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The Role Of Invariant Manifolds In Low Thrust Trajectory Design

Martin W. Lo[*], Rodney Anderson[†], Gregory Whiffen[*], Larry Romans[*]

An initial study of techniques to be used in understanding how invariant manifolds are involved in low thrust trajectory design for the Jovian moon missions was performed using a baseline trajectory from the Europa Orbiter studies. Poincaré sections were used in order to search for unstable resonant orbits. The unstable manifolds of these resonant orbits were computed, and they were found to provide an indication of how the EO trajectory was able to transition between resonances. A comparison with the stable and unstable manifolds of Lissajous orbits around the Jupiter-Europa L_2 Lagrange point provided evidence that the Europa capture utilizes invariant manifolds of quasi-periodic orbits.

INTRODUCTION

Invariant manifold theory and low energy orbits may play a significant role in the design and optimization of low thrust interplanetary trajectories (see Lo[1]). This is the first of a series of papers where we plan to demonstrate first, that invariant manifolds do indeed play such a role, second, explain how the dynamics of low thrust interplanetary trajectories interact with invariant manifolds. Our plan is to analyze and characterize the dynamics of the trajectories of the Europa Orbiter (EO, see Johannessen and D'Amario[2]) and of the Jupiter Galilean Moons Tour (JGMT see Whiffen[3]). The idea is to first study the role of invariant manifolds for a trajectory which uses impulsive maneuvers which is simpler to analyze, then apply the knowledge gained to the low thrust trajectory where the analysis is much more complex. In this paper we will introduce the various concepts we will use and focus on the analysis of the EO trajectory.

The fact that there may be a significant connection between low thrust interplanetary trajectories and invariant manifold theory is suggested by two seminal work in the mid 1990's. The first work,by Bolt and Meiss 1995[4], described an algorithm to construct a very low energy transfer trajectory from the Earth to the Moon using recurrence of chaotic trajectories generated by the invariant manifolds of unstable periodic and quasipeirodic orbits around the Earth-Moon system. Although this approach required many years for the transfer to occur, one can infer from this work that chaotic trajectories may play a role in low thrust interplanetary trajectory design and optimization. The second work continued Bolt and Meiss' work but instead of using the random recurrences of intersecting chaotic trajectories, Shroer and Ott 1997[5] targeted the invariant manifolds of unstable resonant orbits between the Earth and the Moon. This greatly reduced the

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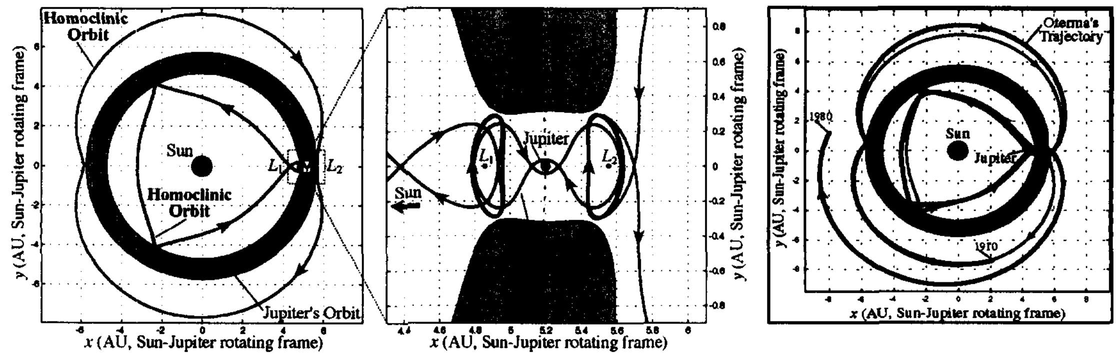

1a. 1 b.

Figure 1.a. A homoclinic-heteroclinic chain within the Jovian system. These are a special set of trajectories linking the S, J, X regions of Jupiter via two of its periodic orbits at L_1 and L_2. 1.b. The orbit of comet Oterma superimposed on the chain showing how closely the comet orbit is guided by the chain.

flight time to less than 1 year. The maneuvers for the targeting were miniscule in both approaches since chaotic orbits were used. These two papers suggested to us that invariant manifolds do play a very important role the dynamics of low thrust interplanetary trajectories.

A second stream of research which feeds into the current series of papers is the understanding of the role of invariant manifolds in resonances and ballistic captures. For this, we take our cue from the heavenly bodies which have exploited this low energy dynamics for eons. It has long been observed that comets frequently change their orbits from one resonance to another. Here resonance refers to that between the orbital period of a comet or asteroid with that of a planet such as Jupiter or Saturn. For example, the Hilda Asteroid Group is in 3:2 resonance with Jupiter, meaning that the asteroid group makes 3 revolutions around the Sun for every 2 revolutions which Jupiter makes around the Sun. The comets Oterma, Gehrals3, and Helin-Roman-Crockett all exhibit this resonance transition phenomena going between the 3:2 and 2:3 resonances with Jupiter.

Some have proposed the weak stability boundary to explain the resonance transition (see Belbruno and B. Marsden[6]), but there have been no computations or proofs known to the authors using the weak stability boundary to demonstrate this conjecture. Lo and Ross[7] proposed the invariant manifolds of unstable orbits around L_1 and L_2 as a mechanism for this transition. Koon, Lo, Marsden, and Ross [8,9], using semi-analytical methods, demonstrated the transition between the 3:2 and 2:3 resonance of Jupiter is effected by heteroclinic orbits between Liapunov orbits around JL_1 (Jupiter L_1) and JL_2 and the intersection of invariant manifolds between an unstable 2:3 resonant orbit with the manifolds of a Liapunov orbit around JL_2, and the intersection of invariant manifolds between an unstable 3:2 resonant orbit with the manifolds of a Liapunov orbit around JL_1 (see Figure 1 above). See Yamato and Spencer[10] for further developments of the Petit Grand Tour concept.These computations were performed in the restricted 3-body problem. Howell, Marchand, and Lo[11] showed that the orbits of the comets mentioned above do indeed closely followed the invariant manifolds of halo orbits around Jupiter's

L_1 and L_2 Lagrange points computed with the JPL ephemeris model of the Solar System. Using interval analysis methods, Wilczak and P. Zgliczy'nski [12] provided a computer assisted proof of the existence of the heteroclinic connections computed in Koon et al [8].

This excursion into celestial mechanics actually has some compelling consequences for new mission concepts. Using these transitions via invariant manifolds, Koon, Lo, Marsden, and Ross [8,9]proposed a "Petit Grand Tour" to serially capture and orbit the Galilean Moons using the invariant manifolds to provide low energy intersatellite transfers and ballistic captures to reduce the total ΔV required. This approach reduced the ΔV from the Hohmann Transfer by roughly 50%. However, the performance of the EO was even better by using repeated resonant flybys. In fact, using repeated resonant flybys for a Petit Grand Tour mission, extending the total mission elapsed time from 10's of days to 10 years, the total ΔV may be reduced to less than 20 m/s (see Ross, Koon, Lo, Marsden[13]) Clearly, resonant orbits are important for lowering the energy required for interplanetary transfers, although it is at the expense of longer flight times. This opens the mission trade space by giving mission designers the freedom to balance the ΔV budget with the total flight time.

We provide a heuristic description of how low thrust trajectories may interact with invariant manifolds. We begin by with the energy surfaces within the 3-body problem which is itself an invariant manifold. A spacecraft with a given energy (Jacobi constant) will move on that energy surface. If we turn on a low thrust engine, the spacecraft will move onto a nearby energy surface, layer after layer, piercing a family of energy surfaces. However, we know much are about the structures in phase aside from energy surfaces. For the spatial circular restricted 3-body problem, the phase space is 6-dimensional and the energy surface is 5-dimensional. But the unstable periodic and quasiperiodic orbits have stable and unstable manifolds of dimensions 2 and 3 respectively. They themselves further partition the energy surface and restrict the motions of the spacecraft. But as the thrust level is low, the spacecraft is traversing these layers of manifolds slowly. The manifolds themselves are also changing slowly locally, assuming we are not close to singularities. Thus overall, the spacecraft will appear to be traveling along the invariant manifolds. This is frequently seen in trajectories optimized by the MYTSTIC tool at JPL, even though neither MYSTIC nor the initial guess solution to start the optimization process do not explicitly incorporate a prior knowledge of invariant manifolds.

Our ultimate goal is to combine our knowledge of invariant manifolds in the Solar System (those generated by unstable orbits) and the theory of transport through resonances and libration orbits, using the heuristic plan above as a guide to first develop an empirical and semi-numerical understanding of the dynamics of low thrust interplanetary trajectories. From this numerical foundation, we may be able to build a more systematic and theoretical approach to understand the dynamics of low thrust interplanetary trajectories.

For low thrust trajectories in the two body problem, the dynamical systems approach is also be very useful. There, the picture is slightly different. Instead of invariant manifolds associated with unstable orbits, the stable orbits are organized into families of concentric tori, hence the spiraling of low thrust trajectories to escape the gravity well of a planet. These tori are also invariant manifolds, albeit stable. It is remarkable that still today, the problem of optimizing spiral escape and capture orbits in the two-body context is an unsolved problem. See for a description of the current research in this deceptively challenging problem.

JGMT AND LOW THRUST TRAJECTORIES

An initial examination was made of the Jupiter Galilean Moons Orbiter trajectory generated by Gregory Whiffen using the MYSTIC optimization software for a spacecraft traveling from Ganymede to Europa in the Jovian system[1]. The trajectory covers a span of approximately 83.5 days (from JED = 2457225.9 to JED = 2457309.4) using the JPL DE405 ephemerides for the planets and moons. A plot of the trajectory including the thrusting directions in Jupiter centered inertial coordinates is given in Figure 2. A plot in rotating frame of the same trajectory is given in Figure 3.

However, we quickly discovered that many of the standard techniques such as Poincaré sections are very difficult to use in the Jovian system due to the strong perturbations of the multiple moons in resonance with one another. Instead, we decided to attack a simpler problem by first examining an impulsive version of a Jovian satellite tour, the Europa Orbiter (EO) trajectory. It is well known that EO uses a ballistic capture into Euorpa orbit. We had long surmised that this capture follows the invariant manifolds

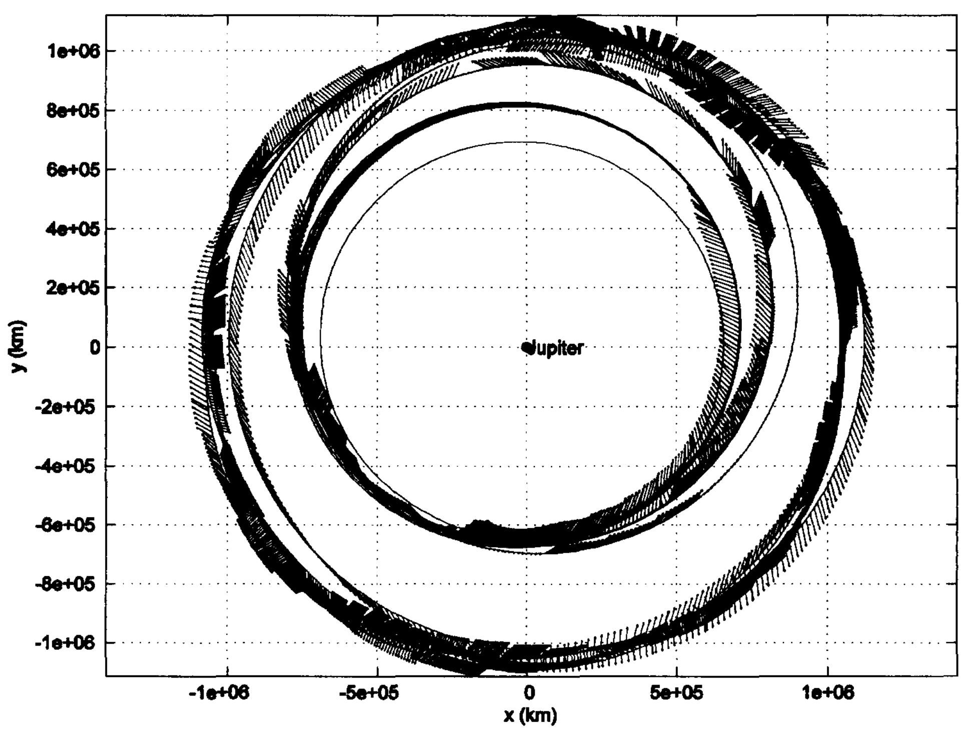

Figure 2 JGMT Trajectory with Thrusting Directions (inertial frame)

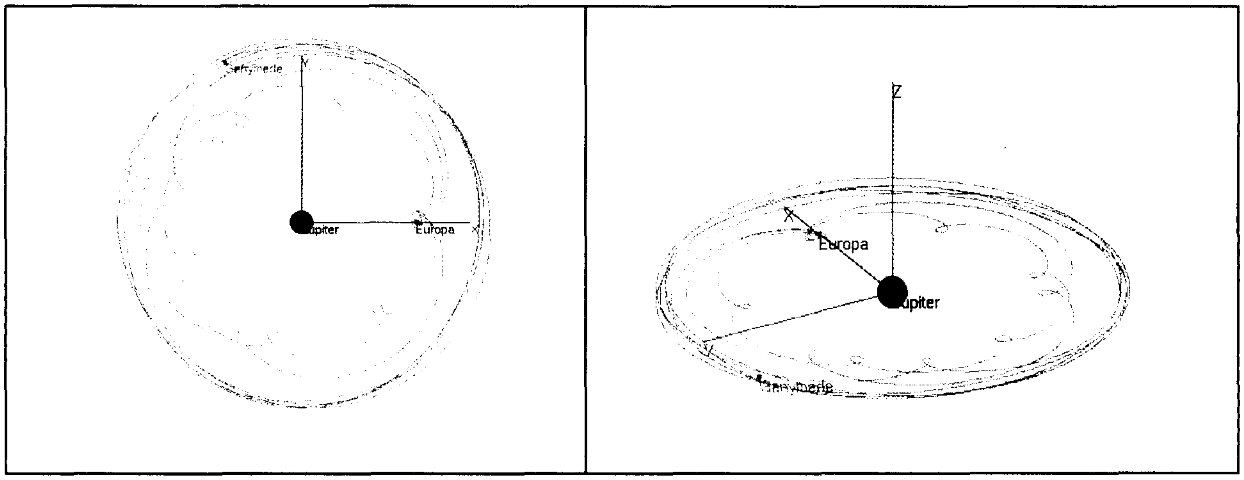

Figure 3. JGMT Trajectory in the Europa Rotating Frame in various projections

of some JL_2 libration orbit. Moreover, the resonant flybys provide EO with an extremely efficient intersatellite transfer. Although we know that for JGMT, the final capture mechanism uses an unstable retrograde orbit instead of a libration orbit. Nevertheless, resonances and invariant manifolds are the keys to the trajectory design of both EO and JGMT's.

MODELING THE EUROPA ORBITER TRAJECTORY IN THE CR3BP

A Europa Orbiter trajectory was obtained from Jennie Johannesen at JPL to use as the baseline for this study[1]. The capture into an orbit around Europa was currently the primary focus, so only the last portion of the trajectory was used here. This portion began with a flyby around Europa on September 12, 2009 and ended with Europa Orbit Insertion (EOI) on October 19, 2009. There were two intermediate ΔVs which divided the trajectory into three segments. A view of the trajectory in the inertial coordinate system is shown in Figure 4., and views in the rotating coordinate system are plotted in Figure 5. A list of events is given in Table 1.

As a first step, the selected portion of the trajectory was imported into the CR3BP in order to eliminate extraneous effects and focus on the dynamics of the problem. Specifically, an understanding of how the trajectory uses invariant manifolds to transition between the different segments was desired. This process required several steps and utilized a differential corrections scheme implemented in JPL's Libration Point Mission Design Tool (LTool). As a first step, the original Europa Orbiter baseline trajectory integrated using the ephemerides for each of the bodies was imported into LTool as an SPK file. Several points on this trajectory were then selected at regular intervals and used as "patchpoints" in the differential corrector. Refer to Roby or Howell for an explanation of the algorithms used in this differential corrector[0]. Using these patchpoints the differential corrector quickly converged on a solution when Jupiter and its four major moons (Io, Europa, Ganymede, and Callisto) were included in the integration with their ephemerides. The differential corrector was unable to converge when the trajectory was imported directly into the CR3BP from this point, so the trajectory was transferred to the

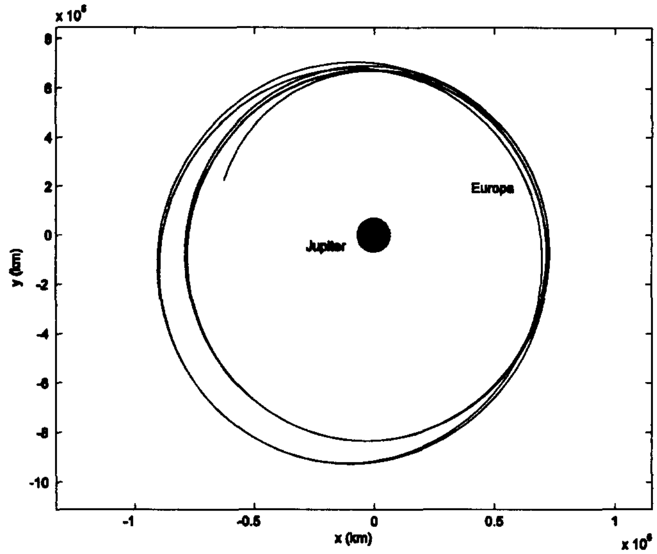

Figure 4. EarthMeanEcliptic2000 Inertial Europa Orbiter Trajectory.

CR3BP in stages. A large number of patchpoints seemed to keep the differential corrector from converging, so code was written to eliminate patchpoints by visual inspection where they appeared to be unnecessary. With these points eliminated, each of the moons, except Europa, were removed one at a time, and the differential corrector was allowed to converge each time before removing the next moon. For some reason, removing Io caused the greatest difficulty, and the trajectory only converged if it was removed last. Once the trajectory had converged using only Jupiter and Europa, the trajectory was imported into the CR3BP. Because the final portion of the trajectory approached Europa, the patchpoints were imported into the CR3BP relative to Europa. Once this procedure was completed, the trajectory finally converged in the CR3BP. It should also be mentioned that the primary consideration in the differential correction process was retaining the shape of the trajectory with the expectation that this would be most likely to preserve the dynamics that were of interest. This made the process more difficult since the differential corrector tended to converge to a very different type of trajectory. The ability to edit the patchpoints was especially important for preserving the shape. As can be seen from comparing Figures 5 and 6, the CR3BP trajectory retained the same general characteristics as the original baseline Europa Orbiter Trajectory.

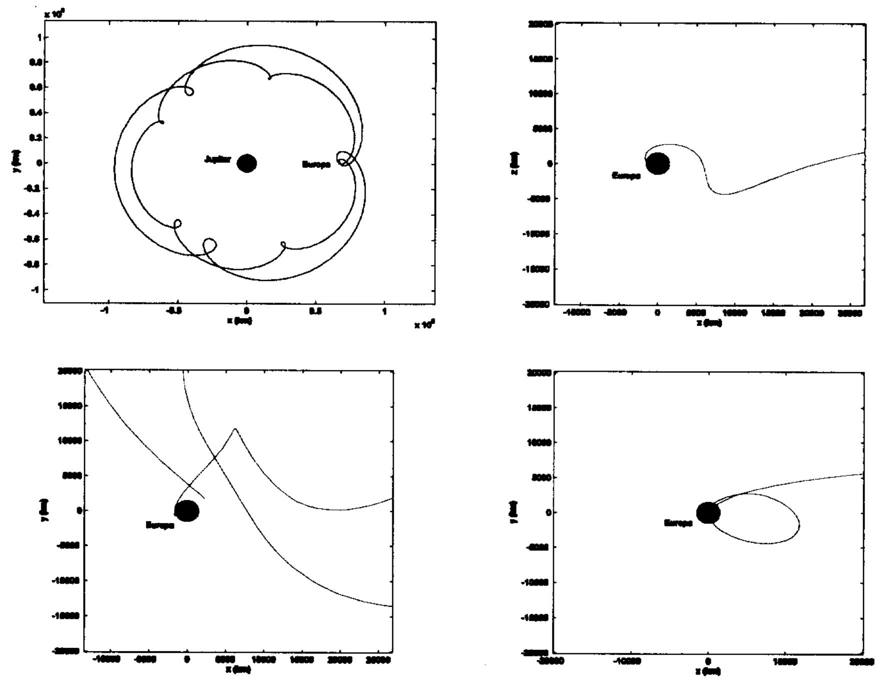

Figure 5. Europa Orbiter Baseline Trajectory in the Jupiter-Europa Rotating Frame (Including Ephemerides)

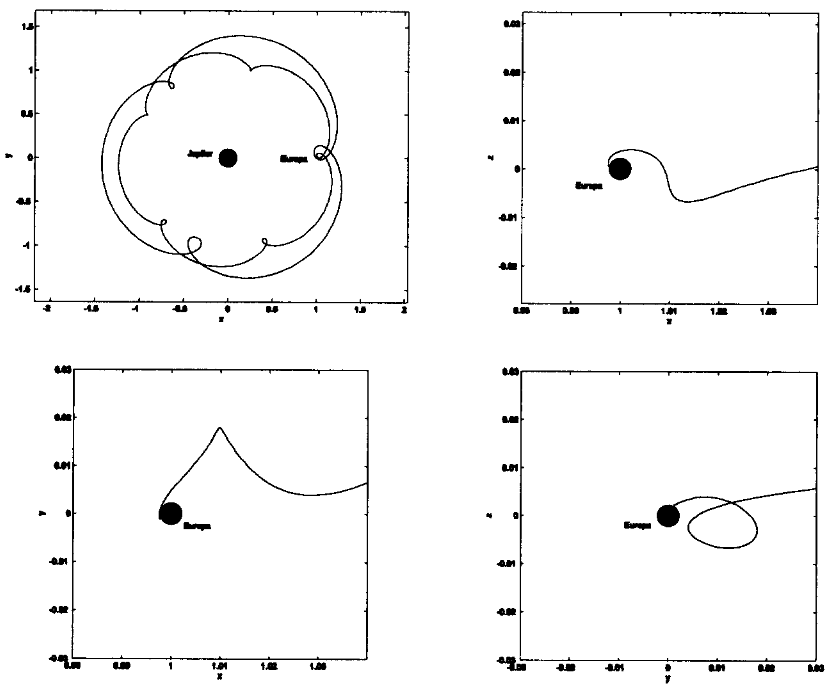

Figure 6. CR3BP Europa Orbiter Trajectory in the Jupiter-Europa Rotating Frame.

A comparison of the ΔVs is provided with the trajectory events in Table 1. Since one area of interest was the final approach to Europa, which moved out of the xy-plane significantly, the trajectory was left as a three-dimensional trajectory. In future studies that will focus on the resonances, an attempt will be made to force the trajectory to remain in the xy-plane since most of the trajectory far from Europa was nearly planar.

Table 1 Comparison of Trajectories Before and After Differential Correction

Date	Event	Original Traj.	CR3BP Traj.
09/12/2009 15:06	Initial Epoch	-	-
09/20/2009 02:14	ΔV_1	118.4 m/s	133.5 m/s
09/27/2009 03:05	Europa Swingby	-	-
10/08/2009 02:46	ΔV_2	91.6 m/s	150 m/s
10/19/2009 13:29	EOI	-	-

RESULTS AND DISCUSSION

Resonant Orbits

In the analyzed portion of the Europa Orbiter Trajectory, there appear to be two primary resonances with Europa. These can be observed by computing the osculating period of the trajectory relative to Jupiter using the usual two-body equations. The results of this calculation, with the period of the trajectory divided by that of Europa, are shown in Figure 7.

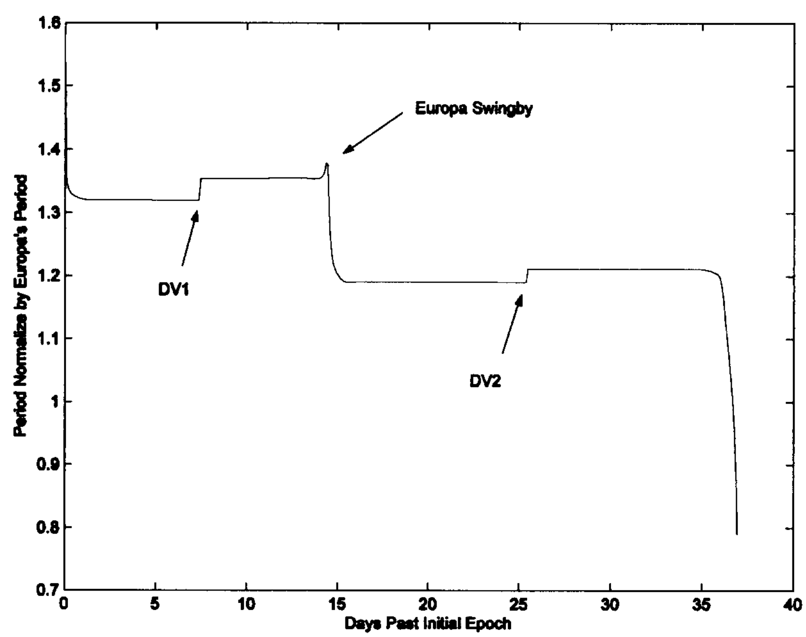

Figure 7. Normalized Period of the Trajectory as a Function of Time.

Both ΔVs in this portion of the trajectory are shown here along with the single swingby of Europa. The largest change in the period of the S/C relative to Jupiter comes as a result of the Europa swingby. The ΔVs cause a change both in the period and the Jacobian constant calculated in the Jupiter-Europa system. The trajectory has been divided into three different segments according to the value of the Jacobian constant as shown in Table 2.

Table 2 Jacobian Constants for Each Trajectory Segment

Segment	Jacobi Constant
1	2.98455692613
2	2.99720409688
3	3.00247993226

By examining the period, it can be seen that there appears to be a 3:4 resonance for the first segment of the trajectory which corresponds to a value of $1.\overline{33}$ for the normalized period in Figure 7. It should be noted that the resonances in this paper will be given in the form [Europa period]:[S/C period]. The next resonance appears to be a 5:6 resonance, which is simply a normalized period of 1.2 in Figure 7.

In order to examine these resonances, a series of Poincaré sections were computed. Each of these Poincaré sections were computed for a specific Jacobian energy in the planar CR3BP using the Europa rotating frame. In each case, a grid of initial conditions was used to initialize the Poincaré calculations. In each grid, x and $\dot{x}$ were incremented across a selected range, and each trajectory was started with $y = 0$. The value of v_y was specified by the Jacobi energy. The first three intersections of the trajectory with the $y = 0$ line were not included so as to remove the effect of the grid. The question of the quantities to use in computing the Poinaré section was also examined. Malhotra[15] uses the Delaunay variables for canonical momentum and mean anomaly (M) as well as simply x and $\dot{x}$. Koon et al. use the quantities $L = \sqrt{a}$ and $\bar{g}$, the argument of periapse relative to the rotating axis[8]. Calculation of L and $\bar{g}$ was ultimately determined to be the preferred method for this study, since the focus was on the resonances or the period, and thereby the semi-major axis. Initially the quantities L and M were used as shown in Figure 8. to search for a 3:4 unstable, resonant orbit.

An unstable orbit was the subject of this search since it was expected that the unstable manifold would provide a means for the spacecraft to move between resonances. Although future work will include the development of techniques to search for these unstable, resonant orbits, a visual search with a trial and error method was used for this initial investigation. In this procedure, a point was visually selected near the area where an unstable orbit of the desired period (L value) was expected to exist. For a given value of L, this point was expected to be π radians away from the stable resonant orbit at the center of the 'open' areas in the Poincaré section (seen Koon et al.[8]). Once a point was selected, the monodromy matrix after one period (in the rotating frame) was computed.

The eigenvalue of the monodromy matrix with the maximum absolute magnitude was examined in order to obtain a measure of stability of the chosen resonant orbit. An orbit for the 3:4 resonance was found in this manner at the point specified by the red dot in Figure 8. It has L = , M = . The resulting orbit is shown in the rotating coordinate frame in Figure 10, and it is shown with its unstable manifold in Figure 11. The stable manifold followed the original orbit to the resolution of the plot. It can be seen that the unstable manifold is gradually drifting away from the 3:4 resonance. Currently, this research is using the computation of L and $\overline{g}$ as shown in Figure 9 to search for additional unstable manifolds.

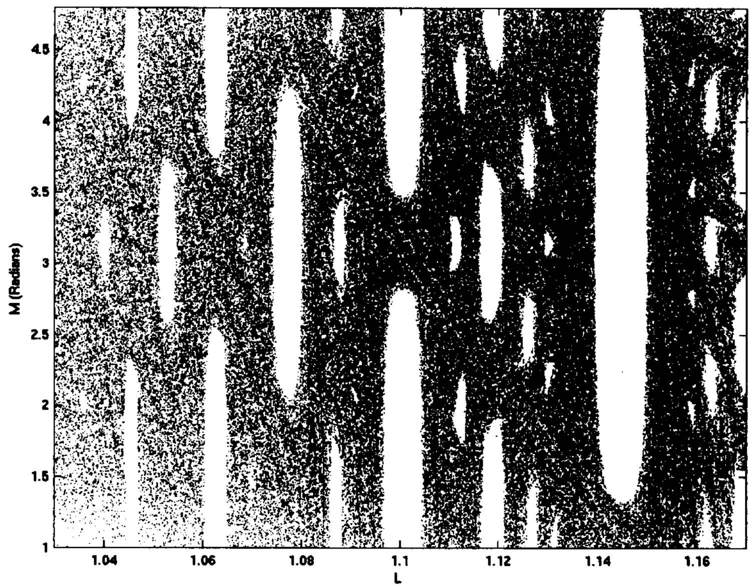

Figure 8. Poincaré Section using L and M for the 3:4 Resonance (red indicates selected orbit)

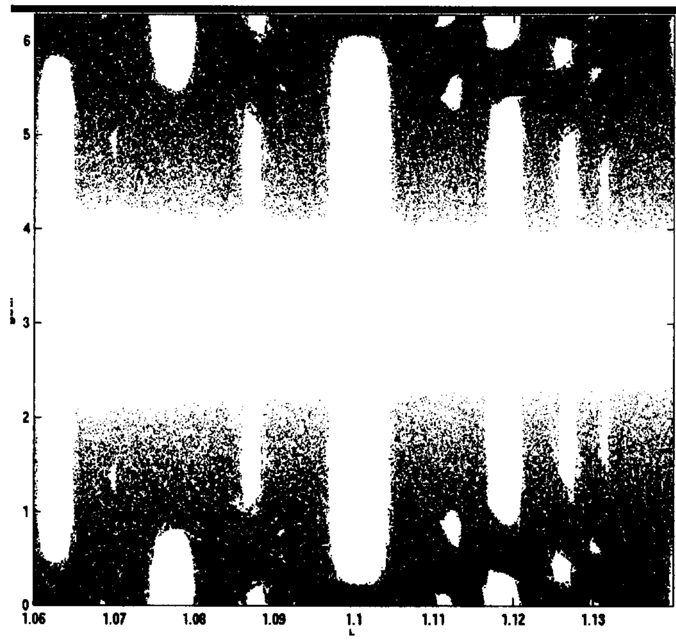

Figure 9. Poincaré Section using L and $\overline{g}$ for the 3:4 Resonance

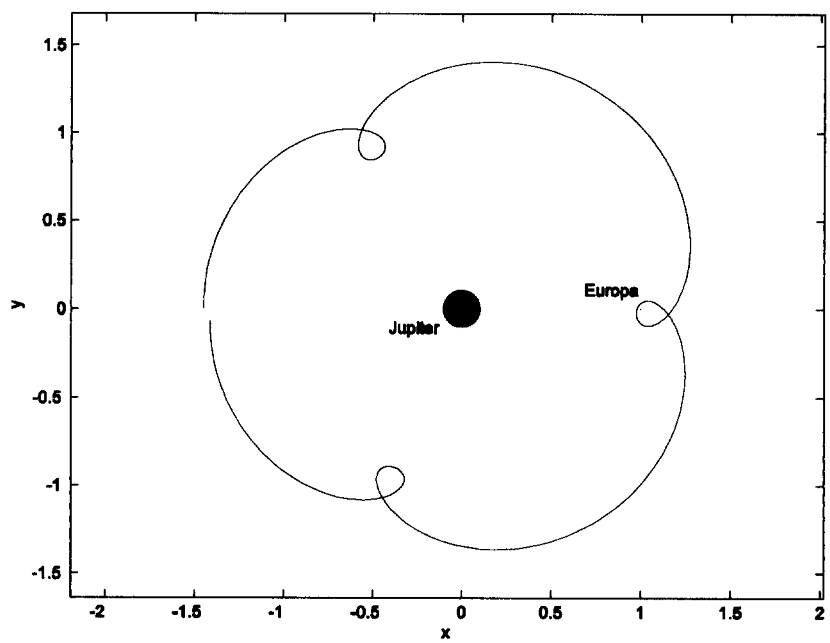

Figure 10. Unstable 3:4 Resonant Orbit Corresponding to Red Point in Figure 8.

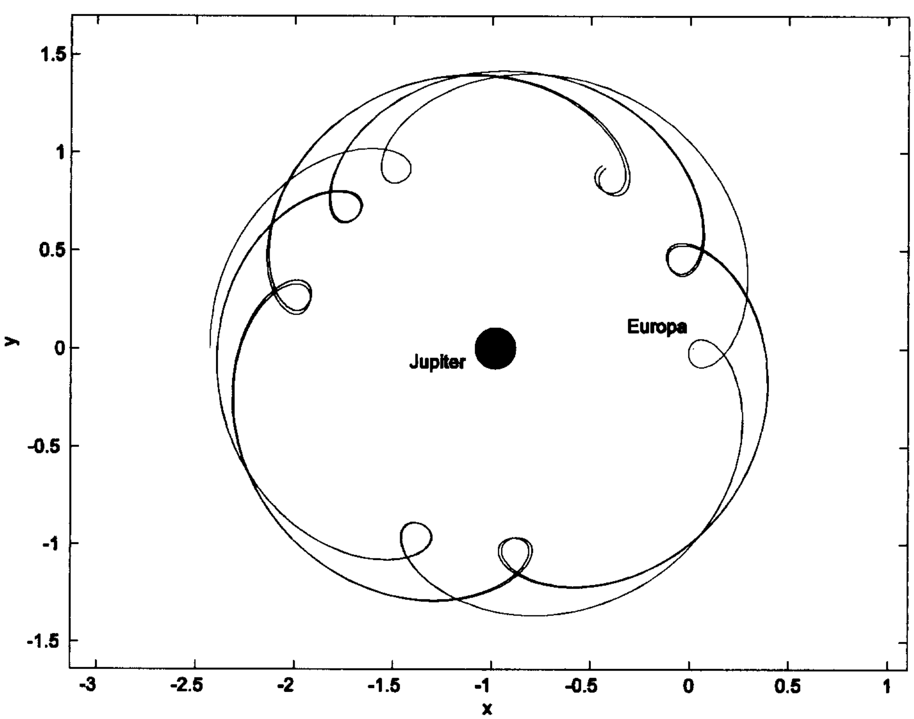

Figure 11. Unstable 3:4 Resonant Orbit (Blue) with Unstable Manifold (Red) (Stable Manifold Follows Resonant Orbit)

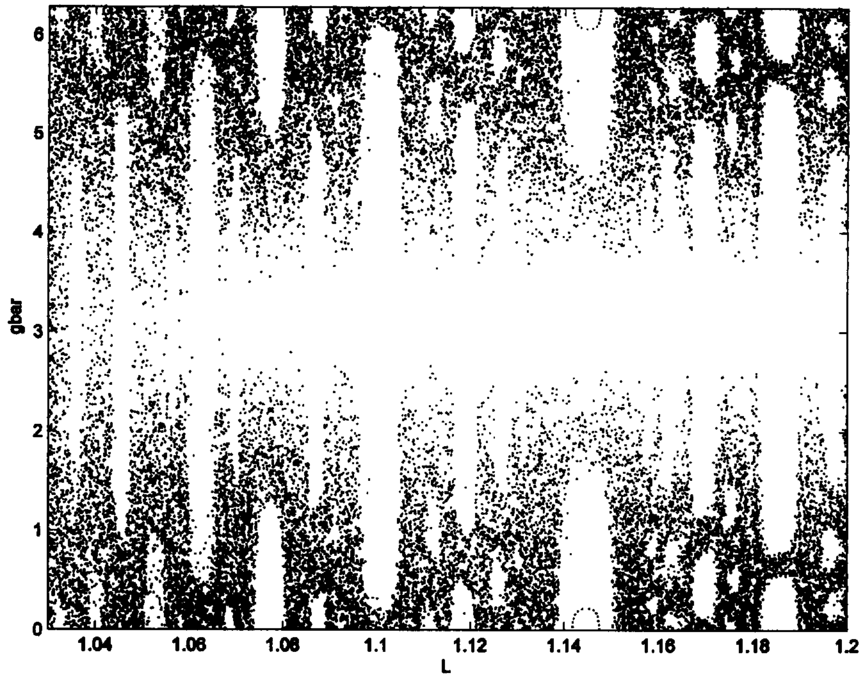

Figure 12. Poincaré Section using L and $\bar{g}$ for the 5:6 Resonance (red indicates selected orbit)

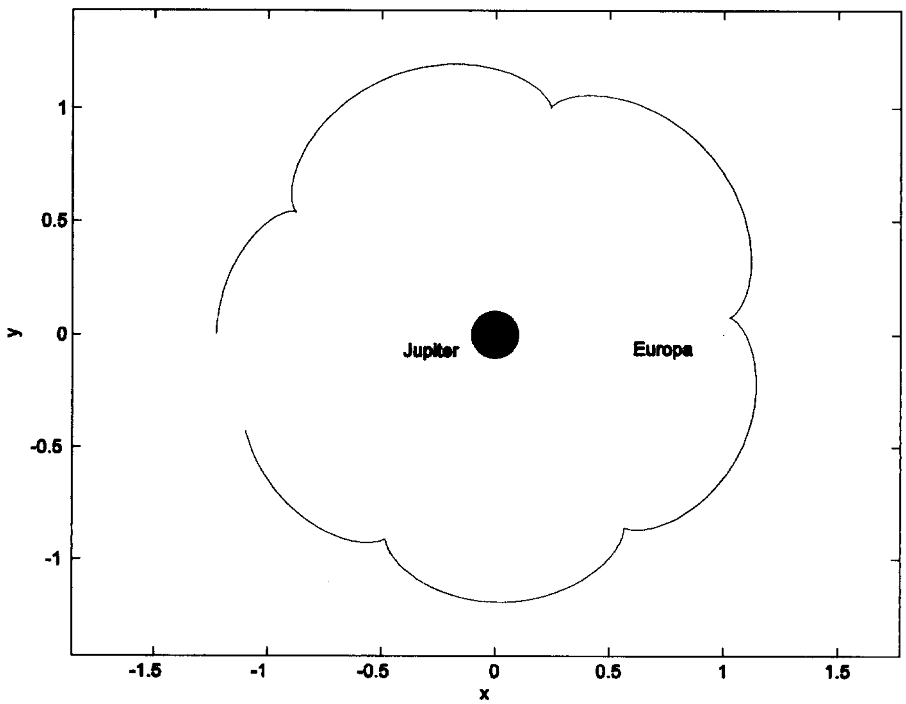

Figure 13. 5:6 Unstable Resonant Orbit in the Jupiter-Europa Rotating Frame

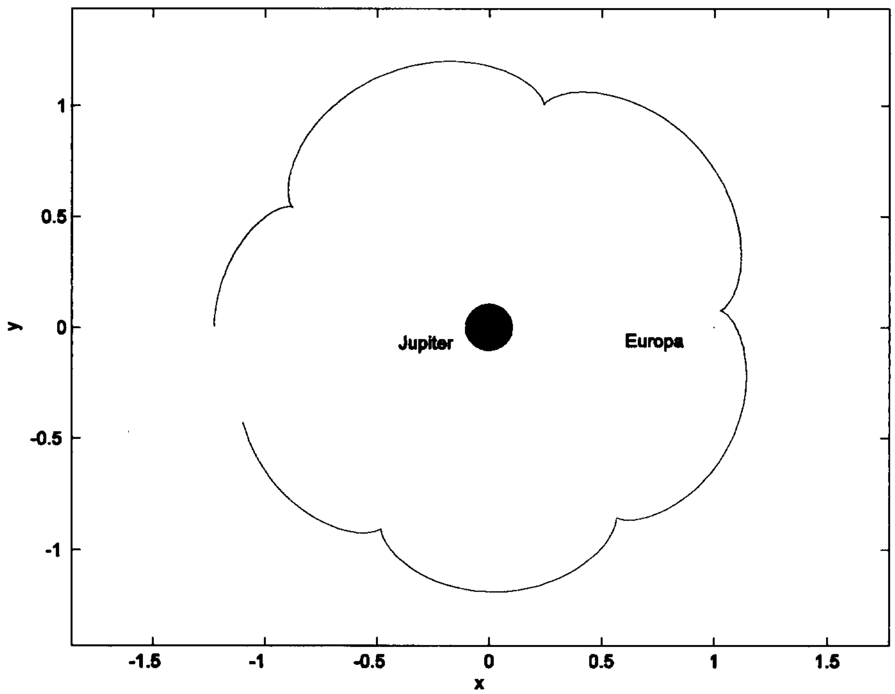

Figure 13. 5:6 Unstable Resonant Orbit in the Jupiter-Europa Rotating Frame

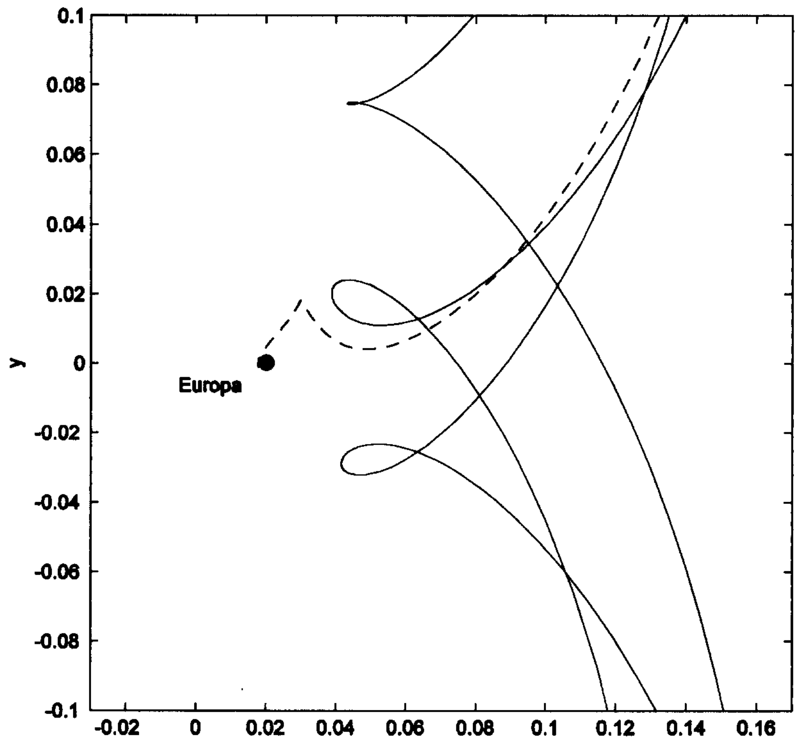

Figure 14. Unstable Manifold of 5:6 Resonant Orbit in the Jupiter-Europa Rotating Frame (solid) and Europa Orbiter Trajectory (dashed)

A procedure similar to that used in finding an unstable orbit for the 3:4 resonance was used to search for an unstable orbit at the 5:6 resonance. The Poincaré section in Figure 12. was computed for this purpose using L and $\bar{g}$.

The selected orbit is shown in the Poincaré section as a red dot and is plotted in Figure 13. This orbit had the largest maximum absolute eigenvalue of the tested orbits, which was important in order to determine whether an unstable manifold of the resonant orbit could be found that would approach the Europa Orbiter trajectory as it came near to Europa. The results for the most unstable resonant orbit currently obtained are shown in Figure 14. The black dashed line is the Europa Orbiter trajectory, and the red line is the unstable manifold of the selected resonant orbit. On the first pass by Europa (the uppermost loop), the unstable manifold has not yet traveled very far from the resonant orbit (not shown). The second pass falls on the other side of Europa, and the third pass comes closest to the Europa Orbiter trajectory. Remember that the trajectories taken from the Poincaré section were planar, so some differences would be expected. This portion of the study is in its early stages, so even closer matches with the Europa Orbiter are anticipated. An additional avenue of research will involve a comparison with a planar version of the Europa Orbiter trajectory.

Europa Capture

A primary interest of this study was to understand the Europa Orbiter's final approach to Europa. After examining the orbit, it was suspected that the Europa Orbiter may closely follow the stable and unstable manifolds of a quasi-periodic orbit at the Jupiter-Europa L2 Lagrange point. This hypothesis was initially tested by choosing a point on the Europa Orbiter trajectory and performing a ΔV. The trajectory that traveled through this new point in phase space was then examined. The ΔV was modified until the trajectory appeared to come from a nearly periodic orbit around L2 as shown in Figures 15 and 16.

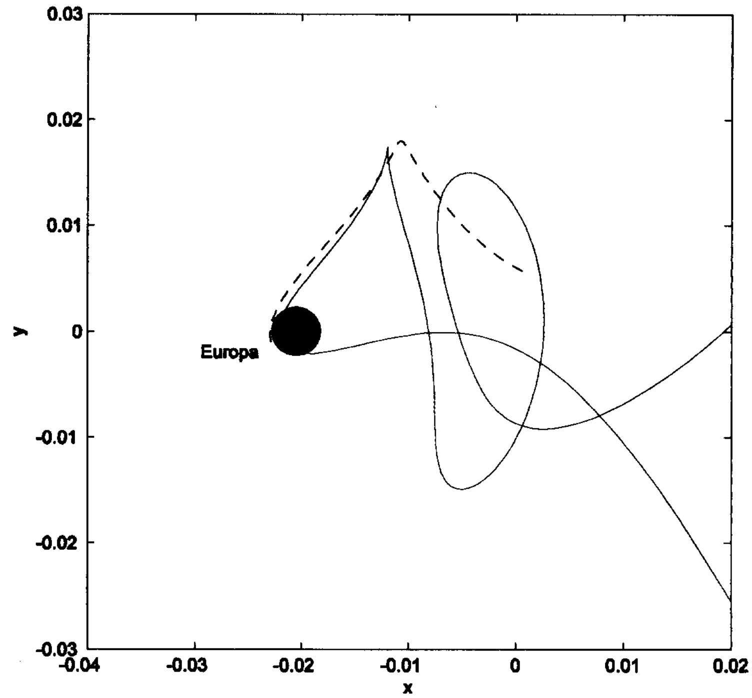

Figure 15. Orbit Obtained with a 49 m/s ΔV (solid) Compared to Europa Orbiter Trajectory (dashed)

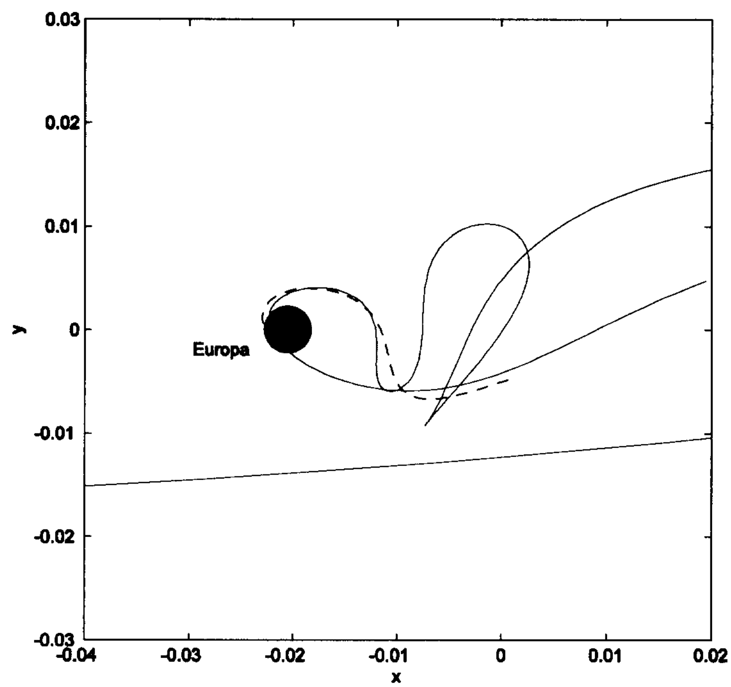

Figure 16. Orbit Obtained with a 49 m/s ΔV (solid) Compared to Europa Orbiter Trajectory (dashed)

The characteristics of this trajectory were then used as initial guesses to find a quasi-periodic Lissajous orbit around L2 with the same energy as the Europa Orbiter trajectory. Several different Lissajous orbits were computed using a Richardson-Cary expansion (put in??) along with their stable and unstable manifolds. A simple interpolation algorithm was used to compute the Lissajous orbit with the desired Jacobian energy once the initial guess had been obtained. First, a comparison of the unstable manifold of the selected Lissajous orbit was made with the incoming Europa Orbiter Trajectory. The best fit obtained is shown in Figures 17 and 18. From the different views it can be seen that the Europa Orbiter trajectory appears to follow the surface of the stable manifold of the Lissajous orbit. A slightly different Lissajous orbit was used to compute the unstable manifold for comparison with the Europa Orbiter trajectory as shown in Figures 19 and 20. The Europa Orbiter trajectory appears to have a shape similar to many of those that comprise the unstable manifold, and although it starts out at some distance from the unstable manifold, it approaches it as it comes closer to Europa. The similarity in shape and difference in position suggests that some other type of quasi-periodic orbit may provide a better fit.

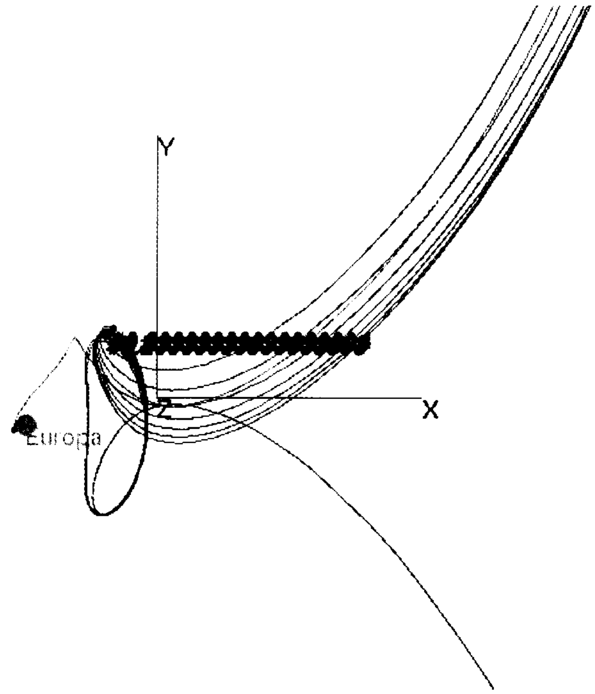

Figure 17. Stable Manifold of L2 Lissajous Orbit (green) Compared to the Incoming Europa Orbiter Trajectory (orange)

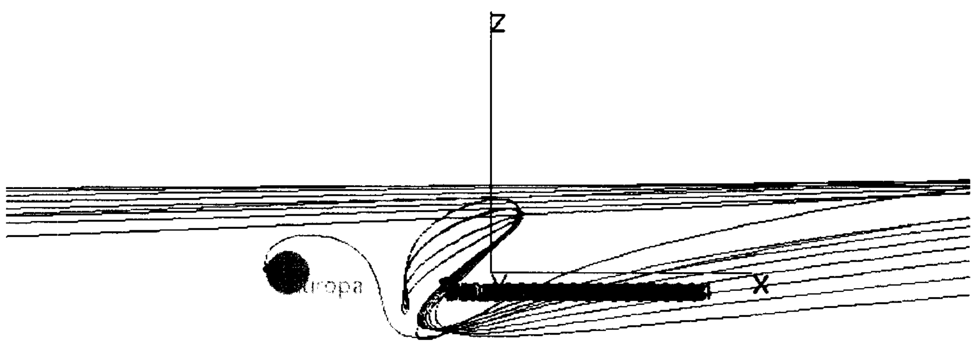

Figure 18. Stable Manifold of L2 Lissajous Orbit (green) Compared to the Incoming Europa Orbiter Trajectory (orange)

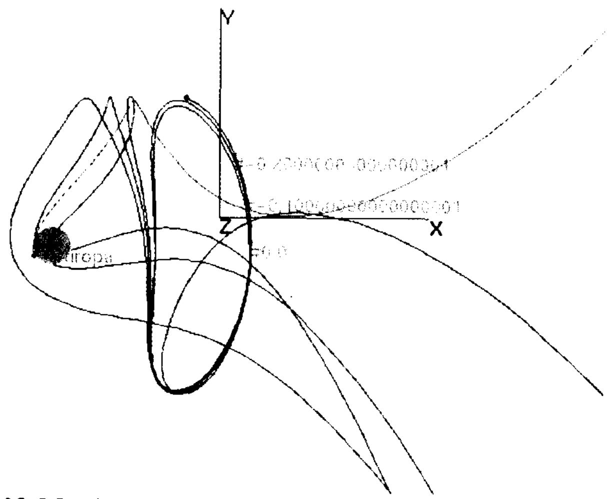

Figure 19. Unstable Manifold of L2 Lissajous Orbit (red) Compared to the Incoming Europa Orbiter Trajectory (orange)

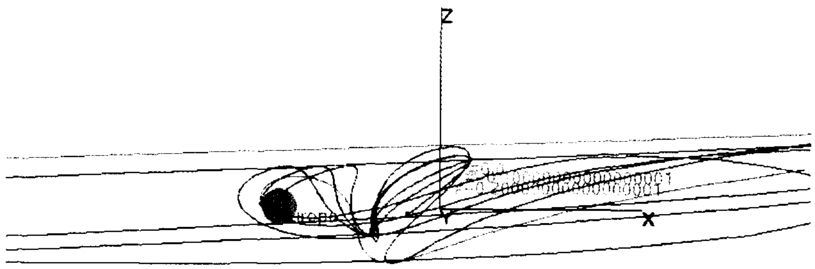

Figure 20. Unstable Manifold of L2 Lissajous Orbit (red) Compared to the Incoming Europa Orbiter Trajectory (orange)

CONCLUSIONS

Examination of the Europa Orbiter trajectory has provided a means to test many of the concepts that it will be necessary to understand in the low thrust problem. It was found that Poincaré sections allowed the determination of unstable resonant orbits, although a more rigorous technique will probably be needed. In combination with the calculation of the eigenvalues of the monodromy matrix, it was possible to compute resonant orbits with varying degrees of instability. By using the most unstable orbits, this study gave an indication that the unstable manifolds might be used to transition to different resonances and capture around Europa. During the approach to Europa, the proximity of the Europa Orbiter trajectory to the stable manifold of an L_2 Lissajous orbit, and the similarity in shape to the unstable manifold of a Lissajous orbit provide insight into how the Europa capture may be using invariant manifolds. This knowledge suggests that we can use pieces of the invariant manifolds associated with Lissajous or halo or unstable resonant orbits as initial guesses for a low thrust trajectory. This is, in part, our approach to providing better initial guess for tools like MYSTIC to optimize complex low thrust trajectory more quickly and provide more control.

FUTURE WORK

The richness of this problem suggests may possible approaches for future work. We will describe a few that we consider the more accessible and potentially more fruitful. An even simpler model is to consider a planar version of the EO trajectory. In the case of the coupled 3-body problems (to model the Gallilean satellite system), the invariant manifolds of the individual 3-body problem are much better understood and simpler to work with due to the lower dimensionality of the problem.

The problem of computing Poincaré sections of highly perturbed systems such as the Gallilean satellite system is very challenging. Is the situation simply chaotic and random, or is there still structures at much smaller timescale than the period of an orbit around the primary? For example, we might think of the invariant manifold tubes of the unstable orbits pulsating with the dynamics instead of being completely destroyed. In other words, can we think of the invariant manifold structure as a time-varying system? This means that tubes have short life time, but their structure still plays a role in the dynamics. What techniques can we use to recover this structure? Perhaps some type of lifting of the problem into higher dimensions to unfold dynamics in some sense. In this regard, the planar problem mentioned above is clearly the place to start such an analysis.

From the purely numerical point of view, in order to accomplish our research goals, we must have access to families of invariant manifolds and the ability to easily work with them, computing intersections, volumes, and surface areas, etc. Parallelism is likely to be required to perform so many calculations. Data structures must be carefully designed to represent these objects so that they can be easily manipulated and operated upon.

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