Physics Letters B 674 (2009) 210–212

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Space–time dimensionality from brane collisions

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ARTICLE INFO

Article history:
Received 3 February 2009
Received in revised form 12 March 2009
Accepted 18 March 2009
Available online 20 March 2009
Editor: A. Ringwald

PACS:
04.60.Kz
04.60.Pp
98.80.Qc

ABSTRACT

Collisions and subsequent decays of higher dimensional branes leave behind three-dimensional branes and anti-branes, one of which could play the rôle of our universe. This process also leads to the production of one-dimensional branes and anti-branes, however their number is expected to be suppressed. Brane collisions may also lead to the formation of bound states of branes. Their existence does not alter this result, it just allows for the existence of one-dimensional branes captured within the three-dimensional ones.

1. Introduction

The advent of string theory, and more recently non-commutative geometry — as an approach to unifying gravity with the other fundamental forces — has led to considerations of spatial dimensions greater than three. If these theories are to have physical merit, then they should provide a mechanism by which our, undoubtfully $(3+1)$-dimensional, universe is produced. By achieving that we will get, with the same token, an answer to the long-standing puzzle of explaining the space–time dimensionality by purely scientific means as opposed to (a strong form of the) anthropic principle.

In string theory, following the Kaluza–Klein approach, space dimensionality is typically explained by assuming that the extra dimensions are tightly curled up, so as to be beyond our experimental reaches. Precisely how this compactification occurs and gets stabilised is a major difficultly in string theory that has yet to be resolved. Within the Kaluza–Klein approach, the *string gas scenario* [1] — based on the target-space duality relating string theories compactified on large and small tori, by interchanging winding and Kaluza–Klein states — has been proposed long ago. According to this scenario, the universe started with all its dimensions being initially of the string scale, $\sqrt{\alpha'}$, while for dynamical reasons only three spatial dimensions were expanded. This can be easily understood, since only in a four-dimensional hyper-surface the world-sheets of winding and anti-winding modes can naturally overlap, and thus annihilate leading to the subsequent expansion of only three spatial dimensions. The string gas scenario has been supported by cosmic string experiments on a lattice [2].

It has also been noted that whilst critical superstring theory requires $(9+1)$ dimensions — at the basic level to ensure that the fermionic world-volume degrees of freedom can be simultaneously Weyl and Majorana — in non-critical strings the dimensionality of space–time is a dynamical parameter. Some progress has been made recently towards understanding how four-dimensional non-critical string theory behaves [3].

In light of large extra dimensions, it has been argued in Ref. [4], that starting with a distribution of branes — of any possible dimension allowed from the theory — embedded in a higher dimensional bulk, brane interactions could naturally lead to the survival of only three-dimensional branes — one of which could play the rôle of our universe — and one-dimensional branes (D-strings) — which could play the rôle of cosmic strings — called cosmic superstrings [5]. Strictly speaking, in Ref. [4] only interactions between branes of the same dimensionality were considered. In what follows, we repeat this argument, whilst extending it to allow for collisions between branes of different dimension. The collision of branes of different dimension leads to complications because of the possibility of forming bound states in which a Dp-brane absorbs a Dq-brane (for $q < p$) to form a bound D(p,q)-brane system. Whilst it has long been known that such a system is described by a Dp-brane with world-volume gauge fields (see, for example Ref. [6]), it is only recently that the tachyonic decay of such branes — due to either their non-BPS nature or the presence of an anti-$\bar{\text{D}}p$-brane — has been explicitly derived [7].

In this note, we derive the conditions under which two branes collide and hence decay, or form a bound state composed of two

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doi:10.1016/j.physletb.2009.03.028

branes of unequal dimension. Our analysis is performed within the context of type IIB string theory.[1] We show that brane collisions are unlikely for Dp-branes, with $p \leqslant 3$, even if bound states were formed during previous brane collisions. We hence show that given an initially random distribution of Dp-branes — where the brane dimensionality p is any odd number from 1 to 9 — embedded in a higher dimensional bulk ($d-1=9$, with $d-1$ the number of spatial dimensions), the end point of the decay chain will generically be D3-branes and possibly D1-branes (and their anti-brane counterparts). We thus firstly confirm the argument of Ref. [4], and secondly address the issue raised in Ref. [8] regarding the rôle of bound states. More explicitly, we show that the possible existence of bound states does not alter the conclusion of Ref. [4], and thus, by a simple *geometric* argument, we address successfully the origin of space–time dimensionality within the realm of string theory.

2. Bound state decays

Consider a Dp-brane, with p the number of spatial dimensions and choose coordinates such that the brane would be aligned along directions $\sigma_\mu = (1, 2, \ldots, p)$, if it were flat. Excitations of the brane geometry are given by world-volume scalars $X^m(\sigma_\mu)$, describing deviations from flatness along the remaining m (with $m = p+1, \ldots, d-1$) directions.

The bosonic part of the Dirac–Born–Infeld (DBI) effective Lagrangian for a Dp-brane in terms of the U(1) gauge field strength and the scalar fields X^m reads

$$\mathcal{L}_{\rm eff} = -\sqrt{-\det\left(\eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X_m + F_{\mu\nu}\right)}; \tag{1}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the world-volume electromagnetic field, with A_μ world-volume gauge fields; $\eta_{\mu\nu}$ is the $(9+1)$-dimensional Minkowski metric. Note that here we are neglecting the fermionic sector of the D-brane action, however all the analysis below can be extended to explicitly include fermions.

If the brane is non-BPS, then there is a tachyon present in its spectrum. The dynamics of the tachyon field on a non-BPS Dp-brane of type IIA or IIB superstring theory is given from the tachyon effective action [9]

$$\mathcal{L}_{\rm eff} = -V(T)\left[-\det\left(\eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X_m + F_{\mu\nu} + \partial_\mu T \partial_\nu T\right)\right]^{1/2}, \tag{2}$$

in terms of the massless gauge fields A_μ (with $0 \leqslant \mu, \nu \leqslant p$) and the transverse scalar fields X^m (with $(p+1) \leqslant m \leqslant 9$) on the world-volume of the non-BPS brane. The tachyon T is a scalar and $V(T)$ is the tachyon potential taken to be non-negative. Note that $V(T)$ has a unique local maximum at the origin ($T=0$) and a unique global minimum — far from the origin — where the potential vanishes.

It has been shown [10] that the tachyonic potential for a non-BPS Dp-brane contains an infinitely thin kink solution of finite tension describing a co-dimension one BPS D-brane, as a topological remnant of the bound states decay. The action and dynamics of this kink can be calculated and precisely agree with those of a D$(p-1)$-brane. This remains true in the presence of world-volume gauge fields [7], provided the tachyonic kink is aligned along the direction of the gauge fields.

If a Dq-brane collides with a Dp-brane, with $q < p$, then it dissolves into the higher dimensional Dp-brane, with its degrees of freedom becoming gauge (magnetic) fields (see, for example Ref. [6]). Since we are specifically looking at the case where $q < p$, these magnetic fields will be aligned along q-directions on the Dp-brane, leaving $(p-q)$-directions (with $(p-q) \geqslant 1$) available for the non-BPS D(p,q)-brane to decay along. The result of this decay process will be either a D$(p-1, q)$-brane, if $q < p-1$, or the same Dq-brane that was initially dissolved, if $q = p-1$.

It is important to note that the presence of such a dissolved brane does not affect the decay mechanism [7]. More precisely, as we have shown in Ref. [7], a bound system composed by two branes of different dimensionality, Dp, Dq with $q < p$, which can be described by just a Dp-brane with world-volume gauge fields, decays exactly as a standard Dp-brane, i.e., by forming a bound state of a D$(p-1)$ and a Dq-brane. Thus, the resulting defect is localised, as expected.

3. Bound state collisions

When a Dp-brane collides with an anti-brane of the same dimension, $\bar{\rm D}p$-brane, there is again a tachyon in the string spectrum. If we take both branes to have the same transverse directions, so that they are parallel, the low energy effective action for this system reads [9]

$$\mathcal{L}_{\rm eff} = -V\left(|T|, \left|X^m_{(1)} - X^m_{(2)}\right|\right) \times \left[\sqrt{-\det \mathcal{M}_{(1)}} + \sqrt{-\det \mathcal{M}_{(2)}}\,\right], \tag{3}$$

where

$$\mathcal{M}_{(\Delta)\mu\nu} = \eta_{\mu\nu} + \partial_\mu X^m_{(\Delta)} \partial_\nu X_{(\Delta)m} + F_{(\Delta)\mu\nu} + \frac{1}{2}(D_\mu T)^\star (D_\nu T) + \frac{1}{2}(D_\nu T)^\star (D_\mu T)$$

and

$$F_{(\Delta)\mu\nu} = \partial_\mu A_{(\Delta)\nu} - \partial_\nu A_{(\Delta)\mu},$$
$$D_\mu T = (\partial_\mu - iA_{(1)\mu} + iA_{(2)\mu})T \tag{4}$$

with $\Delta = 1, 2$. The potential, V, depends only on the magnitude of the tachyon $|T|$ — which in this case is complex — and $\sum_m [X^m_{(1)} + X^m_{(2)}]^2$. Clearly for $T = 0$, the above action reduces to the sum of the DBI action on the individual branes.

As in the non-BPS case, the potential has been shown [10] to contain topological obstructions to reaching the true vacuum, with the defect this time being of co-dimension 2. This vortex-like defect again has precisely the same dynamics as a D$(p-2)$-brane. The situation is similar for the case when either, or both, of the branes contain gauge fields, providing the D$(p-2)$-brane that remains after the decay is aligned parallel to the initial gauge fields [7].

If a Dq-brane dissolves into a Dp-brane, which subsequently decays due to a collision with an anti-$\bar{\rm D}p$-brane, the gauge fields on the Dq-brane should allow — at least two — directions perpendicular, to which the vortex (defect of co-dimension 2) can form. Thus, for this process to occur, the condition $q \leqslant p-2$ must be met.

If this is not the case, then the bound state D$(p, p-1)$-brane and the anti-$\bar{\rm D}p$-brane will not be able to decay. However, one of the two branes that makes up the bound D$(p, p-1)$-brane must be non-BPS. This indicates that the bound state will also not be a BPS-state and hence would not be energetically preferred to an unbound Dp–D$(p-1)$-brane system. In other words, we would not expect such a bound state to form.

Let us relax the condition that the brane and anti-brane in the system are parallel. Allowing the transverse scalars on one of the branes to grow linearly along one direction, we effectively rotate that brane. This may lead to problems at infinity, however since the tachyonic decay is a local process, this issue can safely be ig-

[1] By applying T-duality, one could equally well consider type IIA string theory.

nored.[2] In such a situation, the tachyon decays exactly as before, with the restriction that the vortex defect contains the transverse scalars of the original, rotated brane. Thus, in essence the collision between two non-parallel branes behaves in exactly the same manner as for parallel branes, simply resulting in a rotated brane of co-dimension 2.

4. Collisions of different dimension branes

It was argued in Ref. [4] that the condition for two Dp-branes to generically intersect in d space–time dimensions reads

$$2p + 1 \geqslant d - 1. \tag{5}$$

The argument goes as follows: a Dp-brane is extended in $(p+1)$ directions. Thus, two p-dimensional branes require $(2p+2)$ dimensions to exist, if there is no any special alignment of the branes. For these branes to avoid a collision — in the case that there are no special, non-generic, alignments — there must be at least one additional dimension, namely $d \geqslant 2p + 3$. Hence, one expects to get generically collisions between Dp-branes according to Eq. (5).

The above condition, Eq. (5), is nevertheless not sufficient to explain the space–time dimensionality. As it was shown in Ref. [4], only if the (equal dimensionality, p) intersecting branes are unstable, *evaporation* will eventually take place leading to a population of remnant D3- and D1-branes. Brane evaporation was shown to take place provided the bulk coordinates are compactified on a torus and the branes align themselves so that they intersect in a manifold of dimensionality $(p-1)$ and reconnection can indeed take place. These conditions were shown in Ref. [4] to be satisfied. Certainly, if bound states are not formed, then all arguments of Ref. [4] hold, so we do not repeat them here. If the colliding branes have unequal dimensionality, then either statistically speaking D3-branes survive unaffected from other dimensionality branes, since they will only intersect with D7- and D9-branes which are the ones to evaporate first [4], or they will form bound states. It is the latter case that is addressed below.

The condition given in Eq. (5) readily extends to collisions between Dp- and Dq-branes, where for a generic intersection we require

$$p + q + 1 \geqslant d - 1. \tag{6}$$

For $d = 10$ and $p \geqslant q$ one can easily check that collisions will only occur for $p \geqslant (d-2)/2$. In Table 1 we explicitly give all possible collisions between Dp- and Dq-branes, with $p \geqslant q$. Clearly the cases with $q \geqslant p$ are obtained by simply interchanging p and q.

As we have seen in the previous sections, the collision between, for example, a D3-brane and a D5-brane will result in a bound D(5, 3)-brane system, that is essentially a D5-brane with (three-dimensional) world-volume gauge fields. The collision of such a bound state with an anti-$\bar{\text{D}}$5-brane releases the original D3-brane, in the process of brane annihilation.

If we introduce the notation that a D$^*p$-brane is either a Dp-brane or a bound D(p,q)-brane, then the collision between a D$^*p$-brane and a D$^*q$-brane results to a (different) D$^*p$-brane, whilst the collision between a D$^*p$-brane and an anti-$\bar{\text{D}}p$-brane results is a D$^*(p-2)$-brane. Also we have seen that a non-BPS D$^*p$-brane will decay into a D$^*(p-1)$-brane. In this notation, it is only collisions between similar dimensional branes, or their self decay due

Table 1
A Dp-brane will generically collide with a Dq-brane, with $p \geqslant q$ according to this table.

$p = 9$	$q = 1, 3, 5, 7, 9$
$p = 7$	$q = 1, 3, 5, 7$
$p = 5$	$q = 3, 5$

to being non-BPS, that results in lower dimension branes. One important feature of Table 1 is that a D3-brane does not generically collide with a lower dimension brane and hence D*3-branes are equivalent to D3-branes.

Thus, at the end of a possibly complicated decay chain, the only possible scenario is that a D*5-brane colliding with an anti-$\bar{\text{D}}^*$5-brane forms a D3-brane, which possibly had previously been absorbed by one of the D5-branes. In conclusion, within the context of type IIB string theory, only D3, and possibly D1-branes, would be created from the decay of higher dimensional branes.

We note here that although we have pointed out that D3-branes do not, generically collide with lower dimensional branes, it is still possible that bound state D$(3,q)$-branes, with $q < 3$ can form. If a bound D$(5,q)$-brane were to collide with an anti-$\bar{\text{D}}$5-brane, the result would be a D$(3,q)$-brane. Such a process allows for embedded lower dimensional branes to be present in our universe as relics of earlier, higher dimensional collisions. Their existence does not alter our result, it just allows for three-dimensional branes having absorbed one-dimensional ones.

5. Conclusion

We have shown that the collision and subsequent decay of higher dimensional branes generically leads to D3-branes (and anti-$\bar{\text{D}}$3-branes). The production of (anti-)D1-branes cannot be completely excluded, however such branes are formed only through the annihilation of a pair of non-BPS branes, which would likely be suppressed. The formation of bound states during the collision process does not alter this result, but it just opens up the possibility that embedded branes may be present on the three-dimensional ones, one of which can play the rôle of our universe.

Acknowledgements

The work of M.S. is partially supported by the European Union through the Marie Curie Research and Training Network *UniverseNet* (MRTN-CT-2006-035863).

References

[1] R.H. Brandenberger, C. Vafa, Nucl. Phys. B 316 (1989) 391.
[2] M. Sakellariadou, Nucl. Phys. B 468 (1996) 319, arXiv:hep-th/9511075.
[3] J. Alexandre, J.R. Ellis, N.E. Mavromatos, JHEP 0612 (2006) 071, arXiv:hep-th/0610072.
[4] R. Durrer, M. Kunz, M. Sakellariadou, Phys. Lett. B 614 (2005) 125, arXiv:hep-th/0501163.
[5] M. Sakellariadou, Philos. Trans. R. Soc. A 366 (2008) 2881, arXiv:0802.3379 [hep-th].
[6] C. Johnson, D-Branes, Cambridge Univ. Press, 2003.
[7] W. Nelson, M. Sakellariadou, Phys. Rev. D 78 (2008) 104012, arXiv:0809.3958 [hep-th].
[8] A. Karch, L. Randall, Phys. Rev. Lett. 95 (2005) 161601, arXiv:hep-th/0506053.
[9] A. Sen, JHEP 9910 (1999) 008.
[10] A. Sen, Phys. Rev. D 68 (2003) 066008, arXiv:hep-th/0303057.

[2] Alternatively one can think on transverse scalars varying linearly with one direction, until some specific, large, value where they drop again to zero. Thus, locally the brane would be rotated and at infinity it would become parallel to the anti-brane.