Aberration of light in a uniformly moving optical medium

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(Received 7 August 2003; accepted 19 December 2003)

The transverse Fresnel–Fizeau light drag in the presence of a nondispersive homogeneous optical medium in uniform rectilinear motion is explained by using a simple Huygens' construction. As a consequence of the motion of the medium, the wavefront of every individual secondary wavelet is an ellipse partially dragged by the moving medium. The resulting formula agrees with the experiment by Jones in the early 1970s and with Fresnel's formula for transverse light drag.
[DOI: 10.1119/1.1648683]

I. INTRODUCTION

The first step toward a theoretical understanding of the effects on light propagation in moving media was made by Fresnel in his pioneering work in 1818.[1] By considering light as a mechanical wave propagating through the luminiferous ether, Fresnel predicted that the velocity of light in a moving medium would depend on the angle between the light ray and the direction of motion of the medium. According to Fresnel, in the presence of a dispersionless medium in uniform rectilinear motion at constant speed u, light propagating parallel to the motion of the medium would have a velocity component $\pm u(1-1/n^2)$ in addition to its phase velocity c/n which it would have if the medium were at rest. The sign of this additional component depends on whether the light and the medium are moving in the same or opposite directions. The formula for the light velocity was verified experimentally by Fizeau,[2] and since then the effect is known as the longitudinal Fresnel–Fizeau light drag. Later investigations showed that if the dispersion of the medium is taken into account, an extra term also must be considered.[3]

Almost a century later, Fresnel's naive approach was revised, and the resulting formula was confirmed in the framework of Einstein's theory of special relativity[4] and the modern formulation of electrodynamics of moving media.[5] The theory of electrodynamics of moving media has certain advantages over a purely special relativistic approach. It appears to be an irreplaceable mathematical tool for describing the advancement of the wavefront of the light beam through a uniformly moving boundary separating two media, each of which is in uniform rectilinear motion at a different speed in a different direction (see Fig. 1). This example is one for which the standard Lorentz transformation procedure is not applicable, because there is no reference frame in which the boundary and the optical media divided by it are all at rest at the same time. In other words, there exists no reference frame in which Snell's law of refraction takes place, so there is nothing to be Lorentz transformed.[6]

Of particular recent theoretical and experimental interest is light propagation through a nonuniformly moving medium.[7] In this case, the light actually ''sees'' the moving medium as an effective gravitational field. This optical analog of a curved space-time could provide a laboratory test bed for general relativity. One could simply create an optical black hole[8] in an ordinary glass of rotating water, and then, for example, investigate what would be the equivalent of Hawking radiation.[9] However, precise calculations showed that optical black holes would be observable only if the speed of rotation of the medium exceeded the speed of light in the actual medium. A possible way to overcome this difficulty is to utilize a so-called ''slow light'' medium, in which the speed of the light is drastically reduced to several meters per second.[10] These ''slow light'' materials also can be used for increasing the light drag considerably when the medium is moving at constant velocity. Furthermore, under certain conditions, a vortex flow in a fluid can cause other less dramatic, but equally interesting phenomena, such as an optical analog of the Aharonov–Bohm effect.[11]

The purpose of this paper is to provide additional insight into the motion of light in a moving medium. Although we will restrict our attention to optical medium in uniform rectilinear motion, our analysis should be extendible to more complicated situations, such as the case of a nonuniformly moving ''slow light'' material. The last statement can be clarified by the fact that the curved space–time that the light ''feels'' while propagating through a nonuniformly moving medium can be considered as being locally flat, consisting of a large number of different local inertial reference frames related to each other in a way determined by the curvature tensor.[11]

We will consider a hypothetical case of a nondispersive material medium, which should work well if the dispersion in the real material is so small that correcting terms due to dispersion cannot be detected by the measuring equipment. We will argue that Huygens' construction can still be used as a ray-tracing tool in a dispersionless optical medium in uniform rectilinear motion if it is applied to the distorted secondary wavelets. We will investigate a special case of refraction of light, a situation for which the plane surface of the moving medium, on which the light is incident, is parallel to the motion of the medium. The law of refraction obtained in this manner will be used for describing an experiment by Jones,[12,13] in which a light-beam probe is allowed to pass through a rotating disk made of glass, parallel to its axis of rotation. The setup is an analogue to that of Fizeau,[2] but here the incident light enters the medium perpendicularly to the direction of its motion.

Our procedure will replace the sophisticated mathematical apparatus of the electrodynamics of moving media by the simpler problem of using ordinary plane geometry and elementary analysis to analyze the propagation of light in a uniformly moving optical medium.

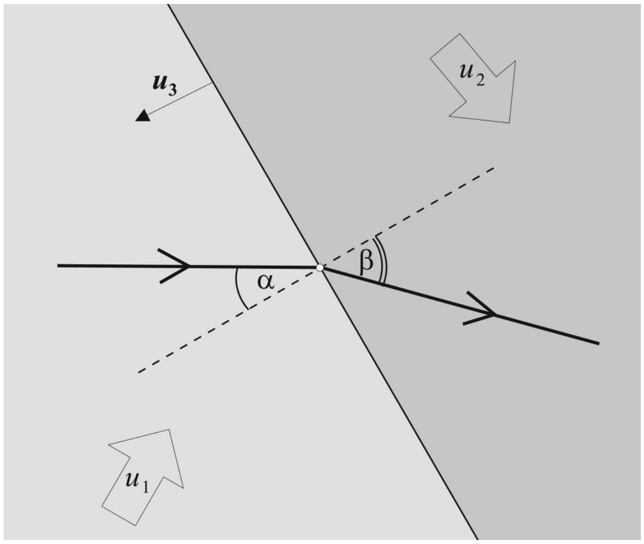

Fig. 1. The general case of light refraction between uniformly moving media. Observe that, in addition to the motion of the media, the boundary also is in uniform rectilinear motion.

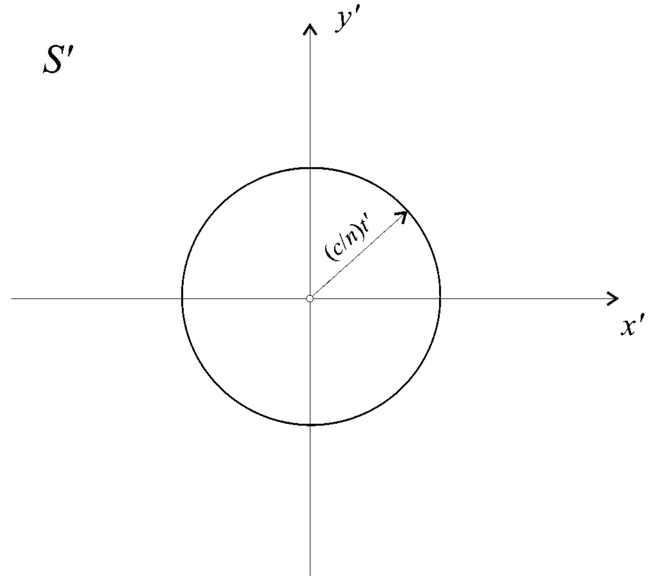

Fig. 2. In the reference frame S' where the medium is at rest, the wavefront of the elementary wavelet represents a circle with radius $(c/n)t'$.

II. HUYGENS' CONSTRUCTION IN A MOVING MEDIUM

To trace the path of an arbitrary light beam through a medium, a Huygens' construction can be employed. It states that every point that belongs to the primary wavefront at some time t_0 serves as an elementary source of secondary wavelets which spread in all directions with the same frequency and velocity as the primary wavefront. The envelope of these wavelets is the wavefront of the light beam at a later time $t_0+\Delta t$. If the medium through which light propagates is made of an optically isotropic substance, then the light ray can be constructed as a line normal to every subsequent wavefront at all times.[14]

If the light propagates through a vacuum, the evolution of the wavefront of an arbitrary secondary wavelet is governed by

$$x^2+y^2+z^2=c^2t^2. \tag{1}$$

The wavefront of the elementary source is an expanding sphere whose radius at time t is ct, where t is the time measured from the beginning of the emission of the wavelet, and c is the speed of light in vacuum. If the propagation of light is observed from a frame of reference in which the optical system is moving in a straight line with constant velocity, the shape of the wavelet remains unchanged because the equation of the elementary wavefront is invariant under Lorentz transformations.

However, a problem arises when we are dealing with light propagating through a medium. In the presence of a homogeneous, isotropic, and nonconducting transparent medium at rest, the evolution of the wavefront of the elementary source is described by

$$x'^2+y'^2+z'^2=\left(\frac{c}{n}\right)^2 t'^2, \tag{2}$$

where n is the refractive index of the medium in its rest frame. It can be easily demonstrated that Eq. (2) is not invariant under Lorentz transformations due to the extra factor of n^2 on the right-hand side. As a result of the noninvariance of Eq. (2), we expect that the shape of the secondary wavelet will become substantially different if viewed from a frame of reference in which the medium is moving at constant velocity.[15]

The analytical description of the distorted elementary two-dimensional wavefront in the reference frame S in which the medium is moving at fixed velocity u along the positive direction of the X axis, can be done by applying the Lorentz transformations formulas,

$$x'=\frac{x-ut}{\sqrt{1-(u/c)^2}}, \quad y'=y, \quad t'=\frac{t-(ux)/c^2}{\sqrt{1-(u/c)^2}}, \tag{3}$$

to the relation

$$x'^2+y'^2=\left(\frac{c}{n}\right)^2 t'^2. \tag{4}$$

Equation (4) describes the evolution of the elementary light pulse observed in the reference frame S' where the medium is at rest (see Fig. 2). By making a transition from S' to S using Eq. (3), we arrive at

$$(x-ut)^2+y^2\left(1-\frac{u^2}{c^2}\right)=\frac{1}{n^2}\left(ct-\frac{ux}{c}\right)^2. \tag{5}$$

The wavefront of the wavelet as viewed from S is no longer a circle, but an ellipse partially dragged by the moving medium (Fig. 3). As long as the speed of the medium u is smaller than c/n, the wavefront will be incompletely dragged and the origin of the elementary wavelet will remain inside the ellipse. Otherwise, if $c/n<u<c$, which is the case of light propagation in a medium moving at superluminal velocity, the dragging is overwhelming and the ellipse no longer encloses the origin (Fig. 4).

By a careful examination of the plot shown in Fig. 3 of Eq. (5) for the case $u<c/n$, we conclude that the speed of light in a uniformly moving optical medium depends on the angle θ between the direction of propagation of the light and the velocity of the medium. In this way, a medium that is optically isotropic in its rest frame, possesses an optical an-

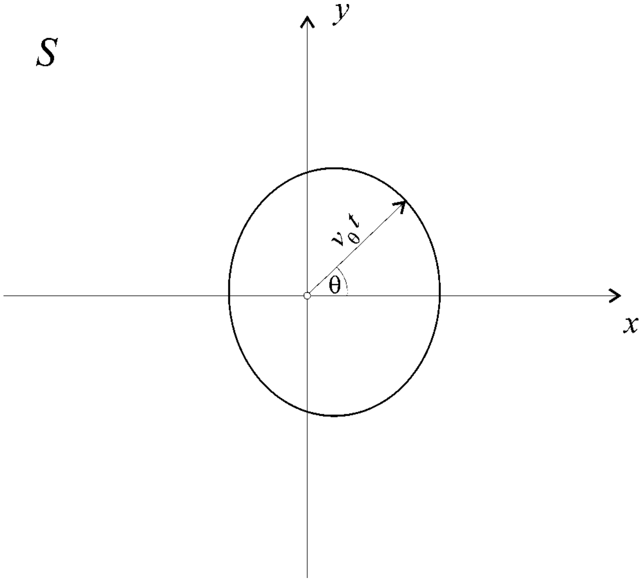

Fig. 3. As a consequence of the motion of the medium, the wavefront of the elementary wavelet is no longer a circle, but an ellipse partially dragged by the moving medium. The figure is a sketch of the situation when the speed of the medium u is less than c/n. Note that the dragged ellipse possesses an axial symmetry with respect to the X axis.

isotropy when observed from the frame in which the medium is in uniform rectilinear motion. The situation becomes more interesting for the case $c/n<u<c$ shown in Fig. 4, where the superluminal motion of the medium causes the existence of a Mach cone, outside of which no light ever reaches.

III. REFRACTION BETWEEN MOVING MEDIA

The idea behind the analysis is to trace the advancement of the wavefront in a moving nondispersive medium by applying the Huygens–Fresnel principle to the distorted secondary wavelets. We will investigate the refraction of an arbitrary light beam on a plane boundary separating two

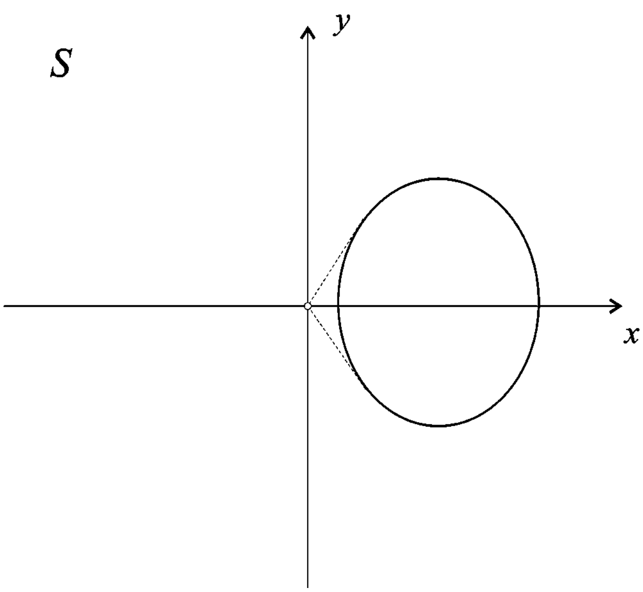

Fig. 4. Illustration of the dragging effect when the optical medium is moving at a superluminal velocity ($c/n<u<c$).

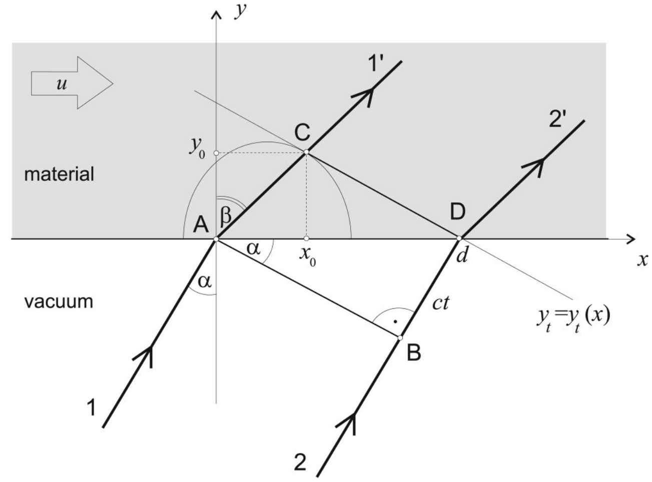

Fig. 5. Huygens' construction of the refracted wavefront when the medium on which the light is incident moves at constant velocity u to the right.

homogeneous transparent media from the reference frame in which the entire system moves to the right at constant velocity u (Fig. 5). We take the medium in which the incident wavefront propagates to be a vacuum, and the boundary between the vacuum and the material medium to coincide with the X axis. We denote by n the refractive index of the medium in its rest frame. The aim is to derive the formula that connects the angle of incidence α to the angle of refraction β.

The wavefront $\overline{AB}$ of the incident plane-polarized light beam sweeps across the boundary starting at point A, causing the atoms along the interface to radiate secondary wavelets. After the time t needed for the incident wavefront to reach the final point D, the wavefront of the secondary wavelet originating from the elementary source at A is a dragged ellipse whose shape is described by Eq. (5). The envelope of all the elementary wavelets whose sources lie along $\overline{AD}$ is the distance $\overline{CD}$, which is the wavefront of the refracted light. The distance $\overline{CD}$ belongs to the line $y_t=y_t(x)$, which is a tangent line of all the elementary wavefronts. Due to the apparent anisotropy of the moving medium, the line segments $\overline{AC}$ and $\overline{CD}$ are not orthogonal to each other, which is readily observable from Fig. 5. As a consequence, the ray and the normal to the wavefront are not identical in this case.

The equation of the tangent line is

$$y_t-y_0=\left(\frac{dy}{dx}\right)_{x_0,y_0}(x-x_0). \tag{6}$$

We have taken into account that the tangent line touches the elementary wavefront emanating from the initially disturbed point A at the point C (x_0,y_0), which implies that the slope of the wavefront at the point C coincides with the slope of the tangent line $y_t=y_t(x)$. By taking the derivative with respect to x of Eq. (5) at the point (x_0,y_0), we obtain

$$\left(\frac{dy}{dx}\right)_{x_0,y_0}=\frac{ut\left(1-\frac{1}{n^2}\right)-x_0\left(1-\frac{u^2}{n^2c^2}\right)}{y_0\left(1-\frac{u^2}{c^2}\right)}. \tag{7}$$

Then, the equation of the tangent line can be written as

$$y_t - y_0 = \frac{ut\left(1-\frac{1}{n^2}\right) - x_0\left(1-\frac{u^2}{n^2c^2}\right)}{y_0\left(1-\frac{u^2}{c^2}\right)}(x - x_0). \tag{8}$$

The tangent line intersects the interface at the point D $(d,0)$. Therefore, the point $(d,0)$ satisfies Eq. (8). Then, by setting $x=d$ and $y_t=0$ in Eq. (8), we have

$$-y_0 = \frac{ut\left(1-\frac{1}{n^2}\right) - x_0\left(1-\frac{u^2}{n^2c^2}\right)}{y_0\left(1-\frac{u^2}{c^2}\right)}(d - x_0). \tag{9}$$

The point of tangency C belongs to the elementary wavefront emitted from A, which means that the coordinate point (x_0, y_0) satisfies Eq. (5), that is,

$$(x_0 - ut)^2 + y_0^2\left(1-\frac{u^2}{c^2}\right) = \frac{1}{n^2}\left(ct - \frac{ux_0}{c}\right)^2. \tag{10}$$

By solving Eqs. (9) and (10) for x_0 and y_0, we obtain

$$x_0 = \frac{(n^2-1)utd + c^2t^2\left(1-\frac{n^2u^2}{c^2}\right)}{n^2d\left(1-\frac{u^2}{n^2c^2}\right) - ut(n^2-1)}, \tag{11}$$

and

$$y_0 = \frac{ct\sqrt{1-\frac{u^2}{c^2}}\sqrt{n^2(d-ut)^2 - \left(ct - \frac{ud}{c}\right)^2}}{n^2d\left(1-\frac{u^2}{n^2c^2}\right) - ut(n^2-1)}. \tag{12}$$

Here we take only the positive solution in y_0, which is the one that corresponds to the actual situation. It is evident from Fig. 5 that $\tan\beta = x_0/y_0$. By taking into account Eqs. (11) and (12), we have

$$\tan\beta = \frac{(n^2-1)\frac{u}{c}\frac{d}{ct} + \left(1-\frac{n^2u^2}{c^2}\right)}{\sqrt{n^2\left(\frac{d}{ct} - \frac{u}{c}\right)^2 - \left(1 - \frac{u}{c}\frac{d}{ct}\right)^2}\sqrt{1-\frac{u^2}{c^2}}}. \tag{13}$$

From the triangle ABD, we have

$$\frac{d}{ct} = \frac{1}{\sin\alpha}. \tag{14}$$

We substitute Eq. (14) into Eq. (13) and find the law of refraction of the wave

$$\tan\beta = \frac{(n^2-1)\frac{u}{c} + \left(1-\frac{n^2u^2}{c^2}\right)\sin\alpha}{\sqrt{n^2\left(1 - \frac{u}{c}\sin\alpha\right)^2 - \left(\sin\alpha - \frac{u}{c}\right)^2}\sqrt{1-\frac{u^2}{c^2}}}. \tag{15}$$

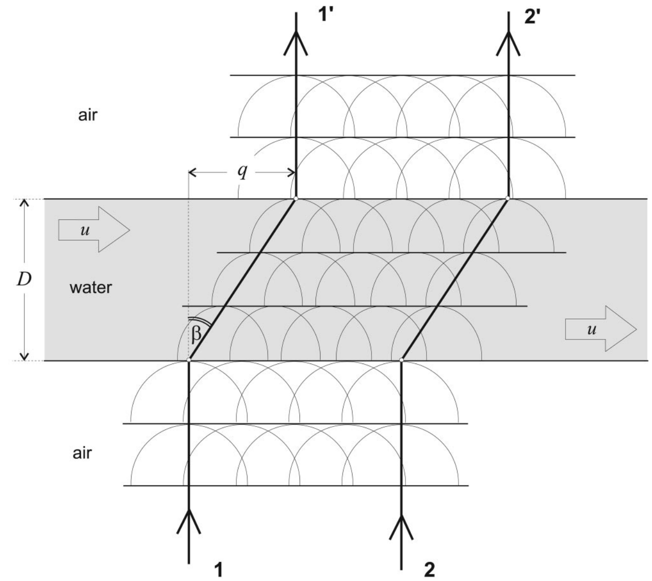

Fig. 6. Modified version of the famous Fizeau's experiment.

If we let $u \ll c$, so that $u/c \approx 0$, we see that Eq. (15) reduces to the usual Snell's law of refraction

$$\sin\alpha = n\sin\beta. \tag{16}$$

It is possible to derive Eq. (15) by using Lorentz transformations (see the Appendix).

IV. ABERRATION OF LIGHT IN A MOVING MEDIUM

In the following we consider a modified version of Fizeau's historic experiment.[2] Unlike the original experiment, we take the incident light to be perpendicular to the direction of the motion of the fluid (see Fig. 6). As a result of the dragging effect, the boundary rays of the incident wavefront will undergo a continual deflection toward the right as the wave propagates through the moving water.

By tracing the path of the light wave through the optical air–water–air assemblage, we find that the outgoing light will emerge parallel to the incoming light and will be displaced to the right by the distance q. From Fig. 6, we have

$$\tan\beta = \frac{q}{D}, \tag{17}$$

where D is the thickness of the layer. The refraction of the light beam from the layer is in agreement with Eq. (15) if we take the incident angle to be zero $(\alpha=0)$. Thus,

$$\tan\beta = \frac{\left(n - \frac{1}{n}\right)\frac{u}{c}}{\sqrt{1-\frac{u^2}{n^2c^2}}\sqrt{1-\frac{u^2}{c^2}}}, \tag{18}$$

where n is the refractive index of the fluid at rest. We substitute Eq. (17) into Eq. (18) to find

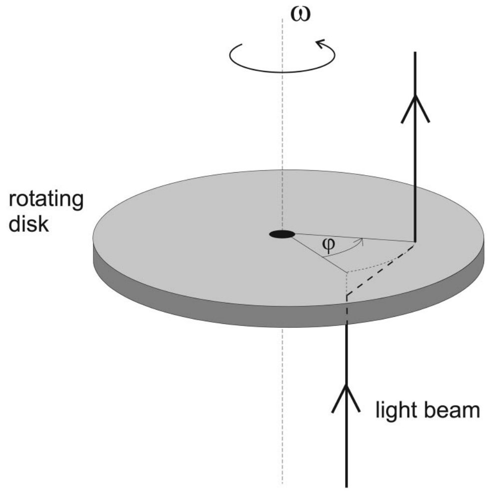

Fig. 7. Simplified presentation of the experiment performed by Jones (Refs. 12 and 13).

$$q=\frac{\dfrac{Du}{c}\left(n-\dfrac{1}{n}\right)}{\sqrt{1-\dfrac{u^2}{n^2c^2}}\sqrt{1-\dfrac{u^2}{c^2}}}. \tag{19}$$

For $u \ll c$, we can expand the right-hand side of Eq. (19) in powers of u/c and neglect all the second and higher order terms to obtain

$$q \approx \frac{Du}{c}\left(n-\frac{1}{n}\right). \tag{20}$$

Equation (20) is identical to the Fresnel formula for the transverse aberration of light.[1] The result was experimentally confirmed by Jones[12,13] for white light passing through a rapidly rotating disk made of a low-dispersive glass, parallel to its axis of rotation (Fig. 7). In this case, q is the tangential displacement of the light beam due to the dragging effect caused by the motion of the disk, u is the tangential speed of the rotating disk in the region of passage, D is the thickness of the disk, and n is the refractive index of the glass.

V. CONCLUDING REMARKS

We have presented an alternative approach to the propagation of light in a nondispersive optical material in uniform rectilinear motion. We have limited our discussion to a specific geometry of light refraction, where we considered the surface of the material, on which the light is incident, to be parallel to the motion of the material. This sort of arrangement gave us an opportunity to compare our derivation with the results of the experiment by Jones to measure the transverse Fresnel–Fizeau light drag effect. Our conclusions are in agreement with the result of the experiment, and matches the one derived by Fresnel for transverse light drag.

However, when the rotating disk used in the experiment is made of a highly dispersive glass material, a significant enhancement of the drag effect was noticed.[16] In this case, the value of the displacement q of the beam was found to be different from the one predicted by Eq. (20). It has been shown[17,18] that the original Fresnel formula is modified by the presence of an additional term due to dispersion, and this modification is in perfect agreement with the experiment. The formula that replaces Eq. (20) for a dispersive material is[17,18]

$$q=\frac{Du}{c}\left(n_g-\frac{1}{n_p}\right) \tag{21}$$

to first order in u/c. Here, n_p is the phase refractive index of the medium, which is a function of the frequency f of the light, and n_g is its group refractive index related to n_p by $n_g = n_p + f\,dn_p/df$. For a low dispersive medium, $n_g \approx n_p = n$, Eq. (21) reduces to Eq. (20).

The derivation of Eq. (21) by a procedure similar to the one used here is an open question, and we leave it as a subject for future study. We expect that the method of analysis in this paper can be applied to more complicated configurations, such as the one shown in Fig. 1, where in addition to the motion of the media, the boundary between them also is moving at constant velocity. We believe that the method also could lead to a simpler and more intuitive approach to the problem of light propagation through a nonuniformly moving material medium.

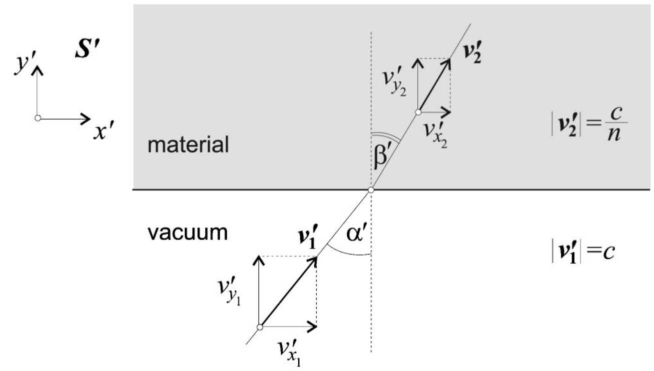

Fig. 8. Refraction of light from a material medium observed in the reference frame S' where the medium is at rest.

APPENDIX

In reference frame S' where the whole system is at rest (Fig. 8), Snell's law of refraction is

$$\sin\alpha' = n\sin\beta', \tag{A1}$$

and the velocity components of the propagating light beam are

$$v'_{x_1} = c\sin\alpha', \tag{A2}$$

$$v'_{y_1} = c\cos\alpha', \tag{A3}$$

$$v'_{x_2} = \frac{c}{n}\sin\beta', \tag{A4}$$

$$v'_{y_2} = \frac{c}{n}\cos\beta'. \tag{A5}$$

In the reference frame S where the medium is in uniform rectilinear motion to the right (Fig. 9), the components of the velocity of the light beam are

$$v_{x_1} = c\sin\alpha, \tag{A6}$$

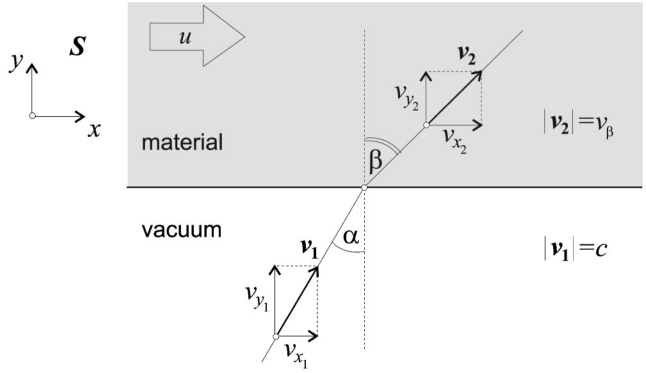

Fig. 9. Refraction observed in the reference frame S where the medium is in uniform rectilinear motion.

$$v_{y_1}=c\cos\alpha, \tag{A7}$$

$$v_{x_2}=v_\beta\sin\beta, \tag{A8}$$

$$v_{y_2}=v_\beta\cos\beta, \tag{A9}$$

where v_β refers to the velocity of the light beam in the moving medium in the direction determined by the angle β. The X components of the velocity of the incident light in S and S' are connected to each other by

$$v'_{x_1}=\frac{v_{x_1}-u}{1-\frac{uv_{x_1}}{c^2}}. \tag{A10}$$

If we use Eqs. (A2) and (A6), we have

$$\sin\alpha'=\frac{\sin\alpha-\frac{u}{c}}{1-\frac{u}{c}\sin\alpha}. \tag{A11}$$

By applying Eq. (A1), we have

$$\sin\beta'=\frac{1}{n}\frac{\sin\alpha-\frac{u}{c}}{1-\frac{u}{c}\sin\alpha}. \tag{A12}$$

Then, by using Eq. (A12), Eqs. (A4) and (A5) become

$$v'_{x_2}=\frac{c}{n^2}\frac{\sin\alpha-\frac{u}{c}}{1-\frac{u}{c}\sin\alpha}, \tag{A13}$$

and

$$v'_{y_2}=\frac{c}{n^2}\frac{1}{1-\frac{u}{c}\sin\alpha}\sqrt{n^2\left(1-\frac{u}{c}\sin\alpha\right)^2-\left(\frac{u}{c}-\sin\alpha\right)^2}. \tag{A14}$$

The components of the velocity of the refracted light beam in both reference frames are related by

$$v_{x_2}=\frac{v'_{x_2}+u}{1+\frac{uv'_{x_2}}{c^2}}, \tag{A15}$$

$$v_{y_2}=v'_{y_2}\frac{\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{uv'_{x_2}}{c^2}}. \tag{A16}$$

By eliminating v_β from Eqs. (A8) and (A9), and using Eqs. (A15) and (A16), we have

$$\tan\beta=\frac{v_{x_2}}{v_{y_2}}=\frac{v'_{x_2}+u}{v'_{y_2}\sqrt{1-\frac{u^2}{c^2}}}. \tag{A17}$$

Finally, we substitute Eqs. (A13) and (A14) into Eq. (A17) and obtain

$$\tan\beta=\frac{(n^2-1)\frac{u}{c}+\left(1-\frac{n^2u^2}{c^2}\right)\sin\alpha}{\sqrt{n^2\left(1-\frac{u}{c}\sin\alpha\right)^2-\left(\sin\alpha-\frac{u}{c}\right)^2}\sqrt{1-\frac{u^2}{c^2}}}. \tag{A18}$$

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VISUALIZING PHYSICS

I found out that the main ability to have was visual, and also an almost tactile, way to imagine the physical situations, rather than a merely logical picture of the problems.

The feeling for problems in physics is quite different from purely theoretical mathematical thinking. It is hard to describe the kind of imagination that enables one to guess at or gauge the behavior of physical phenomena. Very few mathematicians seem to possess it to any great degree. Johnny, [von Neumann] for example, did not have to any extent the intuitive common sense and ‘‘gut’’ feeling or penchant for guessing what happens in given physical situations. His memory was mainly auditory, rather than visual.

Stanislaw M. Ulam, *Adventures of a Mathematician* (Charles Scribner’s Sons, 1983). Reprinted in *The World Treasury of Physics, Astronomy, and Mathematics* (Little, Brown and Company, Boston, MA, 1991), p. 710.