# Relativistic velocity and acceleration transformations from thought experiments

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I derive the three-dimensional velocity and acceleration transformations of special relativity from a set of thought experiments. These derivations are facilitated by the use of an enhanced particle-light clock and employ simple physical arguments based on kinematic considerations and the principle of relativity. The derivations are conceptually simpler, more intuitive, and less abstract, and require significantly less background and preparatory effort than the usual derivations employing the Lorentz transformation. They also serve to emphasize the directness and immediacy of the connection between the principle of relativity and its physical consequences. 

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## I. INTRODUCTION

The use of thought experiments to convey the essence of the basic anti-common sense effects associated with special relativity is an effective and time-honored technique. On the one hand, two venerable special relativity textbooks[1,2] that were discussed over 30 years ago[3] are extreme examples of this use. In these two books, all of the fundamental aspects of special relativity are derived from thought experiments. On the other hand, most university-level introductory physics[4–20] and modern physics[21–31] textbooks, many introductory-level special relativity[1,2,32–43] textbooks, and some higher-level books[44–53] introduce some or all of the three fundamental aspects of special relativity, namely, the relativity of simultaneity, time dilation, and length contraction, via thought experiments. Similar discussions can be found in many articles, of which I cite a few.[54–67] Also, in its ongoing study of student understanding of special relativity, the Physics Education Group at the University of Washington has developed instructional materials that can be viewed, at least in part, as a set of thought experiments designed to explicate the concept of the relativity of simultaneity.[68] Some of the books cited[5,14,24,32,38,53] and several papers[56,66,69–72] also employ a thought experiment to obtain the Einstein relativistic velocity addition rule.[73,74]

The three-dimensional velocity transformation is generally considered only with the use of the Lorentz transformation.[4,6,13–17,20,21,23–47,50–52,75–87] The acceleration transformation, when it is considered at all, also is discussed only with the use of the Lorentz transformation.[20,33–36,39,40,42,43,51,75,77,84]

In this paper, I discuss and analyze a set of thought experiments that utilize an enhanced particle-light clock. The analysis employs straightforward physical arguments that are based on kinematic considerations and the principle of relativity. On the basis of these thought experiments and their analysis, I derive the three-dimensional relativistic transformations for velocity and acceleration. These derivations are conceptually simpler, more intuitive, and considerably less abstract, and require significantly less background and preparatory effort than derivations based on the Lorentz transformation. These derivations also serve to emphasize the directness of the connection between the principle of relativity and its physical consequences. To my knowledge, these derivations of the perpendicular part of the velocity transformation and the acceleration transformation from thought experiments are new.

The principle of relativity can be expressed in the form of two postulates:[27] (1) The laws of physics are the same in all inertial reference frames. (2) The speed of light in free space has the same value in all inertial reference frames. The second statement is redundant,[71,88,89] but I include it, as is traditional, for explicitness.

In Sec. II, I describe the enhanced particle-light clock that I employ in the thought experiments. I also note how this clock can be used to obtain time dilation and length contraction. In Sec. III, I present the thought experiment and its analysis that yields the three-dimensional relativistic velocity transformation. In Sec. IV, I discuss the thought experiment that yields the relativistic acceleration transformation. I conclude the paper in Sec. V with a summary and a brief discussion.

## II. AN ENHANCED PARTICLE-LIGHT CLOCK

The particle-light clock, which I hereafter refer to as the clock, is a generalized version of the timing device employed by Krane[24] in his discussion of the Einstein relativistic velocity addition rule. It has four essential elements, as shown schematically in Fig. 1. The first element, $S_{LPA}$, is a source (S) with three switch settings. On the first switch setting (L), $S_{LPA}$ is a source of light flashes. On its second switch setting (P), it is a source of particles. These two possibilities are included in Krane's timing device.[24] The new feature is activated by the third switch setting (A); then $S_{LPA}$ is a source of what appears to be particles, but are actually miniature powered flying craft capable of maintaining any possible acceleration for at least a short period of time. The device $D_{LPA}$ detects the light flashes, particles, and accelerated miniature flying craft emitted by $S_{LPA}$, and subsequently triggers the emission of a flash of light by the source $S_L$, which is in turn detected by the detector $D_L$.

The light flashes emitted by $S_{LPA}$ and $S_L$ are sufficiently brief that their spatial extent along their direction of propagation is much less than $L_p$, the proper length of the clock, the length of the clock in its rest frame. The delay between $D_{LPA}$'s detection of a flash of light or a particle or an accel-

  

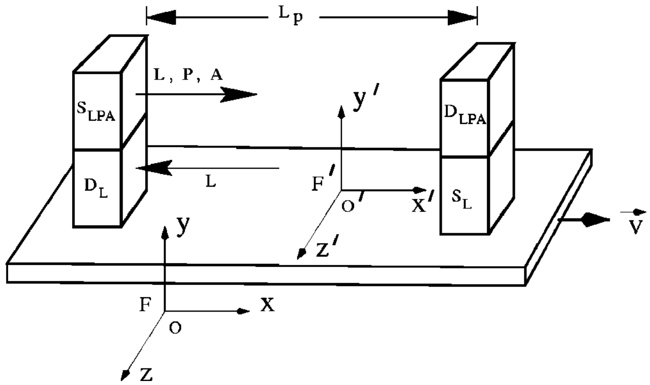

Fig. 1. Schematic diagram of the enhanced particle-light clock. $S_{LPA}$ is a source of brief light flashes (L), particles (P), or miniature flying craft that travel with constant acceleration (A). When the light flash, particle, or craft emitted by $S_{LPA}$ reaches $D_{LPA}$, this detector triggers the emission of a brief flash of light by $S_L$. This flash of light is subsequently detected by $D_L$. The length of the clock in its own rest frame is $L_P$. The frames $F(0xyz)$ and $F'(0'x'y'z')$ are the rest frames of the outside observer and the clock, respectively. Both frames are inertial frames.

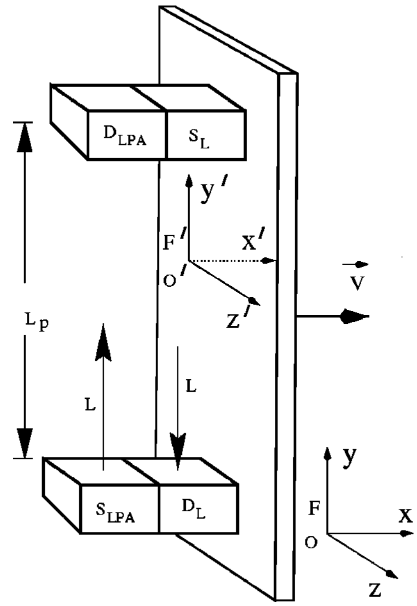

Fig. 2. The clock is oriented perpendicularly to **V** for the time dilation experiment.

erated craft and $S_L$'s emission of a flash of light is negligible compared to the time required for light to travel the length of the clock. In addition, $S_{LPA}$ and $D_L$ are sufficiently compact that, for the purposes of the experiments, they are at the same spatial location, as is also true of $D_{LPA}$ and $S_L$.

The clock is mounted in a vehicle that is capable of traveling with a constant velocity **V** relative to an outside observer. Moreover, the clock can be rotated within this vehicle so that the velocity of the light flash or particle or accelerated craft can be in any direction.

The reference frames $F(0xyz)$ and $F'(0'x'y'z')$, with the $x$ and $x'$ axes parallel, are shown schematically in Fig. 1 and are the rest frame of the outside observer and the rest frame or proper frame of the clock, respectively. These frames are oriented so that **V** is in the common $x$ and $x'$ direction. I assume that they are inertial reference frames.

With the source $S_{LPA}$ on its first (L) setting, we can carry out the usual thought experiments to reveal time dilation and length contraction.[24] For the experiment to measure time dilation, an observer in $F'$ orients the clock perpendicular to **V**, as shown in Fig. 2. The apparent trajectory of the light, as measured by an outside observer, is shown in Fig. 3. An analysis of the thought experiment reveals that time dilation is embodied in the relation

$$T = \gamma T', \tag{1}$$

where

$$\gamma = \left(1 - \frac{V^2}{c^2}\right)^{-1/2}. \tag{2}$$

The tick interval measured by the outside observer, $T$, is larger than the tick interval, $T'$, measured in $F'$, by a factor of $\gamma$. The tick intervals, $T$ and $T'$, are the times for the entire cycle, for the light to travel from $S_{LPA}$ to $D_{LPA}$ and then from $S_L$ to $D_L$.

To measure length contraction, an observer in $F'$ orients the clock parallel to **V**, as shown in Fig. 1. Alternatively, if we wish to derive length contraction so that we do not presume time dilation in the clock in which length contraction is observed, we use two identical clocks, clock 1, oriented perpendicular to **V**, and clock 2, oriented parallel to **V**, as shown in Fig. 4. In either way, Fig. 5 depicts the three key events in the clock cycle as recorded by the outside observer. An analysis of either thought experiment reveals that length contraction is embodied in the relation

$$L = \frac{L_p}{\gamma}. \tag{3}$$

To the outside observer, the length $L$ of a clock oriented parallel to **V** is measured to be smaller by a factor of $1/\gamma$ than the proper length of the clock, $L_p$.

## III. VELOCITY TRANSFORMATION

For the experiment to observe the velocity transformation, an observer in $F'$ orients the clock at an angle $\theta'$ relative to **V**, as shown in Fig. 6. The dimensions of the clock in $F$ in the $x'$ and $y'$ directions are then $W_p = L_p \cos\theta'$ and $H_p = L_p \sin\theta'$, respectively. The subscript $p$ again refers to $F'$, the proper or rest frame of the clock.

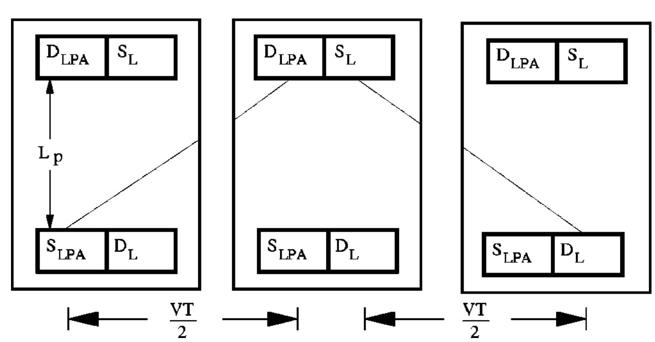

Fig. 3. The three key events in the time dilation experiment appear at distinct spatial locations for the outside observer.

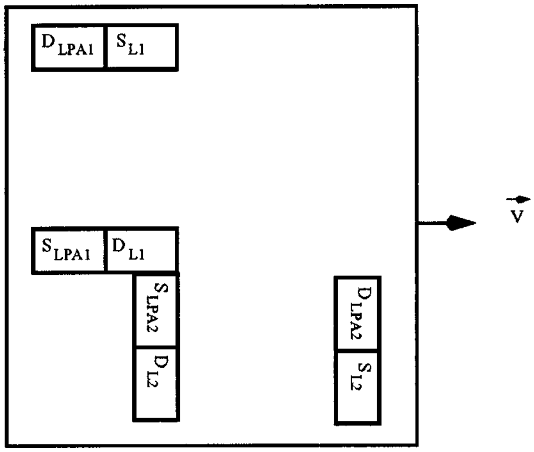


Fig. 4. Clock 1 is oriented perpendicular to **V** and identical clock 2 is oriented parallel to **V** for the length contraction experiment. If the clock elements are sufficiently compact, then to a sufficient approximation for the purposes of the experiment, if the two clocks are synchronized in their rest frame, they are synchronized in all inertial frames.

For this experiment, I use the second switch setting, P, for which $S_{LPA}$ emits particles with velocity $\mathbf{u}'$ relative to $F'$ and $\mathbf{u}$ relative to $F$. In $F'$, the time for a particle to travel from $S_{LPA}$ and reach $D_{LPA}$ is given by

$$t_0' = \frac{W_p}{u_x'} = \frac{H_p}{u_y'}. \tag{4}$$

I use $u_x'$ and $u_y'$ rather than $u_{x'}'$ and $u_{y'}'$, respectively, for simplicity. Because the $x$ and $x'$ and $y$ and $y'$ axes are parallel, there is no ambiguity in the notation. The time for the returning light flash to return to $D_L$ is given by

$$t_b' = \frac{\sqrt{W_p^2 + H_p^2}}{c}. \tag{5}$$

The tick interval in $F'$ is given by

$$T' = t_0' + t_b'. \tag{6}$$

The corresponding times in $F$ are given by

$$t_0 = \frac{W}{u_x - V} = \frac{H}{u_y}, \tag{7}$$

$$(ct_b)^2 = (W - Vt_b)^2 + H^2, \tag{8}$$

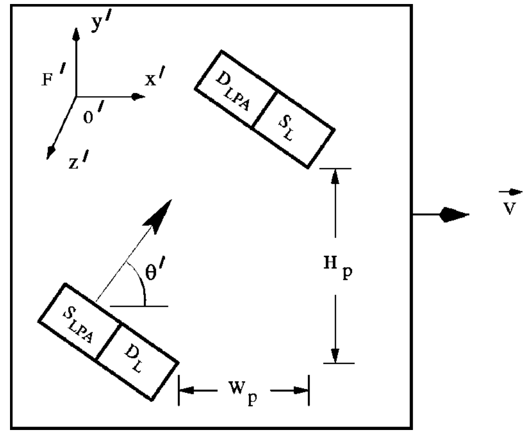


Fig. 6. For the velocity transformation and acceleration transformation experiments, the clock, in its rest frame, is oriented at an angle $\theta'$ relative to **V**. The dimensions of the clock in its rest frame in the $x'$ and $y'$ directions are $W_p$ and $H_p$, respectively.

and

$$T = t_0 + t_b. \tag{9}$$

Given what we know about length contraction, we have

$$W = \frac{W_p}{\gamma}, \quad H = H_p. \tag{10}$$

Equations (8) and (10) readily yield

$$t_b = \frac{\gamma}{c}\left( -W_p \frac{V}{c} + \sqrt{W_p^2 + H_p^2} \right). \tag{11}$$

Equation (1), the first of Eqs. (4) and (7), Eqs. (5), (6), (9)–(11), and a little algebra, yield

$$u_x = \frac{u_x' + V}{1 + \dfrac{u_x' V}{c^2}}. \tag{12}$$

Equation (12) is the Einstein relativistic velocity addition rule.

From Eq. (4) it follows that

$$\frac{u_y'}{u_x'} = \frac{H_p}{W_p}. \tag{13}$$

From Eqs. (7) and (10) we find that

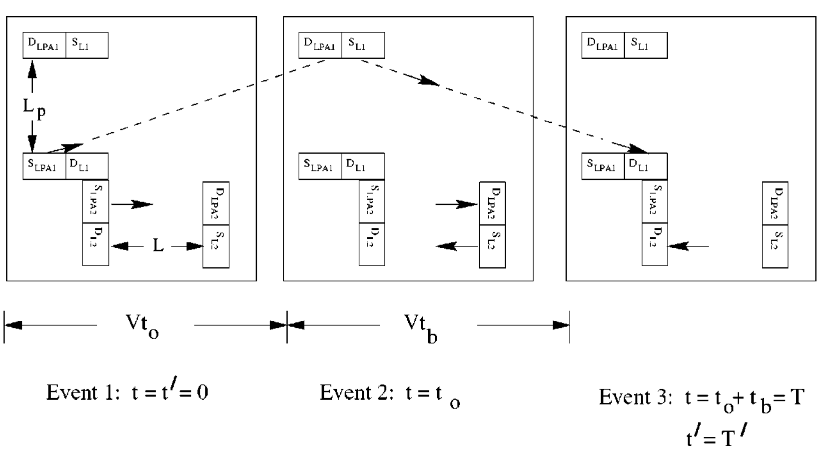


Fig. 5. Situation at the start of a cycle for both clocks, at the end of the “out” portion of the cycle for clock 2, and at the end of a cycle for both clocks, from the viewpoint of the outside observer.

$$\frac{u_y}{u_x - V} = \gamma \frac{H_p}{W_p}. \tag{14}$$

Equations (13) and (14) together with Eq. (12) yield

$$u_y = \frac{u_y'}{\gamma\left(1 + \dfrac{u_x' V}{c^2}\right)}. \tag{15}$$

For velocities $\mathbf{u}$ and $\mathbf{u}'$ that also have $z$ and $z'$ components, respectively, it follows that

$$u_z = \frac{u_z'}{\gamma\left(1 + \dfrac{u_x' V}{c^2}\right)}. \tag{16}$$

Equations (12), (15), and (16) constitute the relativistic velocity transformation. Note that this result follows from Eqs. (1), (2), (4) to (7), and (9) to (11). In other words, given time dilation and length contraction, the velocity transformation follows from kinematic considerations.

## IV. ACCELERATION TRANSFORMATION

To obtain the acceleration transformation, the clock is oriented as for the velocity transformation experiment. I now use the third switch setting, A, on $S_{\mathrm{LPA}}$, so that it emits miniature flying craft that emerge with initial velocity $\mathbf{u}'$ or $\mathbf{u}$ and travel with constant acceleration $\mathbf{a}'$ or $\mathbf{a}$, relative to $F'$ and $F$, respectively.

In $F'$ the time for the accelerated craft to travel out to $D_{\mathrm{LPA}}$ is determined by

$$W_p = u_{x'}' t_0' + \tfrac{1}{2} a_{x'} t_0'^2, \tag{17}$$

and

$$H_p = u_y' t_0' + \frac{1}{2} a_y' t_0'^2. \tag{18}$$

I use $a_x'$ and $a_y'$ rather than $a_{x'}'$ and $a_{y'}'$ in the same spirit that I used $u_x'$ and $u_y'$ instead of $u_{x'}'$ and $u_{y'}'$. The solutions for $t_0'$ are

$$t_0' = \frac{-u_x' + \sqrt{u_x'^2 + 2a_x' W_p}}{a_x'}, \tag{19}$$

and

$$t_0' = \frac{-u_y' + \sqrt{u_y'^2 + 2a_y' H_p}}{a_y'}. \tag{20}$$

The time for the light flash emitted by $S_L$ to travel back to $D_L$ is given by Eq. (5) and the tick interval is given by Eq. (6).

In $F$ we have

$$W + V t_0 = u_x t_0 + \tfrac{1}{2} a_x t_0^2, \tag{21}$$

and

$$H = u_y t_0 + \tfrac{1}{2} a_y t_0^2. \tag{22}$$

The solutions for $t_0$ are

$$t_0 = \frac{-(u_x - V) + \sqrt{(u_x - V)^2 + \dfrac{2a_x W_p}{\gamma}}}{a_x}, \tag{23}$$

and

$$t_0 = \frac{-u_y + \sqrt{u_y^2 + 2a_y H_p}}{a_y}. \tag{24}$$

The time for the light flash emitted by $S_L$ to return to $D_L$ is given by Eq. (11) and the tick interval is given by Eq. (9).

Equations (17), (18), (21), and (22) are a consequence of the kinematic definitions of position, velocity, and acceleration. Of course, Eq. (1) also applies for this experiment.

The next step in the analysis is to explicitly write out Eq. (1). I use Eqs. (5), (6), and (23) to evaluate $T$ and Eqs. (5), (6), and (19) to evaluate $T'$. In this way I obtain

$$\frac{u_x - V}{a_x}(-1 + Q_x) - W_p \gamma \frac{V}{c^2} = \gamma \frac{u_x'}{a_x'}(-1 + Q_x'), \tag{25}$$

where

$$Q_x = \sqrt{1 + \frac{2a_x W_p}{\gamma (u_x - V)^2}}, \quad Q_x' = \sqrt{1 + \frac{2a_x' W_p}{u_x'^2}}. \tag{26}$$

With the use of Eq. (12), which applies because $\mathbf{u}$ and $\mathbf{u}'$ are the initial velocities in $F$ and $F'$, respectively, of the craft emitted by $S_{\mathrm{LPA}}$, I obtain from Eq. (25) and a little algebra,

$$\frac{u_x - V}{a_x}(Q_x - 1)^2 = \gamma \frac{u_x'}{a_x'}(Q_x' - 1)^2. \tag{27}$$

Equation (27) apparently contains the desired connection between $a_x$ and $a_x'$. It also involves $V$, $u_x$, $u_x'$, and, through $Q_x$ and $Q_x'$, $W_p$. That the connection between $a_x$ and $a_x'$ involves $V$ is expected. That the connection involves $u_x$ and $u_x'$ appears to be a problem, inasmuch as $\mathbf{u}$ and $\mathbf{u}'$ are the initial velocities of the craft in $F$ and $F'$, respectively. We would expect the connection between $a_x$ and $a_x'$ at some instant to involve the velocities at that instant (with appropriate care taken in interpreting the meaning of instant), rather than the initial velocities. The presence of $W_p$ implies that the connection between $a_x$ and $a_x'$ involves the dimensions of the clock.

We can arrive at an understanding of the reason for these apparent difficulties and resolve them in a way that elucidates the measurement process by considering the analogy between our clock and a real speedometer or accelerometer. The latter devices measure the speed or magnitude of the acceleration, respectively, averaged over a time interval determined by the instrument. Our clock measures the connection between $a_x$ and $a_x'$, averaged over a tick interval. A real speedometer, real accelerometer, and our clock, would give a better approximation to the instantaneous value of the quantity being measured by averaging over a smaller time interval. We thus see that the way to resolve our apparent problem with the connection between $a_x$ and $a_x'$ is to reduce the time interval over which our clock is measuring this connection. That is, we must reduce the tick interval of the clock. The only way that we can do so is to shrink the clock, that is, make $L_p$, and thus $W_p$ and $H_p$, so small that $\mathbf{u}$ and $\mathbf{u}'$ are good approximations to the velocities of the craft in the frames $F$ and $F'$, respectively, during the time that it takes the craft to traverse the clock.

Taking $L_p$ to be very small poses no fundamental limitation and is only a constraint on the instrument we are using for the experiment, that is, the clock. A constraint of this nature is to be expected for any instrument that carries out a measurement by averaging over a time interval. It would be surprising not to have some such constraint associated with an instrument that produces its results by averaging over a time interval.

I now divide Eq. (27) by $W_p^2$ and take the limit as $W_p$ goes to zero. I thereby obtain

$$a_x = \left( \gamma \frac{u_x - V}{u_x'} \right)^3 a_x' . \tag{28}$$

With the use of Eq. (12), which, as we have noted, applies, we immediately rewrite Eq. (28) as

$$a_x = \frac{1}{\gamma^3 \left( 1 + \frac{u_x' V}{c^2} \right)^3} a_x' . \tag{29}$$

Equation (29) is the longitudinal part of the relativistic acceleration transformation.

To obtain the connection between $a_y$ and $a_y'$, I again explicitly write out Eq. (1), use Eqs. (9), (11), and (24) to evaluate $T$, and use Eqs. (5), (6), and (20) to evaluate $T'$. In this way, I obtain

$$\frac{u_y}{a_y}(Q_y - 1) = \gamma \frac{u_y'}{a_y'}(Q_y' - 1) + \gamma \frac{V}{c^2} W_p , \tag{30}$$

where

$$Q_y = \sqrt{1 + \frac{2 a_y H_p}{u_y^2}}, \quad Q_y' = \sqrt{1 + \frac{2 a_y' H_p}{u_y'^2}}. \tag{31}$$

Just as Eq. (27) contains the connection between $a_x$ and $a_x'$, so Eq. (30) contains the desired connection between $a_y$ and $a_y'$. However, Eq. (30) contains both $W_p$ and $H_p$, which slightly complicates matters. Note that Eqs. (12) and (15) imply that

$$\gamma \frac{V}{c^2} = \left( \frac{u_y'}{u_y} - \gamma \right) \frac{1}{u_x'} , \tag{32}$$

and Eqs. (19) and (20) lead to

$$\frac{W_p}{u_x'} = \frac{H_p}{u_y'} - \frac{1}{2} \left( \frac{a_y'}{u_y'} - \frac{a_x'}{u_x'} \right) \frac{H_p^2}{u_y'^2} + 0(H_p^3). \tag{33}$$

After combining Eqs. (30)–(33), I find that the first-order terms in $H_p$ cancel. I then divide by $H_p^2$ and take the limit as $H_p \to 0$. In this way I obtain

$$a_y = \left( \frac{u_y}{u_y'} \right)^2 a_y' + \left( \gamma - \frac{u_y'}{u_y} \right) \left( \frac{u_y}{u_y'} \right)^3 \left( \frac{u_y'}{u_x'} \right) a_x' . \tag{34}$$

With the use of Eq. (15), Eq. (34) can be reduced to

$$a_y = \frac{1}{\gamma^2 \left( 1 + \frac{u_x' V}{c^2} \right)^2} a_y' - \frac{\frac{u_y' V}{c^2}}{\gamma^2 \left( 1 + \frac{u_x' V}{c^2} \right)^3} a_x' . \tag{35}$$

In exactly the same way, it follows that

$$a_z = \frac{1}{\gamma^2 \left( 1 + \frac{u_x' V}{c^2} \right)^2} a_z' - \frac{\frac{u_z' V}{c^2}}{\gamma^2 \left( 1 + \frac{u_x' V}{c^2} \right)^3} a_x' . \tag{36}$$

Equations (29), (35), and (36) constitute the relativistic acceleration transformation. Note that this transformation follows from Eqs. (1), (2), (5), (6), (9) to (12), (15), (16), (19), (20), (23), and (24). In other words, given time dilation, length contraction, and the velocity transformation, the acceleration transformation follows from kinematic considerations.

## V. SUMMARY AND DISCUSSION

I have derived the three-dimensional velocity and acceleration transformations of special relativity on the basis of thought experiments that employ an enhanced particle-light clock without using the Lorentz transformation. The derivation of the relativistic velocity transformation is based on time dilation and length contraction, as embodied in Eqs. (1), (2), and (10), and kinematic considerations, as summarized by Eqs. (4) to (7), (9), and (11).

The derivation of the relativistic acceleration transformation is based on time dilation and length contraction, as embodied in Eqs. (1), (2), and (10), the velocity transformation, as given by Eqs. (12), (15), and (16), and kinematic considerations, as summarized by Eqs. (5), (6), (9), (11), (19), (20), (23), and (24).

In using Eqs. (5) and (11) with the same value for the speed of light, $c$, I have explicitly invoked the principle of relativity. All that stands between the principle of relativity and the relativistic velocity and acceleration transformations in my treatment is some discussion and some algebra. The discussion is comparatively simple and largely intuitive. The algebra involves nothing more complicated than solving a quadratic equation. Moreover, the insight concerning the constraint on the clock used in the thought experiment to obtain the relativistic acceleration transformation is a bonus that might be illuminating in other considerations of measurement processes. It is these points that I have in mind when I assert that my derivations of the relativistic velocity and acceleration transformations serve to emphasize the directness and immediacy of the connection between the principle of relativity and its physical consequences.

Nearly a century after the emergence of special relativity, we sometimes encounter the assertion that the velocity transformation, particularly the longitudinal part, that is, the Einstein velocity addition rule, violates common sense. The usual accompanying observation is that this violation of common sense stems from the basis of the velocity transformation in the Lorentz transformation, which itself violates common sense. The fact that my thought experiment leads to the same velocity and acceleration transformations as follow from the Lorentz transformation is an effective rebuttal to this assertion. Moreover, my thought experiments and the associated analysis constitute a powerful tool for revealing the core aspects of special relativity inasmuch as an elementary grasp of algebra and a certain amount of patience are the sole requirements for understanding these derivations.

The thought experiments I have discussed could be actual experiments, because the clock that I have described in Sec. II could be built. To be sure, such an instrument would be subject to limitations. However, the limitations would be technological rather than fundamental in nature. Moreover, some of the limitations of the clock could be accounted for in the analysis of the experiments in which it was used. The most important limitations would probably be the maximum attainable speed, that is, the magnitude of **V**, and the minimum attainable gravitational field in the vicinity of the clock. These limitations could be treated, in part, by miniaturization and by placing the clock in a strategically located spacecraft. In other words, it appears that the thought experiments that I have discussed could be turned into real experiments, with the primary limitations being those imposed by the limits of our technological capabilities. Research in materials, miniaturization, nanotechnology, self-assembling systems, self-replicating systems, and novel propulsion systems could perhaps facilitate progress. One place to begin to learn about some of the relevant efforts is the NASA Institute of Advanced Concepts.[90]

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