Reflection of light from a uniformly moving mirror

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(Received 22 April 2003; accepted 10 June 2004)

We derive a formula for the law of reflection of a plane-polarized light beam from an inclined flat mirror in uniform rectilinear motion by applying the Huygens–Fresnel principle. We then use this formula and the postulates of special relativity to show that the moving mirror is contracted along the direction of its motion by the usual Lorentz factor. The result emphasizes the reality of Lorentz contraction by showing that the contraction is a direct consequence of the first and second postulates of special relativity, and is not a consequence of the relativistic measurement of the length.

[DOI: 10.1119/1.1778390]

I. INTRODUCTION

Experiments involving moving mirrors are among the most interesting experiments encountered in physics. Michelson's apparatus for measuring the speed of light with a rotating wheel consisting of mirrored edges, an array of corner mirrors on the Moon's surface for estimating the distance between the Earth and the Moon, the Michelson and Morley interferometer for detecting the ether, and the rotating Sagnac interferometer for determining the angular velocity of the Earth are just a few of the experiments in which moving mirrors have had prominent roles. In most textbooks that discuss these experiments, it is implicitly assumed that the ordinary law of reflection of light is valid, that is, the angles of incidence and reflection are equal. Our goals in this paper are to show that the ordinary law of reflection does not hold when the mirror is moving at a constant velocity and to find a correct relation between the incident and the reflected angle.

A discussion of the reflection of light from a uniformly moving mirror is not new.[1] A particular case of the problem was elaborated by Einstein almost a century ago.[2] Einstein considered the oblique incidence of a plane-polarized electromagnetic wave on a perfectly reflecting mirror whose velocity was directed perpendicularly to its surface. To derive the equations for the angle of reflection and the wave characteristics of the reflected light, Einstein Lorentz-transformed the equations describing the reflection in the reference frame where the mirror was at rest.

In the following we will use a different approach based on elementary principles of wave optics and the postulates of special relativity. Its simplicity could bring this important problem into the undergraduate classroom and stimulate student thinking and intuitive understanding of the basic principles of special relativity.

II. HUYGENS' CONSTRUCTION

The Huygens' construction is usually employed to trace the path of an arbitrary light beam through a medium.[3] Every point that belongs to the primary wave front at some fixed time serves as a source of secondary wavelets which propagate in all directions with the same frequency and velocity as the primary wave front. The envelope of these wavelets is the wave front of the light beam at a later time. If the light propagates in an optically isotropic medium, the light ray can be constructed as a line normal to every subsequent wave front at all times.

We will implement a microscopic version of the Huygens–Fresnel principle to describe the reflection of light by an inclined plane mirror moving at constant velocity v in vacuum (see Fig. 1). The inclination angle φ of the moving mirror determines the orientation of the mirror's surface with respect to the negative x-axis. Let 1 and 2 label the boundary rays of the incident plane-polarized light beam, and the distance $\overline{AB}$ the wave front of the incoming light at some time t_0. The atoms at point A are disturbed by the incident wave front $\overline{AB}$ and begin to radiate a wavelet. The wave front $\overline{AB}$ continues to disturb the atoms along the surface of the moving mirror. The disturbance due to the wave front $\overline{AB}$ stops at time t when the wave front strikes point D. The atoms at point D become a source of wavelets $(t-t_0)$ seconds after the initial disturbance of the atoms at point A. If the mirror is stationary, an elementary wave front emitted from a source on the mirror's surface would be an expanding sphere whose radius would equal $c\tau$, where τ is the time interval from the beginning of the emission, and c is the speed of light in vacuum. If the mirror is in uniform rectilinear motion, the elementary wave front will remain a sphere, expanding equally in all directions at the same constant speed c.[4] This statement is a direct consequence of the second postulate of special relativity that the speed of light in vacuum is a universal constant and its value c is independent of the motion of the source. This statement also follows from the fact that the equation that describes the evolution of an elementary wave front in vacuum is invariant under a Lorentz transformation.[5] Consequently, at the moment when the incident wave front $\overline{AB}$ reaches point D, the elementary wave front emitted from the source at A is a sphere with radius $\overline{AC}=c(t-t_0)$.

The motion of the mirror causes the elementary sources of the secondary wavelets to lie along the straight line connecting points A and D. The envelope of all these elementary wavelets is the distance $\overline{CD}$, which is the reflected wave front, and $1'$ and $2'$ label its boundary rays (see Fig. 1). It is obvious that the optical disturbance of every point along $\overline{CD}$ has the same phase. We denote by α the angle of incidence of the wave front relative to the normal n of the mirror's surface, and by β the angle of its reflection. If $v=0$, we can

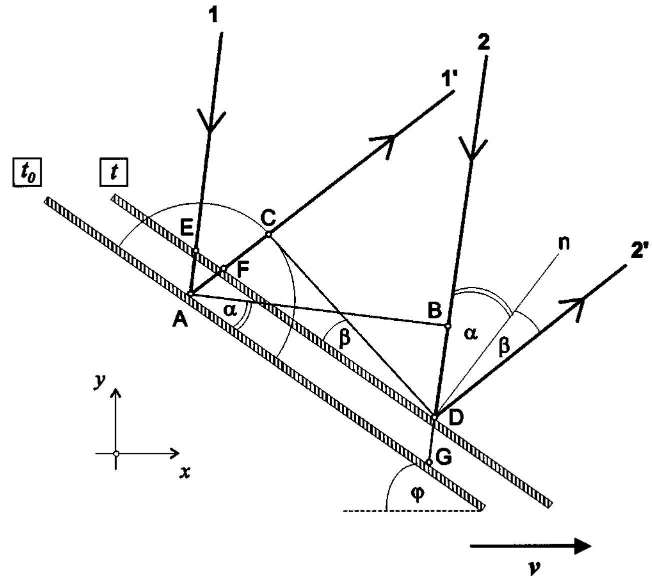

Fig. 1. Huygens' construction of the reflected wave front when the mirror is moving at constant velocity v along the positive direction of the x axis.

prove by using certain triangle similarities the well-known relation $\alpha=\beta$.[6] However, the ordinary law of reflection is not valid when the mirror is moving.

We now give a brief description of a shifting phenomenon, whose appearance is a result of the mirror's motion and the finite profile of the incident light (see Fig. 2). If we take the incident light beam to emerge from a stationary source, then the wave front $\overline{A'B'}$, which is incident on the moving mirror after the initial wave front $\overline{AB}$, will be reflected at the same angle β as the wave front $\overline{AB}$, but from slightly different points on the mirror's surface. Hence, the boundary rays $1''$ and $2''$ of the reflected wave front $\overline{C'D'}$ will not coincide with the boundary rays $1'$ and $2'$ of the initially reflected wave front $\overline{CD}$, but will be shifted an infinitesimal distance from $1'$ and $2'$ (see Fig. 2). We note that the reflected wave fronts will be parallel to each other and will have equal widths. This shift continues with each consecutive wave

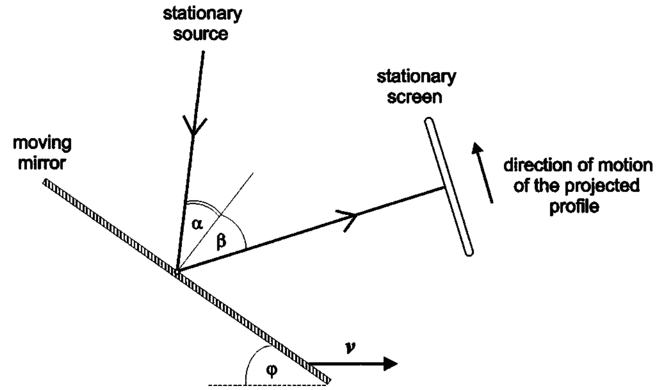

Fig. 3. As a consequence of the motion of the mirror, the profile of the reflected beam projected on a stationary screen will be moving at constant velocity.

front until the moving mirror "escapes" the incident light. If a flat screen is placed against the reflected light, the projected profile of the reflected beam will not be fixed, but, as a consequence of this shift, will move at constant velocity (see Fig. 3).

We now concentrate on the wave front $\overline{AB}$ and its reflected analogue $\overline{CD}$. From Fig. 1, we have

$$\sin\alpha=\frac{\overline{BD}+\overline{DG}}{\overline{AG}}, \tag{1}$$

$$\sin\beta=\frac{\overline{AC}-\overline{AF}}{\overline{AG}-\overline{EF}}. \tag{2}$$

We have taken into account that $\overline{ED}=\overline{AG}$. From the discussion associated with Fig. 1, we have

$$\overline{AC}=\overline{BD}=c(t-t_0). \tag{3}$$

Figure 4 is an enlargement of the area around point A. Observe that $d=\overline{AO}=v(t-t_0)\sin\varphi$ is the shortest distance between the positions of the moving mirror at times t_0 and t. We thus obtain the following relations:

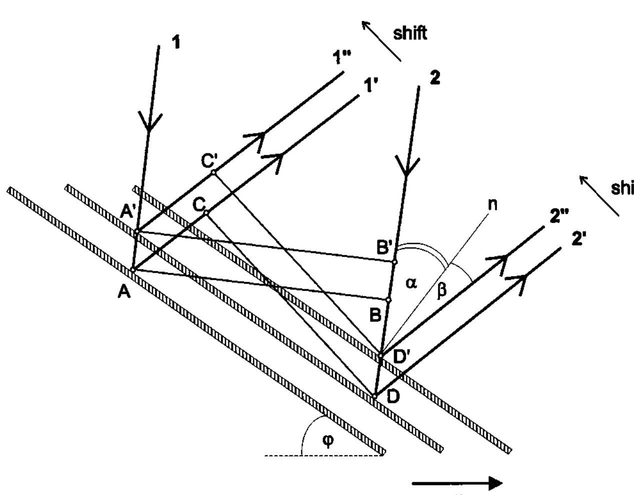

Fig. 2. Schematic description of the shift due to the motion of the mirror. The distances between the consecutive wave fronts and between the boundary rays of the reflected wave fronts are exaggerated for convenience.

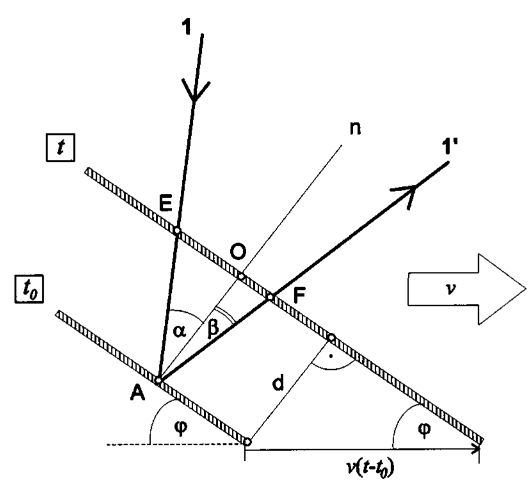

Fig. 4. Magnification of the area around the initially disturbed point A on the moving mirror.

$$\overline{DG}=\overline{AE}=\frac{\overline{AO}}{\cos\alpha}=\frac{v(t-t_0)\sin\varphi}{\cos\alpha}, \tag{4}$$

$$\overline{AF}=\frac{\overline{AO}}{\cos\beta}=\frac{v(t-t_0)\sin\varphi}{\cos\beta}. \tag{5}$$

From the triangles AEO and AFO, we have $\overline{EO}=\overline{AO}\tan\alpha$ and $\overline{OF}=\overline{AO}\tan\beta$. But $\overline{EF}=\overline{EO}+\overline{OF}$, which leads to

$$\overline{EF}=v(t-t_0)\sin\varphi(\tan\alpha+\tan\beta). \tag{6}$$

We substitute Eqs. (3)–(6) into Eqs. (1) and (2) and obtain

$$\sin\alpha=\frac{c+v\,\dfrac{\sin\varphi}{\cos\alpha}}{\dfrac{\overline{AG}}{(t-t_0)}}, \tag{7}$$

$$\sin\beta=\frac{c-v\,\dfrac{\sin\varphi}{\cos\beta}}{\dfrac{\overline{AG}}{(t-t_0)}-v\sin\varphi(\tan\alpha+\tan\beta)}. \tag{8}$$

The straightforward elimination of the term $\overline{AG}/(t-t_0)$ from Eqs. (7) and (8) and a little algebra results in a relation that links the incident angle α and the reflected angle β:

$$\sin\alpha-\sin\beta=\frac{v}{c}\sin\varphi\sin(\alpha+\beta). \tag{9}$$

Equation (9) is the law of reflection of light from an inclined flat mirror in uniform rectilinear motion. Obviously, when the mirror is at rest ($v=0$) or when its inclination angle is zero ($\varphi=0$), the angles of incidence and reflection are equal. If $\alpha=0$, then $\beta=0$ for any φ and v. When the angle of incidence differs from zero ($\alpha\neq 0$), Eq. (9) can be rewritten in the more compact form

$$\frac{\sin\alpha-\sin\beta}{\sin(\alpha+\beta)}=\frac{v}{c}\sin\varphi. \tag{10}$$

By following a similar procedure, we can show that the law of reflection of the light when the mirror is moving in the opposite direction to the one shown in Fig. 1 is

$$\frac{\sin\alpha-\sin\beta}{\sin(\alpha+\beta)}=-\frac{v}{c}\sin\varphi. \tag{11}$$

Equation (11) also can be obtained from Eq. (10) by letting $-v$ instead of v, or, equivalently, $-\varphi$ instead of φ.

At first sight it appears that Eqs. (9)–(11) are transcendental equations. However we can derive an expression for the angle of reflection (incidence) directly in terms of the angle of incidence (reflection), the inclination angle φ, and the velocity v of the moving mirror. The procedure is described in Problem 3 of the Appendix for a special arrangement, but it also can be derived in general when the moving mirror makes an arbitrary angle φ with respect to the negative direction of its velocity vector. The reader is encouraged to show that Eq. (10) leads to the following formula for the reflected angle β:

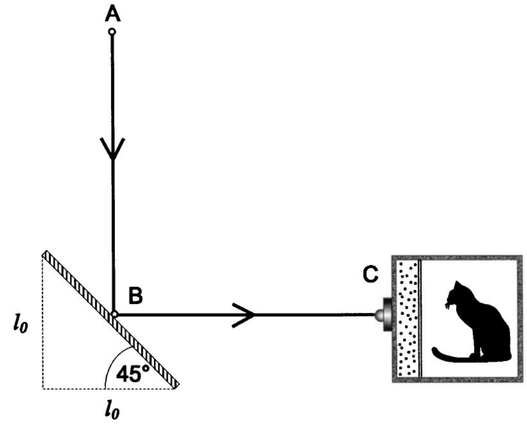

Fig. 5. Einstein's cat experiment.

$$\cos\beta=\frac{2\dfrac{v}{c}\sin\varphi+\left(1+\dfrac{v^2}{c^2}\sin^2\varphi\right)\cos\alpha}{1+2\dfrac{v}{c}\sin\varphi\cos\alpha+\dfrac{v^2}{c^2}\sin^2\varphi}. \tag{12}$$

To make the derivations in the following as simple as possible, we will use the law of reflection given in Eqs. (9)–(11).

We note that the obvious asymmetric treatment of the angles of incidence and reflection in Eq. (10) has an important consequence. Namely, if the reflected light ray becomes incident, it will not be reflected at the angle α, but at a different angle δ that is a solution of

$$\frac{\sin\beta-\sin\delta}{\sin(\beta+\delta)}=\frac{v}{c}\sin\varphi. \tag{13}$$

In other words, the principle of reversibility of the light rays does not hold if the light is reflected by a moving mirror.

We emphasize that the inclination angle φ is an important constituent of the law of reflection given by Eq. (9). Although the derivation of Eq. (9) was based on the second postulate of special relativity, the angle φ is a real physical entity, which, by itself, has nothing to do with relativity. The value of φ is neither a result of an act of measurement, nor a result of an act of seeing.[7] To make this point clear, we urge the reader to recall that while applying the Huygens–Fresnel principle in the derivation of Eq. (9), we assumed that the surface of the moving mirror was made of atoms, each of which was moving at the same constant velocity v as the mirror, and each of which radiates secondary wavelets if disturbed by the incident light. The inclination angle φ was introduced to describe the moving plane on which these atoms are located. Thus, the angle φ is the physical angle that the mirror makes at any time with respect to the negative direction of its velocity. We will utilize this fact in Sec. III to derive an important inherent property of an inclined flat mirror in uniform rectilinear motion.

III. EINSTEIN'S CAT EXPERIMENT

Consider the experimental setup shown in Fig. 5. The light beam emanating vertically downward from the light source at A is reflected by a flat mirror at the point B. The mirror is inclined at $\pi/4$ rad with respect to the negative x-axis. The reflected beam hits point C, which belongs to a chamber

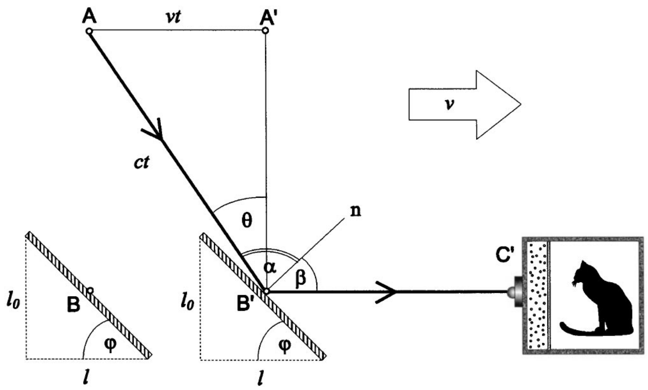

Fig. 6. The experiment shown in Fig. 5, observed in a reference frame where the setup is in uniform rectilinear motion. The bold line $AB'C'$ is not the path of the light beam itself, but a path of a single wave front emitted at the instant of time when the light source was positioned at point A.

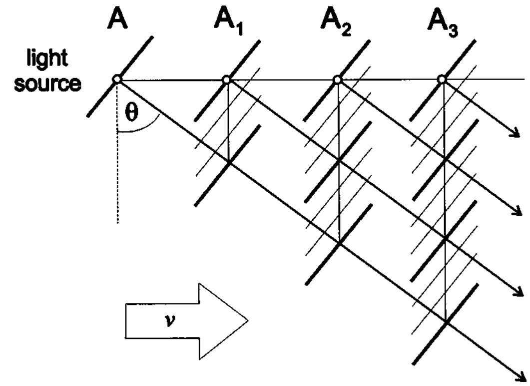

Fig. 7. Several consecutive snapshots of the propagation of the wave fronts from the moving source to the moving mirror. The snapshots were taken at times when the light source was positioned at the points A, A_1, A_2, and A_3, that is, along the line of its motion. At each time, all the wave fronts will be lined up along the vertical line connecting the moving source and the moving mirror. Observe that the plane of every propagating wave front is perpendicular to the wave front's path, but not to the vertical line along which the light beam advances.

consisting of two rooms separated by a surface with electronically controlled permeability. By hitting point C, the beam activates a life-supporting mechanism that prevents the poisonous gas from entering the room where the cat is located by acting on the permeability of the protecting surface.

What happens when the experiment is observed from a frame in which the setup is moving to the right at velocity v? We can follow the traditional discussion in most textbooks and say that Fig. 6 shows the path of the light beam when the setup is in uniform rectilinear motion.[8] However, this answer is not correct, as we shall see in the following. We offer an explanation by using the principles of special relativity.

The reader must be cautious and recognize that the line $AB'C'$ in Fig. 6 is not the path of the light beam itself, but a path traversed by a single wave front, the one emitted at the time when the source of the light beam was at point A. All the wave fronts emitted at later times will be emitted by the same source, but from different points in space. Due to the motion of the source, these points will be located to the right of point A. The plane of every single wave front on its journey from the moving source to the moving mirror will be perpendicular to the wave front's trajectory, and the velocity of the wave front along its trajectory will be constant and equal to c.[9] The latter statement is Einstein's second postulate at work. The mirror obviously has the same velocity v as the source, which means that at any time along the course of their motion, the moving source and the moving mirror will lie on the same vertical line.

By looking at Fig. 6 we see that the distance traversed by the initially emitted wave front from its source at A to the mirror at B' is $\overline{AB'}=ct$, where t is the time required for the wave front to cover the distance $\overline{AB'}$ at velocity c. The distance traveled by the light source, or equivalently, by the mirror, for the same time, is $\overline{BB'}=\overline{AA'}=vt$. Consequently, from the triangle $AA'B'$ we have

$$\sin\theta=\frac{v}{c}, \tag{14}$$

which is the well-known formula for the aberration of light.[10] The path from the moving source to the moving mirror traversed by every subsequently emitted wave front will be parallel to the path $\overline{AB'}$ of the initially emitted wave front. At each instant of time, all the wave fronts propagating from the moving source to the moving mirror will be lined up along the vertical line connecting the moving source and the moving mirror (see Fig. 7). They will be reflected by the moving mirror from the same atoms along its surface as the initially reflected wave front. Therefore, if the experiment in Fig. 5 is observed from a reference frame traveling to the left at constant velocity v, it would appear that the light beam is advancing at velocity $c\sqrt{1-v^2/c^2}$ along the vertical line connecting the moving source and the moving mirror, while, at the same time, the whole setup (including the light beam) is moving at velocity v to the right (Fig. 8).

According to Einstein's first postulate (the principle of relativity), all inertial frames are equivalent (there are no preferred inertial reference frames), and the laws of physics are identical in all of them. In this case, it means that if light is hitting the switch of the chamber in the reference frame where the setup is at rest (Fig. 5), then the light must hit the

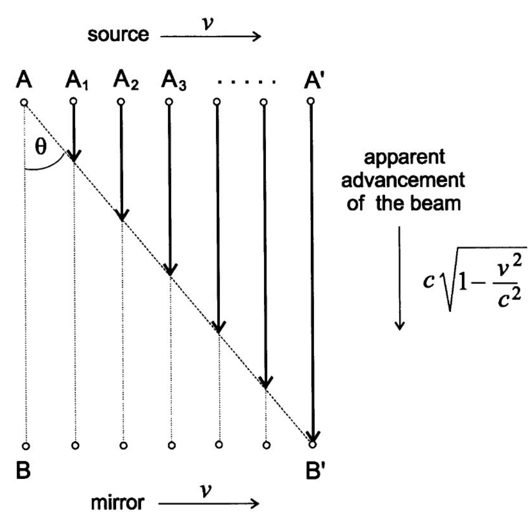

Fig. 8. Several consecutive snapshots of the advancement of the light beam from the source to the mirror, observed from a reference frame traveling to the left at constant velocity v. While the light beam is advancing at a speed $c\sqrt{1-v^2/c^2}$ along the vertical line between the source and the mirror, the beam as a whole is moving at velocity v together with the rest of the setup.

switch of the chamber in every other inertial reference frame, regardless of the direction of motion of the setup. In simple words, if the cat is alive in one inertial reference frame, then the cat must stay alive in every other inertial reference frame. Our previous considerations require that every single wave front must be reflected by the moving mirror, and, in the present case (Fig. 6), must follow the horizontal in order to hit the switch of the chamber. It might be argued that the light beam will hit the mirror at some point other than B', or that the light is somehow hitting C' directly, and not by reflection from the moving mirror. However, we can modify the setup in Fig. 5 by making the point B an on–off switch of another life-supporting mechanism belonging to another cat in a chamber. Then, in the frame of reference where the setup is moving (Fig. 6), the beam will miss the switch at point B', the second life-supporting mechanism will not be activated, and the cat will be dead. Hence, we conclude that the light beam must hit the moving mirror at B'.

Now that we are convinced that the situation shown in Fig. 6 is correct, but represents an individual wave front (the one emitted from the source at A), we can verify the consistency of Eq. (9). We will assume that Eq. (9) correctly describes the reflection of the wave fronts of the incident light from the surface of the moving mirror. We also will assume that the inclination angle φ of the moving mirror may differ from the inclination angle of the stationary mirror (in our case, $\pi/4$ rad). From Fig. 6 we express the incident angle α and the reflected angle β as

$$\alpha=\theta+\varphi, \tag{15}$$

$$\beta=\pi/2-\varphi. \tag{16}$$

By substituting Eqs. (15) and (16) into Eq. (10), we have

$$\frac{\sin(\theta+\varphi)-\sin(\pi/2-\varphi)}{\sin(\theta+\varphi+\pi/2-\varphi)}=\frac{v}{c}\sin\varphi. \tag{17}$$

After some rearrangements, and using Eq. (14), we obtain

$$\tan\varphi=\frac{1}{\cos\theta}=\frac{1}{\sqrt{1-\sin^2\theta}}=\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{18}$$

Equation (18) shows that $\tan\varphi\neq 1$, which means that our assumption was correct and the moving mirror really has a different inclination angle than the angle for the stationary mirror. We emphasize that the lateral dimensions of an object do not change when the object is in uniform rectilinear motion,[11] which implies that the change of the inclination angle of the mirror due to its uniform motion can be caused only by a change of the mirror's dimensions parallel to the direction of its motion. With this assertion in mind, we can express the inclination angle of the moving mirror as

$$\tan\varphi=\frac{l_0}{l}, \tag{19}$$

where l_0 denotes the projection of the mirror's length on the axis perpendicular to its motion, and l is the projection of the mirror's length on the direction of its motion. Then, by substituting Eq. (19) into Eq. (18), we have

$$l=l_0\sqrt{1-\frac{v^2}{c^2}}, \tag{20}$$

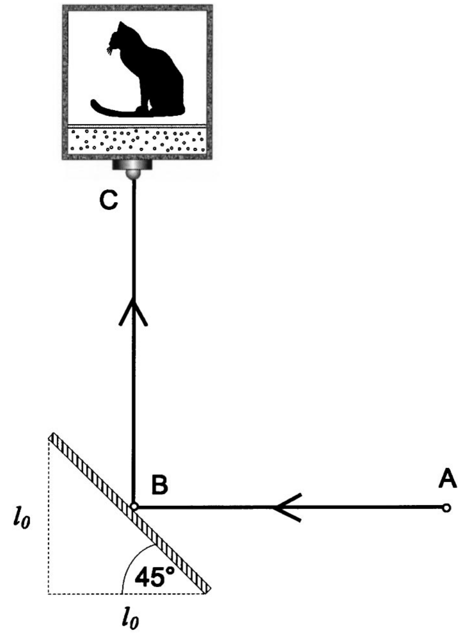

Fig. 9. Einstein's cat experiment 2. The positions of the light source and the chamber are interchanged.

which is a well-known formula in special relativity, commonly called the Lorentz contraction of the length. Equation (20) states that an inclined flat mirror moving at constant velocity v will be Lorentz contracted along the direction of its motion.

Let us explore an identical apparatus, but we will exchange the positions of the light source A and the chamber C. The setup with respect to the reference frame where the mirror is at rest is shown in Fig. 9. If we follow the same arguments as in the previous case, we can demonstrate that Fig. 10 shows the situation (for an individual wave front) ob-

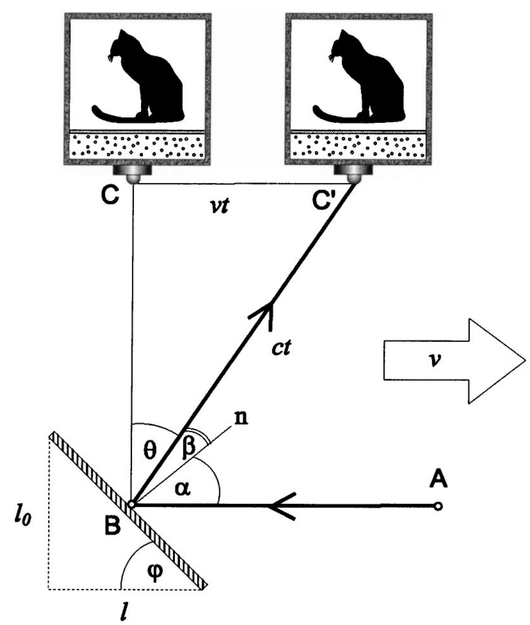

Fig. 10. The experiment shown in Fig. 9, observed in a reference frame where the setup is in uniform rectilinear motion.

served from a frame in which the setup is in motion. In this case,

$$\alpha = \pi/2 - \varphi, \tag{21}$$

$$\beta = \varphi - \theta. \tag{22}$$

We substitute Eqs. (21) and (22) into Eq. (9), take into account Eqs. (14) and (19), and obtain

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}. \tag{20}$$

The mirror will be shortened along the line of its motion in the same manner as in the previous case.

By assuming Eq. (9) is correct and using Einstein's postulates, we were able to derive the relativistic contraction formula for an inclined flat mirror in uniform rectilinear motion, which implies that Eq. (9) correctly describes the propagation of the wave front reflected by a moving mirror. If we assume that instead of Eq. (9), the usual law of reflection is correct (that is, the angle of incidence equals the reflected angle), we would arrive at peculiar results, not just contradictory to the predictions of special relativity (see Problem 1 in the Appendix).

If the ray optics of the original Einstein's cat experiment shown in Fig. 5 is modified, for example, the inclination angle of the stationary mirror is not equal to $\pi/4$ while the position of the source and the angle of the emerging light are adjusted such that the path of the reflected beam remains unchanged, that is, it follows the horizontal in order to hit the switch at the point C, then the effect of relativistic contraction of the moving mirror along its velocity vector would still be in accordance with Eq. (20). Different versions of Einstein's cat experiment can be used as homework problems.

IV. DISCUSSION

We emphasize that the expression for the law of reflection of light from a uniformly moving mirror in Eq. (9) is a consequence of the constant speed of light postulate. While deriving Eq. (9), we argued that as a result of Einstein's second postulate, each elementary wave front originating from a source on the moving mirror's surface will preserve its shape with respect to the situation when the mirror is stationary. The shape of the wave front will remain a sphere, expanding equally in all directions at constant speed c. The rest of the derivation is a standard Huygens' construction.

By making use of the atomic version of the Huygens–Fresnel principle, we showed that the angle of reflection of the light depends on the angle of incidence, the inclination angle φ, and the velocity v of the moving mirror. For a specific thought experiment, we concluded that the inclination angle of the moving mirror would differ from its inclination angle if the mirror were stationary. Because we previously stressed that the inclination angle φ is the physical angle that the mirror makes at any instant with respect to the negative direction of its velocity, we conclude that this tilt of the mirror due to its motion is a real effect. We further showed that this tilt is a consequence of the fact that the physical length of the moving mirror in the direction of its motion is less than the physical length of the same mirror at rest by the usual Lorentz factor.

In the special theory of relativity the length of an object in a given inertial frame is defined as the distance between any two simultaneous events that occur at the object's ends. According to this definition and the Lorentz transformation, an object moving at constant velocity will be Lorentz contracted along the direction of its motion. A common interpretation of the effect of Lorentz contraction is that relativistic contraction is not a real, physical contraction, but an apparent, artificial phenomenon, whose presence is solely due to the process of relativistic measurement of the length.[12] By a relativistic measurement we mean a measurement performed by a stationary observer with a stationary measuring equipment (rulers, clocks, etc.) and interpreted using the notion of simultaneity. According to this interpretation, the length of a uniformly moving object will be measured to be Lorentz contracted along the direction of its motion, while its physical length will remain the same no matter what reference frame one uses to describe it.[13] The effect of Lorentz contraction does not exist apart from the measuring process. It occurs only when the length of an object is measured in a relativistic sense. However, we showed that Lorentz contraction is a real effect and is an inherent physical property of a mirror (and, therefore, of any object) in uniform rectilinear motion.[14] As a physical property of an object in motion, Lorentz contraction is not just a result of the process of relativistic measurement of the object's length.

While describing Einstein's cat experiment, we argued that some important points are usually omitted or not correctly discussed. We showed that the bold line depicted in Fig. 6 is not the path of the light beam itself, but a path of a single wave front emitted at the instant of time when the light source was positioned at point A. The actual advancement of the light beam from the source to the mirror, with respect to the frame of reference in which the setup is in motion, is described in Fig. 8. There should be no confusion with the statement that the light beam is advancing at a speed $c\sqrt{1-v^2/c^2}$ along the vertical line between the source and the mirror. This statement does not contradict Einstein's second postulate. At the same time the beam as a whole is moving at velocity v together with the rest of the setup. The motion of the beam at velocity v in the direction of motion of the setup combined with the advancement of the beam at velocity $c\sqrt{1-v^2/c^2}$ along the vertical line between the moving source and the moving mirror will cause every wave front along the beam to possess a net constant speed c whose direction is determined by the aberration angle θ.

If these considerations are not taken into account, we would encounter a paradoxical situation. Suppose that the path in Fig. 6 is the real path taken by the light beam. Then for the case $v = 0.85c$ for example, the angle of incidence α will be greater than $\pi/2$ even without taking relativistic contraction into account. (Detailed calculations with and without Lorentz contraction are left to the student as an exercise.) Hence, the light beam will strike the back side of the mirror, and, therefore, it will never reach the switch at C'. Consequently the cat would be dead.

The reflection of light from a uniformly moving mirror is commonly neglected in standard textbooks on optics. Surprisingly, the topic appears to be unexplored even by advanced treatments of relativistic electrodynamics. The problem dates back to Einstein's monumental work on special relativity. Einstein correctly solved the problem in the framework of the newly developed theory. We have shown that the

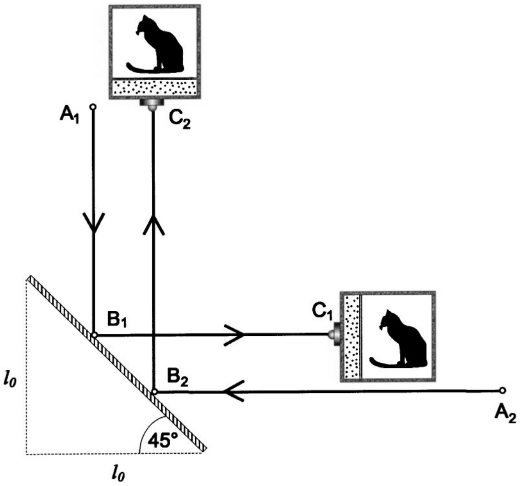

Fig. 11. Modified version of Einstein's cat experiment.

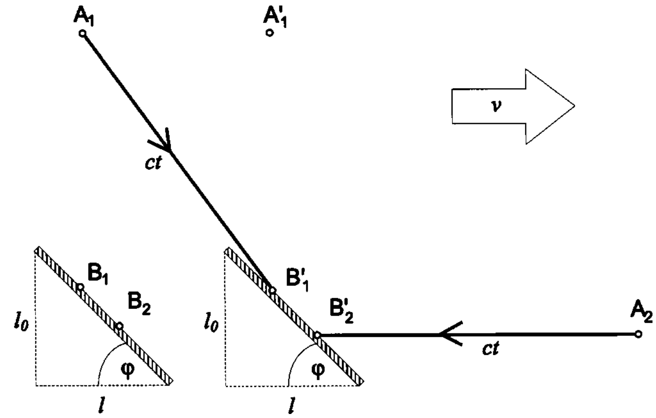

Fig. 12. In the reference frame where the setup is in uniform rectilinear motion, the wave fronts emitted simultaneously from the light sources A_1 and A_2 will hit the moving mirror at the same time.

subject can be approached in a way that is accessible to undergraduate students by using elementary ray-tracing and the postulates of special relativity.

ACKNOWLEDGMENTS

I would like to acknowledge the anonymous referee for a thorough revision process and for a set of very useful, constructive, and thought-provoking comments and suggestions, without which the key idea of this article would not have been written. This work was partially sponsored by Professor Hendrik Ferdinande (University of Ghent) and Professor Viktor Urumov (Sts. Cyril and Methodius University) under TEMPUS JEP #13576-98.

APPENDIX: PROBLEMS

Problem 1. Failure of the classical law of reflection. By using the postulates of special relativity, show that for the situations depicted in Figs. 6 and 10 the angles of incidence and reflection cannot be equal.

Solution. Assume that for the setup shown in Fig. 6, the angles of incidence and reflection are equal, that is, $\alpha=\beta$. Then, by equating the right-hand sides of Eqs. (15) and (16), we obtain $\theta=\pi/2-2\varphi$. From Eq. (14), we have

$$\sin\theta=\sin(\pi/2-2\varphi)$$
$$=\cos 2\varphi=\cos^2\varphi-\sin^2\varphi=\frac{1-\tan^2\varphi}{1+\tan^2\varphi}=\frac{v}{c}. \tag{A1}$$

If we apply Eq. (19), we would conclude that the mirror is elongated in the direction of v according to

$$l=l_0\sqrt{\frac{1+v/c}{1-v/c}}. \tag{A2}$$

If we repeat the entire procedure for the setup in Fig. 10 under the same assumption $\alpha=\beta$, we would conclude that the mirror will be shortened as

$$l=l_0\sqrt{\frac{1-v/c}{1+v/c}}. \tag{A3}$$

We might speculate that the moving mirror is somehow adjusting itself toward the light beam, and its "rotation" depends on the angle of incidence of the incoming wave front. Now let us investigate the setup depicted in Fig. 11. This setup is a combination of the setups shown in Figs. 5 and 9. There are two parallel light beams emerging from different sources and hitting two different chambers after being reflected from a stationary mirror. The position of the second source A_2 is adjusted horizontally toward the mirror such that when the whole experiment is observed from the reference frame where the setup is moving at constant velocity v to the right (Fig. 12), after a simultaneous flash of the light sources A_1 and A_2, the two initially emitted wave fronts will hit the mirror at the same time. In the frame of reference where the setup is in motion, at the instant of time when these two wave fronts will hit the mirror, the mirror will be elongated and shortened simultaneously according to Eqs. (A2) and (A3). The resulting contradiction proves that the assumption $\alpha=\beta$ is incorrect.

Problem 2. Michelson–Morley experiment. Prove that in the Michelson–Morley experiment the light rays leaving the interferometer will meet on parallel paths.[15–18]

Solution. Figure 13 is a schematic of the Michelson–Morley interferometer in uniform rectilinear motion at constant velocity v to the right. The incident light beam is divided into two beams by a half-silvered mirror at point A. After traversing different paths in the apparatus, the wave fronts of these two beams are recombined at point A''. Be-

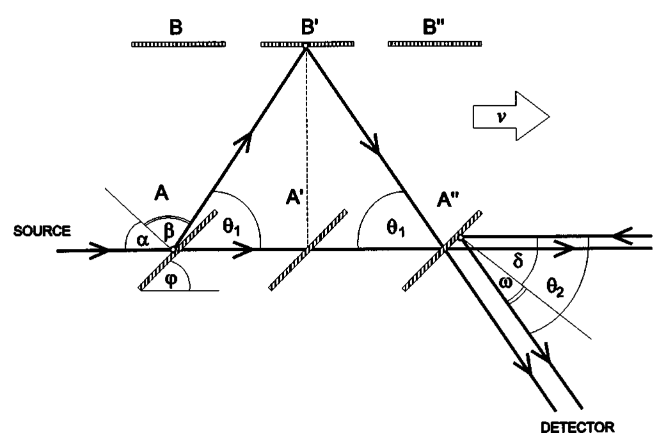

Fig. 13. The setup of Michelson and Morley in uniform rectilinear motion.

fore recombining at A'', the wave front of the second (horizontal) light beam was previously reflected by a third mirror not shown. The law of reflection of the wave front reflected at point A states

$$\frac{\sin\alpha-\sin\beta}{\sin(\alpha+\beta)}=-\frac{v}{c}\sin\varphi. \tag{A4}$$

By looking at Fig. 13 we see that the following relations are valid:

$$\alpha=\pi/2-\varphi, \tag{A5}$$

$$\beta=\pi/2+\varphi-\theta_1. \tag{A6}$$

The substitution of Eqs. (A5) and (A6) into Eq. (A4) would yield a result that can be simplified to give

$$\theta_1=2\arctan\sqrt{\frac{1-v/c}{1+v/c}}. \tag{A7}$$

The wave front reflected at A strikes the second mirror at B'. In this case, the angles of incidence and reflection are equal. This equality follows from Eq. (10) if we take into account that the inclination angle of the mirror at the point B' equals π.

For the wave front reflected at point A'' we have

$$\frac{\sin\delta-\sin\omega}{\sin(\delta+\omega)}=\frac{v}{c}\sin\varphi, \tag{A8}$$

$$\delta=\pi/2-\varphi, \tag{A9}$$

$$\omega=\theta_2-\pi/2+\varphi. \tag{A10}$$

If we substitute Eqs. (A9) and (A10) into Eq. (A8) and do some algebra, we obtain

$$\theta_2=2\arctan\sqrt{\frac{1-v/c}{1+v/c}}. \tag{A11}$$

We conclude that $\theta_1=\theta_2$, which means that the light rays emerging from the apparatus will meet on parallel paths. The reader can show that Eq. (A11), or, equivalently, Eq. (A7), can be simplified to

$$\tan\theta_{1,2}=\frac{c}{v}\sqrt{1-\frac{v^2}{c^2}}. \tag{A12}$$

The result is identical to the one obtained by Schumacher.[15]

Problem 3. Einstein's mirror. Einstein derived the law of reflection of a plane-polarized electromagnetic wave from a flat mirror moving at constant velocity v in vacuum (see Fig. 14).[2] By applying Lorentz transformations to the equations derived in the reference frame where the mirror was at rest, he arrived at

$$\cos\beta=\frac{\left(1+\frac{v^2}{c^2}\right)\cos\alpha-2\frac{v}{c}}{1-2\frac{v}{c}\cos\alpha+\frac{v^2}{c^2}}. \tag{A13}$$

Show that this result also can be obtained from Eq. (9).

Solution. We let $\varphi=\pi/2$ and write $-v$ instead of v, so that Eq. (9) becomes

$$\sin\alpha-\sin\beta=-\frac{v}{c}\sin(\alpha+\beta). \tag{A14}$$

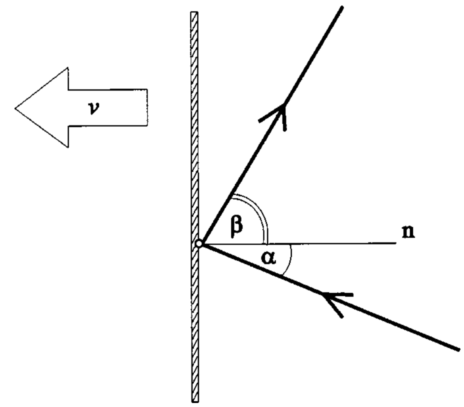

Fig. 14. Reflection of an electromagnetic wave by a vertical mirror moving at constant velocity v to the left.

We use some basic trigonometric identities and some algebra to obtain

$$\left(1+\frac{v}{c}\cos\beta\right)\sin\alpha=\left(1-\frac{v}{c}\cos\alpha\right)\sin\beta. \tag{A15}$$

By taking the square of Eq. (A15) and rearranging the terms, we obtain a quadratic equation in $\cos\beta$,

$$\left(1-2\frac{v}{c}\cos\alpha+\frac{v^2}{c^2}\right)\cos^2\beta+2\frac{v}{c}(1-\cos^2\alpha)$$
$$\times\cos\beta+2\frac{v}{c}\cos\alpha-\left(1+\frac{v^2}{c^2}\right)\cos^2\alpha=0, \tag{A16}$$

whose solutions are

$$(\cos\beta)_1=\frac{2\frac{v}{c}\cos^2\alpha-\left(1+\frac{v^2}{c^2}\right)\cos\alpha}{1-2\frac{v}{c}\cos\alpha+\frac{v^2}{c^2}}, \tag{A17}$$

and

$$(\cos\beta)_2=\frac{-2\frac{v}{c}+\left(1+\frac{v^2}{c^2}\right)\cos\alpha}{1-2\frac{v}{c}\cos\alpha+\frac{v^2}{c^2}}. \tag{A18}$$

In the reference frame where the mirror is stationary ($v=0$), the angles of incidence and reflection must be equal. But, if $\alpha=\beta$, then $\cos\alpha=\cos\beta$. By substituting $v=0$ in Eqs. (A17) and (A18), we obtain

$$(\cos\beta)_1=-\cos\alpha, \tag{A19}$$

and

$$(\cos\beta)_2=\cos\alpha. \tag{A20}$$

It follows that the only physically correct solution is

$$(\cos\beta)_2 = \frac{-2\frac{v}{c} + \left(1 + \frac{v^2}{c^2}\right)\cos\alpha}{1 - 2\frac{v}{c}\cos\alpha + \frac{v^2}{c^2}}, \tag{A18}$$

which is identical to Eq. (A13) derived by Einstein in an alternative way.

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[1]For a beautiful historical overview and a good introductory account of the problem of reflection and refraction of electromagnetic waves from moving boundaries, see B. M. Bolotovskii and S. N. Stolyarov, ''Wave reflection from a moving mirror and related problems,'' Usp. Fiz. Nauk **159**, 155–180 (1989).

[2]A. Einstein, ''Zur Elektrodynamik bewegter Körper,'' Ann. Phys. (Leipzig) **17**, 891–921 (1905); reprinted in *Einstein's Miraculous Year: Five Papers That Changed the Face of Physics*, edited by John Stachel (Princeton U.P., Princeton, 1998).

[3]See, for example, M. Born and E. Wolf, *Principles of Optics* (Cambridge U.P., Cambridge, 1999), 7th ed.

[4]G. P. Sastry and T. R. Ravury, ''Modeling some two-dimensional relativistic phenomena using an educational interactive graphics software,'' Am. J. Phys. **58**, 1066–1073 (1990).

[5]C. Møller, *The Theory of Relativity* (Clarendon, Oxford, 1972), 2nd ed.

[6]See, for example, E. Hecht, *Optics* (Addison–Wesley, Reading, MA, 1987), 2nd ed.

[7]A word of caution! The visual appearance of a high-velocity moving object (that is, the process of seeing) and the measurement of its shape (that is, the process of relativistic measurement) are completely different. See, for example, J. Terrel, ''Invisibility of the Lorentz contraction,'' Phys. Rev. **116**, 1041–1045 (1959), and M. L. Boas, ''Apparent shape of large objects at relativistic speeds,'' Am. J. Phys. **29**, 283–286 (1961).

[8]An illustrative example of this misconception is the discussion associated with Fig. 15-3 in R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison–Wesley, Reading, MA, 1989), Vol. 1, p. 15-6. See in particular, part (c) of the caption, which states ''Illustration of the diagonal path taken by the light beam in a moving 'light clock.' ''

[9]The result follows from the requirement that the phase of a plane-polarized electromagnetic wave must be an invariant quantity under a Lorentz transformation. See, for example, Sec. 2.9 in Ref. 5 and J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed., Sec. 11.3(d).

[10]It is worth noticing that Eq. (14) also can be obtained by a direct application of the addition law for relativistic velocities. See, for example, L. Landau and E. Lifshitz, *The Classical Theory of Fields* (Addison–Wesley, Cambridge, 1951). Still another way to derive the aberration formula is discussed in Ref. 9.

[11]The statement that the lateral dimensions of a uniformly moving object remain unchanged with respect to the lateral dimensions of the same object at rest can be easily proved by using only Einstein's first postulate. See, for example, Problem 6.1 in A. A. Pinsky, *Problems in Physics* (MIR, Moscow, 1988).

[12]For a more detailed discussion on the length of a moving object, the process of measurement, and Lorentz contraction in special relativity, see T. A. Moore, *Six Ideas That Shaped Physics, Unit R: The Laws of Physics Are Frame-Independent* (WCB/McGraw–Hill, Boston, 1998).

[13]The physical length of a uniformly moving object would equal the object's proper length, that is, the object's length measured in the reference frame in which the object is at rest.

[14]The statement that Lorentz contraction of a plane mirror in uniform rectilinear motion implies Lorentz contraction of any uniformly moving object can be clarified by imagining the object's surface to consist of an infinite number of infinitely small plane mirrors having different inclination angles with respect to the object's motion.

[15]R. A. Schumacher, ''Special relativity and the Michelson-Morley interferometer,'' Am. J. Phys. **62**, 609–612 (1994).

[16]A. A. Michelson and E. W. Morley, ''On the relative motion of the earth and the luminiferous ether,'' Am. J. Sci. **34**, 333–345 (1887).

[17]E. H. Kennard and D. E. Richmond, ''On reflection from a moving mirror and the Michelson-Morley experiment,'' Phys. Rev. **19**, 572–577 (1922).

[18]R. S. Shankland, ''Michelson-Morley experiment,'' Am. J. Phys. **32**, 16–35 (1964).