

The Lorentz Electron Theory of Relativity

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This paper traces H. A. Lorentz's work on the electrodynamics of moving bodies from 1887 to 1909. His initial rejection of Michelson's 1881 interferometer experiment and the development of the "electron theory" as a modification and extension of Maxwell's ideas are discussed. The 1892 article in which Lorentz proposes the contraction hypothesis (Lorentz-Fitzgerald contraction) is analyzed, and the manner in which the hypothesis is integrated into the electron theory then, and later in the 1895 *Versuch*, is presented. A discussion of the *Versuch's* introduction of "local time" and the "theorem of corresponding states" follows, and it is then shown that Lorentz introduced *second-order* Lorentz transformations in an 1899 paper. The well-known 1904 paper is analyzed, and is shown to be presenting the latest modifications in transformation equations designed to prove a theorem of corresponding states for *many* electromagnetic phenomena to all orders of v/c . Using Lorentz's 1909 *Theory of Electrons*, it is argued that the Lorentz transformations as understood by Lorentz prior to Einstein's work possessed inverse transformations which entailed length *dilations*, time *contractions*, etc., and which were nonrelativistic and inconsistent with the Lorentz electron theory. Finally a brief discussion of a reason for Einstein's difference of approach is outlined.

In 1887 H. A. Lorentz published an article which surveyed the influence of the earth's motion on optical phenomena.¹ Near the end of the paper, he turned his attention to Michelson's first interferometer experiment which had been performed in 1881.² Michelson believed that the results of this experiment supported Stokes' theory of an aether which was completely dragged along by the earth near the earth's surface.³ Lorentz, disposed to favor Fresnel's theory of the aether⁴ which admitted of only partial aether drag, was able to demonstrate that Michelson had overestimated the expected fringe shift by a factor of 2, and that the corrected estimate fell within the expected error of

the results. In the words of Lorentz, "il reste donc douteux, à mon avis, que l'hypothèse de Fresnel soit réfutée par l'expérience de M. Michelson."⁵

Michelson repeated the experiment in 1887 with the assistance of Morley, and increased the precision of the experiment.⁶ No appreciable fringe shift was detected and the result was clear: unless Fresnel's theory could be modified somehow, it had to be given up.

By 1892 Lorentz had succeeded in developing a theory of electromagnetic phenomena in moving bodies that yielded the Fresnel convection coefficient $[1 - (1/n^2)]$, where n is the absolute index of refraction of the medium in a state of rest.⁷ More will be said concerning this theory, which is in its essentials Lorentz's well-known theory of electrons, in later sections of this paper. Suffice it to say here, that to avoid giving up *not strictly the previously mentioned theory of Fresnel*, but more his own theory of the stagnant aether, Lorentz proposed a bold hypothesis,⁸ which had also been

¹ H. A. Lorentz, "De l'Influence du Mouvement de la Terre sur les Phénomènes Lumineux," Arch. Neerl. **21** (1887); also in H. A. Lorentz, *Collected Papers* [Martinus Nijhoff (Hague, 1937)], Vol. 4, pp. 153-214. Page references to all Lorentz citations are to the *Collected Papers*, except when noted otherwise.

² A. A. Michelson, Amer. J. Sci. **22**, 120 (1881). The Michelson and the later Michelson-Morley experiments have been the subject of two comprehensive studies in recent years. R. S. Shankland's "Michelson-Morley Experiment," Amer. J. Phys. **32**, 16 (1964), is an excellent piece of analysis. For a longer study which incorporates some additional theory, see L. Swenson, "The Ethereal Aether: A Descriptive History of the Michelson-Morley Aether Drift Experiments 1880-1930" (unpublished Ph.D. dissertation, Claremont University, 1962).

³ G. Stokes, Phil. Mag. **27**, 9 (1845).

⁴ A. Fresnel, Ann. Chim. **9**, 57 (1818). Also in A. Fresnel, *Oeuvres* (de Senarmont, Paris, 1866-70), Vol. 2, 627.

⁵ Lorentz, Ref. 3, p. 214.

⁶ A. Michelson and E. W. Morley, Amer. J. Sci. **34**, 333 (1887).

⁷ H. A. Lorentz, "La Théorie Électromagnétique de Maxwell et Son Application aux Corps Mouvements," Arch. Neerl. **25**, 363 (1892); also in *Collected Papers*, Vol. 2, p. 164. See especially p. 319 of this work for the discussion of the Fresnel convection coefficient.

⁸ H. A. Lorentz, "De relatieve beweging van de aarde en den aether," Versl. K. Akad. W. Amsterdam **1**, 74 (1892); also in English in *Collected Papers*, Vol. 4, pp. 219-223.

put forward by Fitzgerald.⁹ (Lorentz, however, still spoke of the stagnant aether as Fresnel's theory.) In stating the contraction hypothesis, Lorentz wrote as follows:

This (Michelson–Morley) experiment has been puzzling me for a long time, and in the end I have been able to think of only one means of reconciling its result with Fresnel's theory. It consists in the supposition that the line joining two points of a solid body, if at first parallel to the direction of the earth's motion, does not keep the same length when it is subsequently turned through 90°. If for example, its length be l in the latter position and $l(1-\alpha)$ in the former, the expression (1) must be multiplied by $(1-\alpha)$. Neglecting $\alpha p^2/V^2$ this gives

$$2(l/V)[1+(p^2/V^2)-\alpha]$$

The difference between this expression and (2), and with it the whole difficulty, would disappear if α were equal to $p^2/2V^2$.

Now, some such changes in the length of the arms in Michelson's first experiment and in the dimensions of the slab in the second one is so far as I can see, not inconceivable.¹⁰

Here p is the velocity of the earth through the aether, and V is the velocity of light.

This Lorentz–Fitzgerald contraction (hereafter referred to as the L–F contraction) has often been misinterpreted by physicists, historians of science, and philosophers of science. It has been characterized as the paradigm of an *ad hoc* hypothesis¹¹ (which it is, but in a largely *unnoticed* sense) and has been considered as explicable by the theory of relativity¹² (which it most decidedly is not). Most of these misinterpretations have rested on a lack of acquaintance with Lorentz's own writings, and the papers of various physicists who performed experimental work around the turn of the twentieth century.

⁹ See A. M. Bork, "The 'Fitzgerald' Contraction," *Isis* **57**, 199–207 (1966), for a discussion of the independence claims of Lorentz and Fitzgerald on the contraction hypothesis. Also see S. G. Brush's, "Note on the History of the Fitzgerald–Lorentz Contraction," *Isis* **58**, 230–232 (1966), for evidence that Fitzgerald published a letter in 1889 suggesting the contraction hypothesis.

¹⁰ Lorentz, "Relative Motion. . .," p. 221. (See Ref. 8).

¹¹ See for example, F. K. Richtmyer, E. H. Kennard, and T. Lauritsen, *Introduction to Modern Physics* (McGraw-Hill Book Co., New York, 1955), 5th ed., p. 56.

¹² See for example, E. Whittaker, *A History of the Theories of Aether and Electricity*, Vol. II: *The Modern Theories* (Harper Torchbooks, New York, 1960), p. 37.

It is the purpose of this paper to reveal the evolving role which the contraction hypothesis played in Lorentz's electron theory, and to indicate the type of support which it enjoyed within that theory. It will become evident that the contraction hypothesis was associated with a transformation of variables between aether rest frame and moving frame, and that moreover, other transformations were later introduced for similar reasons, which have a form which is the same as the transformations of relativity theory, but with a vastly different meaning.

THE PLAUSIBILITY OF THE L–F CONTRACTION

Immediately after proposing the contraction of the interferometer arm, Lorentz goes on to state:

What determines the size and shape of a solid body? Evidently the intensity of the molecular forces; any cause which would alter the latter would also influence the shape and dimensions. Nowadays we may safely assume that electric and magnetic forces act by means of the intervention of the ether. It is not far-fetched to suppose the same to be true of the molecular forces. But then it may make all the difference whether the line joining two material particles shifting together through the ether, lies parallel or crosswise to the direction of that shift. It is easily seen that an influence of the order of p/V is not to be expected, but an influence of the order of p^2/V^2 is not excluded and that is precisely what we need.

Since the nature of the molecular forces is entirely unknown to us, it is impossible to test the hypothesis. We can only calculate—with the aid of more or less plausible suppositions, of course—the influence of the motion of ponderable matter on electric and magnetic forces. It may be worth mentioning that the result obtained in the case of electric forces yields, when applied to molecular forces, exactly the value given above for α .

Let A be a system of material points carrying certain electric charges and at rest with respect to the ether; B the system of the same points while moving in the direction of the x -axis with the common velocity p through the ether. From the equations developed by me, one can deduce which forces the particle in system B exert on one another. The simplest way to do this, is to introduce still a third system C , which just as A , is at rest but differs from the latter as regards the location of the points. System C , namely, can be obtained from system A by a simple extension by which all dimensions in the direction of the x -axis are multiplied by the factor $(1+p^2/2V^2)$ and all dimensions perpendicular to it remain unaltered.

Now the connection between the forces in B and in C amounts to this, that the x -components in C are equal to those in B whereas the components at right angles to the x -axis are $1+p^2/2V^2$ times larger than in B .

We will apply this to molecular forces. Let us imagine a solid body to be a system of material points kept in equilibrium by their mutual attractions and repulsions and let system B represent such a body whilst moving through the ether. The forces acting on any of the material points of B must in that case neutralize. From the above, it follows that the same can not then be the case for system A whereas for system C it can; for even though the transition from B to C is accompanied by a change in all forces at right angles to the axis, this cannot disturb the equilibrium, because they are all changed in the same proportion. In this way it appears that if B represents the state of equilibrium of the body during a shift through the ether then C must be the state of equilibrium when there is no shift. But the dimensions of B in the direction of the x axis are $(1-p^2/2V^2)$ times the corresponding dimensions of C whereas the dimensions along directions at right angles to the x -axis are the same in both systems. One obtains, therefore, exactly an influence of the motion on the dimensions equal to the one which, as appeared above, is required to explain Michelson's experiment.¹³

It is important to note that the *ad hoc* aspect of the contraction hypothesis consists in the extension of the transformation characteristics of electric forces to molecular forces, and not simply in the contraction in the length of the moving body.

It will be useful to show how the contraction spoken of in connection with the electric and magnetic forces is a consequence of Lorentz's electron theory. The reference to the "equations developed by me" is to the 1892 paper on Maxwell's theory.

LORENTZ'S ELECTRON THEORY OF 1892

Let us begin by recounting briefly what Lorentz was able to establish in his 1892 monograph. The scope of this paper will not permit any exhaustive account of the full Lorentz theory, but the discussion will serve to introduce as much of the theory of electrons as is necessary to indicate that Lorentz was not particularly concerned with contraction effects in that paper. Then it is indicated how he reformulated his approach to

obtain the plausibility of the contraction in his better known *Versuch* of 1895.^{14,15}

Lorentz does not introduce the reader to his theory of electrons in the 1892 article until the fourth chapter. The earlier chapters are concerned with Maxwell's contributions, the application of D'Alembert's principle to electromagnetic theory, and some of Hertz's work and Lorentz's criticisms.

Essentially, Lorentz fused Maxwell's equations describing electrical and magnetic phenomena with a hypothesis of electroatomism, which he explicitly admitted he borrowed from Weber and Clausius.¹⁶ Lorentz supposed an aether that was completely at rest and which pervaded all matter—even into the interior of molecules and electrons or "Ions." Though believing in the existence of an aether which was the absolute frame of references and the seat of the \mathbf{d} and \mathbf{h} vectors, Lorentz did not espouse the churning aether of the English school—a notion which culminated in Joseph Larmor's aether model presented in his *Aether and Matter*.¹⁷ Lorentz accepted Maxwell's equations in their simplified Hertz-Heaviside form as the

¹⁴ A. Lorentz, *Versuch einer Theorie der Elektrischen und Optischen Erscheinungen in Bewegten Körpern* (E. J. Brill, Leiden, 1895); also in *Collected Papers*, Vol. 5, pp. 1–138.

¹⁵ Secondary sources on Lorentz's electron theory are not very extensive. The reader can refer to M. Born's *Einstein's Theory of Relativity* (Dover Publications, Inc., New York, 1962), pp. 199–207 and 218–224. Whittaker is an accessible source—cf. Vol. 1 of his *History . . .*, Chap. 13, and also Vol. 2, Chap. 2, but must be read with caution as he tends to conflate later ideas with earlier ones. The accounts of A. D. Fokker and H. B. G. Casimir in *H. A. Lorentz: Impressions of His Life and Work*, G. L. de Haas-Lorentz, Ed. (North-Holland Publ. Co., Amsterdam, 1957), are overly brief. An essay by P. Ehrenfest: "Prof. H. A. Lorentz as Researcher" included in *Paul Ehrenfest Collected Scientific Papers*, M. J. Klein, Ed. (North-Holland Publ. Co., Amsterdam, 1959), pp. 471–478 is helpful though elementary. The best account^{15a} is given by L. Silberstein, *The Theory of Relativity* (MacMillan and Co., Ltd., London, 1914), Chaps. 2 and 3.

¹⁵ (a) (Note added in revised version.) An excellent study of Lorentz's theory after 1892 came to my attention after this paper was completed. This is T. Hirose, "Electrodynamics before the Theory of Relativity, 1890–1905," *Japanese Studies History Sci.* **5**, 1 (1966).

¹⁶ Cf. Lorentz, "La Théorie Électromagnétique . . .," p. 229.

¹⁷ J. Larmor, *Aether and Matter* (Cambridge University Press, Cambridge, England, 1900).

¹³ Lorentz, "Relative Motion . . .," pp. 222–223.

correct equations for the “free”—or charge free aether. These are obtainable from the more general equations which hold for space containing charge by putting the charge density, $\rho=0$. These general equations as given in Lorentz’s 1892 paper are¹⁸:

Lorentz’s 1892 Form	Lorentz’s Later and Contemporary (Except for Units) Form
	$\mathbf{F} = \rho(4\pi V^2 \mathbf{d} + \mathbf{v} \times \mathbf{h})$
$\mathbf{X} = 4\pi V^2 \int \rho f d\tau + \eta \int \rho \gamma d\tau - \zeta \int \rho \beta d\tau$	(in which:
$\mathbf{Y} = 4\pi V^2 \int \rho g d\tau + \zeta \int \rho \alpha d\tau - \xi \int \rho \gamma d\tau$	$\mathbf{F} = \lim_{\tau \rightarrow 0} \frac{\mathbf{X} + \mathbf{Y} + \mathbf{Z}}{\tau}$
$\mathbf{Z} = 4\pi V^2 \int \rho h d\tau + \xi \int \rho \beta d\tau - \eta \int \rho \alpha d\tau \quad (1)$	where τ is a volume element and
$(\partial f / \partial x) + (\partial g / \partial y) + (\partial h / \partial z) = \rho \quad (2)$	$\mathbf{v} = \xi \mathbf{i} + \eta \mathbf{j} + \zeta \mathbf{k}$
$(\partial \alpha / \partial x) + (\partial \beta / \partial y) + (\partial \gamma / \partial z) = 0 \quad (3)$	$\mathbf{d} = f \mathbf{i} + g \mathbf{j} + h \mathbf{k}$
$(\partial \gamma / \partial y) - (\partial \beta / \partial z) = 4\pi [\rho \xi + (\partial f / \partial t)]$	$\mathbf{h} = \alpha \mathbf{i} + \beta \mathbf{j} + \gamma \mathbf{k}.) \quad (1)$
$(\partial \alpha / \partial z) - (\partial \gamma / \partial x) = 4\pi [\rho \eta + (\partial g / \partial t)]$	$\text{div } \mathbf{d} = \rho \quad (2)$
$(\partial \beta / \partial x) - (\partial \alpha / \partial y) = 4\pi [\rho \zeta + (\partial h / \partial t)] \quad (4)$	$\text{div } \mathbf{h} = 0 \quad (3)$
$4\pi V^2 [(\partial g / \partial z) - (\partial h / \partial y)] = \partial \alpha / \partial t$	$\text{curl } \mathbf{h} = 4\pi [\rho \mathbf{v} + (\partial \mathbf{d} / \partial t)] \quad (4)$
$4\pi V^2 [(\partial h / \partial x) - (\partial f / \partial z)] = \partial \beta / \partial t$	$4\pi V^2 \text{curl } \mathbf{d} = -\partial \mathbf{h} / \partial t \quad (5)$
$4\pi V^2 [(\partial f / \partial y) - (\partial g / \partial x)] = \partial \gamma / \partial t \quad (5)$	

Here \mathbf{XYZ} are the force components with which the aether acts on a charged particle; f, g, h are components of the dielectric displacement; α, β, γ , components of the magnetic force; ξ, η, ζ , the velocity components of a moving charge; V , the velocity of light in empty space; and ρ , the electric charge density. These are the equations for ponderable bodies at rest in the aether.

In the succeeding chapters, Lorentz used these equations to obtain Maxwell’s equations for ponderable bodies by a process of integrating over what have been termed “physically infinitesimal regions.”¹⁹ Lorentz also applied the equations to the propagation of light in dielectrics that are at rest and in motion. In the course of developing

these applications, he sometimes found it convenient to introduce “auxiliary functions” which are analogous to the more familiar forms of the scalar and vector potentials which appear in later formulations of his theory.

Specifically I wish to refer to the point where he developed the solution of an auxiliary function χ which is defined (component-wise) by²⁰

$$\square \chi_1 = \rho_0 \mathbf{x} \quad \square \chi_2 = \rho_0 \mathbf{y} \quad \square \chi_3 = \rho_0 \mathbf{z},$$

where \square is understood as short for $V^2 \nabla^2 - [(\partial / \partial t) - \rho(\partial / \partial x)]^2$, in which V is the velocity of light and p the velocity of the moving system. \mathbf{x}, \mathbf{y} , and \mathbf{z} are small displacements of a particle in the moving matter. Lorentz also introduced an S' as an abbreviation of $(\partial \chi_1 / \partial x) + (\partial \chi_2 / \partial y) + (\partial \chi_3 / \partial z)$. A scalar potential ω is defined by $\square \omega = \rho_0$,

¹⁸ Lorentz, “La Theorie Électromagnétique . . .,” p. 246.

¹⁹ Cf. Lorentz, Ref. 18, p. 252 f and also L. Rosenfeld *Theory of Electrons* (Dover Publications, Inc., New York, 1965), pp. 15–16, for a more recent treatment.

²⁰ Cf. Lorentz, Ref. 18, pp. 297 ff, for this discussion.

where ρ_0 is the charge density in the moving system. ρ_0 for Lorentz is time-independent.

With the aid of these auxiliary functions, Lorentz indicated that the electric and magnetic forces were determinable. For example,

$$f = (V^2 - p^2) [\partial(\omega - S')/\partial x] + (\partial^2 \chi_1 / \partial t^2) - p [(\partial^2 \chi_1 / \partial x \partial t) + (\partial S' / \partial t)].$$

Lorentz also notes that since ρ_0 is independent of time, the equation for the auxiliary function ω becomes:

$$\{(V^2 - p^2)(\partial^2 / \partial x^2) + V^2[(\partial^2 / \partial y^2) + (\partial^2 / \partial z^2)]\} \omega = \rho_0.$$

This last equation is most important in later arguments (i.e., in the 1895 *Versuch*), and though it is introduced here, it is not yet exploited in connection with the L-F contraction.

In solving for the function x , Lorentz found it convenient to introduce new variables r and t' , where

$$r = [V / (V^2 - p^2)^{1/2}] x, \quad \text{and} \quad t' = t - (\epsilon / V) r,$$

in which $\epsilon = p / (V^2 - p^2)^{1/2}$. The substitution of variables in this monograph is a mathematical device used to change the differential equation into Poisson's form, which admits of a rather simple solution. Mention is made neither of a contraction effect on a body moving through the aether, nor of a time parameter which is at all different from a normal universal Newtonian time, though Lorentz does treat the t' introduced above as a time parameter of a "retarded" function.

On August 18, 1892 Lorentz wrote to Lord Rayleigh as follows:

I have read this note with much interest (this was Rayleigh's article on "Aberration" in the March 24, 1892 issue of *Nature*) and I gather from it that we agree completely as to the position of the case. Fresnel's hypothesis, taken conjointly with his coefficient $1 - 1/n^2$, would serve admirably to account for all the observed phenomena were it not for the interferential experiment of Mr. Michelson, which has, as you know, been repeated after I published my remarks on its original form, and which seems decidedly to contradict Fresnel's views. I am totally at a loss to clear away this contradiction, and yet I believe if we were to abandon Fresnel's theory, we should have no adequate theory at all, the conditions which Mr. Stokes has imposed on the movement of the ether being irreconcilable to each other.

Can there be some point in the theory of Mr.

Michelson's experiment which has as yet been overlooked?

Lorentz concludes the letter by mentioning that he has endeavored:

To apply the electromagnetic theory to a body which moves through the ether without dragging this medium along with it; my paper is now under the press and I hope, in a few weeks, to be able to offer you a copy of it. Assuming an approach which may appear somewhat startling but which may, as I think, serve as a working hypothesis, I have found the right value $1 - (1/n)^2$ for F[resnel]'s coefficient. I hope to apply to some other problems the equations obtained, as for Fizeau's experiment on the rotation of the plane of polarization by a pack of glass plates.²¹

The "somewhat startling" approach is the hypothesis of electroatomism, and the "equations" referred to were cited above.

It should be clear that at this point Lorentz was not aware that his 1892 theory could account in any way for the null result of the Michelson-Morley experiment. But it was not long after that, in the issue of the Dutch periodical *Verslag van de Vergaderingen der Akademie van Wetenschappen Natuurkunde* dated 26 November 1892, that Lorentz first proposed the contraction hypothesis. I have quoted Lorentz at sufficient length to indicate that he proposed an explanation of the contraction hypothesis that required that intermolecular forces transform in the same way as the electric forces. I turn now to Lorentz's 1895 *Versuch* and to his rather formal "derivation" of the transformation equations for the electric forces.

LORENTZ'S VERSUCH EINER THEORIE DER ELECTRISCHEN UND OPTISCHEN ERSCHEINUNGEN IN BEWEGTEN KÖRPERN

The account of the theory of electrons which appeared in this work is both mathematically

²¹ The first section of this letter, as quoted, appears in R. J. Strutt's *John William Strutt: Third Baron Rayleigh* (Edward Arnold & Co., London, 1924), p. 346. The second section is taken from a microfilm copy made by S. G. Brush of Lorentz's original letter in the Algemeen Rijksarchief at the Hague. A copy of this microfilm is on deposit at the Center of History and Philosophy of Physics of the American Institute of Physics in New York, N. Y. I should like to thank Dr. Charles Weiner, Director, and Mrs. Joan N. Warnow, librarian, for their assistance with bibliographic research on this project.

simpler, because of the use of the vector calculus, and more developed. In this book which first appeared in 1895 and which soon became a classic, Lorentz introduced his notion of a "local time," which he utilized to obtain a "theorem of corresponding states." This theorem indicates that the earth's motion through the aether will have no first-order effect whatever on experiments using terrestrial light sources. Second-order effects still constituted a problem, however, and the Michelson-Morley experiment was relegated to a chapter with the title "Experiments whose results cannot be explained without additions (to the theory)." Again, as in the 1892 paper first proposing the contraction hypothesis, Lorentz asserts that "we are led to (the contraction hypothesis) . . . if we, *firstly*, without taking molecular movement into consideration, assume that in a solid body left to itself the forces, attractions or repulsions acting upon any molecule, maintain one another in equilibrium, and, *secondly*—though to be sure there is no reason for doing so—if we apply to these molecular forces the law which in another place we deduced for electrostatic actions."²²

The transformation law for electrostatic forces is primarily developed in Sec. 23 of the *Versuch*, though reference to the fundamental equations of the theory of electrons is of course supposed.

The equation for the ponderomotive force which the aether exerts on a charged particle is given as²³:

$$\mathbf{F} = 4\pi c^2 \mathbf{d} + (\mathbf{v} \times \mathbf{h}) + (\mathbf{p} \times \mathbf{h}),$$

where \mathbf{d} is the dielectric displacement, \mathbf{h} the magnetic force, c the velocity of light, \mathbf{v} the velocity of the moving body, and \mathbf{p} the velocity of the charged particle. Lorentz shows since \mathbf{d} and \mathbf{h} can be defined as functions of ω , v , and ρ , that the force equation becomes (componentwise):

$$F_x = 4\pi(c^2 - v^2)(\partial\omega/\partial x), \text{ etc.}$$

In a reference system in which the moving body is at rest, this equation becomes

$$F_x' = 4\pi c^2(\partial\omega'/\partial x').$$

It is clear that to get a comparison of F and F' ,

²² Lorentz, *Versuch*, p. 123. The English translation is based on W. Perrett and G. B. Jeffery's translated version of 89-92 of the *Versuch* in the well-known anthology *The Principles of Relativity* (Dover Publications, Inc., New York, 1923), pp. 6-7.

²³ Lorentz, *Versuch*, p. 34.

one must determine the relation between ω and ω' . Lorentz does this as follows:

In order to clarify the meaning of the above formulas, we will compare the aforementioned system S_1 with a second system S_2 . The latter shall *not* be supposed to move, and shall be obtained from an enlargement of all the dimensions of S_1 which are oriented along the x -axis (this holds as well for the x -axis dimension of the electrons [ions]), in the ratio of $(c^2 - v^2)^{1/2}$ to v , or: between the coordinates of x, y, z of a point of S_1 , and the coordinates x', y', z' , of the corresponding point of S_2 , we will postulate the relations:

$$x = x'[1 - (v^2/c^2)]^{1/2} \quad y = y' \quad z = z'.$$

In addition, the elements of volume corresponding to one another, and according the electrons (ions), in S_1 and S_2 should have the same charges.

If we distinguish all quantities that belong to the second system by priming them, then

$$\rho' = \rho[1 - (v^2/c^2)]^{1/2},$$

and

$$\begin{aligned} (\partial^2\omega'/\partial x'^2) + (\partial^2\omega'/\partial y'^2) + (\partial^2\omega'/\partial z'^2) &= \rho' \\ &= \rho[1 - (v^2/c^2)]^{1/2}. \end{aligned}$$

Since the Eq. (23) [which is essentially the 3rd equation given on p. 9 of this (Lorentz, *Versuch*) article (on page 503, Schaffner's article) only in the form:

$$\left(1 - \frac{v^2}{c^2} \right) \frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2} + \frac{\partial^2\omega}{\partial z^2} = \rho, \quad]$$

may be written in the form:

$$\frac{\partial^2\omega}{\partial x'^2} + \frac{\partial^2\omega}{\partial y'^2} + \frac{\partial^2\omega}{\partial z'^2} = \rho.$$

Therefore we have,

$$\omega = \omega'/(1 - v^2/c^2)^{1/2},$$

and since we have in the second system:

$$F_x' = 4\pi c^2(\partial\omega'/\partial x'),$$

we then have:

$$\begin{aligned} F_x &= F_x', & F_y &= F_y'[1 - (v^2/c^2)]^{1/2} \\ F_z &= F_z'[1 - (v^2/c^2)]^{1/2}. \end{aligned}$$

The same relations as hold between the components of F and F' , since the charges in S_1 and S_2 are equal, also hold between the force components, which in both cases act on an electron (ion).

If in the second system at a certain point, $F = 0$, then F disappears in the corresponding place in the first system.²⁴

What Lorentz has done here is to use a transformation which is similar to the transformations

²⁴ Lorentz, *Versuch*, pp. 36-38.

used in his 1892 paper—but to which he now gives a *physical* significance. The spatial transformation is now more than a mathematical change of variable. Assuming that intermolecular forces, which determine the shape of a body, transform in the same manner as the electrical forces, Lorentz has shown that they will be in equilibrium only if the body shrinks in the proportion $[1 - (v^2/c^2)]^{1/2}$ to 1. [This assumes as Lorentz pointed out later (1904) that there is but *one* configuration of equilibrium for any given absolute velocity.]

It is most important to note that the spatial dimensions of S_2 —the system at rest in the aether—are considered to be *enlarged* by the factor $[1 - (v^2/c^2)]^{1/2}$ in comparison with the moving system. It is this *expansion* effect to which we will refer later in showing that Lorentz's interpretation not only of the contraction hypothesis and the "local" time hypothesis, but also of *any* transformation equation (e.g., force, mass, charge density) relating moving and rest systems is a nonreciprocal interpretation and quite antithetical to a theory of relativity.^{24a}

Lorentz used these force transformation equations derived in Sec. 23 and extended by hypothesis to intermolecular forces to do more than just account for the null result of the Michelson-Morley experiment. Lorentz argued that the Michelson-Morley result could be accounted for assuming a change in the arm perpendicular to the direction of motion from 1 to $1 + \epsilon$, and/or a corresponding change in the parallel arm from 1 to $1 + \delta$, in such a manner that

$$\epsilon - \delta = v^2/2c^2.$$

Thus a traverse expansion, *or* a longitudinal contraction, *or* some combination such that the above equation is fulfilled, is required. In Sec. 92 of the *Versuch* on the basis of the argument developed in Sec. 23 as quoted above, Lorentz decided on a purely longitudinal contraction.

^{24a} Some of the differences in meaning between Lorentz's and Einstein's transformations have been discussed by A. D'Abro in his *The Evolution of Scientific Thought* (Dover Publications, Inc., New York, 1927). Recently K. R. Popper commented on the difference in symmetry between the L-F and Einstein contractions, but did not touch on the more general point that such "symmetry" must apply to *all* transformations. See Popper's note in B.J.P.S. **16**, 332 (1966).

Accordingly we see that an extension of the theory of electrons, i.e., making an assumption about the behavior of intermolecular forces, results in the proposal of a purely *longitudinal* contraction—something which the Michelson-Morley experiment did not expressly require.

As noted above, the contraction hypothesis was invoked to account for the second-order null result of the interferometer experiment. But any account of Lorentz's theory of 1895 would be seriously incomplete without a discussion of his notion of "local time" and its relation to the "theorem of corresponding states." These ideas are introduced in the *Versuch* to explain null result experiments of the first order, and Lorentz first introduces the idea of local time in connection with his Chap. V on optical phenomena in bodies in motion with respect to the aether. Local time is the independent time variable in a moving system, and it is *not* equal to the time t in the rest system, but is rather related to it by $t' = t - (1/c^2)(v_x x + v_y y + v_z z)$.²⁵ With the aid of this new time variable Lorentz was able to prove his general *theorem of corresponding states*:

If, for a system of bodies at rest, a state of things is known where

$$P_x, P_y, P_z, E_x, E_y, E_z, H_x, H_y, H_z$$

are certain functions of x, y, z , and t , then in this same system, provided it moves with a velocity v , a state of things can exist in which:

$$P_x', P_y', P_z', E_x, E_y, E_z, H_x', H_y', H_z'$$

are the same functions of x', y', z' , and t' [i.e.

$$t - (1/c^2)(v_x x + v_y y + v_z z)]^{26}$$

Here Lorentz understands the P to represent the dielectric displacement, the E the electric force, and the H the magnetic force. Silberstein has suggested that this theorem of corresponding states was "to a great extent . . . the germ of modern relativistic tendencies."²⁷ It is interesting to note that Lorentz used the theorem to account for the results of stellar aberration experiments using both air and water telescopes. Lorentz also

²⁵ Lorentz, *Versuch*, p. 81.

²⁶ Lorentz, *Versuch*, p. 84.

²⁷ L. Silberstein, *The Theory of Relativity* (MacMillan and Co., Ltd., London, 1914), p. 67.

emphasized that according to this theorem, the earth's motion through the aether will have no first-order effect whatever on experiments using terrestrial light sources: "*Überhaupt wird nach unserer Theorie die Bewegung der Erde nie einen Einfluss erster Ordnung auf Versuche mit terrestrischen Lichtquellen haben*"²⁸ (Lorentz's italics).

THE SIMPLIFIED THEORY OF 1899

Several years later in 1899 Lorentz published a paper in which he proceeded to generalize the approach which he had introduced in the *Versuch*. In the 1895 work, though he had applied the local time hypothesis as a general transformation variable, he had only introduced the spatial transformation in a rather restricted way—i.e., in connection with his electrostatic and intermolecular force transformation equations. The x' which appeared in his theorem of corresponding states was not the x' which appeared in the force transformation derivation of Sec. 23 of the *Versuch*. In the 1899 paper which he titled "Simplified Theory of Electrical and Optical Phenomena in Moving Systems,"²⁹ Lorentz stated:

I shall now show that the theory may be still further simplified if the fundamental equations are immediately transformed in an appropriate manner.³⁰

Lorentz proceeded to introduce the transformations in two steps. First he referred the equations of the theory of electrons to a moving system in accordance with the standard Galilean transformation $x' = x - vt$. He then asserted that "in order to simplify the equations, the following quantities may be taken as independent variables

$$x' = [c/(c^2 - v^2)]^{1/2}x, \quad y' = y, \quad z' = z, \\ t' = t - [v/(c^2 - v^2)]x.$$

The last of these is the . . . *local time* . . ."

Finally introducing the expressions for the electric and magnetic force referred to the moving system, Lorentz obtained his equations for a

moving system. He then applied these to electrostatic phenomena and to optical phenomena, and reasserted, having demonstrated it in a simpler way, his theorem of corresponding states in the following form:

If, in a body or a system of bodies, without a translation, a system of vibrations be given, in which the displacements of the ions and the components of F' and H' are certain functions of the coordinates and the time, then, if a translation be given to the system, there can exist vibrations, in which the displacements and the components of F' and H' are the same functions of the coordinates and the *local* time. This is the theorem, to which I have been led in a much more troublesome way in my "Versuch einer Theorie, etc.," and by which most of the phenomena, belonging to the theory of aberration may be explained.³¹

Here F' is the electric force, and H' the magnetic force. (For a body without translation, that is, in which $v = 0$ where, v is the velocity with respect to the aether, $F' = F$ and $H' = H$.)³²

In Sec. 9 of this 1899 paper, Lorentz speculated for the first time about extending his theorem of corresponding states to the second order of v/c . His initial motivation was Michelson's interferometer experiment, but in a modified form. Lorentz wrote:

Some time ago, M. Liénard has emitted the opinion that according to my theory, the experiment should have a positive result, if it were modified in so far, that the rays had to pass through a solid or a liquid dielectric.

It is impossible to say with certainty what would be observed in such a case. . . . In what follows I shall shew not that the results of the experiment must necessarily be negative but that this might well be the case. At the same time it will appear what would be the theoretical meaning of such a result.³³

At this point Lorentz introduced transformation equations which he claimed would hold good for second-order quantities of v/c . Lorentz suggested:

³¹ See Ref. 30, p. 437.

³² These equations are as follows:

$$F'_x = 4\pi c^2 d_x, \quad F'_y = 4\pi k c^2 d_y - kvH_z, \quad F'_z = 4\pi k c^2 d_z + kvH_y \\ H'_x = kH_x, \quad H'_y = k^2 H_y + 4\pi k^2 v d_x, \quad H'_z = k^2 H_z - 4\pi k^2 v d_y$$

where d is the dielectric displacement, c the velocity of light, v the absolute velocity of the reference system through the aether, and $k = (1 - v^2/c^2)^{-1/2}$. Cf. Lorentz, Ref. 30, p. 429.

³³ See Ref. 30, p. 438.

²⁸ Lorentz, *Versuch*, p. 90.

²⁹ H. A. Lorentz, "Simplified Theory of Electrical and Optical Phenomena in Moving Systems," Koninkl. Akad. Wetenschap. Proc. [(English ed.) 1, 427 (1899)]. There is a French version in *Collected Papers*, V, p. 139. References here are to the English version.

³⁰ Lorentz, "Simplified Theory . . .," p. 427.

Let ϵ be an indeterminate coefficient, differing from unity by a quantity of the order v^2/c^2 , and let us put

$$x = (\epsilon/k)x'', \quad y = \epsilon y'', \quad z = \epsilon z'', \dots \quad (6)$$

and

$$t' = k\epsilon t'', \dots \quad (9)$$

so that t'' is a modified local time....³⁴

The double-prime notation is introduced to distinguish these equations from those presented earlier. The transformation equations for x and t are now in their famous 1904 form. These modified transformations allowed Lorentz to claim that:

The transformation of which I have now spoken, is precisely such a one as is required in my explication of Michelson's experiment. In this explication the factor ϵ may be left indeterminate. We need hardly remark that for the real transformation produced by a translatory motion, the factor should have a definite value. I see, however, no means to determine it.³⁵

Lorentz also speculated in this article on the effect of vibratory movement and translatory motion on the masses of the ions (electrons). Such an effect was required on the basis of his "second-order" transformation equations for electric and molecular forces. Lorentz noted that:

...states of motion, related to each other in the way we have indicated, will only be possible, if in the transformation of S_0 (the rest system) into S (the moving system) the masses of the ions change; even, this must take place in such a way that the same ion will have different masses for vibrations parallel and perpendicular to the velocity of translation.

Such a hypothesis seems very startling at first sight. Nevertheless we need not wholly reject it. Indeed as is well known, the *effective* mass of an ion depends on what goes on in the ether; it may therefore very well be altered by a translation and even to different degrees for vibrations of different directions.

³⁴ See Ref. 30, p. 439. Note that by algebraic substitution for the value of k , and by simplification after referring the motion of the moving system, as Lorentz did implicitly, to the stationary set of axes, one obtains:

$$x'' = \epsilon^{-1}[x - vt / (1 - v^2/c^2)^{1/2}], \quad y'' = \epsilon^{-1}y, \quad z'' = \epsilon^{-1}z,$$

and

$$t'' = \epsilon^{-1}[(t - vx/c^2) / (1 - v^2/c^2)^{1/2}].$$

³⁵ See Ref. 30, p. 440.

If the hypothesis may be taken for granted, Michelson's experiment should always give a negative result, whatever transparent media were placed on the path of the rays of light, and even if one of these went through air, and the other, say, glass.³⁶

EXPERIMENTS TO DETECT THE L-F CONTRACTION—THE INFLUENCE OF EXPERIMENT ON THE LORENTZ ELECTRON THEORY

Lorentz's electron theory with its new hypothesis of local time seemed to offer a fairly good explanation of most of the known electrical and optical phenomena associated with moving bodies. The problem of aberration phenomena for the *first* order of v/c was viewed as totally unproblematical.³⁷ Lorentz's account of the lack of positive aberration effects of the second order, however, was considered somewhat doubtful, and, as Lorentz himself had pointed out many times, somewhat *ad hoc*, as the intermolecular forces for all bodies were not *required* by any known theory to transform in the way stipulated. The Lorentz-Fitzgerald contraction was *not* considered *ad hoc* in the sense that it was thought that *no* additional consequences would follow from it other than an explanation of the negative result of the Michelson-Morley experiment. In fact, new experiments that would test the adequacy of both Lorentz's theory in general, and his contraction hypothesis in particular, were being conceived and performed.

In 1902 Lord Rayleigh³⁸ reasoned that the L-F contraction should cause a strain in moving liquids and solids, and result in double-refraction of the order of v^2/c^2 . This double-refraction caused by the earth's motion through the aether should be directionally associated with the path of the earth's motion around the sun. Rayleigh tested water, carbon bisulfide, and stacks of glass plates, but the only observable effect was less than 1% of what was calculated on the supposition of the L-F contraction being a normal contraction. The

³⁶ See Ref. 30, p. 442.

³⁷ See Lorentz's comments on first- and second-order aberration effects in his contribution to the "Conference on the Michelson-Morley Experiment," published in the *Astrophys. J.* **68**, 341 (1928), esp. pp. 349-350.

³⁸ Lord Rayleigh "Does Motion Through the Aether Cause Double Refraction?" *Phil. Mag.* **4**, 678 (1902).

experiment was repeated with increased accuracy by Brace³⁹ in 1904, again with a null result.

In 1903 Trouton and Noble⁴⁰ undertook an investigation of the motion of the aether on a charged condenser. If the condenser was at an angle with the direction of the earth's motion through the aether, the condenser should experience a turning couple tending to bring it into line with the earth's motion.

This experiment was not specifically conceived of as a test of the Lorentz-Fitzgerald contraction, nor was it considered—at least explicitly—to be a test of the Lorentz electron theory. Trouton and Noble rather considered it a test of Larmor's theory which was similar to Lorentz's in a number of respects.⁴¹ It was a second-order aether drift detection experiment, however, and should, on the basis of the Lorentz electron theory, have yielded a positive result. The experiment showed no resulting torque.

Lorentz was quite sensitive to these failures of his electron theory, and set to work to modify his theory so as to absorb the results of these experiments. These modifications are presented in Lorentz's rather well-known 1904 paper.⁴²

In this paper Lorentz cited the negative results of both the Rayleigh-Brace experiments, and the Trouton-Noble experiment. Noting these experimental difficulties, and also feeling the sting of

Poincaré's criticism that a new hypothesis seemed to be required each time new facts are brought to light, Lorentz proposed to attempt to show

by means of certain fundamental assumptions, and without neglecting terms of one order of magnitude or another, that many electromagnetic actions are entirely independent of the motion of the system. Some years ago, I have already sought to frame a theory of this kind. (Lorentz here refers to his 1899 paper discussed above.) I believe now to be able to treat the subject with a better result.⁴³

The unexpressed but clearly visible intent of Lorentz in this paper is to generalize his theorem of corresponding states so as to be able to apply it to second and higher orders. The paper builds towards a presentation of this theorem in a slightly different form from that which it had in the 1895 and 1899 papers, and then shows that it can account for the Michelson-Morley experiment, the Rayleigh and Brace experiments, and the Trouton-Noble result.⁴⁴ Lorentz proceeded in a way that Einstein later would have called "constructive."⁴⁵ He began with the fundamental equations of the theory of electrons with its supposition of electrons and aether, he introduced a classical or "Galilean" velocity addition formula for electrons moving within the moving system, and then postulated, after applying the transformation $x' = x - vt$, the transformation equations which when taken together with $x' = x - vt$ are the 1899 transformation equations and which equivalent to the well-known form of the *Lorentz transformation equations* when the arbitrary function is determined. The t' was the latest form of the "local time." Transformation equations for the dielectric displacement and the magnetic force (different from the 1899 form) were postulated, as is a transformation equation for electric charge density and a non-Galilean modification in the velocity addition formula. Thus far Lorentz was simply making slight alterations in his earlier transformation equations.

In Sec. 8, however, Lorentz found it necessary to introduce an electron which is spherical in a state of rest with respect to the aether, but which has its "*dimensions changed by the effect of a translation,*

³⁹ D. B. Brace, "On Double Refraction in Matter Moving Through the Aether," *Phil. Mag.* **6**, 317 (1904).

⁴⁰ F. T. Trouton and H. R. Noble, *Phil. Trans. Roy. Soc. London*, **A202**, 165 (1903).

⁴¹ See Ref. 17 above.

⁴² H. A. Lorentz, "Electromagnetic Phenomena in a System Moving with any Velocity less than Light," *Proc. Acad. Sci. Amsterdam* **6**, 809 (1904). Also in *Collected Papers*, Vol. 5, pp. 172-197, and in the well-known *The Principle of Relativity* (Dover Publ. Inc., New York, 1923), pp. 11-34. The Dover reprint is defective in that it omits Sec. 14 of this paper, which discusses an earlier and different (1902) condenser experiment by Trouton. Lorentz's 1904 paper also refers to his influential 1904 review article on "Elektronentheorie" which appeared in second part of Vol. 5 of the *Encyklopädie der Mathematischen Wissenschaften* (B. G. Teubner, Leipzig, 1904), p. 145. Though this article does develop the electron theory in slightly new ways—e.g., the discussion of an "electromagnetic momentum" and proposed equations for magnetized bodies—the most important developments as regards relative motion are not presented. Rather Lorentz refers his readers to the above cited "Electromagnetic Phenomena..." paper.

⁴³ Lorentz, "Electromagnetic Phenomena..." p. 174.

⁴⁴ See Ref. 43, esp. p. 190.

⁴⁵ The term "constructive" and the type of theories to which it applies will be discussed further below when a comparison between Einstein and Lorentz is discussed.

the dimensions in the direction of motion becoming kl times and those in the perpendicular directions l times smaller."⁴⁶ Heretofore Lorentz had not considered a contractile electron, though he had introduced charge density transformation equations. To obtain the correct compensating effects for both the Rayleigh-Brace and Trouton-Noble experimental results, however, the contractile electron is essential.⁴⁷ Lorentz also added his usual hypothesis concerning the intermolecular force transformation laws that ensure the old *macroscopic* contraction effect. He calculated, from a definition of electromagnetic momentum which is based on the foregoing hypotheses, that the longitudinal and transverse mass of the electrons would have certain velocity dependent values. The simple form that Lorentz obtained required that he assume the mass of the electron to be completely "electromagnetic"⁴⁸—but this was a rather reasonable assumption to make at this point in the history of physics. In order to obtain what Lorentz referred to as "a state of the moving system which . . . will really be possible," Lorentz assumed that the "products of the mass m and the acceleration of an electron are to each other in the same relation as the forces."⁴⁹ This further allowed him to deduce that the arbitrary function l which appeared in his transformation equations must be constant and equal to one.⁵⁰ Lorentz was then able to present a generalized version of his theorem of corresponding states, and account for the null results of the troublesome experiments with which the paper began.⁵¹ Before concluding his theoretical investigations Lorentz speculated about molecular motion within contracting bodies, and proposed that one "suppose that *the masses of all particles are*

influenced by a translation to the same degree as the electromagnetic masses of the electrons."⁵²

Finally Lorentz argued that the results of Kaufmann on the motion of electrons in electric and magnetic fields, which appeared to disagree with the contractile electron and instead support Abraham's rigid spherical electron, really supported his contractile electron as much as they did Abraham's. There is reference and a brief discussion of Trouton's 1902 condenser experiment, and an explanation for the lack of a predicted impulse being delivered to it by charging and discharging. This ends the paper.

THE LORENTZ TRANSFORMATION EQUATIONS, THE LORENTZ-FITZGERALD CONTRACTION, AND EINSTEIN'S SPECIAL THEORY OF RELATIVITY

What have come to be called the Lorentz transformation equations are but a subset of a wider set of transformations which Lorentz postulated to account for the lack of influence of the "aether wind." The Lorentz-Fitzgerald contraction hypothesis claims that a contraction of the body occurs *as a result of* the translation *through the aether*. The contraction effect is clearly nonreciprocal for two observers at rest in frames of reference which are moving with respect to one another at a constant velocity. It need not be that one of these observers be at rest in the aether—whichever observer's reference frame is moving with a greater absolute velocity (this recall is the meaning of the v in Lorentz's interpretation of the contraction hypothesis) will have measuring rods, interferometers, etc, shorter than the other observer.

The interpretation of the "local-time" transformation equation is also nonreciprocal. Time will go slower for an observer moving *through the aether* than it will for one at rest, or one moving with a slower *absolute* velocity. The nonreciprocity also holds for the interpretation of the transformation equation stating the mass dependency on velocity, as well as for the charge density, ponderomotive force, and intermolecular force transformation equations. In his 1904 paper, Lorentz does introduce a covert reciprocity for the transformation equations for the dielectric displacement and the magnetic force transformation

⁴⁶ See Ref. 43, pp. 182-183.

⁴⁷ This is implicit, of course, in Lorentz's 1904 paper. For a detailed discussion of the necessity for the contractile electron see: H. A. Lorentz's *The Theory of Electrons* (Columbia University Press, New York, 1909), pp. 210-220. The second edition of this book (1915) has been reprinted by Dover, with the additional notes added in 1915. Page references to the text proper are the same in both additions; in the notes, however, the pagination in the two editions diverges. References are to the second ed.

⁴⁸ Lorentz, "Electromagnetic Phenomena . . .," p. 185.

⁴⁹ See Ref. 48, p. 188.

⁵⁰ See Ref. 48, p. 188.

⁵¹ See Ref. 48, pp. 189-190.

⁵² See Ref. 48, p. 191.

equations (i.e., if $l=1$, as he later proved),⁵³ but the ponderomotive force, which is defined in terms of these quantities and the absolute velocity, is still subject to a nonreciprocal transformation.

The nonreciprocal character of Lorentz's interpretation is closely associated with his notion of a privileged reference frame—the aether. Systems which are at rest in the aether are spatially *dilated* in comparison with a moving system. In comparing ponderomotive forces in systems in his 1904 paper Lorentz notes: "The result may be put in a simple form if we compare the moving system S with which we are concerned, to another electrostatic system S' which remains at rest and into which S is changed, if the dimensions parallel to the axis of x are multiplied by $kl \dots$ {or $[1 - (v^2/c^2)]^{-1/2}$ }"⁵⁴ This interpretation is consistently maintained from 1892 through his 1906 lectures—published in 1909 as the *Theory of Electrons*.⁵⁵ In this latter work Lorentz writes in discussing his theorem of corresponding states that:

It is to be observed that corresponding electrons in the two systems occupy corresponding parts of space, and that while their charges are equal, they are geometrically dissimilar; if the electrons in S (the moving system here) are spheres, those in S_0 (the rest system) are lengthened ellipsoids.⁵⁶

Lorentz refers to Einstein's various papers on the theory of relativity near the close of his *Theory of Electrons*. After he has presented his theorem of corresponding states in essentially the same form as he did in his 1904 paper, and employed the same transformation equations, this time labeling the x' and the t' the "effective coordinate" and "effective time" to distinguish them from the "true coordinate" and "true time," x and t , and the relative coordinate $x_r = x - vt$, he writes:

The denominations "effective coordinates," "effective time", etc. [the "etc." covers notions like effective charge density and effective mass] of which we have availed ourselves for the sake of facilitating our mode of expression, have prepared us for a very interesting interpretation of the above results for which we are indebted to Einstein.⁵⁷

⁵³ See Ref. 48, p. 176.

⁵⁴ See Ref. 48, pp. 182–183.

⁵⁵ See Ref. 47.

⁵⁶ Lorentz, *Theory of Electrons*, p. 200.

⁵⁷ See Ref. 56, p. 223. Lorentz's references to Einstein include the famous *Ann. Physik* paper of 1905, as well as later papers, in 1906 and 1907 in the same Journal. He also cites Einstein's review article which appeared in the *Jahrbuch Rad. Elek.* 4, 411 (1907).

Lorentz commences to show that "when clocks are adjusted by means of optical signals, each of them will indicate the local time t' corresponding to its position."⁵⁸ Furthermore, if observers were to use these "local time" reading clocks, and "shrunk" rods, to build up an electron theory based on measurements made within their system and were to "keep a record of their observations and the conclusions drawn from them, these records would, on comparison, be found to be exactly identical."⁵⁹

It is of importance not to forget that, in doing all that has been said, [i.e., making measurements with clocks and rod *in the moving system*] the [moving] observer would remain entirely unconscious of his system moving (with himself) through the ether, and of the errors of his rod and his clocks. . . .

Attention must now be drawn to a remarkable reciprocity that has been pointed out by Einstein. Thus far it has been the task of the observer A , [at rest in the ether] to examine the phenomena in the stationary system, whereas A' [who is in motion through the ether] has had to confine himself to the system S' . Let us now imagine that each observer is able to see the system to which the other belongs, and to study the phenomena going on in it. . . .

It will be clear by what has been said that the impressions received by the two observers A' , and A would be alike in all respects. It would be impossible to decide which of them moves or stands still with respect to the ether, and there would be no reason for preferring the times and lengths measured by the one to those determined by the other, nor for saying that either of them is in possession of the 'true' times or the 'true' lengths. This is a point which Einstein has laid particular stress on, in a theory in which he starts from what he calls the principle of relativity, i.e., the principle that the equations by means of which physical phenomena may be described are not altered in form when we change the axes of coordinates for others having a uniform motion of translation relatively to the original system.

I cannot speak here of the many highly interesting applications which Einstein has made of this principle. His results concerning electromagnetic and optical phenomena (leading to the same contradiction with Kaufmann's results that was pointed out in Sec. 179) agree in the main with those which we have obtained in the preceding pages, the chief difference being that Einstein simply postulates what we have deduced, with some difficulty and not altogether satisfactorily, from the fundamental equations of the electromagnetic

⁵⁸ Lorentz, *Theory of Electrons*, p. 226.

⁵⁹ See Ref. 58, p. 226.

field. By doing so, he may certainly take credit for making us see in the negative result of experiments like those of Michelson, Rayleigh and Brace, not a fortuitous compensation of opposing effects, but the manifestation of a general and fundamental principle.

Yet, I think, something may also be claimed in favor of the form in which I have presented the theory. I cannot but regard the ether, which can be the seat of an electromagnetic field with its energy and its vibrations, as endowed with a certain degree of substantiality, however different it may be from all ordinary matter. In this line of thought it seems natural not to assume at starting that it can never make any difference whether a body moves through the ether or not, and to measure distances and lengths of time by means of rods and clocks having a fixed position relatively to the ether.⁶⁰

It should be clear from the foregoing where Lorentz stood on the theory of relativity, and by implication what place he merits in the development of the theory of relativity as viewed from the point of view of science as a whole. In terms of the effect of his writings on Einstein, Poincaré, and Minkowski, the reader is referred to other writings.⁶¹

Though a detailed comparison between Einstein's theory of relativity and Lorentz's "absolute" electron theory, will be discussed elsewhere,⁶² I should like to conclude this paper by alluding to the most important points of such a comparison. This also provides an opportunity to discuss some of Einstein's revolutionary advances over Lorentz.

(1) The Lorentz electron theory along with its nonrelativistic interpretation was for a number of years after Einstein's theory of relativity had been proposed, considered to be a "rival" theory to Einstein's. M. von Laue in assessing the reasons for the success of Einstein's theory over Lorentz's

⁶⁰ See Ref. 58, pp. 226-230.

⁶¹ See G. Holton's "On the Origins of the Special Theory of Relativity," *Amer. J. Phys.* **28**, 630 (1960), as well as E. T. Whittaker's *A History of the Theories of Aether and Electricity* (Ref. 12), which Holton justly criticizes. G. H. Keswani's articles on the "Origin and Concept of Relativity" are worth looking at even though they contain a number of misleading statements *B. J. P. S.* **15** (1965), 286; **16** (1965), 19; **16** (1966), 273. Also S. Goldberg's recent study of Poincaré and relativity, *Amer. J. Phys.* **35** (1967) is valuable.

⁶² An article "Crucial Experiments, *Ad Hoc* Hypotheses, and Theory Replacement," is currently in preparation.

and E. Cohn's theories of the electrodynamics of moving bodies noted, that though Cohn's theory was open to serious experimental objection,

one is not able to decide in such an absolute way, by experiment, between the complete Lorentz theory and the theory of relativity.⁶³

N. R. Campbell writing in the *Philosophical Magazine* in 1911 said:

The special laws which it is the business of the Principle of Relativity to explain (that is, those which it is specially important to be able to deduce from the theory) are those which are met within the study of the optical and electrical properties of systems in relative motion, but in this case, as in most cases, it turns out that laws other than those contemplated originally are deducible from the theory. It is important to notice that there is another theory, that of Lorentz, which explains completely all the electrical laws of relatively moving systems; that the deductions from the Principle of Relativity are identical with those from the Lorentzian theory, and that both sets of deductions agree completely with all experiments that have been performed. If then, anyone prefers one theory to the other it must be either on the ground of differences in the laws not contemplated originally which are predicted respectively by the two theories, or because of some general grounds independent of experimental considerations.⁶⁴

Similarly, Paul Ehrenfest argued in his article on "The Crisis in the Hypothesis of the Luminiferous Aether" that the Lorentz and Einstein theories were rival theories which did not then admit of an experimental decision between them.⁶⁵

Now it is clear that Lorentz's theory and Einstein's theory are rather different theories—but it is exceedingly difficult precisely to define the difference. In part, this is because the Lorentz theory is what Einstein termed a "constructive theory"—whereas the theory of special relativity is a "theory of principle."⁶⁶ The electron theory

⁶³ M. Laue, *Das Relativitätsprinzip* (Frederick Vieweg und Sohn, Braunschweig, Germany, 1911), p. 19.

⁶⁴ N. R. Campbell, *Phil. Mag.* **21**, 502 (1911).

⁶⁵ P. Ehrenfest, *Collected Scientific Papers*, M. J. Klein, Ed. (North-Holland Publ. Co., Amsterdam, 1959).

⁶⁶ See Einstein's discussion in his "Autobiographical Notes" in *Albert Einstein: Philosopher-Scientist*, P. A. Schlipp, Ed. (Library of Living Philosophers, Evanston, Ill.), p. 53. Also see his more extensive comparison of the two types of theories in his article "What is the Theory of Relativity," in *A. Einstein The World as I See It* (Covici, New York, 1934), esp., pp. 74-75.

commences with the basic laws of electrical and magnetic phenomena and builds up a theory from Maxwell's equations and its extension to small charged particles. Moreover, this theory went through several stages of complicating evolution, adding hypotheses concerning intermolecular force transformation laws for molecule systems moving through the aether, time dilation, electron contraction, etc. Furthermore, the Lorentz theory possesses an *interpretation* of these transformation equations which is nonreciprocal, i.e., it assumes maximum length, etc., in *one* reference frame at rest in the aether.

What is normally referred to as Einstein's special theory of relativity is not Einstein's kinematics *plus* Einstein's electrodynamics, but rather Einstein's kinematics alone. This part of Einstein's 1905 paper constitutes according to him a "theory of principle." The theory consists of two general principles which in a sense are empirical generalizations. The two principles: the principle of relativity and the principle of the independence of the velocity of light from its source, are reconcilable in the light of a new understanding of simultaneity, and are sufficient for the derivation of the Lorentz transformation equations. *Application* of the transformations, plus a derivable velocity addition theorem, to Maxwell's and also Lorentz's "constructive" equations for a frame of reference at rest, yields a simple and consistent theory of the electrodynamics of moving bodies.

It seems, however, that it is Einstein's full account of the electrodynamics of moving bodies—his revolutionary kinematics *plus* the borrowed electrodynamic equations which can best be compared with Lorentz's theory. It is this comparison that is tacit throughout the following.

The Lorentz theory is presented by Lorentz so as to maintain one reference frame as privileged. In this frame all rest rods are of maximum length, electrons are spherical, and time goes fastest and is the Newtonian universal time. In reference frames which move with respect to this privileged frame, rods are contracted, time goes slower, etc. The transformations that relate the moving frame to the aether—that is those in which the aether frame is taken as the observer's frame—are the same in form as those that appear in Einstein's theory. *The inverse transformations*, however,

relating the aether frame to the moving frame (assuming the observer at rest in the moving frame now) are *not* the inverse Einsteinian transformations. Rather, they are simply the *reciprocals* of the first transformations. If $x' = (x - vt) / (1 - v^2/c^2)^{1/2}$, then $x = x'(1 - v^2/c^2)^{1/2} + vt$. This is what permits Lorentz to talk about *dilations* of the rest system in comparison with a moving system. These inverse transformations are the "aether inverse transformations."

Now what is most curious is that Lorentz did not notice that there was anything wrong with this interpretation—nor did he indicate that there was another type of inverse transformations until near the very end of his *Theory of Electrons*. The long quote from Lorentz above was concerned precisely with this question. The aether inverse transformation interpretation, however, can only be consistently applied if each observer can ascertain his *absolute* velocity, either directly, or even by some complicated indirect process of measurement. But on the basis of the Lorentz electron theory and the Lorentz transformation equations, an observer in a moving system S' must ascribe velocity $v' = -v$ to a system S , at "rest" in the aether, with respect to which he is moving with velocity v . Lorentz shows this, and he also shows that the moving observer can introduce inverse transformations which *only* differ by having a $+v$ term where the "stationary" observer employed a $-v$.⁶⁷ These inverse transformations had *not* previously been used by Lorentz, who had only utilized—largely implicitly—the aether inverse transformations. The use of the Einsteinian inverse transformations introduces a reciprocity interpretation, as Lorentz clearly saw. Moreover, Lorentz also saw that a *complete* reciprocity interpretation would entail the rejection of an aether. The reciprocity interpretation is closely associated with the principle of relativity—the principle of relativity requires it, and *complete* reciprocity of all theories (together with the absence of any experiments which disagreed with these theories) would provide the strongest possible support for Einstein's principle of relativity. Lorentz noted that accepting the principle of relativity would mean giving up the aether, and in 1908 he still had theoretical reservations, and

⁶⁷ Lorentz, *Theory of Electrons*, p. 227–228.

the experimental evidence was still sufficiently confused, that he did not feel he was compelled to take such a radical step.⁶⁸ Furthermore, as the long quote above indicated, though he acknowledged the remarkable character of the reciprocity interpretation of the Lorentz transformation equations, he seemed somewhat nettled by Einstein's approach that simply postulated what he (Lorentz) "had deduced, with some difficulty, and not altogether satisfactorily, from the fundamental equations of the electromagnetic field."

It was apparently not until 1914 or 1915, that Lorentz was able to see the simplicity and utility of Einstein's theory. Lorentz's Haarlem Lectures (1914)⁶⁹ and, especially, the notes added to the 1915 edition of the *Theory of Electrons* are the basis of this opinion.

One can conclude on the basis of the foregoing that two of the really significant differences between Einstein's theory and Lorentz's are the differences in (a) the *interpretations* of the transformation equations, and (b) the status—i.e., privileged or coequal—of different reference non-accelerated frames in motion with respect to each other. There are, to be sure, other differences, but let us reserve treatment of these for the next section.

(2) The second point is to call attention to an important factor in the determination of Einstein's radically different *approach* to the problem of the electrodynamics of moving bodies. Prior to Einstein's revolutionary analysis, the Lorentz theory, as we have seen above, was thought to give a fairly good account for the failure of experiments to detect absolute motion. The theory to be sure seemed rather "Baroque" with its many complicated-looking transformation equations, and might well be rejected in comparison with a "sleeker" theory. Simplicity of this type is comparative, however, and not to be considered significant as a *determining* ground for a scientist's *approach*.

⁶⁸ Comparison of the 1909 and the 1915 eds. of Lorentz's *Theory of Electrons*, reveals the extent to which the experimental situation cleared up in those years. The possibility of gravitational experiments that would reveal aether wind effects was still a live possibility until 1910. See Lorentz's discussion of this in his *Das Relativitätsprinzip: Drei Vorlesungen . . . Haarlem* (B. G. Teubner, Leipzig, 1914), pp. 20–22.

⁶⁹ See Ref. 68.

Complexity within a theory, *per se*, does not suggest how a simpler theory might be constructed.

Consider the following question: why did Einstein not follow Lorentz's approach in developing a "constructive" type of theory which would begin from some fundamental equations of electrodynamics and provide an explanation of the failure to detect motion with respect to the aether. Any complete answer would most probably be exceedingly complex, and most likely would intermingle psychological and logical reasons. Einstein's early conceptual experiment of following a light beam is very relevant⁷⁰; the *complexity* of the Lorentz theory, however, does not seem as important as other issues. One clearly relevant consideration, however, has been too little emphasized and what follows below also indicates why the Einstein theory is of such lasting value.

As noted above, Einstein drew a distinction between two types of theories: theories of principle and constructive theories. He has also stated, unequivocally, that both the special theory of relativity and the general theory were "principle type" theories. A variant of the question which is posed above, then, is, why did Einstein introduce a "principle type" of theory in this domain? The answer to this question is not difficult to find.

Einstein writes in his "Autobiographical Notes" that his work prior to (and in) 1905 was connected with one "major question,"—namely, "what general conclusions can be drawn from the (Planck blackbody) radiation-formula concerning the structure of radiation and even more generally concerning the electro-magnetic foundation of physics."⁷¹

In his paper, *On a Heuristic Point of View about the Creation and Conversion of Light*, which was completed in March of 1905,⁷² Einstein showed that the theoretical basis of Maxwell's theory and the electron theory fails completely for blackbody radiation at short wavelengths and low radiation densities. He proposed, in contradiction with Maxwell's theory which considered energy "to be a continuous function in space for all purely electro-

⁷⁰ See Einstein's "Autobiographical Notes," p. 53.

⁷¹ See Ref. 70, p. 47.

⁷² A. Einstein, *Ann. Physik* **17**, 132 (1905). There is an English translation of this paper in D. ter Haar's *The Old Quantum Theory* (Pergamon Press, Inc., Oxford, 1967), p. 91. Page references are to this version.

magnetic phenomena," that "the observations on 'black-body radiation', photoluminescence, the production of cathode rays by ultraviolet light and other phenomena involving the emission or conversion of light can be better understood on the assumption that the energy of light is distributed discontinuously in space. . . (Light) consists of a finite number of energy quanta localized in space, which move without being divided."⁷³

Einstein stated that "reflections of this type made it clear to me as long ago as shortly after 1900, i.e., shortly after Planck's trailblazing work, that neither mechanics nor thermodynamics (nor electrodynamics as it was understood at that time) could (except in limiting cases) claim exact validity. By and by I despaired of the possibility of discovering the true laws by means of constructive efforts based on known facts. The longer and the more despairingly I tried, the more I came to the conviction that only the discovery of a universal formal principle could lead us to assured results."⁷⁴

This principle of course is the famous "principle of special relativity." It must be emphasized however that the principle must immediately cover, or be associated in a most intimate way, with the principle of the constancy of the velocity of light. This *latter* principle, however, is all that need be assumed from electrodynamics; there is no need to introduce hypotheses of contractile electrons, velocity dependence of electromagnetic mass, intermolecular force transformations, etc., or on the other hand, equations which would account for the *particulate character of light* at the same time that they would account for its wave properties, (Some of these Lorentzian type hypothesis will follow from *application* of Einstein's two principles to *other* laws, as, for example, the contractile electron is relativistically interpreted as a spherical electron viewed from a reference frame which is in motion with respect to the rest frame of the spherical electron. Of course it need not be emphasized that the theory of relativity neither accounts for the *existence of electrons* nor depends in any way on their existence.)

An analysis of Einstein's revolutionary account of simultaneity in order to show how he reconciled his two principles is not within the scope of this

paper, nor can more be said than has been suggested under point (1) above of how Einstein's analysis revealed the true significance of Lorentz's heuristically adopted "local time." At this point we simply wish to argue that Einstein's approach *via* a theory of principle was motivated by his earlier discoveries of the wave-particle duality and the quantum nature of radiation. Furthermore, Einstein's selection of the single postulate of the constancy of the velocity of light resulted not only in a particularly "simple" form of his theory, but also it afforded, both then and in the light of subsequent investigation, a security of foundation that has placed it at the heart of many later physical theories.

This paper is best closed by citing a letter of Einstein's to Carl Seelig, in which he indicates which of Lorentz's works he was familiar with. The letter also supports the point about the reason for the difference of approach of Lorentz and Einstein.

Einstein wrote in 1955 that:

I know only Lorentz's important works of 1895—"La theorie electromagnetique de Maxwell" (this is actually the 1892 work) and "Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern"—but not Lorentz's later work, nor the consecutive investigations by Poincare. In this sense my work of 1905 was independent. The new feature of it was the realization of the fact that the bearing of the Lorentz-transformations transcended their connection with Maxwell's equations and was concerned with the nature of space and time in general. A further new result was that "Lorentz invariance" is a general condition for any physical theory. This was for me of particular importance because I had already previously found that Maxwell's theory did not account for the micro-structure of radiation and could therefore have no general validity—.⁷⁵

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⁷⁵ Letter quoted and translated in M. Born's "Physics and Relativity," "Jubilee of Relativity Theory" volume of *Helvetica Physica Acta Supplementum* IV, 1956.

⁷³ See Ref. 72, p. 92.

⁷⁴ Einstein, "Autobiographical Notes," p. 51-53.