NORDTVEDT'S REMARKS ON GRAVITOMAGNETISM

The literature of any field of study spans the range from awful to outstanding. And once in a long while a paper is published that is a true jewel, one that is written with crystal clarity that goes to the heart of a truly fundamental issue with little or no accompanying clutter to distract your attention from what's really important. Much has been written on the origin of inertia and Mach's principle in the past century, some of it outstandingly good. But there have only been a few jewels. Dennis Sciama's paper in the *Monthly Notices of the Royal Astronomical Society* back in 1953 is one. Another is Derek Raine's 1981 paper in *Reports of Progress in Physics* where he showed that Sciama's account of inertial reaction forces follows from general relativity theory in all isotropic cosmological models. And in the matter of the existence of gravitomagnetism, there is Ken Nordtvedt's 1988 paper in the *International Journal of Theoretical Physics*.

Back in the teens, almost as soon as Einstein published his general relativity theory, Lense and Thirring pointed out that a consequence of the field equations was that "frame dragging" should take place near rotating massive objects like the Earth or the Sun. Frame dragging usually means that space *per se* (and anything that happens to be in it) is moved by the motion of a nearby object. For much of this past century, this has been considered one of the chief predictions of general relativity theory. Detecting such frame dragging has motivated a half-billion dollar research effort spanning now many decades, known presently as "Gravity Probe B", a satellite with ultrasensitive gyroscopes soon to fly it is said. Indeed, in the past several months, several claims of the detection of frame dragging, indirect and direct, have been advanced.

Frame dragging is a clear Machian effect since rotation takes place with respect to the very distant matter in the universe. In Newtonian gravity no frame dragging takes place because there are no gravitomagnetic effects. And the almost universal opinion of even experts has been that gravitomagnetic effects are exceedingly small; so small that they have only very recently been detected. In this connection, Nordtvedt quoted a National Academy of Sciences report of the mid-80s where it was claimed that, "At present there is no experimental evidence arguing for or against the existence of the gravitomagnetic effects predicted by general relativity. . . ." One hears echoes of this view to this day. Nordtvedt's point was that this is, simply put, wrong. Gravitomagnetism is commonplace. As he put it at the end of the introduction of his paper, "In summary, inertial frame 'dragging' -- both linear accelerative dragging and rotational 'Lence[*sic.*]-Thirring' dragging -- are ubiquitous in gravitational phenomena already observed and measured." In section 5 of his paper, he ties these ideas up to Mach's ideas.

At the end of his section on Mach [section 5] Nordtvedt writes down the expression for the frame dragging effect, $\delta\boldsymbol{a}$, of a linear acceleration, ***a***, of a celestial body on the space within it. It is the last [un-numbered] equation of the paper:

$$\delta\boldsymbol{a}(\boldsymbol{r},t) = -(\quad)[U(\boldsymbol{r},t)/c^2]\boldsymbol{a}$$

$U(\boldsymbol{r},t)$ is the scalar Newtonian gravitational potential at the point ***r*** within the body caused by all of the matter within the body. The empty parentheses [( )] on the right hand side of this equation actually contain a sum of "post-Newtonian" parameters of order of unity or zero. How these parameters are chosen allows you to select a variety of different gravity theories. [Nordtvedt was a major contributor to the elaboration of the parametrized post-Newtonian, or "PPN", formalism in the late 60s and early 70s.] If we choose the values of those parameters that correspond to general relativity theory, then the empty parentheses have the value four.

Nordtvedt, uninterested in cosmic scale phenomena, didn't push this business beyond this point. But we can. To see the full Machian implications of general relativity using Nordtvedt's equation above, all we have to do is ask a simple question: What would be the condition needed to make the inertial space within a body move rigidly with the body when it is subjected to accelerations? This condition guarantees that absolute accelerations within the body are completely undetectable -- preserving the principle of relativity. The answer, given the above equation, is obvious. Rewriting the Newtonian potential as ϕ, the condition is:

$$4\phi / c^2 = 1,$$

making $\delta \boldsymbol{a}$ equal to $\boldsymbol{a}$. This, up to the factor of four, is Sciama's condition for inertia to be a gravito-magnetic vector potential effect. Viewed from Nordtvedt's perspective, $4\phi / c^2 = 1$ is the condition that insures that local space *and everything in it* is dragged rigidly with the universe should it be given a translational acceleration (by God, or whatever). Since only relative accelerations matter, we can check to see if this is true by accelerating local objects relative to the rest of the universe to see if they experience inertial reaction forces (with predictable results). The idea, even if only in a "thought experiment", of the rigid acceleration of the universe we've invoked may be bothering you. It's addressed in the context of Sciama's argument in the Subtleties section of "The Origin of Inertia".

Sciama and Nordtvedt weren't the only ones to recover the essence of the results we have looked at here. Long before them, Einstein got the same answers. His results can be found on pages 99 to 108, especially page 102, of *The Meaning of Relativity* (Princeton, 1953; first ed. 1922). Einstein, however, came to the conclusion, because of the dependence of inertial reaction forces on ϕ, that the masses of things would be affected by the presence of nearby objects (sometimes called "spectator masses") that, in this approximation, would seem to contribute to the value of ϕ. In another jewel of a paper Carl Brans showed that this wasn't right ["Mach's Principle and the Locally Measured Gravitational Constant in General Relativity," *Physical Review*, **125**, 388-396 (1962)]. The problem is the "linear order" (in the mass) approximation used to get the field equations into Maxwellian form. This is also the source of epoch dependence -- time variation of the gravitational constant -- problems too. They are resolved by the realization that in the full non-linear theory the *locally measured* total gravitational potential (or, in Brans' terms, the gravitational constant) must be a scalar invariant exactly like the speed of light. If this is not true, then gravitational fields and accelerated frames of reference can be distinguished, and the principle (not theory) of relativity is violated. Since the groundwork of general relativity theory is the principle of equivalence, it follows that this must be true. Raine's demonstration that inertia is gravitationally induced in all isotropic cosmologies confirms this conclusion. Like it or not, inertia is a gravitational phenomenon.

Nordtvedt's effect, now tested to considerable accuracy at the scale of planetary bodies, is a transient change in the mass of an accelerated object. For an object comprised of *n* gravitationally interacting massive particles subjected to an external force that produces an acceleration $\boldsymbol{a}$ of each of its parts, the mass shift of the i^{th} particle, δm_i, Nordtvedt finds to satisfy:

$$\delta m_i \boldsymbol{a} \approx m_i [4U(\boldsymbol{r},t) / c^2] \boldsymbol{a}$$

where $U(\boldsymbol{r},t)$ is the Newtonian gravitational potential at the i^{th} particle due to the other particles in the object, and PPN values for general relativity have been used. [This is Nordtvedt's Equation (14) with only the lowest order term displayed and the Newtonian potential written as *U* instead of as an explicit sum of the individual potentials. Nordtvedt ignores interactions other than gravitational that complicate, but do not change, this business in creating this expression.] In light of our frame dragging discussion, several important inferences follow from this expression. The first is that when we let the accelerated body be the entire universe, then in the linear order approximation we are using $4U(\boldsymbol{r},t) / c^2 = 1$, and Nordtvedt's transient mass shift is precisely the mass of the particle. That is, normal inertial reaction forces are the consequence of Nordtvedt's lowest order transient mass shift stimulated by the relative acceleration of objects and chiefly the most distant matter in the cosmos.

The second important inference that follows from the above expression is that $U(\boldsymbol{r},t)$ [or ϕ as we've been calling it] cannot be rescaled by an arbitrary additive constant. Were that possible, we could set it equal to zero, and then there wouldn't be any inertial reaction forces at all. At the local scale, the Nordtvedt effect wouldn't exist. The third inference is that mass itself arises from the gravitational potential energy, that is, $m_i U(\mathbf{r},t)$, which suggests that "test particles" in an empty universe should experience no inertial reaction forces whatsoever -- as expected on the basis of Mach's principle. The transient mass fluctuation derived in "Transient Mass Fluctuations" isn't the lowest order cosmic Nordtvedt effect -- normal inertial reaction mass that is. It's a higher order effect. But it, like the Nordtvedt effect, leads to the same conclusions we've found here.

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