehat{\mathbf{r}}) - [\mathfrak{K}_a\mathbf{h} - \mathfrak{K}_b(\mathbf{z} + q\widehat{\mathbf{r}})] \text{ by (6.52)}\\
&= \mathfrak{y}_a(\delta_a/h^2)\mathbf{z} + q\mathfrak{y}_a(\delta_a/h^2)\widehat{\mathbf{r}} - \mathfrak{y}_a(\hbar_a/h^2)\mathbf{h} - \varepsilon_b\varphi_f\delta_a\mathfrak{y}_a\mathbf{r} + r\varepsilon_a\varepsilon_b\varphi_f\mathfrak{y}_a\mathbf{h} - \varepsilon_e\varphi_f\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})\\
&\quad + \varphi_a\hbar_b(\mathfrak{y}_d/h^2)\mathbf{h} - r^2\varrho_a\varepsilon_b\varphi_f\mathfrak{y}_d\mathbf{h} + \varrho_a\varepsilon_e\varphi_f\mathfrak{y}_d(\mathbf{r} \times \widehat{\mathbf{p}}) + \rho\mathfrak{K}_g\mathbf{h} - \rho\mathfrak{K}_f\mathbf{z} - q\rho\mathfrak{K}_f\widehat{\mathbf{r}}\\
&\quad + 3\rho\varrho_a\varrho_b\varepsilon_e\varphi_f(\widehat{\mathbf{p}} \times \mathbf{r}) - 3\rho\varrho_a\varrho_b\mathbf{h} - \varepsilon_b\varphi_f\delta_c\mathfrak{K}_c\mathbf{r} + \mathfrak{K}_d\mathbf{h} - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z}) - q\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \widehat{\mathbf{r}})\\
&\quad - \mathfrak{K}_a\mathbf{h} + \mathfrak{K}_b\mathbf{z} + q\mathfrak{K}_b\widehat{\mathbf{r}}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{y}_a(\delta_a/h^2)\mathbf{z} - \rho\mathfrak{K}_f\mathbf{z} + \mathfrak{K}_b\mathbf{z} + \mathfrak{y}_a(\delta_a/h^2)(q/r)\mathbf{r} - \varepsilon_b\varphi_f\delta_a\mathfrak{y}_a\mathbf{r}\\
&\quad - \rho\mathfrak{K}_f(q/r)\mathbf{r} - \varepsilon_b\varphi_f\delta_c\mathfrak{K}_c\mathbf{r} + \mathfrak{K}_b(q/r)\mathbf{r} - \mathfrak{y}_a(\hbar_a/h^2)\mathbf{h} + r\varepsilon_a\varepsilon_b\varphi_f\mathfrak{y}_a\mathbf{h} + \varphi_a\hbar_b(\mathfrak{y}_d/h^2)\mathbf{h}\\
&\quad - r^2\varrho_a\varepsilon_b\varphi_f\mathfrak{y}_d\mathbf{h} + \rho\mathfrak{K}_g\mathbf{h} - 3\rho\varrho_a\varrho_b\mathbf{h} + \mathfrak{K}_d\mathbf{h} - \mathfrak{K}_a\mathbf{h} - \varepsilon_e\varphi_f\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z})\\
&\quad + \varrho_a\varepsilon_e\varphi_f\mathfrak{y}_d(\mathbf{r} \times \widehat{\mathbf{p}}) + 3\rho\varrho_a\varrho_b\varepsilon_e\varphi_f(\widehat{\mathbf{p}} \times \mathbf{r}) - \varepsilon_b\varepsilon_e\varphi_f^2(q/r)(\widehat{\mathbf{p}} \times \mathbf{r})\\
&= [\mathfrak{K}_b - \rho\mathfrak{K}_f + \mathfrak{y}_a(\delta_a/h^2)]\mathbf{z} + [\mathfrak{y}_a\delta_a(\varphi_b/h^2) - \varepsilon_b\varphi_f(\delta_a\mathfrak{y}_a + \delta_c\mathfrak{K}_c) + \varphi_b(\mathfrak{K}_b - \rho\mathfrak{K}_f)]\mathbf{r}\\
&\quad + [\mathfrak{K}_d - \mathfrak{K}_a + h^{-2}(\varphi_a\hbar_b\mathfrak{y}_d - \mathfrak{y}_a\hbar_a) + r\varepsilon_b\varphi_f(\varepsilon_a\mathfrak{y}_a - r\varrho_a\mathfrak{y}_d) + \rho(\mathfrak{K}_g - 3\varrho_a\varrho_b)]\mathbf{h}\\
&\quad - \varepsilon_e\varphi_f\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z}) + \varepsilon_e\varphi_f[\varrho_a(3\rho\varrho_b - \mathfrak{y}_d) - \varepsilon_b\varphi_b\varphi_f](\widehat{\mathbf{p}} \times \mathbf{r}) \text{ by (6.1b)}\\
&= \mathfrak{K}_l\mathbf{z} + \mathfrak{K}_m\mathbf{r} + \mathfrak{K}_n\mathbf{h} - \varepsilon_e\varphi_f\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z}) + \mathfrak{K}_o(\widehat{\mathbf{p}} \times \mathbf{r}) \text{ by (6.51a)}
\end{aligned} \tag{6.53b}
$$

$$
\begin{aligned}
&\mathfrak{S}_t + \mathfrak{S}_u\\
&= \mathfrak{K}_j\mathbf{h} - \rho\mathfrak{Y}\mathfrak{K}_e\delta_a\mathbf{z} - \mathfrak{K}_i\mathbf{r} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) - \mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \mathfrak{Y}\mathfrak{x}_n(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})\\
&\quad - \rho\varrho_a\mathfrak{x}_n(\mathbf{r} \times \widehat{\mathbf{p}}) + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) + \mathfrak{K}_l\mathbf{z} + \mathfrak{K}_m\mathbf{r} + \mathfrak{K}_n\mathbf{h} - \varepsilon_e\varphi_f\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})\\
&\quad - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z}) + \mathfrak{K}_o(\widehat{\mathbf{p}} \times \mathbf{r}) \text{ by (6.53)}\\
&= \mathfrak{K}_j\mathbf{h} + \mathfrak{K}_n\mathbf{h} - \rho\mathfrak{Y}\mathfrak{K}_e\delta_a\mathbf{z} + \mathfrak{K}_l\mathbf{z} - \mathfrak{K}_i\mathbf{r} + \mathfrak{K}_m\mathbf{r} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) - \mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \mathfrak{Y}\mathfrak{x}_n(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})\\
&\quad - \varepsilon_e\varphi_f\mathfrak{y}_a(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \rho\varrho_a\mathfrak{x}_n(\widehat{\mathbf{p}} \times \mathbf{r}) + \mathfrak{K}_o(\widehat{\mathbf{p}} \times \mathbf{r}) + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z})\\
&= (\mathfrak{K}_j + \mathfrak{K}_n)\mathbf{h} + (\mathfrak{K}_l - \rho\mathfrak{Y}\mathfrak{K}_e\delta_a)\mathbf{z} + (\mathfrak{K}_m - \mathfrak{K}_i)\mathbf{r} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) - \mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z})\\
&\quad + (\mathfrak{Y}\mathfrak{x}_n - \varepsilon_e\varphi_f\mathfrak{y}_a)(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + (\mathfrak{K}_o + \rho\varrho_a\mathfrak{x}_n)(\widehat{\mathbf{p}} \times \mathbf{r}) + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z})\\
&= \mathfrak{K}_p\mathbf{h} + \mathfrak{K}_q\mathbf{z} + \mathfrak{K}_r\mathbf{r} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) - \mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \mathfrak{K}_s(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{K}_t(\widehat{\mathbf{p}} \times \mathbf{r})\\
&\quad + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z}) \text{ by (6.51a).}
\end{aligned} \tag{6.54}
$$

Art 27c. *Computation of the magnitude of* $\mathfrak{S}_t + \mathfrak{S}_u$*.*

To compute the magnitude of vector $\mathfrak{S}_t + \mathfrak{S}_u$, we first derive

$$
\begin{aligned}
&\mathbf{h} \cdot (\mathfrak{S}_t + \mathfrak{S}_u)\\
&= \mathbf{h} \cdot [\mathfrak{K}_p\mathbf{h} + \mathfrak{K}_q\mathbf{z} + \mathfrak{K}_r\mathbf{r} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) - \mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \mathfrak{K}_s(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{K}_t(\widehat{\mathbf{p}} \times \mathbf{r})\\
&\quad + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z})] \text{ by (6.54)}\\
&= \mathfrak{K}_p(\mathbf{h} \cdot \mathbf{h}) + \mathfrak{K}_q(\mathbf{h} \cdot \mathbf{z}) + \mathfrak{K}_r(\mathbf{h} \cdot \mathbf{r}) + \mathfrak{K}_k[\mathbf{h} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] - \mathfrak{Y}\varepsilon_b\varphi_f[\mathbf{h} \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{z})] + \mathfrak{K}_s[\mathbf{h} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})]\\
&\quad + \mathfrak{K}_t[\mathbf{h} \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] + \rho\varrho_a\varepsilon_b\varphi_f[\mathbf{h} \cdot (\mathbf{r} \times \mathbf{z})] - \varepsilon_b\varepsilon_e\varphi_f^2[\mathbf{h} \cdot (\widehat{\mathbf{p}} \times \mathbf{z})]\\
&= \mathfrak{K}_p h^2 + \mathfrak{K}_k\varepsilon_e - \mathfrak{Y}\varepsilon_b\varphi_f\varepsilon_g - \mathfrak{K}_s\delta_e + \mathfrak{K}_t\varepsilon_f + r\rho\varrho_a\varepsilon_b\varphi_f\varepsilon_h - \varepsilon_b\varepsilon_e\varphi_f^2\varepsilon_i \text{ by (1.5c), (6.1a) \& (6.10a)}\\
&= \mathfrak{K}_u \text{ by (6.51b)}
\end{aligned} \tag{6.55a}
$$

$$\begin{aligned}
&\mathbf{z}\cdot(\mathcal{S}_t+\mathcal{S}_u)\\
&=\mathbf{z}\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})]\text{ by }(6.54)\\
&=\mathfrak{K}_p(\mathbf{z}\cdot\mathbf{h})+\mathfrak{K}_q(\mathbf{z}\cdot\mathbf{z})+\mathfrak{K}_r(\mathbf{z}\cdot\mathbf{r})+\mathfrak{K}_k[\mathbf{z}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]-\mathcal{Y}\varepsilon_b\varphi_f[\mathbf{z}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[\mathbf{z}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\mathfrak{K}_t[\mathbf{z}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]+\rho\varrho_a\varepsilon_b\varphi_f[\mathbf{z}\cdot(\mathbf{r}\times\mathbf{z})]-\varepsilon_b\varepsilon_e\varphi_f^2[\mathbf{z}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&=\mathfrak{K}_qz^2+r\mathfrak{K}_r\varepsilon_d-r\mathfrak{K}_k\delta_f+\mathfrak{K}_s\varsigma_c-\mathfrak{K}_t\varsigma_b\text{ by }(1.5c),(6.1a),(6.10a)\ \&\ (6.21a)\\
&=\mathfrak{K}_v\text{ by }(6.51b)
\end{aligned}\tag{6.55b}$$

$$\begin{aligned}
&\mathbf{r}\cdot(\mathcal{S}_t+\mathcal{S}_u)\\
&=\mathbf{r}\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})]\text{ by }(6.54)\\
&=\mathfrak{K}_p(\mathbf{r}\cdot\mathbf{h})+\mathfrak{K}_q(\mathbf{r}\cdot\mathbf{z})+\mathfrak{K}_r(\mathbf{r}\cdot\mathbf{r})+\mathfrak{K}_k[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]-\mathcal{Y}\varepsilon_b\varphi_f[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\mathfrak{K}_t[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]+\rho\varrho_a\varepsilon_b\varphi_f[\mathbf{r}\cdot(\mathbf{r}\times\mathbf{z})]-\varepsilon_b\varepsilon_e\varphi_f^2[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&=r\mathfrak{K}_q\varepsilon_d+r^2\mathfrak{K}_r-r\mathcal{Y}\varepsilon_b\varphi_f\delta_f+\mathfrak{K}_s\varsigma_a-\varepsilon_b\varepsilon_e\varphi_f^2\varsigma_b\text{ by }(1.5c),(6.1a),(6.10a)\ \&\ (6.21a)\\
&=\mathfrak{K}_w\text{ by }(6.51b)
\end{aligned}\tag{6.55c}$$

$$\begin{aligned}
&(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\cdot(\mathcal{S}_t+\mathcal{S}_u)\\
&=(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})]\text{ by }(6.54)\\
&=\mathfrak{K}_p[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]+\mathfrak{K}_q[\mathbf{z}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]+\mathfrak{K}_r[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]+\mathfrak{K}_k[(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\quad-\mathcal{Y}\varepsilon_b\varphi_f[(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{K}_t[(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\cdot(\mathbf{r}\times\mathbf{z})]-\varepsilon_b\varepsilon_e\varphi_f^2[(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&=\mathfrak{K}_p\varepsilon_e-r\mathfrak{K}_q\delta_f+\mathfrak{K}_k[r^2-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})^2]-\mathcal{Y}\varepsilon_b\varphi_f[(\mathbf{r}\cdot\mathbf{z})-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\mathfrak{K}_s[(\mathbf{r}\cdot\widehat{\mathbf{p}})-(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{K}_t[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})r^2-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{r}\cdot\widehat{\mathbf{p}})]+\rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{r}\cdot\mathbf{z})-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})r^2]\\
&\quad-\varepsilon_b\varepsilon_e\varphi_f^2[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{r}\cdot\mathbf{z})-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{r}\cdot\widehat{\mathbf{p}})]\text{ by }(6.1a),(6.10a)\ \&\ (A.2)
\end{aligned}$$

$$\begin{aligned}
&=\mathfrak{K}_p\varepsilon_e-r\mathfrak{K}_q\delta_f+\mathfrak{K}_k(r^2-r^2\varepsilon_a^2)-\mathcal{Y}\varepsilon_b\varphi_f(r\varepsilon_d-r\delta_b\varepsilon_a)+\mathfrak{K}_s(r\varepsilon_c-r\varepsilon_b\varepsilon_a)+\mathfrak{K}_t(\varepsilon_br^2-r^2\varepsilon_a\varepsilon_c)\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f(r^2\varepsilon_a\varepsilon_d-\delta_br^2)-\varepsilon_b\varepsilon_e\varphi_f^2(r\varepsilon_b\varepsilon_d-r\delta_b\varepsilon_c)\text{ by }(6.1a)\ \&\ (6.10a)\\
&=\mathfrak{K}_p\varepsilon_e-r\mathfrak{K}_q\delta_f+r^2\mathfrak{K}_k(1-\varepsilon_a^2)-r\mathcal{Y}\varepsilon_b\varphi_f(\varepsilon_d-\delta_b\varepsilon_a)+r\mathfrak{K}_s(\varepsilon_c-\varepsilon_b\varepsilon_a)+r^2\mathfrak{K}_t(\varepsilon_b-\varepsilon_a\varepsilon_c)\\
&\quad+r^2\rho\varrho_a\varepsilon_b\varphi_f(\varepsilon_a\varepsilon_d-\delta_b)-r\varepsilon_b\varepsilon_e\varphi_f^2(\varepsilon_b\varepsilon_d-\delta_b\varepsilon_c)\\
&=\mathfrak{K}_x\text{ by }(6.51b)
\end{aligned}\tag{6.55d}$$

$$\begin{aligned}
&(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\mathcal{S}_t+\mathcal{S}_u)\\
&=(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})]\text{ by }(6.54)\\
&=\mathfrak{K}_p[\mathbf{h}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_q[\mathbf{z}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_r[\mathbf{r}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_k[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\quad-\mathcal{Y}\varepsilon_b\varphi_f[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{K}_t[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\mathbf{r}\times\mathbf{z})]-\varepsilon_b\varepsilon_e\varphi_f^2[(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&=\mathfrak{K}_p\varepsilon_g+r\mathfrak{K}_r\delta_f+\mathfrak{K}_k[(\mathbf{z}\cdot\mathbf{r})-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})]-\mathcal{Y}\varepsilon_b\varphi_f[z^2-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})^2]+\mathfrak{K}_s[(\mathbf{z}\cdot\widehat{\mathbf{p}})-(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\quad+\mathfrak{K}_t[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\mathbf{r})-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})(\mathbf{z}\cdot\widehat{\mathbf{p}})]+\rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})z^2-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{z}\cdot\mathbf{r})]\\
&\quad-\varepsilon_b\varepsilon_e\varphi_f^2[(\widehat{\boldsymbol{\kappa}}\cdot\widehat{\mathbf{p}})z^2-(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})(\mathbf{z}\cdot\widehat{\mathbf{p}})]\text{ by }(6.1a),(6.10a)\ \&\ (A.2)
\end{aligned}$$

$$
\begin{aligned}
&= \mathfrak{K}_p\varepsilon_g + r\mathfrak{K}_r\delta_f + \mathfrak{K}_k(r\varepsilon_d - r\varepsilon_a\delta_b) - \mathcal{Y}\varepsilon_b\varphi_f(z^2 - \delta_b^2) + \mathfrak{K}_s(\delta_d - \varepsilon_b\delta_b) + \mathfrak{K}_t(r\varepsilon_b\varepsilon_d - r\varepsilon_a\delta_d) \\
&\quad + \rho\varrho_a\varepsilon_b\varphi_f(rz^2\varepsilon_a - r\delta_b\varepsilon_d) - \varepsilon_b\varepsilon_e\varphi_f^2(\varepsilon_b z^2 - \delta_b\delta_d) \text{ by (6.1a) \& (6.10a)} \\
&= \mathfrak{K}_p\varepsilon_g + r\mathfrak{K}_r\delta_f + r\mathfrak{K}_k(\varepsilon_d - \varepsilon_a\delta_b) - \mathcal{Y}\varepsilon_b\varphi_f(z^2 - \delta_b^2) + \mathfrak{K}_s(\delta_d - \varepsilon_b\delta_b) + r\mathfrak{K}_t(\varepsilon_b\varepsilon_d - \varepsilon_a\delta_d) \\
&\quad + r\rho\varrho_a\varepsilon_b\varphi_f(z^2\varepsilon_a - \delta_b\varepsilon_d) - \varepsilon_b\varepsilon_e\varphi_f^2(z^2\varepsilon_b - \delta_b\delta_d) \\
&= \mathfrak{K}_y \text{ by (6.51b)}
\end{aligned} \tag{6.55e}
$$

$$
\begin{aligned}
&(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot [\mathfrak{K}_p\mathbf{h} + \mathfrak{K}_q\mathbf{z} + \mathfrak{K}_r\mathbf{r} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) - \mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \mathfrak{K}_s(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{K}_t(\widehat{\mathbf{p}} \times \mathbf{r}) \\
&\quad + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z})] \text{ by (6.54)} \\
&= \mathfrak{K}_p[\mathbf{h} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_q[\mathbf{z} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_r[\mathbf{r} \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_k[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad - \mathcal{Y}\varepsilon_b\varphi_f[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{z})] + \mathfrak{K}_s[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_t[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] \\
&\quad + \rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\mathbf{r} \times \mathbf{z})] - \varepsilon_b\varepsilon_e\varphi_f^2[(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) \cdot (\widehat{\mathbf{p}} \times \mathbf{z})] \\
&= -\mathfrak{K}_p\delta_e + \mathfrak{K}_q\varsigma_c + \mathfrak{K}_r\varsigma_a + \mathfrak{K}_k[(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] - \mathcal{Y}\varepsilon_b\varphi_f[(\widehat{\mathbf{p}} \cdot \mathbf{z}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{z})(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_s[1 - (\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})^2] + \mathfrak{K}_t[(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \mathbf{r}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})] + \rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\boldsymbol{\kappa}} \cdot \mathbf{r})(\widehat{\mathbf{p}} \cdot \mathbf{z}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{z})(\widehat{\mathbf{p}} \cdot \mathbf{r})] \\
&\quad - \varepsilon_b\varepsilon_e\varphi_f^2[(\widehat{\boldsymbol{\kappa}} \cdot \widehat{\mathbf{p}})(\widehat{\mathbf{p}} \cdot \mathbf{z}) - (\widehat{\boldsymbol{\kappa}} \cdot \mathbf{z})] \text{ by (6.10a), (6.21a) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= -\mathfrak{K}_p\delta_e + \mathfrak{K}_q\varsigma_c + \mathfrak{K}_r\varsigma_a + \mathfrak{K}_k(r\varepsilon_c - r\varepsilon_a\varepsilon_b) - \mathcal{Y}\varepsilon_b\varphi_f(\delta_d - \delta_b\varepsilon_b) + \mathfrak{K}_s(1 - \varepsilon_b^2) + \mathfrak{K}_t(r\varepsilon_b\varepsilon_c - r\varepsilon_a) \\
&\quad + \rho\varrho_a\varepsilon_b\varphi_f(r\varepsilon_a\delta_d - r\delta_b\varepsilon_c) - \varepsilon_b\varepsilon_e\varphi_f^2(\varepsilon_b\delta_d - \delta_b) \text{ by (6.1a) \& (6.10a)} \\
&= -\mathfrak{K}_p\delta_e + \mathfrak{K}_q\varsigma_c + \mathfrak{K}_r\varsigma_a + r\mathfrak{K}_k(\varepsilon_c - \varepsilon_a\varepsilon_b) - \mathcal{Y}\varepsilon_b\varphi_f(\delta_d - \delta_b\varepsilon_b) + \mathfrak{K}_s(1 - \varepsilon_b^2) + r\mathfrak{K}_t(\varepsilon_b\varepsilon_c - \varepsilon_a) \\
&\quad + r\rho\varrho_a\varepsilon_b\varphi_f(\varepsilon_a\delta_d - \delta_b\varepsilon_c) - \varepsilon_b\varepsilon_e\varphi_f^2(\varepsilon_b\delta_d - \delta_b) \\
&= \mathfrak{K}_z \text{ by (6.51b)}
\end{aligned} \tag{6.55f}
$$

$$
\begin{aligned}
&(\widehat{\mathbf{p}} \times \mathbf{r}) \cdot (\mathfrak{S}_t + \mathfrak{S}_u) \\
&= (\widehat{\mathbf{p}} \times \mathbf{r}) \cdot [\mathfrak{K}_p\mathbf{h} + \mathfrak{K}_q\mathbf{z} + \mathfrak{K}_r\mathbf{r} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}} \times \mathbf{r}) - \mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}} \times \mathbf{z}) + \mathfrak{K}_s(\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}}) + \mathfrak{K}_t(\widehat{\mathbf{p}} \times \mathbf{r}) \\
&\quad - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}} \times \mathbf{z}) + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r} \times \mathbf{z})] \text{ by (6.54)} \\
&= \mathfrak{K}_p[\mathbf{h} \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] + \mathfrak{K}_q[\mathbf{z} \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] + \mathfrak{K}_r[\mathbf{r} \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] + \mathfrak{K}_k[(\widehat{\mathbf{p}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{r})] \\
&\quad - \mathcal{Y}\varepsilon_b\varphi_f[(\widehat{\mathbf{p}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \mathbf{z})] + \mathfrak{K}_s[(\widehat{\mathbf{p}} \times \mathbf{r}) \cdot (\widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}})] + \mathfrak{K}_t[(\widehat{\mathbf{p}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \mathbf{r})] \\
&\quad - \varepsilon_b\varepsilon_e\varphi_f^2[(\widehat{\mathbf{p}} \times \mathbf{r}) \cdot (\widehat{\mathbf{p}} \times \mathbf{z})] + \rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\mathbf{p}} \times \mathbf{r}) \cdot (\mathbf{r} \times \mathbf{z})] \\
&= \mathfrak{K}_p\varepsilon_f - \mathfrak{K}_q\varsigma_b + \mathfrak{K}_k[(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})r^2 - (\widehat{\mathbf{p}} \cdot \mathbf{r})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] - \mathcal{Y}\varepsilon_b\varphi_f[(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \mathbf{z}) - (\widehat{\mathbf{p}} \cdot \mathbf{z})(\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_s[(\widehat{\mathbf{p}} \cdot \widehat{\boldsymbol{\kappa}})(\mathbf{r} \cdot \widehat{\mathbf{p}}) - (\mathbf{r} \cdot \widehat{\boldsymbol{\kappa}})] + \mathfrak{K}_t[r^2 - (\widehat{\mathbf{p}} \cdot \mathbf{r})^2] - \varepsilon_b\varepsilon_e\varphi_f^2[(\mathbf{r} \cdot \mathbf{z}) - (\widehat{\mathbf{p}} \cdot \mathbf{z})(\mathbf{r} \cdot \widehat{\mathbf{p}})] \\
&\quad + \rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\mathbf{p}} \cdot \mathbf{r})(\mathbf{r} \cdot \mathbf{z}) - (\widehat{\mathbf{p}} \cdot \mathbf{z})r^2] \text{ by (6.1a), (6.21a) \& (A.2)}
\end{aligned}
$$

$$
\begin{aligned}
&= \mathfrak{K}_p\varepsilon_f - \mathfrak{K}_q\varsigma_b + \mathfrak{K}_k(\varepsilon_b r^2 - r^2\varepsilon_c\varepsilon_a) - \mathcal{Y}\varepsilon_b\varphi_f(r\varepsilon_b\varepsilon_d - r\delta_d\varepsilon_a) + \mathfrak{K}_s(r\varepsilon_b\varepsilon_c - r\varepsilon_a) + \mathfrak{K}_t(r^2 - r^2\varepsilon_c^2) \\
&\quad - \varepsilon_b\varepsilon_e\varphi_f^2(r\varepsilon_d - r\delta_d\varepsilon_c) + \rho\varrho_a\varepsilon_b\varphi_f(r^2\varepsilon_c\varepsilon_d - r^2\delta_d) \text{ by (6.1a) \& (6.10a)} \\
&= \mathfrak{K}_p\varepsilon_f - \mathfrak{K}_q\varsigma_b + r^2\mathfrak{K}_k(\varepsilon_b - \varepsilon_c\varepsilon_a) - r\mathcal{Y}\varepsilon_b\varphi_f(\varepsilon_b\varepsilon_d - \delta_d\varepsilon_a) + r\mathfrak{K}_s(\varepsilon_b\varepsilon_c - \varepsilon_a) + r^2\mathfrak{K}_t(1 - \varepsilon_c^2) \\
&\quad - r\varepsilon_b\varepsilon_e\varphi_f^2(\varepsilon_d - \delta_d\varepsilon_c) + r^2\rho\varrho_a\varepsilon_b\varphi_f(\varepsilon_c\varepsilon_d - \delta_d) \\
&= \mathfrak{H}_a \text{ by (6.51c)}
\end{aligned} \tag{6.55g}
$$

$$\begin{aligned}
&(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&\quad=(\widehat{\mathbf{p}}\times\mathbf{z})\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\qquad-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})]\text{ by (6.54)}\\
&\quad=\mathfrak{K}_p[\mathbf{h}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\mathfrak{K}_q[\mathbf{z}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\mathfrak{K}_r[\mathbf{r}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\mathfrak{K}_k[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\qquad-\mathfrak{Y}\varepsilon_b\varphi_f[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{K}_t[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]\\
&\qquad-\varepsilon_b\varepsilon_e\varphi_f^2[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\mathbf{p}}\times\mathbf{z})\cdot(\mathbf{r}\times\mathbf{z})]\\
&\quad=\mathfrak{K}_p\varepsilon_i+\mathfrak{K}_r\varsigma_b+\mathfrak{K}_k[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\mathbf{r})-(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})]-\mathfrak{Y}\varepsilon_b\varphi_f[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})z^2-(\widehat{\mathbf{p}}\cdot\mathbf{z})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\qquad+\mathfrak{K}_s[(\widehat{\mathbf{p}}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\widehat{\mathbf{p}})-(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{K}_t[(\mathbf{z}\cdot\mathbf{r})-(\widehat{\mathbf{p}}\cdot\mathbf{r})(\mathbf{z}\cdot\widehat{\mathbf{p}})]-\varepsilon_b\varepsilon_e\varphi_f^2[z^2-(\widehat{\mathbf{p}}\cdot\mathbf{z})^2]\\
&\qquad+\rho\varrho_a\varepsilon_b\varphi_f[(\widehat{\mathbf{p}}\cdot\mathbf{r})z^2-(\widehat{\mathbf{p}}\cdot\mathbf{z})(\mathbf{z}\cdot\mathbf{r})]\text{ by (6.1a), (6.21a) \& (A.2)}
\end{aligned}$$

$$\begin{aligned}
&=\mathfrak{K}_p\varepsilon_i+\mathfrak{K}_r\varsigma_b+\mathfrak{K}_k(r\varepsilon_b\varepsilon_d-r\varepsilon_c\delta_b)-\mathfrak{Y}\varepsilon_b\varphi_f(z^2\varepsilon_b-\delta_d\delta_b)+\mathfrak{K}_s(\varepsilon_b\delta_d-\delta_b)+\mathfrak{K}_t(r\varepsilon_d-r\varepsilon_c\delta_d)\\
&\quad-\varepsilon_b\varepsilon_e\varphi_f^2(z^2-\delta_d^2)+\rho\varrho_a\varepsilon_b\varphi_f(rz^2\varepsilon_c-r\delta_d\varepsilon_d)\text{ by (6.1a) \& (6.10a)}\\
&=\mathfrak{K}_p\varepsilon_i+\mathfrak{K}_r\varsigma_b+r\mathfrak{K}_k(\varepsilon_b\varepsilon_d-\varepsilon_c\delta_b)-\mathfrak{Y}\varepsilon_b\varphi_f(z^2\varepsilon_b-\delta_d\delta_b)+\mathfrak{K}_s(\varepsilon_b\delta_d-\delta_b)+r\mathfrak{K}_t(\varepsilon_d-\varepsilon_c\delta_d)\\
&\quad-\varepsilon_b\varepsilon_e\varphi_f^2(z^2-\delta_d^2)+r\rho\varrho_a\varepsilon_b\varphi_f(z^2\varepsilon_c-\delta_d\varepsilon_d)\\
&=\mathfrak{H}_b\text{ by (6.51c)}
\end{aligned}\tag{6.55h}$$

$$\begin{aligned}
&(\mathbf{r}\times\mathbf{z})\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&\quad=(\mathbf{r}\times\mathbf{z})\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\qquad-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})]\text{ by (6.54)}\\
&\quad=\mathfrak{K}_p[\mathbf{h}\cdot(\mathbf{r}\times\mathbf{z})]+\mathfrak{K}_q[\mathbf{z}\cdot(\mathbf{r}\times\mathbf{z})]+\mathfrak{K}_r[\mathbf{r}\cdot(\mathbf{r}\times\mathbf{z})]+\mathfrak{K}_k[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\qquad-\mathfrak{Y}\varepsilon_b\varphi_f[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{K}_t[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]\\
&\qquad-\varepsilon_b\varepsilon_e\varphi_f^2[(\mathbf{r}\times\mathbf{z})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\rho\varrho_a\varepsilon_b\varphi_f[(\mathbf{r}\times\mathbf{z})\cdot(\mathbf{r}\times\mathbf{z})]\\
&\quad=r\mathfrak{K}_p\varepsilon_h+\mathfrak{K}_k[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\mathbf{r})-r^2(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})]-\mathfrak{Y}\varepsilon_b\varphi_f[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})z^2-(\mathbf{r}\cdot\mathbf{z})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})]\\
&\qquad+\mathfrak{K}_s[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{z}\cdot\widehat{\mathbf{p}})-(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\widehat{\boldsymbol{\kappa}})]+\mathfrak{K}_t[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{z}\cdot\mathbf{r})-r^2(\mathbf{z}\cdot\widehat{\mathbf{p}})]\\
&\qquad-\varepsilon_b\varepsilon_e\varphi_f^2[(\mathbf{r}\cdot\widehat{\mathbf{p}})z^2-(\mathbf{r}\cdot\mathbf{z})(\mathbf{z}\cdot\widehat{\mathbf{p}})]+\rho\varrho_a\varepsilon_b\varphi_f[r^2z^2-(\mathbf{r}\cdot\mathbf{z})^2]\text{ by (6.1a) \& (A.2)}
\end{aligned}$$

$$\begin{aligned}
&=r\mathfrak{K}_p\varepsilon_h+\mathfrak{K}_k(r^2\varepsilon_a\varepsilon_d-r^2\delta_b)-\mathfrak{Y}\varepsilon_b\varphi_f(rz^2\varepsilon_a-r\varepsilon_d\delta_b)+\mathfrak{K}_s(r\varepsilon_a\delta_d-r\varepsilon_c\delta_b)+\mathfrak{K}_t(r^2\varepsilon_c\varepsilon_d-r^2\delta_d)\\
&\quad-\varepsilon_b\varepsilon_e\varphi_f^2(rz^2\varepsilon_c-r\varepsilon_d\delta_d)+\rho\varrho_a\varepsilon_b\varphi_f(r^2z^2-r^2\varepsilon_d^2)\text{ by (6.1a) \& (6.10a)}\\
&=r\mathfrak{K}_p\varepsilon_h+r^2\mathfrak{K}_k(\varepsilon_a\varepsilon_d-\delta_b)-r\mathfrak{Y}\varepsilon_b\varphi_f(z^2\varepsilon_a-\varepsilon_d\delta_b)+r\mathfrak{K}_s(\varepsilon_a\delta_d-\varepsilon_c\delta_b)+r^2\mathfrak{K}_t(\varepsilon_c\varepsilon_d-\delta_d)\\
&\quad-r\varepsilon_b\varepsilon_e\varphi_f^2(z^2\varepsilon_c-\varepsilon_d\delta_d)+r^2\rho\varrho_a\varepsilon_b\varphi_f(z^2-\varepsilon_d^2)\\
&=\mathfrak{H}_c\text{ by (6.51c)}
\end{aligned}\tag{6.55i}$$

all of which lead to

$$\begin{aligned}
&|\mathfrak{S}_t+\mathfrak{S}_u|^2=(\mathfrak{S}_t+\mathfrak{S}_u)\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&\quad=(\mathfrak{S}_t+\mathfrak{S}_u)\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathfrak{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\qquad-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})]\text{ by (6.54)}\\
&\quad=\mathfrak{K}_p[\mathbf{h}\cdot(\mathfrak{S}_t+\mathfrak{S}_u)]+\mathfrak{K}_q[\mathbf{z}\cdot(\mathfrak{S}_t+\mathfrak{S}_u)]+\mathfrak{K}_r[\mathbf{r}\cdot(\mathfrak{S}_t+\mathfrak{S}_u)]+\mathfrak{K}_k[(\mathfrak{S}_t+\mathfrak{S}_u)\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\qquad-\mathfrak{Y}\varepsilon_b\varphi_f[(\mathfrak{S}_t+\mathfrak{S}_u)\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[(\mathfrak{S}_t+\mathfrak{S}_u)\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{K}_t[(\mathfrak{S}_t+\mathfrak{S}_u)\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]\\
&\qquad-\varepsilon_b\varepsilon_e\varphi_f^2[(\mathfrak{S}_t+\mathfrak{S}_u)\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\rho\varrho_a\varepsilon_b\varphi_f[(\mathfrak{S}_t+\mathfrak{S}_u)\cdot(\mathbf{r}\times\mathbf{z})]\\
&\quad=\mathfrak{K}_p\mathfrak{K}_u+\mathfrak{K}_q\mathfrak{K}_v+\mathfrak{K}_r\mathfrak{K}_w+\mathfrak{K}_k\mathfrak{K}_x-\mathfrak{Y}\varepsilon_b\varphi_f\mathfrak{K}_y+\mathfrak{K}_s\mathfrak{K}_z+\mathfrak{K}_t\mathfrak{H}_a\\
&\qquad-\varepsilon_b\varepsilon_e\varphi_f^2\mathfrak{H}_b+\rho\varrho_a\varepsilon_b\varphi_f\mathfrak{H}_c\text{ by (6.55)}\\
&\quad\therefore|\mathfrak{S}_t+\mathfrak{S}_u|=\mathfrak{H}_d\text{ by (6.51c).}
\end{aligned}\tag{6.56}$$

Art 27d. *Development of equation* (3.24c).

To evaluate the quantities defined by (3.24c), we proceed by first deriving

$$
\begin{aligned}
&\widehat{\boldsymbol{\kappa}}\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&=\widehat{\boldsymbol{\kappa}}\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})]\text{ by (6.54)}\\
&=\mathfrak{K}_p(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h})+\mathfrak{K}_q(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{z})+\mathfrak{K}_r(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r})+\mathfrak{K}_k[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]-\mathcal{Y}\varepsilon_b\varphi_f[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\mathfrak{K}_t[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]+\rho\varrho_a\varepsilon_b\varphi_f[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{r}\times\mathbf{z})]-\varepsilon_b\varepsilon_e\varphi_f^2[\widehat{\boldsymbol{\kappa}}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&=\mathfrak{K}_p\delta_a+\mathfrak{K}_q\delta_b+r\mathfrak{K}_r\varepsilon_a+\mathfrak{K}_t\varsigma_a-r\rho\varrho_a\varepsilon_b\varphi_f\delta_f-\varepsilon_b\varepsilon_e\varphi_f^2\varsigma_c\text{ by (6.1a), (6.10a) \& (6.21a)}\\
&=\mathfrak{H}_e\text{ by (6.51d)}
\end{aligned}\tag{6.57a}
$$

$$
\begin{aligned}
&\mathbf{u}\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&=\mathbf{u}\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})]\text{ by (6.54)}\\
&=\mathfrak{K}_p(\mathbf{u}\cdot\mathbf{h})+\mathfrak{K}_q(\mathbf{u}\cdot\mathbf{z})+\mathfrak{K}_r(\mathbf{u}\cdot\mathbf{r})+\mathfrak{K}_k[\mathbf{u}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]-\mathcal{Y}\varepsilon_b\varphi_f[\mathbf{u}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[\mathbf{u}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\mathfrak{K}_t[\mathbf{u}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]+\rho\varrho_a\varepsilon_b\varphi_f[\mathbf{u}\cdot(\mathbf{r}\times\mathbf{z})]-\varepsilon_b\varepsilon_e\varphi_f^2[\mathbf{u}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&=\mathfrak{K}_q(-r\varrho_b\varphi_b)+r\mathfrak{K}_r(r\varrho_b)-\mathfrak{K}_k[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{u}\times\mathbf{r})]-\mathcal{Y}\varepsilon_b\varphi_f[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{z}\times\mathbf{u})]-\mathfrak{K}_s[\widehat{\boldsymbol{\kappa}}\cdot(\mathbf{u}\times\widehat{\mathbf{p}})]\\
&\quad-\mathfrak{K}_t[\widehat{\mathbf{p}}\cdot(\mathbf{u}\times\mathbf{r})]+\rho\varrho_a\varepsilon_b\varphi_f[\mathbf{z}\cdot(\mathbf{u}\times\mathbf{r})]-\varepsilon_b\varepsilon_e\varphi_f^2[\widehat{\mathbf{p}}\cdot(\mathbf{z}\times\mathbf{u})]\text{ by (1.5c), (6.27) \& (A.4)}
\end{aligned}
$$

$$
\begin{aligned}
&=-r\mathfrak{K}_q\varrho_b\varphi_b+r^2\mathfrak{K}_r\varrho_b-\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\cdot\mathbf{h})-\mathcal{Y}\varepsilon_b\varphi_f[\widehat{\boldsymbol{\kappa}}\cdot(-h^{-2}\mathfrak{y}_z\mathbf{h})]-(\mathfrak{K}_s/h^2)[\widehat{\boldsymbol{\kappa}}\cdot(\hbar_c\mathbf{h}-\delta_c\mathbf{z}-q\delta_c\widehat{\mathbf{r}})]\\
&\quad-\mathfrak{K}_t(\widehat{\mathbf{p}}\cdot\mathbf{h})+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{z}\cdot\mathbf{h})-\varepsilon_b\varepsilon_e\varphi_f^2[\widehat{\mathbf{p}}\cdot(-h^{-2}\mathfrak{y}_z\mathbf{h})]\text{ by (1.5c), (6.11) \& (6.45a)}\\
&=-r\mathfrak{K}_q\varrho_b\varphi_b+r^2\mathfrak{K}_r\varrho_b-\mathfrak{K}_k\delta_a+\varepsilon_b\varphi_f\mathfrak{y}_z\delta_a(\mathcal{Y}/h^2)-(\mathfrak{K}_s/h^2)[\hbar_c\delta_a-\delta_c(\delta_b+q\varepsilon_a)]\\
&\quad-\mathfrak{K}_t\delta_c+\varepsilon_b\varepsilon_e\varphi_f^2\mathfrak{y}_z(\delta_c/h^2)\text{ by (6.10a), (6.1a) \& (1.5c)}\\
&=-r\mathfrak{K}_q\varrho_b\varphi_b+r^2\mathfrak{K}_r\varrho_b-\mathfrak{K}_k\delta_a+\varepsilon_b\varphi_f\mathfrak{y}_z\delta_a(\mathcal{Y}/h^2)-(\mathfrak{K}_s/h^2)(\hbar_c\delta_a-\delta_c\hbar_a)\\
&\quad-\mathfrak{K}_t\delta_c+\varepsilon_b\varepsilon_e\varphi_f^2\mathfrak{y}_z(\delta_c/h^2)\text{ by (6.10b)}\\
&=\mathfrak{H}_f\text{ by (6.51d)}
\end{aligned}\tag{6.57b}
$$

$$
\begin{aligned}
&\widehat{\mathbf{p}}\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&=\widehat{\mathbf{p}}\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\quad+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})]\text{ by (6.54)}\\
&=\mathfrak{K}_p(\widehat{\mathbf{p}}\cdot\mathbf{h})+\mathfrak{K}_q(\widehat{\mathbf{p}}\cdot\mathbf{z})+\mathfrak{K}_r(\widehat{\mathbf{p}}\cdot\mathbf{r})+\mathfrak{K}_k[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]-\mathcal{Y}\varepsilon_b\varphi_f[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[\widehat{\mathbf{p}}\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]\\
&\quad+\mathfrak{K}_t[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]+\rho\varrho_a\varepsilon_b\varphi_f[\widehat{\mathbf{p}}\cdot(\mathbf{r}\times\mathbf{z})]-\varepsilon_b\varepsilon_e\varphi_f^2[\widehat{\mathbf{p}}\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]\\
&=\mathfrak{K}_p\delta_c+\mathfrak{K}_q\delta_d+r\mathfrak{K}_r\varepsilon_c-\mathfrak{K}_k\varsigma_a+\mathcal{Y}\varepsilon_b\varphi_f\varsigma_c-\rho\varrho_a\varepsilon_b\varphi_f\varsigma_b\text{ by (6.1a), (6.10a) \& (6.21a)}\\
&=\mathfrak{H}_g\text{ by (6.51d)}
\end{aligned}\tag{6.57c}
$$

$$
\begin{aligned}
&(\mathbf{r}\times\mathbf{h})\cdot(\mathfrak{S}_t+\mathfrak{S}_u)\\
&=(\mathbf{r}\times\mathbf{h})\cdot[\mathfrak{K}_p\mathbf{h}+\mathfrak{K}_q\mathbf{z}+\mathfrak{K}_r\mathbf{r}+\mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})-\mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})+\mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})+\mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r})\\
&\quad-\varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})+\rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z})]\text{ by (6.54)}\\
&=\mathfrak{K}_p[\mathbf{h}\cdot(\mathbf{r}\times\mathbf{h})]+\mathfrak{K}_q[\mathbf{z}\cdot(\mathbf{r}\times\mathbf{h})]+\mathfrak{K}_r[\mathbf{r}\cdot(\mathbf{r}\times\mathbf{h})]+\mathfrak{K}_k[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{r})]\\
&\quad-\mathcal{Y}\varepsilon_b\varphi_f[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\mathbf{z})]+\mathfrak{K}_s[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}})]+\mathfrak{K}_t[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\mathbf{p}}\times\mathbf{r})]\\
&\quad-\varepsilon_b\varepsilon_e\varphi_f^2[(\mathbf{r}\times\mathbf{h})\cdot(\widehat{\mathbf{p}}\times\mathbf{z})]+\rho\varrho_a\varepsilon_b\varphi_f[(\mathbf{r}\times\mathbf{h})\cdot(\mathbf{r}\times\mathbf{z})]
\end{aligned}
$$

$$
\begin{aligned}
&= -r\mathfrak{K}_q\varepsilon_h + \mathfrak{K}_k[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\mathbf{r}) - r^2(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})] - \mathcal{Y}\varepsilon_b\varphi_f[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\mathbf{z}) - (\mathbf{r}\cdot\mathbf{z})(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})] \\
&\quad + \mathfrak{K}_s[(\mathbf{r}\cdot\widehat{\boldsymbol{\kappa}})(\mathbf{h}\cdot\widehat{\mathbf{p}}) - (\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\widehat{\boldsymbol{\kappa}})] + \mathfrak{K}_t[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\mathbf{r}) - r^2(\mathbf{h}\cdot\widehat{\mathbf{p}})] \\
&\quad - \varepsilon_b\varepsilon_e\varphi_f^2[(\mathbf{r}\cdot\widehat{\mathbf{p}})(\mathbf{h}\cdot\mathbf{z}) - (\mathbf{r}\cdot\mathbf{z})(\mathbf{h}\cdot\widehat{\mathbf{p}})] + \rho\varrho_a\varepsilon_b\varphi_f[r^2(\mathbf{h}\cdot\mathbf{z}) - (\mathbf{r}\cdot\mathbf{z})(\mathbf{h}\cdot\mathbf{r})] \text{ by (6.1a) \& (A.2)} \\
&= -r\mathfrak{K}_q\varepsilon_h + \mathfrak{K}_k(-r^2\delta_a) - \mathcal{Y}\varepsilon_b\varphi_f(-r\varepsilon_d\delta_a) + \mathfrak{K}_s(r\varepsilon_a\delta_c - r\varepsilon_c\delta_a) + \mathfrak{K}_t(-r^2\delta_c) \\
&\quad - \varepsilon_b\varepsilon_e\varphi_f^2(-r\varepsilon_d\delta_c) \text{ by (1.5c), (6.1a) \& (A.2)} \\
&= -r\mathfrak{K}_q\varepsilon_h - r^2\mathfrak{K}_k\delta_a + r\mathcal{Y}\varepsilon_b\varphi_f\varepsilon_d\delta_a + r\mathfrak{K}_s(\varepsilon_a\delta_c - \varepsilon_c\delta_a) - r^2\mathfrak{K}_t\delta_c + r\varepsilon_b\varepsilon_e\varphi_f^2\varepsilon_d\delta_c \\
&= r[-\mathfrak{K}_q\varepsilon_h - r(\mathfrak{K}_k\delta_a + \mathfrak{K}_t\delta_c) + \mathfrak{K}_s(\varepsilon_a\delta_c - \varepsilon_c\delta_a) + \varepsilon_b\varepsilon_d\varphi_f(\mathcal{Y}\delta_a + \varepsilon_e\varphi_f\delta_c)] \\
&= \mathfrak{H}_h \text{ by (6.51d)} \qquad (6.57\text{d})
\end{aligned}
$$

from which we obtain

$$
\begin{aligned}
\Re_1 &= \widehat{\boldsymbol{\kappa}}\cdot(\boldsymbol{\mathfrak{S}}_t + \boldsymbol{\mathfrak{S}}_u) \text{ by (3.24c)} \\
&= \mathfrak{H}_e \text{ by (6.57a)} \qquad (6.58\text{a}) \\
\Re_2 &= \mathbf{a}\cdot(\boldsymbol{\mathfrak{S}}_t + \boldsymbol{\mathfrak{S}}_u) \text{ by (3.24c)} \\
&= (-\varrho_a\mathbf{r})\cdot(\boldsymbol{\mathfrak{S}}_t + \boldsymbol{\mathfrak{S}}_u) \text{ by (1.5a) \& (6.21b)} \\
&= -\varrho_a\mathfrak{K}_w \text{ by (6.55c)} \qquad (6.58\text{b})
\end{aligned}
$$

$$
\begin{aligned}
\Re_3 &= \dot{\mathbf{a}}\cdot(\boldsymbol{\mathfrak{S}}_t + \boldsymbol{\mathfrak{S}}_u) \text{ by (3.24c)} \\
&= (-\varrho_a\mathbf{u} + 3\varrho_a\varrho_b\mathbf{r})\cdot(\boldsymbol{\mathfrak{S}}_t + \boldsymbol{\mathfrak{S}}_u) \text{ by (6.26a)} \\
&= -\varrho_a\mathfrak{H}_f + 3\varrho_a\varrho_b\mathfrak{K}_w \text{ by (6.55c) \& (6.57b)} \qquad (6.58\text{c}) \\
\Re_4 &= \ddot{\mathbf{a}}\cdot(\boldsymbol{\mathfrak{S}}_t + \boldsymbol{\mathfrak{S}}_u) \text{ by (3.24c)} \\
&= (6\varrho_a\varrho_b\mathbf{u} - \varrho_a\varrho_l\mathbf{r})\cdot(\boldsymbol{\mathfrak{S}}_t + \boldsymbol{\mathfrak{S}}_u) \text{ by (6.26b)} \\
&= 6\varrho_a\varrho_b\mathfrak{H}_f - \varrho_a\varrho_l\mathfrak{K}_w \text{ by (6.55c) \& (6.57b)} \qquad (6.58\text{d})
\end{aligned}
$$

$$
\begin{aligned}
\Re_5 &= \ddot{\mathbf{e}}\cdot(\boldsymbol{\mathfrak{S}}_t + \boldsymbol{\mathfrak{S}}_u) \text{ by (3.24c)} \\
&= [\mathfrak{x}_o\widehat{\mathbf{p}} + 2\kappa\varepsilon_b\mathfrak{x}_k\mathbf{z} + 2\kappa q\varepsilon_b\mathfrak{x}_k\widehat{\mathbf{r}} + \mathfrak{x}_p(\mathbf{r}\times\mathbf{h})]\cdot(\boldsymbol{\mathfrak{S}}_t + \boldsymbol{\mathfrak{S}}_u) \text{ by (6.36b)} \\
&= \mathfrak{x}_o\mathfrak{H}_g + 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{K}_v + 2\kappa(q/r)\varepsilon_b\mathfrak{x}_k\mathfrak{K}_w + \mathfrak{x}_p\mathfrak{H}_h \text{ by (6.55) \& (6.57)} \\
&= \mathfrak{x}_o\mathfrak{H}_g + 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{K}_v + 2\kappa\varphi_b\varepsilon_b\mathfrak{x}_k\mathfrak{K}_w + \mathfrak{x}_p\mathfrak{H}_h \text{ by (6.1b).} \qquad (6.58\text{e})
\end{aligned}
$$

We have also that

$$
\begin{aligned}
&\ddot{\mathcal{Y}}\Re_1 + \ddot{\rho}\Re_2 + b_2\Re_3 + \rho\Re_4 + \Re_5 \\
&= \mathfrak{y}_b\Re_1 + \mathfrak{x}_i\Re_2 + \mathfrak{y}_e\Re_3 + \rho\Re_4 + \Re_5 \text{ by (6.38b), (6.32b) \& (6.39b)} \\
&= \mathfrak{y}_b\mathfrak{H}_e + \mathfrak{x}_i(-\varrho_a\mathfrak{K}_w) + \mathfrak{y}_e(-\varrho_a\mathfrak{H}_f + 3\varrho_a\varrho_b\mathfrak{K}_w) + \rho(6\varrho_a\varrho_b\mathfrak{H}_f - \varrho_a\varrho_l\mathfrak{K}_w) \\
&\quad + \mathfrak{x}_o\mathfrak{H}_g + 2\kappa\varepsilon_b\mathfrak{x}_k\mathfrak{K}_v + 2\kappa\varphi_b\varepsilon_b\mathfrak{x}_k\mathfrak{K}_w + \mathfrak{x}_p\mathfrak{H}_h \text{ by (6.58)} \\
&= \mathfrak{y}_b\mathfrak{H}_e + \mathfrak{x}_o\mathfrak{H}_g + \mathfrak{x}_p\mathfrak{H}_h - \varrho_a[\mathfrak{x}_i\mathfrak{K}_w + \mathfrak{y}_e(\mathfrak{H}_f - 3\varrho_b\mathfrak{K}_w) + \rho(\varrho_l\mathfrak{K}_w - 6\varrho_b\mathfrak{H}_f)] \\
&\quad + 2\kappa\varepsilon_b\mathfrak{x}_k(\mathfrak{K}_v + \varphi_b\mathfrak{K}_w) \\
&= \mathfrak{H}_i \text{ by (6.51d)} \qquad (6.59\text{a})
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{Y}\widehat{\boldsymbol{\kappa}} + \rho\mathbf{a} - \mathbf{u} + \mathbf{e} \\
&= \mathcal{Y}\widehat{\boldsymbol{\kappa}} + \rho(-\varrho_a\mathbf{r}) - [h^{-2}(\mathbf{z}\times\mathbf{h} + q\widehat{\mathbf{r}}\times\mathbf{h})] + f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}} \text{ by (1.5) \& (6.21b)} \\
&= \mathcal{Y}\widehat{\boldsymbol{\kappa}} - \rho\varrho_a\mathbf{r} - h^{-2}(\mathbf{z}\times\mathbf{h}) - h^{-2}(q/r)(\mathbf{r}\times\mathbf{h}) + f_1(\mathbf{r}\times\mathbf{h}) + f_2\widehat{\mathbf{p}} \\
&= \mathcal{Y}\widehat{\boldsymbol{\kappa}} + f_2\widehat{\mathbf{p}} - \rho\varrho_a\mathbf{r} - h^{-2}(\mathbf{z}\times\mathbf{h}) + (f_1 - h^{-2}\varphi_b)(\mathbf{r}\times\mathbf{h}) \text{ by (6.1b)} \\
&= \mathcal{Y}\widehat{\boldsymbol{\kappa}} + \varepsilon_e\varphi_f\widehat{\mathbf{p}} - \rho\varrho_a\mathbf{r} - h^{-2}(\mathbf{z}\times\mathbf{h}) + (\varepsilon_b\varphi_f - h^{-2}\varphi_b)(\mathbf{r}\times\mathbf{h}) \text{ by (6.2g) \& (6.2h)} \\
&= \mathcal{Y}\widehat{\boldsymbol{\kappa}} + \varepsilon_e\varphi_f\widehat{\mathbf{p}} - \rho\varrho_a\mathbf{r} - (1/h^2)(\mathbf{z}\times\mathbf{h}) + \mathfrak{H}_j(\mathbf{r}\times\mathbf{h}) \text{ by (6.51d).} \qquad (6.59\text{b})
\end{aligned}
$$

Art 27e. *Results of the computations.*

By substituting (6.59), (6.56) and (6.54) into (3.26), we finally get

$$\overline{\mathbb{K}} = \frac{\mathfrak{H}_d}{c^3\mathcal{R}^3}, \qquad \overline{\mathbb{T}} = \frac{\mathfrak{H}_i}{(\mathfrak{H}_d)^2} \tag{6.60a}$$

$$\begin{aligned}
\overline{\boldsymbol{\ell}}_t &= \frac{1}{c\mathcal{R}}\Big[\mathcal{Y}\widehat{\boldsymbol{\kappa}} + \varepsilon_e\varphi_f\widehat{\mathbf{p}} - \rho\varrho_a\mathbf{r} - (1/h^2)(\mathbf{z}\times\mathbf{h}) + \mathfrak{H}_j(\mathbf{r}\times\mathbf{h})\Big] \\
\overline{\boldsymbol{\ell}}_b &= \frac{1}{\mathfrak{H}_d}\Big[\mathfrak{K}_p\mathbf{h} + \mathfrak{K}_q\mathbf{z} + \mathfrak{K}_r\mathbf{r} + \mathfrak{K}_k(\widehat{\boldsymbol{\kappa}}\times\mathbf{r}) - \mathcal{Y}\varepsilon_b\varphi_f(\widehat{\boldsymbol{\kappa}}\times\mathbf{z}) + \mathfrak{K}_s(\widehat{\boldsymbol{\kappa}}\times\widehat{\mathbf{p}}) + \mathfrak{K}_t(\widehat{\mathbf{p}}\times\mathbf{r}) \\
&\quad + \rho\varrho_a\varepsilon_b\varphi_f(\mathbf{r}\times\mathbf{z}) - \varepsilon_b\varepsilon_e\varphi_f^2(\widehat{\mathbf{p}}\times\mathbf{z})\Big]
\end{aligned} \tag{6.60b}$$

as the complete set of equations describing the apparent geometry of obliquated rays for a gravitating observer.

Part III: Illustrations

The beauty of nature lies in detail; the message in generality. Optimal appreciation demands both and I know of no better tactic than the illustration of exciting principles by well-chosen particulars.

Stephen J. Gould (1941 - 2002)

7 Effects of constant acceleration

Art 28. *On the apparent direction to a light source.*

When an observer translates with a constant acceleration $\mathbf{a} = \dot{\mathbf{u}}$, the velocity $\mathbf{u}$ and the position $\mathbf{r}$ of the observer at any instant t are given by

$$\mathbf{u} = \mathbf{a}t + \mathbf{u}_o, \quad \mathbf{r} = \frac{1}{2}\mathbf{a}t^2 + \mathbf{u}_o t + \mathbf{r}_o \tag{7.1a}$$

where $\mathbf{u}_o$ and $\mathbf{r}_o$ are respectively the observer's velocity and position at instant $t = 0$. By squaring the second of these equations and rearranging its terms, we get

$$a^2t^2 + 2(\mathbf{a} \cdot \mathbf{u}_o)t + 2\mathbf{a} \cdot (\mathbf{r}_o - \mathbf{r}) = 0 \tag{7.1b}$$

while by squaring the second equation in (7.1a) and taking (7.1b) into account, we get

$$u^2 = u_o^2 + 2atu_o \cos\theta_o + a^2t^2, \quad u^2 = u_o^2 + 2\mathbf{a} \cdot (\mathbf{r} - \mathbf{r}_o) \tag{7.1c}$$

where θ_o is the angle between $\mathbf{a}$ and $\mathbf{u}_o$. Consequently, if $\beta_o = u_o/c$, then by (1.6) and (7.1c),

$$\beta(t) = [\beta_o^2 + 2\sigma t\beta_o \cos\theta_o + \sigma^2t^2]^{1/2}, \quad \beta(\mathbf{r}) = [\beta_o^2 + 2(\boldsymbol{\sigma}/c) \cdot (\mathbf{r} - \mathbf{r}_o)]^{1/2}. \tag{7.1d}$$

If we solve the first equation in (7.1c) for at and take the second equation into account, we shall obtain

$$at = -u_o \cos\theta_o \pm (u^2 - u_o^2 \sin^2\theta_o)^{1/2} = -u_o \cos\theta_o \pm [u_o^2 \cos^2\theta_o + 2\mathbf{a} \cdot (\mathbf{r} - \mathbf{r}_o)]^{1/2}. \tag{7.1e}$$

Also, by multiplying the first equation in (7.1a) vectorwise by $\mathbf{a}$, one may show that

$$u \sin\theta = u_o \sin\theta_o \tag{7.1f}$$

where θ is the angle between $\mathbf{a}$ and $\mathbf{u}$ (cf. Figure 1). These results have been known since Galileo brought them forcefully to the attention of philosophers; our task will be to study their kineoptical consequences in some detail.

The acceleration $\mathbf{a}$ and the wave vector $\boldsymbol{\kappa}$ being constant, the angle λ between them at any instant does not change in the course of the observer's motion, whence

$$\lambda(t) = \lambda(\mathbf{r}) = \lambda_o \tag{7.2a}$$

where λ_o is the angle between these vectors at instant $t = 0$. From (7.1c) and (7.1f), we have [11]

$$\theta(t) = \arcsin\left\{\frac{u_o \sin\theta_o}{\sqrt{u_o^2 + 2atu_o \cos\theta_o + a^2t^2}}\right\}, \quad \theta(\mathbf{r}) = \arcsin\left\{\frac{u_o \sin\theta_o}{\sqrt{u_o^2 + 2\mathbf{a} \cdot (\mathbf{r} - \mathbf{r}_o)}}\right\}. \tag{7.2b}$$

[11] We remind the reader that throughout this work and in accordance with (1.6), the case $\mathbf{u} = \mathbf{0}$ is to be excluded from consideration.

By multiplying through the first equation in (7.1a) scalarwise by $\widehat{\boldsymbol{\kappa}}$ and taking (7.1e) into consideration, one may show that

$$\begin{gathered}\phi(t) = \arccos\left\{\frac{u_o\cos\phi_o + at\cos\lambda_o}{\sqrt{u_o^2 + 2atu_o\cos\theta_o + a^2t^2}}\right\}\\ \phi(\mathbf{r}) = \arccos\left\{\frac{u_o(\cos\phi_o - \cos\lambda_o\cos\theta_o) \pm \cos\lambda_o\sqrt{u_o^2\cos^2\theta_o + 2\mathbf{a}\cdot(\mathbf{r}-\mathbf{r}_o)}}{\sqrt{u_o^2 + 2\mathbf{a}\cdot(\mathbf{r}-\mathbf{r}_o)}}\right\}\end{gathered} \tag{7.2c}$$

where ϕ is the angle between $\widehat{\boldsymbol{\kappa}}$ and $\mathbf{u}$, and ϕ_o is the angle between $\widehat{\boldsymbol{\kappa}}$ and $\mathbf{u}_o$.

Art 28a. *Translation with zero acceleration.*

When the observer translates without accelerating ($\mathbf{a} = \mathbf{0}$), we have $\sigma = 0$ and $\mu = 0$ by (1.6). The various quantities defined by (4.2d) become

$$\rho = 0, \quad \pi = 0, \quad d = 1, \quad \vartheta = 0, \quad \mathcal{Y} = c. \tag{7.3a}$$

Putting $\mathbf{a} = \mathbf{0}$ into (7.2c) and $\sigma = 0$ into (7.1d) leads to

$$\phi = \phi_o, \quad \beta = \beta_o \tag{7.3b}$$

so that, in view of (7.3a) and (7.3b), the quantities defined by (4.2b) reduce to

$$\mathcal{G} = -\beta_o + \cos\phi_o, \quad \mathcal{R}^2 = 1 + \beta_o^2 - 2\beta_o\cos\phi_o, \quad \mathcal{F}^2 = \sin^2\phi_o. \tag{7.3c}$$

Substituting the value of $\mathcal{R}$ from (7.3c) into (3.5a) gives the ray speed as

$$\upsilon = c(1 + \beta_o^2 - 2\beta_o\cos\phi_o)^{1/2} \tag{7.4a}$$

while by putting the values of $\mathcal{F}$ and $\mathcal{G}$ from (7.3c) into (4.2c), we get the angular displacement of the light source as

$$\psi = \arctan\left\{\frac{\sin\phi_o}{-\beta_o + \cos\phi_o}\right\}. \tag{7.4b}$$

This well known result [29] is equivalent to (2.6a). We conclude that when an observer translates without accelerating, polarization and dispersion have no effect on the apparent displacement of a light source or on the speed of a light ray.

Art 28b. *Rectilinear translation with nonzero acceleration.*

If the initial velocity $\mathbf{u}_o$ of the observer is directed parallel to the acceleration $\mathbf{a}$, the first equation in (7.1a) shows that the instantaneous velocity $\mathbf{u}$ will remain parallel to the acceleration at all times. Indeed, substituting $\theta_o = 0$ and $\phi_o = \lambda_o$ into (7.2b) and (7.2c) gives (cf. Figure 1)

$$\theta = \theta_o = 0, \quad \phi = \phi_o = \lambda_o = \lambda \tag{7.5a}$$

where we have taken advantage of (7.2a). Substituting (7.5a) into (7.1d) leads to

$$\beta(t) = \beta_o + \sigma t, \quad \beta(\mathbf{r}) = [\beta_o^2 + 2(\sigma/c)|\mathbf{r}-\mathbf{r}_o|]^{1/2}. \tag{7.5b}$$

Again in view of (7.5a), we have from (4.2d) that

$$\begin{gathered}\vartheta = \mu\cos\phi_o, \quad d = \left\{\frac{1+\sqrt{1+\vartheta^2}}{2}\right\}^{1/2}\\ \pi = \vartheta(1+\vartheta^2)^{-1/2}, \quad \rho = \pi/(4d\omega_o), \quad \mathcal{Y} = c(d - 2\rho\sigma\cos\phi_o)\end{gathered} \tag{7.5c}$$

while (4.4a) becomes

$$\begin{aligned} &\mathcal{F} = |d - 2\rho\sigma\cos\phi_o|\sin\phi_o, \quad \mathcal{G} = d\cos\phi_o - \beta - \rho\sigma\cos 2\phi_o \\ &\mathcal{R}^2 = d^2 + \beta^2 - 2d\beta\cos\phi_o + \rho\sigma(\rho\sigma - 2d\cos\phi_o + 2\beta\cos 2\phi_o). \end{aligned} \tag{7.5d}$$

Substituting the value of $\mathcal{R}$ from (7.5d) into (3.5a) gives the ray speed as

$$\upsilon = c[d^2 + \beta^2 - 2d\beta\cos\phi_o + \rho\sigma(\rho\sigma - 2d\cos\phi_o + 2\beta\cos 2\phi_o)]^{1/2} \tag{7.6a}$$

while by putting the values of $\mathcal{F}$ and $\mathcal{G}$ from (7.5d) into (4.2c), we get the angular displacement of the light source as

$$\psi = \arctan\left\{\frac{|d - 2\rho\sigma\cos\phi_o|\sin\phi_o}{d\cos\phi_o - \beta - \rho\sigma\cos 2\phi_o}\right\}. \tag{7.6b}$$

We conclude that for an observer in accelerated rectilinear translation, both acceleration and dispersion will be observed to affect the speed of a light ray as well as the angular displacement of a light source.

Example 7-A. *Transverse line of incidence.* If the light ray is incident [12] at right angles to the observer's velocity, so that $\phi_o = 90°$, then by (7.5c),

$$\vartheta = 0, \quad d = 1, \quad \pi = 0, \quad \rho = 0, \quad \mathcal{Y} = c. \tag{7.7a}$$

From (7.6a) and (7.6b), we obtain

$$\upsilon = c(1 + \beta^2)^{1/2}, \quad \psi = \arctan(-1/\beta) \tag{7.7b}$$

which shows that the ray speed and the angular displacement of the light source depend only on the instantaneous velocity of the observer. We conclude that there are indeed situations in which obliquation is determined by the instantaneous velocity of the observer and is independent of the observer's acceleration as postulated by the so-called clock or locality hypothesis of aclassical physics [24, 30]. Generally speaking, however, obliquation depends explicitly on the observer's acceleration although its effects are easy to eliminate by an appropriate choice of geometry.

Example 7-B. *Semi-transverse line of incidence.* If the light ray is incident at 45° to the observer's velocity, so that $\phi_o = 45°$ and $\cos\phi_o = \sin\phi_o = 1/\sqrt{2} = \sqrt{2}/2$, then by (7.5c),

$$\vartheta = \mu/\sqrt{2}, \quad \mathcal{Y} = c(d - \rho\sigma\sqrt{2}) \tag{7.8a}$$

while from (7.6), we get

$$\upsilon = c[d^2 + \beta(\beta - d\sqrt{2}) + \rho\sigma(\rho\sigma - d\sqrt{2})]^{1/2}, \quad \psi = \arctan\left\{\frac{|d - \rho\sigma\sqrt{2}|}{d - \beta\sqrt{2}}\right\} \tag{7.8b}$$

which shows the effects of acceleration (via σ) and dispersion (via ρ) on the ray speed υ and on the angular displacement ψ of the light source. We conclude in view of (7.5c) and (7.8a) that since $\rho \neq 0$ and $\sigma \neq 0$ unless $a = 0$, the effects of acceleration and dispersion cannot be strictly eliminated in this case unless the observer ceases to accelerate.

Example 7-C. *Ultragamma approximation for semi-transverse loi.* When the quantity μ is so small that expressions containing its third and higher powers can be neglected ($\mu \ll 1$), we have

[12] We remind the reader of the importance of distinguishing the line of incidence (loi) of a ray from its line of sight (los) since insufficient attention to this distinction has proved to be a rich source of errors for careless minds.

from (7.8a) that the third and higher powers of ϑ can also be neglected. Accordingly, by (7.5c) and (7.8a), we have the following approximations

$$d \approx 1 + \frac{\mu^2}{16}, \quad \pi \approx \frac{\mu}{\sqrt{2}}, \quad \rho \approx \frac{\mu}{4\omega_o\sqrt{2}}, \quad \mathcal{Y} \approx c\left\{1 - \frac{3\mu^2}{16}\right\} \tag{7.9a}$$

where we have used the fact that $\mu = \sigma/\omega_o$ by (1.6). Bearing this fact in mind in addition to (7.8a) and (7.9a), it is easy to show that for $d - \rho\sigma\sqrt{2} \geq 0$, (7.8b) takes the form

$$v \approx c\left\{1 + \beta(\beta - \sqrt{2}) - \frac{\mu^2}{8}\left(1 + \frac{\beta}{\sqrt{2}}\right)\right\}^{1/2}, \quad \psi \approx \arctan\left\{1 + \frac{\mu^2}{4} - \beta\sqrt{2}\left(1 + \frac{3\mu^2}{16}\right)\right\}^{-1}. \tag{7.9b}$$

We conclude that in this approximation, the effects of acceleration and dispersion on the ray speed and on the angular displacement of the light source are of second and higher orders in μ.

Example 7-D. *Infraradio approximation for semi-transverse loi.* When the quantity μ is so large that expressions containing its first and higher powers are much greater than $\sqrt{2}$, we have from (7.8a) that the first and higher powers of ϑ are much greater than 1. Accordingly, by (7.5c) and (7.8a), we have the following approximations

$$d \approx \left\{\frac{\mu}{\sqrt{8}}\right\}^{1/2}, \quad \pi \approx 1, \quad \rho \approx \frac{1}{4\omega_o}\left\{\frac{\sqrt{8}}{\mu}\right\}^{1/2}, \quad \mathcal{Y} \approx 0. \tag{7.10a}$$

Equation (7.8b) takes the form

$$v \approx c\left\{\beta^2 + b\left(\frac{b}{4} - \beta\right)\right\}^{1/2}, \quad b = \left\{\frac{\mu}{\sqrt{2}}\right\}^{1/2}, \quad \psi \approx \arctan 0 \tag{7.10b}$$

from which we conclude that, in this approximation, the effects of acceleration and dispersion on the ray speed manifest at first order in μ while the light source suffers an angular displacement that places it in a direction parallel or antiparallel to the observer's velocity.

Scholium 7-A. *Prosaic character of superluminal velocities.* Each expression for the ray speed v in the foregoing examples imposes a constraint on β for a given μ since the radicand in the expression must be positive in order for the expression to hold. These constraints are however geometric and specific to each example. We may therefore not regard any of them as a fundamental statement applicable to all obliquation phenomena. This is particularly important when the constraint is such as to require that $\beta < 1$, because in this case those who have not learnt to give due diligence to the demands of epistemological completeness may be tempted to suppose (in view of the claim of many proponents of aclassical physics to this effect) that this constraint is an Act of Nature. We emphasize therefore that in this work we neither require $\beta < 1$ nor place any apriori constraint on β. This is illustrated by the foregoing examples which admit $\beta \geq 1$ without contradictions.

Art 28c. *Coradial translation with nonzero acceleration.*

When an observer translates with a constant acceleration directed along the line of incidence of a light ray, we have $\lambda_o = 0, \theta_o = \phi_o$ which upon substitution into (7.2) yields (cf. Figure 1)

$$\theta_o = \phi_o, \quad \theta = \phi, \quad \lambda_o = \lambda = 0 \tag{7.11a}$$

on account of which, by (7.2b) and (7.2c),

$$\phi(t) = \arcsin\left\{\frac{u_o^2\sin^2\phi_o}{u_o^2 + 2atu_o\cos\phi_o + a^2t^2}\right\}^{1/2}, \quad \phi(\mathbf{r}) = \arcsin\left\{\frac{u_o^2\sin^2\phi_o}{u_o^2 + 2\mathbf{a}\cdot(\mathbf{r} - \mathbf{r}_o)}\right\}^{1/2}. \tag{7.11b}$$

Substituting (7.11a) into (7.1d) leads to

$$\beta(t) = [\beta_o^2 + 2\sigma t\beta_o \cos\phi_o + \sigma^2 t^2]^{1/2}, \quad \beta(\mathbf{r}) = [\beta_o^2 + 2(\boldsymbol{\sigma}/c)\cdot(\mathbf{r}-\mathbf{r}_o)]^{1/2}. \tag{7.11c}$$

In view of (7.11a), we have from (4.2d) that

$$\vartheta = \mu, \quad d = \left\{\frac{1+\sqrt{1+\vartheta^2}}{2}\right\}^{1/2}$$
$$\pi = \vartheta(1+\vartheta^2)^{-1/2}, \quad \rho = \pi/(4d\omega_o), \quad \mathcal{Y} = c(d - 2\rho\sigma) \tag{7.12a}$$

while according to (4.5a),

$$\mathcal{F} = |d-\rho\sigma|\sin\phi, \quad \mathcal{G} = -\beta + (d-\rho\sigma)\cos\phi$$
$$\mathcal{R}^2 = (d-\rho\sigma)^2 + \beta^2 - (2d+\rho\sigma)\beta\cos\phi. \tag{7.12b}$$

Substituting the value of $\mathcal{R}$ from (7.12b) into (3.5a) gives the ray speed as

$$\upsilon = c[\beta^2 - \beta(2d+\rho\sigma)\cos\phi + (d-\rho\sigma)^2]^{1/2}. \tag{7.13a}$$

Furthermore, by putting the values of $\mathcal{F}$ and $\mathcal{G}$ from (7.12b) into (4.2c), we get the angular displacement of the light source as

$$\psi = \arctan\left\{\frac{|d-\rho\sigma|\sin\phi}{-\beta + (d-\rho\sigma)\cos\phi}\right\} \tag{7.13b}$$

which, while similar in form to (4.5b), differs from (4.5b) in that ϕ is required to satisfy (7.11b). We conclude that for an observer in accelerated coradial translation, both acceleration and dispersion will be observed to affect the speed of a light ray as well as the angular displacement of a light source.

Example 7-E. *Coradial translation with transverse loi.* It may seem that one can obtain an interesting and useful result by putting $\phi = 90°$ into (7.13). One difficulty with this circumstance is that since ϕ is a function of time by (7.11b), the condition $\phi = 90°$ can be established only for a brief instant. A more serious difficulty is that since the observer's acceleration is parallel to the line of incidence of the light ray, the condition $\phi = 90°$ requires the observer's velocity to be perpendicular to the acceleration. The acceleration being constant, however, this requirement can be satisfied only at the instant when the observer's velocity is zero. We conclude that putting $\phi = 90°$ into (7.13) does not lead to a very useful result.

Art 29. *On the apparent drift of a light source.*

For an observer translating with constant acceleration, the velocity and acceleration of the observer are independent quantities, the acceleration being constant while the velocity varies, so that the formulae of Art 17 are applicable. In view of (7.1), we have from (4.8) that

$$\mathfrak{X} = \imath_0\widehat{\boldsymbol{\kappa}} + \imath_1\mathbf{u}_o + (\imath_1 t - \imath_2)\mathbf{a} \tag{7.14a}$$

$$\imath_4 = \frac{\beta(\mathcal{R}^2 + \beta\mathcal{G})}{\mathcal{F}\mathcal{R}^2 u^2}, \quad \imath_3 = \frac{\beta + \mathcal{G}}{\mathcal{F}u^2}, \quad \imath_2 = \imath_4\rho, \quad \imath_1 = \imath_3 + \imath_4, \quad \imath_0 = -\imath_4\mathcal{Y}. \tag{7.14b}$$

Furthermore, by (3.7c), (4.9) and (7.14a), we get

$$\dot\psi = a[\imath_0\cos\lambda + \imath_1 u_o\cos\theta_o + a(\imath_1 t - \imath_2)], \quad \Theta = \dot\psi/a$$
$$\mathfrak{X}^2 = \imath_0^2 + 2\imath_0\imath_1 u_o\cos\phi_o + \imath_1^2 u_o^2 + a(\imath_1 t - \imath_2)[a(\imath_1 t - \imath_2) + 2\imath_0\cos\lambda + 2\imath_1 u_o\cos\theta_o] \tag{7.14c}$$

which gives the drift $\dot{\psi}$ and the magnitude of the obliquation gradient $\mathfrak{X}$ as functions of time t, the variation of obliquation Θ being defined only when the observer has a nonzero acceleration.

Art 29a. *Translation with zero acceleration.*

When the observer translates without accelerating, the obliquation gradient $\mathfrak{X}$ is well defined and can be calculated by first substituting (7.3) into (7.14b) to get

$$\imath_4 = \frac{\beta_o(1-\beta_o\cos\phi_o)}{u_o^2(1+\beta_o^2-2\beta_o\cos\phi_o)\sin\phi_o}, \quad \imath_3 = \frac{\cos\phi_o}{u_o^2\sin\phi_o}, \quad \imath_2 = 0 \tag{7.15a}$$

$$\imath_1 = \frac{\beta_o+(1-2\beta_o\cos\phi_o)\cos\phi_o}{u_o^2(1+\beta_o^2-2\beta_o\cos\phi_o)\sin\phi_o}, \quad \imath_0 = \frac{\beta_o\cos\phi_o-1}{u_o(1+\beta_o^2-2\beta_o\cos\phi_o)\sin\phi_o} \tag{7.15b}$$

and then putting $\mathbf{a}=\mathbf{0}$ into (7.14a) and (7.14c) to get

$$\mathfrak{X} = \imath_0\widehat{\boldsymbol{\kappa}}+\imath_1\mathbf{u}_o, \quad \mathfrak{X}^2 = \imath_0^2+2\imath_0\imath_1u_o\cos\phi_o+\imath_1^2u_o^2, \quad \dot{\psi}=0. \tag{7.15c}$$

We conclude that when an observer translates without accelerating, the angular displacement of a light source will not change in the course of the observer's motion regardless of the line of incidence of a light ray from the source to the observer.

Scholium 7-B. *Interpretation of obliquation gradient.* So long as the obliquation angle ψ depends on the observer's velocity $\mathbf{u}$, the gradient of ψ with respect to $\mathbf{u}$ is well defined even for a constant $\mathbf{u}$ because the gradient gives the amount by which ψ changes if and when $\mathbf{u}$ changes without explicitly requiring $\mathbf{u}$ to be variable. When $\mathbf{u}$ does vary, as it must when an observer accelerates, the obliquation gradient gives the amount by which the apparent position of a light source changes for a unit change in the observer's velocity at any given instant. But when $\mathbf{u}$ is fixed, as it must be when an observer does not accelerate, the obliquation gradient gives the amount by which the apparent position of a light source differs for two observers, each moving with a constant velocity, for every unit difference in the observers' velocities. This case deserves clarification because those familiar with the prevailing mode of expression in aclassical physics may be tempted to interprete it by introducing a space filled with observers, all moving with different but constant velocities, and may therefore be led to suppose that obliquation gradient by its very nature requires a multiplicity of observers.

Art 29b. *Rectilinear translation with nonzero acceleration.*

When an observer translates rectilinearly with a constant acceleration, the results of Art 28b are applicable. Introducing

$$\delta = \frac{d(d-\beta\cos\phi_o)+\rho\sigma(\rho\sigma-2d\cos\phi_o+\beta\cos2\phi_o)}{d^2+\beta^2-2d\beta\cos\phi_o+\rho\sigma(\rho\sigma-2d\cos\phi_o+2\beta\cos2\phi_o)} \tag{7.16}$$

we have by substituting (7.5d) and the value of $\mathfrak{Y}$ from (7.5c) into (7.14b),

$$\imath_4 = \frac{\beta\delta}{u^2(d-2\rho\sigma\cos\phi_o)\sin\phi_o}, \quad \imath_3 = \frac{d\cos\phi_o-\rho\sigma\cos2\phi_o}{u^2(d-2\rho\sigma\cos\phi_o)\sin\phi_o} \tag{7.17a}$$

$$\imath_2 = \imath_4\rho, \quad \imath_1 = \imath_3+\imath_4, \quad \imath_0 = -\delta/(u\sin\phi_o). \tag{7.17b}$$

Also, by using (7.5a) in (7.14c), we get

$$\begin{gathered}\dot{\psi} = a[\imath_0\cos\phi_o+\imath_1u_o+a(\imath_1t-\imath_2)], \quad \Theta = \dot{\psi}/a \\ \mathfrak{X}^2 = \imath_0^2+2\imath_0\imath_1u_o\cos\phi_o+\imath_1^2u_o^2+a(\imath_1t-\imath_2)[a(\imath_1t-\imath_2)+2\imath_0\cos\phi_o+2\imath_1u_o]\end{gathered} \tag{7.18}$$

from which we conclude that the apparent displacement of a light source will in general vary with time when an observer translates rectilinearly with a constant nonzero acceleration.

Example 7-F. *Transverse line of incidence.* If a light ray is incident at right angles to the observer's velocity, so that (7.7a) applies, then (7.17) will reduce to

$$\begin{gathered}\delta = \frac{1}{1+\beta^2}, \quad \imath_4 = \frac{\beta}{u^2(1+\beta^2)}, \quad \imath_3 = 0, \quad \imath_2 = 0 \\ \imath_1 = \frac{\beta}{u^2(1+\beta^2)}, \quad \imath_0 = -\frac{1}{u(1+\beta^2)}\end{gathered} \tag{7.19a}$$

on account of which (7.18) will become

$$\dot{\psi} = \frac{\sigma}{1+\beta^2}, \quad \Theta = \frac{1}{c(1+\beta^2)}, \quad \mathfrak{X}^2 = \frac{1}{u^2(1+\beta^2)} \tag{7.19b}$$

where β satisfies (7.5b). We conclude that in this scenario the apparent drift of the light source is affected by acceleration but not by dispersion.

Art 30. *On the apparent path of a light source.*

To calculate the apparent path of a light source for an observer translating with a constant acceleration, we note that for this observer, (4.10a) reduces to

$$\begin{gathered}\varsigma_a = 0, \quad \varsigma_b = 0, \quad \varsigma_c = 0, \quad \varsigma_d = 0, \quad \varsigma_e = 0, \quad \varsigma_f = 0, \quad \varsigma_g = 0, \quad \varsigma_h = 0 \\ \varsigma_i = 0, \quad \varsigma_j = 0, \quad \varsigma_k = 0, \quad \varsigma_l = 0, \quad \varsigma_m = 0, \quad \varsigma_n = 0, \quad \varsigma_o = 0, \quad \varsigma_p = 0 \\ \varsigma_q = 0, \quad \varsigma_r = 0, \quad \varsigma_s = 0, \quad \varsigma_t = 0, \quad \varsigma_u = 0, \quad \varsigma_v = 0.\end{gathered} \tag{7.20a}$$

Ignoring $\varrho_a, \varrho_b, \varrho_c$ because they will not be needed in the calculations, we also have by (4.10),

$$\begin{gathered}\varrho_d = -1, \quad \varrho_e = 0, \quad \varrho_f = 0, \quad \varrho_g = 0, \quad \varrho_h = 0, \quad \varrho_i = 0, \quad \varrho_j = 0, \quad \varrho_k = 0 \\ \varrho_l = 0, \quad \varrho_m = 0, \quad \varrho_n = 0, \quad \varrho_o = 0, \quad \varrho_p = 1, \quad \varrho_q = a, \quad \varrho_r = 0, \quad \varrho_s = 0 \\ \varrho_t = 0, \quad \varrho_u = 0, \quad \varrho_v = 0, \quad \varrho_w = 0, \quad \varrho_x = 0, \quad \varrho_y = 0.\end{gathered} \tag{7.20b}$$

Substituting these values into (4.25) gives [13]

$$\mathbb{K} = 0, \quad \mathbb{T} = 0, \quad \boldsymbol{\ell}_t = -\mathbf{a}/a, \quad \boldsymbol{\ell}_b = \perp \tag{7.21}$$

from which we conclude that the apparent path of the light source is a straight line in a direction antiparallel to the observer's acceleration whatever maybe the line of incidence of the light ray from the source to the observer. We conclude further that if the observer's motion is not accelerated, the light source will not have an apparent path ($\boldsymbol{\ell}_t = \perp$) and will therefore be apparently stationary [31].

Art 31. *On the apparent geometry of rays.*

To study the apparent geometry of obliquated rays for an observer translating with a constant acceleration, we note that for this observer, (4.26a) reduces to

$$\begin{gathered}\mathfrak{H}_a = \widehat{\boldsymbol{\kappa}} \cdot (\mathbf{a} \times \mathbf{u}), \quad \mathfrak{H}_b = \mathbf{0}, \quad \mathfrak{H}_c = \mathbf{0} \\ \mathfrak{H}_d = \mathbf{0}, \quad \mathfrak{H}_e = \mathbf{0}, \quad \mathfrak{H}_f = \mathbf{0}, \quad \mathfrak{H}_g = \mathbf{0}, \quad \mathfrak{H}_h = \mathbf{0}\end{gathered} \tag{7.22a}$$

so that, with (7.20), the remaining quantities defined in (4.26) are given by

$$\begin{gathered}\mathfrak{K}_a = -\mathfrak{Y}, \quad \mathfrak{K}_b = -\mathfrak{H}_a, \quad \mathfrak{K}_c = 0, \quad \mathfrak{K}_d = 0, \quad \mathfrak{K}_e = 0 \\ \mathfrak{K}_f = a^2\mathfrak{Y}^2 \sin^2\lambda, \quad \mathfrak{K}_g = a^2u^2\sin^2\theta, \quad \mathfrak{K}_h = \mathfrak{Y}ua^2(\cos\lambda\cos\theta - \cos\phi).\end{gathered} \tag{7.22b}$$

[13] We use $\perp$ to denote an indeterminate or undefined scalar or vector.

We substitute (7.22) into (4.31) to get, in view of (7.20b) and (7.1a),

$$\begin{gathered}\overline{\mathbb{K}} = \Pi/(c\mathcal{R}^3), \quad \overline{\mathbb{T}} = 0, \quad \overline{\boldsymbol{\ell}}_t = \frac{\mathcal{Y}\widehat{\boldsymbol{\kappa}} - \mathbf{u}_o + (\rho - t)\mathbf{a}}{c\mathcal{R}}, \quad \overline{\boldsymbol{\ell}}_b = \frac{\mathbf{a} \times (\mathcal{Y}\widehat{\boldsymbol{\kappa}} - \mathbf{u}_o)}{c^2\Pi} \\ \Pi = \sigma[\beta^2 \sin^2\theta + (\mathcal{Y}/c)^2 \sin^2\lambda + 2\beta(\mathcal{Y}/c)(\cos\lambda\cos\theta - \cos\phi)]^{1/2}.\end{gathered} \tag{7.23}$$

We conclude that for an observer translating with a constant acceleration, the light ray is curved but lies entirely in a plane whatever the line of incidence of the ray maybe.

Example 7-G. *Rectilinear translation.* When the observer's motion is rectilinear, the quantities featured in (7.23) have the values calculated in Art 28b. Substituting (7.5a) into (7.23) and using the value of $\mathcal{Y}$ from (7.5c) gives

$$\Pi = \sigma|d - 2\rho\sigma\cos\phi_o|\sin\phi_o. \tag{7.24a}$$

In particular, when the light ray is incident at right angles to the observer's initial velocity $\mathbf{u}_o$, we have $\phi_o = 90°$ which reduces (7.5d) to $\mathcal{R}^2 = 1 + \beta^2$ and (7.24a) to $\Pi = \sigma$. It follows by putting these values into (7.23) that

$$\overline{\mathbb{K}} = \frac{\sigma}{c(1+\beta^2)^{3/2}} \approx \begin{cases} \sigma/c & \text{if } \beta \ll 1 \\ (\sigma/c)\beta^{-3} & \text{if } \beta \gg 1 \end{cases} \tag{7.24b}$$

where β satisfies (7.5b). We conclude that dispersion has no effect on the curvature of the ray. However, unlike the corresponding obliquation angle given by (7.7b), the ray curvature depends explicitly on the observer's acceleration and not only on the instantaneous velocity of the observer. Hence those who wish to uphold the socalled locality hypothesis mentioned in Example 7-A ought to bear in mind that while the obliquation angle may depend only the instantaneous velocity of the observer, the curvature of the ray does in fact depend on the observer's acceleration, on account of which one may not claim, even in aclassical physics, that acceleration has no effect on obliquation.

Example 7-H. *Contigency of obliquated light rays.* If P and Q are two points on a ray, and if the tangent vectors to the ray at these points are respectively $\overline{\boldsymbol{\ell}}_t^{(P)}$ and $\overline{\boldsymbol{\ell}}_t^{(Q)}$, the angle between these vectors in the limit as Q approaches P is called the contigency of the ray at P ([32], pg 14). By definition, then,

$$\mathrm{d}\varphi = \overline{\mathbb{K}}\,\mathrm{d}\overline{s} \tag{7.25a}$$

where $\mathrm{d}\overline{s}$ is an element of the ray and $\mathrm{d}\varphi$ is an element of the contigency angle. Observing that $\mathrm{d}\overline{s}/\mathrm{d}t = \upsilon$ is the ray speed, the above equation can be rewritten as

$$\dot{\varphi} = \overline{\mathbb{K}}\upsilon \tag{7.25b}$$

where $\dot{\varphi}$ may be called the variation of contigency. From (7.7b), (7.24b) and (7.25b) we obtain

$$\dot{\varphi} = \frac{\sigma}{(1+\beta^2)^3} \tag{7.26a}$$

as the variation of contigency for a light ray incident at right angles to the initial velocity of an observer translating rectilinearly with constant acceleration. Moreover, comparing (7.26a) with (7.19b), we get

$$\sigma^2\dot{\varphi} = \dot{\psi}^3 \tag{7.26b}$$

which relates the variation of obliquation at any instant to the corresponding variation of contigency at that instant.

Scholium 7-C. *Deviation of obliquated light rays.* Consider a point moving along a light ray with the ray speed υ, and let this point be at position P at time t_P and at position Q at time t_Q.

If we integrate the variation of contigency $\dot{\varphi}$ from $t = t_P$ to $t = t_Q$, the result will be the angle $\Delta\varphi$ between the tangent vectors to the ray at P and Q. This so-called bending angle or deviation constitutes a global measure of the ray curvature. It is a useful physical quantity when the ray lies entirely in one plane between P and Q. If the ray does not lie entirely in one plane between these positions, the physical usefulness of the bending angle is questionable. And even when the ray lies entirely in one plane between these positions, the usefulness of the bending angle can still be questioned on the grounds that $\Delta\varphi = 0$ does not necessarily imply zero curvature everywhere between the two positions, which may be interpreted as showing that the bending angle is too crude a measure of curvature. Since the problems we are investigating in this work may admit rays that are both curved and twisted, we shall not in general be concerned with the bending angle in this work. The variation of contigency will be found to be much more suitable for our purposes because it places more stringent constraints on conflicting kineoptical theories.

Art 32. *On the apparent frequency of rays.*

The apparent frequency of a light ray can be calculated for an observer translating with a constant acceleration by substituting the appropriate value of $\mathcal{R}$ into (3.30). In this way we shall find that the apparent ray frequency ω' is affected in general by both acceleration and dispersion.

Art 32a. *Apparent ray frequency for nonaccelerated observers.*

When an observer translates without accelerating, $\mathcal{R}$ has the value given by (7.3c). Using this value in (3.30) gives

$$(\omega'/\omega_o)^2 = 1 + \beta_o^2 - 2\beta_o \cos\phi_o \tag{7.27}$$

so that for $\cos\phi_o = 0$, which corresponds to the situation shown in Figure 5(a), we have

$$\omega' = \omega_o\sqrt{1 + \beta_o^2} \tag{7.28a}$$

and for $\cos\phi_o = \beta_o$, corresponding to the situation shown in Figure 5(b), we have

$$\omega' = \omega_o\sqrt{1 - \beta_o^2}. \tag{7.28b}$$

We conclude that in either of the transverse situations shown in Figure 5, the apparent ray frequency depends on second and higher order terms in β_o when the observer translates without accelerating [33, 34, 35, 36]. Also, for a light ray incident in a direction parallel to the observer's velocity ($\cos\phi_o = 1$), we get

$$\omega' = \omega_o|1 - \beta_o| \tag{7.29a}$$

while for a light ray incident in a direction antiparallel to the observer's velocity ($\cos\phi_o = -1$), we get

$$\omega' = \omega_o(1 + \beta_o). \tag{7.29b}$$

We conclude that in either of these so-called longitudinal or radial situations, the apparent ray frequency satisfies the usual classical formulae attributed to Doppler.

Art 32b. *Apparent ray frequency for accelerated observers.*

When an observer translates rectilinearly with a constant acceleration, $\mathcal{R}$ has the value given by (7.5d). Substituting this value into (3.30) gives

$$(\omega'/\omega_o)^2 = d^2 + \beta^2 - 2d\beta\cos\phi_o + \rho\sigma(\rho\sigma - 2d\cos\phi_o + 2\beta\cos 2\phi_o) \tag{7.30}$$

where the various quantities have the values calculated in Art 28b.

Example 7-I. *Transverse line of incidence.* For a light ray incident at right angles to the observer's initial velocity ($\cos\phi_o = 0$), using (7.7a) in (7.30) gives

$$\omega' = \omega_o\sqrt{1 + \beta^2} \tag{7.31}$$

where β satisfies (7.5b). Comparing this result with (7.28a), we conclude that in this situation, the apparent frequency of the ray depends only on the instantaneous velocity of the observer and not explicitly on the observer's acceleration (cf. Example 7-A). We may also consider the situation corresponding to $\cos\phi_o = \beta$. But in this case we must bear in mind that since β varies with time by (7.5b), the condition $\cos\phi_o = \beta$ can be established only for a brief instant. Designating the value of β at the instant when this condition holds good by β_t, (7.5c) and (7.30) give

$$(\omega'/\omega_o)^2 = d^2 + \beta_t^2 - 2d\beta_t^2 + \rho\sigma[\rho\sigma - 2d\beta_t + 2\beta_t(2\beta_t^2 - 1)] \tag{7.32a}$$

$$d = \left\{ \frac{1 + \sqrt{1 + \mu^2\beta_t^2}}{2} \right\}^{1/2}, \quad \rho = \frac{1}{4d\omega_o} \left\{ \frac{\mu\beta_t}{\sqrt{1 + \mu^2\beta_t^2}} \right\} \tag{7.32b}$$

from which we conclude that, at this particular instant, the apparent ray frequency is affected by both dispersion and acceleration.

Example 7-J. *Longitudinal line of incidence.* For a light ray incident in a direction parallel to the initial velocity of the observer ($\cos\phi_o = 1$), we have by (7.5c) and (7.30) that

$$(\omega'/\omega_o)^2 = d^2 + \beta^2 - 2d\beta + \rho\sigma(\rho\sigma - 2d + 2\beta), \quad \vartheta = \mu \tag{7.33a}$$

while for a light ray incident in a direction antiparallel to the initial velocity of the observer ($\cos\phi_o = -1$), we have

$$(\omega'/\omega_o)^2 = d^2 + \beta^2 + 2d\beta + \rho\sigma(\rho\sigma + 2d + 2\beta), \quad \vartheta = -\mu \tag{7.33b}$$

where d and ρ are given in both cases by

$$d = \left\{ \frac{1 + \sqrt{1 + \vartheta^2}}{2} \right\}^{1/2}, \quad \rho = \frac{1}{4d\omega_o} \left\{ \frac{\vartheta}{\sqrt{1 + \vartheta^2}} \right\}. \tag{7.33c}$$

We conclude that acceleration and dispersion have effects on the apparent frequency of a ray incident in a direction parallel or antiparallel to the observer's velocity.

Example 7-K. *Ultragamma approximation for longitudinal loi.* When the quantity μ is so small that expressions containing its third and higher powers can be neglected ($\mu \ll 1$), we have by (7.33a) or (7.33b) that the same condition holds good for the quantity ϑ. Equation (7.33c) then becomes

$$d \approx 1 + \frac{\vartheta^2}{8}, \quad \rho \approx \frac{\vartheta}{4\omega_o} \tag{7.34a}$$

which upon substitution into (7.33a) gives

$$(\omega'/\omega_o)^2 \approx (1 - \beta)^2 - \frac{\mu^2(1 - \beta)}{4} \tag{7.34b}$$

and upon substitution into (7.33b) gives

$$(\omega'/\omega_o)^2 \approx (1 + \beta)^2 - \frac{\mu^2(1 + \beta)}{4}. \tag{7.34c}$$

We conclude that in this approximation, the effects of acceleration and dispersion on the apparent ray frequency are of second and higher orders in μ. Moreover, when the observer's motion is such that $\beta < 1$ at all instants, we see that the effect of acceleration is such as to reduce the ray frequency regardless of whether the line of incidence of the ray is parallel or antiparallel to the observer's velocity.

Example 7-L. *Infraradio approximation for longitudinal loi.* When the quantity μ is so large that expressions containing its first and higher powers are much greater than 1 ($\mu \gg 1$), we have by (7.33a) or (7.33b) that the same condition holds good for ϑ. In this case we have by (7.33c) that

$$d \approx \sqrt{\frac{|\vartheta|}{2}}, \quad \rho \approx \frac{\operatorname{sgn}\vartheta}{4\omega_o}\sqrt{\frac{2}{|\vartheta|}} \tag{7.35a}$$

which upon substitution into (7.33a) gives

$$\omega' \approx \omega_o(b - \beta), \quad b = \sqrt{\mu/8} \tag{7.35b}$$

and upon substitution into (7.33b) gives

$$\omega' \approx \omega_o(b + \beta), \quad b = \sqrt{\mu/8}. \tag{7.35c}$$

We conclude that in this approximation, the effects of acceleration and dispersion on the apparent ray frequency manifest at first order in the square root of μ. We conclude also that when the observer's motion is such that $\mu > 8$, the effect of acceleration is such as to enhance the ray frequency regardless of whether the line of incidence of the ray is parallel or antiparallel to the observer's velocity.

8 Effects of centripetal acceleration

Art 33. *On the apparent direction to a light source.*

Throughout this section we consider a light ray incident in the plane of motion of an observer moving with constant linear speed u_o and constant angular speed Ω_o in a circle of radius r_o. Then at any instant t, the observer's position $\mathbf{r}$, velocity $\mathbf{u}$ and acceleration $\mathbf{a}$ are given by

$$\begin{gathered}\mathbf{r} = r_o(\hat{\boldsymbol{\kappa}}\cos\nu + \hat{\boldsymbol{\jmath}}\sin\nu), \quad \mathbf{u} = \Omega_o r_o(-\hat{\boldsymbol{\kappa}}\sin\nu + \hat{\boldsymbol{\jmath}}\cos\nu) \\ \mathbf{a} = -\Omega_o^2\mathbf{r} = -\Omega_o(\mathbf{u}\times\hat{\boldsymbol{\imath}}), \quad \nu(t) = \Omega_o t\end{gathered} \tag{8.1a}$$

where $\hat{\boldsymbol{\jmath}}$ is a unit vector in the plane of the observer's motion satisfying $\hat{\boldsymbol{\kappa}}\cdot\hat{\boldsymbol{\jmath}} = 0$, while $\hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\kappa}}\times\hat{\boldsymbol{\jmath}}$ is a unit vector perpendicular to the plane of the observer's motion, and ν is the phase or azimuth of the observer's position with respect to $\hat{\boldsymbol{\kappa}}$. Observing from these equations that

$$\begin{gathered}\hat{\boldsymbol{\kappa}}\cdot\mathbf{u} = -\Omega_o r_o\sin\nu, \quad \hat{\boldsymbol{\kappa}}\times\mathbf{u} = \hat{\boldsymbol{\imath}}\,\Omega_o r_o\cos\nu \\ \mathbf{u}\cdot\mathbf{a} = 0, \quad \hat{\boldsymbol{\kappa}}\cdot\mathbf{a} = -\Omega_o^2 r_o\cos\nu, \quad \hat{\boldsymbol{\kappa}}\times\mathbf{a} = -\hat{\boldsymbol{\imath}}\,\Omega_o^2 r_o\sin\nu\end{gathered} \tag{8.1b}$$

we obtain (cf. Figure 1)

$$\cos\theta = 0, \quad \cos\phi = \sin\lambda = -\sin\nu, \quad \sin\phi = -\cos\lambda = \cos\nu. \tag{8.1c}$$

Consequently, (4.2b) becomes

$$\begin{gathered}\mathcal{F} = |d\cos\nu + \rho\sigma\cos 2\nu|, \quad \mathcal{G} = -\beta - d\sin\nu - \rho\sigma\sin 2\nu \\ \mathcal{R}^2 = d^2 + \beta^2 + 2d\beta\sin\nu + \rho\sigma(\rho\sigma + 2d\cos\nu + 2\beta\sin 2\nu)\end{gathered} \tag{8.2a}$$

while (4.2d) becomes

$$\vartheta = -\mu \cos\nu, \quad d = \left\{ \frac{1 + \sqrt{1+\vartheta^2}}{2} \right\}^{1/2} \tag{8.2b}$$
$$\pi = \vartheta/(1+\vartheta^2)^{1/2}, \quad \rho = \pi/(4d\omega_o), \quad \mathcal{Y} = c(d + 2\rho\sigma\cos\nu).$$

Moreover, substituting the values of $\mathcal{F}$ and $\mathcal{G}$ from (8.2a) into (4.2c) gives the instantaneous obliquation angle of the light source as

$$\tan\psi = \frac{|d\cos\nu + \rho\sigma\cos 2\nu|}{-\beta - d\sin\nu - \rho\sigma\sin 2\nu}. \tag{8.2c}$$

We shall find it convenient to say that the observer is at new phase when $\cos\nu = 1$, at first quarter when $\sin\nu = 1$, at full phase when $\cos\nu = -1$, and at last quarter when $\sin\nu = -1$.

Art 33a. *Apparent direction for observers at new phase.*

When an observer is at new phase, (8.2a) reduces to

$$\mathcal{F} = |\rho\sigma + d|, \quad \mathcal{G} = -\beta, \quad \mathcal{R}^2 = d^2 + \beta^2 + \rho\sigma(\rho\sigma + 2d) \tag{8.3a}$$

while (8.2b) reduces to

$$\vartheta = -\mu, \quad d = \left\{ \frac{1 + \sqrt{1+\mu^2}}{2} \right\}^{1/2}, \quad \pi = \frac{-\mu}{\sqrt{1+\mu^2}} \tag{8.3b}$$
$$\rho = \frac{1}{4d\omega_o} \left\{ \frac{-\mu}{\sqrt{1+\mu^2}} \right\}, \quad \mathcal{Y} = c(d + 2\rho\sigma)$$

where we have taken advantage of (1.6). The ray speed υ and the angle ψ of obliquation are obtained by substituting the value of $\mathcal{R}$ from (8.3a) into (3.5a) and putting $\cos\nu = 1$ into (8.2c) to get

$$\upsilon = c[\beta^2 + (\rho\sigma + d)^2]^{1/2}, \quad \tan\psi = -|\rho\sigma + d|/\beta. \tag{8.4}$$

We conclude that when the observer is at new phase, acceleration and dispersion affect both the ray speed and the angular displacement of the light source.

Example 8-A. *Ultragamma approximation.* If $\mu \ll 1$, then to a second order accuracy in μ, we have from (8.3b) that

$$d \approx 1 + (\mu^2/8), \quad \rho \approx -\mu/(4\omega_o) \tag{8.5a}$$

which reduces (8.4) to

$$\upsilon \approx c\left\{1 + \beta^2 - \frac{\mu^2}{4}\right\}^{1/2}, \quad \tan\psi \approx -\frac{1}{\beta}\left\{1 - \frac{\mu^2}{8}\right\}. \tag{8.5b}$$

We conclude that in this approximation, the effects of acceleration and dispersion on the ray speed and on the apparent position of the light source are of second and higher orders in μ.

Example 8-B. *Infraradio approximation.* If $\mu \gg 1$, then to all orders of accuracy in μ, (8.3b) gives

$$d \approx \sqrt{\mu/2}, \quad \rho \approx -(4\omega_o)^{-1}\sqrt{2/\mu} \tag{8.6a}$$

which reduces (8.4) to

$$\upsilon \approx c\left\{\beta^2 + \frac{\mu}{8}\right\}^{1/2}, \quad \tan\psi \approx -\frac{1}{\beta}\left\{\frac{\mu}{8}\right\}^{1/2}. \tag{8.6b}$$

We conclude that in this case the effects of acceleration and dispersion on the ray speed and on the apparent position of the light source manifest at first order in μ.

Example 8-C. *Relevance to low frequency radio astronomy.* To put some numbers into the above calculations, let $c \approx 3 \times 10^8 \mathrm{ms}^{-1}$ and take $a \approx 6 \times 10^{-2} \mathrm{ms}^{-2}$ to represent the earth's acceleration towards the sun. Then we have $\mu\omega_o \approx 200\mathrm{pHz}$. For nanohertz infraradio waves with $\omega_o \approx 1\mathrm{nHz}$, we get $\mu \approx 0.2 \ll 1$ which indicates that (8.5b) gives a good approximation for ψ. But for picohertz infraradio waves with $\omega_o \approx 1\mathrm{pHz}$, we get $\mu \approx 200 \gg 1$ which indicates that (8.6b) gives a better approximation for ψ. These numbers suggest that if the earth's orbital motion around the sun can be assumed to be circular and with constant speed, then the effect of the earth's acceleration on obliquation can be detected with reasonable certainty if an infraradio survey of the sky is performed in the picohertz range, for in this range we should find that $\tan\psi$ is about five times larger than what would be expected on the basis of other theories. Thus we have reasons to look forward to technological advances in extremely low frequency radio astronomy which may one day make such surveys possible.

Art 33b. *Apparent direction for observers at first quarter.*

When an observer is at first quarter, (8.2b) reduces to

$$\vartheta = 0, \quad d = 1, \quad \pi = 0, \quad \rho = 0, \quad \mathcal{Y} = c \tag{8.7a}$$

while (8.2a) reduces to

$$\mathcal{F} = 0, \quad \mathcal{G} = -(1+\beta), \quad \mathcal{R} = 1+\beta. \tag{8.7b}$$

Substituting the value of $\mathcal{R}$ from (8.7b) into (3.5a) and putting $\sin\nu = 1$ into (8.2c) gives

$$\upsilon = c(1+\beta), \quad \tan\psi = 0. \tag{8.8}$$

We conclude that when the observer is at first quarter, the ray speed is not affected by acceleration and dispersion while the light source suffers no angular displacement.

Art 33c. *Apparent direction for observers at full phase.*

When an observer is at full phase, (8.2b) reduces to

$$\begin{gathered}\vartheta = \mu, \quad d = \left\{\frac{1+\sqrt{1+\mu^2}}{2}\right\}^{1/2}, \quad \pi = \left\{\frac{\mu}{\sqrt{1+\mu^2}}\right\} \\ \rho = \frac{1}{4d\omega_o}\left\{\frac{\mu}{\sqrt{1+\mu^2}}\right\}, \quad \mathcal{Y} = c(d - 2\rho\sigma)\end{gathered} \tag{8.9a}$$

while (8.2a) reduces to

$$\mathcal{F} = |\rho\sigma - d|, \quad \mathcal{G} = -\beta, \quad \mathcal{R}^2 = d^2 + \beta^2 + \rho\sigma(\rho\sigma - 2d). \tag{8.9b}$$

Substituting the value of $\mathcal{R}$ from (8.9b) into (3.5a) and putting $\cos\nu = -1$ into (8.2c) gives

$$\upsilon = c[\beta^2 + (\rho\sigma - d)^2]^{1/2}, \quad \tan\psi = -|\rho\sigma - d|/\beta. \tag{8.10}$$

We conclude that when the observer is at full phase, acceleration and dispersion affect both the ray speed and the angular displacement of the light source.

Art 33d. *Apparent direction for observers at last quarter.*

When an observer is at last quarter, (8.2b) reduces to

$$\vartheta = 0, \quad d = 1, \quad \pi = 0, \quad \rho = 0, \quad \mathcal{Y} = c \tag{8.11a}$$

while (8.2a) reduces to

$$\mathcal{F} = 0, \quad \mathcal{G} = 1 - \beta, \quad \mathcal{R} = |1 - \beta|. \tag{8.11b}$$

Substituting the value of $\mathcal{R}$ from (8.11b) into (3.5a) and putting $\sin\nu = -1$ into (8.2c) gives

$$\upsilon = c|1 - \beta|, \quad \tan\psi = 0. \tag{8.12}$$

We conclude that when the observer is at last quarter, the ray speed is not affected by acceleration and dispersion while the light source suffers no angular displacement.

Example 8-D. *Obliquation as a vector transport problem.* The foregoing calculations show that when the observer is at new phase, the light source is displaced at an angle ψ to the observer's velocity in accordance with (8.4). As the observer progresses to first quarter, the displacement of the light source reduces and finally vanishes when the observer reaches first quarter, at which point the apparent direction to the light source coincides with the true direction. As the observer progresses further to full phase, the light source again suffers a displacement which reaches a maximum value when the observer is at full phase. This displacement in turn diminishes as the observer advances to last quarter, at which point the apparent direction to the light source coincides again with the true direction. As the observer finally moves from last quarter to new phase, the light source again suffers a displacement that reaches a maximum value when the observer is at new phase. It appears therefore that one may treat obliquation as a problem of transporting the ray velocity vector along the path of the observer. But while this mode of expression may be more convenient for mathematicians, it seems to add nothing to the physics of the phenomena being investigated, and will therefore not be considered further in this work.

Art 34. *On the apparent drift of a light source.*

For an observer translating with centripetal acceleration, the velocity and acceleration of the observer are dependent quantities, the acceleration being related to the velocity as in (8.1a), so that the formulae of Art 17 are not applicable. Since the applicable general formulae for this case were not derived in that article, we shall not study the apparent drift of a light source for an observer translating with centripetal acceleration in the present article. The results which ought to have been derived here can however be obtained as a special case of the results obtained in Art 39 where we study the same problem for a rotating or precessing observer.

Art 35. *On the apparent path of a light source.*

To calculate the apparent path of the light source, we first note that by (8.1a),

$$\dot{\mathbf{a}} = -\Omega_o^2\mathbf{u}, \quad \ddot{\mathbf{a}} = \Omega_o^4\mathbf{r}, \quad \dddot{\mathbf{a}} = \Omega_o^4\mathbf{u} \tag{8.13a}$$

$$\begin{aligned}
&\widehat{\boldsymbol{\kappa}}\cdot\mathbf{r} = r_o\cos\nu, \quad \widehat{\boldsymbol{\kappa}}\times\mathbf{r} = \widehat{\boldsymbol{\imath}}\,r_o\sin\nu, \quad \mathbf{u}\cdot\mathbf{r} = 0, \quad \mathbf{u}\times\mathbf{r} = -\widehat{\boldsymbol{\imath}}\,\Omega_o r_o^2\\
&\widehat{\boldsymbol{\kappa}}\cdot\mathbf{a} = -a_\parallel, \quad \widehat{\boldsymbol{\kappa}}\times\mathbf{a} = -\widehat{\boldsymbol{\imath}}\,a_\perp, \quad \widehat{\boldsymbol{\kappa}}\cdot\dot{\mathbf{a}} = \Omega_o a_\perp, \quad \widehat{\boldsymbol{\kappa}}\times\dot{\mathbf{a}} = -\widehat{\boldsymbol{\imath}}\,\Omega_o a_\parallel\\
&\widehat{\boldsymbol{\kappa}}\cdot\ddot{\mathbf{a}} = \Omega_o^2 a_\parallel, \quad \widehat{\boldsymbol{\kappa}}\times\ddot{\mathbf{a}} = \widehat{\boldsymbol{\imath}}\,\Omega_o^2 a_\perp, \quad \dot{\mathbf{a}}\cdot\ddot{\mathbf{a}} = 0, \quad \dot{\mathbf{a}}\times\ddot{\mathbf{a}} = \widehat{\boldsymbol{\imath}}\,a^2\Omega_o^3\\
&\mathbf{a}\cdot\dot{\mathbf{a}} = 0, \quad \mathbf{a}\times\dot{\mathbf{a}} = \widehat{\boldsymbol{\imath}}\,a^2\Omega_o, \quad \mathbf{a}\cdot\ddot{\mathbf{a}} = -\Omega_o^2 a^2, \quad \mathbf{a}\times\ddot{\mathbf{a}} = \mathbf{0}
\end{aligned} \tag{8.13b}$$

where we have introduced the convenient quantities

$$\begin{aligned} a_{\|} &= a\cos\nu, \quad a_{\perp} = a\sin\nu, \quad \sigma_{\|} = \sigma\cos\nu \\ \sigma_{\perp} &= \sigma\sin\nu, \quad \mu_{\|} = \mu\cos\nu, \quad \mu_{\perp} = \mu\sin\nu. \end{aligned} \tag{8.13c}$$

Substituting (8.13b) into (4.10a) and taking (8.1) into account, we get

$$\varsigma_a = \omega_o\Omega_o\sigma_{\perp}, \quad \varsigma_b = \omega_o\Omega_o^2\sigma_{\|}, \quad \varsigma_c = -\omega_o\Omega_o^3\sigma_{\perp} \tag{8.14a}$$

$$\begin{aligned} &\varsigma_d = 0, \quad \varsigma_e = -a^2\Omega_o^2, \quad \varsigma_f = 0, \quad \varsigma_g = a^2\Omega_o^2, \quad \varsigma_h = 0, \quad \varsigma_i = -a^2\Omega_o^4 \\ &\varsigma_j = a^2\Omega_o^4, \quad \varsigma_k = 0, \quad \varsigma_l = a^2\Omega_o^6, \quad \varsigma_m = 0, \quad \varsigma_n = 0, \quad \varsigma_o = 0 \\ &\varsigma_p = 0, \quad \varsigma_q = 0, \quad \varsigma_r = 0, \quad \varsigma_s = 0, \quad \varsigma_t = 0, \quad \varsigma_u = 0, \quad \varsigma_v = 0 \end{aligned} \tag{8.14b}$$

where we have used the fact that $\omega_o = c\kappa$, $\sigma = a/c$, $\mu = \sigma/\omega_o$ by (1.2b) and (1.6). Putting these values into (4.10g) gives $\varrho_y = 0$ which upon substitution into (4.25) yields $\mathbb{T} = 0$. We conclude that the apparent path of the light source is a plane curve. Moreover, since the various vectors in the expression for $\boldsymbol{\ell}_b$ in (4.25) are parallel or antiparallel to $\widehat{\boldsymbol{\imath}}$ by (8.13b), we conclude also that the apparent path of the light source lies in the plane of the observer's motion.

Art 35a. *Apparent path for observers at new phase.*

When an observer is at new phase, we have $a_{\|} = a$, $a_{\perp} = 0$ which reduces (8.14a) to

$$\varsigma_a = 0, \quad \varsigma_b = \omega_o\Omega_o^2\sigma, \quad \varsigma_c = 0. \tag{8.15a}$$

The various quantities defined in (4.10) become, in view of (8.3b), (8.14b) and (8.15a),

$$\begin{aligned} &\varrho_a = (1+\mu^2)^{1/2}, \quad \varrho_b = 1 - 4\mu^2, \quad \varrho_c = (1/\varrho_a^3) - (\pi/2d)^2, \quad \varrho_d = -1 \\ &\varrho_e = -\pi(\Omega_o/\varrho_a)^2, \quad \varrho_f = 0, \quad \varrho_g = -y\sigma\Omega_o, \quad \varrho_h = 0, \quad \varrho_i = \Omega_o xy, \quad \varrho_j = 0, \quad \varrho_k = 0 \\ &\varrho_l = -ya\Omega_o(2x-1), \quad \varrho_m = 0, \quad \varrho_n = \varrho_l, \quad \varrho_o = -ay^2(2x-1), \quad \varrho_p = 1 + xy^2 \\ &\varrho_q = a(1+y^2)^{1/2}, \quad \varrho_r = 0, \quad \varrho_s = -a^3\Omega_o^2(1+3xy^2), \quad \varrho_t = 0, \quad \varrho_u = a^4\Omega_o^2(1+3xy^2) \\ &\varrho_v = 0, \quad \varrho_w = a^4\Omega_o^4(1+3xy^2), \quad \varrho_x = a^2\Omega_o|1+3xy^2|, \quad \varrho_y = 0 \end{aligned} \tag{8.15b}$$

where we have introduced the quantities

$$x = x_o\left\{x_o - \frac{\mu^2}{4d^2}\right\}\begin{cases} \geq 0 & \text{if } \mu^2 \leq 3 \\ < 0 & \text{if } \mu^2 > 3 \end{cases}, \quad y = x_o y_o, \quad \varepsilon_1 = \frac{1+3xy^2}{|1+3xy^2|} \tag{8.16a}$$

with d, x_o and y_o given by

$$d = \left\{\frac{1+\sqrt{1+\mu^2}}{2}\right\}^{1/2}, \quad x_o = \frac{1}{\sqrt{1+\mu^2}}, \quad y_o = \frac{\mu\Omega_o}{4d\omega_o}. \tag{8.16b}$$

In view of (8.15), (8.13) and (8.1), (4.25) becomes

$$\begin{aligned} &\mathbb{K} = \frac{|1+3xy^2|}{u\big(1+y^2\big)^{3/2}}, \quad \mathbb{T} = 0, \quad \boldsymbol{\ell}_t = \frac{\widehat{\boldsymbol{\kappa}} + \widehat{\boldsymbol{\jmath}}\,y}{\sqrt{1+y^2}}, \quad \boldsymbol{\ell}_b = \varepsilon_1\widehat{\boldsymbol{\imath}} \\ &\boldsymbol{\ell}_n = \frac{\varepsilon_1(\widehat{\boldsymbol{\jmath}} - \widehat{\boldsymbol{\kappa}}\,y)}{\sqrt{1+y^2}}, \quad \boldsymbol{\ell}_c = u\left\{\frac{\big(1+y^2\big)^{3/2}}{|1+3xy^2|}\right\}\boldsymbol{\ell}_n. \end{aligned} \tag{8.17}$$

We conclude that at the instant when the observer is at new phase, the apparent path of the light source will be a curved line if $\varepsilon_1 \neq \perp$ and a straight line if $\varepsilon_1 = \perp$.

Example 8-E. *Apparent path for small accelerations.* When the acceleration of the observer is so small as to be negligible ($\mu \approx 0$), we have in (8.16) that

$$x \approx 1, \quad y \approx 0 \tag{8.18a}$$

which upon substitution into (8.17) gives

$$\mathbb{K} = \frac{1}{u}, \quad \boldsymbol{\ell}_t = \widehat{\boldsymbol{\kappa}}, \quad \boldsymbol{\ell}_n = \widehat{\boldsymbol{\jmath}}, \quad \boldsymbol{\ell}_c = u\boldsymbol{\ell}_n. \tag{8.18b}$$

We conclude that if the true position of the light source is considered to be at the center of curvature of its apparent path, a vector drawn from its apparent position to its true position will be of the same magnitude and direction as the observer's velocity, a result that was first established with the greatest authority and originality by Hamilton [37]. We conclude further that at the instant in question, the sense of the apparent motion of the light source is parallel to the line of incidence of a light ray from the source to the observer, and therefore directed towards the observer.

Example 8-F. *Apparent path in the ultragamma limit.* When the observer's acceleration is not entirely negligible but the quantities μ and Ω_o/ω_o are small enough that their third and higher powers may be neglected, we have that (8.5a) holds. Substituting this equation into (8.16) gives

$$x \approx 1 - \frac{5\mu^2}{4}, \quad y \approx y_o \tag{8.19a}$$

which reduces (8.17) to

$$\mathbb{K} \approx \frac{1}{u}\left\{1 + \frac{3y_o^2}{2}\right\}, \quad \boldsymbol{\ell}_t \approx \left\{1 - \frac{y_o^2}{2}\right\}\widehat{\boldsymbol{\kappa}} + y_o\,\widehat{\boldsymbol{\jmath}} \tag{8.19b}$$

$$\boldsymbol{\ell}_n \approx \left\{1 - \frac{y_o^2}{2}\right\}\widehat{\boldsymbol{\jmath}} - y_o\,\widehat{\boldsymbol{\kappa}}, \quad \boldsymbol{\ell}_c \approx u\left\{1 - \frac{3y_o^2}{2}\right\}\boldsymbol{\ell}_n. \tag{8.19c}$$

We conclude that, strictly speaking, the vector from the apparent position of the light source to its true position has a magnitude and direction which differ from those of the observer's velocity by small but finitesimal measures.

Example 8-G. *Apparent path in the infraradio limit.* When the observer's acceleration is not entirely negligible and the quantity μ is large enough that its first and higher powers dominate any expression containing them, we have that (8.6a) holds. Substituting this equation into (8.16) gives

$$x \approx -\frac{1}{2}, \quad y \approx \frac{\Omega_o}{4\omega_o}\sqrt{\frac{2}{\mu}} \tag{8.20a}$$

on account of which (8.17) becomes, provided Ω_o/ω_o is not too large,

$$\mathbb{K} \approx \frac{1}{u}\left\{1 - \frac{3\Omega_o^2}{8\mu\omega_o^2}\right\}, \quad \boldsymbol{\ell}_t \approx \left\{1 - \frac{\Omega_o^2}{16\mu\omega_o^2}\right\}\left\{\widehat{\boldsymbol{\kappa}} + \left(\frac{\Omega_o}{4\omega_o}\sqrt{\frac{2}{\mu}}\right)\widehat{\boldsymbol{\jmath}}\right\} \tag{8.20b}$$

$$\boldsymbol{\ell}_n \approx \left\{1 - \frac{\Omega_o^2}{16\mu\omega_o^2}\right\}\left\{\widehat{\boldsymbol{\jmath}} - \left(\frac{\Omega_o}{4\omega_o}\sqrt{\frac{2}{\mu}}\right)\widehat{\boldsymbol{\kappa}}\right\}, \quad \boldsymbol{\ell}_c \approx u\left\{1 + \frac{3\Omega_o^2}{8\mu\omega_o^2}\right\}\boldsymbol{\ell}_n. \tag{8.20c}$$

We conclude that in this approximation also, the magnitude and direction of a vector from the apparent position of the light source to its true position differ from those of the observer's velocity by small but finitesimal measures.

Art 35b. *Apparent path for observers at first quarter.*

When an observer is at first quarter, we have $a_\parallel = 0$, $a_\perp = a$ which reduces (8.14a) to

$$\varsigma_a = \sigma\omega_o\Omega_o, \quad \varsigma_b = 0, \quad \varsigma_c = -\sigma\omega_o\Omega_o^3. \tag{8.21a}$$

In view of (8.7a), (8.14b) and (8.21a), the various quantities defined in (4.10) become

$$\begin{aligned}
&\varrho_a = 1, \quad \varrho_b = 1, \quad \varrho_c = 1, \quad \varrho_d = y_o - 1, \quad \varrho_e = 0, \quad \varrho_f = -\mu\Omega_o^3(1+3\mu^2)\\
&\varrho_g = 4d\omega_o^2 y_o^2, \quad \varrho_h = 0, \quad \varrho_i = 0, \quad \varrho_j = -y_o\Omega_o^2(1+3\mu^2) - 12y_o^3\omega_o^2, \quad \varrho_k = 0\\
&\varrho_l = -3ay_o\Omega_o, \quad \varrho_m = 0, \quad \varrho_n = 3ay_o\Omega_o(y_o-1), \quad \varrho_o = 0, \quad \varrho_p = (y_o-1)(2y_o-1)\\
&\varrho_q = a|y_o - 1|, \quad \varrho_r = a^3\Omega_o(y_o^2-1), \quad \varrho_s = 0, \quad \varrho_t = -a^3\Omega_o^3(y_o^2-1)\\
&\varrho_u = -a^4\Omega_o^2(y_o^2-1), \quad \varrho_v = 0, \quad \varrho_w = -a^4\Omega_o^4(y_o^2-1), \quad \varrho_x = a^2\Omega_o|y_o^2-1|, \quad \varrho_y = 0
\end{aligned} \tag{8.21b}$$

where y_o is given by (8.16), and we shall introduce for convenience [14]

$$\varepsilon_2 = \mathrm{ugn}(y_o - 1). \tag{8.21c}$$

In view of (8.21), (8.13) and (8.1), (4.25) becomes

$$\mathbb{K} = \frac{|y_o^2-1|}{u|y_o-1|^3}, \quad \mathbb{T} = 0, \quad \boldsymbol{\ell}_t = -\varepsilon_2\,\widehat{\boldsymbol{\jmath}}, \quad \boldsymbol{\ell}_b = -\varepsilon_2\,\widehat{\boldsymbol{\imath}} \tag{8.22a}$$

$$\boldsymbol{\ell}_n = -\varepsilon_2^2\widehat{\boldsymbol{\kappa}}, \quad \boldsymbol{\ell}_c = u\left\{\frac{|y_o-1|^3}{|y_o^2-1|}\right\}\boldsymbol{\ell}_n. \tag{8.22b}$$

We conclude that at the instant when the observer is at first quarter, the apparent motion of the light source may be prograde ($\varepsilon_2 = -1$), retrograde ($\varepsilon_2 = +1$) or stationary ($\varepsilon_2 = \perp$).

Art 35c. *Apparent path for observers at full phase.*

When an observer is at full phase, we have $a_\parallel = -a$, $a_\perp = 0$ which reduces (8.14a) to

$$\varsigma_a = 0, \quad \varsigma_b = -\omega_o\Omega_o^2\sigma, \quad \varsigma_c = 0. \tag{8.23a}$$

The various quantities defined in (4.10) become, in view of (8.9a), (8.14b) and (8.23a),

$$\begin{aligned}
&\varrho_a = (1+\mu^2)^{1/2}, \quad \varrho_b = 1 - 4\mu^2, \quad \varrho_c = (1/\varrho_a^3) - (\pi/2d)^2, \quad \varrho_d = -1\\
&\varrho_e = -\pi(\Omega_o/\varrho_a)^2, \quad \varrho_f = 0, \quad \varrho_g = -y\sigma\Omega_o, \quad \varrho_h = 0, \quad \varrho_i = -xy\Omega_o, \quad \varrho_j = 0, \quad \varrho_k = 0\\
&\varrho_l = ya\Omega_o(1+2x), \quad \varrho_m = 0, \quad \varrho_n = \varrho_l, \quad \varrho_o = -ay^2(1+2x), \quad \varrho_p = 1 + xy^2\\
&\varrho_q = a(1+y^2)^{1/2}, \quad \varrho_r = 0, \quad \varrho_s = a^3\Omega_o^2(1-xy^2), \quad \varrho_t = 0, \quad \varrho_u = a^4\Omega_o^2(1-xy^2)\\
&\varrho_v = 0, \quad \varrho_w = a^4\Omega_o^4(1-xy^2), \quad \varrho_x = a^2\Omega_o|1-xy^2|, \quad \varrho_y = 0
\end{aligned} \tag{8.23b}$$

where x and y are given by (8.16), and we shall find it convenient to introduce

$$\varepsilon_3 = \mathrm{ugn}(1 - xy^2). \tag{8.23c}$$

[14] We define the function $\mathrm{ugn}(x)$ to be such that

$$\mathrm{ugn}(x) = \begin{cases} +1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ \perp & \text{if } x = 0 \end{cases}$$

In view of (8.23), (8.13) and (8.1), (4.25) becomes

$$\begin{gathered}\mathbb{K}=\frac{|1-xy^2|}{u(1+y^2)^{3/2}},\quad \mathbb{T}=0,\quad \boldsymbol{\ell}_t=\frac{-\widehat{\boldsymbol{\kappa}}+\widehat{\boldsymbol{\jmath}}\,y}{\sqrt{1+y^2}},\quad \boldsymbol{\ell}_b=\varepsilon_3\widehat{\boldsymbol{\imath}}\\ \boldsymbol{\ell}_n=\frac{-\varepsilon_3(\widehat{\boldsymbol{\jmath}}+\widehat{\boldsymbol{\kappa}}\,y)}{\sqrt{1+y^2}},\quad \boldsymbol{\ell}_c=u\left\{\frac{(1+y^2)^{3/2}}{|1-xy^2|}\right\}\boldsymbol{\ell}_n.\end{gathered}\tag{8.24}$$

We conclude that at the instant when the observer is at full phase, the apparent path of the light source will be a curved line if $\varepsilon_3 \neq \perp$ and a straight line if $\varepsilon_3 = \perp$.

Art 35d. *Apparent path for observers at last quarter.*

When an observer is at last quarter, we have $a_\parallel = 0$, $a_\perp = -a$ which reduces (8.14a) to

$$\varsigma_a=-\sigma\omega_o\Omega_o,\quad \varsigma_b=0,\quad \varsigma_c=\sigma\omega_o\Omega_o^3.\tag{8.25a}$$

The various quantities defined in (4.10) become, in view of (8.11a), (8.14b) and (8.25a),

$$\begin{aligned}&\varrho_a=1,\quad \varrho_b=1,\quad \varrho_c=1,\quad \varrho_d=-1-y_o,\quad \varrho_e=0,\quad \varrho_f=\mu\Omega_o^3(1+3\mu^2)\\ &\varrho_g=4d\omega_o^2y_o^2,\quad \varrho_h=0,\quad \varrho_i=0,\quad \varrho_j=y_o\Omega_o^2(1+3\mu^2)+12\omega_o^2y_o^3,\quad \varrho_k=0\\ &\varrho_l=-3ay_o\Omega_o,\quad \varrho_m=0,\quad \varrho_n=-3ay_o\Omega_o(1+y_o),\quad \varrho_o=0,\quad \varrho_p=(1+y_o)(1+2y_o)\\ &\varrho_q=a(1+y_o),\quad \varrho_r=-a^3\Omega_o(y_o^2-1),\quad \varrho_s=0,\quad \varrho_t=a^3\Omega_o^3(y_o^2-1)\\ &\varrho_u=-a^4\Omega_o^2(y_o^2-1),\quad \varrho_v=0,\quad \varrho_w=-a^4\Omega_o^4(y_o^2-1),\quad \varrho_x=a^2\Omega_o|1-y_o^2|,\quad \varrho_y=0\end{aligned}\tag{8.25b}$$

where y_o is given by (8.16), and we shall find it convenient to introduce

$$\varepsilon_4=\mathrm{ugn}(1-y_o).\tag{8.25c}$$

In view of (8.25), (8.13) and (8.1), (4.25) becomes

$$\begin{gathered}\mathbb{K}=\frac{|y_o-1|}{u(y_o+1)^2},\quad \mathbb{T}=0,\quad \boldsymbol{\ell}_t=-\widehat{\boldsymbol{\jmath}},\quad \boldsymbol{\ell}_b=\varepsilon_4\widehat{\boldsymbol{\imath}}\\ \boldsymbol{\ell}_n=\varepsilon_4\widehat{\boldsymbol{\kappa}},\quad \boldsymbol{\ell}_c=u\left\{\frac{(y_o+1)^2}{|y_o-1|}\right\}\boldsymbol{\ell}_n.\end{gathered}\tag{8.26}$$

We conclude that at the instant when the observer is at last quarter, the apparent path of the light source will be a curved line if $\varepsilon_4 \neq \perp$ and a straight line if $\varepsilon_4 = \perp$.

Example 8-H. *Elongation in the apparent path of a light source.* Let $\varepsilon_3 > 0$ and $\varepsilon_1 > 0$. Then when the observer is at full phase or new phase, we have by (8.24) and (8.17) that

$$\boldsymbol{\ell}_n^F=\frac{-\widehat{\boldsymbol{\jmath}}-\widehat{\boldsymbol{\kappa}}\,y}{\sqrt{1+y^2}},\quad \boldsymbol{\ell}_n^N=\frac{\widehat{\boldsymbol{\jmath}}-\widehat{\boldsymbol{\kappa}}\,y}{\sqrt{1+y^2}}\tag{8.27a}$$

where $\boldsymbol{\ell}_n^F$ and $\boldsymbol{\ell}_n^N$ are unit vectors drawn from the apparent position of the light source to the center of curvature of its apparent path at full phase and new phase respectively. It follows that, unless $y = 0$, the angle φ between these vectors will be different from 180°. To calculate the amount by which φ differs from 180°, we take the scalar product of the vectors to get

$$\cos\varphi=\frac{y^2-1}{y^2+1}\tag{8.27b}$$

so that to a third order accuracy in μ, (8.16) permits us to write

$$\cos\varphi\approx\frac{y_o^2-1}{y_o^2+1},\quad y_o\approx\frac{\mu\Omega_o}{4\omega_o}\tag{8.27c}$$

from which the elongation $180° - \varphi$ can be easily calculated. We conjecture that when the light source is not stationary but moving, either inherently or as a result of observational errors, elongation may induce a precession in the apparent path of the source since the center of curvature of the path will no longer be fixed in space [15].

Art 36. *On the apparent geometry of rays.*

To study the apparent geometry of obliquated rays for an observer in circular motion, we substitute (8.1) and (8.13) into (4.26) to get, in view of (8.14),

$$\begin{gathered}\mathfrak{H}_a = 0, \quad \mathfrak{H}_b = 0, \quad \mathfrak{H}_c = 0, \quad \mathfrak{H}_d = 0, \quad \mathfrak{H}_e = 0 \\ \mathfrak{H}_f = 0, \quad \mathfrak{H}_g = \Omega_o a_\perp, \quad \mathfrak{H}_h = -\Omega_o^2 u^2\end{gathered} \tag{8.28a}$$

$$\begin{gathered}\mathfrak{K}_a = \mathcal{Y}\varrho_d, \quad \mathfrak{K}_b = 0, \quad \mathfrak{K}_c = 0, \quad \mathfrak{K}_d = 0, \quad \mathfrak{K}_e = 0 \\ \mathfrak{K}_f = [\mathcal{Y}\varrho_d a_\perp + \rho\Omega_o(\mathcal{Y}a_\parallel - \rho a^2)]^2, \quad \mathfrak{K}_g = a^2u^2\varrho_d^2 \\ \mathfrak{K}_h = au\varrho_d[\mathcal{Y}\varrho_d a_\perp + \rho\Omega_o(\mathcal{Y}a_\parallel - \rho a^2)]\end{gathered} \tag{8.28b}$$

where we have used the fact that in all the cases we shall be considering, $\varrho_k = 0$ by (8.15b), (8.21b), (8.23b) and (8.25b).

Art 36a. *Apparent ray geometry for observers at new phase.*

When an observer is at new phase, we have $a_\parallel = a, a_\perp = 0$ by definition while $\varrho_d = -1$ by (8.15b). Equation (8.28b) becomes, in view of (8.3b) and (8.16),

$$\begin{gathered}\mathfrak{K}_a = -c(\delta - \beta y), \quad \mathfrak{K}_b = 0, \quad \mathfrak{K}_c = 0, \quad \mathfrak{K}_d = 0, \quad \mathfrak{K}_e = 0 \\ \mathfrak{K}_f = (cay\delta)^2, \quad \mathfrak{K}_g = (au)^2, \quad \mathfrak{K}_h = au(cay\delta)\end{gathered} \tag{8.29a}$$

where we have introduced, with y and d given by (8.16),

$$\delta = d - \beta y, \quad \varepsilon_5 = \mathrm{ugn}(\beta + \delta y) \tag{8.29b}$$

so that by using (8.3), (8.29) and (8.15b) in (4.31), we get

$$\overline{\mathbb{K}} = \Pi/(c\mathcal{R}^3), \quad \overline{\mathbb{T}} = 0, \quad \overline{\boldsymbol{\ell}}_t = (\delta\,\widehat{\boldsymbol{\kappa}} - \beta\,\widehat{\boldsymbol{\jmath}})/\mathcal{R}, \quad \overline{\boldsymbol{\ell}}_b = \varepsilon_5\widehat{\boldsymbol{\imath}} \tag{8.30a}$$

$$\Pi = \sigma|\beta + \delta y|, \quad \mathcal{R}^2 = \beta^2 + \delta^2. \tag{8.30b}$$

We conclude that at the instant when the observer is at new phase, the ray is curved but lies entirely in a plane.

Art 36b. *Apparent ray geometry for observers at first quarter.*

When an observer is at first quarter, we have $a_\parallel = 0, a_\perp = a$ by definition while $\varrho_d = y_o - 1$ by (8.21b). Equation (8.28b) becomes, in view of (8.7a) and (8.16),

$$\begin{gathered}\mathfrak{K}_a = c(y_o - 1), \quad \mathfrak{K}_b = 0, \quad \mathfrak{K}_c = 0, \quad \mathfrak{K}_d = 0, \quad \mathfrak{K}_e = 0 \\ \mathfrak{K}_f = c^2a^2(y_o - 1)^2, \quad \mathfrak{K}_g = a^2u^2(y_o - 1)^2, \quad \mathfrak{K}_h = ca^2u(y_o - 1)^2.\end{gathered} \tag{8.31}$$

Using (8.7), (8.31) and (8.21) in (4.31), we get

$$\overline{\mathbb{K}} = \Pi/(c\mathcal{R}^2), \quad \overline{\mathbb{T}} = 0, \quad \overline{\boldsymbol{\ell}}_t = \widehat{\boldsymbol{\kappa}}, \quad \overline{\boldsymbol{\ell}}_b = -\varepsilon_2\widehat{\boldsymbol{\imath}} \tag{8.32a}$$

[15] To examine this question more rigorously, it is necessary to first consider the problem of whether or not the velocity of a light ray may depend on the velocity of its source, a problem that is outside the scope of this work but will be treated in detail when we study the kineoptics of intrinsic redshifts.

$$\Pi = \sigma|y_o - 1|, \quad \mathcal{R} = 1 + \beta \tag{8.32b}$$

where ε_2 is given by (8.21c). We conclude that at the instant when the observer is at first quarter, the ray is curved even though the observer is moving along the line of incidence of the ray.

Art 36c. *Apparent ray geometry for observers at full phase.*

When an observer is at full phase, we have $a_\| = -a, a_\perp = 0$ by definition while $\varrho_d = -1$ by (8.23b). Equation (8.28b) becomes, in view of (8.9a) and (8.16),

$$\begin{gathered}\mathfrak{K}_a = -c(\delta - \beta y), \quad \mathfrak{K}_b = 0, \quad \mathfrak{K}_c = 0, \quad \mathfrak{K}_d = 0, \quad \mathfrak{K}_e = 0 \\ \mathfrak{K}_f = (cay\delta)^2, \quad \mathfrak{K}_g = a^2u^2, \quad \mathfrak{K}_h = au(cay\delta)\end{gathered} \tag{8.33}$$

where δ is given by (8.29b), and by using (8.9), (8.33) and (8.23b) in (4.31), we get with ε_5 given by (8.29b),

$$\overline{\mathbb{K}} = \Pi/(c\mathcal{R}^3), \quad \overline{\mathbb{T}} = 0, \quad \overline{\boldsymbol{\ell}}_t = (\delta\,\widehat{\boldsymbol{\kappa}} + \beta\,\widehat{\boldsymbol{\jmath}})/\mathcal{R}, \quad \overline{\boldsymbol{\ell}}_b = \varepsilon_5\widehat{\boldsymbol{\imath}} \tag{8.34a}$$

$$\Pi = \sigma|\beta + \delta y|, \quad \mathcal{R}^2 = \beta^2 + \delta^2. \tag{8.34b}$$

We conclude that at the instant when the observer is at full phase, the ray is curved to the same extent as when the observer is at new phase.

Art 36d. *Apparent ray geometry for observers at last quarter.*

When an observer is at last quarter, we have $a_\| = 0, a_\perp = -a$ by definition while $\varrho_d = -(y_o + 1)$ by (8.25b). Equation (8.28b) becomes, in view of (8.11a) and (8.16),

$$\begin{gathered}\mathfrak{K}_a = -c(y_o + 1), \quad \mathfrak{K}_b = 0, \quad \mathfrak{K}_c = 0, \quad \mathfrak{K}_d = 0, \quad \mathfrak{K}_e = 0 \\ \mathfrak{K}_f = c^2a^2(y_o + 1)^2, \quad \mathfrak{K}_g = a^2u^2(y_o + 1)^2, \quad \mathfrak{K}_h = -a^2uc(y_o + 1)^2.\end{gathered} \tag{8.35a}$$

Introducing the quantity

$$\varepsilon_6 = \mathrm{ugn}(1 - \beta) \tag{8.35b}$$

and using (8.11), (8.35) and (8.25) in (4.31), we get

$$\overline{\mathbb{K}} = \Pi/(c\mathcal{R}^3), \quad \overline{\mathbb{T}} = 0, \quad \overline{\boldsymbol{\ell}}_t = \varepsilon_6\widehat{\boldsymbol{\kappa}}, \quad \overline{\boldsymbol{\ell}}_b = -\varepsilon_6\widehat{\boldsymbol{\imath}} \tag{8.36a}$$

$$\Pi = \sigma\mathcal{R}(y_o + 1), \quad \mathcal{R} = |1 - \beta|. \tag{8.36b}$$

We conclude that at the instant when the observer is at last quarter, the ray is curved provided $\beta \neq 1$. For $\beta = 1$, we see that the ray will not propagate relative to the observer and the light source will therefore be imperceptible to the observer at last quarter.

Art 37. *On the apparent frequency of rays.*

To calculate the apparent frequency of a light ray for an observer in circular motion with constant speed, we substitute the value of $\mathcal{R}$ from (8.2a) into (3.30). In this way we shall obtain

$$(\omega'/\omega_o)^2 = \mathcal{R}^2, \quad \mathcal{R}^2 = d^2 + \beta^2 + 2d\beta\sin\nu + \rho\sigma(\rho\sigma + 2d\cos\nu + 2\beta\sin 2\nu) \tag{8.37}$$

which shows that the apparent ray frequency ω' is affected in general by both acceleration and dispersion.

Example 8-I. *Ray frequency for observers at new phase.* For an observer at new phase, $\mathcal{R}$ has the value given by (8.30b). Putting this value into (8.37) yields

$$\omega' = \omega_o\sqrt{\delta^2 + \beta^2} \tag{8.38}$$

where δ is given by (8.29b). We conclude that at the instant when the observer is at new phase, the effect of the observer's motion is to enhance the ray frequency.

Example 8-J. *Ray frequency for observers at first quarter.* For an observer at first quarter, $\mathcal{R}$ has the value given by (8.32b), and by substituting this value into (8.37), we get

$$\omega' = \omega_o(1+\beta) \tag{8.39}$$

from which we conclude that at the instant when the observer is at first quarter, the apparent frequency of the ray depends only on the instantaneous speed of the observer and not on the observer's acceleration.

Example 8-K. *Ray frequency for observers at full phase.* For an observer at new phase, $\mathcal{R}$ has the value given by (8.34b). Substituting this value into (8.37) leads to

$$\omega' = \omega_o\sqrt{\delta^2+\beta^2} \tag{8.40}$$

where δ is given by (8.29b). We conclude that at the instant when the observer is at full phase, the effect of the observer's motion is to enhance the ray frequency by the same amount as when the observer is at new phase.

Example 8-L. *Ray frequency for observers at last quarter.* For an observer at last quarter, $\mathcal{R}$ has the value given by (8.36b). Using this value of $\mathcal{R}$ in (8.37), we get

$$\omega' = \omega_o|1-\beta| \tag{8.41}$$

from which we conclude that at the instant when the observer is at last quarter, the effect of the observer's motion is to reduce the ray frequency if $\beta < 1$ and to enhance the ray frequency if $\beta > 1$. For $\beta = 1$, the ray does not propagate relative to the observer and the ray frequency therefore vanishes.

9 Effects of precessional acceleration

Art 38. *On the apparent direction to a light source.*

Throughout this section we consider an observer rotating with a constant angular speed Ω about an axis which precesses at a constant rate $|\Gamma|$ about a fixed direction $\widehat{\mathbf{\Gamma}}$ so that

$$\mathbf{\Lambda} = \mathbf{\Gamma}\times\mathbf{\Omega}, \quad \dot{\mathbf{\Lambda}} = (\mathbf{\Gamma}\cdot\mathbf{\Omega})\mathbf{\Gamma} - \Gamma^2\mathbf{\Omega}, \quad \ddot{\mathbf{\Lambda}} = -\Gamma^2(\mathbf{\Gamma}\times\mathbf{\Omega}), \quad \dddot{\mathbf{\Lambda}} = -\Gamma^2(\mathbf{\Gamma}\cdot\mathbf{\Omega})\mathbf{\Gamma} + \Gamma^4\mathbf{\Omega} \tag{9.1}$$

where $\mathbf{\Gamma}$ is a vector with magnitude $|\Gamma|$ parallel to $\widehat{\mathbf{\Gamma}}$ for $\Gamma > 0$ and antiparallel to $\widehat{\mathbf{\Gamma}}$ for $\Gamma < 0$. Considering a light source whose rays are incident in a direction $\widehat{\boldsymbol{\kappa}}$ parallel to $\widehat{\mathbf{\Gamma}}$ and are linearly polarized in a direction $\widehat{\mathbf{p}}$ perpendicular to $\widehat{\mathbf{\Gamma}}$, we introduce a coordinates system with $\widehat{\boldsymbol{\kappa}}$, $\widehat{\mathbf{p}}$ and $\widehat{\boldsymbol{\jmath}} = \widehat{\mathbf{p}}\times\widehat{\boldsymbol{\kappa}}$ as basis vectors. Then we have

$$\begin{gathered} \widehat{\mathbf{\Gamma}} = \widehat{\boldsymbol{\kappa}}, \quad \mathbf{\Omega} = \Omega(\widehat{\boldsymbol{\jmath}}\sin\varepsilon\cos\varpi + \widehat{\mathbf{p}}\sin\varepsilon\sin\varpi + \widehat{\boldsymbol{\kappa}}\cos\varepsilon) \\ \mathbf{r} = r(\widehat{\boldsymbol{\jmath}}\sin\nu\cos\varphi + \widehat{\mathbf{p}}\sin\nu\sin\varphi + \widehat{\boldsymbol{\kappa}}\cos\nu) \end{gathered} \tag{9.2a}$$

$$0^\circ \le \varpi < 360^\circ, \quad 0^\circ \le \varphi < 360^\circ, \quad 0^\circ \le \varepsilon \le 180^\circ, \quad 0^\circ \le \nu \le 180^\circ \tag{9.2b}$$

where ε is the angle between $\widehat{\boldsymbol{\kappa}}$ and $\mathbf{\Omega}$, ν is the angle between $\widehat{\boldsymbol{\kappa}}$ and $\mathbf{r}$, ϖ is the angle between $\widehat{\boldsymbol{\jmath}}$ and the projection of $\mathbf{\Omega}$ on the plane of $\widehat{\boldsymbol{\jmath}}$ and $\widehat{\mathbf{p}}$, while φ is the angle between $\widehat{\boldsymbol{\jmath}}$ and the projection of $\mathbf{r}$ on the same plane. In this coordinates system (9.1) becomes

$$\begin{gathered} \mathbf{\Lambda} = \Omega\Gamma(\widehat{\mathbf{p}}\sin\varepsilon\cos\varpi - \widehat{\boldsymbol{\jmath}}\sin\varepsilon\sin\varpi), \quad \dot{\mathbf{\Lambda}} = -\Omega\Gamma^2(\widehat{\boldsymbol{\jmath}}\sin\varepsilon\cos\varpi + \widehat{\mathbf{p}}\sin\varepsilon\sin\varpi) \\ \ddot{\mathbf{\Lambda}} = -\Omega\Gamma^3(\widehat{\mathbf{p}}\sin\varepsilon\cos\varpi - \widehat{\boldsymbol{\jmath}}\sin\varepsilon\sin\varpi), \quad \dddot{\mathbf{\Lambda}} = \Omega\Gamma^4(\widehat{\boldsymbol{\jmath}}\sin\varepsilon\cos\varpi + \widehat{\mathbf{p}}\sin\varepsilon\sin\varpi) \end{gathered} \tag{9.3a}$$

while the angle A between $\mathbf{r}$ and $\mathbf{\Omega}$ is given by the cosine law

$$\cos A = \cos\varepsilon\cos\nu + \sin\varepsilon\sin\nu\cos B, \quad B = \varpi - \varphi, \quad 0^\circ \le A \le 180^\circ. \tag{9.3b}$$

These equations are however much too general to be useful for the purposes of illustration. We shall suppose therefore in the sequel that

$$\cos\varepsilon = \cos A = 0, \quad \cos B = 0, \quad \cos\varphi = \sin\varpi, \quad \sin\varphi = -\cos\varpi \tag{9.3c}$$

since the loss of generality incurred with this supposition is more than balanced by the instructiveness of its results.

Art 38a. *Effects of polarization.*

It was noted in Section §1 that the propagation condition $\boldsymbol{\kappa}\cdot(\widehat{\mathbf{p}} \times \mathbf{\Omega}) \neq 0$ needs to hold in order for the light rays to propagate relative to the rotating observer as regular rays. When this condition is violated, the rays degenerate into a mode where they either cease to propagate or propagate with a velocity that is no longer given by (1.1). In terms of the particular situation being considered here, this condition requires that the angular velocity $\mathbf{\Omega}$ be noncollinear with the polarization direction $\widehat{\mathbf{p}}$. More generally, as the axis of rotation of the observer precesses about the line of incidence of the rays, it will be carried into the plane of $\widehat{\boldsymbol{\kappa}}$ and $\widehat{\mathbf{p}}$ at two instants corresponding to $\varpi = 90^\circ$ and $\varpi = 270^\circ$. At these instants the rays will no longer propagate as regular rays. We conclude that if light rays from the cat's eye nebula are linearly polarized and incident at right angles to the ecliptic, then observational astronomers will do well to study the photometry of light from this nebula when the earth's axis of rotation lies in a plane determined by the polarization direction of the rays and the normal to the ecliptic.

Art 38b. *Effects of acceleration.*

To study the effects of acceleration on the obliquation angle of the light source, we use (9.2) and (9.3) to get

$$\begin{aligned} \widehat{\mathbf{p}} \times \mathbf{\Omega} &= -\widehat{\boldsymbol{\kappa}}\Omega\cos\varpi, \quad \widehat{\mathbf{p}} \times \mathbf{\Lambda} = \widehat{\boldsymbol{\kappa}}\Omega\Gamma\sin\varpi \\ \mathbf{\Lambda} \times \mathbf{r} &= r\Omega\Gamma(\widehat{\boldsymbol{\jmath}}\cos\varpi\cos\nu + \widehat{\mathbf{p}}\sin\varpi\cos\nu) \\ \mathbf{\Omega} \times \mathbf{r} &= r\Omega(-\widehat{\boldsymbol{\kappa}}\sin\nu - \widehat{\mathbf{p}}\cos\nu\cos\varpi + \widehat{\boldsymbol{\jmath}}\cos\nu\sin\varpi) \end{aligned} \tag{9.4}$$

which upon substitution into (5.1) yields

$$\begin{aligned} &\varepsilon_a = 0, \quad \varepsilon_b = 0, \quad \varepsilon_c = \Omega\sin\varpi, \quad \varepsilon_d = r\cos\nu, \quad \varepsilon_e = 0 \\ &\varepsilon_f = r\sin\nu\sin\varphi, \quad \varepsilon_g = 0, \quad \varepsilon_h = -r\Omega\Gamma\sin\nu, \quad \varepsilon_i = \Omega\Gamma\cos\varpi \\ &\varepsilon_j = -\Omega\cos\varpi \neq 0, \quad \varepsilon_k = -r\Omega\sin\nu, \quad \varepsilon_l = -r\Omega\cos\nu\cos\varpi \\ &\varepsilon_m = r\Omega^2\Gamma\cos\nu, \quad \varepsilon_n = 0, \quad \varepsilon_o = r\Omega\Gamma\sin\varpi\cos\nu \end{aligned} \tag{9.5a}$$

$$\begin{aligned} &\varphi_a = r\Omega, \quad \varphi_b = r\Pi\Omega^2, \quad \varphi_c = 0, \quad \varphi_d = -r\Omega^2\cos\nu, \quad \varphi_e = 0, \quad \varphi_f = 0, \quad \varphi_g = 0 \\ &\varphi_h = -\mho/(2\omega_o^2\Omega^2\cos^2\varpi), \quad \varphi_i = (\mho\Gamma)/(2\omega_o^2), \quad \varphi_j = 0, \quad \varphi_k = -2\rho r\Omega^2\cos\nu \\ &\varphi_l = 1, \quad \varphi_m = 0, \quad \varphi_n = 0, \quad \varphi_o = 0, \quad \varphi_p = (r\Gamma^2\Omega^2\mho\cos\nu)/(2\omega_o^2) \\ &\varphi_q = -(\mho\Gamma\Omega)/(2\omega_o^2), \quad \varphi_r = -4\rho\Omega\sin\nu\cos\nu, \quad \varphi_s = 2\rho\Omega(\mho + cd)\sin\nu\cos\nu \\ &\varphi_t = -\sin\nu, \quad \varphi_u = -(\cos\nu)/\Pi, \quad \varphi_v = 0 \end{aligned} \tag{9.5b}$$

where, by using (9.5) in (5.12d), we have

$$\begin{aligned} &\tilde{\Omega} = \Omega/\omega_o, \quad \tilde{\Gamma} = \Gamma/(2\omega_o), \quad \Upsilon = \Gamma/\Omega, \quad \Pi = (1 + \Upsilon^2\cos^2\nu)^{1/2} \\ &\vartheta = -\beta\tilde{\Omega}\cos\nu, \quad \alpha = -\beta\Omega\omega_o\cos\nu, \quad \gamma = 1, \quad \beta = (r\Omega)/c, \quad \sigma = \Pi\Omega\beta \end{aligned} \tag{9.6a}$$

$$\pi = \frac{\vartheta}{\sqrt{1+\vartheta^2}}, \quad d = \left\{ \frac{1+\sqrt{1+\vartheta^2}}{2} \right\}^{1/2}, \quad \rho = \frac{\pi}{4d\omega_o} \tag{9.6b}$$
$$\hbar = \rho\Omega\beta\cos\nu, \quad \mathfrak{d} = d + 2\hbar, \quad \mathcal{Y} = c\mathfrak{d}, \quad \mathcal{S} = \tilde{\Gamma}(4\mathfrak{d}\hbar + \tilde{\Omega}^2\mathfrak{d}^2)^{1/2}.$$

Accordingly, (5.12a) and (5.12b) become

$$\begin{gathered} \mathcal{L} = -2\hbar\sin\nu, \quad \mathcal{P} = \mathcal{S}^2 + 4\hbar(d + \beta\sin\nu) \\ \mathcal{N} = \mathcal{S}^2 + 4\hbar(d\cos^2\nu - \hbar\sin^2\nu) \end{gathered} \tag{9.7a}$$

$$\begin{aligned} &\mathcal{G} = -\beta - \mathfrak{d}\sin\nu \\ &\mathcal{R}^2 = \mathcal{S}^2 + \rho^2\sigma^2 + d^2 + \beta^2 + 4\hbar(d + \beta\sin\nu) - 2d(\hbar - \beta\sin\nu) \\ &\mathcal{F}^2 = \mathcal{S}^2 + \rho^2\sigma^2 + d^2\cos^2\nu + 4\hbar(d\cos^2\nu - \hbar\sin^2\nu) - 2d\hbar \end{aligned} \tag{9.7b}$$

which upon substitution into (5.12c) yields

$$\tan\psi = \frac{[\mathcal{S}^2 + \rho^2\sigma^2 + d^2\cos^2\nu + 4\hbar(d\cos^2\nu - \hbar\sin^2\nu) - 2d\hbar]^{1/2}}{-\beta - \mathfrak{d}\sin\nu}. \tag{9.8}$$

We conclude that the obliquation angle of the light source depends in general on the precession of the observer and the frequency of the light rays.

Example 9-A. *Selfconsistency of classical kineoptics.* When the observer's axis of rotation does not precess ($\Gamma = 0$), we have by (9.6) that $\tilde{\Gamma} = 0$, $\Upsilon = 0$, $\Pi = 1$, $\sigma = \Omega\beta$, and $\mathcal{S} = 0$. Under these conditions, (9.7) reduces — as it should — to the corresponding equation for an observer in centripetal motion that was treated in Section §8. Considering that the formulae which describe the propagation of light rays for observers in translation and rotation differ greatly from one another as described in Section §1, we have here a fine indication of the selfconsistency of our treatment of classical kineoptics.

Art 38c. *Obliquation at transverse position.*

When the observer's position is at right angles to the axis of precession ($\sin\nu = \pm 1$), we have by (9.6) and (9.7) that

$$\begin{gathered} \Pi = 1, \quad \vartheta = 0, \quad \alpha = 0, \quad \sigma = \beta\Omega, \quad \pi = 0, \quad d = 1 \\ \rho = 0, \quad \hbar = 0, \quad \mathfrak{d} = 1, \quad \mathcal{Y} = c, \quad \mathcal{S} = \tilde{\Gamma}\tilde{\Omega} \end{gathered} \tag{9.9a}$$

$$\mathcal{G} = -\beta \mp 1, \quad \mathcal{R}^2 = \tilde{\Gamma}^2\tilde{\Omega}^2 + (1 \pm \beta)^2, \quad \mathcal{F}^2 = \tilde{\Gamma}^2\tilde{\Omega}^2 \tag{9.9b}$$

where the upper signs correspond to $\sin\nu = +1$ and the lower signs correspond to $\sin\nu = -1$. By (9.8), (3.5a) and (9.9b), we obtain

$$\upsilon = c\left[\tilde{\Gamma}^2\tilde{\Omega}^2 + (1 \pm \beta)^2\right]^{1/2}, \quad \tan\psi = -\left\{\frac{\tilde{\Omega}}{\beta \pm 1}\right\}\left|\tilde{\Gamma}\right| \tag{9.10}$$

from which we conclude that the apparent angular displacement of the light source depends on the precession of the observer as well as on the frequency of the light rays.

Art 38d. *Obliquation at longitudinal position.*

When the observer's position is parallel or antiparallel to the axis of precession ($\cos\nu = \pm 1$), we have by (9.6) and (9.7) that

$$\Pi = \sqrt{1+\Upsilon^2}, \quad \vartheta = \mp\beta\tilde{\Omega}, \quad \alpha = \mp\beta\Omega\omega_o, \quad \sigma = \beta\Omega\Pi \tag{9.11a}$$

$$\pi = \mp\beta(\tilde{\Omega}/x), \quad d = \sqrt{(1+x)/2}, \quad \rho = \pi/(4d\omega_o)$$
$$\hbar = \pm\rho\Omega\beta, \quad \mathfrak{d} = d + 2\hbar, \quad \mathcal{Y} = c\mathfrak{d}, \quad \mathcal{S} = \tilde{\Gamma}(4\mathfrak{d}\hbar + \tilde{\Omega}^2\mathfrak{d}^2)^{1/2} \tag{9.11b}$$

$$\mathcal{G} = -\beta, \quad \mathcal{R}^2 = \beta^2 + y^2 + (d+\hbar)^2, \quad \mathcal{F}^2 = y^2 + (d+\hbar)^2 \tag{9.11c}$$

where, with the upper signs corresponding to $\cos\nu = +1$ and the lower signs corresponding to $\cos\nu = -1$, we have introduced

$$x = \sqrt{1 + \beta^2\tilde{\Omega}^2}, \quad y = \sqrt{\mathcal{S}^2 + \hbar^2\Upsilon^2}. \tag{9.11d}$$

Thus by (9.8), (3.5a) and (9.11c), we obtain

$$\upsilon = c\left[\beta^2 + y^2 + (d+\hbar)^2\right]^{1/2}, \quad \tan\psi = -\frac{\sqrt{y^2 + (d+\hbar)^2}}{\beta} \tag{9.12}$$

from which more convenient approximations may be obtained.

Art 39. *On the apparent drift of a light source.*

To calculate the apparent drift of the light source, we assume for simplicity that $\sin\nu = +1$ so that (9.9) holds. Substituting (9.2) through (9.7) into (5.13) under this assumption then gives

$$\delta_a = 0, \quad \delta_b = \Omega\Gamma\sin\varpi, \quad \delta_c = 0, \quad \delta_d = c\Omega\tilde{\Omega}\tilde{\Gamma}, \quad \delta_e = 0, \quad \delta_f = 0, \quad \delta_g = 1$$
$$\delta_h = 1/(4\omega_o^3), \quad \delta_i = 0, \quad \delta_j = -\Omega^2\Gamma\cos^2\varpi, \quad \delta_k = 0, \quad \delta_l = 0, \quad \delta_m = 0 \tag{9.13a}$$
$$\delta_n = 0, \quad \delta_o = 0, \quad \delta_p = 0, \quad \delta_q = 0, \quad \delta_r = 0, \quad \delta_s = 0, \quad \delta_t = 0, \quad \delta_u = 0$$

$$\hbar_a = r, \quad \hbar_b = 0, \quad \hbar_c = r\Omega\tilde{\Gamma}\tilde{\Omega}, \quad \hbar_d = 0, \quad \hbar_e = 0, \quad \hbar_f = 0. \tag{9.13b}$$

Substituting (9.13) into (5.22b) in view of (3.7c) and (3.13b) leads to

$$\imath_0 = -c\mathcal{X}, \quad \imath_1 = 0, \quad \imath_2 = 0, \quad \imath_3 = 0, \quad \imath_4 = -c\mathcal{Y}$$
$$\imath_5 = 0, \quad \imath_6 = 0, \quad \imath_7 = 0, \quad \imath_8 = 0, \quad \imath_9 = \mathcal{Z} \tag{9.14a}$$

where we have introduced, with $\mathcal{F}$ given by (9.9b),

$$\mathcal{X} = \mathcal{F} + \frac{1+\beta}{\mathcal{F}}, \quad \mathcal{Y} = \frac{\Gamma}{2\omega_o^2}\left\{\mathcal{F} + \frac{1+\beta}{\mathcal{F}}\right\}, \quad \mathcal{Z} = -\frac{(1+\beta) + \mathcal{F}^2(1-\beta)}{\beta\mathcal{F}}. \tag{9.14b}$$

Taking (9.2) and (9.4) into account, we substitute (9.14) into (5.22a) and (5.22c) to get

$$\mathfrak{X} = -\frac{1}{\beta c\mathcal{R}^2}\left[(\mathcal{X} + \beta\mathcal{Z})\widehat{\boldsymbol{\kappa}} + \mathcal{Y}\Omega(\widehat{\boldsymbol{\jmath}}\cos\varpi + \widehat{\mathbf{p}}\sin\varpi)\right], \quad \Theta = 0, \quad \dot{\psi} = 0 \tag{9.15}$$

from which we conclude that at this particular instant, the apparent angular displacement ψ of the light source is not varying with time.

Art 40. *On the apparent path of a light source.*

The contents of this article have been omitted in so far as I have not been able to put the formulae into a form that is sufficiently simple, general, and illustrative of some pertinent principle.

Art 41. *On the apparent geometry of rays.*

The contents of this article have been omitted in so far as I have not been able to put the formulae into a form that is sufficiently simple, general, and illustrative of some pertinent principle.

Art 42. *On the apparent frequency of rays.*

To study the apparent ray frequency for the precessing observer, we substitute the value of $\mathcal{R}$ from (9.7b) into (3.30) to get

$$(\omega'/\omega_o)^2 = \mathcal{S}^2 + \rho^2\sigma^2 + d^2 + \beta^2 + 4\hbar(d + \beta \sin\nu) - 2d(\hbar - \beta \sin\nu) \tag{9.16}$$

where $\mathcal{S}, \rho, \sigma, d, \beta$ and $\hbar$ are given by (9.6). We conclude that the apparent ray frequency ω' is affected in general by both dispersion and precession.

Art 42a. *Apparent ray frequency at transverse position.*

When the observer's position is at right angles to the axis of precession ($\sin\nu = \pm 1$), the various quantities defined in (9.6) have the values given by (9.9a). Putting these values into (9.16) gives

$$\omega' = \omega_o \sqrt{\tilde{\Gamma}^2\tilde{\Omega}^2 + (1 \pm \beta)^2} \tag{9.17}$$

as the apparent ray frequency for the observer. In particular, if the observer were to rotate without precessing, the problem will reduce to that of an observer in centripetal motion at first or last quarter, and the apparent ray frequencies given by (9.17) will coincide with those obtained in Art 37.

Art 42b. *Apparent ray frequency at longitudinal position.*

When the observer's position is collinear with the axis of precession ($\cos\nu = \pm 1$), the quantities defined in (9.6) have the values given by (9.11). Using these values in (9.16) gives

$$\omega' = \omega_o \sqrt{\beta^2 + y^2 + (d + \hbar)^2} \tag{9.18}$$

as the apparent ray frequency for the observer. If the observer rotates without precessing, the problem reduces to that of an observer in centripetal motion at new or full phase, and it is not difficult to show that the apparent ray frequencies obtained from (9.18) agree with those obtained previously in Art 37.

10 Effects of gravitational acceleration

Art 43. *On the apparent direction to a light source.*

Throughout this section we consider a gravitating observer and a light source whose rays are linearly polarized in a direction that lies in the orbital plane of the observer's motion and are incident in a direction perpendicular to this plane. Thus if $\widehat{\mathbf{z}}$ is a unit vector directed from the dynamical focus of the orbit towards perihelion while $\widehat{\mathbf{y}}$ is a unit positive normal to the plane of the orbit and $\widehat{\mathbf{x}} = \widehat{\mathbf{y}} \times \widehat{\mathbf{z}}$, then

$$\begin{gathered} \mathbf{z} = z\widehat{\mathbf{z}}, \quad \mathbf{h} = -h\widehat{\mathbf{y}}, \quad \mathbf{r} = r(\widehat{\mathbf{z}}\cos\nu + \widehat{\mathbf{x}}\sin\nu) \\ \boldsymbol{\kappa} = -\kappa\widehat{\mathbf{y}}, \quad \widehat{\mathbf{p}} = (\widehat{\mathbf{z}}\cos\nu_o + \widehat{\mathbf{x}}\sin\nu_o) \end{gathered} \tag{10.1a}$$

$$\begin{gathered} \mathbf{r} \times \mathbf{h} = rh(\widehat{\mathbf{x}}\cos\nu - \widehat{\mathbf{z}}\sin\nu), \quad \mathbf{z} \times \mathbf{h} = zh\widehat{\mathbf{x}}, \quad \widehat{\boldsymbol{\kappa}} \times \mathbf{z} = -z\widehat{\mathbf{x}} \\ \widehat{\boldsymbol{\kappa}} \times \widehat{\mathbf{p}} = \widehat{\mathbf{z}}\sin\nu_o - \widehat{\mathbf{x}}\cos\nu_o, \quad \widehat{\boldsymbol{\kappa}} \times \mathbf{r} = r(\widehat{\mathbf{z}}\sin\nu - \widehat{\mathbf{x}}\cos\nu), \quad \widehat{\mathbf{p}} \times \mathbf{z} = \widehat{\mathbf{y}}z\sin\nu_o \end{gathered} \tag{10.1b}$$

where the true anomaly ν is the instantaneous angle between $\widehat{\mathbf{r}}$ and $\widehat{\mathbf{z}}$, and the polarization anomaly ν_o is the constant angle between $\widehat{\mathbf{p}}$ and $\widehat{\mathbf{z}}$.

Art 43a. *Effects of polarization.*

We have remarked earlier in Section §1 that the propagation condition $\widehat{\mathbf{p}} \cdot (\mathbf{r} \times \mathbf{h}) \neq 0$ must hold in order for the light rays in consideration to propagate relative to a gravitating observer. If the rays are polarized in a direction that lies in the orbital plane of the observer, then there are two positions in the observer's orbit at which the vectors $\widehat{\mathbf{r}}$ and $\widehat{\mathbf{p}}$ are parallel or antiparallel. When the observer is at either of these positions, the propagation condition is violated, the light rays will not propagate relative to the observer and the light source will be imperceptible to the observer. Assuming therefore that light rays from the cat's eye nebula are linearly polarized and incident nearly perpendicularly to the earth's orbital plane, their polarization direction will lie in this plane and there should be two positions on the earth's orbit at which these rays will not propagate relative to an observer on the earth. Observational astronomers are therefore advised to pay close attention to the photometry of light from this nebula as a function of the earth's position in its orbit.

Example 10-A. *Permanently imperceptible light rays.* For light rays that are linearly polarized in a direction normal to the orbital plane of a gravitating observer, the propagation condition is violated at all instants. Hence at no instant will the light rays propagate relative to the observer, from which we conclude that the light source will be permanently imperceptible to the observer, all other things being equal.

Example 10-B. *Transmission of radio signals to artificial satellites.* When a linearly polarized radio signal is transmitted to an artificial satellite, it is essential that the propagation condition holds in order for the satellite to receive the signal. If this condition is violated, say by making the signal's polarization direction to be nearly perpendicular to the satellite's orbital plane, then the satellite should find it quite difficult to receive the signal, all other things being equal.

Art 43b. *Effects of acceleration.*

To study the effects of acceleration on obliquation for a gravitating observer, we observe that on account of (10.1), the various quantities defined in (6.1) become

$$\begin{gathered}\varepsilon_a = 0, \quad \varepsilon_b = 0, \quad \varepsilon_c = \cos\varpi, \quad \varepsilon_d = z\cos\nu, \quad \varepsilon_e = 0 \\ \varepsilon_f = -rh\sin\varpi \neq 0, \quad \varepsilon_g = 0, \quad \varepsilon_h = zh\sin\nu, \quad \varepsilon_i = zh\sin\nu_o\end{gathered} \tag{10.2a}$$

$$\begin{gathered}\varphi_a = \Phi/r, \quad \varphi_b = \Phi, \quad \varphi_c = 0, \quad \varphi_d = -1, \quad \varphi_e = 1, \quad \varphi_f = c/(2rh\sin\varpi) \\ \varphi_g = 0, \quad \varphi_h = \tilde{q}\tilde{\varepsilon}, \quad \varphi_i = 0, \quad \varphi_j = (\tilde{e}\sin\nu)/\tilde{\varepsilon}, \quad \varphi_k = 0 \\ \varphi_l = 0, \quad \varphi_m = 0, \quad \varphi_n = 0, \quad \varphi_o = 0\end{gathered} \tag{10.2b}$$

where we have introduced [38, 39]

$$\begin{gathered}\Phi = q/r, \quad a = q/r^2, \quad \varpi = \nu - \nu_o, \quad \tilde{e} = z/q, \quad \tilde{q} = q/h, \quad \tilde{z} = z/h \\ \tilde{p} = h^2/q = r(1+\tilde{e}\cos\nu), \quad \tilde{\varepsilon} = (\tilde{e}^2 + 2\tilde{e}\cos\nu + 1)^{1/2}\end{gathered} \tag{10.3a}$$

and used the fact that by (6.9c),

$$\begin{gathered}\alpha = 0, \quad \gamma = 1, \quad \vartheta = 0, \quad \beta = (\tilde{q}\tilde{\varepsilon})/c \\ \sigma = \Phi/(rc), \quad \mathcal{Y} = c, \quad \pi = 0, \quad \rho = 0, \quad d = 1.\end{gathered} \tag{10.3b}$$

In view of the foregoing equations, the quantities featured in (6.9a) reduce to

$$\mathcal{L} = 0, \quad \mathcal{P} = 0, \quad \mathcal{N} = 0, \quad \mathcal{G} = -\beta, \quad \mathcal{R} = (1+\beta^2)^{1/2}, \quad \mathcal{F} = 1 \tag{10.4a}$$

which upon substitution into (6.9b) and (3.5a) yields

$$\upsilon = c(1+\beta^2)^{1/2}, \quad \tan\psi = -\frac{1}{\beta}. \tag{10.4b}$$

We conclude that the obliquation angle of the light source depends only on the instantaneous speed of the observer and not explicitly on the observer's acceleration.

Art 44. *On the apparent drift of a light source.*

To calculate the apparent drift of the light source, we substitute (10.1), (10.2), (10.3) and (10.4a) into (6.10) to get

$$\delta_a = h, \quad \delta_b = 0, \quad \delta_c = 0, \quad \delta_d = z\cos\nu_o, \quad \delta_e = 0, \quad \delta_f = -z\sin\nu \tag{10.5a}$$

$$\begin{gathered}\hbar_a = 0, \quad \hbar_b = q + z\cos\nu, \quad \hbar_c = z\cos\nu_o + q\cos\varpi, \quad \hbar_d = c/h, \quad \hbar_e = 0, \quad \hbar_f = 0 \\ \hbar_g = -q, \quad \hbar_h = 0, \quad \hbar_i = 1, \quad \hbar_j = 1/\omega_o, \quad \hbar_k = 1/(4\omega_o^3), \quad \hbar_l = 0, \quad \hbar_m = 0\end{gathered} \tag{10.5b}$$

$$\begin{gathered}\hbar_n = 0, \quad \hbar_o = 0, \quad \hbar_p = 1/(\beta c^2), \quad \hbar_q = 1/h^2, \quad \hbar_r = -(\tilde{q}^2\tilde{\varepsilon}^2)/c \\ \hbar_s = (\Phi\tilde{z}\sin\nu)/r, \quad \hbar_t = 0, \quad \hbar_u = 0, \quad \hbar_v = 0, \quad \hbar_w = 0, \quad \hbar_x = 0.\end{gathered} \tag{10.5c}$$

Using (10.4a) in (6.20c) leads to

$$\mathcal{X}_1 = 0, \quad \mathcal{X}_2 = 1, \quad \mathcal{X}_3 = -\beta^2, \quad \mathcal{X}_4 = 1 + \beta^2 \tag{10.6a}$$

while using (10.2), (10.3), (10.5) and (10.6a) in (6.20b) yields

$$\begin{gathered}\imath_0 = 0, \quad \imath_1 = 0, \quad \imath_2 = 0, \quad \imath_3 = 0 \\ \imath_4 = 1/\tilde{p}, \quad \imath_5 = (\Phi\tilde{z}\sin\nu)/(4r^3\omega_o^3), \quad \imath_6 = h^{-2}.\end{gathered} \tag{10.6b}$$

By substituting (10.2a), (10.3b), (10.4a), (10.5) and (10.6) into (6.20a) and (6.20d), we obtain

$$\mathfrak{X} = \frac{\widehat{\mathbf{x}}\,(\tilde{z} + \tilde{q}\cos\nu) + \widehat{\mathbf{y}}c - \widehat{\mathbf{z}}\,\tilde{q}\sin\nu}{\beta c^2(1+\beta^2)}, \quad \Theta = -\frac{\tilde{e}\sin\nu}{\tilde{\varepsilon}c(1+\beta^2)}, \quad \dot{\psi} = -\frac{\sigma\tilde{e}\sin\nu}{\tilde{\varepsilon}(1+\beta^2)} \tag{10.7}$$

from which we conclude that dispersion and polarization have no effects on the slope and variation of obliquation for the observer.

Art 45. *On the apparent path of a light source.*

To study the effects of acceleration, dispersion and polarization on the apparent path of the light source, we substitute (10.2), (10.3) and (10.5) into (6.21) to get [16]

$$\begin{aligned}&\varsigma_a = -r\sin\varpi, \quad \varsigma_b = 0, \quad \varsigma_c = -z\sin\nu_o \\ &\varrho_a = q/r^3, \quad \varrho_b = (\tilde{z}\sin\nu)/r, \quad \varrho_c = 0, \quad \varrho_d = q^2(\cos\varpi + \tilde{e}\cos\nu_o - 3\tilde{e}\sin\nu\sin\varpi)/r^3 \\ &\varrho_e = 0, \quad \varrho_f = 0, \quad \varrho_g = 0, \quad \varrho_h = 0, \quad \varrho_i = 0, \quad \varrho_j = 0, \quad \varrho_k = (\tilde{q}\tilde{\varepsilon})/r \\ &\varrho_l = (2\Phi + 15\tilde{z}^2\sin^2\nu - 3\tilde{q}^2\tilde{\varepsilon}^2)/r^2, \quad \varrho_m = q(9\tilde{q}\tilde{\varepsilon}r^2 - 8q - 45r\tilde{z}^2\sin^2\nu)/r^6 \\ &\varrho_n = 15(q\tilde{z}\sin\nu)(2q - 3\tilde{q}\tilde{\varepsilon}r^2 + 7r\tilde{z}^2\sin^2\nu)/r^7, \quad \varrho_o = 0, \quad \varrho_p = 0, \quad \varrho_q = 0 \\ &\varrho_r = 0, \quad \varrho_s = 0, \quad \varrho_t = 0, \quad \varrho_u = 0, \quad \varrho_v = 0, \quad \varrho_w = 0, \quad \varrho_x = 0\end{aligned} \tag{10.8a}$$

$$\begin{aligned}&\mathfrak{x}_a = 0, \quad \mathfrak{x}_b = 0, \quad \mathfrak{x}_c = 0, \quad \mathfrak{x}_d = 0, \quad \mathfrak{x}_e = 0, \quad \mathfrak{x}_f = 0, \quad \mathfrak{x}_g = 0, \quad \mathfrak{x}_h = 0, \quad \mathfrak{x}_i = 0 \\ &\mathfrak{x}_j = 0, \quad \mathfrak{x}_k = \top, \quad \mathfrak{x}_l = \top, \quad \mathfrak{x}_m = \top, \quad \mathfrak{x}_n = 0, \quad \mathfrak{x}_o = 0, \quad \mathfrak{x}_p = 0, \quad \mathfrak{x}_q = 0 \\ &\mathfrak{x}_r = 0, \quad \mathfrak{x}_s = 0, \quad \mathfrak{x}_t = 0, \quad \mathfrak{x}_u = 0, \quad \mathfrak{x}_v = 0 \\ &\mathfrak{y}_a = 0, \quad \mathfrak{y}_b = 0, \quad \mathfrak{y}_c = 0, \quad \mathfrak{y}_d = -1, \quad \mathfrak{y}_e = -1, \quad \mathfrak{y}_f = -1, \quad \mathfrak{y}_g = 0, \quad \mathfrak{y}_h = 0 \\ &\mathfrak{y}_i = 1, \quad \mathfrak{y}_j = q^2(2\Phi - 3\tilde{z}^2\sin^2\nu - 3\tilde{q}^2\tilde{\varepsilon}^2)/r^8, \quad \mathfrak{y}_k = 0, \quad \mathfrak{y}_l = 0, \quad \mathfrak{y}_m = 0 \\ &\mathfrak{y}_n = 0, \quad \mathfrak{y}_o = 0, \quad \mathfrak{y}_p = 0, \quad \mathfrak{y}_q = 0, \quad \mathfrak{y}_r = 0, \quad \mathfrak{y}_s = 0, \quad \mathfrak{y}_t = 1, \quad \mathfrak{y}_u = 0 \\ &\mathfrak{y}_v = 0, \quad \mathfrak{y}_w = 0, \quad \mathfrak{y}_x = 0, \quad \mathfrak{y}_y = \varrho_a^2, \quad \mathfrak{y}_z = qz(\tilde{e} + \cos\nu)\end{aligned} \tag{10.8b}$$

[16] We use $\top$ to denote values that are not needed for the calculations and are therefore not evaluated for convenience.

$$
\begin{aligned}
&\varkappa_a = 0, \quad \varkappa_b = 0, \quad \varkappa_c = 0, \quad \varkappa_d = 0, \quad \varkappa_e = 0, \quad \varkappa_f = -q/r^3, \quad \varkappa_g = 0 \\
&\varkappa_h = -q^2/r^6, \quad \varkappa_i = q/r^2, \quad \varkappa_j = 0, \quad \varkappa_k = -h^2q^2/r^6, \quad \varkappa_l = 0, \quad \varkappa_m = 0 \\
&\varkappa_n = 0, \quad \varkappa_o = -(q^2zh\sin\nu_o)/r^6, \quad \varkappa_p = -(q^2h\sin\varpi)/r^5, \quad \varkappa_q = -(q^2zh\sin\nu)/r^5 \\
&\varkappa_r = 0, \quad \varkappa_s = q^3(\cos\varpi + \tilde{e}\cos\nu_o)/r^6, \quad \varkappa_t = zq^3(\tilde{e} + \cos\nu)/r^6, \quad \varkappa_u = (hq^2)/r^6 \\
&\varkappa_v = -(hq^2)/r^6, \quad \varkappa_w = 0, \quad \varkappa_x = 0, \quad \varkappa_y = 0, \quad \varkappa_z = 0 \\
&\mathfrak{v}_a = 0, \quad \mathfrak{v}_b = 0, \quad \mathfrak{v}_c = 0, \quad \mathfrak{v}_d = 0, \quad \mathfrak{v}_e = 0, \quad \mathfrak{v}_f = -q^2/r^6, \quad \mathfrak{v}_g = 0.
\end{aligned}
\tag{10.8c}
$$

Substituting (10.8) into (6.50) and taking (10.1), (10.2), (10.3) and (10.5) into account, we obtain

$$
\begin{gathered}
\mathbb{K} = 1/\tilde{q}, \quad \mathbb{T} = 0, \quad \boldsymbol{\ell}_t = \widehat{\mathbf{z}}\cos\nu + \widehat{\mathbf{x}}\sin\nu, \quad \boldsymbol{\ell}_b = \widehat{\mathbf{y}} \\
\boldsymbol{\ell}_n = \widehat{\mathbf{x}}\cos\nu - \widehat{\mathbf{z}}\sin\nu, \quad \boldsymbol{\ell}_c = \tilde{q}\boldsymbol{\ell}_n
\end{gathered}
\tag{10.9}
$$

from which we conclude that the apparent path of the light source is a circle with a radius equal in magnitude to the quantity $\tilde{q}$ and lying in a plane parallel to the orbital plane of the observer.

Scholium 10-A. *Interpretation of the true position of a light source.* If we consider the center of curvature of the apparent path of a light source to represent the true position of the light source, it follows from (10.9) that a vector drawn from the apparent position of the light source to its true position does not have the same magnitude as the speed $\tilde{q}\tilde{\varepsilon}$ of the observer. On the other hand, if we consider the true position of a light source to be such that a vector drawn from the apparent position of the light source to its true position necessarily has the same magnitude as the speed of the observer, as Hamilton seemed to have supposed, then it will follow from (10.9) that although the apparent path of the light is an exact circle, the true position of the light source will not be at the center of this circle unless the observer's orbit is exactly circular ($\tilde{\varepsilon} = 1$), a conclusion that was first established by Hamilton [37].

Art 46. *On the apparent geometry of rays.*

To study the apparent geometry of the light rays, we substitute (10.2), (10.3), (10.5) and (10.8) into (6.51) to get

$$
\begin{aligned}
&\mathfrak{K}_a = 0, \quad \mathfrak{K}_b = 0, \quad \mathfrak{K}_c = 0, \quad \mathfrak{K}_d = 0, \quad \mathfrak{K}_e = \tilde{a}/h^2, \quad \mathfrak{K}_f = 0, \quad \mathfrak{K}_g = 0 \\
&\mathfrak{K}_h = -c, \quad \mathfrak{K}_i = 0, \quad \mathfrak{K}_j = 0, \quad \mathfrak{K}_k = \tilde{a}c, \quad \mathfrak{K}_l = 0, \quad \mathfrak{K}_m = 0, \quad \mathfrak{K}_n = -\tilde{a} \\
&\mathfrak{K}_o = 0, \quad \mathfrak{K}_p = -\tilde{a}, \quad \mathfrak{K}_q = 0, \quad \mathfrak{K}_r = 0, \quad \mathfrak{K}_s = 0, \quad \mathfrak{K}_t = 0, \quad \mathfrak{K}_u = -\tilde{a}h^2 \\
&\mathfrak{K}_v = acz\sin\nu, \quad \mathfrak{K}_w = 0, \quad \mathfrak{K}_x = \Phi c, \quad \mathfrak{K}_y = acz\cos\nu, \quad \mathfrak{K}_z = ac\cos\varpi
\end{aligned}
\tag{10.10a}
$$

$$
\begin{gathered}
\mathfrak{H}_a = a\tilde{p}\tilde{q}\sin\varpi, \quad \mathfrak{H}_b = -\Phi a\tilde{p}\tilde{z}\sin\nu_o, \quad \mathfrak{H}_c = -az\tilde{p}\tilde{q}\sin\nu, \quad \mathfrak{H}_d = ac\mathfrak{K}, \quad \mathfrak{H}_e = -h\tilde{a} \\
\mathfrak{H}_f = -hc\tilde{a}, \quad \mathfrak{H}_g = ac\sin\varpi, \quad \mathfrak{H}_h = -\Phi hc, \quad \mathfrak{H}_i = -hc\tilde{a}^2, \quad \mathfrak{H}_j = -\Phi/h^2
\end{gathered}
\tag{10.10b}
$$

where we have introduced the convenient quantities

$$
\tilde{a} = a/r, \quad b = h/c, \quad \mathfrak{K} = \left\{1 + \frac{b^2}{r^2}\right\}^{1/2}.
\tag{10.11}
$$

Substituting (10.10) into (6.60) and taking (10.1), (10.2), (10.3), (10.5) and (10.8) into account, we obtain

$$
\overline{\mathbb{K}} = \frac{\sigma\mathfrak{K}}{c\mathcal{R}^3}, \quad \overline{\mathbb{T}} = -\frac{b}{r^2\mathfrak{K}^2}
\tag{10.12a}
$$

$$
\overline{\boldsymbol{\ell}}_t = \frac{1}{c\mathcal{R}}\Big[\widehat{\mathbf{z}}\,\tilde{q}\sin\nu - \widehat{\mathbf{x}}\,(\tilde{z} + \tilde{q}\cos\nu) - \widehat{\mathbf{y}}\,c\Big], \quad \overline{\boldsymbol{\ell}}_b = \frac{1}{r\mathfrak{K}}\Big[\widehat{\mathbf{z}}\,r\sin\nu - \widehat{\mathbf{x}}\,r\cos\nu + \widehat{\mathbf{y}}\,b\Big]
\tag{10.12b}
$$

where $\mathcal{R}$ is given by (10.4a). We conclude that the light ray has both a curvature and a torsion neither of which is affected by polarization or dispersion.

Example 10-C. *Apparent ray geometry for observers in circular orbits.* When an observer is in a circular orbit, the quantities r and β have constant values by (10.3). The curvature and torsion of the light ray then have constant nonzero values by (10.12a), and we conclude therefore that the light ray is a circular helix. Moreover, since $b > 0$ by (10.11), we have $\overline{\mathbb{T}} < 0$ by (10.12a). We conclude also that the light ray is a left handed helix.

Example 10-D. *Some clouds may be kineoptical illusions.* When we consider a multitude of light rays each of which is in the form of a circular helix, it would seem that when the light source is sufficiently distant from the observer, that these rays may give the illusion of a cloud surrounding the light source. If this be so, then perhaps it may not be entirely without merit to conjecture that such apparent clouds do exist, and that such a cloud can in fact be perceived in popular images of the cat's eye nebula.

Scholium 10-B. *Acceleration-induced light bending is kineoptical.* It is often said by those who ought to have reasoned better that when an observer in accelerated motion perceives a light ray to be curvilinear instead of rectilinear, that the curvature of the ray is indeed due the observer's acceleration, but that since gravitational and nongravitational accelerations are indistiguishable, that the curvature of the ray can be ascribed to gravity. These careless minds spare no effort to assure us that gravity and the curvature of the ray are a manifestation of the geometry of a "spacetime" continuum, nor do they show kind regards towards anyone who dares to voice an objection to their immaculate doctrines, however reasoned such objections may be. It will suffice for us to say that the curvature of the ray is a kineoptical effect due to the acceleration of the observer, and that it makes no difference whatsoever to the phenomena whether this acceleration is caused by the action of gravity or by the action of fairies [17]. From the kineoptical viewpoint, therefore, to ascribe the curvature of the ray in principle to a particular cause of acceleration is to indulge oneself in profound nonsense.

Art 47. *On the apparent frequency of rays.*

To study the apparent ray frequency for the observer, we substitute the value of $\mathcal{R}$ from (10.4a) into (3.30) to get

$$\omega' = \omega_o \sqrt{1 + \beta^2} \tag{10.13}$$

where β is given by (10.3b). We conclude that the apparent ray frequency ω' depends only on the instantaneous speed of the observer and not explicitly on the observer's acceleration.

Example 10-E. *Effects of eccentricity on apparent ray frequency.* Substituting the value of β from (10.3b) into (10.13), we obtain

$$\omega' = \omega_o \sqrt{1 + \frac{\Gamma\Phi}{c^2}}, \quad \Gamma = \frac{qr\tilde{\varepsilon}}{h^2}. \tag{10.14a}$$

Observing now that by (10.3),

$$\tilde{\varepsilon} = \frac{h^2}{q}\left\{\frac{2}{r} - \frac{1 - \tilde{e}^2}{\tilde{p}}\right\}, \tag{10.14b}$$

we substitute this value of $\tilde{\varepsilon}$ into (10.14a) to get

$$\Gamma = 2 - \left\{\frac{1 - \tilde{e}^2}{1 + \tilde{e}\cos\nu}\right\} \tag{10.14c}$$

[17] So long as the dependence of the acceleration on time and position remains the same.

which permits the effects of eccentricity on the apparent frequency of the light ray to be determined.

Example 10-F. *Apparent ray frequency at perihelion.* When the observer is at perihelion, we have $\nu = 0°$ so that by (10.14) and (10.3), the apparent ray frequency satisfies

$$\omega'_p = \omega_o \sqrt{1 + \frac{\Phi_p(1+\tilde{e})}{c^2}}, \quad \Phi_p = \tilde{q}^2(1+\tilde{e}) \tag{10.15}$$

where ω'_p and Φ_p are respectively the values of ω' and Φ when the observer is at perihelion.

Example 10-G. *Apparent ray frequency at aphelion.* When the observer is at aphelion, we have $\nu = 180°$ so that by (10.14) and (10.3), the apparent ray frequency satisfies

$$\omega'_a = \omega_o \sqrt{1 + \frac{\Phi_a(1-\tilde{e})}{c^2}}, \quad \Phi_a = \tilde{q}^2(1-\tilde{e}) \tag{10.16}$$

where ω'_a and Φ_a are respectively the values of ω' and Φ when the observer is at aphelion.

Example 10-H. *Frequency shift for observers in eccentric orbits.* From (10.15) and (10.16), we find that the apparent ray frequencies at aphelion and perihelion differ by a small amount given to a second order accuracy in $\tilde{q}/c$ by

$$\Delta\omega = \left\{\frac{q\tilde{e}}{hc}\right\}^2 \omega_o \tag{10.17}$$

where $\Delta\omega = \omega'_p - \omega'_a$. We conclude therefore that if the frequency of light rays from the cat's eye nebula is measured when the earth is at perihelion and when the earth is at aphelion, the two measurements should differ by the amount given by (10.17), all other things being equal.

A Appendices

A.1 Vector Identities (Algebra)

Let $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ be vectors. Then,

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b}) \tag{A.1}$$

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}) \tag{A.2}$$

$$(\mathbf{a} \times \mathbf{b})^2 + (\mathbf{a} \cdot \mathbf{b})^2 = a^2 b^2 \tag{A.3}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) \tag{A.4}$$

$$(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = \mathbf{c}[\mathbf{d} \cdot (\mathbf{a} \times \mathbf{b})] - \mathbf{d}[\mathbf{c} \cdot (\mathbf{a} \times \mathbf{b})] \tag{A.5}$$

A.2 Vector Identities (Calculus)

Let $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ be vector fields with respect to a vector $\mathbf{u}$. Let ϕ, ψ be scalar fields with respect to $\mathbf{u}$. Let n be an integer. Then,

$$\nabla_u \cdot \mathbf{u} = 3 \tag{A.6}$$

$$\nabla_u \cdot (\nabla_u \times \mathbf{a}) = 0 \tag{A.7}$$

$$\nabla_u \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla_u \times \mathbf{a}) - \mathbf{a} \cdot (\nabla_u \times \mathbf{b}) \tag{A.8}$$

$$\nabla_u \times \mathbf{u} = \mathbf{0} \tag{A.9}$$

$$\nabla_u \times (\nabla_u \phi) = \mathbf{0} \tag{A.10}$$

$$\nabla_u \times (\nabla_u \times \mathbf{a}) = \nabla_u (\nabla_u \cdot \mathbf{a}) - {\nabla_u}^2 \mathbf{a} \tag{A.11}$$

$$\nabla_u \times (\phi \mathbf{a}) = \phi (\nabla_u \times \mathbf{a}) - \mathbf{a} \times (\nabla_u \phi) \tag{A.12}$$

$$\nabla_u \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a}(\nabla_u \cdot \mathbf{b}) - (\mathbf{a} \cdot \nabla_u)\, \mathbf{b} + (\mathbf{b} \cdot \nabla_u)\, \mathbf{a} - \mathbf{b}(\nabla_u \cdot \mathbf{a}) \tag{A.13}$$

$$\nabla_u (u^n) = n u^{n-2} \mathbf{u} \tag{A.14}$$

$$\nabla_u [\phi(\psi)] = (\mathrm{d}\phi / \mathrm{d}\psi)(\nabla_u \psi) \tag{A.15}$$

$$\nabla_u (\phi \psi) = \phi (\nabla_u \psi) + \psi (\nabla_u \phi) \tag{A.16}$$

$$\nabla_u (\mathbf{a} \cdot \mathbf{u}) = \mathbf{a} + (\mathbf{u} \cdot \nabla_u)\, \mathbf{a} + \mathbf{u} \times (\nabla_u \times \mathbf{a}) \tag{A.17}$$

$$a (\nabla_u a) = \mathbf{a} \times (\nabla_u \times \mathbf{a}) + (\mathbf{a} \cdot \nabla_u)\, \mathbf{a} \tag{A.18}$$

$$\nabla_u (\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \times (\nabla_u \times \mathbf{b}) + (\mathbf{a} \cdot \nabla_u)\, \mathbf{b} + (\mathbf{b} \cdot \nabla_u)\, \mathbf{a} + \mathbf{b} \times (\nabla_u \times \mathbf{a}) \tag{A.19}$$

$$|\mathbf{a} \times \mathbf{u}| [\nabla_u |\mathbf{a} \times \mathbf{u}|] = u^2 a (\nabla_u a) + a^2 \mathbf{u} - (\mathbf{a} \cdot \mathbf{u}) \nabla_u (\mathbf{a} \cdot \mathbf{u}) \tag{A.20}$$

$$(\mathbf{a} \cdot \nabla_u)\, \mathbf{u} = \mathbf{a} \tag{A.21}$$

$$(\mathbf{a} \cdot \nabla_u)\, (\phi \mathbf{b}) = \mathbf{b} [\mathbf{a} \cdot (\nabla_u \phi)] + \phi [(\mathbf{a} \cdot \nabla_u)\, \mathbf{b}] \tag{A.22}$$

$$(\mathbf{a} \cdot \nabla_u)\, (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \times [(\mathbf{a} \cdot \nabla_u)\, \mathbf{c}] - \mathbf{c} \times [(\mathbf{a} \cdot \nabla_u)\, \mathbf{b}] \tag{A.23}$$

A.3 Important Riders

For the convenience of the reader, we state here the conditions under which (1.1) was derived [1]. This equation assumes that the strengths or intensities of electric and magnetic fields are characterized by vector quantities, that magnetic and electric fluxes (or charges) and currents are defined and measured in accordance with Hertz's theory, and that in the space region of interest,

- the Gauss-Ostrogradsky divergence theorem holds for the field vectors,
- there is conservation of magnetic and electric fluxes or charges,
- there is a stationary, homogeneous and isotropic medium with no boundaries,
- the light waves are plane, monochromatic, linearly polarized, have a complex frequency with a real wave vector,
- the observer is in accelerated translational, rotational or gravitational motion.

When any or some of these conditions are not satisfied in a space region, the calculations in this monograph may need to be reworked in the appropriate manners.

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