

Einstein, Mach's principle, and the Unification of Gravity and Inertia

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The first two chapters of *Making Starships and Stargates (MSAS)* were the hardest chapters to write. They deal with a body of material which is generally known to all those who have studied relativity theory, though the usual formal treatments of general relativity theory have not had much to say about Mach's principle and the origin of inertia for many years now. Placing that material into the framework of standard relativity is not as simple a job as it might appear that it should be. As I allowed in the acknowledgements of *MSAS*, email exchanges with Paul Zielinski and Jack Sarfatti helped identify some of the issues that needed to be addressed with some care in treating that material. That conversation continued beyond the publication of *MSAS* (in December of 2012). And Paul, whose inclination is to go to "primary" sources when available, tracked down some Einstein documents to support his claims – documents that had not been available years ago when I read the available literature for the first time. I am pleased to say that those documents support the position that Einstein regarded general relativity theory as encompassing inertia as a gravitational phenomenon, as elaborated in chapters 1 and 2 of *MSAS*. After a first draft of this essay was written in November of 2014, Bruce Camber brought a then recently published essay by John Stachel to my attention.¹ In addition to calling Einstein's field equations the "inertio-gravitational" field equations, he employed several Einstein quotes on the matters addressed here that I was not familiar with – now included below. This essay recounts Einstein's evolving views on inertia and how they fit into general relativity, extending the material in chapters 1 and 2 of *MSAS*. As elaborated below, we will see that Einstein's conviction that inertia and inertial forces are gravitational in origin never wavered after he identified the Equivalence Principle in 1907, notwithstanding that he was willing to abandon "Mach's principle" when he was challenged by Willem de Sitter shortly after the publication of general relativity theory in 1915.

I

1905, in the history of science, is known as Einstein's "miracle year".² In that year he published "On the Electrodynamics of Moving Bodies", wherein the "aether" of Maxwell's theory of electrodynamics was banished in his special relativity theory shortly after he gave the correct explanation for Phillip Lenard's "photoelectric" effect in terms of Planck's recently introduced "quantum" hypothesis. And then, seemingly almost as an

¹ Stachel was the first editor of the Einstein papers and Director of the Center for Einstein Studies at Boston University for many years. His paper, "The Hole Argument and Some Physical and Philosophical Implications" can be found online at *Living Reviews in Relativity*.

² Miracle years got their start with Newton, who allegedly figured out light, mechanics, and the law of gravity in 1666 while at home from school because of an outbreak of the plague.

afterthought, he wrote: “Does the Inertia of a Body Depend on Its Energy Content?” Likely, most historians of science would say that the chronological order of publication correlates with the importance of each work. Special relativity theory and quantum mechanics being most important and energy and inertia being the least. But other considerations suggest that this ranking would be a mistake. For example, the formal content of special relativity theory – the Lorentz transformations – had already been published by Lorentz a year and more earlier. And Henri Poincaré already had a lengthy paper in the process of being published (in the *Rendiconti di Circolo Mathematico di Palermo*) that would appear in 1906 where the physics of relativity was all disclosed. Poincaré and Lorentz, however, did not take the radical step that Einstein did in denying the presence of an “aether” in spacetime, and adopted the stance that the Lorentz transformations were applicable only to “local” spacetime. They persisted in the belief that there was an absolute background spacetime, although they understood that the background spacetime was unobservable in practice. Despite their belief in an absolute background spacetime, there can be no question but that they both understood the principle of relativity and physics involved.

In the matter of the quantum explanation of the photo-electric effect, Einstein himself regarded this as a lesser contribution. But it eventually won him the Nobel Prize for physics – and the undying hatred of Phillip Lenard who regarded the effect as his personal property as he had discovered it. In the matter of energy and inertia, we are talking about the most famous equation in human experience.³

$$E = mc^2 \tag{1}$$

Vast numbers of words have been written about this relationship. Einstein has been pilloried for allegedly never having given a correct accounting of how this relationship is obtained. But aside from literary invocations of the equation, almost all of the attention lavished on it has focused on the enormous magnitude of c^2 and concomitantly, the enormous amount of energy locked up in even very modest amounts of mass.

Einstein, however, was after something else. He wanted to know what contributed to the resistance of bodies to changes in their state of motion. It’s in the title of his paper. He wrote $m = E/c^2$. Already in 1905, he had identified inertia as a foundational concept wanting explanation. Since mass is both the manifestation of inertia and the source of the gravitational field (and the thing acted upon by the gravitational field), suspecting some sort of inter-relationship is hardly surprising. Doubtless, his interest had been piqued by reading Mach’s critiques of classical mechanics. But speculative conjectures in the context of a critique are a horse of a very different color from a fully elaborated theory that explicitly corrals the concept of inertia. If you suspect, as Einstein evidently did, that inertia is due to “some sort of interaction”, as Mach had suggested, with other bodies in the universe, the obvious candidate

³ Rivalled perhaps only by the mathematical statement of Newton’s second law [$\mathbf{F} = m\mathbf{a}$] and the Pythagorean theorem [$a^2 + b^2 = c^2$].

interaction is gravity. Like inertia, gravity is a universal property of mass, and after 1905, energy too.

Einstein's first success in wedding gravity and inertia came in 1907 in the form of the Equivalence Principle. First identified by Galileo and a central piece of Newtonian mechanics, the Equivalence Principle has been the subject of endless speculation since Einstein identified and named it – and made it a cornerstone of general relativity theory. Numerous different forms of the Principle have been articulated: the Weak Equivalence Principle (WEP), the Strong Equivalence Principle (SEP), and the Einstein Equivalence Principle (EEP) to name but a few. While it is widely acknowledged in the gravitational physics community that the EEP is part of the foundations of general relativity, there's a camp that denies that the Equivalence Principle in any form is a cornerstone of general relativity. They would have you believe that although the Equivalence Principle served a heuristic purpose in the creation of general relativity theory, it is not a physically correct description of the world of our experience. (See, for example, Hans Ohanian's *Einstein's Mistakes*.) But as Einstein understood it, it is a cornerstone of general relativity. As John Norton, in a recent essay on Einstein's path to general relativity notes, Einstein always adhered to a particular formulation of the Equivalence Principle. One that avoids the contentions and confusions that often attend other formulations. Einstein's version consists of the assertion that in an accelerating frame of reference (in deep outer space far from local concentrations of matter), a "gravity-like" field appears that acts on all objects attached to the accelerating frame of reference *exactly* as a real homogenous gravity field would.⁴

This formulation, Einstein's "happiest thought" as he later characterized it, leads immediately to the well-known example of the indistinguishability of a chamber in a rocket sitting at rest on the Earth and a rocket accelerating at one "gee" in deep outer space. It also explains immediately why local inertial frames of reference in the vicinity of the Earth, free fall frames of reference, are those in which the Earth's gravity field is exactly cancelled by the "gravity-like" field that appears in accelerating frames of reference, since the free fall frames are accelerating with respect to the local cosmic rest frame.⁵ Since the cancelling "gravity-like" field behaves in all respects like a regular gravitational field, the obvious surmise is that it *is* a gravitational field. Why didn't Einstein make this claim? Well, there's a little problem. Fields have sources, and Einstein couldn't identify any plausible sources that would give rise to his "gravity-like"

⁴ This version of the Equivalence Principle was modified in the wake of the work of Carl Brans in the early 1960s to explicitly exclude the localization of gravitational potential energy, for as has already been noted in *MSAS*, if gravitational potential energy can be localized, gravity fields and accelerating reference frames can always be distinguished from each other by local observations only. General relativity theory, specifically the relativity of inertia, and thus the principle of relativity prohibits such distinguishability.

⁵ Some people object to the use of the "local cosmic rest frame", arguing that this constitutes the selection of a preferred frame of reference, and thus constitutes a violation of the relativistic prohibition against preferred frames of reference. This is a mistake that proceeds from a faulty understanding of what a preferred frame of reference is. A preferred frame of reference is one that can be singled out **by local observations alone**. The mean cosmic rest frame clearly does not violate this stipulation, for it can only be identified by **non-local** observations. That is, by looking out into the universe to see what stuff that's far, far away is doing. In all other regards, the mean cosmic rest frame is just another inertial frame of reference (in the absence of local concentrations of matter of course).

field. A field that would vanish (exert no forces) in all inertial frames of reference, but become manifest in frames of reference accelerating with respect to local inertial frames of reference. Note, by the way, that by definition a local inertial frame of reference is one in which the vector sum of all gravitational and “gravity-like” fields vanishes. But just because the vector sum of such fields is zero does *not* mean that no field – one that cannot be “gauged” away – is present.⁶

Einstein’s earliest attempt to merge gravity and inertia appears in a short paper published in 1912, a year after his (incorrect by a factor of 2) prediction of the deflection of starlight passing close to the Sun. He had used a variable speed of light, dependent on the local gravitational field strength, to calculate the predicted deflection. Notwithstanding that he had made the speed of light (in vacuum) a constant in creating special relativity theory, he persisted in assuming that the speed of light was lower in strong gravity fields than it is in spacetime far from local concentrations of matter. At the time it appears that he did not see how these conflicting assumptions could be reconciled. But his physical intuition told him this was right. It was. The speed of light in vacuum in a region of strong gravity due to local matter concentrations is slower than that far from gravity producing bodies – *as viewed by distant observers*. Locally, it is always measured to have the same value. That is, it is a locally measured invariant. At least at an intuitive level, he must have understood this.

Einstein’s chief aim in 1912 was to show that inertia was an “inductive” gravitational effect.⁷ In Newtonian mechanics and special relativity theory, inertia is a property of mass-energy conferred on it by its presence in space – independent of the presence or absence of other material bodies in that space. Since space was thought to be a physical entity independent of its contents and their interactions, and it was believed to be the inducer of inertia, Einstein was proposing a fundamental change in the understanding of physical reality by suggesting that the action of gravity was the origin of inertia and inertial effects. To do this he asks us to consider a mass point P at the center of a thin spherical shell of matter K – and what happens when the shell is accelerated. Drawing on his then recent exposition of his second law:

$$m = E / c^2 \tag{2}$$

he notes that the masses of the point and shell are each modified by the presence of the other owing to the gravitational potential energy conferred on each by the other. These contributions are

⁶ In particular, the scalar part of the gravitational field is present (and equal to the square of the speed of light). Unlike in electrodynamics, where a scalar field can be “gauged” away at any point by a global rescaling of the scalar electric potential, this is not possible in general relativity theory, for the theory is not invariant under global transformations of the Poincaré group. [General relativity theory is invariant under the group of arbitrary coordinate transformations; that is, physics is unaffected by your choice of coordinates. This property is called “general covariance”.]

⁷ As Pfister and King note in their *Inertia and Gravitation* (Springer, 2015), inductive effects of the sort encountered in electrodynamics (a vector field theory) are not possible in the scalar gravity theory of the sort that Einstein was using in 1912. But he was on the right track.

$$\Delta m = m' - m = GmM / Rc^2 \quad (3)$$

where m is the mass of the mass point P , M the mass of the shell K , and R the distance from P to K . This relationship also obtains for M with the roles of m and M interchanged. Having written down the gravitational contribution to the mass of P , Einstein goes on to remark:

Th[is] result is of great interest in itself. It shows that the presence of the inertial shell K increases the inertial mass of the material point P inside the shell.¹ This suggests that the *entire* inertia of a mass point is an effect of the presence of all other masses, which is based on a kind of interaction with the latter. The degree to which this conception is justified will become known when we will be fortunate enough to have come into possession of a serviceable dynamics of gravitation.

Einstein's footnote:

1. This is exactly the same point of view that E. Mach advanced in his astute .investigation on this subject. . . .

We see that already in 1912 Einstein had tentatively identified the *origin* of inertia: the gravitational interaction. Absent a “serviceable dynamics of gravitation” however, he could not simply show that the action of the observable universe accelerating in some direction would be the production of a gravity field that would account for the third law reaction force experienced by an object accelerating in the opposite direction when viewed in the “cosmic” rest frame (or one moving inertially with respect thereto). His way of dealing with this problem was to propose an *ansatz* relating the forces on K and P , and then using relationships of the sort above involving the masses of K and P in each other's presence, solve for the forces. He found that:

In the case where only K is accelerated, but P kept fixed, the second of equations (4) assumes the form, using the value of α that was just found:

$$(-k) = \frac{3}{2} \frac{GmM}{Rc^2} \Gamma .$$

k is here the force that must be exerted on the material point P in order for it to remain at rest; thus $(-k)$ is the force exerted (induced) on P by the spherical shell K which possess acceleration Γ . This force has the same sign as the acceleration, in contrast to the corresponding interaction between equivalent electrical masses.

Ignoring the factor of $3/2$, we see that were $GMm/Rc^2 = 1$, that is, if the condition for the entire inertia of the point mass m to be due to the gravitational interaction Einstein had identified obtains, then the gravitational force induced would just be the inertial reaction force involved. Did Einstein see this? Does the Sun rise in the east?

Being fortunate enough to have such an insight as a young man, alas, does not necessarily mean that you will remember it always. As John and Mary Gribbin relate, in their biography of Feynman (pp. 266-7),

This curious fact – the balance between mass-energy and gravitational energy – had been known (as a mere curiosity) for about 20 years by the time Feynman gave his lectures on gravitation [in the 1960s]. Back in the 1940s, on a visit to Einstein in Princeton, the pioneering cosmologist George Gamow casually mentioned, while they were out walking, that a colleague, Pasqual Jordan, had realized that a star might be made out of nothing, since at the point zero its negative gravitational energy is numerically equal to its positive rest mass energy.

Einstein stopped in his tracks, and, since we were crossing a street, several cars had to stop to avoid running us down.

In spite of its impact on Einstein, Jordan's idea was regarded as no more than a curiosity, and probably Feynman had never heard of it. Certainly nobody had thought of applying it to the Universe as a whole. . . . All of this requires that the amount of matter in the Universe should be just enough to match the so-called 'critical' density, for which spacetime is described as being [spatially] flat. . .

Actually, Einstein had applied it to the Universe as a whole – in 1912. One wonders if Einstein stopped “in his tracks” because he remembered his conjecture of 1912.

After inventing general relativity theory, Einstein had “the serviceable dynamics” of the gravitational field he needed to address the deficiencies of his arguments of 1912. But there were two problems with general relativity, at least from the point of view of the issue of inertia. The first was that the field equations of general relativity admitted solutions that were in conflict with any reasonable interpretation of inertia as a gravitational phenomenon. The second was that taking the Galaxy to be the extent of the known universe, GM/R was less than a millionth of the value of c^2 . Since astronomers at the time were just beginning to get a sense of what the universe was really like, the second problem could be dismissed on the grounds of ignorance. The first problem was a different matter. In a series of exchanges with Willem deSitter, Einstein discovered just how much of a problem this turned out to be. Even the introduction of the cosmological constant term in his field equations was insufficient to suppress solutions clearly at variance with any reasonable version of “Mach's principle”.

It seems to be widely thought that in some sense Einstein “lost” the debate with deSitter on the Machian nature of cosmological solutions of the field equations of general relativity. After all, some years later, near the end of his life, Einstein allowed that it would be better not to talk about Mach's principle any more. But to believe that deSitter had persuaded Einstein to abandon the gravitational origin of inertia and inertial forces would be a very serious mistake. deSitter's arguments forced Einstein to accept that his field equations of general relativity theory had solutions that were not consistent with the notion that the distribution and motion of matter in the universe determined the local

inertial properties and behavior of matter. In a sense, it is perhaps unreasonable to expect a local field theory to do this without additional constraints. After all, Faraday had invented the field concept specifically to make it possible to treat interactions as local phenomena independent of the sources that produce the fields. Mach and Einstein were just asserting that the sources that produce the field are important too, at least when it comes to inertia. More to the point is the question: did deSitter's arguments cause Einstein to abandon his conviction that general relativity theory accounted for inertial phenomena? Simple answer: no. But Einstein complicated all this in the fall of 1920.

Hendrick Lorentz arranged an annual visiting professorship in 1920, a brief stay each fall, for Einstein at his home institution: the University of Leiden. Einstein took up this post in the fall of that year. In his traditional inaugural address upon taking up the post, Einstein complicated everything having to do with general relativity by making remarks about the "aether". They have given those who would turn back the clock even on special relativity solace. And those who would see gravity as a (quantum) phenomenon in spacetime, rather than spacetime *per se* have likewise seized on them as precursory thinking on Einstein's part partial to their aspirations. Most, however, think Einstein was just sucking up to Lorentz, whom he deeply admired, and who had never abandoned the aether of Maxwell. I think the historical record supports the sucker upper hypothesis.

Aside from the sucker upper aspect of this, why would Einstein do this? Well, as the geometrical interpretation of general relativity theory, made possible by the fact that both gravity *and* inertia satisfy the Equivalence Principle, sank in after 1915, Einstein saw that space and time – spacetime in fact – would have to be conceptualized as a real physical substance independent of "matter" that resides in and distorts spacetime. Since general relativity theory is "background independent", spacetime *is* the gravitational field. It is not something passive in which gravity fields exist.

Einstein seems to have been trying to kill two birds with one stone. First, making the point of the dynamical physicality of spacetime by calling it "aether", thereby evoking the substantial nature of the aether of Maxwell. This notwithstanding that he had banished the aether of Maxwell (and Lorentz) with his special relativity theory. And, two, playing up to Lorentz's known prejudices. Whatever his motivation, it didn't work. Max Born, for example, was convinced that Lorentz never accepted relativity theory. And Einstein's critics were not bamboozled. They knew their enemy; and they gave him no quarter.

In May of 1921, Einstein gave a series of lectures at Princeton University on general relativity theory. In those lectures he gave his then present thinking on Mach's ideas on inertia, as mentioned already in chapter 2 of *MSAS*. Armed with the dynamical theory of the gravitational field that he lacked in 1912, he could be more explicit about the behavior of matter due to the action of gravity. To do this he calculated the gravitational action on a (unit mass) test particle due to the presence of surrounding matter. (Einstein's introductory remarks are quoted near the beginning of chapter 2 [p. 31] of *MSAS* and his results are discussed by Carl Brans in the excerpt of his 1962 article

at the end of chapter 2 [pp. 58 – 59].) The equation of motion he found, at linear order in small quantities using the weak field approximation, is:

$$\frac{d}{dt}[(1 + \bar{\sigma})\mathbf{v}] = \nabla\bar{\sigma} + \frac{\partial\mathbf{A}}{\partial t} + [\nabla \times \mathbf{A}, \mathbf{v}] \quad (4)$$

The left hand side of this equation is just the derivative with respect to local [proper] time of the [unit] mass of the test particle times the local velocity that results from the action of the field on the right hand side. The scalar and vector potentials of the field are given by:

$$\bar{\sigma} = \frac{\kappa}{8\pi} \int \frac{\sigma dV_o}{r} \quad (5)$$

whereas the vector potential is

$$\mathbf{A} = \frac{\kappa}{2\pi} \int \frac{\sigma \frac{dx_\alpha}{dt} dV_o}{r} \quad (6)$$

where the integrations extend over all causally connected spacetime (out the past light cone to the particle horizon). Einstein wrote this solution of his field equations out for the gravitational action of “spectator” masses, a collection of nearby masses (notwithstanding the range of the integrations), on the point mass under consideration. He went on to say:

Although all of these effects are inaccessible to experiment, because of κ [his symbol for the gravitational constant] being so small, nevertheless they certainly exist according to the general theory of relativity. We must see in them a strong support for Mach’s ideas as to the relativity of all inertial actions. If we think these ideas consistently through to the end we must expect the *whole* inertia, that is the *whole* $g_{\mu\nu}$ -field, to be determined by the matter of the universe, and not mainly by the boundary conditions at infinity.

Although he was speaking of spectator matter, clearly, he was thinking of the universe. Brans 40 years later would point out that the gravitational potential cannot enter the equation of motion (4) on the left hand side as written here above, for it is a violation of the Equivalence Principle. But Einstein was doubtless thinking of his 1912 conjecture that the potential, if equal to c^2 , could by itself completely account for the origin of the test particle mass. And the $\partial\mathbf{A}/\partial t$ term in the equation of motion could account for inertial forces (up to the factor of 4 difference in the coefficient of the scalar potentials in two equations for the potentials anyway). But there was still the problem of the actual value of GM/R for the universe. . . .

Having reintroduced the aether into discourse about spacetime in 1920, Einstein found himself writing about it again in 1924, likely an attempt to clean up the mess he had created several years earlier. He wasted no time getting to this, for he opened his essay with,

When we speak here of aether, we are, of course, not referring to the corporeal aether of mechanical wave-theory that underlies Newtonian mechanics, whose individual points each have a velocity assigned to them. . . . Instead of ‘aether’, one could equally well speak of ‘the physical qualities of space’. Now, it might be claimed that this concept covers all objects of physics, for according to consistent field theory, even ponderable matter, or its constituent elementary particles, are to be understood as fields of some kind or particular ‘states of space’. But it must be admitted that such a view would be premature, since, thus far, all efforts directed toward this goal have foundered. [and still have]

He then went on to talk about “absolute” aether – aether [space] that is not acted upon by its material contents. For example, the absolute space/aether of Newtonian mechanics. To make his point, he remarked that, “The occurrence of centrifugal effects with a (rotating) body, whose material points do not change their distances from one another, shows that this aether is not to be understood as a mere hallucination of the Newtonian theory, but rather it corresponds to something real that exists in nature.” Einstein continues:

We see that, for Newton, ‘space’ was something physically real, in spite of the curiously indirect way this real thing reaches our awareness. Ernst Mach, the first after Newton to subject the foundations of mechanics to a deep analysis, perceived this clearly. He sought to escape this hypothesis of the ‘mechanical aether’ by reducing inertia to immediate interaction between the perceived mass and all other masses of the universe. This view was certainly a logical possibility but, *as a theory involving action at a distance, cannot be taken seriously today*. The mechanical aether – which Newton called ‘absolute space’ – must remain for us a physical reality. . . .

When Newton referred to the space of physics as ‘absolute’, he was thinking of yet another property of what we call here aether. Every physical thing influences others and is, in its turn, generally influenced by other things. This does not however apply to the aether of Newtonian mechanics. For the inertia-giving property of this aether is according to classical mechanics, not susceptible to any influence, neither from the configuration of matter nor anything else. Hence the term ‘absolute’.

In this passage, Einstein does several things. First, he identifies “Mach’s principle” as demanding action at a distance – understood as instantaneous propagation of physical effects across finite distances – since the inertial effects of distant matter are detected instantly when local forces are applied. This he rejects, and distinguishes it from “the relativity of inertia”, a non-action at a distance [*i.e.*, pre-existing field produced by

sources lying along the past light cone out to the particle horizon] version of the same proposition that the inertia of objects is determined by their interaction with the gravity field of the rest of the matter in the universe. Second, he prepares the way for the novelty of general relativity theory that matter acts directly on the spacetime, in general, causing it to be distorted from whatever state it might have in the absence of all matter. Since, as in Newtonian mechanics, space is the cause of inertia, the novelty of general relativity is that inertia can be influenced by distant matter through its ability to distort local spacetime through the gravitational interaction. [And because the distribution of matter and its motions can change in time, inertia ceases to be the “absolute” property that it is in classical mechanics and special relativity theory.] In Einstein’s words:

. . . It was recognized that the equations of electromagnetism did not, in fact, single out one particular state of motion, but rather that, in accordance with these equations, just as with those of classical mechanics, there exists an infinite multitude of coordinate systems in mutually equivalent states of motion, providing the appropriate transformation formulas are used for the spatial and temporal coordinates. . . . No longer was a special state of motion to be ascribed to the electromagnetic aether. Now, like the aether of classical mechanics, it resulted not in favoring of a particular state of motion, only the favoring of a particular state of acceleration. . . . According to special relativity too, the aether was absolute, since its influence on inertia and the propagation of light was thought of as being itself independent of physical influence. . . .

The general theory of relativity rectified a mischief of classical dynamics. According to the latter, inertia and gravity appear as quite different, mutually independent phenomena, even though they both depend on the same quantity, mass. The theory of relativity resolved this problem by establishing the behavior of the electrically neutral point-mass by the law of the geodetic line, according to which inertial and gravitational effects are no longer considered as separate. In doing so, it attached characteristics to the aether which vary from point to point, determining the metric and dynamical behavior of material points, and determined, in their turn, by physical factors, namely the distribution of mass/energy.

The aether of general relativity differs from those of classical mechanics and special relativity in that it is not ‘absolute’ but determined, in its locally variable characteristics, by ponderable matter. This determination is a complete one if the universe is finite and closed. That there are, in general relativity, no preferred spacetime coordinates uniquely associated with the metric is more characteristic of its mathematical form than its physical framework.

Einstein’s last remark is worth noting. It’s his way of saying that “general covariance” is more a matter of mathematical formalism, less one of physics. By implication, the physics of general relativity is its treatment of gravity *and* inertia. Or, as he put it in 1918, the Principle of Equivalence and Mach’s principle.

The remainder of Einstein's 1924 article on the aether was devoted to remarks about the unification of gravity and electromagnetism. In this connection he mentioned Arthur Schuster's speculations on planetary and stellar magnetic fields (without attribution), later pursued by Patrick M.S. Blackett in the '40s and '50s.⁸ Also mentioned are the Compton effect and the work of Bose on the statistics of now named bosons. His conclusion, however, bluntly addresses his core belief about gravity and inertia at the time:

... we will not be able to do without the aether in theoretical physics, that is, a continuum endowed with physical properties; for general relativity, to whose fundamental viewpoints physicists will always hold fast, rules out direct action at a distance. But every theory of local action assumes continuous fields, and thus also the existence of an 'aether'.

Einstein's comment about general relativity "ruling out" action at a distance can be taken as his abandonment of Mach's principle, which in full-blown form requires instantaneous propagation action at a distance. Presumably, he did this because he was convinced that plain general relativity encompassed both gravity and inertia, so full-blown action at a distance was not needed. As he would discover later, many physicists did not share his understanding of his theory.

In Newtonian mechanics space, and special relativity theory spacetime, is absolute. It confers inertia on its material contents, but is not acted upon by them in any way. General relativity is different. The structure of spacetime is not "absolute", but determined by "ponderable matter". And if the universe is finite and closed, then the determination is complete. This difference merits comment. Especially in light of de Sitter's demonstration that empty universes (those without "ponderable matter") are consistent with Einstein's field equations. Einstein meant more than his words above convey. He meant that in his vision, spacetime – as it **is** the gravitational field, and fields must have sources – simply does not exist in the absence of any ponderable matter sources. Notwithstanding deSitter's argument. As John Stachel, in a recent article relates, in 1921 (after the exchange with de Sitter), when asked by a reporter what would happen to space and time were matter "destroyed", he replied, "Then there would be no time and space." And in 1931, when asked to sum up his theory in one sentence, he replied, "Before my theory, people thought that if you removed all the matter from the universe, you would be left with empty space. My theory says that if you remove all the matter, space disappears too!" What he meant by this is that when there is no matter (or mass-energy if you prefer), then there is no spacetime – because matter is the source of the gravitational field, and spacetime **is** the gravitational field.

⁸ This conjecture proposes that a rotating electrically neutral massive body generates a (electro)magnetic field and implies that the gyromagnetic (or magnetogyro) ratio for such a body should remain fixed as the object spins down. When pulsars were discovered in the late '60s and understood to be highly magnetized neutron stars, a new way of testing this conjecture became available. See: J.F. Woodward, articles on pulsar evolution in the *Astrophysical Journal*, **225**, 574 (1978); **256**, 617 (1982); **279**, 802 (1984); **316**, 743 (1987).

Did Einstein abandon Mach's principle, defined as an action at a distance proposition, by the mid-'20s as the above quote suggests? Well, no, not really. How do we know? Because in 1940 when John Wheeler and Richard Feynman (at Princeton) went to see Einstein (at the Institute for Advanced Study in Princeton) to get his take on their action at a distance theory of electrodynamics, Einstein immediately told them of the work of Hugo Tetrode (in 1921) and Adriaan Fokker (in 1929) on action at a distance theory. A skeptic might argue that this is only evidence for Einstein's familiarity with the literature. The action at a distance theories of Tetrode, Fokker, and Wheeler and Feynman, however, were not the instantaneous propagation type. Not long thereafter, Feynman gave a talk (his first) on the theory at a Physics Department colloquium. At the end of the talk, Feynman relates that,

Wolfgang Pauli who was sitting next to Einstein, said: 'I do not think this theory can be right because of this, that and the other thing.' . . . At the end of this criticism, Pauli said to Einstein, 'Don't you agree, Professor Einstein? I don't believe this is right, don't you agree, Professor Einstein?' Einstein said, 'No,' in a soft German voice that sounded very pleasant to me, very polite. 'I find only that it would be very difficult to make a corresponding theory [*i.e.*, an action at a distance theory] for gravitational interactions.'

Einstein didn't need to develop an action at a distance version of general relativity theory, for he was certain that the weaker version of Mach's principle – the origin and relativity of inertia – was already built into the very foundations of his theory. And that was good enough. But general relativity theory had acquired a life of its own. And others that appropriated it did not necessarily understand its physical content on the same terms as Einstein. By the early '50s there was already a budding "modernist" interpretation of general relativity – where "fictitious" forces like inertia are never "real" and the full physical content of the theory is nothing more than the metric and the geodesics derivable therefrom; that in the absence of curvature, no gravitational field is present; and later, the Equivalence Principle is excess baggage of historical interest only. Some, who fancied themselves true general relativists, even suggested that Einstein didn't even understand his own theory, at any rate, not as well as they. Some still do.

One may ask, had Einstein's understanding of general relativity theory changed in any significant way since the 1920s? And how did the "modernist" interpretation of general relativity theory come about? [You may want to take the modernist interpretation as better than Einstein's. I will not.] The answer to the first question is simple. No. Einstein's understanding of his theory had not changed. This is evidenced in his understanding of the content and significance of the Equivalence Principle (which, as John Norton has noted, did not change from its inception in 1907 to his death in 1955). We know because his friend and colleague Max von Laue queried him on precisely this point in 1950.⁹ Laue, who had written books on relativity theory, sought Einstein's

⁹ Laue won the 1914 Nobel Prize for physics for the theory of X-ray diffraction. He and Einstein had been esteemed friends and colleagues for decades. Indeed, when Einstein refused to return from travels to

views on claims then circulating that curvature was the key to gravitation; that in the absence of any curvature, no gravity field was present in spacetime. Perhaps he smelled a rat, because Einstein had said for years that spacetime *is* the gravitational field; and were that true, spacetime without gravity wouldn't make much sense. Einstein's answer:

It is true that in the case the R^i_{klm} [the Riemann curvature tensor components] vanish, so that one could say: "There is no gravitational field present." However, what characterizes the existence of a gravitational field from the empirical standpoint is the non-vanishing of the Γ^l_{ik} [the connection components], not the vanishing of the R^i_{klm} . If one does not think intuitively in such a way, one cannot grasp why something like a curvature should have anything at all to do with gravitation. In any case, no reasonable person would have hit upon such a thing. The key for the understanding of the equality of inertial and gravitational mass [the Equivalence Principle] is missing.

It's worth remarking that Laue later allowed as how he didn't really understand general relativity theory until 1950.

Einstein's view on spacetime and its relation to the gravitational field toward the end of his life is also found in his published writings of that time, as John Stachel has recently pointed out. One finds in Appendix II of *The Meaning of Relativity*, written and published in the 1950s, as quoted by Stachel,

It is the essential achievement of the general theory of relativity that it freed physics from the necessity of introducing the "inertial system" (or inertial systems) . . . The development . . . of the mathematical theories essential for the setting up of general relativity had the result that at first the Riemannian metric was considered the fundamental concept on which the general theory of relativity and thus the avoidance of the inertial system were based. Later, however, [Tulio] Levi-Civita rightly pointed out that the element of the theory that makes it possible to avoid the inertial system is rather the infinitesimal displacement field Γ

Stachel goes on to comment that, "Einstein's vision can be summed up in the sentence: 'Space-time does not claim existence on its own, but only as a structural quality of the field.' The two main elements of this vision are:

1. If there is no field, there can be no space-time manifold.
2. The spatio-temporal 'structural qualities' of the field include the affine connection, which is actually of primary significance compared to the metric tensor field.

Up until quite recently [2014], the standard formulations of general relativity did not incorporate this vision."

Germany in the early '30s after the ascension of the Nazis, he allowed as the only two colleagues he would really miss were Laue, and Planck.

The issue here is so fundamental that perhaps it is worth a small digression, aided by 20 – 20 hindsight. General relativity theory can be thought of as consisting of three levels of phenomena. The first level is that of the field equations, and metric and geodesics that are determined thereby. The metric and geodesics are objective, invariant physical facts. Inertial (geodesic) motion in one frame of reference is inertial motion in all frames of reference, notwithstanding that it may appear accelerated in some frames of reference chosen to be accelerating with respect to the geodesics, for no *net* gravitational or inertial force acts on the object in inertial motion. That is, the metric and geodesics are completely independent of observers and any motions they may be engaged in. In the weak field, slow motion approximation, this level is that of Newtonian gravity. Normally, when considering the gravitational field of, say, a spherical object, one assumes that at this level it is reasonable to assume that spacetime is spatially flat, actually, pseudo-Euclidean and gravity free at asymptotic infinity. That is, the boundary condition of Minkowski spacetime is imposed. Already, we have a problem. Minkowski spacetime is pre-general relativity. The fact of our existence is that there is no observable place where spacetime exists in the absence of gravity. And there is no reason whatsoever to believe that were gravityless spacetime even possible, it would possess full inertial structure. Imposing Minkowski spacetime as a boundary condition is the merest speculation. For the implicit assumption that the spacetime possesses inertial structure cannot be verified (or falsified) by any experiment.

What we do know about reality is that far from local concentrations of matter, spacetime is spatially flat. And since temporal cosmic evolution is so sedate, deviation from zero curvature due to cosmic expansion can be ignored. So, in deep outer space, spacetime is essentially exactly pseudo-Euclidean. But the spatial flatness is a consequence *not* of the absence of gravity. Rather, it results from critical cosmic matter density. And that produces a scalar gravitational potential (*i.e.*, $g_{00} \approx 1 - 2GM/Rc^2$) where $GM/R \sim c^2$, an enormous potential by any measure. As long as we restrict our attention to inertial/geodesic motion, we never see any evidence of this potential, for as a locally measured invariant, the potential is the same everywhere/when in local measurements. What the potential does do, however, as Einstein speculated as early as 1912, is confer inertia on objects to which it couples, in particular, non-gravitational mass-energy. (If you're thinking that the Higgs field, or some variant thereof, does this, ask yourself: what gives the Higgs particles their enormous masses?)

The second level is that of non-vanishing connection coefficients – completely irrespective of whether the Riemann curvature vanishes or not, as Einstein pointed out to Laue in the above quote. This is the level of the “post-Newtonian”, or “linear order”, or “weak field”, or “vector” approximation of general relativity theory. The connection encodes all things that satisfy the Equivalence Principle, and as such can be dealt with by a clever choice of geometry – in principle at least. This is the level of general relativity theory that is required to calculate equation of motion for any behavior that is not strictly inertial/geodesic, for inertial forces arise in such systems as in general they involve non-gravitational forces and the proper accelerations those forces induce. As Nordtvedt noted in 1988, this level is also required whenever first level behavior is observed from a frame

in motion with respect to carefully selected frames where all gravimagnetic effects vanish. In those moving frames (which may be inertial) the gravimagnetic vector potential may not vanish. And it (and its time-derivative) may be required to bring prediction into conformity with observation.

At a more fundamental level, this level is the source of almost all of the serious interpretational issues besetting general relativity theory. This has manifested itself in several ways. One, for example, is the very large literature that has been generated relating to whether a strictly homogenous gravitational field (one of constant field strength everywhere/when with exactly zero Riemann curvature) is even possible. The underlying motivation for these discussions seems to be that if such a field cannot ever be actualized, then the Equivalence Principle can be dispensed with as a hopelessly idealized fiction of no relevance to real gravity theory. Another example is the extensive literature on various versions of the Equivalence Principle. The odd thing about that literature is that essentially none of it discusses the principle in the terms that Einstein himself understood it: as a statement about the appearance of a “gravity-like” force in certain frames of reference that in some cases acts to cancel real gravitational forces.

At the linear order post Newtonian approximation with its weak field, it is easy to illustrate what Einstein was trying to get at. (It’s even easier with Sciama’s vector approximation to general relativity.) Recall that in the weak field approximation, the metric is written as:

$$g_{\mu\nu} \cong \eta_{\mu\nu} + h_{\mu\nu} \tag{7}$$

where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}$ a small perturbation of the Minkowski metric due to some local concentration of matter. (Were we not working at linear order with a small perturbation, this simple superposition of these metrics would not be possible.) The first thing to note is that it is a mistake to take eta mew gnu as the Minkowski metric. It should be in fact the spatially flat, essentially pseudo-Euclidean metric of critical cosmic matter density cosmology, with its scalar gravitational potential that is roughly equal to c^2 . If you assume that spacetime in inter-galactic space is Minkowskian – and thus gravity free – you will get wrong answers to dynamical questions, for there will be no acceleration-dependent “inertial” forces to counter any non-gravitational forces that may act (unless the inertial forces are arbitrarily put in by hand).

We first ignore the perturbation metric, and ask only what happens if we accelerate some test object with respect to the local pseudo-Euclidean metric (produced by the homogenous and isotropic distribution of cosmic matter)? Einstein has already done the calculation for us (in 1921). Leaving out his spectator matter term on the left hand side of his equation of motion and the rotational and Newtonian terms on the right hand side, we find that:

$$\frac{d}{dt}[m \mathbf{v}] = m \frac{\partial \mathbf{A}}{\partial t} \tag{8}$$

And

$$\mathbf{A} = \frac{\kappa}{2\pi} \int \frac{\sigma \frac{dx_\alpha}{dt} dV_o}{r} = \frac{\kappa}{2\pi} \frac{dx_\alpha}{dt} \int \frac{\sigma dV_o}{r} = \frac{\kappa \mathbf{v}}{2\pi} \int \frac{\sigma dV_o}{r} \quad (9)$$

We are using Einstein's units, and I have used Sciama's "trick" to remove the relative velocity of the test particle and cosmic matter from the integral for the vector potential \mathbf{A} .¹⁰ And the test particle mass m is written explicitly in the equation of motion. We also now know that when the integration is carried out to get \mathbf{A} , \mathbf{A} just turns out to be \mathbf{v} (when rest of the integration returns the value 1), and its time derivative is just the [proper] acceleration. That is, in the accelerating the frame of the test particle, a gravitational field arises, owing to the presence of the enormous scalar gravitational potential present due to cosmic matter, that produces the inertial reaction force that balances the non-gravitational accelerating force. Einstein's "gravity-like" field in the accelerating frame is not "like", it *is* gravitational (as he no doubt expected). This acceleration dependent gravity field arises in all accelerating frames of reference, the strength of the field depending on the magnitude of the acceleration. The appearance of the absence of a gravity field in inertial frames in deep space is just that: an appearance. It is the enormous scalar field present that causes the appearance of the acceleration dependent field in general relativity. When a non-zero proper acceleration characterizes the motion of an object or a frame of reference, the gravitational field that appears and acts does not spring into existence from nothing.

¹⁰ Calling Sciama's observation about the appearance of the relative velocity of a test particle and the rest of the universe here a "trick" has proved unfortunate. This is not a trick in any meaningful sense. Consider some convenient volume of space at some fairly cosmological distance. The gravitating stuff in that volume may have some random motions (called "peculiar" by astrophysics types), small by comparison with the speed of light, within that volume. They however, will average to zero for a sufficiently large local volume. Our cosmological volume element will have some radial velocity with respect to us by virtue of the expansion of the universe, but when this motion is integrated over a sphere, the velocity will average to zero (but the velocity may contribute to relativistic mass increase of the contents of the spherical shell). The only velocity of the contents of a spherical shell that will not average to zero over the shell is the relative velocity produced by the velocity of our test particle with respect to the local cosmological frame of rest. Moreover, that relative velocity will be retarded as the propagation speed of gravity is the same as that of light. And when we see the matter in the shell moving at some velocity, it will have been moving with that velocity at just the right time in the past. Perhaps not, "intuitively obvious to the most casual observer" as my then undergrad math major brother would have said, mimicking his instructors. But not a "trick".

To be fair, one has to be careful about "intuitive obviousness". Years ago, physics grad students were often told a joke in this regard (and perhaps still are). An eminent physicist (Dirac or Pauli if memory serves, probably Dirac) gave a talk at Princeton, writing equations on the blackboard as he spoke. Between two equations, he remarked that the second was an intuitively obvious consequence of the first. Someone in the audience challenged him. He looked at the equations and after a few moments, excused himself and left the room. When he didn't return after about 5 minutes, someone in the audience went looking for him. They found him in an empty classroom across the hall – writing furiously on the backboard (with an eraser in his other hand, erasing the equations he had just written almost as quickly as he wrote them down). After a half an hour, the speaker returned to the lecture room and said, "yes, intuitively obvious," and without further comment on this issue, continued with his talk.

What about the case of a test body moving in the gravity field of a local concentration of matter? Say the field of the Earth? In this case, the Newtonian term in the equation of motion does not vanish, and the test particle is accelerated toward the Earth, just as in Newtonian gravity. But the local perturbation field produced by the Earth does not make the enormous scalar field due to cosmic matter disappear. And the acceleration induced by the Earth, just as in the case of an accelerating rocket in deep outer space, causes an acceleration dependent gravity field due to cosmic matter to appear in the frame of reference of our test particle accelerating under the action of the Earth's gravity. How large is that acceleration-dependent **real** gravity field produced by cosmic matter? Exactly equal and opposite to the acceleration induced by the local gravity field of the Earth. So the total gravitational action on the freely falling test particle is exactly zero – and the motion, as a result, is inertial and the path of the test particle a geodesic. Einstein, of course, was not armed with our knowledge of cosmology, so he only claimed that the cancelling field produced by the acceleration due to the local concentration of matter was “gravity-like”. We know better.

Einstein was right. The most important fundamental physics in general relativity theory is contained in the second level, Equivalence Principle dependent part of the theory. It is that part of the theory that shows gravity and inertia to be unified in much the same way that electricity and magnetism are unified in Maxwell's electrodynamics. (It also leads to the geometric interpretation of the theory, as Einstein mentioned to Laue in the above quote.) This is more than a vague verbal analogy. Our analysis of the gravitational response to the proper acceleration of a test particle in outer space shows that the inertial reaction force *always* arises from the time derivative of the gravimagnetic vector potential, and *never* from the Newtonian type gravitational action. So, inertia plays the same role in gravitation as magnetism does in electrodynamics. The analogy between Newtonian gravity and simple electrical phenomena is already commonplace. It seems fair to say that Einstein with general relativity theory affected the unification of gravity and inertia, a major accomplishment for which he is almost never credited as it is widely believed that he failed to incorporate Mach's principle in general relativity theory. And he deserves at least as much credit for doing so as Maxwell is accorded for unifying electricity and magnetism.

Before passing to (brief) consideration of the third level, a word on Equivalence Principles is in order. The “weak” Equivalence Principle is normally taken to be the assertion that the inertial and passive gravitational masses of objects are identically the same, so that all objects, regardless of mass or composition, fall with the same acceleration in a gravity field (due to a local concentration of matter). The “strong” Equivalence Principle extends this identity to “active” gravitational mass, the mass-energy that is the source of the field that acts on passive gravitational mass. Note that these customary definitions say nothing about “gravity-like” fields. And Einstein's Equivalence Principle is usually taken to be the weak version of the Principle, the identity of inertial and passive gravitational mass. In the early '60s, Robert Dicke introduced a new version of the Principle: the Einstein Equivalence Principle. Why? Because the weak Equivalence Principle does not accommodate Carl Brans' argument on spectator

matter. It is always possible to distinguish an accelerating frame of reference in deep outer space from the (homogenous) gravity field of a local concentration of matter by examining the charge to mass ratios of elementary particles – unless you demand the non-localization of gravitational potential energy, the key additional element of the Einstein Equivalence Principle. That general relativity theory does this correctly is obvious if you understand the Equivalence Principle as did Einstein. If you think curvature to be the essential element of general relativity theory, you will likely miss this. You might even think the Equivalence Principle is not the essential core of the theory.

The third level of general relativity theory is that of appreciable curvature when the simple approximation of the second level that permits the superposition of a pseudo-Euclidean and perturbation metric breaks down. This is the strong field domain. Gravity in the vicinity of large and/or compact concentrations of matter. That is, black hole physics. The realm of the “information paradox” and the “firewall paradox”, and the gravity wave physics of coalescing black holes. Trendy and intriguing as all this is, it is irrelevant to the issues of concern to us, so I will resist the temptation to comment on this level of the theory.

II

If Einstein got the gravitational origin and relativity of inertia right, as recounted above, why isn't this a commonplace of physics today? In part, the reason why is that by the 1940s and '50s, Einstein had been relegated to the sidelines of the physics community because of his refusal to go along with the mainstream regarding quantum mechanics and his insistence on pursuing a unified theory of the gravitational and electromagnetic fields. While, after about 1920, due to the success of his prediction of the deflection of starlight by the Sun's gravity field, measured the year before by Arthur Eddington, Einstein appeared to be a heroic genius to the general public, those who knew him saw him differently. This is captured by David Hilbert's well-known remark that, “Every boy in the streets of Göttingen understands more about four-dimensional geometry than Einstein. Yet, in spite of that, Einstein did the work and not the mathematicians.” [Ohanian, *Einstein's Mistakes*, p. 221] Not exactly familiarity breeds contempt. After all, we are talking about Einstein. But something along those lines.¹¹

When asked by his former assistant (in the 1930s), Leopold Infeld, how he was getting along at the Institute for Advanced Studies in the early '50s, Einstein responded, “Sie denken das Ich bin einer alter trottel.” His old friend Laue knew better. And others were laying the foundations for the eventual ascendance of gravitational physics into the mainstream. But in the 1960s, Jeremy Bernstein, an excellent physicist who occasionally wrote “popular” articles for the *New Yorker*,¹² remarked in a piece on John Wheeler and

¹¹ Einstein's self-image, as one might expect, was not affected by the public hype. As he remarked to Levi Civita, complimenting him on an elegant calculation, “it must be nice to ride through these fields on the horse of true mathematics while the like of us make our way laboriously on foot,”:

¹² My favorite opening for an article is one of Bernstein's where he recounts the story of the medieval scribe being interrogated by the abbot of his monastery regarding a manuscript he had recently produced. The abbot suspected the scribe of making up (rather than simply copying) some of the material relating to

gravitational physics at Princeton University, that many of Wheeler's colleagues were displeased with Wheeler for snagging some of their best grad students and seducing them into working on general relativity, which they regarded as a stagnant backwater.

From the inception of general relativity in 1915 through the early part of the 1950s, general relativity was the preserve of a handful of physicists, likely no more than a dozen or two world-wide at any given time. While it was widely hailed as the profound and elegant theory that it in fact is, the experimental confirmations of the theory were few in number, and less than compellingly accurate. The mathematics was (is) difficult, and courses in general relativity graced the curricula of almost no universities, at any level. The chief areas of physics with high visibility were nuclear (high energy particle) physics based on steadily improving accelerators, and solid state (condensed matter) physics, both based on extensions of quantum mechanics developed in the '20s and '30s. As I've noted elsewhere, this is nicely recounted in Crease and Mann's *The Second Creation* and other sources. The story of the development of general relativity in the '50s and '60s is less well documented. I had thought to provide a sketch of those developments here. But that, done justice, is a task that would lead us far astray from our chief concern. Instead, let me mention several sources that tackle that story.

John Archibald Wheeler has written an autobiography that is relevant as he was the creator of the tradition of general relativity at Princeton University. By 1960, he had been joined by both graduate students and several colleagues. Kip Thorne, a central character in the business of wormholes, was one of those grad students. His *Black Holes and Time Warps: Einstein's Outrageous Legacy* recounts many of the developments of this era. And recently, he has published *The Science of Interstellar*, a companion to the movie *Interstellar*, where some of the developments of the past 25 years are mentioned. One of Thorne's former grad students, Daniel Kennefick, has written an outstanding history of gravity wave physics, *Traveling at the Speed of Thought: Einstein and the Quest for Gravity Waves*. Jean Eisenstadt (a student of Anne Marie Tonnelat at the University of Paris) has also written a history of the development of general relativity at this time. Alan Lightman and Roberta Braver collected oral histories of most of the outstanding cosmologists of that era (which means general relativity of course) published as *Origins: the Lives and Worlds of Modern Cosmologists* by Harvard University Press. And there is a serial, *Einstein Studies*, devoted to the history and foundations of general relativity. Hans Ohanian's *Einstein's Mistakes* does a very nice job on many historical issues relating to the development of general relativity, especially the Einstein-Hilbert priority dispute, though it does not treat developments after Einstein's death in 1955 in any detail. Doubtless, many more sources have become available in the past decade or so with which I am not familiar.

In very broad brush strokes, the '50s saw the formation of what amounted to "schools" of general relativity around charismatic teachers, notably John Wheeler at Princeton and Dennis Sciama at Cambridge. Major research universities started hiring a general relativist or two as the importance of the field became increasingly apparent –

miracles. The scribe defended himself by pointing out that he wasn't smart enough to make up the material being challenged.

especially after the discovery of quasars in the mid-‘60s and the advent of “relativistic astrophysics”. Advances in radio and radar astronomy in the ‘50s and ‘60s also played a role in the surge of interest in general relativity,¹³ as did the experimental programs of Robert Dicke at Princeton (Equivalence Principle and other experiments) and Joseph Weber at Maryland (gravity wave detection). Eventually, Eric Adelberger instituted a first-rate experimental program at the University of Washington focused on Eötvös-type (Equivalence Principle) experiments.

Organizationally, Andre Mercier at Berne organized the International Committee on General Relativity and Gravitation in 1955, which in the early ‘70s morphed into the International Society on General Relativity and Gravitation. The first major conference on general relativity took place in 1957 at Bryce deWitt’s home institution, the University of North Carolina, and Mercier’s Committee set in motion major conferences at three year intervals thereafter that continue to this day. New texts on general relativity were written too. In the mid-‘60s, Adler, Bazin, and Schiffer’s text came out; Wolfgang Rindler too wrote texts on both special and general relativity, and not long thereafter texts by Steven Weinberg (1973) and Misner, Thorne, and Wheeler (1973 also) were published. Hawking and Penrose’s *Large Scale Structure of Spacetime* should not pass un-noticed. Mercier’s Committee convinced Plenum Publishers to create a journal devoted to general relativity and gravitation, indeed, so named, in the ‘60s; and the Institute of Physics in Great Britain followed suit in the ‘70s with *Classical and Quantum Gravity*.

Mach’s principle was a topic of high visibility and attention in the ‘50s and ‘60s, especially after Dennis Sciama published his first paper on it in 1953 and then wrote popular books and more technical articles on the subject in subsequent years. Discussion around the topic became heated and, at times, polemical. Indeed, Abraham Pais recounts (in *Subtle is the Lord*, p. 288) that the *Zeitschrift für Physik* stopped taking papers on general relativity because those that mentioned Mach’s principle evoked such hostile replies. But Mach’s principle was hardly the only subject that excited passions. Gravity wave physics, especially after Joe Weber reported the detection of gravity waves with his “Weber bar” antennae, also became publicly contentious after Kip Thorne argued that Weber’s bars were not sufficiently sensitive to detect gravity waves from any of the anticipated sources. (Thorne, not long thereafter, hired Ronald Drever away from Glasgow and instituted a gravity wave project at Cal Tech that eventually morphed into LIGO. Drever had done superb experimental work before moving to Cal Tech showing that mass is a scalar, not a tensor quantity as some had speculated.) As I write this in 2014, notwithstanding an investment of more than a billion dollars by NSF, gravity waves are still convincingly undetected.

When Carl Brans published part of his doctoral work, excerpted *in extenso* at the end of chapter 2 of *MSAS*, it pretty much put an end to arguments about the localizability of gravitational energy in general relativity. Since Einstein had argued that Mach’s ideas

¹³ See especially the work of Irwin I. Shapiro on a fourth test of general relativity, a time-delay observed in the radar ranging of the inner planets, and radio frequency measurements of the deflection of strong radio sources as they were/are occulted by the Sun.

(but not explicitly “Mach’s principle” which demanded action at a distance) on the origin and relativity of inertia predicted that spectator matter should change the masses of local objects, Brans’ correction of this mistake on Einstein’s part was taken to mean that Mach’s principle, however defined, was not built into general relativity theory. Oddly enough, it seems that neither Sciama nor Wheeler, though they accepted Brans’ argument (in Wheeler’s case, see the excerpt from Misner, Thorne and Wheeler on gravitational energy localization at the end of chapter 1 in *MSAS* and below), ever really accepted that general relativity had nothing to say about the origin and nature of inertia.

Arguments about Mach’s principle continued. But Bob Dicke and Carl Brans inferred (as did many others) that Mach’s principle was not to be found in general relativity, and went off and reinvented scalar-tensor gravity as a way of trying to bring Mach’s principle back into gravity theory. They did this by noting that Mach’s principle requires that $GM/Rc^2 = 1$, or, $1/G = M/Rc^2$. Since R is a function of time, it would seem to follow that G should also be a function of time. Brans and Dicke, already aware that Paul Dirac had proposed that G might be a function of time, took this as the basis for proposing a universally coupled, time-dependent scalar field that would supplement general relativity (but not replace it). As they noted, they were merely reviving a theory already explored by Pasqual Jordan more than a decade earlier. (Brans has nicely recounted these developments in a 2010 article found on <Einstein.online>. A series of exacting experiments to test scalar-tensor gravity by Dicke and others over the next decade and a half followed. They showed that the proposed scalar field is either inconsequential or non-existent. All this is recounted in Clifford Will’s excellent book, *Was Einstein Right? Putting General Relativity to the Test* (Basic Books, New York, 1986).

The irony in all this is that Brans’ critique of Einstein’s 1921 work on Mach’s principle was dead on correct. But the inference that he and Dicke, and most everyone else took away from this was, to be blunt, wrong. This notwithstanding that they had clearly understood that Machian inertia demands that $GM/Rc^2 = 1$, or $\Phi = c^2$, so that the coefficient of $[\partial\mathbf{A}/\partial t = (\Phi/c^2)\partial\mathbf{v}/\partial t]$, that is, Φ/c^2 is one. The factor of 4 that appears in the general relativistic version is a complicating distraction that, alas, can lead one astray. (And if this is true everywhere/when, as it is in critical cosmic matter density FRW cosmology, then the origin and behavior of inertia are built into general relativity.) Given the context of the times and the variable nature of the coordinate speed of light in general relativity, this is perhaps not too surprising. But as the speed of light is a locally measured invariant in general relativity, it follows that anything that is *physically* equal to the speed of light, or its square, with no dependence on other parameters, must also be a locally measured invariant. Note that it is Φ in the simple vector approximation [or 4Φ in the weak field approximation], not just G as Brans argued, that is the locally measured invariant that preserves the Equivalence Principle in general relativity theory.

A complication, from this perspective, is that while Brans’ argument is correct, it doesn’t mean that spectator matter has no influence at all on objects in local shielded laboratories. It is obvious from the equation of motion calculated by Einstein in 1921 (and that calculated by Nordtvedt in 1988) that should substantial amounts of spectator

matter be accelerated in the vicinity of our shielded lab, it will exert a gravitational force on the lab and its contents. Since the force is gravitational, the resulting accelerations of the lab and its contents will all be exactly the same irrespective of the mass or composition of the stuff being accelerated. For that reason, we can just say that the accelerating spectator matter produces local frame dragging. And from the perspective of an observer in the lab, it is impossible to say, on the basis of local experiments only, whether the acceleration of the lab has changed, or the change in acceleration is due to nearby accelerating spectator matter.

The waters were further muddied some years later by the movement to view general relativity as only about Riemann curvature, a movement already underway in 1950 as Laue's question to Einstein noted above shows. Better yet, from the perspective of believers in gravity = curvature, is to get rid of anything that might suggest that inertia is gravity (because inertia is present in pseudo-Euclidean spacetime with its zero Riemann curvature). This can be done by asserting that the $\partial\mathbf{A}/\partial t$ term in the equation of motion is zero. Edward Harris, in an *American Journal of Physics* article published in 1991 comparing the vector approximation of general relativity to Maxwell's equations, by dubious choice of "gauge", did this. This could be argued to be the case on the basis of energy non-localizability, for the integral over matter currents can be reduced by Sciama's "trick" to the integral over sources that returns the scalar potential, a gravitational energy. If this potential is *not* taken to be a locally measured invariant (by virtue of it being the square of the speed of light), then spectator matter should change the masses of objects in isolated labs. And the Equivalence Principle is false, even if all objects, regardless of mass or composition, respond to a gravitational field with the same acceleration. Nonetheless, only a few moment's reflection should be needed to ascertain that any gauge condition that leads to the suppression of the $\partial\mathbf{A}/\partial t$ term in the gravelectric field equation is *obviously* wrong as gauge invariance of the "first kind" (global rescaling of the scalar potential) clearly does *not* apply to gravitation, so \mathbf{A} and its time derivative(s) are likely to be large even in the weak field limit because of its implicit Minkowski or spatially flat FRW background (that is usually ignored) in simple situations as the weak field perturbation of the flat metric is the focus of attention. As Steven Weinberg remarked in talking about the cosmological constant many years ago, just because something is (for all practical purposes) infinite doesn't mean you can set it to zero.

Harris's mistake was picked up by Hans Ohanian and Remo Ruffini in their textbook (*Gravitation and Spacetime*) in 1994.¹⁴ It was corrected in 2000 by Jose Pasqual-Sanchez (and is the reason why his paper was included in the technical articles section of the bibliography of *MSAS*). I am only now exploring the scuttlebutt of the general relativity community, assisted by Pfister and King's new book, *Inertia and Gravitation*, to be able to say how far this mistake penetrated that community and has become part of the "modern" interpretation of general relativity. But it certainly didn't mislead John Wheeler, as his remarks in his last serious book on gravitation (with Ignazio Ciufolini), *Gravitation and Inertia* show beyond any doubt.

¹⁴ Ohanian is a supporter of the view that the Equivalence Principle has only historical heuristic value; that it is in fact wrong, and thus not a proper foundation for general relativity. See his *Einstein's Mistakes*.

Indeed, the title of their book is a dead give-away. But Wheeler hit upon a way of implementing Einstein’s vision in the ‘90s very different from those treatments based on the techniques used by Sciama. Instead of doing integrations out the past light cone to some suitably chosen horizon, Wheeler chose to stipulate initial data on some suitably chosen spatial hypersurface and then look at the dynamical evolution of the system. Instead of using hyperbolic field equations with their finite propagation speed, he chose to use elliptic “constraint” equations because the propagation speed of their solutions is infinite. This way, he was able to introduce action at a distance without the curiosity of retarded waves *and* advanced waves propagating backward in time. Donald Lynden-Bell, a many medaled general relativist known for his prediction of super massive black holes at the centers of galaxies, had the same idea, independently, at about the same time.¹⁵

Did Wheeler appreciate that the hypersurface/constraint equations approach was just a stand-in for an integration out the past light cone? Does the Sun set in the west? In chapter 5 we find:

5.6.3. [An Integral Equation to Give Spacetime Geometry, and Therefore Inertia, Here and Now, in Terms of the Density and Flow of Material Mass, There and Then?]

After appealing to the analogy of the Coulomb gauge in electrodynamics [wherein the electric field “propagates” instantaneously] and asserting that the wild and chaotic local motions of matter out there in the cosmos that presumably beset any attempt at an integration out the past light cone with insuperable difficulties, Wheeler said,

However, no one will choose that route to knowledge who has in initial-value data and dynamic equations a simpler way to follow what is going on – simpler because the initial-value equations operate on a space-like hypersurface rather than on the past light “cone.” We forego here any integral over the past light “cone” in favor of **initial-value data on a spacelike hypersurface plus evolution by dynamic equation.**

The counter argument to Wheeler’s position is simple. We know what the mean matter density in the universe is. It is the critical cosmic matter density needed to produce the *observed* spatial flatness out there. As for chaotic local motions, any damned fool can get into an open car on a country road on a moonless night, fix his/her local cosmic rest frame, and then drive down the road watching – out the past lightcone (because that’s all you can see) – the universe move past in the opposite direction – seemingly rigidly – with minus their speed. And because the speed of propagation of gravity is that of light, you know that the relative velocity of the stuff on the lightcone and you takes place at the correct retarded time if you choose to invoke Leinard-Wiechert

¹⁵ Lynden-Bell used to list “Mach’s principle” as one of his areas of research interest on his web-page at Cambridge University. In the summer of 2014, he changed that to “the relativity of inertia”. Seems Mach’s principle has gotten a bad name again.

potentials, Green's functions, and all that. This is the reason why Wheeler "foregoes" any integral over the past lightcone. It is *not* because he thought that inertia is not a gravitational phenomenon, or because an integration out the past light cone done correctly would return incorrect results. As he says in section 6.2:

Gravitomagnetism may be thought of as a manifestation of the way inertia originates in Einstein geometrodynamics [that is, general relativity theory]; "*mass-energy there rules inertia here*" (chap 5). The measurement of the gravitomagnetic field will be the experimental evidence of this interpretation of the origin of the local inertial forces, that might be called a weak general relativistic interpretation of the Mach principle.

The emphasis is Wheeler's. Note that his famous characterization of general relativity – "matter there tells spacetime here how to curve; spacetime here tells matter here how to move" – has been changed to, "mass-energy there **rules inertia** here." This is followed immediately by a reprinting of Einstein's 1921 comments on Mach's principle already quoted here and in chapter 2 of *MSAS*. Worthy of note is the footnote that Wheeler attached to Einstein's comment on the influence of spectator matter: "However, regarding this point it has to be stressed that in general relativity, because of the very strong equivalence principle, the inertial mass and the gravitational mass, or G , do not change in the field of other masses. . . . Indeed, in 1962, Brans had shown that in general relativity this change is a mere coordinate effect." It also turns out to be crucial to the gravitational origin of inertia and inertial forces in general relativity. Don't let anyone tell you that John Wheeler didn't understand Einstein's position on the gravitational origin of inertia.

I read Ciufolini and Wheeler's book when it first came out, now the best part of 20 years ago. At the time, I was disappointed to find that the authors had given others, Sciama and Derek Raine in particular, less credit for their contributions than I thought reasonable. But, hey, I'm just an old experimentalist doing experiments not sanctioned by mainstreamers. Then, around 2000, someone came up with the idea that the origin of mass – that is, the origin of inertia – was the Higgs particle, something that is *obviously* wrong for anyone who actually understands general relativity as Einstein did. Almost immediately thereafter, Frank Wilczek pointed out that such claims for the Higgs mechanism and its particle were serious overstatements of the facts, for almost all of the mass of normal matter resides in the energy of zero rest mass gluons – to which the Higgs field does not directly couple. I've watched, as an innocent bystander, with admiration, Wilczek try to get his colleagues to be reasonable for the past decade and a half. And then, in my little world of advanced propulsion, I had the Higgs business thrown up as a counter argument to the correct understanding of inertia in general relativity. I went back and re-read *Gravitation and Inertia* with better appreciation of things. Wheeler's explanations of most things are outstandingly clear. But on a few things, not so much so. However, it is clear to me now that Wheeler deserves to be included with those I named as the originators of the gravitational laws of inertia spelled out at the end of chapter 2.

They should be called the Mach-Einstein-Wheeler-Sciama laws of inertia.¹⁶ Credit where credit is due.

III

In retrospect, it isn't too hard to figure out what went wrong when – after you've figured out what's right. And it is almost always easy to assign blame for mistakes made. But, as a general rule, it's very, very much harder than almost anyone appreciates to figure out what's right in the first place. Even when what's right turns out to be almost trivially obvious once it's identified. While Bacon was certainly right when he remarked that, “truth emerges more readily from error than from confusion,” that needs to be qualified with truth, no matter how it is ascertained, if it is worth knowing, is always very hard to discover.

IV

As a coda of sorts to this essay, I note that interest in action at a distance, or “absorber” theory of the Wheeler-Feynman type is again in evidence. From my perspective, the chief problem with absorber theory was the absorber part. In electrodynamics envisioning an absorber that eventually perfectly absorbs electromagnetic “radiation”, creating the needed conditions for the theory to work is not difficult. The problem is that gravity and electrodynamics are not exactly analogous. Where electromagnetic signals are attenuated by net-charge-neutral matter, gravity is not so attenuated. This is a consequence of there being no naturally occurring negative masses to act as sinks for the “field” produced by positive masses. This means that as gravitational disturbances propagate out the future lightcone and interact with matter they encounter along the light cone, while energy in disturbances may be communicated to encountered matter, the underlying gravity “field” and changes therein continue unabated. That is, in electrodynamics there is no invariant $\Phi = c^2$ field as there are both positive and negative charges as sources *and sinks*. In positive mass gravity there are sources, but no sinks, just other positive masses to act on. The result of this peculiar situation is that gravity disturbances continue to propagate indefinitely unless they are cut off by some physical process other than simple recessional velocity as in most standard FRW cosmologies. In these cosmologies, light signals can propagate infinitely far into to the future, and wherever they can get, the advanced signal from anything they encounter can get back to the source, seemingly instantaneously. This bothered me so much that I wrote a paper, “Killing Time”, with it as the central theme back in 1996 [see the technical articles section of the bibliography of *MSAS*].

Years later, I discovered that Stephen Hawking had seen the same problem which he characterized as “divergence of the advanced solution” in Hoyle and Narlikar's action

¹⁶ I have not used alphabetical order. The good gnus is that in this order, we have the MEWS laws of inertia.

at a distance version of general relativity.¹⁷ Hawking suggested that negative mass might help with this problem (just as negative electric charge solves the problem that would otherwise arise in electrodynamics) – a suggestion not taken seriously in the age of the positive energy theorem. But there is another way to solve this problem: accelerating expansion of the universe. This didn't dawn on me until I read Brian Greene's *The Fabric of the Cosmos*, in particular, footnote 10 for chapter 8, where he mentions that accelerating expansion produces a "cosmic horizon" that renders the spacetime beyond the horizon forever inaccessible to light signals emitted at the origin of coordinates, no matter how long they propagate into the future. This is the "cut-off" that keeps the action at a distance interaction finite. Were I a clever theoretician, I could have predicted accelerating cosmic expansion before it was discovered. Instead, I kept on working the experimental stuff.

But in the fall of 2014, I convinced my colleague, Heidi Fearn, who had joined me on the Mach effects project a few years earlier, to fill in the formalism and write this up. She sent it to the "gr-qc" section of the arXiv operation. Her first submission there. They told her she needed to get endorsers. She asked two world famous general relativists, who immediately begged off, and Jayant Narlikar, who allowed that he had never used the arXiv and so could not be an endorser. But the next day it was up on the server. She had submitted it to *the Journal of Modern Physics* for a special issue on gravitation and cosmology. It was accepted in less than a week. And friends have brought serious articles in prestigious journals on action at a distance gravity to our attention. Perhaps there is more to Einstein's outrageous legacy than hitherto has been widely thought. And perhaps there is a way to manipulate the spatiotemporal continuum that is the inertio-gravitational field to get around spacetime quickly.

¹⁷ Hawking detected this problem while reading Hoyle and Narlikar's manuscript overnight before Hoyle presented the work at the Royal Society meeting the following day. It took me a lot longer to zero in on this problem. But hey, I'm just an experimentalist.