Feynman's "Paradox" and Electromagnetic Moments of Inertia

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In Sect. 17_4 of **The Feynman Lectures on Physics, V2**, Feynman presents a "paradox." A superconducting solenoid is mounted on a plastic disc that is free to rotate on a frictionless axle. Around the periphery of the disc are embedded small, charged spheres. A constant current initially circulates in the solenoid. The disc is initially at rest. There is a magnetostatic field.

As the solenoid temperature rises toward room temperature, the coil loses its superconductivity. At some temperature the current begins to drop toward zero. d$\underline{B}$/dt is nonzero as the current drops, and a circulating $\underline{E}$ field is induced. Each charged ball experiences a tangential electric force. There is a nonzero torque, and the disc begins to rotate.

According to Feynman, here is the paradox: the initial angular momentum is supposedly zero, and hence the final angular momentum should also be zero.

The article on the electromagnetic mass of a solenoid reveals the source of the confusion. The initial solenoidal current has a nonzero moment of inertia. Hence the initial angular momentum is not zero! The final angular momentum of the entire apparatus is partly due to the torque exerted by the solenoid on the disc as its current drops, and partly due to the induced emf on the peripheral charges.

We can simplify Feynman's apparatus by dispensing with the charged balls. Again we suppose that the solenoid is initially superconducting and has a constant current per unit length (or constant, nonzero $I_{enclosed}$). Approximating the electromagnetic mass per unit length with that of an infinitely long solenoid, we can estimate the initial angular momentum. When the superconductivity is lost and the current begins to drop, the disc will begin to rotate.

Maxwell was aware of the inertia-like character of electric currents. In Article 547 of **A Treatise on Electricity and Magnetism** he quotes Faraday: "…the first thought that arises in the mind is that the electricity circulates with something like momentum or inertia in the wire."

An important aspect of the induction phenomenon pertains to the "self" (or reactive) force/torque experienced when the momentum of an electric current changes in time. In the variation on Feynman's apparatus suggested above it is the "self" torque, experienced by the dropping solenoidal current in its own, (angular) acceleration-induced electric field, that physically accounts for the torque passed on to the plastic disc. Lenz's law is, in one respect, a more general consequence of reactive forces and torques.

It is noteworthy that the angular momentum of a solenoid is not simply the sum of the angular momenta of the individual, circulating charges (or, as we now know, conduction electrons). For when $I_{enclosed}$ is varied in time, the electrons experience interactive forces from the other electrons, in addition to their own "self" forces. The relative magnitude of these interactive forces depends upon the circuit's geometry. A solenoid consisting of a kilometer of wire has significantly greater electromagnetic kinetic energy than does the same wire stretched out in a straight line.

The interactive forces are most dramatic when N charges, each with electromagnetic mass

$m_{ElecMag}$, are superimposed. In this extreme (and idealized) case, the apparent electromagnetic mass of the collection is not simply $Nm_{ElecMag}$. For each charge experiences not only its own, acceleration-induced electric field; it experiences the acceleration-induced electric fields of all the other charges as well. The apparent electromagnetic mass of any one charge is not simply $m_{ElecMag}$; it is $Nm_{ElecMag}$. And the apparent electromagnetic mass of all N charges is N times this amount, or $N^2m_{ElecMag}$!

Maxwell appears not to have appreciated the role of interactive forces in a current's momentum, perhaps because the particulate nature of electricity hadn't yet been discovered. In his mind, 'momentum' is an intrinsic property of a physical entity, independent of other entities in the environment. Thus he believed that the total momentum of a system is the sum of the momenta of its constituent parts. In Article 549 of his Treatise he states, "…if the phenomena are due to momentum, the momentum is certainly not that of the electricity in the wire, because the same wire, conveying the same current, exhibits effects which differ according to its form…"