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THE PROBLEM OF INERTIA IN FRIEDMANN UNIVERSES

JOSEPH SULTANA
Department of Mathematics, Faculty of Science, University of Malta
Msida MSD2080, Malta
joseph.sultana@um.edu.mt

DEMOSTHENES KAZANAS
Astrophysics Science Division, NASA/Goddard Space Flight Center
Greenbelt, Maryland 20771, USA
demos.kazanas@nasa.gov

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In this paper we study the origin of inertia in a curved spacetime, particularly the spatially flat, open and closed Friedmann universes. This is done using Sciama's law of inertial induction, which is based on Mach's principle, and expresses the analogy between the retarded far fields of electrodynamics and those of gravitation. After obtaining covariant expressions for electromagnetic fields due to an accelerating point charge in Friedmann models, we adopt Sciama's law to obtain the inertial force on an accelerating mass m by integrating over the contributions from all the matter in the universe. The resulting inertial force has the form $F = -kma$, where $k < 1$ depends on the choice of the cosmological parameters such as Ω_M, Ω_Λ, and Ω_R and is also red-shift dependent.

Keywords: Inertia; Friedmann Universes; Mach's Principle.

1. Introduction

The concept of inertia has been one of the most debated topics of classical physics, starting with Newton's ideas of absolute space and that of inertia as an intrinsic property of matter devoid of any external influence. The notion of absolute space was criticized by Leibniz and later Bishop Berkeley who claimed that it is metaphysical. They were followed by Ernst Mach [1] who in 1872 rejected the existence of absolute space in favour of relative motion with respect to a "fixed" frame provided by the matter distribution in the universe, and claimed that it is the acceleration relative to this frame that determines the inertial properties of matter. This is the essence of Mach's Principle, a term coined by Albert Einstein in 1918, which says that "the inertial force which acts on an accelerating object is due to its interaction with all matter present in the rest of the universe". There are indeed several interpretations of Mach's principle and arguments on whether the general relativity theory is Machian or not.[2–3] Besides Mach's principle there are other approaches towards the explanation for the orgin of inertia which describe inertia as a local instead of a global phenomenon. Among these one can mention the suggestion that inertial forces result from the interaction of matter with electromagnetic fluctuations of the zero point field, [4] (which is assumed to be homogeneous and isotropic, a property likely also related to the large scale distribution of matter in the universe and thus to the same frame envisioned by Mach) or the attribution of inertia to the interaction of a particle with its own field, namely the self-force.[5]

The implementation of Mach's principle as a quantitative law was done by D.W. Sciama [6] in 1953, by applying the formalism of electrodynamics to gravitation. According to Sciama the gravitational field of a moving particle in the universe can be expressed in terms of a "gravoelectric" part **E** and a "gravomagnetic" part **B**. These can be written in terms of the retarded scalar and vector potentials ϕ and **A** by

$$\begin{aligned} \mathbf{E} &= -\mathrm{grad}\phi - \left(\tfrac{1}{c}\right)\tfrac{\partial \mathbf{A}}{\partial t} \\ \mathbf{B} &= \mathrm{curl}\mathbf{A}, \end{aligned} \tag{1}$$

where ϕ is taken to be the Newtonian potential and $\mathbf{A} = (\phi/c)\mathbf{v}(t)$; $\mathbf{v}(\mathbf{t})$ being the velocity of the particle.

According to this law, the gravitational force between two masses m_1 and m_2 with relative acceleration a can be expressed by

$$F = \frac{Gm_1m_2}{r^2} + \frac{Gm_1m_2a}{c^2r}, \tag{2}$$

in analogy with the Lorentz force, where the acceleration dependent term is referred to as "inertial induction" and is the result of acceleration in fields that obey retardation. Since the universe can be taken to be isotropic on the large scale, the $1/r^2$ term will cancel when integrating over all masses in the universe to find the resulting inertial force on an accelerating test particle of mass m. This force is therefore given by [7]

$$F = \sum_{\mathrm{all}\,M} \frac{GmMa}{c^2r} = \frac{Gma\tilde{\rho}}{c^2}\iiint_{R_U}\frac{1}{r}\mathrm{d}\mathbf{r}, \tag{3}$$

where the region R_U is taken by Berry (see Ref. 7) to be the observable universe having density $\tilde{\rho}$ given by that part of the universe which is contained within the Hubble sphere with radius c/H, representing the distance where galaxies are receding with the speed of light. Hence

$$F = \frac{4\pi Gma\tilde{\rho}}{c^2}\int_0^{c/H} r\,\mathrm{d}r = \left(\frac{2\pi G\tilde{\rho}}{H^2}\right)ma. \tag{4}$$

Substituting values for $\tilde{\rho}$ and H in the coefficient of ma in equation (4), Berry obtained a value of $\frac{1}{25}$ instead of unity. He explained that this discrepancy results from the uncertainty in $\tilde{\rho}$ which does not include dark matter and the increase in masses of distant galaxies by the relativistic factor $(1 - v^2/c^2)^{-1/2}$.

An attempt to generalize Mach's principle to curved spaces, particularly the Friedmann universes, was done by R. L. Signore,[8–9] by using Sciama's law with a potential

$$\phi = \int \left(\frac{\ddot{R}}{R} \right) r \, \mathrm{d}r, \tag{5}$$

where $R(t)$ is the scale factor, instead of the above Newtonian potential. Moreover instead of limiting the volume of the universe to the Hubble sphere, the above integration is extended to the causal sphere of the accelerating test particle represented by the particle horizon of the particle. In the case of the spatially flat Einstein-de Sitter model this potential takes the simple value of $-c^2$, and then (1) gives the result $F = ma$, in accordance with the equivalence principle. In the case of a spatially curved Friedman model the calculation leads to $F = \lambda ma$ where $\lambda > 1$ is time dependent. This means that either the time dependent inertial mass is larger than the gravitational mass, or else, the gravitational constant G changes with time to keep the ratio of the gravitational to inertial forces constant. However Signore's work is based on two assumptions which in our opinion limit the validity of the obtained results. The first one involves the integration in (5), which as in Berry's calculation (4), assumes that the density of the universe ρ is constant over its entire history. Moreover Signore claimed that in a curved space the field acting on an accelerating test particle is still given by the non-covariant flat space expressions in (1).

In this article we relax these two assumptions and use Sciama's approach to find the inertial force on an accelerating test particle arising from the inertial induction of all matter in the universe. As done by Signore we consider the particle horizon instead of the Hubble sphere to mark the region of the universe that influences our local dynamics.[10] This represents the distance to the farthest objects that can be observed at any particular time, and the redshift becomes infinite for emitters on the particle horizon.[a] However unlike previous calculations by Berry and Signore we use a variable denisty ρ when integrating over the entire universe. Moreover instead of the expressions for the retarded potentials and fields in (1) which are valid in flat space, we use general covariant formulations valid in a curved spacetime. The inertial force on an accelerating particle for the different types of Friedmann models is obtained in a form which is covariant with respect to coordinate transformations in the 3-space of these models. In section 2, we derive covariant expressions for the retarded potentials and fields for an accelerating test particle in the three cases of flat, open and closed Friedmann models. Then in section 3 we obtain expressions for the inertial forces and the results are summerized and discussed in section 4.

[a]We can receive photons emitted by receding emitters beyond the Hubble sphere ($v_{\rm rec} > c$), because the Hubble sphere expands and overtakes these photons which end up inside the Hubble sphere where $v_{\rm rec} < c$.

2. Covariant Fields

The study of electromagnetism in a general curved spacetime was done by DeWitt and Brehme.[11] Later Hobbs[12] obtained a formal solution for the electromagnetic potentials[b] $A^{\mu} = (\phi, \mathbf{A})$ in the case of a conformally flat spacetime, and showed that in this case, the electromagnetic signals propagate only on the light cone without scattering off the Riemann tensor or radiation tail as in general curved spacetimes, thereby simplifying the solutions.

Let us write the metric for the Friedmann universes in the form

$$ds^2 = -dt^2 + R^2(t)[d\rho^2 + f_k^2(\rho)d\Omega^2], \tag{6}$$

where

$$f_k(\rho) = \begin{cases} \sin\rho, & \text{closed } (k=1) \\ \rho, & \text{flat } (k=0) \\ \sinh\rho, & \text{open } (k=-1) \end{cases} \tag{7}$$

$R(t)$ is the scale factor obtained by solving the Friedmann equations and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Then the potentials due to an accelerating point charge q moving along the path $\mathbf{x}' = \mathbf{z}(t')$, where $(t', \mathbf{x}')$ represent the retarded position, are given by[13]

$$\begin{aligned} A_0(\mathbf{x}, t) &= \frac{qF(\Psi)}{R(t)\sigma}, \\ A_k(\mathbf{x}, t) &= -\frac{q[G(\Psi)d_{km'} + H(\Psi)n_k n_{m'}]v^{m'}(t')}{R(t)\sigma}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} F(\Psi) &= \cot\Psi, & k &= 1, \\ &= 1/\Psi, & k &= 0, \\ &= \coth\Psi, & k &= -1, \\ G(\Psi) &= \csc\Psi, & k &= 1, \\ &= 1/\Psi, & k &= 0, \\ &= \mathrm{csch}\Psi, & k &= -1, \\ H(\Psi) &= \tan\tfrac{1}{2}\Psi, & k &= 1, \\ &= 0, & k &= 0, \\ &= -\tanh\tfrac{1}{2}\Psi, & k &= -1, \end{aligned} \tag{9}$$

and

$$n_k = R(t)\Psi_{;k}; \quad n_{k'} = R(t')\Psi_{;k'}, \tag{10}$$

$$d_{km'} = R(t)R(t')g_{km'}, \tag{11}$$

[b] Greek indices take values from 0 to 3; latin indices represent the spatial components 1 to 3.

$$\sigma = 1 + n_{k'} v^{k'}; \quad v^{k'} = dz^k(t')/dt'. \tag{12}$$

The function $\Psi(\mathbf{x}, \mathbf{x}')$ is the bi-scalar distance[c] in 3-space measured along a geodesic joining the points $\mathbf{x}$ and $\mathbf{x}'$. This can be expressed as $L(\mathbf{x}, \mathbf{x}', t)/R(t)$, where L is the distance between $\mathbf{x}$ and $\mathbf{x}'$ at time t. The vectors n_k and $n_{k'}$ in (10) are unit tangent vectors at $\mathbf{x}$ and $\mathbf{x}'$ to the geodesic joining these two points, and the bi-vector parallel propagator[14] $d_{km'}$ gives a relation between the components of a vector A^i at $\mathbf{x}$ and the components of the same vector parallel transported along the geodesic joining $\mathbf{x}$ and $\mathbf{x}'$,

$$A_k = d_{km'} A^{m'}, \quad A_{m'} = d_{m'k} A^k. \tag{13}$$

The components of the electromagnetic field tensor $F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}$ are given by[13]

$$F_{0k}(\mathbf{x}, t) = \tfrac{1}{R^2(t)} \left\{ \tfrac{G(\Psi)F(\Psi)}{R(t')} (d_{km'} + n_k n_{m'}) J^{m'} + \tfrac{d}{ds} \left[\tfrac{G(\Psi)}{R(t')} (d_{km'} + n_k n_{m'}) J^{m'} \right] \right.$$
$$\left. + \tfrac{G^2(\Psi)}{R(t')} (n_k n_{m'} J^{m'} - n_k J^{0'}) \right\}, \tag{14}$$

$$F_{km}(\mathbf{x}, t) = \tfrac{1}{R^2(t)} \left\{ \tfrac{G(\Psi)F(\Psi)}{R(t')} (d_{kl'} n_m - d_{ml'} n_k) J^{l'} \right.$$
$$\left. + \tfrac{d}{ds} \left[\tfrac{G(\Psi)}{R(t')} (d_{kl'} n_m - d_{ml'} n_k) J^{l'} \right] \right\}, \tag{15}$$

where

$$s = \int_t^{t'} \frac{dt''}{R(t'')} + \Psi(\mathbf{x}, \mathbf{x}'), \tag{16}$$

expresses the relation between the retarded time t' and the time t; obtained by setting $s = 0$, and the four-current density is given by

$$J^{m'}(\mathbf{x}', t') = q \frac{dz^{m'}}{ds} = q \frac{R(t') v^{m'}}{\sigma}. \tag{17}$$

The Lorentz force on a comoving test charge $\tilde{q}$ having four-velocity u^α in the Friedmann models (6), is given by the space part of the four vector

$$f^\mu = \tilde{q} F^{\mu\nu} u_\nu, \tag{18}$$

i.e.,

$$\mathbf{F} = -\tilde{q} F^{i0} = \tilde{q} F^{0i}. \tag{19}$$

In order to obtain expressions for the fields in (14) for different Friedmann universes represented by (7), consider a point charge q at the retarded position (t', ρ') moving

[c]In general two-point tensors, or bi-tensors, are used in the description of physical processes in which a cause at a point P' brings about an effect at a point P. Their indices (or arguments) refer to the points P' or P and are written with or without a prime over the index. For more details on bi-tensors see Synge (1960) in Ref. 14.

in the $\theta = 0$ direction such that $\mathbf{v}' = (v\cos\theta/(R(t'), -v\sin\theta/R(t'), 0)$. To find the components of the parallel propagator $d_{km'}$ in (14) consider the change in the velocity vector as it is parallel transported radially along a geodesic in 3-spaces of constant curvature given in (6). Then

$$dv^i = -\Gamma^i_{j1} v^j d\rho. \tag{20}$$

2.1. *Case k=0*

In this case the radial component of the velocity vector is unchanged under parallel transport and hence $v^1 = v^{1'}$ so that $g_{11'} = 1$. Also $v^2 = -v\sin\theta/R(t)\rho$ such that $g_{22'} = \rho$ and $v^3 = v^{3'} = 0$. Taking the limit $v << c$ the expression for the field in (14) with $\Psi = \rho$ becomes

$$F_{0k} = \frac{qR^2(t')}{\rho R(t)}(g_{km'} + \rho_{;k}\rho_{;m'})a^{m'} - \frac{q\rho_{;k}}{R(t)\rho^2}, \tag{21}$$

where $a^{m'}$ are the components of the acceleration of the point charge at the retarded coordinates (t', ρ'). This expression reduces to $-\mathbf{E}$ given by (1) in the case of flat spacetime where $R(t) = R(t') = 1$. The total force on the accelerating point charge q from a charge distribution, can then be obtained by using (19) and integrating over all contributions. If the distribution is isotropic the $1/\rho^2$ term in (21) vanishes, and one only needs to consider the component of the force along the direction of motion, given by

$$F = \int \tilde{\rho}(x^n)F^{0i}\frac{a_i}{|\mathbf{a}|}\, d^3x^n = -\int \tilde{\rho}(x^n)F_{0i}\frac{a^i}{|\mathbf{a}|}\, d^3x^n, \tag{22}$$

where $\tilde{\rho}(x^n)$ is the charge density of the distribution and

$$F_{0i}\frac{a^i}{|\mathbf{a}|} = \frac{qR(t')}{\rho R^2(t)}a\sin^2\theta. \tag{23}$$

2.2. *Case k =1*

In this case $v^1 = v^{1'}$ and $v^2 = -v\sin\theta/R(t)\sin\rho$. Therefore $g_{11'} = 1$ and $g_{22'} = \sin\rho$. Also $\Psi = \rho$ and the expression for the field (14) in the limit $v << c$ is given by

$$F_{0k} = \frac{qR^2(t')}{R(t)\sin\rho}(g_{km'} + \rho_{;k}\rho_{;m'})a^{m'} - \frac{q\rho_{;k}}{R(t)\sin^2\rho}. \tag{24}$$

The component of the field in the direction of motion is then given by

$$F_{0i}\frac{a^i}{|\mathbf{a}|} = \frac{qR(t')}{R^2(t)\sin\rho}a\sin^2\theta. \tag{25}$$

2.3. *Case k =-1*

Again there is no change in the radial component of the velocity vector under parallel transport and so $g_{11'} = 1$. The parallel transported angular component is given by $v^2 = -v \sin\theta / R(t) \sinh\rho$, and so $g_{22'} = \sinh\rho$. The previous expressions for the field and its component along the direction of motion are given (for $v << c$) by

$$F_{0k} = \frac{qR^2(t')}{R(t)\sinh\rho}(g_{km'} + \rho_{;k}\rho_{;m'})a^{m'} - \frac{q\rho_{;k}}{R(t)\sinh^2\rho}, \tag{26}$$

and

$$F_{0i}\frac{a^i}{|\mathbf{a}|} = \frac{qR(t')}{R^2(t)\sinh\rho} a \sin^2\theta. \tag{27}$$

Sciama's law of inertial induction can be generalized to the curved spacetime of the Friedmann models by using (21), (24) and (26) to represent the gravitational field of an accelerating particle and (22) for the gravitational force on this particle from an isotropic matter distribution instead of the previous expressions in (1) and (2) which are only valid in flat space. This is done in the next section.

3. Inertial Force

Using the coordinate transformation $r = R_0 f_k(\rho)$, where R_0 represents the scale factor at the present time $t = t_0$ the Friedmann models in (6) can be written in the form

$$ds^2 = -dt^2 + A^2(t)\left[\frac{dr^2}{1-\kappa r^2} + r^2 d\Omega^2\right], \tag{28}$$

where $A(t)$ is the dimensionless scale factor $A(t) = R(t)/R_0$ and $\kappa = k/R_0^2$. In these coordinates the expressions for the component of the field along the direction of motion given by (23), (25) and (27) are given by

$$F_{0i}\frac{a^i}{|\mathbf{a}|} = \frac{qA(t')}{A^2(t)r} a \sin^2\theta, \tag{29}$$

for all models. Then using Sciama's law of inertial induction, the total inertial force on an accelerating particle of mass m in the Friedmann models is given by

$$F = -\frac{Gma}{c^2}\int\int\int \frac{\tilde{\rho}A(t')}{r}\sin^2\theta\,\sqrt{g}\,dr\,d\theta\,d\phi, \tag{30}$$

where $\tilde{\rho}$ is the density of the universe, and we set $A(t) = 1$ because the force on the accelerating particle due to matter at the retarded position $(t', \mathbf{x}')$ is calculated at the present time $t = t_0$. Instead of the radial coordinate in (30) it is more convenient to use the redshift factor $z = 1/A(t') - 1$ which is related to r by[15]

$$r = \frac{cH_0^{-1}}{\sqrt{|\Omega_{c0}|}} f_k\left[\sqrt{|\Omega_{c0}|}\int_0^z \frac{dz'}{E(z')}\right], \tag{31}$$

where f_k is given by (7) and H_0, Ω_{c0} represent the Hubble's constant and the current density parameter associated with spatial curvature respectively. Also for the metric (28)

$$\sqrt{g} = \frac{A^3(t')r^2 \sin\theta}{\sqrt{1-\kappa r^2}}; \quad \kappa = -\frac{H_0^2 \Omega_{c0}}{c^2}, \tag{32}$$

and

$$E(z) = \left[\sum_{i(c)} \Omega_{i0}(1+z)^{n_i} \right]^{1/2}, \tag{33}$$

where n_i depends on the source of the density parameter according to Table 1. Assuming that source of the energy density of the Friedmann models consists of

Table 1. Values of n_i for different sources in Eq. (33)

Source	Parameter	n_i
matter	Ω_M	3
radiation	Ω_R	4
curvature	Ω_c	2
vacuum	Ω_Λ	0

matter and radiation, the density $\tilde{\rho}$ in (30) can be expressed in terms of the redshift by

$$\tilde{\rho}(z) = \frac{3H_0^2}{8\pi G} \left(\Omega_{M0}(1+z)^3 + \Omega_{R0}(1+z)^4 \right). \tag{34}$$

Substituting (31)-(34) in (30) we get the inertial force on the accelerating particle for the three Friedmann models

$$k = 0; \qquad F = -ma \int_0^\infty \frac{\Omega_{M0} + \Omega_{R0}(1+z)}{(1+z)E(z)} \left[\int_0^z \frac{dz'}{E(z')} \right] dz, \tag{35}$$

$$k = 1; \; F = -\frac{ma}{\sqrt{|\Omega_{c0}|}} \int_0^\infty \frac{\Omega_{M0} + \Omega_{R0}(1+z)}{(1+z)E(z)} \sin\left(\sqrt{|\Omega_{c0}|} \int_0^z \frac{dz'}{E(z')} \right) dz, \tag{36}$$

$$k = -1; \;\; F = -\frac{ma}{\sqrt{\Omega_{c0}}} \int_0^\infty \frac{\Omega_{M0} + \Omega_{R0}(1+z)}{(1+z)E(z)} \sinh\left(\sqrt{\Omega_{c0}} \int_0^z \frac{dz'}{E(z')} \right) dz. \tag{37}$$

Unlike (4) where the integration is limited to the Hubble sphere, in our case the integration is extended over the causal sphere bounded by the particle horizon of the accelerating particle where $z = \infty$. These includes the inertial contribution from all matter and radiation in the universe.

4. Results and Discussion

In this note we have tried to compute the inertial mass of particles following the approach of Sciama, i.e. the analogy between electromagnetic and gravitational forces, assuming that the latter, just like the former are subject to retardation. As a result, we assumed that they yield, just like electromagnetism, a far-field force between material particles of masses m_1, m_2 proportional to Gm_1m_2a/c^2r; note that because of the $1/r$ (rather than the $1/r^2$ appropriate for non-accelerating particles) force dependence, a spherically symmetric mass distribution yields a non-zero force at its center. Based on this analogy we followed Ref. 13 to compute the electric fields of accelerated charges in an arbitrary FRW space time and replaced the coupling constant by Gm_1 to mimic the effects of gravitational force. Finally, by integrating this force over all matter in the universe we computed the gravitational force of the latter on a body of mass m in relative acceleration (with the universe!). This force is proportional to the acceleration a with a coefficient that depends on the cosmological parameters, in (philosophical) agreement with Mach's principle.

Starting with the spatially flat Friedman model ($k = 0$), the inertial force on the accelerating particle given by (35) in a matter dominated Einstein-de Sitter universe ($\Omega_{M0} = 1$, $\Omega_{\Lambda 0} = \Omega_{R0} = 0$) is given by $F = -(1/3)ma$. Although this coefficient of ma is larger than that obtained by Berry, it is still far from unity, and this implies that the total force on the particle cannot be explained only in terms of the inertial contribution from all the matter in the universe. In a radiation dominated universe ($\Omega_{R0} = 1$, $\Omega_{\Lambda 0} = \Omega_{M0} = 0$) $F = -(1/2)ma$, while if we choose the currently favored cosmological parameters $\Omega_{M0} = 0.3, \Omega_{\Lambda 0} = 0.7$ we get $F = -0.23\,ma$.

Having said this, we have to point out that the coefficient of ma in all three cases given by (35) - (37) depends on the *current* values of the cosmological parameters. Since these parameters depend on the redshift z according to the expression

$$\Omega_j = \frac{\Omega_{j0}(1+z)^{n_j}}{\left[\sum_{i(c)} \Omega_{i0}(1+z)^{n_i}\right]} \tag{38}$$

where n_i are given in Table 1, this coefficient will be different at different epochs in the evolution of the universe. Therefore if $\Omega_{M0} = 0.3$, $\Omega_{\Lambda 0} = 0.7$, then at z = 1 $\Omega_{M1} = 0.77$, $\Omega_{\Lambda 1} = 0.23$ and the inertial force is $F = -0.31ma$. This higher value of the inertial force is due to a higher percentage of matter at earlier times. For very large red-shifts the universe is radiation dominated with $\Omega_R \to 1$ and $\Omega_\Lambda \to 0$, and the inertial force thus calculated becomes $F \to (1/2)ma$. It is not obvious at this point whether such a variation can lead to observable effects or inconsistencies with observations; we hope to be able to look at these issues in the future.

For a spatially closed ($k = 1$) universe the effect of positive curvature leads, as expected, to a higher value for the inertial force on the accelerating particle. So for a slightly curved universe with $\Omega_{c0} = -0.1$, $\Omega_{M0} = 0.3$, $\Omega_{\Lambda 0} = 0.8$ we get $F = -0.24ma$. As in the flat case the coefficient of ma is higher in the presence of radiation. So if $\Omega_{M0} = 0.3$, $\Omega_{\Lambda 0} = 0.7$, $\Omega_{c0} = -0.1$, $\Omega_{R0} = 0.1$, then $F = -0.33ma$.

On the other hand in the spatially open ($k = -1$) case, the negative spatial curvature reduces the inertial force. Taking $\Omega_{M0} = 0.2$, $\Omega_{\Lambda 0} = 0.8$, $\Omega_{c0} = 0.1$, we get $F = -0.19ma$. The previous remark about the z-dependence of our results applies also to the curved Friedmann models.

To summarize, in this article we have used the formalism of electrodynamics in curved spacetime to generalized Sciama's law of inertial induction to a curved spacetime represented by the Friedmann models, and obtained covariant expressions for the inertial force on a accelerating particle in these models. This extends previous work by Berry and later by Signore on the subject of inertia in Friedmann universes. As in Berry's case we have showed that the total inertial force $F = -ma$ on an accelerating particle cannot be explained only in terms of inertial induction from the total matter and radiation in the current universe. We have also seen from (38) that the inertial force is redshift dependent and increases for earlier times until it reaches the asymptotic value of $-(1/3)ma$ in the matter dominated case or $-(1/2)ma$ in the radiation dominated case.

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