Recent Developments in Quantum Gravity

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INTRODUCTION

I would like to divide this report into three parts. In the first, I would like to explain, in general terms, what the basic questions of quantum gravity are; in the second, I would like to point out the main difficulties that we face; and, in the third, I would like to discuss some recent results. The last part is a summary of the present status of a program that was first developed during a six-month workshop on quantum gravity, held at the Institute of Theoretical Physics at Santa Barbara in 1986. Since then, about two dozen individuals have made substantial contributions to this program. Unfortunately, because my space is somewhat limited, I will be able to discuss only a few of these contributions and I apologize in advance for the omissions. Also, because this is a "general interest" report rather than a technical presentation, discussion will be somewhat qualitative. I hope the experts will excuse me for repeating some of the well-known points and for glossing over references to the older material.

The central messages are the following:

(i) Nonperturbative quantum gravity (say, quantum general relativity) is feasible and may well be a viable theory.
(ii) It is very likely that the microstructure of space-time is radically different from the one suggested by perturbation theory. In particular, the space-time metric may not be a good variable to discuss this microstructure.
(iii) Because, a priori, there is no space-time in quantum gravity, quantum mechanical notions such as the Schrödinger equation and the resulting unitary evolution, which are normally regarded as fundamental, are now derived concepts that emerge when "time" is appropriately identified from among the basic mathematical variables used in the theory.

BASIC QUESTIONS OF QUANTUM GRAVITY

In broad terms, the aim of quantum gravity is to unify the principles underlying general relativity and quantum mechanics. General relativity has been successful in explaining the large-scale structure of the universe, whereas quantum mechanics seems indispensible in the microscopic domain. Quantum gravity would be a theory applicable in both domains; general relativity and quantum mechanics, as we know it, are to emerge as limiting cases of this deeper theory.

Because general relativity is not only a theory of gravity, but also of space-time structure, the task of constructing a quantum theory of gravity leads us to a number of novel, conceptual questions. To see how they arise, let us briefly recall the evolution of the notion of space-time. The first mathematical model of space-time was given by Newton in the *Principia*. In that model, there is an absolute time: space-time is sliced by a preferred family of 3-surfaces representing "space", that is, events that are simultaneous with each other. This model provided a basis for all of physics and astronomy for more than two centuries. It was only when Maxwell predicted that the speed of light is a universal constant, independent of the choice of a rest frame, that the model received a jolt. The verification of this prediction by Michelson and Morley and the formulation of special relativity by Einstein were the dramatic climaxes of the tremors that followed. Special relativity provided us with a new model of space-time. In this model, space-time is a four-dimensional continuum, equipped with a flat Lorentzian metric. Thus, the preferred slicing of space-time disappears and the distant simultaneity loses its absolute meaning. The new model brought with it new concepts, such as light cones and causal propagation of signals, and all physical theories were now subject to new viability criteria. The revision of the notion of space-time is indeed radical. Yet, the new model shares a feature with the old one: in both models, space-time is fixed once and for all, unaffected by matter. This changed with Einstein's general relativity. The notion of space and time now underwent an even more drastic alteration. Here, space-time is no longer an inert background. It is sensitive to matter: matter curves space-time according to Einstein's field equations. The geometry of space-time is a dynamical entity, with degrees of freedom of its own. Thus, geometry is very much on the same footing as matter. Now, although matter, such as this sheet of paper, appears to form a continuum at first sight, it in fact has constituents. The closer we look, the richer is the substructure. Because general relativity taught us that geometry of space-time is also a physical entity, it is natural to examine its microstructure. To do so, we must bring in quantum mechanics; we must have a quantum theory of gravity. Hence, some of the basic questions of quantum gravity are: What is space-time made of? How do its microscopic "constituents" fit together to give us the continuum picture on an everyday scale? When does the continuum approximation break down? What are the physical concepts that survive this breakdown? How could we do physics in absence of a space-time?

Let us now turn to the evolution of quantum mechanics. Both the mathematical structure and the measurement theory of quantum mechanics make a crucial use of the background space-time in which the system under consideration resides. Here, because of the page limit, I will restrict myself to the mathematical structure. In nonrelativistic quantum mechanics, the presence of an absolute time features prominently in the Schrödinger equation,

$$(i\hbar)\partial\Psi/\partial t = H \cdot \Psi;$$

it is with respect to this time that the evolution is unitary. In the transition to special relativity, this equation generalizes nicely: on the left-hand side of the Schrödinger equation, we can use a time variable adapted to any one Lorentz frame and, on the right-hand side, we can use the corresponding component of the 4-momentum of the system for the Hamiltonian. On the other hand, in general relativity, we do not have a preferred set of (Lorentz) rest frames and we begin to see the tension between general covariance and quantum time-evolution in quantum field theory in curved space-times.

In quantum gravity, the situation becomes much more dramatic. Because there is no background space-time at all, it is hard to see even the meaning of the "t" that appears in the Schrödinger equation. Therefore, we are now led to another set of questions: Is there a notion of "time" and "dynamics" in full quantum gravity? Or, are they only approximate concepts? If so, when does the approximation break down? Does this breakdown lead to a loss of unitarity in some intermediate regime? Can we do meaningful physics without the notion of time-evolution?

As all these questions indicate, quantum gravity would have to do much more than provide us just with amplitudes for graviton-graviton scattering or corrections to quantities that we have been computing in quantum field theories. Because the gravitational field is intertwined with the very structure of space-time, the burden of quantum gravity is vastly heavier than that of, say, quantum electrodynamics or quantum chromodynamics. It must provide a deeper framework for all of fundamental physics.

FAILURE OF PERTURBATION THEORY

As we saw, difficult conceptual problems arise in quantum gravity because the space-time metric $g_{\mu\nu}$ plays a dual role in the classical theory: on the one hand, it is the gravitational dynamical variable, analogous to the Newtonian potential ϕ, and, on the other, it determines the space-time geometry. Therefore, an obvious strategy to simplify the situation would be to "split" the two roles played by the $g_{\mu\nu}$ by introducing a background (say, flat) metric $\eta_{\mu\nu}$ and defining a "dynamical variable" $h_{\mu\nu}$ via $g_{\mu\nu} = \eta_{\mu\nu} + Gh_{\mu\nu}$, where G is Newton's constant. The geometrical role of the metric can be assigned to $\eta_{\mu\nu}$ and the role of the gravitational potential can be allotted to $h_{\mu\nu}$. We can now use the standard perturbative methods that have been so successful in quantum field theories. Quantization of the dynamical field $h_{\mu\nu}$ on the background metric $\eta_{\mu\nu}$ is rather straightforward and leads to massless, spin-2 quanta—the gravitons. The idea now is to study the interactions of these gravitons amongst themselves, as well as with other elementary particles, using perturbative techniques. Unfortunately, this simple strategy fails by its own criteria.

Let me summarize the situation. If we use, as our point of departure, the Einstein-Hilbert Lagrangian, the theory turns out to be nonrenormalizable at two loops for pure gravity and at one loop for gravity interacting with matter fields. This means that the quantum theory contains an infinite number of unknown parameters. Consequently, it has no predictive power at all. It is possible to imagine that the trouble arises because of the use of general relativity as a starting point for quantum theory. After all, we only know that this theory is in excellent agreement with experiments only macroscopically. At high energies (or short distances), the true dynamics may be quite different. Therefore, we may add higher-derivative terms to the Einstein-Hilbert action, terms whose effects may be negligible on macroscopic scales, but that dominate the microscopic behavior. The addition of such terms does indeed improve the short-distance behavior. The theory becomes renormalizable (in fact, even asymptotically free). However, the quantum theory is no longer unitary. In fact, the Hamiltonian is unbounded below, signaling a dramatic instability.

It then was felt that one should consider a supersymmetric extension of general relativity. One hoped that the new boson-fermion symmetry would come to the rescue and that the infinities from the bosonic sector would be precisely canceled by the infinities in the fermionic sector. The new theory, supergravity, is indeed better behaved. Unlike higher-derivative theories, it is unitary and has a positive-definite Hamiltonian. Furthermore, even in the presence of (supersymmetric) matter, the theory is renormalizable to two loops. Unfortunately, though, renormalizability fails at the third loop.

A more radical revision is suggested by the superstring theory. Here, one abandons local field theories altogether and considers extended objects—strings—as fundamental. It is an especially attractive idea because, unlike in any previous attempt, all of the matter couplings are now determined automatically; very little is fed in by hand. There also is a tremendous technical improvement. The theory is unitary and, in the standard terminology, perturbatively finite. This means that none of the individual terms in the perturbation theory diverges. However, when summed, the series diverges and does so rather badly.[a] Thus, even in the superstring theory, perturbative calculations of the (total) physical amplitudes produce infinities. Now, in familiar theories (e.g., quantum electrodynamics), such infinities are generally regarded as "acceptable" because, by themselves, these theories are incomplete. It is possible to believe that if we understood the true nature of the microstructure of space-time and took into account all interactions that occur at high momenta, these infinities would go away. A theory of "everything", on the other hand, cannot use such excuses.

By now, it is generally accepted that the root of the problem is the use of perturbative treatments. More precisely, it is felt that all the imaginative attempts listed above fail because they assume that, even at small distance scales, where, by simple dimensional arguments, quantum gravity effects should dominate, space-time geometry is smooth. To obtain a viable theory, we must drop this assumption and let the theory itself tell us what the microstructure of space-time is like. For this, we must face the problem of quantum gravity nonperturbatively.

Are there indications that nonperturbative gravity would indeed be qualitatively different? Let me first consider Einstein's theory in $2+1$ dimensions. This theory has no "local" degrees of freedom (i.e., no gravitons) because, in three dimensions, the vanishing of the Ricci tensor implies the vanishing of the total curvature. Consequently, the system has only global, topological degrees of freedom. Nonetheless, this is a very useful "toy model" because, structurally, it is very similar to $3+1$–dimensional Einstein gravity. Now, it was believed for a long time that this theory was perturbatively nonrenormalizable. However, recently, Witten[2] has shown that a nonperturbative quantization is possible and leads to an interesting and viable quantum theory. The exact solution also reveals that the "ground state" of the theory is very different from the naive guess used in the perturbation theory. In particular, in this state, the expectation value of the metric operator vanishes rather than giving the flat, $2+1$–dimensional Minkowski metric.

What is the situation in $3+1$ dimensions? Let me give a simple example that brings

[a]For the case of the bosonic string, the divergence is discussed in reference 1. The situation is expected to be essentially the same for the heterotic string (D. Gross, private communication, 1988).

out the limitation of perturbation expansions in powers of Newton's constant already in the classical theory and that also illustrates a key feature characteristic of theories in which the space-time metric is a dynamical variable rather than a part of the background structure. Consider the problem of self-energy of a point charge. It is convenient to think of a point charge as a limit, as ϵ goes to zero, of thin shells of radius ϵ with uniform charge and mass densities. Now, if we ignore gravity altogether, the total energy is given by

$$m(\epsilon) = m_0 + \frac{e^2}{\epsilon},$$

with m_0 being the rest mass. The electromagnetic self-energy, of course, diverges as ϵ goes to zero. Let us bring in Newtonian gravity. Here, the mass, including the gravitational self-energy, is

$$m(\epsilon) = m_0 + \frac{e^2}{\epsilon} - \frac{Gm_0^2}{\epsilon},$$

which again diverges in the limit (unless e and m_0 are fine-tuned by hand). Let us now bring in general relativity. The key idea here is that everything couples to gravity including gravity itself. Therefore, in the expression of the gravitational self-energy, we have to replace m_0 by m. The resulting equation,

$$m(\epsilon) = m_0 + \frac{e^2}{\epsilon} - \frac{Gm^2}{\epsilon},$$

is quadratic in $m(\epsilon)$ and thus has two roots. Let me just appeal to physical requirements and choose the positive root:

$$m(\epsilon) = -\frac{\epsilon}{2G} + \frac{1}{G}\sqrt{\frac{\epsilon^2}{4} + Gm_0\epsilon + Ge^2},$$

which, in the limit as ϵ tends to zero, yields a finite result,

$$m(\epsilon = 0) = \frac{e}{\sqrt{G}}.$$

Note that we did not have to fine-tune any of the parameters. If we had done a perturbation expansion in powers of Newton's constant, as is clear from the formula for $m(\epsilon)$ above, each term in the series would have diverged even though the result is perfectly finite. Can this argument be made rigorous? This was achieved by Arnowitt, Deser, and Misner already in the 1960s using the exact framework of general relativity. Of course, the model itself is too simple (in particular, it ignores all quantum effects) to provide realistic values of mass of observed particles. However, it does suggest that general relativity has certain "built-in" regulating mechanisms that, unfortunately, are lost if we insist on using perturbation expansions in powers of Newton's constant. A detailed examination shows that this is a feature shared by all theories in which there is no background space-time metric. More precisely, the

Hamiltonian structure of theories changes dramatically once the metric itself becomes dynamical, and it is this change that is at the root of the "regulating mechanism".

RECENT DEVELOPMENTS

Once we accept the premise that the problem of quantum gravity should be faced nonperturbatively, the rationale for abandoning general relativity as the point of departure loses its force. This does not mean that quantum general relativity would necessarily be the correct physical theory. Rather, the viewpoint is that it is as good a starting point as any if we want to gain insight into the type of questions that were raised in the second section of this report.

Let us then focus on the problem of nonperturbative quantization of general relativity. The approach that seems most promising for an exact treatment is the canonical quantization method because, in this program, we do not need to fix a background geometry or build strong fields by superposing weak fluctuations; we can deal directly with highly curved geometries. The major obstacle that had blocked progress in this program had been the fact that Einstein's equations take on a rather complicated form in terms of the traditionally used canonical variables, making it very difficult to solve their quantum analogues. This obstacle was removed recently:[3] it was shown, by performing a canonical transformation on the gravitational phase-space, that we can introduce new canonical variables in terms of which all equations of the theory become polynomial. Furthermore, their use brings out a hidden relation between Einstein's theory and Yang-Mills theory, a relation that had hitherto gone unnoticed. This enables us to import ideas and techniques into general relativity and quantum gravity that have been successful in Yang-Mills theory and QCD. These two features—polynomialization of the equations and the close relation to Yang-Mills theory—seem to be quite robust; they are not tied to the specific form of the Einstein action. In particular, they continue to exist if we allow for a nonzero cosmological constant,[4] couple the gravitational field to matter (i.e., Klein-Gordon, Dirac, and Yang-Mills fields),[4,5] or consider a supersymmetric extension of the theory.[6] As a result, there has been significant activity in nonperturbative quantum gravity in the last two years. As mentioned in the INTRODUCTION, though, I will not be able to present a comprehensive survey of these developments here; I will only provide a brief summary of the basic ideas and a guide to the literature.[b] Nonetheless, as is perhaps inevitable in a discussion of recent work, the material that follows is a little more technical than the broad introduction to the field contained in the first three sections.

For simplicity, let me restrict myself to the source-free Einstein theory. It is convenient to begin with complex general relativity—that is, with a complex metric satisfying Einstein's equation on a real 4-manifold—and then take the appropriate "real" sections of the resulting complex phase-space to recover real Lorentzian or Euclidean general relativity, depending on the application we have in mind. The canonically conjugate pair then consists of complex fields $\tilde{E}_{\mathrm{i}}^{a}$ and A_{a}^{i} on a real

[b] For a detailed review of the work prior to January 1988, see reference 7.

3-manifold Σ, where "a" is the (co)vector index and "i" is the (triad or) $SO(3)$ internal index. (A tilde over a letter denotes a density of weight one and a tilde below a letter denotes a density of weight minus one.) In any solution to field equations, $\tilde{E}_i^a$ represents a (density-weighted, complex) triad, whereas the connection 1-form A_a^i is a potential for the self-dual part of the Weyl curvature. However, because of the mathematical structure of these fields, we also can think of them as the canonical variables of Yang-Mills theory: there is a natural isomorphism between the two phase-spaces. The diffeomorphism and frame-rotation invariance of Einstein's theory leads to first-class constraints. They now have the form,

$$\mathcal{D}_a \tilde{E}^a = 0, \qquad \mathrm{tr}\, \vec{E} \times \vec{B} = 0, \qquad \text{and} \qquad \mathrm{tr}\, \vec{E} \cdot \vec{E} \times \vec{B} = 0.$$

Here, $\vec{E}$ is a short notation for the vector density $\tilde{E}_i^a$ and $\vec{B}$ is the vector density representing the magnetic field of the connection 1-form A_a^i. (Note that, in general, the Yang-Mills electric and magnetic fields first arise as vector densities of weight one. If we have a background space-time metric, we can "de-densitize" them using the determinant of the metric.) The first of these constraints is precisely the Gauss law, which now ensures that the internal triad rotations are gauge motions. Thus, every initial datum of Einstein's theory is also an initial datum for Yang-Mills theory, which happens to satisfy four additional constraints (one vectorial and one scalar) that are algebraic in the field strengths. Hence, we have an embedding of the Einstein constraint surface into Yang-Mills'. Note that the degrees of freedom match: $SO(3)$ Yang-Mills theory has $3 \times 2 = 6$ degrees of freedom, which, due to the four new constraints, reduce to the two degrees of freedom of the Einstein theory. The embedding allows us to go back and forth between the two theories. For example, Renteln and Smolin[8] have used this embedding to borrow techniques from Hamiltonian lattice QCD to quantum gravity, whereas Samuel[9] has shown that the instanton solution to the Euclidean Yang-Mills equation can now be interpreted, via this embedding, as a gravitational instanton with cosmological constant (namely, the 4-sphere).

Let us now consider dynamics. For this, we need to introduce a lapse and a shift field that will determine what we mean by "time-evolution". For simplicity, let me set the shift to zero. Then, the Hamiltonian generating time-evolution defined by a lapse $\underset{\sim}{N}$ (which, in the present framework, is a density of weight minus one) is given by

$$H(A,E) = \int_\Sigma d^3x \underset{\sim}{N}\, \mathrm{tr}\, \vec{E} \cdot \vec{E} \times \vec{B} + \oint_{\partial\Sigma} \underset{\sim}{N}\, \mathrm{tr}\{(E^a A_a - A_a E^a)\vec{E}\} \cdot d\vec{S}.$$

Note that the integrand of the volume piece is a density of weight one, so the integral is well defined. Contrary to appearances, the surface integral is gauge-invariant because the gravitational boundary conditions are different from those used in the Yang-Mills theory: now, $\tilde{E}_i^a$ tends to the constant triad at infinity as $1/r$, whereas A_a^i goes to zero as $1/r^2$. (Thus, in particular, the triads in the surface integral can be replaced by their asymptotic constant value.) As in the standard canonical formulation of general relativity, the integrand of the volume piece is just a constraint. It is clear by inspection that the constraints and the Hamiltonian are polynomial in the basic variables. Finally, we have to impose the "reality conditions" to ensure that we are dealing with real (Euclidean or Lorentzian) general relativity. It turns out that these require that the (densitized) metric, tr $E^a E^b$, constructed from the triads, and its Poisson brackets with the constraints should be real.[4] Because the metric as well as the constraints are polynomial in the basic canonical variables, so are the reality conditions. Thus, all equations of the theory are polynomial in E_i^a and A_a^i. Note, in particular, that the inverse of the triad (or the 3-metric) never enters any of the field equations, so all our equations continue to be meaningful even if the triads E_i^a become degenerate. Therefore, we have obtained a slight generalization of Einstein's equations. This fact may be significant to analysis of singularities in classical theory and topology change in quantum theory. We shall return to this feature of equations at the end of this report. To conclude the present discussion, we note that the entire framework can be obtained starting from a manifestly covariant, four-dimensional Lagrangian formulation. (See references 10 and 11 and the appendix in reference 12.)

Let us now turn to the problem of quantization. Work is in progress on three fronts. The first deals with the problem of finding exact solutions to quantum constraints. Recall, in the standard canonical formulation, that the scalar constraint depends nonpolynomially on the metric through the scalar curvature term. Due largely to this complication, not a single solution to this quantum constraint was known. Indeed, even in quantum cosmology, where all but a finite number of degrees of freedom are ignored, the problem of finding exact solutions has proved to be difficult as soon as we go beyond the simplest of models and allow, for example, anisotropies. A comprehensive solution was available only in the extreme strong coupling limit of the theory where the nonpolynomial scalar curvature term disappears. On the other hand, in terms of the new variables, all constraints are polynomial. Therefore, we may now hope to obtain exact solutions to quantum constraints. This hope has borne out. In the context of "mini-superspaces", Kodama[13] has obtained exact solutions to all constraints for Bianchi type IX models that allow anisotropies.[c] More significantly, in the full theory, the simplicity of the constraints was exploited by Jacobson and Smolin[14] to obtain a large class of quantum states that are annihilated by the difficult scalar constraint. These states may be thought of as Wilson loops of Yang-Mills theory. Using this result, Rovelli and Smolin[15] recently have obtained an infinite-dimensional space of solutions to all quantum constraints. The solutions are given in the "loop-space representation", that is, as functions on the loop space of the 3-manifold Σ. These results have generated excitement because they bring out an unexpected relation between knot theory and quantum gravity.

These exact solutions represent the physical states of full quantum gravity. Therefore, it is clear that they contain a wealth of information about the microstructure of space-time at the Planck length. How can we extract this information? For this, we have to introduce a suitable inner-product on the space of physical states and construct physically meaningful observables. It is at this stage that the reality conditions will play a key role: the inner-product has to be such that the self-adjointness relations implied in quantum theory by the classical reality conditions are satisfied on the Hilbert space of physical states. (These steps have been completed in the weak field limit where we now obtain[16] a loop-space formulation of spin-2 gravity

[c]His analysis of the physical interpretation of these solutions is, however, incomplete. I believe that a correct treatment of the issue of appropriate inner-product is feasible in this case and is likely to show that at least some of his solutions are physically interesting.

that can be shown to be equivalent to the standard Fock-space description.) Once this mathematical framework is completed, we must try to develop physical intuition for various "natural" operators on the physical Hilbert space, that is, for their action, their eigenstates, their eigenvalues, etc. Are there analogues of "creation" and "annihilation" operators? What is the character of the knot and link excitations they create or annihilate? In the asymptotically flat context, how is the Hamiltonian operator represented on this Hilbert space? Is there a natural candidate for the ground state? Most of the notions we use in quantum field theory and general relativity make sense only on a background space-time geometry. Now, we are in an entirely new regime in which there is no such geometry. Therefore, we have to learn to ask new, interesting questions. We have to learn to do physics in absence of space and time. The exact framework constructed by Jacobson, Rovelli, and Smolin seems well suited to probe this entirely uncharted Planck regime.

In itself, however, the framework seems to be too "pristine" and "abstract" to make contact with the conceptual structure of physics as we know and use it. There is, for example, no notion of "time" and, hence, of "time evolution". How does the physics that we are familiar with then arise? This is the second broad category of questions that is being pursued. It turns out that the scalar (and, similarly, the vector) constraint can be recast using A_a^i and its complex conjugate as follows:

$$\frac{2}{G} \operatorname{tr} D_a(E^{[a}E^{b]}A_b) - A_a^b A_b^{*a} + AA^* = 0,$$

where D is the derivative operator compatible with E^a, $A_a^b = \operatorname{tr} E^b A_a$, and $A = A_a^a$. (Contrary to appearances, this equation is also polynomial in the basic canonical pair.) In quantum theory, physical states are annihilated by the quantum analogue of this constraint (the complex conjugate of A_a^i being replaced by its Hermitian adjoint). The idea now is to think of A_a^i as containing some "heavy" degrees of freedom that are to be regarded as "clocks" with respect to which the other, "light" degrees of freedom of A_a^i evolve. The term with inverse powers of Newton's constants as coefficients refers to the heavy degrees and is to be interpreted as the "time displacement" operator, whereas the remaining terms are to be thought of as the Hamiltonian density generating these displacements. With this interpretation, the quantum scalar constraint would take on the form,

$$i \frac{\delta\Psi(A)}{\delta t(\vec{x})} = \mathbf{H}(\vec{x}) \cdot \Psi(A).$$

It is interesting that the Hamiltonian density, $\mathbf{H}(\vec{x})$, has a rather simple form; it is a linear combination of terms of the form $(A^* \cdot A)$. The broad idea of extracting time from the argument of the wave function is rather old in the canonical quantization program. However, if we use the traditional "metric representation", this appears impossible to achieve in practice even in the weak field limit.[17] In contrast, in terms of the new variables, constraints split up nicely even in the full theory. In the weak field limit, we can carry out the idea in detail. If we keep terms up to second order in the expressions of quantum constraints, the role of time is, in essence, played by the trace of A_a^i (with respect to the background triad) and the scalar constraint reduces to the correct Schrödinger equations.[d] Thus, by appropriately isolating "time" in the argument A_a^i of the wave function, we can derive the Schrödinger equation. Because the constraints have the form needed for the "split" into the heavy and light degrees even in the full theory, it is tempting to hope that we can repeat this procedure in more general contexts, perhaps even in the exact theory. Whether we can do so will determine if the validity of the Schrödinger equation is an exact or only an approximate consequence of nonperturbative quantum gravity.

The issues discussed above are conceptual and therefore somewhat formal in character. Can we make any "phenomenological" predictions that the experimentalists can, at least in principle, test directly? This is the third category of issues being investigated. The basic idea here is to exploit the close connection between the present formulation of general relativity and Yang-Mills theory to incorporate "particle physics type ideas" into quantum gravity. One example of these is the analysis of θ-vacua and the associated CP violation in quantum gravity.[12] It turns out, due to the nontriviality in the topology of the effective configuration space, as in Yang-Mills theory, that there is a one-parameter (θ-) ambiguity in the quantization of the classical theory and there is CP violation if θ fails to be a multiple of π. The effect itself is small and will be probably dwarfed by other CP-violating processes. However, it is interesting that we can venture outside the "formalistic realm" to which investigations in canonical quantization have been generally confined. It is of great interest to see if we can carry over to gravity other qualitative predictions of Yang-Mills theory, such as the constraints imposed on permissible models of "fundamental" particle multiplets by the anomaly cancellation requirement.

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[d] In the weak field limit, to first order, the true degrees of freedom are represented by the symmetric, transverse, traceless part of A_a^i, the decomposition being carried out using the background metric. These fields produce a second-order "coulombic" field that has longitudinal and trace parts. It is these parts that correct the notion of space and time displacements to first order. Incorporation of matter sources is straightforward. Note, incidentally, that, in the full theory as well as in the weak field limit, the factors of Newton's constant get shifted nontrivially in the passage from the usual metric representation to the present self-dual representation, where the wave functions are holomorphic functions of A_a^i.

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