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Foundation of relativistic astrophysics:

Curvature of Riemannian Space versus Relativistic Quantum Field in Minkowski Space

Abstract Modern relativistic astrophysics deals with compact relativistic objects (neutron and quark stars), candidates for black holes of stellar and galactic masses, gravitational radiation and its detection, massive supernova explosions, gamma ray bursts, jets from active galactic nuclei and cosmological models of the Universe. The common basis for all this observed phenomena is the theory of gravitation, for which in modern theoretical physics there are two main directions: Einstein's geometrical and Feynman's nonmetric field approaches for description of gravitational interaction. Though classical relativistic effects have the same values in both approaches, there are dramatically different effects predicted by GRT and FGT for relativistic astrophysics. Crucial observational tests which allow to test the physics of the gravitational interaction are discussed, including detection of gravitational waves by advanced LIGO-Virgo antennas and Event Horizon Telescope observations of active galactic nuclei. Forthcoming relativistic astrophysics can open new ways in physical understanding of gravitation and the unification of forces and substance.

Keywords Relativistic astrophysics · Gravitation · Quantum field theory · Conceptual bases · Lagrangian formalism · Cosmology · Crucial experiments-observations

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1 Introduction

1.1 Surprises of modern relativistic astrophysics

Since the first paper on Relativistic Astrophysics, published by Hoyle et al. 1964 [100], where crucial role of relativistic gravity in studies of extremal astrophysical objects was discussed, more than fifty years passed by. Gravity is really a cosmic force, so the true basis of relativistic astrophysics is the theory of gravitational interaction. Modern relativistic astrophysics deals with compact relativistic objects (neutron and quark stars), candidates for black holes of stellar and galactic masses, gravitational radiation and its detection, massive supernova explosions, gamma ray bursts, jets from active galactic nuclei and cosmological models of the Universe. The common basis for interpretation of all these observed phenomena is the general relativity theory (GRT), which now achieves 100 years from its birthday (Einstein 1915 [65]; Hilbert 1915 [96]).

Though the success of GRT in explanation of classical relativistic gravity effects is generally recognized (Will 2014 [200]), there are some puzzling theoretical and observational problems, which stimulate to search for alternative

gravitation theories. [1] Here I emphasize several observational and conceptual problems of relativistic astrophysics, which can be considered as a signal for more wide study alternatives.

First, recently gravitational-wave signals were detected by using Advanced LIGO interferometric antennas [1], [2], [3]. This means that the gravitational field energy carried by gravitational waves, was localized by a GW detector, i.e. free gravitational field energy can be transformed to the kinetic energy of the moving LIGO mirrors. However according to [127] (§20.4, p.467): "...gravitational energy... is not localizable. The equivalence principle forbids", and (§35.7, p.955): "...the stress-energy carried by gravitational waves cannot be localized inside a wavelength". Hence one can talk only about smeared-out amount of stress-energy within a region of several wavelengths size, while the LIGO gw-detector has length 4 km and localizes the gravitational wave well inside the gw-wavelength $\lambda_{GW} \approx 4000$ km. [2] The localization of the positive gravitational field energy is also consistent with firm observations of the energy loss via gravitational wave radiation from binary system with pulsar PSR 1913+16 [3] (recently summarized in [193]). So gravitational waves carry positive energy density, which would be detected (localized) also from many other collapsing cosmic objects.

Second, very recent surprising observational facts come from studies of the black hole (BH) candidates at the centers of luminous Active Galactic Nuclei and stellar mass Black Hole Binaries. Analysis of the iron K_α line profiles and luminosity variability gave amazing result: the estimated radius of the inner edge (R_{in}) of the accretion disk around central relativistic compact objects (RCO) is about $(1.2-1.4)R_g$, where $R_g = GM/c^2 = R_{Sch}/2$, i.e. less than the Schwarzschild radius R_{Sch} of corresponding central mass (Fabian 2015 [73], Wilkins & Gallo 2015 [202], King et al.2013 [112]). This points to a suspicion that in the nature there is no Schwarzschild black holes. For example, in the case of Seyfert 1 galaxy Mrk335 $R_{in} \approx 1.23R_g$, which means that BH should be a Kerr BH rotating with linear velocity about 0.998c. What is more, the emissivity profile sharply increases to smaller radius of the disk (Wilkins 2015 [202]).

Another kind of observations close to horizon of supermassive BH candidates comes from mm wavelength VLBI Event Horizon Telescope (EHT), see Doeleman et al.2009 [58]. Event-horizon-scale structure in the supermassive black hole candidate at the Galactic Centre (SgrA*) and M87 can be achievable directly with submm EHT in the near future and this will give possibility at first time to test relativistic and quantum gravity theories at the gravitational radius (Doeleman et al.2008 [59], Doeleman et al.2009 [58], Doeleman et al.2012 [60], Falcke & Markoff 2013 [74], Johannsen et al.2015 [109]). The first results of EHT observations at 1.3mm surprisingly demonstrate that for the RCO in SgrA* there are no expected for BH the light

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[1] Last version of Will's review [200] contain list of seven alternative metric gravitation theories.

[2] Localization of the electromagnetic field energy carried by electromagnetic wave does not depend on the wavelength, e.g. waves having 1 kilometer wavelength are localized by antenna having length about 1 meter.

[3] R.A. Hulse and J.H. Taylor won in 1993 Nobel Prize in physics for this discovery.

ring at radius $5.2R_{Sch}$ (Doeleman et al.2008 [59]). These observations have opened a new page in study of RCO. In particular, EHT has been designed to answer the crucial questions: Does General Relativity hold in the strong field regime? Is there an Event Horizon? Can we estimate Black Hole spin by resolving orbits near the Event Horizon? How do Black Holes accrete matter and create powerful jets? (Doeleman et al.2009 [58]).

Conceptual obstacles of GRT, which are directly related to these observations, include well-known "energy localization" and "horizon" problems. The energy localization problem is that within GRT there is no tensor characteristics of the energy-momentum for the gravity field [180], [118], [127], [22]. Landau & Lifshitz 1971 [118] called this quantity pseudo-tensor of energy-momentum and noted that covariant divergence of the total energy-momentum tensor (right side of the Einstein's field equations Eq.15) does not express the energy-momentum conservation for matter plus gravity field. The "pseudo-tensor"(meaning non-tensor) character of the gravitational energy-momentum in GRT has been discussed from time to time for a century (see a review Baryshev 2008 [22]), causing surprises for each new generation of physicists. However rejecting the Minkowski space inevitably leads (according to Noether theorem) to deep difficulties with the definition and conservation of the energy-momentum for the gravitational field (see Sec.1.4 and Sec.2.5).

There are several paradoxes of black hole horizon theory (e.g. discussions in Hawking 2014 [94], 2015 [95], 't Hooft 2015 [179]). Also, there is paradox of the infinite time formation of the classical BH event horizon (in the distant observer coordinates, so for us) and finite time of BH quantum evaporation, so a BH should evaporate before its formation (Chowdhury & Krauss 2014 [46]). The situation is confusing and the Stephen Hawking claimed in [94] that though there is no escape from a black hole in classical theory, but in quantum theory, however, energy and information can escape from a black hole. Hawking's conclusion is that an explanation of the process requires a theory that successfully merges gravity with the other fundamental forces of nature.

Modern cosmological observations is well described by the standard cosmological LCDM model based on Friedmann's solutions of GRT field equations. However there are both observational and conceptual difficulties which also stimulate analysis of alternative gravitation theories and cosmological models (Clifton et al. 2012 [47], Baryshev 2015 [26]). Among them: the cold dark matter crisis on galactic and sub-galactic scales (Kroupa 2012 [117]); the LCDM crisis at super-large scales (Sylos Labini 2011 [170], Clowes et al.2013 [48], Horvath et al.2015 [98]; Shirokov et al. 2016 [163]); the Newtonian character of the exact Friedmann equation (Baryshev 2008 [24]); violation of the energy-momentum conservation within any comoving local volume (Harrison 1995 [92], Baryshev 2008 [24]); violation of the velocity of light by space expansion velocity for galaxies observed at high redshifts (Harrison 1993 [91], 2000 [93], Baryshev & Teerikorpi 2012 [32] Baryshev 2015 [26]). Modern state of the standard cosmological model and possible alternatives will be discussed in Section 5.

1.2 The quest for unification of fundamental forces

The success of the Standard Model of electromagnetic, weak and strong interactions was achieved on the way of unification of the fundamental physical forces in the frame of the quantum field theory (QFT). Now it has reached a respectable status as an accurate and well-studied description of sub-atomic forces and particles, though difficult conceptual and technical problems remain to be solved (Bogolubov & Shirkov 1993 [39]; Wilczek 1999 [195], 2015a [197], 2015b [198]; 't Hooft 2004 [178]).

It is expected, that future "Core Theory" of physics will unify all fundamental forces (electromagnetic, weak, strong and gravitation) and also deliver unification of force (bosons) and substance (fermions) via transformations of supersymmetry (Wilczek 2015a,b [197] [197]).

One obstacle for unification of fundamental forces with the geometrical gravitation theory (general relativity theory − GRT) is that the conceptual basis of GRT principally different from the Standard Model. As it is well-known, gravity in the frame of GRT is not a force (de Sitter 1916 [53]), so quantization must be applied to the curved Riemannian space-time, where energy of gravity field cannot be properly (tensorial) defined ([180], [118], [127], [22]). Also the problem of the non-renormalizability of the gravitational interaction requires new ideas in QFT, e.g. quantization of space-time (Rovelli 2004 [155]).

The QFT reconciled Quantum Mechanics with the Relativistic Field Theory by construction of interacting substances via material fields that does obey the laws of Lorentz invariance, gauge invariance and causality. The concept of a field energy has crucial meaning in the QFT, because of the energy in a quantized field comes in quantized energy packages, which in all respects behave like elementary particles.

The association of forces (or, more generally, interactions) with exchange of particles is a general feature of quantum field theory. Electric and magnetic forces between charged particles are explained as due to one particle acting as a source for electric and magnetic fields, which then influence others. With the correspondence of fields and particles, as it arises in quantum field theory, Maxwells ED corresponds to the existence of photons, and the generation of forces by intermediary fields via the exchange of real and virtual photons.

The first step for constructing quantum electrodynamics (QED) is to develop relativistic classical vector field ($A^i(x^k)$) - classical electrodynamics (ED). In this paper the ED theory will be used as a primary example for preparation of the classical part of the QFT. So bellow I emphasize the crucial points of ED (Landau & Lifshitz 1971 [118]) [4] , which will be compared with geometrical and field gravitation theories.

The basic principles of ED is:

- *the inertial reference frames;*
- *the flat Minkowski space-time;*
- *the relativistic vector field $A^i(t, \mathbf{x})$;*

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[4] We use main definitions and notations similar to Landau & Lifshitz [118], so the Minkowski metric η_{ik} has signature $(+, -, -, -)$, 4-dimensional tensor indices are denoted by Latin letters $i, k, l...$ which take on the values 0, 1, 2, 3, and Greek letters $\alpha, \beta, \mu, \nu...$ take the values 1,2,3.

- *the Least (Stationary) Action Principle;*
- *the conservation of charges;*
- *the gauge invariance principle;*
- *the localizable energy-momentum tensor of the field.*

The action S for the system, consisting of an electromagnetic field with charged particles, must consist of three parts:

$$S = S_{(\text{em})} + S_{(\text{int})} + S_{(\text{p})} = -\frac{1}{c} \int \left(\frac{1}{16\pi} F_{ik} F^{ik} + \frac{1}{c} A_i j^i + \eta_{ik} T^{ik}_{(\text{p})} \right) d\Omega. \qquad (1)$$

The notations (em), (int), (p) refer to the actions for the electromagnetic field, the interaction, and the particles. The physical dimension of each part of the action is

$$[S] = [energy\ density] \times [volume] \times [time],$$

meaning that the definition of energy density of the field should exists within the conceptual bases of the principle of stationary action. j^i - 4-current, A^i - 4-potential, and F^{ik} - electromagnetic field tensor

$$A^i = (\varphi, \mathbf{A}), \quad F_{ik} = A_{k,i} - A_{i,k} \qquad (2)$$

From the Least Action Principle ($\delta S = 0$) by means of the variation of 4-potentials A^i with fixed sources j^i we get field equations with conserved sources

$$(A^{k,i} - A^{i,k})_{,k} = -\frac{4\pi}{c} j^i \quad where \quad j^i_{,i} = 0. \qquad (3)$$

Following Schwingers "source theory" ([160], [161]) in ED the electromagnetic field source is 4-current $j_i(x^k) = (c\rho_e, \mathbf{j})$ which together with the Lorentz invariant law of charge conservation (scalar restriction $j^i_{,i} = 0$) excludes the scalar source of the 4-vector field, i.e. the scalar photons. In fact the logic of spin 0 particle exclusion is following: *current conservation $\Rightarrow$ scalar source exclusion $\Rightarrow$ gauge invariance $\Rightarrow$ constraint field.*

The left side of the field equations eq.(3) allows the gauge invariance in the form:

$$A^i \rightarrow A^i + \xi^{,i} \qquad (4)$$

which allows to use the Lorentz gauge condition

$$A^i_{,i} = 0 \quad i.e. \quad \frac{1}{c} \frac{\partial \varphi}{\partial t} + \text{div} \mathbf{A} = 0 \qquad (5)$$

and the final field equations has ordinary wave equation form

$$\left(\triangle - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) A^i = -\frac{4\pi}{c} j^i \ . \qquad (6)$$

The gauge invariance eq.(4) is consistent with the conservation of the source of the field eq.(3) and with the deleting of the "scalar" photons. Indeed, the 4-vector field can be decomposed under the Lorentz group into a direct sum of two subspaces, one of which has 3 components (spin 1 particles) and another has 1 component (spin 0) [80], [11]. 4-potential A^i has 4 independent components, which correspond to one spin 1 and one spin 0 representations, then current conservation law allows to exclude the source of the spin 0 particles, so only photon with spin 1 is real:

$$\{A^i\} = \{1\} \oplus \{0\} \ \Rightarrow \ current\ conservation \ \Rightarrow \ \{A^i\} = \{1\}. \qquad (7)$$

The canonical energy-momentum tensor (EMT) of the electromagnetic field, after symmetrization, has the form:

$$T^{ik} = \frac{1}{4\pi}(-F^{il}F_l^k + \frac{1}{4}\eta^{ik}F_{lm}F^{lm}) \ , \tag{8}$$

which has following important features:

- $T^{ik} = T^{ki}$ -*symmetry condition;*
- $T^{00} = (E^2 + H^2)/8\pi > 0$ - *localized field energy density, positive for both static and wave field, corresponding to the positive photon energy $E_{photon} = h\nu$;*
- $T = \eta_{ik}T^{ik} = 0$ -*trace of the EMT is zero for mass-less particles (photons);*
- *the EMT from S is defined not uniquely;*
- *the EMT is gauge invariant.*

Variation in action S the trajectory of a particle with fixed 4-potentials gives the 4-equations of motion for charged particle:

$$mc\frac{du_i}{ds} = \frac{e}{c}F_{ik}u^k \ , \tag{9}$$

or in 3-d form it gives the Lorentz force $(i = \alpha)$ and its work $(i = 0)$:

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E} + \frac{e}{c}[\mathbf{v} \times \mathbf{H}] \ , \tag{10}$$

and

$$\frac{dE_{kin}}{dt} = e\mathbf{E} \cdot \mathbf{v} \ , \tag{11}$$

Adding to ED the quantum physical requirements - the uncertainty principle, the principle of superposition, quanta of the field energy as mediators of force, and so on, the QED was constructed in the frame of QFT, and then unified into electro-weak theory.

1.3 Einstein's geometrical and Feynman's field gravitational physics

Two ways in gravity physics. Since the beginning of the 20th century two really alternative approaches were put forwarded for the description of the gravitational interaction in theoretical physics.

The first one is the geometrical Einstein's *general relativity theory* (GRT), which is based on the geometrical concept of curved Riemannian (actually pseudo-Riemannian) space and reject the ordinary physical concept of force in application to gravitation. GRT was founded by Einstein 1915 [65]; 1916 [66] and Hilbert 1915 [96], and gives an example of geometrical way in construction of gravity theory. GRT operates with such concepts as metric tensor g_{ik}, geodesics, curvature, equivalence of the free fall to the inertial motion. Wheeler termed this approach *geometrodynamics*, underlining the fact that geometry is not a passive background but becomes a dynamical physical entity that may be deformed, stretched and even spread in the form of gravitational waves. Geometrical gravity treats the gravitational interaction as the curvature of space and has a singular position among other physical interactions, which are based on the physical concept of the force caused by the exchange of the quanta of the field.

During one hundred years GRT was developed and successfully applied to many gravity phenomena in the Solar System, galactic and extragalactic astronomy (Will 2014[200]; Kopeikin, Efroimsky, Kaplan 2011 [116]; Brumberg 1991 [42]; Zeldovich & Novikov 1984 [206]; Misner, Thorne & Wheeler 1973 [127]; Weinberg 1972 [188], 2008 [190]). However general relativity is not a quantum theory and many attempts to construct geometrical quantum gravity theory have not yet brought generally accepted convincing solution of the "Quantum theory's last challenge" (Amelino-Camelia 2000 [7]; Wilczek 2015a,b [197], [198]).

Another approach for understanding gravity was already suggested by Poincaré, who considered gravitation as a fundamental force in relativistic space-time. As early as 1905, Poincaré in his work "On the dynamics of the electron" put forward an idea about relativistic theory for all physical interactions, including gravity, in flat 4-d space-time (now called Minkowski space). He pointed out that analogously to electrodynamics, gravitation should propagate with the velocity of light, and there should exist mediators of the interaction – gravitational waves, *l' onde gravifique*, as he called them (Poincaré 1905 [150]; 1906 [151]). A few years later in his lecture on "New concepts of matter" Poincaré wrote about inclusion Planck's discovery of the quantum nature of electromagnetic radiation into the framework of future physics for all fundamental interactions. Poincaré thus could be rightfully regarded as the visionary of that approach to gravity which describes it as the relativistic quantum force.

According to Feynman's *Lectures on Gravitation* (1971 [76], 1995 [77]) the *field gravitation theory* (FGT) must be relativistic and quantum, which is described by symmetric second rank tensor field ψ_{ik} in Minkowski spacetime. So as in the theory of electromagnetic interaction we have electrodynamics (ED) and quantum electrodynamics (QED), in the case of FGT we should consider "*gravidynamics*" (GD) and "*quantum gravidynamics*" (QGD). Within FGT, as in ED, general concepts of force and localizable positive field energy density naturally exist, and FGT should be included in the list of the field theories of fundamental physical interactions.

Due to great success of general relativity in explanation of existing experimental and observational facts in gravity physics, the field gravitation theory up to now has been outside general attention. However the field approach to gravitation was partly developed by number of famous physicists, among them Birkhoff 1944 [37]; Moshinsky 1950 [131]; Thirring 1961 [177]; Kalman 1961 [110]. Attempts for a quantum description of the gravitational field were made by Bronstein 1936 [41]; Fierz & Pauli 1939 [78]; , Ivanenko & Sokolov 1947 [106]; Gupta 1952a,b [89],[90]; Feynman 1963 [75]; 1971 [76]; Weinberg 1965 [187]; Zakharov 1965 [205]; Ogievetsky & Polubarinov 1965 [137] and others. The fundamental role of the scalar part (omitted in Feynman's *Lectures on Gravitation*) of the symmetric second rank tensor field (i.e. its trace $\psi = \eta_{ik}\psi^{ik}$) was found and developed by Sokolov and Baryshev (e.g. [167]; [164]; [166];[13]; [14]; [21]; [26]). As a result, a consistent field gravity theory has been developed, where the central role belongs to the inertial frames, Minkowski space and localizable positive energy of the gravitational field, including its scalar part.

1.4 Why FGT is principally different from GRT

The relation between GRT and FGT was discussed in the literature with many misleading claims. The spectrum of the opinions is very wide, from the identity of the field gravity and general relativity, which are "just different languages" for the same physical phenomenon, and up to the claim, that the geometric theory delivers the only possible way to construct the gravitation theory, and the field gravity is completely wrong. Let us consider the real state of the art of the problem.

Special features of the geometrical approach. Within geometrical approach the gravitational interaction is described as a curvature of space-time. GR is a relativistic non-quantum theory of the gravitational interaction and based on the following fundamental concepts:

- *the non-inertial reference frames;*
- *the equivalence principle and geometrization of gravity;*
- *the curved Riemannian space-time with metric g_{ik};*
- *the geodesic motion of matter and light;*
- *the general covariance;*
- *the geometrical extension of Stationary Action Principle.*

Note, that the equivalence principle (EP) played an important role in the history of the general relativity formulation. EP has many forms – from non-relativistic to philosophical, which are not equivalent and difficult to test. Actually in experiment one tests the *universality of the free fall*, which is expected to be independent on the structure and motion of a test body (also known as the "effacing principle" [116]). Another form of the EP is the *geometrization* principle, i.e. geodesic motion in Riemannian space, which now is considered as the primary initial assumption of the geometrical approach. However the geometrical quantum gravity approach predicts violation of the EP and Lorentz invariance (Amelino-Camelia et al. 2005 [8]; Bertolami et al.2006 [36]).

On the bases of these initial principles general relativity was developed and successfully applied to the number of experiments and observations in the weak gravity conditions (Will 2014 [200]). Strong gravity GR predictions, like gravitational collapse to singularity, black hole existence, and global space expansion, may be observed only within astrophysical conditions where interpretation of the data allows different possibilities due to specific passive character of the astronomical observations and the dominance of distortion and selection effects which influence real astronomical data. So, in spite of many claims about proved existence of black holes and space expansion, up to now there is no real **experimental/observational proof** of the GR strong gravity effects, which are still hypothetical models for the observed astrophysical phenomena (see discussion in Sec.3 and Sec.4).

In conditions of the weak gravity general relativity is a well verified theory. It has passed all available tests in the Solar System and binary pulsars. Nevertheless, more accurate and conceptually new tests in the weak-field regime are still needed as well as tests of strong-gravity effects (Fabian 2015 [73]; Wilkins 2015 [202]; Baryshev 2015 [26]; Sokolov 2015 [166]; Will 2014 [200]; Doeleman 2009 [58]; Baryshev 2008 [23]; Bertolami et al.2006 [36]).

Problem of gravitational field energy-momentum in GRT. The most puzzling feature of general relativity is the absence of the tensor character of the energy-momentum "tensor" for the gravity "field". This was clearly exposed already by Einstein 1918 [68]; Schrödinger 1918 [159]; Bauer 1918 [35], and more recently discussed by Landau & Lifshitz 1971 [118]; Logunov & Folomeshkin 1977 [120]; Strauman 2000 [169]; Pitts & Schieve 2001 [147]; Xulu 2003 [203]; Baryshev 2008 [22].

The problem of the energy of the gravity field in general relativity has a long history, it was, in fact, born together with Einstein's equations. Hilbert 1917 [97] was the first who noted that "I contended ... in general relativity ... no equations of energy ... corresponding to those in orthogonally invariant theories". Here "orthogonal invariance" refers to theories in the flat Minkowski space. Emmy Noether 1918 [136], a pupil of Hilbert, proved that the symmetry of Minkowski space is the cause of the conservation of the energy-momentum tensor of all physical fields. Many results of modern relativistic quantum field theories are based on this theorem. So the "prior geometry" of the Minkowski space in the field theories has the advantage of guarantee the tensor character of the energy-momentum and its conservation for the fields.

In fact, Einstein & Grossmann 1913 [70] came close to Noether's result when they wrote: "remarkably the conservation laws allow one to give a physical definition of the straight line, though in our theory there is no object or process modeling the straight line, like a light beam in ordinary relativity theory". In other words, they stated that the existence of conservation laws implies the flat Minkowski geometry. In the same article Einstein & Grossmann also emphasized that the gravity field must have an energy-momentum tensor as all other physical fields. However, in the final version of general relativity Einstein rejected this requirement in order to have a generally covariant gravity theory with no prior Minkowski geometry.

Schrodinger 1918 [159] showed that the mathematical object t^{ik} suggested by Einstein in his final general relativity for describing the energy-momentum of the gravity field may be made vanish by a coordinate transformation for the Schwarzschild solution if that solution is transformed to Cartesian coordinates. Bauer 1918 [35] pointed out that Einstein's energy-momentum object, when calculated for a flat space-time but in a curvilinear system of coordinates, leads to a nonzero result. In other words, t^{ik} can be zero when it should not be, and can be nonzero when it should.

Einstein 1918 [68] replied that already Nordstrom informed him about this problem with t^{ik}. Einstein noted that in his theory t^{ik} is not a tensor and also it is not symmetric. He also withdrew his previous demand of the necessity to have an energy-momentum tensor: "There may very well be gravitational fields without stress and energy density".

The "pseudo-tensor" (meaning "non-tensor") character of the gravity field in GR has simple mathematical cause. As emphasized by Landau & Lifshitz 1971 ([118] sec.101 p.304) due to Bianchi identity the *covariant divergence* of the right part of Einstein's equation (which is the EMT of matter $T^{ik}_{(m)}$) is equal to zero, i.e. $T^{ik}_{(m)\,;k} = 0$. However for conserved quantity one should have the ordinary partial divergence: $((\sqrt{-g})T^{ik}_{(m)})_{,k} = 0$. So Landau & Lifshitz

suggested to consider pseudo-tensor (non-tensor) of energy-momentum of gravitational field which should be added to the EMT of matter and allow to fulfill the needed equation $((\sqrt{-g})(T^{ik}_{(m)} + t^{ik}))_{,k} = 0$.

There are many different expressions for pseudo-tensors but the problem of coordinate dependent (non-physical) definition of the gravity energy-momentum still exists at fundamental level − gravitational field is not a matter within GRT. This also demonstrate, that rejecting the Minkowski space inevitably leads to deep difficulties with the definition and conservation of the energy-momentum for the gravity field.

Attempts to resolve the gravitational energy-momentum problem in geometrical approach. The main question of the gravity physics is the role of the global Minkowski space in the gravitation theory. Within the geometrical approach Minkowski space is a tangent space at each point of curved space (the local Lorentz invariance). The field approach utilizes the global Minkowski space to describe all four fundamental physical interactions as material fields in space.

According to Noether's theorem in the relativistic field theory the conservation of the energy-momentum relates to the flat global Minkowski space. However in general relativity there are no conservation laws for the energy-momentum of the matter plus gravity field, just because of the absence of the global Minkowski space. The energy problem has deep roots in the geometrical approach, which uses curved space and non-inertial reference frames, while the field approach based on the Minkowski space with inertial reference frames naturally contains local EMT for the gravity field.

Within GR there are attempts to construct of a "quasi-local" energy-momentum and angular momentum to save the physical concept of the energy for the gravity "field" (Szabados 2009 [173]). There are also several suggestions how to overcome the energy-momentum problem by a modification of general relativity, or by postulating additional constraints on the metric of the Riemannian space, or by introducing together Minkowski and Riemannian metrics. This has led to different "field-geometrical" gravity theories with different equations and predictions (e.g. Logunov & Mestvirishvili 1989 [121]; Yilmaz 1992 [204]; Babak & Grishchuk 2000 [10]; Pitts & Schieve 2001 [147]; Xulu 2003 [203]).

All these theories are geometrical (use geometrization principle) and they predict some differences with GRT only in the case of the strong gravity field, which is not directly observed yet. However, as we shall show below, the results of the consistent Poincare-Feynman field approach has led to predictions which differ from GRT even in the weak field conditions, which can be tested by experiments in the Earth laboratories and by observations using terrestrial and space observatories.

Attempts to unify general relativity with field approach. There are many attempts to reconcile GRT with ordinary relativistic field physics. For a "proof" of identity of GRT and FGT approaches some physicists tried to use two opposite ways, so called "top-down" and "down-top" argumentations.

The first, top-down approach starts from the "top" full non-linear Einstein equations and goes to the "down" - linear weak field approximation. In this case the GRT field equation is similar to the field equations for the spin 2 particles in Minkowski space for the quantities h^{ik}, defined by the relation $g^{ik} = \eta^{ik} + h^{ik}$. So in this linear approximation one can consider h^{ik} as ordinary tensor field and introduce its quanta $-$ spin 2 massless particles: (Bronstein 1936 [41]; Fierz & Pauli 1939 [78]; Ivanenko & Sokolov 1947 [106]; Gupta 1952a,b [89]; [90]; Feynman 1963 [75]; Zakharov 1965 [205]).

The second, down-top approach starts from the "down" linear equations for material symmetric tensor potentials ψ^{ik} in Minkowski space and goes to a derivation of the "top" Einstein equations for the "metric tensor" $f^{ik} = \eta^{ik} + \psi^{ik}$ of an "effective" Riemannian space (Weinberg 1965 [187]), Ogievetsky & Polubarinov 1965 [137]; Deser 1970 [52]; Feynman 1971 [76]; 1995 [77]; Misner, Thorne, Wheeler 1973 [127]).

However there is a number of fundamental obstacles of such transformation of material field ψ^{ik} in Minkowski space into non-material curved space itself with true metrics $g^{ik} = \eta^{ik} + h^{ik}$. First, the quantities $f^{ik} = \eta^{ik} + \psi^{ik}$, as the sum of two tensors, is a tensor of Minkowski space. Hence its covariant components are $f_{ik} = \eta_{ik} + \psi_{ik}$ and mixed components $f_k^i = \delta_k^i + \psi_k^i$, and its trace is $f^{ik} f_{ik} = 4 + 2\psi + O(\psi_{ik}^2)$, while for a true metric tensor one should have the exact relation $g^{ik} g_{ik} \equiv 4$. So in the geometrical approach the trace of the tensor field ψ^{ik} (scalar part $\psi = \eta_{ik} \psi^{ik}$ of the gravitational potentials) is lost. Also for true metric tensor of any Riemannian space its mixed components are exactly $g_k^i \equiv \delta_k^i$, hence geometric theory has $h_k^i \equiv 0$.

As we discuss below the incompatibility of geometrical (GRT) and quantum field (FGT) approaches exists on the level of the adopted conceptual principles. Conceptual tensions between quantum mechanics and general relativity were noted by some physicists (e.g. Wigner in 1957) and have recently again attracted attention (e.g. Amelino-Camelia 2000 [7]; Chiao 2003 [45]; Yilmaz 1992 [204]). The most pressing problem in present-day theoretical physics is how to unify quantum theory with gravitation, i.e. "quantum gravity problem". The standard scheme of quantization applied to general relativity gives a theory that is not renormalizable (i.e. leads to inevitable infinities in physical quantities), though the quantization of space-time is now under construction (Rovelli 2004 [155]). Other attempts are based on the string/M theory, canonical/loop quantum gravity, and non-commutative geometry. However the difficulties on this way so large that after all attempts there is still no quantum geometrical gravity theory (Amelino-Camelia 2000 [7]; Amelino-Camelia et al. 2005 [8]).

Note that, if in a physical theory the energy-momentum tensor of the field is not defined, then also the energy of the field quanta can not be defined properly. General relativity is not quantizable in ordinary physical sense because it has no energy-momentum tensor for the gravity field. Also important that properly defined energy of the gravity field also exclude an appearance of singularity and horizon (Sec.4.2).

Special features of the Poincare-Feynman approach. Feynman discussed the strategy of the FGT and suggested to construct "theory of gravitation as the

31st field to be discovered" ([77] p.15). He analyzed basic principles of the FGT and emphasized, that "geometrical interpretation is not really necessary or essential for physics"([77] p. 113).

So the natural relativistic quantum field approach to gravitational interaction should be developed on the way where other fundamental interactions already have been constructed. Feynman emphasized that the "world cannot be one-half quantum and one-half classical" and "it should be impossible to destroy the quantum nature of fields" ([77] p.12).

Modern physics deals with four presently known fundamental interactions: the electromagnetic, the weak, the strong and the gravitational. The first three interactions are described by using Lagrangian formalism of the relativistic quantum field theory in Minkowski space. The FGT theory also should be based on the same Lagrangian concepts, including also specific scalar-tensor character of the gravitational field:

- *the inertial reference frames;*
- *the flat Minkowski space-time wih metric η^{ik};*
- *the symmetric tensor potentials $\psi^{ik}(x^m)$ with trace $\psi(x^m) = \psi^{ik}\eta_{ik}$;*
- *the universality of gravitational interaction;*
- *the Stationary Action Principle (Lagrangian formalism);*
- *the conservation law of energy-momentum;*
- *the gauge invariance principle;*
- *the localizable energy-momentum tensor of the gravitational field;*
- *the gravitational field energy quanta as mediators of the gravity force;*
- *the uncertainty principle and other quantum postulates.*

In Sec.3 we discuss how to construct the consistent Poincare-Feynman field gravity theory based on these initial principles. The energy of the gravitational field plays the central role in a reasonable theory of gravitational interaction. Feynman's notorious words in a letter to his wife "Remind me not to come to any more gravity conferences" ([77] Foreword p.xxvii) are related to this very issue, he did not wish to discuss the question of whether there is energy of the gravitational field. For him gravitons were particles carrying the energy-momentum of the field: "the situation is exactly analogous to electrodynamics − and in the quantum interpretation, every radiated graviton carries away an amount of energy $\hbar\omega$" ([77] p. 220).

Nowadays, when the Nobel Prize in Physics (1993) was given for the discovery of the binary pulsar PSR 1913+16, which is loosing orbital energy due to gravitational radiation, and the Advanced LIGO gw-antennas have detected the gravitational waves (i.e. have localized gw-energy), it is clear that Feynman was right when insist on the necessity to have proper concept of the energy of the gravitational field. In the field gravity approach there is usual localizable energy-momentum tensor (EMT) of the gravity field, while in geometrical approach there is well-known problem of pseudo-tensor character of the EMT of the gravitational field.

A note on the history of the field gravity approach. The history of the field gravity approach is characterized by many contraversal opinions and misleading claims. From time to time at a gravity conference a physicist appeared who announced that he ultimately had just derived the full non-linear Einstein's equations from the spin 2 field approach and he will demonstrate it

at the next conference. However at the next conference the situation was repeated.

There are many papers and discussions about the derivation of Einstein's field equations from the spin 2 theory (another name for the field theory), and hence about the identity of general relativity and field approach. Feynman in his lectures on gravitation also tried to derive the full nonlinear Einsteinian Lagrangian by iterating the Lagrangian of the spin 2 field. However many physicists feel a tenuity of such derivation and this is why each eminent physicist try to demonstrate his personal derivation of the geometry from field approach.

Misner, Thorne & Wheeler [127] (chapter 7, p.178) wrote that "tensor theory in flat spacetime is internally inconsistent; when repaired, it becomes general relativity". They refereed to papers by Feynman [75], Weinberg [187], and Deser [52] on a "field" derivation of Einstein's equations. Actually in all this derivations they get first the spin-2 field equations which are identical with Einstein's equations in the linear approximation . To perform the next step to get nonlinear equations one needs to fix the EMT of the gravitational field, so to use the physical concept of the field energy (including its necessary properties), which disappear in the "top" non-linear Einstein's equations. Just this crucial step is still a controversial subject.

The internal inconsistence of such attempts to derive Einstein's equations from the spin 2 field theory, was noted by Straumann 2000 [169] (p.16), who pointed out that:

- *general relativity having black hole solutions violates the simple topological structure of the Minkowski space of the quantum field theory;*
- *general relativity has lost the energy-momentum tensor of the gravity field together with the conservation laws, while in the Standard Model the EMT and its conservation is the direct consequences of the global symmetry of the Minkowski space.*

Padmanabhan 2004 [141] gave a comprehensive review of all such attempts and demonstrated that all derivations of general relativity from a spin 2 field are based on some additional assumptions that are equivalent to the geometrization of the gravitational interaction.

Indeed, as we noted above, general relativity and field gravity rest on incompatible physical principles, such as non-inertial frames and Riemann geometry of curved space on the one side, and material tensor field in inertial frames with Minkowski geometry of flat space on the other side. Geometrical approach eliminates the gravity force, as already de Sitter [53] noted: "Gravitation is thus, properly speaking, not a 'force' in the new theory". This however leads to the problem of energy just because the work done by force changes the energy.

Within the field approach the gravity force is directly defined in an ordinary sense as the fourth interaction and has quantum nature (Feynman [76]; [77]). The question may be formulated as following. Which is more general description of gravitation: geometry of curved space (so a property of space-time itself) or relativistic quantum field (so a kind of matter) in space?

Astrophysical tests of the nature of gravitational interaction. The relation between GRT and FGT is still an open question. Intriguingly, due to different predictions for observations, this question can be answered by means of astrophysical observations and lab experiments, so to test which theory has wider region of applicability?

In physics any mathematical theory has restricted region of applicability, i.e. exact mathematical equations and it's solutions actually have only approximative physical sense. This is why in physics the last word belong to experiments, and especially to the crucial experiments and observations, when rival theories predict different results for certain clearly stated experiment. The geometrical and field approaches are not equivalent experimentally, though the classical relativistic gravity effects in the weak field are identical in both theories. Because of common region of experimentally tested effects, it is possible that geometrical approach can be an approximation of the true quantum field gravity or vise verse.

Geometrical approach of the classical general relativity predicts such specific objects as singularities, black holes, and expanding space of Friedmann cosmological model.

The consistent field approach predicts that the gravity force has an ordinary quantum nature. Actually the gravity force is the sum of the attraction (spin 2) and repulsion (spin 0))(as will be shown in Sec.3 and Sec.4). This prediction of the FGT theory opens possibilities for novel type of experiments in gravity physics. Spin 2 plus spin 0 contribution to the gravity force, scalar gravitational waves, the translational motion of rotating bodies, the atmosphere and the magnetic field of the relativistic compact objects in "black hole candidates" are specific effects of the field gravity which may distinguish FGT from general relativity. In cosmology within the frame of FGT there is a possibility of infinite flat static Minkowski space filled by ordinary baryonic matter and having linear Hubble law of cosmological redshift as the global gravitational redshift effect (see Sec.5).

It is a remarkable result of our considerations that the choice between two conceptually different gravity theories may be founded on the results of experiments/observations in physical laboratory. For example, the problem of gravity quantization is directly linked to the choice of the nature of gravitational interaction. Indeed, if gravity is geometrical in nature (a property of curved space), then one should develop methods of space-time quantization (i.g. Rovelli [155]). But if gravity is a force mediated by gravitons (quanta of the tensor relativistic field), then one should find methods based on the concept of the energy of the gravity field and develops new principles for overcoming the non-renormalizability problem. It will be shown below, that future astrophysical observations of the compact relativistic objects, space experiments in the Solar System and cosmologically relevant observations of the Local and High Redshift Universe may distinguish between these two cardinally different (though having similar predictions within common region of applications) approaches to the theory of gravitational interaction.

2 Einstein's geometrical gravitation theory

The final mathematical formulation of the main equations of general relativity was done by Einstein [65], [66] and derived by Hilbert [96] from geometrical extension of the stationary action principle. It is a mathematically exact non-linear theory without any inner limitations to its physical applications and this is why in GRT singularities and black holes exist. Below we consider shortly the basic steps in construction GRT and its main predictions for experiments/observations, which we shall compare with corresponding equations and predictions of the FGT. We use designations as in the textbook by Landau & Lifshitz [118]. The fundamental physical constants c, G, h are used explicitly.

2.1 Basic principles

The principle of geometrization. General relativity is based on the *principle of geometrization*, which states that all gravitational phenomena can be described by the metric of the Riemannian space. This means that Einstein's gravity theory has no "prior geometry", such as the flat Minkowski space in other fundamental interaction theories. Gravity is not a material field in space, but is a property of the curved space itself. The role of the gravitational "potential" is played by the metric tensor g^{ik} which determines the 4-interval of the corresponding Riemannian space:

$$ds^2 = g_{ik}dx^i dx^k \tag{12}$$

A test particle moves along a *geodesic line* of the Riemannian space.

Note that *geodesic motion* is a form of the *equivalence principle*, which actually has many "non-equivalent" formulations like universality of free fall or philosophical equivalence of the inertial reference frames to the reference frames accelerated by homogeneous gravity field. Equivalence principle played an important role when general relativity was born, while now the basic principle is the *principle of geometrization*, having clear physical and mathematical formulation. The most clear and concise presentation of GR is the textbook by Landau & Lifshitz [118]. The most comprehensive description of the geometrical view on gravity is the textbook by Misner, Thorne & Wheeler [127].

The principle of least action. Einstein's field equations are obtained from the principle of least action by the variation of the metric tensor g_{ik} in the action S of the system matter + gravitational field. It is very important that the action contains only two parts (there is no interaction Lagrangian, while other fields contain the interaction part S_{fg}) :

$$S = S_{(\mathrm{m})} + S_{(\mathrm{g})} = \frac{1}{c} \int \left(\Lambda_{(\mathrm{m})} + \Lambda_{(\mathrm{g})} \right) \sqrt{-g}d\Omega \,, \tag{13}$$

where S_{m} and S_{g} are the actions for the matter and gravitational field, $\Lambda_{(\mathrm{m})}$ is the Lagrangian for the matter, and the Lagrangian for the field is

$$\Lambda_{(\mathrm{g})} = -\frac{c^4}{16\pi G}\Re\,. \tag{14}$$

Here $\Re$ is the scalar curvature of the Riemannian space.

2.2 Basic equations of general relativity

Einstein's field equations. Variation δg_{ik} in (13) gives $\delta(S_{(\mathrm{m})} + S_{(\mathrm{g})}) = 0$ for the field equations:

$$\Re^{ik} - \frac{1}{2}\,g^{ik}\,\Re = \frac{8\,\pi\,G}{c^4}\,T_{(m)}^{ik}\,, \tag{15}$$

where $\Re^{ik}$ is the Ricci tensor. $T_{(\mathrm{m})}^{ik}$ is the energy-momentum tensor (EMT) of the matter, which includes all kinds of material substances, such as particles, fields, radiation, dark energy including the vacuum $T_{(\mathrm{vac})}^{ik} = g^{ik}\Lambda$ (where Λ is the Einstein's cosmological constant).

Note that $T_{(m)}^{ik}$ **does not contain** the energy-momentum tensor of the gravity field itself, because gravitation is not a material field in general relativity (as also discussed below).

The equation of motion of test particles. A mathematical consequence of the field equations (15) is that due to Bianchi identity the covariant derivative of the left side equals zero, so for the right side we also have

$$T_{(\mathrm{m})\,;\,i}^{ik} = 0\,. \tag{16}$$

This continuity equation also gives the equations of motion for a considered matter. [5] It implies the geodesic equation of motion for a test particle:

$$\frac{du^i}{ds} = -\Gamma_{kl}^i u^k u^l\,. \tag{17}$$

$u^i = dx^i/ds$ is the 4-velocity of the particle and Γ_{kl}^i is the Christoffel symbol.

2.3 The weak field approximation

All relativistic gravity effects that have been actually tested by observations, relate to the weak field, where the Newtonian potential $|\varphi| << c^2$. This is why the weak field approximation has an important role in gravity physics.

---

[5] It is not a conservation of the energy-momentum of the system because $T_{(m)}^{ik}$ does not contain the energy-momentum of the gravity field (Landau & Lifshitz [118]p.304). We shall discuss this in sec.2.5.

The metric tensor. In the case of a weak gravity field the metric tensor usually is expressed in the form

$$
\begin{aligned}
g_{ik} &= \eta_{ik} + h_{ik} \\
g^{ik} &= \eta^{ik} - h^{ik} \\
g^i_k &= \delta^i_k
\end{aligned}
\tag{18}
$$

Here eq.(18) means that a tensor of Riemannian space g^{ik} is presented by the sum of two non-tensor quantities, because η^{ik} and $|h_{ik}| << 1$ are not tensors of the curved space, e.g. in the third identity of eq.(18) the components $h^i_k \equiv 0$. Indeed, the different signs of the quantities $\hat{\mathbf{h}}$ for covariant and contravariant components of the metric tensor $\hat{\mathbf{g}}$ are caused by the exact identity valid for the *trace of the metric tensor* of any Riemannian 4-space:

$$
g_{ik} \cdot g^{ik} = 4
\tag{19}
$$

As we shall see below this is an essential difference with the consistent field approach where the sum of two tensors is a tensor of Minkowski space.

The field equations. In the linear approximation $|h_{ik}| << 1$ and Einstein's equations (15) become

$$
\left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right) h^{ik} = \frac{16\pi G}{c^4} \left[T^{ik}_{(m)} - \frac{1}{2}\eta^{ik} T_{(m)} \right] .
\tag{20}
$$

In the important case of a static spherically symmetric weak gravitational field the solution of these equations gives the metric tensor, expressed in isotropic coordinates:

$$
\begin{aligned}
g_{ik} &= \eta_{ik} + \frac{2\varphi_N}{c^2} diag(1,1,1,1) \\
g^{ik} &= \eta^{ik} - \frac{2\varphi_N}{c^2} diag(1,1,1,1) \\
g^i_k &= \delta^i_k, \qquad g_{ik}g^{ik} = 4
\end{aligned}
\tag{21}
$$

where $\varphi_N = -GM/r$ is the Newtonian potential.

The equation of motion in the weak field. The post-Newtonian approximation of the weak field takes into account all terms of order v^2/c^2 and φ_N/c^2. PN-geodesic equations are frequently used in relativistic celestial mechanics. The 3-acceleration of a test particle in the static spherically symmetric weak gravity field (e.g. a planet around the Sun) is given by the equation (Brumberg 1991[42]; Kopeikin et al. 2011 [116]):

$$
\begin{aligned}
(\frac{d\mathbf{v}}{dt})_{\text{GR}} = &-\{1 + (1+\alpha)\frac{v^2}{c^2} + (4-2\alpha)\frac{\varphi_{\text{N}}}{c^2} - 3\alpha(\frac{\mathbf{r}}{r} \cdot \frac{\mathbf{v}}{c})^2\}\boldsymbol{\nabla}\varphi_{\text{N}} \\
&+ (4-2\alpha)\frac{\mathbf{v}}{c}(\frac{\mathbf{v}}{c} \cdot \boldsymbol{\nabla}\varphi_{\text{N}}),
\end{aligned}
\tag{22}
$$

where $\mathbf{v} = d\mathbf{r}/dt$, $\varphi_{\text{N}} = -GM/r$, and $\boldsymbol{\nabla}\varphi_{\text{N}} = GM\mathbf{r}/r^3$. An important quantity here is the parameter α. It is determined by the choice of the coordinate

system: $\alpha = 2$ for the Painleve coordinates, $\alpha = 1$ for the Schwarzschild coordinates, and $\alpha = 0$ for harmonic and isotropic coordinates. Hence *the orbit of a particle* will depend on the chosen coordinates. To avoid this non-physical result it is suggested that *observable* physical quantities should not depend on the coordinate parameter α by taking into account an ad hoc procedure of measuring quantities involved in the orbital motion phenomenon. [6]

It should be emphasized that directly from equation of motion(22) follows the dependence of gravitational acceleration from the value and direction of the test particle velocity. So this result contradicts to that form of the equivalence principle where one asserts the independence of the free fall on the velocity of a test particles.

2.4 Major predictions for experiments/observations

Number of predictions of general relativity for both weak and strong fields were derived from Einstein's field equations and the equations of motion. The triumph of GR in physics and astronomy is caused by experimental and observational confirmation of Einstein's equations with high accuracy. Application of GR to cosmology will be considered in Sec.5.

The classical relativistic gravity effects in the weak field. The classical weak gravity effects have been tested with accuracy of about $0.1 \div 1\%$ (Will [200]; Kopeikin et al.[116]). Among these experimentally verified effects are:

- *Universality of free fall for non-rotating bodies,*
- *The deflection of light by massive bodies,*
- *Gravitational frequency-shift,*
- *The time delay of light signals,*
- *The perihelion shift of a planet,*
- *The Lense-Thirring effect,*
- *The geodetic precession of a gyroscope,*
- *The emission and detection of the quadrupole gravitational waves.*

In the next sections we shall show that all this effects can be explained also within the field gravity approach with the same formulas for the observed quantities, hence they can not distinct between GRT and FGT. However in FGT there are additional weak gravity effects which can be used as crucial tests for GR and FG theories: e.g. free fall of rotating bodies, attraction and repulsion components of the gravitational force, and additional scalar gravitational radiation. Recently detected GW signals by Advanced LIGO antennas also disccused in Sec.4.2 .

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[6] The problem of the physical meaning of coordinates in general relativity has been debated for a long time and up to now there is no commonly accepted solution. E.g., Misner, Thorne & Wheeler 1973 [127], p.1097) wrote that Schwarzschild coordinates are "wrong" because "physicists, astronomers and other celestial mechanics have adopted the fairly standard convention of using 'isotropic coordinates' rather than 'Schwarzschild coordinates' when analyzing the solar system".

Strong gravity effects in GRT: Schwarzschild metric. General relativity predictions for the strong gravity includes an exact solution of Einstein's equations (15) for any centrally symmetric mass distribution which is called the Schwarzschild metric. It has the following form for the 4-interval in the Schwarzschild system of coordinates (t, r, θ, ϕ):

$$ds^2 = (1 - \frac{r_{\text{Sch}}}{r})c^2 dt^2 - \frac{dr^2}{1 - \frac{r_{\text{Sch}}}{r}} - r^2(\sin^2\theta \, d\phi^2 + d\theta^2) \,. \qquad (23)$$

In other coordinate systems this interval has different form for the singular term. The metric in eq.(23) depends only on the total mass M of the gravitating body. The quantity r_{Sch} is called the Schwarzschild radius for the mass M:

$$r_{Sch} = \frac{2GM}{c^2} = 3 \text{ km} \frac{M}{M_\odot} \,. \qquad (24)$$

This metric shows that at $r = r_{\text{Sch}}$ the 00-component is equal to zero and the 11-component is infinite. They say that the gravity "force" becomes so strong that nothing, not even light, can escape a body whose whole mass M is inside r_{Sch} (a definition of the **black hole**).

An external observer within a static system of coordinates will see matter collapsing eternally on the black hole. But if one chooses a non-static free-falling system of coordinates, one finds that a co-falling matter will within a finite (and rather short) proper time cross the gravitational radius. Then the matter inevitably falls into the center of the field ($r = 0$), the true singularity of the metric. This again demonstrates the crucial role of coordinate transformations in general relativity.

Tolman-Oppenheimer-Volkoff equation. Another important exact result within general relativity is the equation of hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G(\rho + p/c^2)(M + 4\pi p r^3/c^2)}{r^2(1 - r_{\text{Sch}}/r)} \,. \qquad (25)$$

According to this Tolman-Oppenheimer-Volkoff equation the factor $1/(1 - r_{\text{Sch}}/r)$ leads to an infinite pressure gradient for $r \to r_{\text{Sch}}$. This has a deep consequence: there is an upper limit for the mass of static compact relativistic stars, around 2 - 3 $M_\odot$. According to the standard GR, compact objects with larger masses may exist only as black holes.

2.5 Conceptual problems of geometrical approach

In spite of the great success of the geometrical approach to description of existing experiments/observations in gravitation physics, there are some conceptual problems of GRT which should be noted in our paper. Among them the most important are:

- *the physical sense of the energy-momentum of the space curvature,*
- *the physical sense of the black hole horizon and singularity,*
- *the physical sense of the space creation in the expanding Universe.*

Geometrical approach without black holes? In recent literature there is intriguing discussion about physical impossibility of black holes, horizons and singularities. Logunov & Mestvirishvili[121]; Kisilev et al.[113]; Mitra[128]; Gershtein et al.[83] emphasized the important role of additional physical conditions which should be used for physically reasonable solution of Einstein's equations. Fore example the Hilbert's causality principle leads to elimination of horizons and singularities [83].

A re-analysis of the physical meaning of the coordinate transformation in general relativity led Mitra[128];[129] to the conclusion that a black hole should have zero mass. Considering both the 4-velocity and the physical 3-velocity of a co-moving observer he concluded that instead of genuine black holes there is a solution of Einstein's equations describing an "eternally collapsing object" (ECO) with a size close to r_{Sch} and all the time radiating energy so that an event horizon never originates.

Robertson & Leiter [154] introduced the strong principle of equivalence requiring that "special relativity must hold locally for all time-like observers in all of space-time". They found solutions of Einstein's equations which satisfy the requirement for time-like world line completeness and introduced "magnetospheric eternally collapsing objects". Such MECOs possess an intrinsic magnetic moment and they do not have any event horizon and curvature singularity.

If a substance has an unusual equation of state $p = p(\rho)$, like that of the physical vacuum and dark energy, it is possible to obtain non-singular static GR solutions for arbitrary large masses, which are stable, and have no singularity, no event horizon and no information paradox (Dymnikova[62]; Mazur & Mottola[126]; Chapline[44]).

These works show that additional conditions on the equation of state or coordinate transformations or the metric tensor of Riemannian space can change the physical contents of the geometrical gravity theory.

The energy-momentum of the space curvature? As we already mentioned, together with Einstein's equations the conceptual problem of the energy of the gravitational field was born. The "pseudo-tensor" (actually non-tensor) character of the EMT of the gravity "field" in GRT has been discussed in many papers, where different ways to avoid this obstacle were suggested.

A mathematical consequence of the Einstein's equation (15) is that the covariant divergence of the matter energy-momentum tensor equals zero:

$$T^{ik}_{(\mathrm{m}) \; ; \; i} = 0\,. \tag{26}$$

One is tempted to see in this expression a usual conservation law, but let us cite the famous, but often ignored statement by Landau & Lifshitz ([118], sect.101 p.304): "however, this equation does not generally express any conservation law whatever. This is related to the fact that in a gravitational field the four-momentum of the matter alone must not be conserved, but

rather the four-momentum of matter plus gravitational field; the latter is not included in the expression for $T^{ik}_{(m)}$". [7]

To define a conserved total four-momentum for a gravitational field plus the matter within it, Landau & Lifshitz [118] suggested the expression

$$\frac{\partial}{\partial x^k}(\sqrt{-g})(T^{ik}_{(m)} + t^{ik}_{(g)}) = 0 \,. \tag{27}$$

Here $t^{ik}_{(g)}$ is called the energy-momentum pseudo-tensor. It is important that the quantities $t^{ik}_{(g)}$ do not constitute a tensor, i.e. they depend on the choice of the system of coordinates. There are many variants of the expressions suggested for the pseudo-tensor, among them Einstein's (non-symmetric) and Landau & Lifshitz's (symmetric) pseudo-tensors. Unfortunately all these expressions do not satisfy to the necessary conditions for EMT of real field (see Sec.1.2).

Moreover, this way of introducing the energy for the geometrized gravity field within GRT is conceptually inconsistent, as discussed in detail by Logunov & Folomeshkin (1977)[120] and Logunov & Mestvirishvili (1989)[121]. Also Yilmaz (1992)[204] has shown that for any pseudo-tensor due to the Freud identity one has $\partial_i(\sqrt{-g}\, t^i_k) = 0$, which leads to a difficulty with the definition of the gravitational acceleration.

Non-localizability of gravity energy. There are attempts to give a physical reason for the non-tensor character of the energy of a gravity field due to the non-localizability of the gravitational field in the geometrical approach (Misner, Thorne & Wheeler [127], p.467): "It is not localizable. The equivalence principle forbids." They also noted the following properties of the pseudo-tensor: "There is no unique formula for it, ... , 'local gravitational energy-momentum' has no weight. It does not curve space. It does not serve as a source term ... It does not produce any relative geodesic deviation of two nearby world lines ... It is not observable." The problem is also clearly seen in the case of the gravitational wave detection (will be discussed in Section 4.2). So the actual cause of the absence of the gravity field energy, i.e. the pseudo-tensor character of the EMT of the gravitational field in general relativity, is the principle of geometrization.

In Friedmann cosmology the problem of the energy of the gravity field leads to the paradox of continous creation/annihilation of matter within any finite comoving volume of an expanding space (will be discussed Sec.5).

Attempts to overcome the energy problem by using simultaneously Minkowski and Riemannian spaces. In the literature there are attempts to construct a gravity theory which based on both flat and curved spaces by accepting some Lorentz-covariant properties of Minkowski space in "effective" Riemannian

---

[7] Mathematically this is because the integral $\int T^{ik}_{(m)}\sqrt{-g}\,dS_k$ is conserved only if the condition $\partial(\sqrt{-g}T^{ik}_{(m)})/\partial x^k = 0$ is fulfilled, while eq.(26) gives $T^{ik}_{(m)\,;\,k} = (1/\sqrt{-g})(\partial(\sqrt{-g}T^{ik}_{(m)})/\partial x^k) - (1/2)(\partial g_{kl}/\partial x_i)T^{kl}_{(m)} = 0$.

space (this is comprehensively reviewed by Pitts & Schieve [147]). As an example of such works we mention three "field-geometrical" theories developed by Logunov, Yilmaz, Grishchuk and their collaborators.

Logunov & Mestvirishvili [121] developed a field-geometrical gravity theory, called the relativistic theory of gravitation (RTG), where they accept "geometrization principle" for matter, while conserve Minkowski flat space for gravitation field. They introduce the metric tensor g^{ik} of the effective Riemann space, and also accept a "causality principle" as an additional restriction on g^{ik}. Due to these assumptions there is no black hole solution in RTG. The scalar part of gravitational tensor potentials exists only in a static field and can not be radiated. The cosmological solution is the Friedmann expanding space with the critical matter density. Recent development of the RTG includes also non-zero rest mass of the gravity field.

Yilmaz [204] constructed a field-geometrical theory where the right-hand side of the field equation contains the EMT of the gravity field in the effective Riemann space with the background Minkowski space. The metric of the effective Riemann space has an exponential form and excludes the event horizon and singularity. The existence of the EMT of the gravity field allows one to consider N-body solutions in this theory.

Baback & Grishchuk [10] claimed that they constructed a field approach which is completely identical to general relativity : "GR may be formulated as a strict non-linear field theory in flat space-time. This is a different formulation of the theory, not a different theory." They introduce the metric tensor $g^{ik}(x^l)$ of a curved space-time via the field variables $h^{ik}(x^l)$ in the form $g^{ik} = (\eta^{ik} + h^{ik})\sqrt{\gamma/g}$ with the condition $g^{ik}g_{il} = g_l^k = \delta_l^k$. Hence the tensor of Riemannian space is presented as a sum of two non-tensors, because the Minkowski metric η^{ik} is not a tensor of curved space. Then they developed a Lagrangian theory where there are energy-momentum tensor of the gravitational field (close to LL-pseudotensor), black holes, quadrupole radiation and expanding space of Friedmann's cosmology. However the internal inconsistency of this approach follows from incompatibility of the initial principles of the geometrical and field theories which leads in GR to different observable consequences. E.g. above we have discussed the notes by Straumann[169] and Padmanabhan [141] who emphasized that in GR there is a non-trivial topology of black holes, while Minkowski space has a trivial one. Also the expanding space of GR violates energy conservation, which is impossible for the field in Minkowski space.

Absence the required physical properties of the EMT in the metric gravity theories. Besides the true tensor character of the EMT there are additional properties of energy-momentum known from the quantum relativistic field theories of other physical interactions. For example the EMT of the boson fields must have the following features:

- *symmetry, $T^{ik} = T^{ki}$;*
- *positive localizable energy density, $T^{00} > 0$;*
- *zero trace for massless fields, $T = 0$.*

The above considered attempts to introduce the EMT of the gravity field within geometrical and effective "field" approaches, though could obey the

symmetry condition, do not possess the other two necessary features of the EMT, i.e. a positive localizable energy density and zero trace. These properties of EMT must be fulfilled within the consistent field approach for both static and free fields, as in the case of the electromagnetic field (Sec.1.2).

The energy problem may be demonstrated with the simplest case of a spherically symmetric weak static gravity field. Indeed, for this case, like in a terrestrial laboratory, one can easily calculate the predicted value of the energy density of the gravitational field for different pseudo-tensors. For instance, in harmonic coordinates the Landau-Lifshiz symmetric pseudotensor gives negative energy density of the gravity field

$$t_{\mathrm{LL}}^{00} = -\frac{7}{8\pi G} \left(\boldsymbol{\nabla}\varphi_{N} \right)^2 . \tag{28}$$

Also the "final" energy-momentum tensor of the gravity field, which was derived by Grishchuk, Petrov & Popova [87], has a negative energy density of the weak static field : $t_{\mathrm{GPP}}^{00} = -\frac{11}{8\pi G} \left(\boldsymbol{\nabla}\varphi_{\mathrm{N}} \right)^2 .$

Hence, according to the LL-pseudo-tensor and the GPP-tensor the energy density of the static gravitational field is negative, which conflicts with the quantum field theories of other fundamental interactions. Also the traces of all these EMTs do not vanish for static fields.

The physical sense of the space creation in the expanding Universe. In cosmology GRT predicts that the homogeneous matter distribution expands together with space. The linear Hubble law of the space-expansion velocity $V_{exp} = H \times R$ is the strict consequence of the matter homogeneity. The physics of the space expansion (increasing distances between galaxies with increasing time) contains several paradoxes. Harrison [93,91,92] demonstrated that the cooling of homogeneous hot gas (including photon gas of CMBR) in the standard cosmological model (SCM) actually means the violation of energy conservation in the expanding space. In modern version of SCM the term "space expansion" actually means continuous creation of vacuum, something that leads to conceptual problems. Recent discussion by Francis et al. [79] on the physical sense of the **increasing distance** to a receding galaxy **without motion** of the galaxy is just a particular consequence of the arising paradoxes [26].

In the Sec.5 we present analysis of the conceptual problems of the SCM: the violation of energy conservation for local comoving volumes, the exact Newtonian form of the Friedmann equation (no direct relativistic effects of expanding substances, e.g. the absence of an upper limit on the receding velocity of galaxies which can be greater than the speed of light), and the presence of the linear Hubble law deeply inside very inhomogeneous large scale galaxy distribution of the Local Universe.

Conclusion. The above discussion demonstrates that all "field-geometrical" theories which introduce a metric of an "effective" Riemannian space in the form $g_{ik} \approx \eta_{ik} + h_{ik}$ must obey an exact equalities $g_k^i \equiv \delta_k^i$ and $g^{ik}g_{ik} \equiv 4$ and so eliminate exactly the internal scalar part - the trace of the true tensor potential. Hence such metric gravity theories lose some essential properties of

the field approach and receive some unphysical properties of the geometrical approach, e.g. non-tensor character of the gravity field energy, the negative energy of the static field, the event horizon and singularity etc.

So one can conclude that all attempts to derive "geometry" from "gravitons" explicitly or implicitly contain propositions that reduce the field approach to geometrical one ([169]; [141]). Hence, the question is how to construct a consistent field gravitation theory (quantum gravidynamics) based on relativistic quantum principles and which, only as an approximation to reality, contains geometrical interpretation, like geometrical optics in electrodynamics.

3 Poincaré-Feynman's field approach to gravitation theory

In Sec.1.3 – 1.4 we have emphasized that the field gravitation theory has its roots in papers by Poincare, Birkhoff, Thirring, Kalman, Feynman and some other eminent physicists. The field approach offers a natural solution to the energy problem, because Minkowski space implies the invariance under the Poincaré group transformation and hence the usual definition of the energy-momentum tensor of the gravitational field and conservation laws, as it follows from the Noether's theorem.

We stress that the construction of the field gravity theory has not completed yet and important questions are still open. For example, the quantization of the gravity field needs to take into account the conservation of the gravitational energy and the finiteness of the gravity force, in order to overcome the problem of non-renormalizability. The main strategy of the consistent field approach is not to write down the final non-linear exact equations, but to control the physical sense of the energy-momentum of the gravitational field in the description of the gravitational interaction. A construction of the field gravity theory was started in the works by Fierz & Pauli 1939 [78]; Birkhoff 1944 [37]; Moshinsky 1950 [131]; Thirring 1961 [177]; Kalman 1961 [110]; Feynman 1971, 1995 [76], [77]; Bowler 1976 [40]. The fundamental role of the scalar part of the symmetric tensor potentials $\psi^{ik}(\mathbf{r}, t)$, i.e. its trace $\psi(\mathbf{r}, t) = \eta_{ik}\psi^{ik}$, was carefully studied by Sokolov and Baryshev (see e.g. references in [167]; [164]; [21]; [22]; [23]).

Up to now, within the field gravity theory only the weak field approximation at the post-Newtonian level has been studied in detail, though some results for strong field regime also exist. The modern development of FGT is enough to show the feasibility of the field approach and to give predictions, which distinguish FGT and GRT. Hence, in contrast to many claims, the field gravity theory is experimentally different from the geometrical general relativity.

3.1 Initial principles

The unity of the fundamental interactions. As Feynman [77] emphasized the gravitational interaction can be described as a non-metric quantum relativistic symmetric second rank tensor field in Minkowski space which is based on

the Lagrangian formalism of the field theory. He discussed a quantum field approach to the gravity just as the next fundamental physical interaction and claimed that "the geometrical interpretation is not really necessary or essential to physics" ([77], p.113).

The FGT is constructed on the common bases with other fundamental physical interactions plus several additional features specific for gravitational interaction. As we noted in Sec.1.4 these basic principles include:

- *the inertial reference frames and Minkowski space with metric η^{ik};*
- *the symmetric second rank tensor potential $\psi^{ik}(x^m)$ and especially its trace $\psi(x^m) = \psi^{ik}\eta_{ik}$ describe gravitational interaction;*
- *the Lagrangian formalism and Stationary Action principle;*
- *the principle of consistent iterations;*
- *the universality of gravitational interaction;*
- *the conservation law of the energy-momentum;*
- *the gauge invariance of the linear field equations;*
- *the positive localizable energy density and zero trace of the gravity field EMT;*
- *the quanta of the field energy as the mediators of the gravity force;*
- *the uncertainty principle and other quantum postulates.*

These elements are the basis of the consistent field approach to gravitation and form a natural starting point for understanding the physics of gravity phenomenon similarly to other fundamental forces.

The principle of consistent iterations. The gravity field has a positive energy density and this energy, in turn, becomes a new source of an additional gravity field and so on. This non-linearity is taken into account by the iteration procedure. It is usual in physics to consider first a linear approximation and then add non-linearity by means of iterations.

The field gravity theory is constructed step by step using an iteration procedure so that at each step all physical properties of the EMT of the gravity field are under control. Each step of iteration is described by linear gauge-invariant field equations with fixed sources in the right-hand side, as it is assumed in the derivation of field equations from the stationary action principle. An important outcome of this procedure is that the superposition principle also can be reconciled with the non-linearity of the gravity field.

The principle of stationary action. The mathematical tool is the Lagrangian formalism of the relativistic field theory. To derive the equations of motion for the gravity field and for the matter one uses the principle of stationary action, which states that for the true dynamical behaviour of the field and matter the variation of the action $\delta S = 0$.

The action integral for the whole system of a gravitational field plus particles (matter) consists of the three parts (instead of two parts in GR eq.(13)):

$$S = S_{(g)} + S_{(int)} + S_{(m)} = \frac{1}{c} \int \left(\Lambda_{(g)} + \Lambda_{(int)} + \Lambda_{(m)} \right) d\Omega . \qquad (29)$$

The notations (g), (int), (m) refer to the actions for the gravity field, the interaction, and the matter (particles or other sources), $d\Omega = dV\, dt$. The physical dimension of each part of the action is [S]= [energy density]×[volume]×[time],

meaning that the definition of energy density of the field should be defined in the theory at the conceptual level.

In general relativity the action integral (13) has only two parts S_g and S_m. There is no interaction part in GR, because of the principle of geometrization.

Lagrangian for the gravitational field. Within the Feynman's field approach, the gravity field is presented by symmetric 2nd rank tensor potentials $\psi^{ik}(\mathbf{r}, t)$ in Minkowski space with metric η^{ik}. The Lagrangian for the gravitational field, related to considered fixed source, we take in the form:

$$\Lambda_{(g)} = -\frac{1}{16\pi G}\left[\left(2\psi_{nm}^{;n}\psi_{,l}^{lm} - \psi_{lm,n}\psi^{lm,n}\right) - \left(2\psi_{ln}^{;l}\psi^{,n} - \psi_{,l}\psi^{,l}\right)\right]. \quad (30)$$

This differs from Thirring's [177] choice by a divergent term, which does not change the field equations, but has the advantage that the canonical energy momentum tensor is symmetric. Here $\psi_{,l}^{ik} = \partial\psi^{ik}/\partial x^l$ is the ordinary partial derivative of the symmetric second rank tensor potential.

Lagrangian for matter. The Lagrangian for matter depends on the physical problem in question (particles, fields, fluid or gas). Gravity is also a kind of matter and at each iteration step it is considered as a source fixed by the preceding step.

For relativistic point (structureless) particles the Lagrangian is

$$\Lambda_{(p)} = -\eta_{ik}T_{(p)}^{ik}, \quad (31)$$

where $T_{(p)}^{ik}$ is the EMT of the particles

$$T_{(p)}^{ik} = \sum_a m_a c^2 \delta(\mathbf{r} - \mathbf{r}_a)\{1 - \frac{v_a^2}{c^2}\}^{1/2}u_a^i u_a^k. \quad (32)$$

Here m, v, u^i are the rest mass, 3-velocity, and 4-velocity of a particle.

For a relativistic macroscopic body the EMT is

$$T_{(m)}^{ik} = (\varepsilon + p)\, u^i u^k - p\eta^{ik}. \quad (33)$$

Here ε and p are the energy density and pressure of a comoving volume element.

The principle of universality and Lagrangian for interaction. In the field approach the *principle of universality* states that the gravitational field ψ^{ik} interacts with all kinds of matter via their energy-momentum tensor T^{ik}, so the Lagrangian for the interaction has the form:

$$\Lambda_{(int)} = -\frac{1}{c^2}\psi_{ik}T^{ik} \quad (34)$$

The principle of universality, eq.(34), was introduced by Moshinsky 1950 [131]. It replaces the equivalence principle used in the geometrical approach. From the principle of *universality of gravitational interaction* (UGI) and the

stationary action principle one can derive those consequences of the equivalence principle, which do not create paradoxes. As it will be shown below, according to UGI the free fall acceleration of a body does not depend on its total mass, but does depend on the direction and value of its velocity.

The equivalence principle of GRT cannot be a basis of the field gravity, because it eliminates the gravity force and accepts the equivalence between the inertial motion and the accelerated motion. E.g., the equivalence principle creates a puzzle in a gedanken experiment with the electric charge resting in the gravity field on a laboratory table (which was debated in the literature for a long time). [8] In the frame of GRT, due to the equivalence of the laboratory frame (with gravity) and the accelerated free falling frame with "$a = g$", the charge at rest on the table is equivalent with an accelerated free falling charge. But a charge with a constant acceleration "a" should radiate energy according to the relation $P = (2/3)(e^2/c^3)a^2$ ergs/s while resting charge in the lab has $a = 0$ so radiated energy P should be zero.

In the field gravity theory the charge at rest on the table does not radiate and the free falling charge does radiate, just because the inertial frame is not equivalent to the accelerated frame. The concept of an inertial frame is fundamental for all physical interactions and it is preserved in field gravity theory.

Instead of the equivalence principle, FGT is based on the principle of universality of gravitational interaction, according to which gravity "see" only the energy momentum tensor of any matter. This point is also different from all "effective geometry" theories where the universality of gravity is understood as geodesic motion in Riemannian space.

3.2 Basic equations of the Field Gravity Theory

Field equations. Using the variation principle to obtain the field equations from the action (29) one must assume that the sources T^{ik} of the field are fixed (or the motion of the matter given) and vary only the potentials ψ^{ik} (serving as the coordinates of the system). On the other hand, to find the equations of motion of the matter in the field, one should assume the field to be given and vary the trajectory of the particle (matter). So keeping the **total EMT of matter in (34) fixed** and varying $\delta\psi^{ik}$ in (29) we get

$$-\psi^{ik,l}_{l} + \psi^{il,k}_{l} + \psi^{kl,i}_{l} - \psi^{,ik} - \eta^{ik}\psi^{lm}_{,lm} + \eta^{ik}\psi^{,l}_{l} = \frac{8\pi G}{c^2}T^{ik}\,. \qquad (35)$$

The trace of the field equations (35) gives the scalar equation for generating the scalar part of the symmetric second rank tensor – its trace $\psi(\mathbf{r}, t)$, in the form

$$2\psi^{,l}_{l} - 2\psi^{lm}_{,lm} = \frac{8\pi G}{c^2}T\,. \qquad (36)$$

The field equations (35) are identical to the linear approximation of Einstein's field equations and that is why there are many similarities between

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[8] A discussion of the puzzle of the electron resting in the lab gravitational field presented in [123]. Actually this effect is related to more general Unruh effect [84]

GRT and FGT in the weak field regime. However, very important difference is that $\psi^{ik}(\mathbf{r}, t)$ and η^{ik} (both and their sum too) are true tensors of the Minkowski space and the trace $\psi(\mathbf{r}, t)$ is a true scalar of the Minkowski space. But in GRT the quantities h^{ik} and η^{ik} are not tensors of a general Riemannian space. In consistent geometrical approach this quantities should obey the following relations: $h^{ik} = -h_{ik}$, $h^i_k = 0$ and $h = 0$ due to the exact formula for the 4-scalar trace $g^{ik}g_{ik} = 4$ valid for any metric tensor. In FGT the second rang symmetric tensor should obeys relations for their components: $\psi^{ik} = \psi_{ik}$, and $\psi^i_k = \psi^k_i$, and $\psi = \eta_{ik}\psi^{ik}$, where the 4-scalar trace $\psi = \psi(\mathbf{r}, t)$ is a function of coordinates.

Remarkable features of the field equations. First, the divergence of the left side of the field equations (35) is zero, implying the conservation law

$$T^{ik}_{,k} = 0\,, \tag{37}$$

in the approximation **corresponding to the step in the iteration** procedure. In the zero approximation it does not include the EMT of the gravity field, but the first approximation contains the gravity field of the zero approximation and expresses the conservation laws and the equations of motion at the post-Newtonian level. It is important that the conservation law (37) does not restricts the trace of the energy-momentum tensor.

Second, equation (35) (and its consequence (36)) is gauge invariant, i.e. it is not changed under the following transformation of the potentials:

$$\psi^{ik} \Rightarrow \psi^{ik} + \lambda^{i,k} + \lambda^{k,i}\,, \tag{38}$$

and corresponding transformation of the trace

$$\psi \Rightarrow \psi + 2\lambda^m_{,m}\,. \tag{39}$$

The four arbitrary functions λ^i are consistent with the four restrictions on the source EMT due to energy-momentum conservation. Note that the conservation law is a property of the field equations which does not depend on the choice of a gauge transformation. Also the conservation law (37) does not restrict the equation (36) for the scalar part of the source, while it restricts the equation (35) for the tensor part of the source.

An important difference between this gauge transformation of the gravitational potentials and the general covariant transformation of coordinates in GRT (Lee derivative), is that (38) (and its consequence (39)) are performed in a fixed inertial reference frame. [9] The gauge freedom (38) and (39) allows one to put four additional conditions on the potentials, in particular a Lorentz invariant gauge – the Hilbert-Lorentz gauge [10] ([177]):

$$\psi^{ik}_{,k} = \frac{1}{2}\psi^{,i}\,. \tag{40}$$

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[9] The gage transformation (38) of the gravitational potentials can also be written as $\psi^{ik} \Rightarrow \psi^{ik} + \lambda^{i,k} + \lambda^{k,i} + 2\gamma^{,ik}$ which however does not change the number of arbitrary functions because the arbitrary function γ can be included in 4 arbitrary new functions $\lambda'_i = \lambda_i + \gamma_{,i}$.

[10] Also called as the de Donder gauge

With the gauge (40) the field equations get the form of wave equations:

$$\left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi^{ik} = \frac{8\pi G}{c^2}\left[T^{ik} - \frac{1}{2}\eta^{ik}T\right], \tag{41}$$

and for the trace component this gives the field equation for the scalar part $\psi = \eta^{ik}\psi_{ik}$ of the gravitational potentials:

$$\left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi(\mathbf{r},t) = -\frac{8\pi G}{c^2}T(\mathbf{r},t). \tag{42}$$

Note the opposite signs in the right-hand sides of eqs.(41, 42). This corresponds to the important fact that the pure tensor part of the field represents attraction, while the **scalar part gives repulsion**. This result is caused by the fact that in the Lagrangian (30) the tensor and scalar parts have opposite signs, which does not mean negative energy of the scalar field but reflects the opposite signs of the pure tensor and pure scalar forces.

Scalar and traceless tensor are dynamical fields in FGT. The multi-component structure of the symmetric tensor potential $\psi^{ik}(\mathbf{r},t)$ is a most important thing in the quantum field theory. It is well known (Barnes [11]; Fronsdal [80]) that the symmetric 2nd rank tensor field ψ^{ik} can be decomposed under the Lorentz group into a direct sum of subspaces: one spin-2, one spin-1, and two spin-0 representations. The decomposition and the appropriate projection operators are exhibited explicitly in Barnes [11]. It corresponds to the dividing the reducible symmetric tensor into a nine-component traceless part and a single-component part which is diagonal. The single component subspace is a spin-0 representation of the Lorentz scalar – the trace of the symmetric tensor ψ_{ik}. In the symbolic form we have:

$$\{\psi^{ik}\} = \{2\} \oplus \{1\} \oplus \{0^{'}\} \oplus \{0\}. \tag{43}$$

The symmetric tensor ψ^{ik} contains $n = 10$ independent components. The relation between the number of components n and the value of the spin s ($n = 2s+1$) is fulfilled for the four particles as $10 = 5+3+1+1$ in eq.(43).

Following to the Schwinger's source theory [160] the real particles corresponds to the source components after taking into account conservation laws. (four additional conditions to delete the four source components). Hence, after four restrictions from the conservation laws of the energy-momentum tensor, which delete four source components corresponding to particles with spin-1 and spin-0', the field equations (35) will describe only two real sources of the gravitational potentials ψ^{ik} as the mixture of two fields with spin-2 and spin-0 (there is no restriction on the trace component), which generated by two corresponding parts of the source T^{ik} of the gravity field:

$$\{T^{ik}\} = \{2\} \oplus \{0\} \Longrightarrow \{\psi^{ik}\} = \{2\} \oplus \{0\} \tag{44}$$

Now we can present the EMT of the source and the initial symmetric tensor potentials as the sum of pure tensor spin-2 and pure scalar spin-0 parts:

$$T^{ik} = T^{ik}_{\{2\}} + T^{ik}_{\{0\}} = (T^{ik} - \frac{1}{4}\eta^{ik}T) + \frac{1}{4}\eta^{ik}T. \tag{45}$$

$$\psi^{ik} = \psi^{ik}_{\{2\}} + \psi^{ik}_{\{0\}} = (\psi^{ik} - \frac{1}{4}\eta^{ik}\psi) + \frac{1}{4}\eta^{ik}\psi\,, \tag{46}$$

where $\eta_{ik}T^{ik}_{\{2\}} = 0$, $\eta_{ik}T^{ik}_{\{0\}} = T$ and $\psi^{ik}_{\{2\}} = \phi^{ik}$, $\eta_{ik}\psi^{ik}_{\{2\}} = 0$, $\eta_{ik}\psi^{ik}_{\{0\}} = \psi$. Both equations(35) and (36) are gauge invariant, hence for the Hilbert-Lorentz gauge (40) they can be written in the form

$$\left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi^{ik}_{\{2\}} = \frac{8\pi G}{c^2}T^{ik}_{\{2\}} \text{ or } \left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\phi^{ik} = \frac{8\pi G}{c^2}\left[T^{ik} - \frac{1}{4}\eta^{ik}T\right]$$
$$\tag{47}$$

and

$$\left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi^{ik}_{\{0\}} = -\frac{8\pi G}{c^2}T^{ik}_{\{0\}} \text{ or } \left(\triangle - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi\frac{1}{4}\eta^{ik} = -\frac{8\pi G}{c^2}T\frac{1}{4}\eta^{ik}$$
$$\tag{48}$$

This means that the field gravity theory is actually a scalar-tensor theory, where the scalar part of the field is simply the trace of the tensor potentials $\psi = \eta_{ik}\psi^{ik}$ generated by the trace of the energy-momentum tensor of the matter $T = \eta_{ik}T^{ik}$. According to the wave equations (47, 48) for spin-2 and spin-0 fields, both kinds of gravitons are massless particles, moving with velocity of light (Sokolov & Baryshev [167]).

Zakharov 1965 [205] noted that the interacting gravitational field ψ^{ik} in eq.(35) describes spin-2 and spin-0 gravitons. However absence of the scalar part of the metric in geometric gravity theory ($g^{ik}g_{ik} = 4$) leads him to rejection of the scalar waves in GR. However, in the frame of FGT, one should take into account both quadrupole tensor and monopole scalar gravitational radiation. Because the conservation law (37) does not restrict the scalar part of the source the spin-0 field is real and not a constraint field. [11]

The source of the scalar wave is the variable trace of the EMT source, e.g. for particles $T = mc^2(1 - v^2/c^2)^{1/2}$ and variation of the particles kinetic energy will generate the scalar gravitational radiation, e.g. via spherical pulsations of a gravitating system. The radiated scalar gravitational wave is monopole and has a longitudinal character in the sense that a test particle in the wave moves along the direction of the wave propagation (GWs are considered in Sec.4.2).

The energy-momentum tensor of the gravity field. The standard Lagrangian formalism and the Lagrangian of the gravity field (30) give the following expression for the canonical energy-momentum tensor:

$$T^{ik}_{(g)} = \frac{1}{8\pi G}\left\{(\psi^{lm,i}\psi_{lm}{}^{,k} - \frac{1}{2}\eta^{ik}\psi_{lm,n}\psi^{lm,n}) - \frac{1}{2}(\psi^{,i}\psi^{,k} - \frac{1}{2}\eta^{ik}\psi_{,l}\psi^{,l})\right\}$$
$$\tag{49}$$

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[11] In the case of electrodynamics the conservation of 4-current lead to exclusion of 1 component of the source of the 4-vector potentials, though constraint equations exist. The conservation law of EMT in FGT restrict only four components and leaves pure tensor and trace-scalar fields real. In the metric gravity theories there is an additional condition that the trace of the metric tensor equals to constant, so the scalar wave is absent.

Two important remarks should be made about this expression.

First, the EMT has an ordinary tensor character and so it is conceptually well defined. However, the Lagrangian formalism cannot give a unique expression for an EMT of any field (e.g. Bogolyubov & Shirkov [39]; Landau & Lifshitz [118]) because a term with zero divergence can always be added. For the final determination of the EMT of the field additional physical requirements must be used, like the positive energy density, the symmetry, and zero value for the trace in the case of a massless field.

Second, the negative sign of the scalar part (the 2nd term in (30)) does not mean that the spin 0 field has negative energy. It reflects the repulsive force produced by the scalar when the field interacts with the sources.

For the free field the energy is positive for the pure tensor (spin 2) and scalar (spin 0) components. Indeed, the total Lagrangian (30) of the interacting gravity field can be divided into two independent parts that correspond to two independent particles with spin 2 (ϕ^{ik}) and spin 0 (ψ). These have the following free field Lagrangians

$$\Lambda_{\{2\}} = \frac{1}{16\pi G}\phi_{lm,n}\phi^{lm,n}, \quad \text{and} \quad \Lambda_{\{0\}} = \frac{1}{64\pi G}\psi_{,n}\psi^{,n}. \tag{50}$$

Both signs are positive due to the positive energy density condition for interger spin free particles. Corresponding EMTs for the tensor and scalar free fields are

$$T^{ik}_{\{2\}} = \frac{1}{8\pi G}\phi_{lm}{}^{,i}\phi^{lm,k}, \quad \text{and} \quad T^{ik}_{\{0\}} = \frac{1}{32\pi G}\psi^{,i}\psi^{,k} . \tag{51}$$

These are symmetric, have a positive energy density and a zero trace for the case of plane monochromatic waves.

The retarded potentials. In the frame of FGT the solution of the field equation (41) for the case of the weak field and slow motion can be presented in the form of retarded potentials:

$$\psi^{ik}(\mathbf{r},t) = -\frac{2G}{c^2}\int\frac{\hat{T}^{ik}(\mathbf{r}',t-R/c)}{R}dV' + \psi^{ik}_0 , \tag{52}$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$ - is the radius vector from the volume element $dV' = dx'dy'dz'$ to the point $\mathbf{r}$, the source has the form $\hat{T}^{ik} = T^{ik} - (1/2)T\eta^{ik}$ in corresponding approximation, ψ^{ik}_0 is the free field solution.

There is important possibility for the scalar field equation (42) which in the case of zero trace for the field and interaction EMTs has exact solution:

$$\psi(\mathbf{r},t) = \frac{2G}{c^2}\int\frac{T_{(m)}(\mathbf{r}',t-R/c)}{R}dV' + \psi_0 , \tag{53}$$

where $T_{(m)}$ is the trace of the matter EMT, and ψ_0 is the solution of equation (42) without the right-hand side. In particular, for the case of a moving test particle along trajectory $\mathbf{r}_0 = \mathbf{r}_0(t)$, having the trace of EMT in the form

$$T = mc^2\sqrt{1 - v^2/c^2}\delta(\mathbf{r} - \mathbf{r}_0) , \tag{54}$$

one gets from Eq.(53)

$$\psi(\mathbf{r}, t) = \frac{2Gm\sqrt{1 - v^2/c^2}}{R - (\mathbf{v} \cdot \mathbf{R})/c} = \frac{2R_m c^2}{R}\mathcal{D} , \tag{55}$$

where $R_m = Gm/c^2$, $R = |\mathbf{r} - \mathbf{r}_0|$, $v = v(t')$, $R = R(t')$ - in the particle point at the moment $t' = t - R(t')/c$, and $\mathcal{D} = \sqrt{1 - v^2/c^2}/(1 - v\cos\theta/c)$ - the relativistic Doppler-factor.

Equations of motion for test particles. Let us consider the motion of a relativistic test particle with rest-mass m_0, 4-velocity u^i, 3-velocity $\mathbf{v}$ in the gravitational field described by the symmetric tensor potential ψ^{ik} on flat Minkowski space-time, where the Cartesian coordinates always exist and the metric tensor $\eta^{ik} = diag(1, -1, -1, -1)$.

To derive the equation of motion in FGT we use the stationary action principle in the form of the sum of the free particles and the interaction parts:

$$\delta S = \delta(\frac{1}{c}\int(\Lambda_{(p)} + \Lambda_{(int)})d\Omega) = 0 \tag{56}$$

where $d\Omega$ is the element of 4-volume and the variation of the action is made with respect to the particle trajectories δx^i for *fixed gravitational potential* ψ^{ik}. The free particle Lagrangian is

$$\Lambda_{(p)} = -\eta_{ik}T_{(p)}^{ik} \tag{57}$$

and the interaction Lagrangian in accordance with principle of universality of the gravitational interaction Eq.(34) is

$$\Lambda_{(int)} = -\frac{1}{c^2}\psi_{ik}T_{(p)}^{ik} \tag{58}$$

where the energy-momentum tensor (EMT) of the point particle is

$$T_{(p)}^{ik} = m_0 c^2 \delta(\mathbf{r} - \mathbf{r_p})\{1 - \frac{v^2}{c^2}\}^{1/2}u^i u^k \tag{59}$$

Below we use Landau & Lifshitz ([118]) method of 4-coordinates variation for the derivation of the equation of motion. Inserting Eqs.(59),(58) into Eq.(56) and taking into account that $ds^2 = dx_l dx^l$ we get

$$\int(m_0 c\delta(\sqrt{dx_l dx^l}) + \frac{m_0}{c}\delta(\psi_{ik}\frac{dx^i dx^k}{\sqrt{dx_l dx^l}})) = 0 \tag{60}$$

Performing the variation and integrating by parts, and taking into account that the variation is made for the fixed values of the integration limits, we find

$$\int(m_0 c\, du_i \delta dx^i + \frac{2m_0}{c}d(u^k\psi_{ik})\delta x^i -$$
$$\frac{m_0}{c}d(\psi_{lk}u^l u^k u_i)\delta x^i - \frac{m_0}{c}u^k\delta\psi_{ik}dx^i) = 0 \tag{61}$$

Consider also that

$$du^i = \frac{du^i}{ds}ds; \qquad dx^i = u^i ds; \qquad \delta\psi_{ki} = \psi_{ki,l}\delta x^l;$$

$$d(u^l u^k u_i \psi_{lk}) = u^l u_i u^k \psi_{lk,n} dx^n + u^l u^k \psi_{lk} du_i + 2\psi_{lk} u^l u_i du^k;$$

finally we get the following equation of motion for test particles in the field gravity theory (Baryshev [13]):

$$A^i_k \frac{d(m_0 c u^k)}{ds} = -m_0 c B^i_{kl} u^k u^l \,, \tag{62}$$

where $m_0 c u^k = p^k$ is the 4-momentum of the particle, and

$$A^i_k = \left(1 - \frac{1}{c^2}\psi_{ln}u^l u^n\right)\eta^i_k - \frac{2}{c^2}\psi_{kn}u^n u^i + \frac{2}{c^2}\psi^i_k \,, \tag{63}$$

$$B^i_{kl} = \frac{2}{c^2}\psi^i_{k,l} - \frac{1}{c^2}\psi_{kl}{}^{,i} - \frac{1}{c^2}\psi_{kl,n}u^n u^i \,, \tag{64}$$

The equation (62) is identical to the equation of motion derived by Kalman [110] in another way, by considering the relativistic Lagrange function L defined as $S = \int L\frac{ds}{c}$ and relativistic Euler equation:

$$\frac{d}{ds}\left(\frac{\partial L}{\partial u^k}u^k - L\right)u_i + \frac{\partial L}{\partial u^i} = -\frac{\partial L}{\partial x^i} \tag{65}$$

Inserting in Eq.(65) the expression for the relativistic Lagrange function

$$L = -m_0 c^2 - m_0 \psi_{ik}\frac{dx^i}{ds}\frac{dx^k}{ds} \tag{66}$$

one gets Kalman's equations of motion (Kalman [110]), which may be also presented in the form of Eq.(62).

Static spherically symmetric weak field. For a spherically symmetric static weak field of a body with mass density ρ_0 and total mass M, the zero (Newtonian) approximation of the total EMT equals that of the matter

$$T^{ik}_{(m)} = \text{diag}(\rho_0 c^2,\, 0,\, 0,\, 0) \tag{67}$$

and the solution of the field equations (eq. 41) is the Birkhoff's [37] potential

$$\psi^{ik} = \varphi_{\text{N}}\,\text{diag}(1,\, 1,\, 1,\, 1), \tag{68}$$

where $\varphi_{\text{N}} = -GM/r$ is the Newtonian potential outside the SSS gravitating body. We note again that ψ^{ik} is a true tensor quantity in Minkowski space.

The Birkhoff's gravitational potential (68) according to equation (46), can be expressed as the sum of the pure tensor and scalar parts $\psi^{ik} = \psi^{ik}_{\{2\}} + \psi^{ik}_{\{0\}}$, so that

$$\psi^{ik} = \frac{3}{2}\varphi_{\text{N}}\,\text{diag}(1,\, \frac{1}{3},\, \frac{1}{3},\, \frac{1}{3}) \; - \; \frac{1}{2}\varphi_{\text{N}}\,\text{diag}(1,\, -1,\, -1,\, -1)\,. \tag{69}$$

This corresponds to attraction by spin 2 and repulsion by spin 0 parts of the Birkhoff potential. Indeed, inserting potential (69) to the equation of motion (62) in the considered approximation we get expression for 3-force in the form:

$$\mathbf{F}_N = \mathbf{F}_{(2)} + \mathbf{F}_{(0)} = -\frac{3}{2}m\boldsymbol{\nabla}\varphi_N + \frac{1}{2}m\boldsymbol{\nabla}\varphi_N = -m\boldsymbol{\nabla}\varphi_N \qquad (70)$$

Hence the Newton force of gravity is the sum of attraction due to the spin 2 tensor field and repulsion due to the spin 0 scalar field. Thus FGT is, strictly speaking, a scalar-tensor theory. But in contrast to the Brans-Dicke theory that introduces additional scalar field with coupling constant ω, in FGT the scalar field is the trace $\psi = \eta_{ik}\psi^{ik}$ of the tensor potential ψ^{ik} and has the same coupling constant G.

In the first (post-Newtonian) approximation the total EMT of the system is equal to the sum of the EMT for the matter, interaction and gravity field (Kalman [110]; Thirring [177]; Baryshev [14]):

$$T_{(\Sigma)}^{ik} = T_{(\text{p/m})}^{ik} + T_{(\text{int})}^{ik} + T_{(\text{g})}^{ik}. \qquad (71)$$

From the solution (eq. 68) and accepted the expressions for the interaction EMT

$$T_{(\text{int})}^{ik} = \rho_0\varphi_N\text{diag}(1,\ 1/3,\ 1/3,\ 1/3) \qquad (72)$$

and the EMT of the gravity field

$$T_{(\text{g})}^{ik} = +\frac{1}{8\pi G}\left(\nabla\varphi_N\right)^2\text{diag}(1,\ 1/3,\ 1/3,\ 1/3) \qquad (73)$$

we find the total energy density for the system gas + gravity in the form

$$T_{(\Sigma)}^{00} = T_{(\text{p/m})}^{00} + T_{(\text{int})}^{00} + T_{(\text{g})}^{00} = \left(\rho_0 c^2 + e\right) + \rho_0\varphi_N + \frac{1}{8\pi G}\left(\nabla\varphi_N\right)^2. \qquad (74)$$

Here $(\rho_0 c^2 + e)$ gives the rest mass and kinetic (or thermal) energy densities, $\rho_0\varphi_N$ is the negative interaction energy density, and $(\nabla\varphi_N)^2/8\pi G$ is the positive and localizable energy density of the gravitational field.

Relativistic physical sense of the potential energy. The total energy of the system in PN approximation will be

$$E_{(\Sigma)} = \int T_{(\Sigma)}^{00}dV = E_0 + E_k + E_p \qquad (75)$$

where $E_0 = \int(\rho_0 c^2)\,dV$ is the rest-mass energy, $E_k = \int(e)\,dV$ is the kinetic energy, and E_p is the classical potential energy that equals the sum of the interaction and gravitational field energies:

$$E_p = E_{(\text{int})} + E_{(\text{g})} = \int\left(\rho_0\varphi_N + \frac{1}{8\pi G}(\nabla\varphi_N)^2\right)dV = \frac{1}{2}\int\rho_0\varphi_N\,dV \qquad (76)$$

The PN correction due to the energy of the gravity field. In the field approach a gravitating body is surrounded by a material gravitational field ψ^{ik} whose mass-energy density is given by the 00-component of the EMT of the gravity field in eq.(73). In the PN approximation this leads to a nonlinear correction for the gravitational potential.

Considering the energy density of the gravitational field (the last term in eq. 74) as the source in the field equation of the second order, we get a nonlinear addition to Birkhoff's ψ^{00} component

$$\psi^{00} = \varphi_{\mathrm{N}} + \frac{1}{2}\frac{(\varphi_{\mathrm{N}})^2}{c^2} \; . \tag{77}$$

Corrections to other components do not influence the motion of particles in this approximation.

The PN equations of motion. Substituting Birkhoff's potential (68) into the equation of motion (62) and taking into account the nonlinear PN correction (77) one gets the 3-acceleration for a test particle:

$$\left(\frac{d\mathbf{v}}{dt}\right)_{\mathrm{FG}} = -\left(1 + \frac{v^2}{c^2} + 4\frac{\varphi_{\mathrm{N}}}{c^2}\right)\boldsymbol{\nabla}\varphi_{\mathrm{N}} + 4\frac{\mathbf{v}}{c}\left(\frac{\mathbf{v}}{c}\cdot\boldsymbol{\nabla}\varphi_{\mathrm{N}}\right) \tag{78}$$

This equation coinsides with the PN equation of motion in GRT only when in eq.(22) the isotropic or harmonic coordinates are used (i.e. $\alpha = 0$). It is important that within the FGT the equation of motion does not depend on the choice of the coordinate system. Also it is important that in the frame of FGT the energy of the gravitational field is an observable quantity which may be measured by observations of the orbital motion of a test particle.

The Newtonian limit. Substituting (69), which gives the gravitational potential as the sum of the pure tensor and scalar parts, into eq.(62) and neglecting all terms of the order v^2/c^2 we get the Newtonian force as the sum of two parts: the attractive force $\mathbf{F}_{(attr)}$ due to the spin 2 part and the repulsive force $\mathbf{F}_{(repuls)}$ due to the spin 0 part:

$$\mathbf{F}_N = \mathbf{F}_{(attr)} + \mathbf{F}_{(repuls)} = -\frac{3}{2}\frac{GmM}{r^3}\mathbf{r} + \frac{1}{2}\frac{GmM}{r^3}\mathbf{r} = -\frac{GmM}{r^3}\mathbf{r}\,. \tag{79}$$

This calculation shows that even on the Newtonian level the physics of the field gravity theory dramatically differs from general relativity.

The role of the scalar part of the field. The scalar ψ is an intrinsic part of the gravitational tensor potential ψ^{ik} and is not related to extra scalar fields such as was introduced in the Jordan-Brans-Dicke theory. So the observational constraints existing for this extra scalar field do not restrict the scalar part ψ of the tensor field ψ^{ik}. Moreover, without the scalar ψ it is impossible to explain the classical relativistic gravity effects.

Repulsive force of the scalar part. Inserting to the equation of motion (62) the scalar part of the gravitational potential in the form $\psi^{lm}_{\{0\}} = (1/4)\psi\eta^{lm}$,

we get the equation of motion of a test particle in the scalar gravity field $\psi = \psi_{lm}\eta^{lm}$ as

$$\left(1 + \frac{1}{4}\frac{\psi}{c^2}\right)\frac{dp^i}{ds} = \frac{m}{4c}\left(\psi^{,i} - \psi_{,l}u^l u^i\right) \ . \tag{80}$$

In the case of a weak field ($\psi/c^2 << 1$) this equation gives for spatial components ($i = \alpha$) the expression for the gravity 3-force

$$\frac{d\mathbf{p}}{dt} = -\frac{m}{4}\boldsymbol{\nabla}\psi \ . \tag{81}$$

In the case of the weak static field (68) the trace of the tensor gravitational potential is equal to $\psi = -2\varphi_{\text{N}}$, hence we get for the gravity 3-force

$$\frac{d\mathbf{p}}{dt} = +\frac{1}{2}m\boldsymbol{\nabla}\varphi_{N} \ . \tag{82}$$

This means that the scalar spin 0 part of the tensor field leads to a repulsive force and only together with the attractive force from the pure tensor spin 2 part the result is the Newtonian force (79).

The most intriguing consequence of the field gravity theory is that the scalar part (spin 0) corresponds to a repulsive force, while the pure tensor part (spin 2) corresponds to attraction. This explains the "wrong" sign for the scalar part in the EMT of the gravity field (eq. 49), because total Lagrangian describes simultaneously attractive and repulsive parts of the total field.

Poincaré force and Poincaré acceleration in PN approximation. In the post-Newtonian approximation we keep terms down to an order of $v^2/c^2 \sim \varphi_N/c^2 \ll 1$ in Eq.(62). For the PN accuracy we need calculations of the ψ^{00} component with the same order, while other components of the tensor gravitational potential ψ^{ik} can be calculated in the linear approximation. Under these assumptions from Eq.(62) for ($i = \alpha$) we get the expression for the PN 3-dimensional gravity force (which we shall call the Poincaré gravity force remembering his pioneer work in 1905 on the relativistic gravity force in flat space-time):

$$\mathbf{F}_{Poincare} = \frac{d\mathbf{p}}{dt} = -m_0\{(1 + \frac{3}{2}\frac{v^2}{c^2} + 3\frac{\phi}{c^2})\boldsymbol{\nabla}\phi - 3\frac{\mathbf{v}}{c}(\frac{\mathbf{v}}{c} \cdot \boldsymbol{\nabla}\phi)\}$$
$$-m_0\{3\frac{\mathbf{v}}{c}\frac{\partial\phi}{c\,\partial t} - 2\frac{\partial\boldsymbol{\Psi}}{c\,\partial t} + 2(\frac{\mathbf{v}}{c} \times rot\boldsymbol{\Psi})\} \tag{83}$$

where $\phi = \psi^{00}$, $\boldsymbol{\Psi} = \psi^{0\alpha} = -\psi_{0\alpha}$.

Taking into account the expression Eq.(77) for the PN 00-component of the gravitational potential, we get the corresponding Poincare 3-acceleration of the test particle :

$$\frac{d\mathbf{v}}{dt} = -(1 + \frac{v^2}{c^2} + 4\frac{\varphi_N}{c^2})\boldsymbol{\nabla}\varphi_N + 4\frac{\mathbf{v}}{c}(\frac{\mathbf{v}}{c} \cdot \boldsymbol{\nabla}\varphi_N)$$
$$+ 3\frac{\mathbf{v}}{c}\frac{\partial\varphi_N}{c\,\partial t} - 2\frac{\partial\boldsymbol{\Psi}}{c\,\partial t} + 2(\frac{\mathbf{v}}{c} \times rot\boldsymbol{\Psi}) \tag{84}$$

From the $(i = 0)$ component of Eq.(62) follows the expression for the work of the Poincaré force:

$$\frac{dE_k}{dt} = \mathbf{v} \cdot \mathbf{F}_{Poincare} =$$

$$-m_0\mathbf{v} \cdot \{(1 - \frac{3}{2}\frac{v^2}{c^2} + 3\frac{\phi}{c^2})\boldsymbol{\nabla}\phi - 3\frac{\mathbf{v}}{c}\frac{\partial\phi}{c \, \partial t} + 2\frac{\partial\boldsymbol{\Psi}}{c \, \partial t}\} \qquad (85)$$

An important particular case is the static spherically symmetric weak gravitational field for which $\boldsymbol{\Psi} = 0$, $\partial\phi/\partial t = 0$, $\psi^{ik} = diag(\phi, \varphi_N, \varphi_N, \varphi_N)$ hence the PN 3-acceleration will have the simple form:

$$(\frac{d\mathbf{v}}{dt})_{FGT} = -(1 + \frac{v^2}{c^2} + 4\frac{\varphi_N}{c^2})\boldsymbol{\nabla}\varphi_N + 4\frac{\mathbf{v}}{c}(\frac{\mathbf{v}}{c} \cdot \boldsymbol{\nabla}\varphi_N) \qquad (86)$$

From the equation of motion (86) it is clear that the acceleration of a test particle depends on the value and the direction of its velocity and gravitational potential and this is a coordinate independent relativistic gravity effect.

For circular motion $\mathbf{v} \perp \boldsymbol{\nabla}\varphi_N$, hence the PN 3-acceleration is

$$(\frac{d\mathbf{v}}{dt})_{FGT}^{\perp} = -(1 + \frac{v^2}{c^2} + 4\frac{\varphi_N}{c^2})\boldsymbol{\nabla}\varphi_N \qquad (87)$$

For radial motion $\mathbf{v} \uparrow\downarrow \boldsymbol{\nabla}\varphi_N$ the 3-acceleration is

$$(\frac{d\mathbf{v}}{dt})_{FGT}^{\parallel} = -(1 - 3\frac{v^2}{c^2} + 4\frac{\varphi_N}{c^2})\boldsymbol{\nabla}\varphi_N \qquad (88)$$

In GRT, as we noted above, PN equation of motion Eq.(22) is dependent on the choice of a coordinate system (due to parameter α) and this is why one can not directly use this equation for a derivation of observable effects. In contrast to Eq.(22), in FGT the Eq.(86) is valid for all coordinate systems in an inertial frame related to the center of mass of the main gravitating body. This allows one in FGT to calculate observable effects from coordinate independent equations (83) and (86).

Lagrange function in Post-Newtonian approximation According to expression (29) for the action in the case of a gravitating test particle the Lagrange function has the form

$$L = -mc^2\sqrt{1 - \frac{v^2}{c^2}} - \frac{mc^2}{\sqrt{1 - v^2/c^2}}\left(\frac{\Phi}{c^2} - \frac{2\boldsymbol{\Psi}}{c^2} \cdot \frac{\mathbf{v}}{c} + \frac{\theta_{\alpha\beta}}{c^2}\frac{v^\alpha v^\beta}{c^2}\right), \qquad (89)$$

where $\Phi = \psi^{00}$, $\boldsymbol{\Psi} = (\psi^{0\alpha})$, $\theta^{\alpha\beta} = \psi^{\alpha\beta}$.

Taking into account small parameters (v/c) we get

$$L_{(4)} = -mc^2 + \frac{mv^2}{2} + \frac{mv^4}{8c^2} - m\Phi - \frac{1}{2}m\Phi\frac{v^2}{c^2}$$

$$+2m\left(\boldsymbol{\Psi} \cdot \frac{\mathbf{v}}{c}\right) - m\left(\theta_{\alpha\beta} \cdot \frac{v^\alpha v^\beta}{c^2}\right). \qquad (90)$$

For a system of N gravitationally interacting particles we get

$$L_{(PN)} = \sum_a \frac{m_a v_a^2}{2} + \sum_a \frac{m_a v_a^4}{8c^2} + \sum_a \sum_b \frac{Gm_a m_b}{2r_{ab}} + \sum_a \sum_b \frac{3Gm_a m_b v_a^2}{2c^2 r_{ab}}$$

$$- \sum_a \sum_b \sum_c \frac{G^2 m_a m_b m_c}{2c^2 r_{ab} r_{ac}} - \sum_a \sum_b \frac{Gm_a m_b}{4c^2 r_{ab}} \left[7(\mathbf{v}_a \mathbf{v}_b) + (\mathbf{v}_a \mathbf{n}_{ab})(\mathbf{v}_b \mathbf{n}_{ab}) \right] \quad (91)$$

where $\mathbf{n}_{ab}$ is the unite vector in direction $(\mathbf{r}_a - \mathbf{r}_b)$.

Note, that this Lagrange function in the frame of FGT does not depends on coordinate system. It coincides with the corresponding expression in GRT, just because in the frame of GRT the harmonic coordinates was used to derive eq.(91) . This coincidence also explains many similarities in predictions of FGT and GRT.

4 Relativistic gravity experiments/observations in FGT

4.1 Classical relativistic gravity effects

The field equations (41) and equations of motion (62), which in the frame of FGT is contained in the expression of the conservation of total energy-momentum $T^{ik}_{(\Sigma),k} = 0$ in corresponding iteration, lead to various observable consequences of the field gravity. It is important that the classical weak-field relativistic gravity effects are the same in both FG and GR theories, hence they can not distinct between GRT and FGT. These common predictions are following:

- *Universality of free fall for non-rotating bodies,*
- *The deflection of light by massive bodies,*
- *Gravitational frequency-shift,*
- *The time delay of light signals,*
- *The perihelion shift of a planet,*
- *The Lense-Thirring effect,*
- *The geodetic precession of a gyroscope,*
- *The quadrupole gravitational radiation.*

Universality of free fall . The rest mass m_0 of a structureless test particle appears in both sides of the equation of motion (62), hence it is canceled off. This demonstrate that within FGT the universality of the free fall is a direct consequence of the principles of stationary action and universality of gravitational interaction. Hence the universality of free fall is not a new "principle of equivalence" but is a particular case of the old stationary action principle. The motion of a test particle in the gravity field of a massive body does not depend on the rest mass m_0 of the test particle and in fact it checks **the universality of the rest mass** of a particle.

For the case of an macroscopic extended body, when one probes the free fall of a real body, which includes internal structure and contributions from all interactions, contribution from thermal energy and pressure, and also rotation of a body, should be analyzed separately (this will be done below).

Light in the gravity field. Within the field gravity theory the deflection of light and the time delay of light signals are consequences of the interaction Lagrangian $L_{\text{int}} = \psi_{ik} T^{ik}_{(\text{elm})}$, taken in the form corresponding to the universality of gravitational interaction (UGI). This gives the "effective" refraction index in the PN approximation [177], [131]:

$$n(r) = 1 + \frac{2GM}{c^2 r} \ . \tag{92}$$

Hence the velocity of a light signal will have the value

$$c_g(r) = \frac{c}{n} = c\left(1 - \frac{2GM}{c^2 r}\right) \ , \tag{93}$$

so the direction of light ray is changed and the time delay appears, both with the same amount as actually observed.

For a photon moving at the impact distance b from a point mass M, in the weak field approximation the asymptotic deflection angle is :

$$\theta_{\text{FG}} = \theta_{\text{GR}} = 2\theta_{\text{N}} = \frac{4GM}{c^2 b} \ . \tag{94}$$

where $\theta_{\text{N}} = 2GM/v^2 b$ is the Newtonian deflection angle. One of the most spectacular success of the GRT was the observed value of the light bending for the Sun, which according to eq.(94) equals to $\theta_{\text{GR}} = 1.75$". The same value of light bending FGT unambiguously predicts.

Interestingly, using a lift analogy (equivalence principle), Einstein 1911 [64] first derived for the deflection angle a value that was a half of really observed value, i.e. Newtonian result θ_{N}. Later it was claimed that additional contribution from the curvature of space should be taken into account.

From equation of motion (78) one gets that a particle passing with velocity v the central mass M at the impact distance b, will experience a small deflection angle:

$$\theta_{\text{FG}} = (1 + \frac{v^2}{c^2})\,\theta_{\text{N}}\,. \tag{95}$$

Hence for the particle velocity $v = c$ one gets the same result as eq.(94).

One may verify from the equation of motion in GRT (22), that in order to derive these formulae one should use isotropic or harmonic coordinates (i.e. $\alpha = 0$), while in FGT the equation of motion (78) and deflection angle does not depend on coordinates.

The time delay of light signals. In the frame of FGT, the time delay phenomenon, or the Shapiro effect, is caused by the change of velocity of light according to eq.(93). If an emitter at a distance r_1 sends a light signal to a mirror at a distance r_2 from a gravitating mass, and R is the distance between the emitter and the mirror, then the additional travel time is

$$(\varDelta t)_{\text{FG}} = \frac{4GM}{c^3}\,ln(\frac{r_1 + r_2 + R}{r_1 + r_2 - R})\,. \tag{96}$$

For the case of the Sun the value of $4GM_\odot/c^3$ is about $20\mu s$.

Though the time delay has the same value for both FGT and GRT, but the physical interpretation of this effect in the field gravity theory has different explanation without geometrical space-time properties. Again in the frame of FGT this effect does not depend on the coordinates.

Atom in gravity field and gravitational frequency-shift. The gravitational redshift of spectral lines in the frame of FGT is a consequence of the shift of atomic levels. It is universal, because gravitation changes the total energy and all energy levels of an atomic system. In the PN approximation $E_{obs} = E_0(1 + \varphi_{_N}/c^2)$ and hence $h\nu_{ik}^{obs} = \Delta E_{ik}^0(1 + \varphi_{_N}/c^2)$.

Moshinsky [131] was the first who consider the interaction Lagrangian (eq. 34) for the case of the interaction between the gravity field and the spinor and electromagnetic fields of a hydrogen atom. A spectral line with frequency ν_{em} radiated by an atom at the distance r from the surface of a massive body with radius R and mass M, will be observed at infinity from the body to have a frequency ν_{obs}. This gravitational redshift in the weak field approximation ($R >> R_g$) is given by

$$z_{grav} = \left(\frac{\nu_{em} - \nu_{obs}}{\nu_{obs}}\right) = \frac{GM}{c^2 r} . \tag{97}$$

For the Sun the value of $GM_\odot/R_\odot c^2$ is 1.9×10^{-10}. A more general formula for the gravitational redshift is:

$$1 + z_{grav} = \frac{1}{\sqrt{1 + \frac{2\Phi}{c^2}}} \tag{98}$$

where $\Phi = \psi^{00}$, which gives the correct PN result $z_{grav} \approx |\varphi_{_N}|/c^2$.

In GRT the observed frequency shift is due to the clock that runs faster when it is farther from the gravitating body. GRT general relation is $dt = d\tau/\sqrt{g_{00}}$, so the Einstein's gravitational redshift is

$$1 + z_{GR} = \frac{1}{\sqrt{1 - \frac{2GM}{c^2 r}}} . \tag{99}$$

Note that in GRT there is an acute discussion about a correct interpretation of the gravitational redshift. According to Will [199] and Okun, Selivanov & Telegdi [138] [139] the energy and frequency of the photon does not change during its radial motion in the gravity field, i.e. the photon does not lose or gain energy. Some times the gravitational redshift effect is considered as a foundation of the strong equivalence principle of GRT [200].

The pericenter shift and positive energy density of gravity field. It is well-known in celestial mechanics [42], [116], that additional terms to Newtonian equation of motion in the form of the eq.(78) leads to the formula for the rate of the pericenter shift $\dot\omega$ of the orbit of a test particle (planet), having semi-major axis a, eccentricity e and period P, in the form:

$$(\dot\omega)_{\text{FG}} = \frac{6\pi GM}{c^2 a(1 - e^2)P} . \tag{100}$$

This effect is derived in the frame of the FGT as an analysis of the small terms of the equation of motion (78) by using ordinary mechanics without geometrical concepts of GRT. For Mercury this gives 43"/century, while for the binary pulsar PSR1913+16 the effect is much larger, about 4°/year.

The formula (100) is the same as in GRT, but the interpretation is different. E.g. the nonlinear contribution (the 2nd term in eq.77) due to $T^{00}_{(g)}$ provides 16.7% of the total value (100). Therefore in the field gravity theory the pericenter shift is directly contains the positive energy density of the gravity field, making this physical quantity experimentally measurable.

The Lense-Thirring effect. The Lense-Thirring (LT) precession is a direct consequence of the ordinary mechanics for the system having additional terms in corresponding Lagrangian without the geometrical concept of "dragging of inertial frames".

In the frame of FGT the LT effect is a direct consequence of the Lagrange function (90). An elliptical orbit of a non-rotating test particle, moving in the field of a central massive *rotating* body, will revolve as a whole about the direction of the rotation axis of the central body with the rate [118]

$$\Omega_{\text{LT}} = \frac{2GJ}{c^2 a^3 (1 - e^2)^{3/2}} (\mathbf{j} - 3\mathbf{l}(\mathbf{l} \cdot \mathbf{j})), \qquad (101)$$

where $\mathbf{j} = \mathbf{J}/J$, $\mathbf{l} = \mathbf{L}/L$, $\mathbf{L}$ is the orbital angular momentum of the particle, and $\mathbf{J}$ is the angular momentum of the central body.

Resent Gravity Probe B experiment [86] confirmed that for an Earth-orbiting satellite LT effect is about 0.1"/year, meaning that the orbit will make a whole rotation in about 13 million years. In the case of pulsars in binary systems and accreting RCO this precession is much larger.

The relativistic precession of a gyroscope. The rate of precession of a gyroscope orbiting a rotating massive body is the sum of two independent parts, one due to the gravitational potential of the central body, effectively non-Newtonian (the Weyl-effect), and the second due to its rotation (the Schiff-effect). This effect can be calculated as the contribution from the Lagrange function of the system of gravitating point masses in the second approximation (eq. 91). It does not contain any reference on the geometrical concepts (see [118] sec.106) Because this Lagrange function is the same for GRT and FGT, then the result is the same also:

$$\Omega_{\text{WS}} = \frac{3GM}{2c^2 R_o^2} \mathbf{n} \times \mathbf{V_o} + \frac{GJ}{c^2 R_o^3} (3\mathbf{n}(\mathbf{n} \cdot \mathbf{j}) - \mathbf{j}). \qquad (102)$$

Here $\mathbf{R_o}$ is the radius vector of the center of inertia of the gyroscope, $\mathbf{n} = \mathbf{R_o}/R_o$, $\mathbf{V_o}$ is the orbital velocity, $\mathbf{J}$ and M are the angular momentum and the mass of the central body, and $\mathbf{j} = \mathbf{J}/J$.

For a gyroscope orbiting the Earth over the poles this precession amounts to about 7"/year. Recent measurement of the precession effects by using the drag-free satellite Gravity Probe B [72] gave the value of the gyroscope precession which is the same in GRT and FGT and cannot distinguish between them.

The quadrupole gravitational radiation. The gravitational radiation is a natural consequence of the Field Gravity approach because relativistic gravitational field obeys the wave equation (41). In the weak field approximation by means of the usual retarded potentials solution (52) of the wave field equations one can infer ([21]) that a system of moving bodies will radiate energy in the form of tensor (spin 2) gravitational waves.

Let us consider the generation of gravitational wave by means of a system of gravitating bodies which have slow motion ($v << c$). In the wave zone ($R >> \lambda$), where the distance R to the field point much larger than the size b of the system ($R >> b$), the retarded potentials (52) can be presented in the form of $(\mathbf{n} \cdot \mathbf{r}')/c$ series

$$\psi^{ik}(\mathbf{r}, t) \approx -\frac{2G}{c^2 r} \int \hat{T}^{ik}(\mathbf{r}', t) d^3 r' - \frac{2G}{c^3 r} \frac{\partial}{\partial t} \int (\mathbf{n} \cdot \mathbf{r}') \hat{T}^{ik}(\mathbf{r}', t) d^3 r'$$
$$-\frac{G}{c^4 r} \frac{\partial^2}{\partial t^2} \int (\mathbf{n} \cdot \mathbf{r}')^2 \hat{T}^{ik}(\mathbf{r}', t) d^3 r' + \dots \quad (103)$$

The standard calculations, which takes into account the traceless character of the pure tensor free wave ϕ^{ik}, gives

$$\phi_{23} = \phi_{32} = -\frac{G}{3c^2 r} \ddot{D}_{23} , \quad \phi_{22} - \phi_{33} = -\frac{G}{3c^2 r} \left(\ddot{D}_{22} - \ddot{D}_{33} \right) , \quad (104)$$

where $D_{\alpha\beta} = \int \varrho (3x^\alpha x^\beta - r^2 \delta^{\alpha\beta} dV$ is the reduced tensor of quadrupole mass moment. So the tensor gravitational radiation (spin 2 field) is quadrupole (the third term in (103)).

According to the equation (51) the positive and localizable energy density of the tensor quadrupole gravitational wave is

$$T^{00}_{\{2\}} = \frac{G}{36\pi c^6 r^2} \left[\dddot{D}^2_{23} + \frac{1}{4} \left(\dddot{D}_{22} - \dddot{D}_{33} \right)^2 \right] \ \frac{\text{ergs}}{\text{cm}^3} . \quad (105)$$

The total radiation in all directions gives the quadrupole luminosity $cT^{00}_{\{2\}}$:

$$L^{\text{FG}}_{\{2\}} = \frac{G}{45c^5} \dddot{D}_{\alpha\beta}^{\ \ 2} \ \frac{\text{ergs}}{\text{sec}} . \quad (106)$$

Tensor gravitational waves in the frame of FGT are transversal and correspond to a particle with spin 2. The quadrupole luminosity (106) is identical to the corresponding formula in GRT. A binary system will lose orbital energy via quadrupole gravitational radiation with luminosity $L_{\{2\}} \approx 2 \times 10^{32} (M_1/M_\odot)^2 (M_2/M_\odot)^2 (M_1 + M_2/2M_\odot)(a/R_\odot)^{-5}$ ergs/sec. M_i is the mass of a component, a is the semi-major axis. [12]

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[12] It is important to note that to calculate the loss of energy (106) one should use in GRT an expression for the energy-momentum "pseudotensor" of the gravitational field, ill-defined in general relativity. This difficulty originated a long-time discussion about the reality of gravitational waves in GRT (Trautman 1966 [180]).

4.2 New FGT predictions different from GRT

The structure of the Newtonian force. As we derived above from the equation of motion (62) for the case of a test particle in the symmetric tensor potential ψ^{ik}, the Newtonian limit gives the usual Newtonian force as the sum of the attractive (spin 2 part) and the repulsive (spin 0 part) force (see eq. 79):

$$\mathbf{F}_N = (\mathbf{F}_{\{attr\}} + \mathbf{F}_{\{repuls\}}) = \frac{3}{2}\mathbf{F}_N - \frac{1}{2}\mathbf{F}_N \ . \qquad (107)$$

This new understanding of the Newtonian force and potential opens new ways for experiments on the nature of the gravitational interaction, e.g. to measure the scalar "antigravity" even in weak-field laboratory conditions. A change of balance between the scalar and tensor parts of the gravitational potential could in principle explain the (debated) gravity-shielding experiments with high-critical-temperature ceramic superconductors reported by Podkletnov & Nieminen[148] and Podkletnov[149]. Modanese[130] concluded that there is no convincing physical understanding of the experiments. Recently an analogous effect of a small change in the weight of a rotating superconducting disc was detected by Tajmar et al.[174].

Translational motion of rotating test body. The gravity force acting on a rotating test body was considered by Baryshev [19] in the frame of FGT. From Eq.(83) in the case of a gyroscope motion in a static spherically symmetric gravitational field it follows the expression for the elementary Poincaré force dF_P acting on each elementary mass dm of the gyroscope:

$$d\mathbf{F}_P = -\{(1 + \frac{3}{2}\frac{v^2}{c^2} + 4\frac{\varphi_N}{c^2})\boldsymbol{\nabla}\varphi_N - 3\frac{\mathbf{v}}{c}(\frac{\mathbf{v}}{c} \cdot \boldsymbol{\nabla}\varphi_N)\}dm \qquad (108)$$

For a rotating rigid body the total gravity force is the sum of elementary forces acting on elementary masses:

$$\mathbf{F}_P = \int d\mathbf{F}_P \qquad (109)$$

Taking into account that the velocity $\mathbf{v}$ of an element dm may be presented in the form

$$\mathbf{v} = \mathbf{V} + [\boldsymbol{\omega}\mathbf{r}] \qquad (110)$$

where $\mathbf{V}$ is the translational velocity of the body, $\boldsymbol{\omega}$ is the angular velocity, $\mathbf{r}$ is the radius vector of an element dm relative to its center of inertia, so that $\int \mathbf{r}dm = 0$.

Inserting Eq.(110) into Eq.(108) and Eq.(109) we get

$$\mathbf{F}_P = -M\{(1 + \frac{3}{2}\frac{V^2}{c^2} + 4\frac{\varphi_N}{c^2} + \frac{3}{2}\frac{I\omega^2}{Mc^2})\boldsymbol{\nabla}\varphi_N$$
$$-3\frac{\mathbf{V}}{c}(\frac{\mathbf{V}}{c} \cdot \boldsymbol{\nabla}\varphi_N) - \frac{3}{Mc^2}\int[\boldsymbol{\omega}\mathbf{r}]([\boldsymbol{\omega}\mathbf{r}] \cdot \boldsymbol{\nabla}\varphi_N)dm\} \qquad (111)$$

where $M = \int dm = M_0$ is the total rest mass of the body, I is its moment of inertia. Note that the assumption of rigid rotation of the test body Eq.(110)

is used by Landau & Lifshitz ([118],p.331) for a post-Newtonian derivation of rotational relativistic effects in GR. The non-rigidity of a body does not play an important role in our case.

The relativistic relation between force $\mathbf{F}$ acting on a body and momentum $\mathbf{p}$ of the body is:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = m_I \frac{d(\mathbf{V}/(1 - V^2/c^2))}{dt} \qquad (112)$$

where m_I is the inertial mass of the body, $\mathbf{V}$ is the translational velocity of the body. From this relation it follows that the 3-acceleration is given by:

$$\frac{d\mathbf{V}}{dt} = \frac{\sqrt{1 - V^2/c^2}}{m_I}(\mathbf{F} - \frac{\mathbf{V}}{c}(\frac{\mathbf{V}}{c} \cdot \mathbf{F})) \qquad (113)$$

The inertial mass m_I of the rigidly rotating test body may be found from the relation

$$m_I = \frac{1}{c^2} \int T^{00} d^3x = M_0 + E_{rot}/c^2 \qquad (114)$$

where T^{00} is the 00-component of the energy momentum tensor of the rotating body, M_0 is its rest mass, and the last equality is obtained by using Eq.(59) for energy momentum tensor of particles composed the body. For rigidly rotating ball $E_{rot} = (1/2)I\omega^2$. Note that in the general case of a self-gravitating macroscopic body the energy density is $T^{00} = T^{00}_{(m)} + T^{00}_{(int)} + T^{00}_{(g)}$ which gives also the correct contribution from classical potential energy (see Thirring [177]; Baryshev [14]).

Under the gravity force Eq.(111) the rotating body will get the 3-acceleration according to the general relation of Eq.(113) where the inertial mass is given by Eq.(114). Hence the acceleration may be written in the form:

$$\frac{d\mathbf{V}}{dt} = -(1 + \frac{V^2}{c^2} + 4\frac{\varphi_N}{c^2} + \frac{I\omega^2}{M_0 c^2})\boldsymbol{\nabla}\varphi_N$$
$$+4\frac{\mathbf{V}}{c}(\frac{\mathbf{V}}{c} \cdot \boldsymbol{\nabla}\varphi_N) + \frac{3}{M_0 c^2} \int [\boldsymbol{\omega}\mathbf{r}]([\boldsymbol{\omega}\mathbf{r}] \cdot \boldsymbol{\nabla}\varphi_N) dm \qquad (115)$$

The equation (115) of motion of a small rotating body having the angular velocity $\boldsymbol{\omega}$ and the rest mass M_0 around of the central mass M shows that the translational orbital velocity of the body will have additional perturbations due to its rotation. Note that one should also add conditions of energy and angular momentum conservation of the rotating body. The last term in Eq.(115) depends on the direction and value of the angular velocity $\boldsymbol{\omega}$ of the body and has an order of magnitude v_{rot}^2/c^2. In principle this effect may be measured in laboratory experiments and astronomical observations including Lunar Laser Ranging and pulsars in binary systems[19].

Testing the equivalence and effacing principles. Important conceptual problem in discussion of the equivalence principle (EP) is how to give proper relativistic definitions for inertial and gravitational masses without refer to the non-relativistic Newtonian equation of motion and without non-verifiable statements.

In the frame of general relativity the definition of the EP based on consideration of an inertial frame of Newtonian dynamics where the equation of motion of a test body in a Newtonian gravity field with the potential φ_N is

$$(\frac{d\mathbf{v}}{dt})_N = -\frac{m_G}{m_I}\boldsymbol{\nabla}\varphi_N \tag{116}$$

where m_I is the inertial mass and m_G is the gravitational (passive) mass of the body, $d\mathbf{v}/dt = \mathbf{a}$ is the Newtonian acceleration of the body under the action of the Newtonian gravity force $\mathbf{F}_N = -m_G\boldsymbol{\nabla}\varphi_N$.

The ratio

$$\frac{m_G}{m_I} = 1 + \eta \tag{117}$$

is not generally restricted by Newtonian mechanics, and the statement that $\eta = 0$ is called the weak equivalence principle. According to the Newtonian equation of motion (Eq.116) an extended body may play the role of the test particle and for $\eta = 0$ the acceleration under gravity force does not depend on the velocity and internal structure of the body.

From modern tests, which use bodies having different compositions the achieved precision in the inferred equality of the inertial and gravitational masses (m_I, m_G) is about 10^{-13}. Several new high-accuracy tests of the equivalence principle have been suggested in last years (Haugan & Lämmerzahl 2001; Bertolami, Paramos & Turyshev 2006).

Within the field gravity theory the basic concept is the universality of gravitational interaction (UGI), which is determined by the relativistic interaction Lagrangian (Eq.34), from which a certain form of equations of motion is derived and can be tested by experiment/observations. In FGT, according to the relativistic PN equation of motion (78) for test body in a static field, the 3-acceleration is

$$\frac{d\mathbf{V}}{dt} = -(\frac{m_0}{m_0})\left\{(1 + \frac{V^2}{c^2} + 4\frac{\varphi_N}{c^2}+)\boldsymbol{\nabla}\varphi_N + 4\frac{\mathbf{V}}{c}(\frac{\mathbf{V}}{c} \cdot \boldsymbol{\nabla}\varphi_N)\right\} \tag{118}$$

where $\mathbf{V}$ is the velocity of the body, φ_N is the Newtonian gravitational potential. In the right side of Eq.(118) the rest mass of the body m_0 is canceled due to the same energy-momentum tensor in both (interaction and free particle) Lagrangian. Hence gravitational acceleration

- *does not depend on the rest mass m_0 of the test body,*
- *does depend on its velocity $\mathbf{V}$ (both on value and direction) and on the value of the gravitational potential φ_N at the location of the particle.*

This means that there are different ways in relativistic regime to define the inertial m_I and the gravitational m_G masses.

For example, in the case of rotating test body, according to GRT equivalence principle the free fall acceleration of a body does not depend on its internal structure (effacing principle) and composition. Hence differently rotating bodies will have the same gravitational acceleration (if one neglects the tidal effect).

However in FGT according to equation of translational motion of rotating body (115) there is an orientation-dependent contribution in the free fall

acceleration. Hence rotating body can give new tests of possible violation of EP due to rotational $E_{rot} = (1/2)I\omega^2$ of the body at the level of E_{rot}/M_0c^2 which for radius R_0 and angular velocity ω is

$$\frac{E_{rot}}{Mc^2} \approx 2 \times 10^{-12}(\frac{R_0}{10cm})^2(\frac{\omega}{10^4 rad/sec})^2 \qquad (119)$$

The most straightforward application of Eq.(115) is to perform a "Galileo-2000" experiment (which is an improved 21st century version of the famous Stevinus-Grotius-Galileo experiment with freely falling bodies in the Earth's gravity field) just taking into account rotation of the bodies. Instead of the Newtonian equation of motion Eq.(116) in frame of FGT we have Eq.(115) and the motion of a rotating body differs from that of non-rotating one.

Indeed let us consider three balls on the top of a tower (like the 110-m Drop Tower of the Bremen University). The first ball is non-rotating and according to Eq.(115) its free fall acceleration is:

$$\mathbf{g}_1 = (\frac{d\mathbf{V}}{dt})_1 = -(1 - 3\frac{V^2}{c^2} + 4\frac{\varphi_N}{c^2})\boldsymbol{\nabla}\varphi_N \qquad (120)$$

Let the rotation axis of the second ball be parallel to the gravity force, i.e. $\boldsymbol{\omega}\|\boldsymbol{\nabla}\varphi_N$, hence its free fall acceleration is:

$$\mathbf{g}_2 = \mathbf{g}_1 \times (1 + \frac{2}{5}\frac{R_0^2\omega^2}{c^2}) \qquad (121)$$

where one takes into account that for a homogeneous ball with radius R_0 and mass M_0 the moment of inertia is $I = \frac{2}{5}M_0R_0^2$.

Let the rotation axis of the third ball be orthogonal to the gravity force, i.e. $\boldsymbol{\omega} \perp \boldsymbol{\nabla}\varphi_N$, hence its free fall acceleration is:

$$\mathbf{g}_3 = \mathbf{g}_1 \times (1 - \frac{1}{5}\frac{R_0^2\omega^2}{c^2}) \qquad (122)$$

Equations (120),(121),(122) imply that the considered three balls will reach the ground at different moments. True, the difference is very small, for example if the radius of the ball is $R = 10$ cm and its angular velocity $\omega = 10^3$ rad/sec, then the expected difference in the falling time from 110 m tower will be $\Delta t \approx (1/2)(\Delta g/g)t \approx 2.5 \times 10^{-13}$ sec.

For NASA's Gravity Probe-B experiment [72] with the drag-free gyroscope orbiting the Earth the expected in FG perturbation of the acceleration of the translational orbital motion of the gyroscope (having radius 2 cm and rotational speed about 80 Hz) is about $\delta g/g \propto 10^{-15}$, which is yet too small for detection the signal in this experiment.

Another type of laboratory experiment for direct testing the velocity dependence of the Poincaré gravity force (111) is to weigh rotating bodies. If two bodies are at the balance and at a moment they start to rotate with different orientations of the rotation axes then the balance will be violated and hence measured by a scale. The expected difference in forces is again about the value given by Eq.(119).

In these laboratory experiments there are no problem with choosing a co-ordinate system at all. The height of a tower and the moments of the contact of the rotating bodies with the ground, and also readings of a balance scales are directly measurable quantities. Hence the equation of motion Eq.(86) in the field gravity theory gives a uniquely defined value for these laboratory experiments. Note that at microscopic level the spin orientation dependence of the gravity force also should be tested. Materials composed with regularly oriented spins of particles or regularly directed internal motion of particles can have different free fall accelerations, which in principle may be tested by experiments.

Scalar and tensor gravitational radiation. Gravitational field equations (47, 48) describe the radiation of two types – pure tensor (traceless, spin 2) "gravitons" and scalar (trace of the tensor potential, spin 0) "levitons" [13].

In the frame of FGT the scalar wave generation can be calculated from the Eq.(53) for retarded potentials, which gives in the case of the wave zone approximation the following expression:

$$\psi(\mathbf{r},\,t) \approx \frac{2GM_0}{r} - \frac{2GE_k}{c^2 r} + \frac{2GM_0}{cr}\left(\mathbf{n}\cdot\dot{\mathbf{R}}\right) + \frac{G}{c^2 r}n_\alpha n_\beta \ddot{I}_{\alpha\beta} + \dots\,, \quad (123)$$

where $M_0 = \sum m_a$, $E_k = 1/2\sum m_a v_a^2$, $\mathbf{R} = \sum m_a\mathbf{r}_a/\sum m_a$, $I_{\alpha\beta} = \sum m_a x_a^\alpha x_a^\beta$. Taking derivative of (123) over time (at fixed point r) and excluding non-contributing terms, we get following equation for the time derivative of the scalar potential:

$$\dot{\psi}(\mathbf{r},\,t) = -\frac{2G\dot{E}_k}{c^2 r}\,. \quad (124)$$

It means that the scalar gravitational radiation is the second order monopole radiation, and there is no first order monopole, dipole and quadrupole scalar radiation. Using the expression (51) for the energy density in the scalar wave, we get

$$T_{\{0\}}^{00} = \frac{G\dot{E}_k^2}{8\pi c^6 r^2}\quad\frac{\text{ergs}}{\text{cm}^3}\,. \quad (125)$$

The energy flux is $cT_{\{0\}}^{00}$, so the additional loss of energy (in 4π steradian) due to the scalar monopole radiation [16] is

$$L_{\{0\}} = \frac{G}{2c^5}\dot{E}_k^2\quad\frac{\text{ergs}}{\text{sec}}\,. \quad (126)$$

so the scalar gravitational (actually "anti-gravitational") radiation has the same order $1/c^5$ as the tensor quadrupole radiation.

A test of the validity of the gravitational radiation formulae is offered by binary pulsar systems. For a binary system the loss of energy due to the pure tensor gravitational radiation is given by the quadrupole luminosity Eq.(106) (which is the same in FGT and GRT) $L_{(2)\text{FG}} = (G/45c^5)\,\dddot{D}_{\alpha\beta}^{\,2}$ (ergs/sec),

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[13] The name "leviton" was suggested by V.V. Sokolov for spin 0 scalar gravitons which corresponds to the repulsive force

where $D_{\alpha\beta}$ is the quadrupole moment of the system. Tensor gravitational wave in the frame of FGT is transversal and has localizable positive energy.

For a binary system the quadrupole luminosity is

$$< \dot{E} >_{\{2\}} = \frac{32G^4 m_1^2 m_2^2 (m_1 + m_2) \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)}{5c^5 a^5 \left(1 - e^2\right)^{7/2}} \quad \frac{\text{ergs}}{\text{sec}} , \qquad (127)$$

here m_1, m_2 are masses of the two stars, a is the semimajor axis and e is the eccentricity of the relative orbit.

For a binary star system the orbital additional energy loss via scalar waves (according to Eq.(126)) is

$$< \dot{E} >_{\{0\}} = \frac{G^4 m_1^2 m_2^2 (m_1 + m_2) \left(e^2 + \frac{1}{4}e^4\right)}{4c^5 a^5 \left(1 - e^2\right)^{7/2}} \quad \frac{\text{ergs}}{\text{sec}} . \qquad (128)$$

Hence the ratio of the scalar to tensor luminosity is

$$\frac{< \dot{E} >_{\{0\}}}{< \dot{E} >_{\{2\}}} = \frac{5}{128} \cdot \frac{\left(e^2 + \frac{1}{4}e^4\right)}{\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)}. \qquad (129)$$

The value of this ratio lies in the interval 0 - 1.1 % depending on the value of the eccentricity 'e', and for a circular orbit equals zero. However for a pulsating spherically symmetric body there is no quadrupole radiation and the scalar radiation becomes dominating.

According to Weisberg et al. 2003 [191], 2010 [192], 2016 [193] the observed rate of change of the orbital period was measured more and more precisely and now is $\dot{P}_b^{obs} = (-2.423 \pm 0.001)10^{-12}$ [193]. The GRT and FGT predict for the corresponding change of binary period due to positive energy loss in quadrupole gravitational waves the very precise value: $\dot{P}_b^{quad} = (-2.40263 \pm 0.00005)10^{-12}$ [193]. Hence the observed excess of energy loss (relative to spin-2 radiation) is $\Delta_{obs}^{\{2\}} = (+0.848 \pm 0.041)\%$.

The orbit of the binary pulsar PSR1913+16 has an eccentricity $e = 0.6171334(5)$, hence the expected additional energy loss due to scalar gravitational radiation (Eq.129) is $\Delta_{scalar} = 0.735\%$ [16]. Hence the remain observed excess relative to spin-2 plus spin-0 gravitational radiation is $\Delta_{obs}^{\{2\}+\{0\}} = (+0.113 \pm 0.041)\%$.

It has been shown by Damour & Taylor 1991 [51] that the observed rate of the orbital period change $\dot{P}_b^{obs}$ must take into account the kinematic "Galactic effect" of the relative acceleration of the pulsar and the Sun in the Galaxy. In the model of the planar circular motion of the Sun and the pulsar the Galactic contribution is given by the relation [51]:

$$\left(\frac{\dot{P}_b}{P_b}\right)^{Gal} = -\frac{V_0^2}{cR_0}\cos l - \frac{V_1^2}{cR_1}\left[\frac{R_0}{R_1}\left(\frac{d}{R_0} - \cos l\right)\right] + \mu^2\frac{d}{c} \qquad (130)$$

were V_0, V_1 are the circular velocities at the Sun's R_0 and the pulsar's R_1 positions in the Galaxy, $l = 49.97°$ is the pulsar's galactic longitude, μ is the

proper motion of the pulsar, d is the distance to the pulsar, c is the velocity of light.

However there is large uncertainty in the Galactic effect due to adopted model of the Sun-pulsar relative motion, adopted distance to the pulsar and errors in the proper motion of the pulsar. The distance d to the pulsar PSR1913+16 is a critical parameter in the calculation of the Galactic effect. Unfortunately, the line of sight to the pulsar passes through a complex region of our Galaxy, and one must be very careful, when derives the value of the distance to the pulsar from the dispersion measure. For example according to Weisberg et al. 2003 [191] the distance to the pulsar $d = 5.9 \pm 0.94$ kpc and the proper motion $\mu = 2.6 \pm 0.3$ mas/yr while Weisberg et al.2016 [193] adopted $d = 9.0 \pm 3$ kpc and the proper motion $\mu = 1.48 \pm 0.04$ mas/yr. So it is possible to choose such Galaxy model parameters which allow to compensate the observed excess of $\dot{P}_b^{obs}$.

In particular the distance to the pulsar PSR1913+16 requires further careful determination by using different methods. [14] A direct determination of the pulsar distance may be regarded as a test of fundamental physics, related to the nature of gravitation. Also distances to other binary pulsars will be crucial for gravity physics. According to [193] now the rate of orbital period change has been measured for other eight binary pulsars with accuracy about 5% so in the near future the scalar GW contribution will be tested more reliably.

Detection of GW signals by Advanced LIGO. Recently the detection of GW signals by Advanced LIGO antennas was announced [1], [2], [3]. This breakthrough discovery has opened new possibility for study the fundamental physics of the gravitational interaction.

The LIGO Scientific Collaboration and the Virgo Collaboration team presents the interpretation of the detected GW events GW150914, GW151226 and LVT151012 as the binary black hole merger signals with total masses up to 100 $M_\odot$ [3]. They used models of the waveform covering the inspiral, merger and ringdown phases based on combining post-Newtonian theory, the effective-one-body formalism and numerical relativity simulations. As a result they found that the observed GW signals corresponds to coalescence of the binaries with masses 36.2 and 29.1, 14.2 and 7.5, 23 and 13 (in $M_\odot$), at distances 420, 440 and 1000 (in Mpc) respectively.

However, in the frame of the GRT there is an important conceptual obstacle (Trautman 1966 [180]) which forbids the localization of the GW energy due to the pseudo-tensor character of the energy-momentum of the gravitational field in all metric gravity theories. According to Misner, Thorne and Wheeler 1973 [127] in §20.4.*"Why the energy of the gravitational field cannot be localized"* on p.467 they wrote: *"...gravitational energy... is not localizable. The equivalence principle forbids"*. And in §35.7.*"The stress-energy carried by a gravitational wave"* on p.955 they wrote: *"...the stress-energy carried by gravitational waves cannot be localized inside a wavelength"*. Hence one

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[14] If the distance to the pulsar PSR 1913+16 has the critical value $d = d_{crit} = 5.4$ kpc, then the second term in eq.(130) equals zero. For distances $d < d_{crit}$ this term even changes its sign.

can talk only about smeared-out amount of stress-energy within a region of several wavelengths size. But the LIGO gw-detectors (arm length 4 km) have localized the gw-energy by measuring the oscillating wave-form of the gw-signal well inside the gw-wavelength ($\lambda_{GW} \simeq 3000$ km).

In the FGT the energy-momentum of the gravitational field is the true tensor, so the GW energy is localizable and carries positive energy, which is given by Eq.(51). Hence the detection of the GW signals can be considered as a new confirmation of the Feynmans field gravitation theory, which is based on the fundamental concept of localizable positive energy of the gravitational field.

The radiation of the quadrupole GW in the FGT has the same value as in GRT, so the interpretation of the observed oscillating form of the aLIGO signals can be presented as the coalescence of the binary Relativistic Compact Objects (instead of black holes). But in FGT also the scalar radiation exists and this can be tested in the near future by means of modern GW detectors. The scalar gw-radiation arises from the spherical pulsations of the collapsing bodies (in massive core collapse SN) and also can be detected and identified by related optical SN explosion.

According to the eq.(51) the flux of the gw-energy in the flat monochromatic scalar GW is given by

$$S_{\{0\}} = cT_{\{0\}}^{00} = \frac{c^3}{32\pi G}(\dot{h})^2 \quad \frac{\text{erg}}{\text{sec cm}^2} \tag{131}$$

where $h = A/c^2$ is the dimensionless gravitational potential A of the wave. Let us consider the "standard" gw-puls introduced by Amaldi & Pizzella 1979 [5], which is a sinusoidal wave $A(t,x) = A_0 \cos(\omega t - kx)$ with amplitude A_0, frequency $\omega_0 = 2\pi\nu_0$ and duration τ_g. For the scalar GW, the amplitude $h_0 = A_0/c^2$ of the signal on the Earth due to the gw-puls that occurs at a distance r, with total energy E_{gw} is

$$h_0 = 1.95 \times 10^{-21} \left(\frac{100 Mpc}{r}\right) \left(\frac{10^2 Hz}{\nu_0}\right) \left(\frac{E_{gw}}{1M_\odot c^2}\right)^{1/2} \left(\frac{0.5s}{\tau_g}\right)^{1/2}. \tag{132}$$

Expected rate of gravitational wave signals from core collapse SN explosions and binary RCO coalescence within inhomogeneous Local Universe ($r \leq 100$ Mpc) was considered by Bayshev & Paturel 2001 [29]. Sensitivity $h \simeq 10^{-22}$ is enough for detection of such gw-events from the Virgo galaxy cluster and the Great Attractor.

Interaction of gravitational wave with gw-antenna has different physics in FGT and GRT ([16], [29]). If we substitute the expression for the scalar plane monochromatic gravitational wave

$$\psi_{\{0\}}^{ik} = A(t,x)\eta^{ik} = A_0 \cos(\omega t - kx)\eta^{ik} \tag{133}$$

into equation (80) and leave the main terms we get the following equations of motion of test particle in scalar wave

$$\frac{dv_y}{dt} = \frac{dv_z}{dt} = 0\,, \tag{134}$$

$$\frac{dv_x}{dt} = \frac{1}{4c}\frac{\partial \psi}{\partial t} \, . \tag{135}$$

and the equation for the work produced by the scalar wave ($i = 0$ in Eq.(80)):

$$\frac{dE_{kin}}{dt} = \frac{m_0}{4c}v_x\frac{\partial \psi}{\partial t} = -\frac{m_0 c^2}{32c^4}\,h_0^2\,\omega\,\sin(2\omega t + \alpha)\,, \tag{136}$$

the kinetic energy of the test particle in the scalar wave is

$$E_{kin}(t) = \frac{m_0 c^2}{64}\,h_0^2\,(1 + \cos(2\omega t + \alpha))\,. \tag{137}$$

Hence according to eqs.(134, 135, 136) the scalar wave accelerates the test particle and the gravitational force produces the work which change the kinetic energy of the particle.

Also the scalar wave is longitudinal and the test particle oscillates along the direction of the wave propagation around initial position with the velocity amplitude Δv and the distance amplitude Δx

$$\Delta v = \frac{cA}{c^2} \quad \text{and} \quad \Delta x = \frac{A}{kc^2} = \frac{\lambda A}{2\pi c^2}. \tag{138}$$

For two test particles at a distance $l_0 \ll \lambda$ along x-axis we get the dimensionless amplitude of oscillation in the form $\Delta l_0/l_0 = A/c^2 = h$.

It is important to note that scalar gravitational wave does not interact with the electromagnetic field because the interaction Lagrangian equals zero

$$\Lambda_{(int)} = -\frac{1}{c^2}\psi_{\{0\}}^{ik} T_{ik}^{(em)} = -\frac{1}{4c^2}\psi\,T_{(em)} = 0\,. \tag{139}$$

It means that detection of the scalar wave can be achieved by means of laser interferometric antenna, because the GW affects only the test masses and has no action on the photon beam. Also very important to know the position of the GW source on the sky for testing the longitudinal and transversal character of the GW.

However, up to now there is no optical-x-ray-gamma identification at the sky the probable sources of GW radiation though the astrophysical black hole candidates are the most bright sources of optical-Xray radiation (due to real astrophysical gas environment). As it was demonstrated many times in the history of astronomy (e.g. radio sources, x-ray sources, gamma bursts) it is impossible to get correct model of radiation process without identification of source at least in two different wave-bands – too many concurrent models are possible. In the near future the Advanced Virgo antenna will start to operate and the localization of the GW sources will achieve about 1 square degree, so optical identification of this violent astrophysical event will be possible with accuracy about 1 arcsec. Only after such identifications one can speak about correct understanding the GW events detected by gw-antennas.

This is why the suggested by LIGO group interpretation of the observed events as black holes coalescence is actually a preliminary model. For example instead of black holes it can be two orbiting relativistic compact objects (RCO) possible in FGT which emit gravitational waves during the late inspiral, merger and pulsations of the resulting RCO. The oscillating form of

the GW signal is possible within FGT as the inspiral, merger and amplitude decay during coalescence of binary RCO. Also oscillating profile is possible for the core pulsations during supernova collapse.

The riddle of Core Collapse Supernova explosions. The problem of supernova explosion is one of the most intriguing in modern relativistic astrophysics. Expected amplitudes and forms of gravitational wave (GW) signals from supernovae explosions detected on the Earth by gravitational antennas essentially depend on the adopted scenario of core-collapsed explosion of massive stars and relativistic gravity theory. This is why the expected detections of GW signals from SN will give for the first time experimental limits on possible theoretical models of gravitational collapse including the strong field regime and even quantum nature of the gravity force. For the estimates of the energy, frequency and duration of supernova GW emission one needs a realistic theory of SN explosion which can explain the observed ejection of massive envelope. Unfortunately, for the most interesting case of SNII explosion such a theory does not exist now, but there is a hope that future 3D supercomputer's calculations will bring a solution of the SNII explosion. However, as it was sadly noted by Paczynski 1999 [140] if there were no observations of SNII it would be impossible to predict them from the first principles.

Modern theories of the core collapse supernova are able to explain all stages of evolution of a massive star before and after the explosion. However, the theory of the explosion itself, which includes the relativistic stage of collapse where a relativistic gravity theory should be applied for the calculation of gravitational radiation, is still controversial and unable to explain the mechanism by which the accretion shock is revitalized into a supernova explosion (see discussion by Burrows 2013 [43], Imshennik 2010 [105]).

Burrows [43] in his review "Perspectives on Core-Collapse Supernova Theory" emphasized that one of the most important, yet frustrating, astronomical question is What is the mechanism of core-collapse supernova explosions? Fifty-years history of CCSN theory, which uses advanced hydrodynamics and shock physics, convection theory, radiative transfer, nuclear physics, neutrino physics, particle physics, statistical physics, thermodynamics and gravitational physics have not definitively answered that question. Intriguingly up to now there is no theoretical understanding how to extract such energy from relativistic collapse of the iron core and produce observed kinetic energy of the expanding stellar envelope [43], [61], [105], though there is a hope that future supercomputer calculations can resolve the problem of CCSN explosion in the frame of GRT.

According to the review [43] for all trustworthy models of the core-collapse SN (CCSN) the energy of the explosion is never higher than a few tenths of Bethe (1 Bethe = 10^{51} ergs), which is not enough to overcome the gravitational binding energy of the canonical neutron star mass $\sim 1.5 M_{sun}$. Many years theorists have been presented with a stalled accretion shock at a radius near $\sim 100 - 200$ km and have been trying to revive it (a review of the literature see [43], [105]). This bounce shock should be the CCSN explosion. However, both simple theory and detailed numerical simulations universally

indicate that due to neutrino burst and photodissociation of the in-falling nuclei debilitate the shock wave into accretion within ~ 5 milliseconds of bounce. What is more, if the shock is not revived and continues to accrete, all cores will collapse to black holes, which contradict observations of NS in SN remnants. Rapid rotation with magnetic fields (e.g. [38]) and 3D MGD simulations taking into account different instabilities need to be studied more carefully in future. The true model should explain also such observational properties of the CCSN as two stage collapse and simultaneous burst of gravitational waves [105] (as it was in the case of SN1987A [82]). However up to now though many different revival mechanisms were considered there is no successful model yet, because the problem of CCSN explosion exists at a very fundamental level.

A possibility to revive the bounce shock essentially depends on the gravity force acting within the pre-neutron star (pre-NS), where at least post-Newtonian relativistic gravity effects should be taken into account [186]. Within the field approach to gravitation besides the tensor (spin 2) waves there is the scalar (spin 0) ones, generated by the trace of the energy-momentum tensor of considered matter. Though in the field gravity theory, there is no detailed calculations of the relativistic stages of the core collapse, but, in principle, the repulsive scalar part of gravitational potential could could lead to revive the bounce shock. Also the released energy of the scalar GW may reach values of about one solar rest mass, with characteristic frequency $100 - 1000$ Hz and durations up to several seconds (Baryshev [18]; Baryshev & Paturel [29]).

According to FGT the general physical concepts of force, energy-momentum, energy-quanta are working as in other theories of fundamental physical interactions, so that gravity force and positive energy density of the gravitational field exist inside and outside a massive body. An important new element of the FGT is the principal role of the scalar part of the symmetric tensor field, which is its trace and actually present repulsive force, which was missed in Feynman's lectures on gravitation [76], [77].

The CCSN explosion within FGT has essentially different scenario than in GRT. Post-Newtonian equations of relativistic hydrodynamics in the frame of FGT were derived in Baryshev [14], according to which the gravity force essentially depends on the value and direction of the gas flow. For example according to PN equation of a test particle radial motion (88) there is a critical value of the radial velocity $v_{rad} \simeq c/\sqrt{3} \approx 0.577\,c$. For $v > v_{crit}$ the gravitational acceleration goes to zero. This gives possibility for pulsation of the inner core of the pre-NS star and formation of main explosion shock wave together with jet-like outflow along the rotation axis. The PN equations of the gas motion was derived in [14] from the conservation laws of the total energy-momentum for the system gas plus gravitational field found in the first iteration:

$$\left(T^{ik}_{(gas)} + T^{ik}_{(int)} + T^{ik}_{(g)} \right)_{,\,i} = 0 \,. \tag{140}$$

From Eq.140 we get

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v} = -(1 + \frac{v^2}{c^2} + 3\frac{\Phi}{c^2})\boldsymbol{\nabla}\Phi + 4\frac{\mathbf{v}}{c}(\frac{\mathbf{v}}{c} \cdot \boldsymbol{\nabla}\Phi)$$

$$+3\frac{\mathbf{v}}{c}\frac{\partial\Phi}{c\,\partial t} - 2\frac{\partial\mathbf{\Psi}}{c\,\partial t} + 2(\frac{\mathbf{v}}{c}\times rot\,\mathbf{\Psi}) - \frac{1}{\varrho_0}\frac{\mathbf{v}}{c}\frac{\partial p}{c\,\partial t}$$

$$-\frac{1}{\varrho_0}(1 - \frac{e+p}{\varrho_0 c^2} - \frac{v^2}{c^2} + \frac{\Phi}{c^2})\mathbf{\nabla}p\,, \quad (141)$$

For the static configuration the gas velocity $v = 0$ and the post-Newtonian equation of hydrostatic equilibrium of a spherically symmetric body in FGT will be:

$$\frac{dp}{dr} = -\frac{G(\varrho_0 + \delta\varrho)\,M_r^*}{r^2}\,, \quad (142)$$

where

$$\delta\varrho = \frac{e+p}{c^2} + 2\varrho_0\frac{\Phi}{c^2}\,, \quad (143)$$

$$M_r^* = \int_0^r 4\pi r'^2(\varrho_0 + \frac{e+3p}{c^2} + 2\frac{\varrho_0\Phi}{c^2} + 2\frac{(d\Phi/dr)^2}{8\pi G c^2})dr'\,. \quad (144)$$

The most important difference between equation Eq.(142) of hydrostatic equilibrium in FGT and the Tolman-Oppenheimer-Volkoff equation (Eq.25) in GRT, is that within FGT the relativistic gravity corrections lead to a decrease of the gravitating mass (and so gravitational force) relative to its Newtonian value (due to the negative value of the gravitational potential ($\Phi = \psi^{00} < 0$)). According to Eq.(142) a hydrostatic equilibrium is possible for any large mass. Another important prediction of the FGT is that the supermassive stars (suggested as a possible source of energy in quasars) are stable to small adiabatic pulsations ([15], [32]).

Core-collapse supernova explosions, gamma-bursts, neutrino and gravitational bursts have common origin. Hence a direct test of the strong gravity effects would be the detection of a gravity wave signal from the relativistic collapse. The absence of black holes in the FGT makes dramatic changes in the physics of supernova explosions. The collapse of the iron core of massive pre-supernovae stars will have a pulsation character and leads to long duration gravitational signals, comparable with neutrino signals and gamma ray bursts, i.e. several seconds. The relation of the gamma-ray-burst (GRB) phenomenon to relativistic core-collapse supernovae has become a generally accepted interpretation of the GRBs (Paczynski [140], Sokolov [165]). If the compact GRB model suggested by Sokolov et al. [168] obtains further confirmation, then there should be a correlation of the gamma-x-ray signal with neutrino and gravitational bursts.

Note, that the gravitational antenna GEOGRAV had observed a signal from SN1987A together with the neutrino signal observed by the Mont Blanc Underground Neutrino Observatory (Amaldi et al. 1987 [6]; Aglietta et al. 1987 [4], Imshennik 2010 [105], Galeotti & Pizzella 2016 [82]). This has been interpreted by Baryshev 1997 [18] as a possible detection of the scalar gravitational radiation (if the bar changes its length) from the spherical core-collapse of the supernova. Another possibility to explain the gw-signal from SN1987A in metallic bar antenna (fixed to the ground) is to take into account the relative difference in motion of free electrons and proton lattice under the action of the scalar GW.

An observational strategy to distinct between scalar and tensor gravitational waves by using sidereal time analysis was considered by Baryshev & Paturel 2001 [29]. There is an evidence for possible detections of gravitational signals by Nautilus and Explorer antennas (Astone et al. 2002 [9]). Though they was not confirmed by later observations (after "improving" sensitivity, which actually excluded resonance), it is needed to develop detectors of GW signals based on working principles compatible with FGT.

Relativistic compact objects instead of black holes. In the case of strong gravity the predictions of FGT and GRT diverge dramatically, mainly because of the positive localizable energy density of the gravitational field and the crucial role of the scalar potential component (trace of the symmetric tensor potential) generated by the trace of EMT of the gravitational field sources. The scalar field is attraction and only in combination with pure tensor part (which is attraction) gives the classical Newtonian gravitation.

In FGT there is no black holes, horizons and singularities, and no such limit as the Oppenheimer-Volkoff mass. This means that compact massive objects in binary star systems and active galactic nuclei are good candidates for testing GRT and FGT theories. According to FGT for a static weak field conditions the positive energy density of the gravitational field around an object with mass M and radius R is

$$\varepsilon_g = \frac{(\boldsymbol{\nabla}\varphi_{\text{N}})^2}{8\pi G} = \frac{G\,M^2}{8\pi r^4} \ \ \text{ergs/cm}^3 \ . \tag{145}$$

So around a neutron star there is a "cloud" of gravitational field with mass density

$$\varrho_g = 1.1 \times 10^{13} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{10\ km}{R}\right)^4 \text{g/cm}^3 \ . \tag{146}$$

It is positive, localizable, and does not depend on a choice of the coordinate system. On the surface of a neutron star the mass density of the gravity field is about the same as the mass density of the nuclear matter.

A very general mass-energy argument shows that in FGT there is the *limiting radius* of any self-gravitating body and there is no singularities. This argument is a precise analogue to that of the classical radius of electron. Indeed, the total mass-energy of the gravitational field existing around a body is given by

$$E_f = \int_{R_0}^{\infty} \frac{(\boldsymbol{\nabla}\varphi_{\text{N}})^2}{8\pi G} 4\pi r^2 dr = \frac{G\,M^2}{2R_0} \ . \tag{147}$$

This energy should be less than the rest mass-energy of the body, which includes the energy of the gravity field. From this condition it follows that:

$$E_f < Mc^2 \quad \Rightarrow \quad R_0 > \frac{G\,M}{2c^2} \ . \tag{148}$$

If one takes into account the non-linearity of the gravity field and the internal energy-part inside the object, then the value of the limiting radius further

increases, because "the energy of the field energy" should be added. As the *gravitational radius* R_g for any massive body in the field gravity we define the radius, where mass-energy of the gravitational field equals to half of its mass-energy measured at infinity,so:

$$R_g = \frac{G\,M}{c^2} = \frac{1}{2}R_{Sch} \ , \qquad (149)$$

As we have discussed in Introduction, very recent surprising observational fact is that the estimated radius of the inner edge (R_{in}) of the accretion disk around black hole candidates has sizes about $(1.2 - 1.4)R_g = (0.6 - 0.7)R_{Sch}$ (Fabian 2015 [73], Wilkins & Gallo 2015 [202], King et al.2013 [112]). This points to a suggestion, that instead of a Kerr BH rotating with velocity about 0.998c, we observe ordinary RCO having radius close to its limiting FGT value R_g (Eq.149).

Also VLBI observations, using submm wavelength Event Horizon Telescope (EHT), will have unique angular resolution which will achieve event-horizon-scale structure in the supermassive black hole candidate at the Galactic Centre (SgrA*) and M87. The first results of EHT observations at 1.3mm surprisingly has demonstrated that for the RCO in SgrA* there are no expected for BH the light ring at radius $5.2R_{Sch}$ (Doeleman 2008 [59]). Again this may points to a possibility the existence of limiting FGT RCO having finite gravity force at its surface which does not produce light rings. So in the frame of FGT there is prediction, that forthcoming EHT observations at 0.6mm will discover a combination of radiation from a central RCO, accretion disc and the origin of relativistic jet from the surface of the RCO (without black hole in the center).

Observations of the stellar mass BH candidates surprisingly discovered a preferred value of RCO mass about $7M_{sun}$ [166]. Intriguingly a quantum consideration of the macroscopic limiting high density quark-gluon bag gives self-gravitating configurations with preferred mass $6.7M_{sun}$ and radius 10 km [166]. So, quantum gravidynamics predicts two peaks in mass distribution of the stellar-mass relativistic compact objects: $1.4M_{sun}$ for neutron stars and $6.7M_{sun}$ for quark stars.

An important consequence of the positive energy density ε_g of gravitational field distributed around a body is the existence of the *limiting gravity force* for objects having limiting size $\sim R_g$. It can be shown [16], that for $\varepsilon_g > 0$ the solution of the corresponding Poisson equation $\Delta\varphi = +(\boldsymbol{\nabla}\varphi)^2/c^2$, gives for potential gradient and hence for the force, that

$$F_g = m\frac{d\varphi}{dr} = \frac{GmM}{r^2}\frac{1}{(1 + GM/rc^2)} \ , \qquad (150)$$

hence for $r \to R_g$ the gravity force decreases relative to its Newtonian value. And for the limiting size $r = R_g$ gravitational acceleration and gravitational force are restricted by:

$$g_{max} \leq \frac{c^4}{GM} = \frac{c^2}{R_g} \ , \ \ and \ \ F_{g(max)} \leq \frac{mc^4}{GM} = \frac{mc^2}{R_g} < \frac{c^4}{G} \ . \qquad (151)$$

where the last inequality is written for the case m = M . For negative energy density $\varepsilon_g < 0$ one would got infinite acceleration and force at finite radius $r = R_g$, as it happens in GRT. Importantly, in the frame of FGT the super-massive RCO have very small gravitational acceleration (and force for unite mass m) at their surface.

Other predictions for testing. Existing variants of quantum geometry predict violation of the equivalence principle, possible violation of the Lorentz invariance, and time-varying fundamental physical constants at such a level that their detection may be realistic in near future (Amelino-Camelia et al.[8]; Bertolami et al.[36]).

However, up to now increasingly strong limits have been derived on variations of fundamental constants (Uzan [184]; Pit'eva & Pit'ev [146]). Also first observations of sharp images of a very distant supernova did not confirm the predicted quantum structure of space-time at Planck scales (Ragazzoni et al. [152]). There is also no deflection from the Newtonian gravity law at small distances down to μm scales (Nesvizhevsky & Protasov [133]).

An evidence on the similarity of the gravity force to other physical forces was obtained in recent experiments by Nesvizhevsky et al. [134];[135]. Using freely falling ultra-cold neutrons they showed that the gravity force acts similarly to the usual electric force producing quantum energy levels for the micro-particles moving in the gravity field (Westphal at al. [194]).

5 Cosmology in GRT and FGT

Modern cosmology is considered as an extension of the laboratory physics to the whole observable Universe. In fact, cosmological model is a particular application of the gravitation theory to the infinite mass distribution. This is why observational cosmology is so important for testing possible approaches to the physics of gravitational interaction.

5.1 General principles of cosmology

Cosmology is a science on the infinite spatial matter distribution and its evolution in time. Cosmology as a physical science is based on observations, experiments and theoretical interpretations. Sandage 1995 [157] used the term "Practical Cosmology" to denote the study of the largest achievable scales of the Universe and the search for the world model which best describes it. Our understanding the Universe is growing with gradually deepening sample of its observable part which delivers possibilities for testing alternative hypotheses in the bases of cosmological models.

In the book Baryshev & Teerikorpi 2012 [32] the "practical cosmology" is presented as wider based than any specific cosmological model. This is because its methods are especially aimed at testing the initial assumptions and basic predictions of different world models.

Very important new aspect of cosmological physics is that we study a realization of infinite mass distribution — N-body system with $N = \infty$. Also

the time retardation effect should be taken into account in studies of the deep parts of the observable Universe.

In cosmology the usual lab suggestion about isolation of a local system does not valid at all, because there is no external empty space. In the infinite mass distribution all problems are internal and division on local and global physics should be studied carefully. New specific physical relativistic quantum effects can appear at cosmological distance and time scales.

For example the definition of an inertial reference frame in cosmology can be made on the hypothesis of exact cancelation of all external forces $\sum \mathbf{F}_i = 0$. A practical realization of such globally rest inertial frame can be based on the observation of the cosmic microwave background radiation isotropy.

Cosmology deals with a number of empirical facts among which one hopes to find fundamental laws. This process is complicated by great limitations and even under the paradigmatic grip of any current standard cosmology. One should distinguish between two kinds of cosmological laws:

 − experimentally measured empirical laws,
 − logically inferred theoretical laws.

The major empirical steps in modern cosmology are connected with advances in instrumentation during the 20th century. The logically inferred theoretical laws (theoretical interpretations) are made on the basis of an accepted cosmological model, e.g. the standard or an alternative cosmological model.

Three fundamental cosmological empirical laws were then unveiled:

 − the cosmological redshift-distance law $cz = Hr$,
 − the thermal law of isotropic cosmic background radiation $B_\nu(T)$,
 − the fractal power-law correlation of galaxy clustering $\Gamma(r) \sim r^{-\gamma}$.

The empirical laws, being based on repeatable observations, are independent of existing or future cosmological models. The theoretical laws are valid only in the frame of a specific model. Good examples are the empirical Hubble's redshift-distance ($z \propto r$) law and the corresponding theoretical linear velocity-distance ($V \propto r$) law within the Friedmann model.

5.2 Friedmann's homogeneous model as the basis of the SCM

Initial assumptions. The geometrical approach of general relativity leads to the Friedmann cosmological model, the frame for modern cosmological research. The expanding homogeneous universe explains all available data, though there are some paradoxes, which are discussed in the next section.

Nowadays the expanding Big Bang cosmological model is generally accepted as the standard cosmological model (SCM) for description of the structure and evolution of the physical Universe (Peebles [143], Weinberg [190], Baryshev & Teerikorpi [32]). SCM is based on the geometrical gravity theory (general relativity) and uses the description of all *physical processes in expanding space.* The fundamental assumptions of the SCM are:

- *General relativity can be applied to the whole Universe (g_{ik}; $\Re_{iklm}$; $T^{ik}_{(\mathrm{m+de})}$).*
- *Homogeneous and isotropic matter distribution in the expanding Universe ($\rho = \rho(t)$; $p = p(t)$; $g^{ik} = g^{ik}(t)$).*
- *Laboratory physics works in the expanding space.*
- *Inflation in the early universe is needed for explanation of the flatness, isotropy and initial conditions of large scale structure formation.*

Einstein's cosmological principle. The fundamental basic element of the SCM is the Einstein's Cosmological Principle, which states that **the universe is spatially homogeneous and isotropic at enough "large scales"**. The term "enough large scales" relates to the fact that the universe is obviously inhomogeneous at scales of galaxies and clusters of galaxies. [15] The hypothesis of homogeneity and isotropy of the matter distribution in space means that starting from certain scale r_{hom}, for all scales $r > r_{\mathrm{hom}}$ we can consider the total energy density $\varepsilon = \rho c^2$ and the total pressure p as a function of time only, i.e. $\varepsilon(r,t) = \varepsilon(t)$ and $p(r,t) = p(t)$. Here the total energy density and the total pressure are the sum of the energy densities for matter and dark energy : $\varepsilon = \varepsilon_{\mathrm{m}} + \varepsilon_{\mathrm{de}}$, and $p = p_{\mathrm{m}} + p_{\mathrm{de}}$.

An ideal fluid equation of state $p = \gamma \varrho c^2$ is usually considered for cosmological fluid, where usual matter and dark energy have following partial equations of state: $p_{\mathrm{m}} = \beta \varepsilon_{\mathrm{m}}$ with $0 \leq \beta \leq 1$, and $p_{\mathrm{de}} = w \varepsilon_{\mathrm{de}}$ with $-1 \leq w < 0$. Recently values $w < -1$ also were considered for description the "fantom" energy.

Expanding space paradigm. An important consequence of homogeneity and isotropy is that the line element $ds^2 = g_{ik}dx^i dx^k$ may be presented in the Robertson-Walker form:

$$ds^2 = c^2 dt^2 - S^2(t)d\chi^2 - S^2(t)I_k^2(\chi)(d\theta^2 + sin^2\theta d\phi^2)\,, \qquad (152)$$

where χ, θ, ϕ are the "spherical" comoving space coordinates, t is synchronous time coordinate, and $I_k(\chi) = (sin(\chi),\ \chi,\ sinh(\chi))$, corresponding to curvature constant values $k = (+1,\ 0,\ -1)$ respectively. $S(t)$ is the scale factor, which determines the time dependence of the metric.

The *expanding space* paradigm states that the proper (internal) metric distance r to a galaxy with fixed co-moving coordinate χ from the observer is given by relation $r(t) = S(t) \cdot \chi$ and increases with time t as the scale factor $S(t)$. Note that physical dimension of metric distance [r] = cm , hence, if physical dimension [S] = cm, then χ is the dimensionless comoving coordinate distance. In direct mathematical sense χ is the spherical angle and S(t) is the radius of the sphere (or pseudosphere) embedded in the 4-dimensional Euclidean space. It means that the "cm" (the measuring rod) itself is defined as unchangeable unit of length in the embedding 4-d Euclidean space.

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[15] There is more general Mandelbrot's Cosmological Principle which state the fractality of matter distribution together with isotropy. Fractal cosmological models can be build on the basis of MCP also in the frame of GRT.

It is important to point out that the hypothesis of homogeneity and isotropy of space implies that for a given galaxy the expansion (recession) velocity is proportional to distance (**exact linear velocity-distance relation for all RW metrics Eq. (152)**):

$$V_{exp}(r) = \frac{dr}{dt} = \frac{dS}{dt}\chi = \frac{dS}{dt} \cdot \frac{r}{S} = H(t)r = c\frac{r}{r_H} \qquad (153)$$

where $H = \dot{S}/S$ is the Hubble constant (also is a function of time) and $r_H = c/H(t)$ is the Hubble distance at the time t. Note that for $r > r_H$ one gets expansion velocity more than velocity of light $V_{exp}(r) > c$.

The dark energy as a matter. In SCM the dark energy is included in the Einstein's field equations in the form:

$$\Re^{ik} - \frac{1}{2}\,g^{ik}\,\Re = \frac{8\,\pi\,G}{c^4}\,(T^{ik}_{(\mathrm{m})} + T^{ik}_{(\mathrm{de})})\,, \qquad (154)$$

where $\Re^{ik}$ is the Ricci tensor, $T^{ik}_{(\mathrm{m})}$ is the energy-momentum tensor (EMT) of the matter, which includes all kinds of material substances, such as particles, fields, radiation, and $T^{ik}_{(\mathrm{de})}$ is the EMT of dark energy, in particular, the cosmological vacuum is described by $T^{ik}_{(\mathrm{vac})} = g^{ik}\Lambda$, where Λ is Einstein's cosmological constant. Usually $T^{ik}_{(\mathrm{m})}$ and $T^{ik}_{(\mathrm{de})}$ are considered as independent quantities, though there are models with interacting matter and dark energy [88]. Note that $T^{ik}_{(\mathrm{m})}$ does not contain the energy-momentum tensor of the gravity field itself, because gravitation in general relativity is a property of space and is not a material field.

A mathematical consequence of the field equations (Eq. (154)) is that the covariant divergence of the left side equals zero (due to Bianchi identity), so for the right side we also have

$$(T^{ik}_{(\mathrm{m})} + T^{ik}_{(\mathrm{de})})_{\,;\,i} = 0\,. \qquad (155)$$

The continuity equation (Eq. (155)) also gives the consistency relation with other equations.

Friedmann's equations. In comoving coordinates the total EMT has the form $T^i_k = diag(\varepsilon, -p, -p, -p)$ and for the case of unbounded homogeneous matter distribution given by metric Eq. (152), the Einstein's equations (Eq. (154)) are directly reduced to the Friedmann's equations (FLRW model). From the initial set of 16 equations we have only two independent equations for the (0,0) and (1,1) components, to which we must add the continuity equation (Eq. (155)) which has the form

$$3\dot{S}/S = -\dot{\varepsilon}/(\varepsilon + p).$$

Using the definition of the Hubble constant $H = \dot{S}/S$, the Friedmann's equations get the form:

$$H^2 - \frac{8\pi G}{3}\varrho = -\frac{kc^2}{S^2}\,, \quad \text{or} \quad 1 - \Omega = -\Omega_k\,, \qquad (156)$$

and

$$\ddot{S} = -\frac{4\pi G}{3}\left(\varrho + \frac{3p}{c^2}\right)S\,, \quad \text{or} \quad q = \frac{1}{2}\Omega\left(1 + \frac{3p}{\varrho c^2}\right)\,, \qquad (157)$$

where $\Omega = \varrho/\varrho_{crit}$, $\varrho_{crit} = 3H^2/8\pi G$, $\Omega_k = kc^2/S^2H^2$ $q = -\ddot{S}S/\dot{S}^2$, and Ω, p, ϱ are the total quantities, i.e. the sum of corresponding components for matter and dark energy.

Note that the Friedmann's equations Eq. (157, 156) in terms of the metric distance $r(t) = S(t)\cdot\chi$ get the exact Newtonian form:

$$\ddot{r} = -\frac{GM_{\mathrm{g}}(r)}{r^2}\,, \quad \text{and} \quad \frac{V_{\exp}^2}{2} - \frac{GM}{r} = \mathrm{const}\,, \qquad (158)$$

where $M_{\mathrm{g}}(r) = -\frac{4\pi G}{3}\left(\varrho + \frac{3p}{c^2}\right)r^3$ is the gravitating mass of a comoving ball with radius $r(t)$.

Solving the Friedmann's equations one finds the dependence on time the scale factor $S(t)$ or the metric distance $r(t)$, which is the mathematical presentation of the space expansion.

Fundamental conclusions of the SCM. There are many explained astrophysical phenomena in the frame of the SCM, such as cosmological redshift of distant objects, cosmic microwave background radiation, Big Bang nucleosynthesis of light elements, large scale structure formation, chemical composition of matter and other. The main observational conclusions of the SCM are:

- *Cosmological redshift $(1+z) = \lambda_0/(\lambda_1) = S_0/S_1$, and the linear velocity-distance relation $V_{exp} = H \times r$ is the consequence of the space expansion $r(t) = S(t) \times \chi$ of the homogeneous Universe.*
- *Cosmic microwave background radiation is the result of the photon gas cooling in the expanding space $T(z) = T_0(1 + z)$.*
- *Small anisotropy $\Delta T/T(\theta)$ of the CMBR is determined by the initial spectrum of density fluctuations which are the source of the large scale structure of the Universe.*
- *The physics of the expanding Universe is described by the LCDM model which predicts the following matter budget at present epoch: 70% of unobservable in lab dark energy, 25% unknown nonbaryonic cold dark matter, 5% ordinary matter . Visible galaxies contribution is less than 0.5% .*

Observational puzzles of the SCM. The mentioned above fundamental results of the SCM interpretations of the observational data rise new problems for the basis of the SCM. We emphasize here several such problems which were discussed recently in the literature.

- **Absurd Universe.** *The visible matter of the Universe, the part which we can actually observe, is a surprisingly small (about 0.5%) piece of the predicted matter content and this looks like an "Absurd Universe"[182]. What is more, about 95% of the cosmological matter density, which determine the dynamics of the whole Universe has unknown physical nature. Turner[183] emphasized that modern SCM predicts with high precision the values for dark energy and nonbaryonic cold dark matter, but "we have to make sense to all this".*

- ***The cosmological constant problem.*** *One of the most serious problem of the LCDM model is that the observed value of the cosmological constant Λ is about 120 orders of magnitude smaller than the expectation from the physical vacuum (as discussed by Weinberg[189] and Clifton et al.[47]). In fact the critical density of the $\Omega = 1$ universe is $\varrho_{crit} = 0.853 \times 10^{-29} g/cm^3$, while the Planck vacuum has $\varrho_{vac} \approx 10^{+94} g/cm^3$..*

- ***The cold dark matter crisis on galactic and subgalactic scales.*** *There are number of problems with predicting behavior of baryonic and nonbaryonic matter within galaxies. It was discussed by Kroupa [117] that there are discrepancies between observed and predicted galaxy density profiles (the cusp problem), small number of observed satellites galaxies (missing satellites problem), and observed tight correlation between dark matter and baryons in galaxies, which is not expected within LCDM galaxy formation theory.*

- ***The LCDM crisis at super-large scales.*** *The most recent observational facts which contradict the LCDM picture of the large scale structure formation, come from: the 2MASS, 2dF and SDSS galaxy redshift surveys (Sylos Labini [170]), problems with observations of baryon acoustic oscillations (Sylos Labini et al.[?]), existence of structures with sizes $\sim$ 400 Mpc/h in the local Universe (Gott et al.[85], Tully et al.[181])and $\sim$ 1000 Mpc/h structures in the spatial distribution of distant galaxies, quasars and gamma-ray bursts (Nabokov & Baryshev [132], Clowes et al.[48], Einasto et al.[63], Horvath et al.[98]), alternative interpretation of the shape of the CMBR correlation function (Lopez-Corredoira & Gabrielli [125]), lack of CMBR power at angular scales larger 60 degrees and correlation of CMBR quadrupole with ecliptic plain (Copi et al. [49]).*

Conceptual paradoxes of the SCM. The existence of the mentioned above observational puzzles in the SCM interpretations of the astrophysical data rises a question: Does the contemporary standard cosmological model present the ultimate physical picture of the Universe?

Contemporary fundamental assumptions in the basis of SCM have led us to the serious observational puzzles which stimulate to search for a new cosmology . As it was emphasized by Turner [183] for making new cosmology one has to answer a new set of questions and the future world model will reveal deep connection between fundamental physics and cosmology: *"There may even be some big surprises: time variation of the constants or a new theory of gravity that eliminates the need for dark matter and dark energy"* [183].

Intriguingly, besides the mentioned above observational puzzles there are several deep conceptual problems in the foundation of the SCM. Their solution could open the door to construction more firmly established future cosmology. Below we present several such conceptual difficulties/paradoxes of the SCM, which already have been discussed in the literature:

- ***Vacuum energy paradox:*** *in the framework of the Einstein's geometrical gravity theory (GRT) there is the paradox of too small value of the Lambda term, considered as the physical vacuum [189].*

- **1st Harrison's paradox (energy-momentum non-conservation):** *physics of space expansion contains such puzzling phenomena as continuous creation of vacuum and violation of energy-momentum conservation for matter in any comoving volume, including photon gas of cosmic background radiation [92], [26].*
- **2nd Harrison's paradox ("motion without motion"):** *the galaxy cosmological velocity is conceptually different from the galaxy peculiar velocity, in particular the cosmological redshift in expanding space is not the Doppler effect, but the Lemaitre effect applicable to a receding galaxy having velocity larger than the velocity of light (so cosmological redshift is a new physical phenomenon which includes the global gravitational cosmological redshift) [91], [93], [26].*
- **Hubble-deVaucouleurs' paradox:** *in the expanding space the linear Hubble law is the fundamental consequence of the assumed homogeneity, however modern observations reveal existence of strongly inhomogeneous (power-law correlated) large-scale galaxy distribution at interval of scales $1 \div 100$ Mpc, where the linear Hubble law is firmly established, i.e. just inside inhomogeneous global spatial galaxy distribution [34], [27].*

5.3 Fractal cosmological model in the frame of FGT

A Field Gravity Fractal (FGF) cosmological model was suggested by Baryshev 1981 [12] and further developed in Baryshev 2008d [25]. It is true that the Standard Cosmological Model (LCDM) has been developing more than 30 years by many physicists before it gets the modern form with many important results. However possible cosmological models in the frame of FGT have not been developed yet because of absence of published foundations of the FGT approach. It is too early to make detail comparison between SCM and FGF. The field gravity fractal cosmological model has now preliminary qualitative character, but it contains also several quantitative results. The modern status of FGF cosmology allows one to formulate the really crucial observational tests of those basic interpretations of fundamental cosmological facts, such as linearity of cosmological redshifts together with strong inhomogeneity of large scale spatial galaxy distribution at distances less than 400 Mpc ($z < 0.1$) where superclusters of galaxies exist.

The FGF model is based on the two assumptions:

- *the matter gravitational interaction is described by the Poincare-Feynman's field gravity theory,*
- *the actual baryonic matter distribution is described by a fractal density law with critical fractal dimension $D_{crit} = 2$.*

Within FGF framework a new qualitative picture of the Universe has emerged, with some quantitative results that may be tested by current and forthcoming observations. The field gravity theory allows one to consider infinite matter distribution in Minkowski space without the gravitational potential paradox. A global evolution of matter is possible without space expansion and initial singularity. Cosmological redshift could have global gravitational nature.

The energy-momentum tensor of the interaction plays the role of an effective cosmological Lambda-term.

Instead of assumed in SCM homogeneous non-baryonic dark matter and dark energy, the fractal distribution of *dark + luminous* baryonic matter from the scales of galactic halos up to the Hubble radius, with the fractal dimension of the total (luminous and dark) matter equals to $D = 2$, can explain the observed linear Hubble law as the global gravitational redshift.

Universal cosmological solution for infinite matter distribution. In the frame of GRT the weak-field approximation corresponds to the smallness of two quantities simultaneously: the gradient of gravitational potential $\nabla \varphi \to 0$ and the gravitational potential itself $|\varphi| \to 0$. However a specific feature of the field gravitation theory is that there is the case of a weak force (small gradient of the field) while $|\varphi| \to c^2/2$. This is what happens in the cosmological problem and we can obtain some quantitative results even from the post-Newtonian equations.

Let us consider the case of a static homogeneous ($\varrho = const$) dust-like cold ($p = 0, e = 0$) infinite matter distribution within infinite space. Using expressions for the post-Newtonian total EMT (Eqs.71,74) and taking into account the traceless of the field and interaction EMTs, we get from Eq.41 the equation for the $\psi^{00} = \varphi$ component in the form

$$\Delta \varphi = 4\pi G \left(\varrho + \frac{2}{c^2} \varrho \varphi + \frac{2}{8\pi G c^2} (\nabla \varphi)^2 \right) . \tag{159}$$

In our case the main terms in the right-hand side of this equation are the positive rest mass density ϱ and the negative interaction mass density $(2\varrho\varphi/c^2)$. The last term can be neglected, because for large mass the force goes to zero. Hence we have the simple equation

$$\Delta \varphi - \frac{8\pi G \varrho}{c^2} \varphi = 4\pi G \varrho . \tag{160}$$

Note that Eq.160 is equivalent to the Einstein's cosmological equation with Lmbda-term $\Lambda = 8\pi G \varrho / c^2$.

The cosmological solution of Eq.160 is

$$\varphi = -\frac{c^2}{2} \tag{161}$$

which means that the net force from the infinite mass distribution equals zero for any place in the Universe.

Cosmological global gravitational redshift. In the case of the finite ball having fractal matter distribution with $D = 2$, i.e. the rest mass density law is

$$\varrho = \frac{\varrho_0 r_0}{r} \tag{162}$$

the solution of Eq.160 inside the ball has the form

$$\frac{\varphi(x)}{c^2} = -\frac{1}{2} + \frac{I_1(4\sqrt{x})}{4\sqrt{x} I_0(4\sqrt{x})} \tag{163}$$

where $x = r/R_H$, and $R_H = c^2/(2\pi G\varrho_0 r_0)$ is the Hubble radius for the $D = 2$ fractal universe, where $\beta = \varrho_0 r_0 = const$ is the new fundamental fractal constant.

For distances $r \ll R_H$ the gravitational potential is a linear function of distance between a source and observer $\varphi(r) \propto r^1$ and the cosmological gravitational redshift $z_{cos-grav}(r) = \delta\varphi/c^2 = (\varphi(r) - \varphi(0))/c^2$ will be

$$z_{cos-grav} = \frac{2\pi G\varrho_0 r_0}{c^2} \ r = \frac{H_g}{c} \ r \qquad (164)$$

where $\delta\varphi$ is the gravitational potential difference between the surface (observer) and the center of the ball (source).

Why does the cosmological gravitational effect give the redshift? From the causality principle it follows that the event of emission of a photon (or a spherical wave) by the source, which marks the centre of the ball, must precede the event of detection of the photon by an observer. The latter event marks the spherical edge where all potential observers are situated after the transition time t = r/c. Therefore to calculate the cosmological gravitational shift within the cosmologically distributed matter one should cut a material ball with the center in the source and with the radius of the ball equal to the distance between the source and an observer. In this case the cosmological gravitational shift is towards red.

Note that in some discussions of the global cosmological gravitational shift [206] they put the observer to the center of the ball and hence get a blueshift instead of de Sitter's and Bondi's redshift. However, such a choice of the reference frame violates the causality in the process considered: the ball with the source on its surface has no causal relation to the emission of the photon.

Moreover the full explanation of the global gravitational redshift will be obtained only in the frame of future relativistic quantum field gravity theory. Indeed, from Eq.(164) one get the expression for the gravitational Hubble constant:

$$H_g = 2\pi\varrho_0 r_0 \ \frac{G}{c} \ , \qquad (165)$$

which can be viewed a production of the fundamental constants only. This is because for a structure with fractal dimension D = 2 the constant $\beta = \varrho_0 r_0$ may be actually viewed as a new fundamental physical constant which determining the value of the gravitational Hubble constant. If the value of the fractal constant is $\beta = 1/(2\pi) \, g/cm^2$ as it happens for an ordinary galaxy, where e.g. one can take $\varrho_0 = 5.2 \times 10^{-24} (g/cm^3$ and $r_0 = 10 \, kpc$, then $H_g = 2\pi\beta G/c = 68.7 (km/s)/Mpc$.

Intriguingly, for the fractal matter distribution with fractal dimension $D = 2$, the mass density – radius relation ($\varrho r \sim 1 \, g/cm^2$) looks universal: starting from the nuclear scales ($\varrho \sim 10^{12} g/cm^3$; $r \sim 10^{-12} cm$) continues at galactic scale ($\varrho \sim 10^{-24} g/cm^3$; $r \sim 10^{24} cm$) and holds up to the Hubble radius ($\varrho \sim 10^{-28} g/cm^3$; $r \sim 10^{28} cm$). So the universal linear gravitational redshift law within the fractal structure with D=2 would have deep roots in the fundamental physics and H_g can be expressed as a combination of fundamental constants of microphysics via expressions ϱ_0 and r_0 for nuclear

matter (h, c, m_p, m_e) (Baryshev & Raikov 1988 [14], Baryshev et al. [33]). So, to understand the global gravitational cosmological redshift one also requires to construct the *G-h-c* gravitation theory.

The total mass-radius relation. For distances $r << R_H$ the gravitating mass is given by the relation

$$M(r) = 2\pi\varrho_0 r_0\, r^2 = 4.8 \times 10^{11} M_{sun} \left(\frac{r}{10\,kpc}\right)^2 \;, \qquad (166)$$

The interesting coincidence that this mass is close to a total galaxy mass (including dark matter) within the radius r about 10 kpc, and also to the critical value of the total mass of the Universe within the Hubble radius $r = R_H$.

However an obstacle appears from estimation (Eq.166) of the gravitating mass. To produce the gravitational Hubble law on scales of about 10 Mpc the total mass within such balls should be $M(10\,Mpc) = 4.8\,10^{16}\,M_{sun}$. Such values much exceed the mass of the luminous matter and this is why the FGF model is compelled to assume that a sufficient amount of dark matter has the fractal distribution with $D = 2$. Also, to have sufficiently small fluctuations in the Hubble law in different directions around an observer the fractal should be a special class: isotropic with small lacunarity.

The observed distribution of luminous matter (galaxies) on scales from 10 kpc up to 100 Mpc is well approximated by a fractal distribution with $D = 2$ ([170], [32], [176]). This means that within the FGF model both dark and luminous matter is similarly distributed on these scales. The nature of the fractal dark matter has to be determined from future observations. Current restrictions on possible dark matter candidates leave room for cold dead stars, neutron and quark stars, Jupiters, planet size objects, asteroids and comets, Pfeniger's hydrogen cloudlets, and also macroscopic quark dust [107], [50].

For large distances $(r >> R_H)$ the total gravitating mass is $M(r) = (c^2/2G)\,r$ for both $D = 2$ and $D = 3$ fractal structures. For scales larger than R_H the fractal dimension of dark matter may become D=3, corresponding to a homogeneous distribution.

The evolution of the Universe. In Minkowski space-time filled by matter there is a special frame of reference, namely the one where the matter is at rest on the average relative to the cosmic background radiation. This frame of reference allows one to speak also about a universal time and the arrow of time is determined by the growth of the local entropy. Initial fluctuations in the homogeneous gas of primordial hydrogen exponentially grow into large scale structures according to the classical scenarios by Jeans 1929 [108] and Hoyle 1953 [99]. The fractal structure of matter distribution with $D = 2$ could naturally originate as the result of the evolution of the initial fluctuations within the explosion scenario (Schulman & Seiden 1986 [162]). The fractal structure with critical dimension $D_{crit} = 2$ is also preferred in the dynamical evolution of self-gravitating N-body system (Perdang 1990 [145]; de Vega et al., 1996 [56]; 1998 [57]).

Within the $D_{crit} = 2$ fractal structure the gravity force acting on a particle from other particles is constant because of $M \propto r^2 \Rightarrow F \propto M/r^2 \propto const$. The positive energy density of the gravity field within $D = 2$ fractal structure is also constant: $\varepsilon_g = T_g^{00} \propto (d\varphi/dr)^2 \propto const$. There is an interesting suggestion by Raikov & Orlov 2008 [153] that the Pioneers effect in solar system may be caused by these cosmological drag-force.

The time-scale of the structure evolution is determined by the characteristic Hubble time: $t_H \approx R_H/c \propto (\varrho_H)^{-1/2} \approx 10^{10} yrs$. The total evolution time of the Universe may be several orders of magnitude larger, which could be tested by observations at high redshifts and by numerical simulations of the large-scale structure and galaxy formation in static space but dynamically evolving matter.

According to the classical argument by Hoyle (1982 [101], 1991 [102]) the cosmic microwave background radiation could be a remnant of the evolution of stars because the CMBR energy density equals to the energy released by the nuclear reactions in stars of all generations during the Hubble time. The optical photons radiated by stars could be thermalized by scattering and gravitational deflections by structures of different masses and scales. The fractal dark matter is also a product of the process of stellar evolution and large scale structure formation. Hence in the frame of the FGF cosmological model all three phenomena - the cosmic background radiation, the fractal large scale structure, and the Hubble law, - could be consequences of a unique evolution process of the initially homogeneous cold hydrogen gas.

5.4 Crucial cosmological tests of alternative models

Philosophical, methodological and sociological aspects of the development of the science on the whole Universe was recently analyzed by Lopez-Corredoira [124], who emphasized the important role of alternative ideas in cosmology, though usually they have small funding in modern cosmological society. The mathematical and physical basis for the construction of alternative cosmological models was discussed by Baryshev & Teerikorpi [32].

For distinction between alternative cosmological models one should develop the crucial observational tests, which compare different predicted results for different models. For this it is very important to understand specific physical processes which characterized the key properties of a certain model.

Physics of space expansion. Mathematically space expansion is a continuous increasing with time of the distance $r(t)$ between galaxies. It is given by relation $r(t) = S(t) \cdot \chi$ where $S(t)$ is the scale factor from Eq. 152. But what does space expansion mean physically? And how can one test its reality?

Cosmological physics of the expanding space is essentially different from the lab physics and even contains deep paradoxes which should be studied carefully [24]. Physically, expansion of the universe means the continues creation of space together with physical vacuum. Real Universe is not homogeneous, it contains atoms, planets, stars, galaxies. In fact bounded physical objects like particles, atoms, stars and galaxies do not expands. So inside

these objects there is no space creation. This is why the creation of space is a new cosmological phenomenon, which has not been tested yet in physical laboratory.

The first puzzling feature of the space expansion physics is that the Friedmann's equations Eq. (157, 156), in terms of the metric distance $r(t) = S(t) \cdot \chi$, actually have the exact Newtonian form Eq.(158). So according to general relativity the dynamics of the whole universe is determined by the exact Newtonian acceleration and Newtonian kinetic plus potential energy conservation (here velocity of light c does not change the Newtonian character of the equations).

The second puzzling fact of the space expanding universe is that in the case of the equation of state $p = \gamma \varrho c^2$ the mass-energy of any local comoving ball (having radius $r(t)$) is changing with time as:

$$M_{\mathrm{g}}(r) = -\frac{4\pi G}{3}\left(1 + 3\gamma\right)\varrho r^3 \qquad \propto \qquad S^{-3\gamma}(t). \qquad (167)$$

For example, for photon gas $\gamma = 1/3$ and the initially hot radiation is cooling proportional to the scale factor $S(t)$.

In cosmology Eq.(167) gives us a possibility to calculate of how much the energy increases or decreases inside any finite comoving volume but it does not tell us where the energy comes from or where it goes. As Harrison emphasized: *"The conclusion, whether we like it or not, is obvious: energy in the universe is not conserved"* (Harrison [93]).

Another puzzling consequence of the Friedmann's equations Eq. (158) is that in exact general relativistic expansion dynamics of the universe there is no relativistic effects due to the velocity of a receding galaxy. The expansion velocity is larger than the velocity of light for distances larger than the Hubble distance: $V_{\mathrm{exp}} > c$ for $r > R_H$, where $R_H = c/H$ (see also Eq. (153)).

The nature of cosmological redshift. Hubble & Tolman 1935 [104] were the first who tried to constract an observational tests for testing the nature of the cosmological redshift. In the Sandage's list of the "23 astronomical problems" [157] the number fifteen (the first in the cosmological section) sounds intriguingly:*"Is the expansion real?"*.

In fact the literature on the SCM contains acute discussion on the nature of the cosmological redshift [32], subject which constantly produces "common big bang misconceptions" or the "expanding confusions". A summary of such discussions was done by Francis et al.[79] who confronts Rees & Weinberg claim: *how is it possible for space, which is utterly empty, to expand? How can nothing expand? The answer is: space does not expand. Cosmologists sometimes talk about expanding space, but they should know better*, with the state by Harrison[93]: *expansion redshifts are produced by the expansion of space between bodies that are stationary in space.*

In mathematical language within FLRW space expanding model the cosmological redshift is a new physical phenomenon where due to the expansion of space the wave stretching of the traveling photons occurs via the Lemaitre's equation $(1 + z) = \lambda_0/\lambda_1 = S_0/S_1$, which is different from the familiar in lab the Dopplers effect. One can also see this if one compares the relativistic

Doppler and cosmological FLRW velocity-redshift $V(z)$ relations. The relativistic Doppler relation has the form $V_{\mathrm{Dop}}(z) = c(2z + z^2)/(2 + 2z + z^2)$ and the velocity always less than c, while expanding space velocity V_{exp} can be arbitrary large[32].

Intriguingly in modern cosmology there is no direct observational testing of the reality of the space expansion (Sandage [157]). However it is important to note that on the verge of modern technology there are direct observational tests of the physical nature of the cosmological redshift. First crucial test of the reality of the space expansion was suggested by Sandage [156], who noted that the observed redshift of a distant object (e.g. quasar) in expanding space must be changing with time according to relation $dz/dt = (1 + z)H_0 - H(z)$. In terms of radial velocity, the predicted change $dv/dt \sim 1$ cm s^{-1}/yr. This may be within the reach of the future ELT telescope [119], [142].

Even within the Solar System there is a possibility to test the global expansion of the universe. According to recent papers by Kopeikin [114], [115] the equations of light propagation used currently by Space Navigation Centers for fitting range and Doppler-tracking observations of celestial bodies contain some terms of the cosmological origin that are proportional to the Hubble constant H_0. Such project as PHARAO may be an excellent candidate for measuring the effect of the global cosmological expansion within Solar System, which has a well-predicted frequency drift magnitude $\Delta\nu/\nu = 2H_0\Delta t \approx 4 \times 10^{-15}(H_0/70kms^{-1}Mpc^{-1})(\Delta t/10^3s)$, where H_0 is the Hubble constant Δt is the time of observations. In the case of the non-expanding Universe the frequency drift equals zero.

Fractality of large-scale galaxy distribution. Modern observations of the 3-dimensional galaxy distribution, obtained from huge redshift surveys (such as 2MASS, 2dF and SDSS), demonstrate (e.g. [171], [170], [172], [176], [27]) that at least for interval of scales $1 \div 100$ Mpc/h there is a power law relation between the average galaxy number density $n(R)$ and the radiuses of test spheres R, so that $n(R) \propto R^{-\gamma}$ (see reviews by Sylos Labini [170], Baryshev & Teerikorpi [32], Baryshev [27]). Such power law behavior is known as the **de Vaucouleurs law** [54], [55]. Note that the power law correlation function is the characteristic feature of the discrete stochastic fractal structures in physics (phase transitions, strange attractors, structure growth) and has clear mathematical presentation (e.g. Gabrielli et al.[81]).

The observed linearity of the **Hubble law** [103] in the local Universe was confirmed by modern studies based on Cepheid distances to local galaxies, supernova distances, Tully-Fisher distances and other distance indicators, which demonstrate that the linear Hubble law is well established within interval of scales $1 \div 100$ Mpc/h (Ekholm et al. [71], Karachentsev et al.[111], Sandage [158], Baryshev & Teerikorpi [32], Baryshev [27]). A puzzling conclusion is that the Hubble law, i.e. the strictly linear redshift-distance relation, is observed just inside strongly inhomogeneous galaxy distribution, i.e. deeply inside fractal structure at scales $1 \div 100$ Mpc/h.

This empirical fact, called **"Hubble-deVaucouleurs paradox"**, presents a profound challenge to the standard model where the homogeneity is the basic explanation of the Hubble law, and *"the connection between homogeneity*
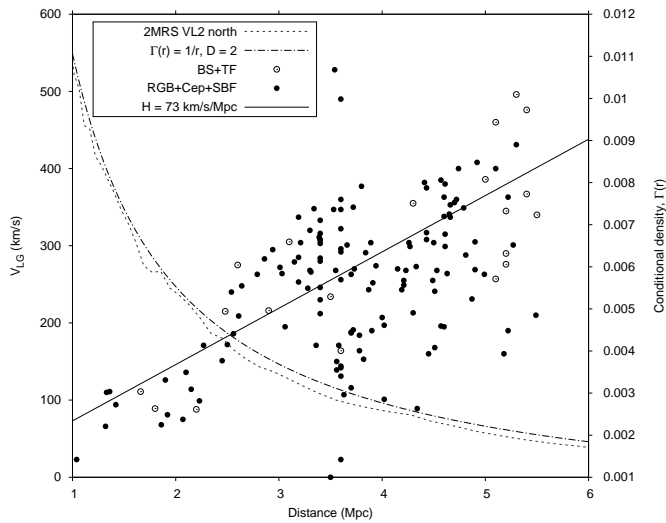
Fig. 1 Demonstration of the Hubble-deVaucouleurs paradox. The observed "apparent" radial velocity-distance relation (V_{app} vs R) for 156 Local Volume galaxies is shown from [111] (filled and empty dots corresponds to different methods of distance estimation). The theoretical linear Hubble law $V_{app} = cz = H_{loc}R$ (straight line) also is shown. The observed conditional density-radius relation (Γ vs r) for VL2N sample from 2MRS survey is shown by dash-line from [176]. The theoretical power-law $\Gamma(r) \propto r^{-\gamma}$ with exponent $\gamma = 1$ is shown (dash-and-dot line) at the same scales ($1 \div 6$ Mpc), which corresponds to the stochastic fractal structure having the critical fractal dimension $D = 2$. For the whole interval of scales $1 \div 100$ Mpc see [27].

and Hubble's law was the first success of the expanding world model" (Peebles et al.[144]).

However, contrary to this expectation, modern data show a good linear Hubble law even for very inhomogeneous spatial distribution of nearby galaxies (see Fig.1 and [27] for whole interval of scales $1 \div 100$ Mpc). It leads to a new conceptual puzzle that the linear Hubble law is not a consequence of the homogeneity of spatial galaxy distribution.

6 Conclusion

Detailed comparison of basic concepts and of experimental/observational effects for Einstein-Hilbert's geometrical general relativity (GRT) and alternative non-metric Poincare-Feynman field gravity theory (FGT) is presented. At the Post-Newtonian (PN) level all really measured classical relativistic gravity effects have the same values in both GRT and FGT approaches. However FGT also predicts, even in the weak field approximation, radically different from GRT effects, which can be tested by experiments/observations.

This opens possibility for formulation and performing conceptually new experiments/observations for testing the gravity physics based on the field

theoretical description of the gravitational interaction similarly to field theories of other fundamental interactions in Minkowski space. The most important difference is that according FGT the principle of universality of the gravitational interaction together with the total energy-momentum conservation and gauge invariance of the symmetric tensor gravitational potential lead to the force of gravity which is produced by the sum of the two real massless positive energy fields – the attraction due to traceless part of the symmetric tensor potential (spin 2 "tensor gravitons") plus the repulsion due to intrinsic scalar part corresponding to the trace of the symmetric tensor potential (spin 0 "scalar gravitons" — "levitons"). FGT theory predicts testable differences from GRT, such as: the translational motion of a rotating body, which may be tested by Lunar Laser Ranging and high accuracy orbit observations of pulsars in binary systems; the detection of the scalar gravitational radiation, generated by the trace of the energy-momentum tensor of the source which can be tested by forthcoming advanced LIGO-Virgo observations; the absence of singularities and horizons for relativistic compact objects (RCO), which can be tested by forthcoming EHT observations of the SgrA* and M87; the detection of quark stars and surface magnetic field of RCO; and the global gravitational nature of the cosmological redshift, which may be verified by the Sandage's dz/dt and Kopeikin's $\Delta\nu/\nu$ effects and other crucial observational tests of reality of systematic increasing distances between galaxies.

Much more observational work is needed, including future observations with LOFAR, SKA, ALMA and also other infrared, optical and x-gamma ray facilities, which will bring new tests for cosmological models both at small and large redshifts. These new crucial experiments/observations will be performed in the near future and can be used to distinguish between the alternative conceptual bases for the theory of the gravitational interaction and cosmology, which is important for development of the relativistic astrophysics.

References

1. Abbott B. et al.(LIGO Scientific Collaboration and Virgo Collaboration), Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Letters 116 , 061102 (2016)
2. Abbott B. et al.(LIGO Scientific Collaboration and Virgo Collaboration), GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence, Phys. Rev. Letters 116, 241103 (2016)
3. Abbott B. et al.(LIGO Scientific Collaboration and Virgo Collaboration), Binary Black Hole Mergers in the first Advanced LIGO Observing Run, arXiv:1606.04856 [gr-qc] (2016)
4. Aglietta M. et al., On the event observed in the Mont Blanc Underground Neutrino Observatory during the occurrence of supernova 1987a, Europhys. Letters 3 , 1315 (1987)

5. Amaldi E. & Pizzella G., The search for gravitational waves. in *Relativity, Quanta and Cosmology in the development of the scientific thought of Albert Einstein*, p. 241, Academic Press, p.241 (1979)

6. Amaldi E. et al., Data recorded by the Rome room temperature gravitational wave antenna, during the supernova SN 1987a in the Large Magellanic Cloud, Europhys. Lett. 3, 1325 (1987)

7. Amelino-Camelia G., Quantum theory's last challenge. Nature, 408, 661 (2000)

8. Amelino-Camelia G., Lammerzahl C., Macias A., Muller H., The search for quantum gravity signals. gr-qc/0501053 (2005)

9. Astone P. et al., Study of the coincidences between the gravitational wave detectors EXPLORER and NAUTILUS in the year 2001, Class. Quant. Grav. 19, 5449 (2002).

10. Babak S.V., Grishchuk L.P., The energy-momentum tensor for the gravitational field. Phys.Rev. D61, 024038 (2000)

11. Barnes K.J., Lagrangian theory for the second-rank-tensor field, J.Math.Phys., 6, 788 (1965)

12. Baryshev, Yu.V., The hierarchical structure of metagalaxy – a review of problems, Reports of Special Astrophysical Observatory of the Russian Academy of Sciences 14, 24 (1981) (English translation: 1984 Allerton Press)

13. Baryshev Yu.V., Equations of motion of test particles in Lorentz-covariant Tensor theory of gravity, Vest. Leningrad. Univ. ser.1, vyp.4, 113 (1986)

14. Baryshev, Yu. V., (1988) Conservation laws and equations of motion in the field gravitation theory, Vest. Leningrad. Univ. ser.1, vyp.2, 80 (1988)

15. Baryshev Yu.V., Pulsation of supermassive star in the tensor field Gravitation theory, in *Variability of Blazars*, Cambridge Univ.Press, 52 (1992)

16. Baryshev, Yu. V., On a possibility of scalar gravitational wave detection from the binary pulsar PSR1913+16, in the Proc. of the *First Amaldi Conference on Gravitational Wave Experiments*, eds. E.Coccia, G.Pizzella, F.Ronga, World Sci.Publ.Co., p. 251 (gr-qc/9911081)(1995)

17. Baryshev Yu.V., Field theory of gravitation: desire and reality, Gravitation, 2, 69 (gr-qc/9912003)(1996)

18. Baryshev Yu.V., Signals from SN1987A in Amaldi-Weber antennas as Possible detection of scalar gravitational waves, Astrophysics, 40, 377 (1997)

19. Baryshev Yu.V., Translational motion of rotating bodies and tests of the equivalence principle, Gravitation & Cosmology, 8, 232 (gr-qc/0010056) (2002)

20. Baryshev Yu.V., Generalized Nordtvedt effect and tests of the equivalence principle for rotating bodies, IAA Transections, vyp.8, Celestial Mechanics, SPb (2002)

21. Baryshev Yu.V., Spatial distribution of galaxies and tests of the relativistic cosmology, Dissertation on doctor of physical-mathematical sciences degree, Saint Petersburg State University, St.-Petersburg (in Russian)(2003)

22. Baryshev, Yu. V., Energy-momentum of the gravitational field: crucial point for gravitation physics and cosmology, in *"Practical Cosmology"*, Proceedings of the International Conference held at Russian Geographical Society, 23-27 June, 2008, Vol.1, p.276 (2008), (arXiv:0809.2323)

23. Baryshev, Yu. V., New possibilities for observational distinction between geometrical and field gravity theories, in *"Practical Cosmology"*, Proceedings of the International Conference held at Russian Geographical Society, 23-27 June, 2008, Vol.1, p.347 (2008), (arXiv:0809.2328)

24. Baryshev, Yu. V. Expanding space: the root of conceptual problems of the cosmological physics, in *"Practical Cosmology"*, Proceedings of the International Conference held at Russian Geographical Society, 23-27 June, 2008, Vol.2, p.20 (2008), (arXiv:0810.0153)

25. Baryshev, Yu. V. Field fractal cosmological model as an example of practical cosmology approach, in *"Practical Cosmology"*, Proceedings of the International Conference held at Russian Geographical Society, 23-27 June, 2008, Vol.2, p.60 (2008), (arXiv:0810.0162)

26. Baryshev, Yu. V. Paradoxes of the Cosmological Physics in the Beginning of the 21-st Century, invited report at XXX-th International Workshop on *High Energy Physics - Particle and Astroparticle Physics, Gravitation and Cosmology -*

Predictions, Observations and New Projects, June 23-27, 2014, Protvino, Moscow region, Russia (2015), (arXiv:1501.01919)

27. Baryshev, Yu. V., Two fundamental cosmological laws of the Local Universe, Proceedings of the International Conference *Cosmology on Small Scales*, Michal Krizek and Yurii Dumin (Eds.), Institute of Mathematics CAS, Prague, p.9 (2016) (arXiv: 1610.05943)

28. Baryshev, Yu. V., Kovalevski M.A. Homogeneous ball in the field gravitation theory, *Vestn. Leningrad. Univ., ser.1, vyp.1* 86 (1991)

29. Baryshev Yu.V., Paturel G., Statistics of the detection rates for tensor and scalar gravitational waves from the local galaxy Universe, Astron.Astrophys., 371, 378 (2001)

30. Baryshev Yu. V., Raikov A.A., Note on characteristic quantities in cosmology. Astrofizika, 28, 689 (in Russian) (1988)

31. Baryshev, Yu. & Teerikorpi, P., The fractal analysis of the large-scale galaxy distribution. Bull. Spec. Astrophys. Obs., 59, 92 (2006)

32. Baryshev Yurij and Teerikorpi Pekka, *Fundamental Questions of Practical Cosmology: Exploring the Realm of Galaxies*, Astrophysics and Space Science Library, Volume 383. ISBN 978-94-007-2378-8. Springer Science+Business Media B.V., pp.328 (2012)

33. Baryshev Yu.V., Raikov A.A., Sergeev A.G., Tron A.A., 1994, A new approach to the large cosmological numbers coincidences. Astron.Astrophys.Transactions 5, 27 (1994)

34. Baryshev, Yu., Sylos-Labini, F., Montuori, M., Pietronero, L., Facts and Ideas in Modern Cosmology, Vistas in Astronomy 38, 419 (1994)

35. Bauer H., On the energy components of the gravitational field. Phyz.Z., 19, 163 (1918)

36. Bertolami O., Paramos J., Turyshev S., General theory of relativity: will it survive the next decade? gr-qc/0602016 (2006)

37. Birkhoff G.D, Flat space-time and gravitation, Proc. Nat. Acad. Sci. U.S. 30, 324 (1944)

38. Bisnovatyi-Kogan G.S., S.G.Moiseenko, N.V.Ardeljan, Magnetorotational explosions of core-collapse supernovae, in Proc. Frascati Workshop 2013, Acta Polytechnica Supplement, arXiv:1408.2395 (2014)

39. Bogolubov N.N., Shirkov D.V., *Quantum Fields*. Nauka,Moscow (1976)

40. Bowler M. G., *Gravitation and relativity*, Pergamon Press, Oxford, 1976

41. Bronstein, M. D. Quantization of gravitational waves *Journ.Exp.Theor.Phys.*, 6, 195 (1936)

42. Brumberg V.A., *Essential relativistic celestial mechanics*, Adam Hildger IOP Publ. Ltd, N.Y. (1991)

43. Burrows A., Colloquium: Perspectives on core-collapse supernova theory, Rev. Mod. Phys., 85, 245 (2013)

44. Chapline G., Dark energy stars, astro-ph/0503200 (2005)

45. Chiao R., Conceptual tensions between quantum mechanics and general relativity: are there experimental consequences? gr-qc/0303100 (2003)

46. Chowdhury B. and Krauss L., Hawking Evaporation is Inconsistent with a Classical Event Horizon at r = 2M, arXiv: 1409.0187 (2014).

47. Clifton T. et al., Modified gravity and cosmology, Phys. Rep., 513, 1 (2012).

48. Clowes R. et al., A structure in the early Universe at z 1.3 that exceeds the homogeneity scale of the R-W concordance cosmology, Month. Not. Royal Astron. Soc. 429 , 2910 (2013).

49. Copi C. et al., Large-Angle Anomalies in the CMB, Advances in Astronomy, 2010, 847541 (2010).

50. Del Popolo A., Non-Baryonic Dark Matter in Cosmology, Int.J.Mod.Phys.,D23, N.3, 1430005 (2014)

51. Damour T., Taylor J., On the orbital period change of the binary pulsar PSR 1913+16. Astrophys. J., v. 366, 501, (1991).

52. Deser, S. Self-interaction and gauge invariance, *Gen. Rel. and Grav.*, **1**, 9 (1970)

53. de Sitter W., On Einstein's Theory of Gravitation and its Astronomical Consequences.I. MNRAS 76, 699 (1916)

54. de Vaucouleurs G., The case for a hierarchical cosmology, Science, 167, 1203 (1970)

55. de Vaucouleurs G., The large-scale distribution of galaxies and clusters of galaxies Publ. Astron. Soc. Pac., 83, 113 (1971)

56. De Vega, H., Sanches, N., Combes, F., Self-gravity as an explanation of the fractal structure of the interstellar medium, Nature 383, 56 (1996)

57. De Vega, H., Sanches, N., Combes, F., The fractal structure of the universe: A new field theory approach. Astrophys. J. 500, 8 (1998)

58. Doeleman S. et al., Imaging an Event Horizon: submm-VLBI of a Super Massive Black Hole, Science White Paper submitted to the ASTRO2010 Decadal Review Panels, arXiv:0906.3899 (2009)

59. Doeleman S. et al., Event-horizon-scale structure in the supermassive black hole candidate at the Galactic Centre, Nature, 455, 78 (2008) arXiv:0809.2442

60. Doeleman S. etal., Jet Launching Structure Resolved Near the Supermassive Black Hole in M87 Science, 338, 355 (2012) arXiv:1210.6132

61. Dolence J., A. Burrows, W. Zhang, Two-dimensional core-collapse supernova models with multi-dimensional transport, Ap.J, 800, id.10 (2015)

62. Dymnikova I., The cosmological term as a source of mass, Class. Quant. Grav., 19, 725 (2002)

63. Einasto M. et al., Tracing a high redshift cosmic web with quasar systems, Astron. Astrophys., 568, 46 (2014).

64. Einstein A., Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes, Ann. d. Phys., 35, 898 (1911)

65. Einstein, A., Die Feldgleichungen der Gravitation. Preuss. Akad. Wiss. Berlin, Sitzber., 844 (1915)

66. Einstein, A., Die Grundlagen der allgemeinen Relativitätstheorie. Ann. d. Phys., 49, 769 (1916)

67. Einstein, A. Kosmologiche Betrachtungen zur allgemeinen Relativitatstheorie, Sitzungsber. d. Berl. Alead., 1, 142 (1917)

68. Einstein A., Note on E.Schrodinger paper, Phys.Z., 19, 115 (1918)

69. Einstein, A. On a stationary system with spherical symmetry Consisting of many gravitating masses, Ann. of Phys., 40, 922 (1939)

70. Einstein, A. & Grossmann M., Entwerf einer verallgemeinerten Relativitats-theorie und einer Theorie der Gravitation, Zeit. Math. Phys., 62, 225 (1913)

71. Ekholm, T., Baryshev, Yu., Teerikorpi, P., Hanski,M., Paturel, G.: On the quiescence of the Hubble flow in the vicinity of the local group: A study using galaxies with distances from the Cepheid PL-relation. Astron. Astrophys. 368, L17 (2001)

72. Everitt C.W.F et al., Gravity Probe B: Final Results of a Space Experiment to Test General Relativity, Phys.Rev.Lett., 106, 221101 (2011)

73. Fabian A., The Innermost Extremes of Black Hole Accretion, arXiv: 1511.08596 (2015)

74. Falcke H., Markoff S., Toward the event horizon—the supermassive black hole in the Galactic Center, Class. Quant. Grav.,30, iss. 24, id. 244003 (2013) arXiv:1311.1841

75. Feynman R., Quantum theory of gravitation. Acta Phys. Pol., XXIV, 697 (1963)

76. Feynman R., *Lectures on Gravitation*, California Institute of Technology (1971)

77. Feynman R., Morinigo F., Wagner W., *Feynman Lectures on Gravitation*, Addison-Wesley Publ. Comp. (1995)

78. Fierz M., Pauli W., On relativistic wave equations for particles of arbitrary spin in an electromagnetic field, Proc. Roy. Soc. (London), A173, 211 (1939)

79. Francis M.J., Barnes L.A., James J.B., Lewis G.F., Expanding Space: the Root of all Evil? Publ. Astron. Soc. Australia, 24, 95 (2007) (astro-ph/0707.0380)

80. Fronsdal C., On the theory of higher spin fields, Il Nuovo Cimento, 9, 416 (1958)

81. Gabrielli A., Sylos Labini F., Joyce M., Pietronero L., *Statistical Physics for Cosmic Structures* , Springer, Berlin, (2005).

82. Galeotti P. & Pizzella G., New analysis for the correlation between gravitational wave and neutrino detectors during SN1987A, Eur. Phys. J., C 76, 426 (2016)

83. Gershtein, S. S.; Logunov, A. A.; Mestvirishvili, M. A. Hilbert's causality principle and the impossibility of gravitational collapse of a nonstatic spherical body, Doklady Physics, Volume 56, Issue 2, pp.65 (2011)

84. Ginzburg V. L. & Frolov V. P., Vacuum in a homogeneous gravitational field and excitation of a uniformly accelerated detector, Sov. Phys. Usp., 30, 1073 (1987)

85. Gott J. et al., A map of the universe, Astrophys. J., 624, 463 (2005).

86. Gravity Probe B: Testing Einsteins Universe, project homepage, Stanford University. URL (accessed 1 June 2014): http://einstein.stanford.edu/.

87. Grischuk, L.P., Petrov, A.N., Popova, A.D., Exact theory of the (Einstein) gravitational field in an arbitrary background space-time, Commun. Math. Phys., 94, 379 (1984)

88. Gromov A., Y. Baryshev, P. Teerikorpi, Two-fluid matter-quintessence FLRW models: Energy transfer and the equation of state of the universe, Astron. Astrophys., 415, 813 (2004).

89. Gupta S., Quantization of gravitational field. I. Proc. Phys. Soc. A65, 161 (1952a)

90. Gupta S., Quantization of gravitational field. II. Proc. Phys. Soc. A65, 608 (1952b)

91. Harrison, E. R., The redshift-distance and velocity-distance laws. ApJ 403, 28 (1993)

92. Harrison, E. R., Mining energy in an expanding universe. ApJ 446, 63-66, 1995

93. Harrison, E. R., *Cosmology: the science of the universe*, Cambridge University Press, 2nd edition (2000)

94. Hawking S., Information Preservation and Weather Forecasting for Black Holes arXiv:1401.5761 (2014)

95. Hawking S., The Information Paradox for Black Holes arXiv:1509.01147 (2015)

96. Hilbert D., Die Grundlagen der Physik, Konigl. Gesell. d. Wiss. Gottingen, Nachr. Math.-Phys. Kl. 395 (1915)

97. Hilbert D., Gottingen Nachrichten, **4**, 21 (1917)

98. Horvath I. et al., New data support the existence of the Hercules-Corona Borealis Great Wall, Astron. Astrophys., 584, A48 (2015)

99. Hoyle F., On the Fragmentation of Gas Clouds Into Galaxies and Stars, Ap.J., 118, 513 (1953)

100. Hoyle F., Fowler W., Burbidge G., Burbidge E., On Relativistic Astrophysics, Astrophys.J., 139, 909 (1964)

101. Hoyle F., The Universe: past and present reflections, Ann. Rev. Astron. Astrophys., 20, 1 (1982)

102. Hoyle F., In: Modern Cosmology in Retrospect, Eds. Bertotti, et al., p.221, Cambridge Univ. Press (1991)

103. Hubble E., A relation between distance and radial velocity among extra-galactic nebulae, Proceed.Nat.Acad.Sci., 15, 168-173 (1929)

104. Hubble E., Tolman R., C., Two methods of investigating the nature of the nebular red-shift. Astrophys. J., 82, 302-337 (1935)

105. Imshennik V.S., Rotational explosion mechanism for collapsing supernovae and the two-stage neutrino signal from supernova 1987A in the Large Magellanic Cloud, Physics - Uspekhi 53 (11), 1121 (2010)

106. Ivanenko, D. D., Sokolov, A., Quantum gravitation theory. *Trans. Moskov. University.*, N 8, 103 (1947)

107. Jacobs D., Starkman G., Lynn B., Macro Dark Matter, MNRAS, 450, 3418 (2015)

108. Jeans J., *Astronomy and Cosmogony*, Cambridge (1929)

109. Johannsen T. et al., Testing general relativity with the shadow size of Sgr A*, arXiv: 1512.02640 (2015)

110. Kalman G., Lagrangian formalism in relativistic dynamics Phys.Rev. 123, 384 (1961)

111. Karachentsev, I., Makarov, D.I., Sharina, M.E., et al.: Local galaxy flows within 5 Mpc. Astron. Astrophys. 398, 493 (2003)

112. King A., et al., What is on Tap? The Role of Spin in Compact Objects and Relativistic Jets Ap.J.,771, 84 (2013)

113. Kiselev, V. V.; Logunov, A. A.; Mestvirishvili, M. A., The physical inconsistency of the Schwarzschild and Kerr solutions Theoretical and Mathematical Physics, Volume 164, Issue 1, pp.972-975 (2010)

114. Kopeikin S., Celestial Ephemerides in an Expanding Universe, Phys. Rev. D, 86, 064004 (2012).

115. Kopeikin S., Local gravitational physics of the Hubble expansion, Eur. Phys. J. Plus (2015) 130, 11 (2015) (arXiv:1407.6667)

116. Kopeikin S., Efroimsky M., Kaplan G., *Relativistic Celestial Mechanics of the Solar System*, WILEY-VCH Verlag GmbH & Co. KGaA, Germany, (2011)

117. Kroupa P., Pawlowski M., Milgrom M., The Failures of the Standard Model of Cosmology Require a New Paradigm, Int. J. Mod. Phys. D 21 , 1230003 (2012).

118. Landau, L.D., Lifshitz, E.M., 1971, *The Classical Theory of Fields* (Pergamon, Oxford) (1971)

119. Liske, J., Grazian, A., Vanzella, E., et al., Cosmic dynamics in the era of extremely large telescopes, Mon. Not. R. Astron. Soc. 386, 1192 (2008)

120. Logunov A. A., Folomeshkin V.N., Problem of energy-momentum and gravity theory. Theor. Math. Phys., 32, 291 (1977)

121. Logunov A. A., Mestvirishvili M.A., *The Relativistic Theory of Gravitation.* Mir, Moscow (1989)

122. Logunov A. A., Mestvirishvili M.A., The causality principle in the field theory of gravitation. gr-qc/0106055 (2001)

123. Logunov A. A., Mestvirishvili M. A., Chugreev Yu. V., On incorrect formulations of the equivalence principle, Phys. Usp., 39 ,73 (1996)

124. Lopez-Corredoira M., Non-standard models and the sociology of cosmology, Studies in History and Philosophy of Modern Physics, 46-A, 86 (2014).

125. Lopez-Corredoira M. & Gabrielli A., Peaks in the CMBR power spectrum. I. Mathematical analysis of the associated real space structures, Physica A, 392, 474 (2013).

126. Mazur P., Mottola E., Gravitational vacuum condensate stars, Proc. Nat. Acad. Sci., 111, 9545 (2004)

127. Misner, C., Thorne, K., Wheeler, J. *Gravitation*. Freeman, San Francisco (1973)

128. Mitra A., On the Final State of Spherical Gravitational Collapse. Found.Phys.Lett., 15, 439 (2002)

129. Mitra A., Radiation pressure supported stars in Einstein gravity: eternally collapsing objects. MNRAS, 369, 492 (2006)

130. Modanese G., Gravitational anomalies by HTC superconductors: a 1999 theoretical status report. physics/9901011 (1999)

131. Moshinsky M., On the interacting Birkhoff's gravitational field with the electromagnetic and pair fields, Phys.Rev., 80, 514 (1950)

132. Nabokov N. & Baryshev Yu., Method for analyzing the spatial distribution of galaxies on gigaparsec scales. II. Application to a grid of the HUDF-FDF-COSMOS-HDF surveys, Astrophysics, 53, 101 (2010).

133. Nesvizhevsky V.V., Protasov K.V., Constrains on non-Newtonian gravity from the experiment on neutron quantum states in the Earth's gravitational field. Class. Quant. Grav., 21, 4557 (2004)

134. Nesvizhevsky V.V., Borner H.G., Petukhov A.K. et al., Quantum states of neutrons in the Earth's gravitational field. Nature, 415, 297 (2002)

135. Nesvizhevsky V.V., Petukhov A.K., Borner H.G., et al., Study of the neutron quantum states in the gravity field. Eur. Phys. J., C40, 479 (2005)

136. Noether E., Invariante Variationsprobleme, Nachr. v. d. Ges. d. Wiss. zu Gottingen,p.235 (1918)

137. Ogievetsky V.I., Polubarinov I.V., Interacting Field of Spin 2 and the Einstein Equations, Ann. Phys. 35, 167 (1965)

138. Okun L. B., Selivanov K. G., Telegdi V. L., Gravitation, photons, clocks, Phys. Usp. 42, 1045 (1999)

139. Okun L.B., Selivanov K.G., Telegdi V.L., 2000, On the interpretation of the redshift in a static gravitational field, Am.J.Phys., 68, 15 (2000) (physics/9907017)

140. Paczynski B., in the Proceedings of the Space Telescope Science Institute 1999 May Symposium (13): *Supernovae and Gamma Ray Bursts; The Largest Explosions Since the Big Bang*, p. 1; Eds.: M. Livio, N. Panagia and K. Sahu; Cambridge University Press, (2001) (astro-ph/9909048)

141. Padmanabhan T., From gravitons to gravity: myth and reality. gr-qc/0409089 (2004)

142. Pasquini L. et al., CODEX: measuring the expansion of the Universe. The Messenger 122, December 2005, p.10 (2005)

143. Peebles, P.J.E., *Principles of Physical Cosmology*, Princeton Univ.Press, (1993)

144. Peebles P. et al., The case for the relativistic hot big bang cosmology , Nature 352, 769 (1991).

145. Perdang, J., Self-gravitational fractal configuration, Vistas Astron. 33, 371 (1990)

146. Pit'eva E, Pit'ev N., Variation of the Solar mass and gravitational constant according to modern observations of planets and cosmic apparatus. Astron.Vestnik, (2011 in press)

147. Pitts J.B., Schieve W.C., Null cones in Lorentz-covariant general relativity. gr-qc/0111004 (2001)

148. Podkletnov E., Nieminen R., A possibility of gravitational force shielding by bulk YBa2Cu3O7-x superconductor. Physica C 203, 441 (1992)

149. Podkletnov E., Weak gravitational shielding properties of composite bulk YBa2Cu3O7-x superconductor below 70K under e.m. field. cond-mat/9701074 (1997)

150. Poincarè H., Sur la dynamique de l'èlectron. Comptes rendus de l'Acadèmie des Sciences 140, p.1504 (June 5)(1905)

151. Poincarè H., Sur la dynamique de l'èlectron. Rendiconti del Circolo matematico di Palermo 21, p.129 (1906)

152. Ragazzoni R. et al., Lack of observational evidence for quantum structure of space-time at Plank scales. Astrophys.J. 587, L1 (2003)

153. Raikov A.A. & Orlov V.V., On the cosmological nature of "Pioneers" anomalous acceleration, in *"Practical Cosmology"*, Proceedings of the International Conference held at Russian Geographical Society, 23-27 June, 2008, Vol.1, p.226 (2008)

154. Robertson S.L., Leiter D.J., The magnetospheric eternally collapsing object (MECO) model of galactic black hole candidates and active galactic nuclei, in *"New Developments in Black Hole Reseach"*, ed. P.V.Kreitler, Nova Science Publ., astro-ph/0602453 (2005)

155. Rovelli C., *Quantum Gravity*, Cambridge Monographs on Mathematical Physics, (Cambridge University Press, Cambridge; New York, (2004). [

156. Sandage, A., The change of redshift and apparent luminosity of galaxies due to the deceleration of the expanding universes. ApJ 136, 319 (1962)

157. Sandage A. , Astronomical problems for the next three decades. In *Key Problems in Astronomy and Astrophysics*, Mamaso A. and Munch G. eds., Cambridge University Press (1995)

158. Sandage A., Reindl B. and Tammann G., The Linearity of the Cosmic Expansion Field from 300 to 30,000 km s-1 and the Bulk Motion of the Local Supercluster with Respect to the Cosmic Microwave Background, Astrophys. J. 714, 1441 (2010).

159. Schrödinger E., The energy components of the gravitational field. Phys. Zeitschr., **19**, 4 (1918)

160. Schwinger J., *Particles, Sources and Fields*, Addison - Wesley, V. I, II (1973)

161. Schwinger J., Gravitons and photons: the methodological unification of the source theory. GRG, 7, 251 (1976)

162. Schulman, L.S., Seiden, P.E., Hierarchical structure in the distribution of galaxies, Astrophys. J., 311, 1 (1986)

163. Shirokov S. I. et al., Large-Scale Fluctuations in the Number Density of Galaxies in Independent Surveys of Deep Fields, Astronomy Reports, Vol. 60, No. 6, 563 (2016)

164. Sokolov V.V., Works on gravitation theory 1992 - 2011, http://w0.sao.ru/hq/grb/team/vvs/vvs.html (2011)
165. Sokolov V.V., The gamma-ray bursts and core-collapse supernovae - global star forming rate peaks at large redshifts, in Proceedings of the XXIX International Workshop on High Energy Physics: *New Results and Actual Problems in Particle & Astroparticle Physics and Cosmology* (HEPFT2013), 26-28 June 2013, Protvino, Moscow region, Russia, (2014)
166. Sokolov V.V., On the Observed Mass Distribution of Compact Stellar Remnants in Close Binary Systems and Localizability of Gravitational Energy, International Journal of Astronomy, Astrophysics and Space Science 2(6): 51-58 (2015).
167. Sokolov V.V., Baryshev Yu.v., Field-theoretical approach to gravitation: energy-momentum tensor of the field, Gravitation and Relativity Theory, Kazan State University, vyp.17, 34 (1980)
168. Sokolov V.V., Bisnovatyi-Kogan G.S., Kurt V.G., Gnedin Yu.N.,Baryshev Yu.V., Observational Constraints on the Angular and Spectral Distributions of Photons in Gamma-Ray Burst Sources, Astron.Rep., 50, 612 (2006)
169. Straumann N., Reflections on gravity. astro-ph/0006423 (2000)
170. Sylos Labini F., Inhomogeneous Universe, Class. Quant. Grav. 28 , 4003 (2011).
171. Sylos Labini, F., Montuori, M., Pietronero, L.: Scale-invariance of galaxy clustering. Phys. Rep. 293, 61 (1998)
172. Sylos Labini F., Tekhanovich D., and Baryshev Yu., Spatial density fluctuations and selection effects in galaxy redshift surveys, J. of Cosmol. and Astropart. Phys. 7, 035 (2014).
173. Szabados L.B., Quasi-Local Energy-Momentum and Angular Momentum in General Relativity, Living Rev. Relativity, 12, 4 (2009)
174. Tajmar M., Plesescu F., Marhold K., de Matos C.J., 2006, Experimental detection of the gravitomagnetic London moment. gr-qc/0603033 (2006)
175. Taylor J.H. et al., Experimental constraints on strong-field relativistic gravity. Nature 355, 132 (1992)
176. Tekhanovich D. I. & Baryshev Yu. V., Global Structure of the Local Universe according to 2MRS Survey, Astrophys. Bulletin, Vol. 71, No. 2, p. 155 (2016)
177. Thirring W. E., An alternative approach to the theory of gravitation. Ann. of Phys. 16, 96 (1961)
178. 't Hooft G., The conceptual basis of quantum field theory, in Handbook of the Philosophy of Science, Elsevier. December 23, (2004)
179. 't Hooft G., Singularities, horizons, firewalls, and local conformal symmetry, Plenary lecture presented at the *2nd Karl Schwarzschild Meeting on Gravitational Physics*, Frankfurt, July 23, (2015) arXiv:1511.04427
180. Trautman A., General Relativity Theory, Sov. Phys. Usp., 9(3), 319 (1966)
181. Tully R. et al., The Laniakea supercluster of galaxies, Nature, 513, 71 (2014).
182. Turner M., Absurd Universe, Astronomy, November 03, 44 (2003).
183. Turner M., Making sense of the new cosmology, Int. J. Mod. Phys. A, 17, 180 (2002).
184. Uzan J.-P., Varying constants, Gravitation and Cosmology, arXiv:1009.5514 (2010)
185. Unzicker A., Why do we Still Believe in Newton's Law ? Facts, Myths and Methods in Gravitational Physics, arXiv: gr/qc 0702009 (2007)
186. Wallace J., Burrows A., Dolence J., Detecting the Supernova Breakout Burst in Terrestrial Neutrino Detectors, ApJ, 817, 182 (2016)
187. Weinberg S., Photons and gravitons in perturbation theory: Derivation of Maxwell's and Einstein's equations, Phys.Rev., B138, 988 (1965)
188. Weinberg, S. E. *Gravitation and Cosmology*. Wiley, New York (1972)
189. Weinberg, S. E. The cosmological constant problem, Rev.Mod..Phys., 61, 1 (1989)
190. Weinberg S., *Cosmology*, Oxford University Press Inc., New York, (2008).
191. Weisberg J. M., Taylor J.H., The Relativistic Binary Pulsar B1913+16, *Radio Pulsars*, ASP Conference Proceedings, Vol. 302, Edited by Matthew Bailes, David J. Nice and Stephen E. Thorsett. San Francisco: Astronomical Society of the Pacific, p.93, (2003)

192. Weisberg J. M., Nice D.J., Taylor J.H., Timing measurements of the relativistic binary pulsar PSR B1913+16 Astrophys.J., 722, 1030 (2010)
193. Weisberg J. M., Huang Y., Relativistic Measurements from Timing the Binary Pulsar PSR B1913+16, Astrophys.J., June 1 (2016)
194. Westphal A., Abele H., Baebler S., et al., A quantum mechanical description of the experiment on the observation of gravitationally bound states. hep-ph/0602093 (2006)
195. Wilczek F., Quantum Field Theory, Rev.Mod.Phys., 71, S85 (1999)
196. Wilczek F., Origins of Mass, Cent. Euro. J. Phys., 10, 1021 (2012)
197. Wilczek F., Physics in 100 Years, (arXiv:1503.07735)
198. Wilczek F., Unification of Force and Substance, Talk at Royal Society Symposium, "Unifying Physics and Technology in the Light of Maxwell's Equations", November (2015), arXiv:1512.02094
199. Will C.M., 1993, *Theory and experiment in gravitational physics*, Cambridge University Press, (1993)
200. Will C.M., The confrontation between general relativity and experiment, Living Rev. Relativity 17, 4 (2014) arXiv:1403.7377
201. Williams J.G., Turyshev S.G., Boggs D.H., Lunar Laser Ranging Tests of the Equivalence Principle with the Earth and Moon, Int.J.Mod.Phys.D18:1129-1175,(2009)
202. Wilkins D. and Gallo L. Driving extreme variability: The evolving corona and evidence for jet launching in Markarian 335, MNRAS, 454, 4440 (2015)
203. Xulu S.S., The Energy-Momentum Problem in General Relativity, arXiv:hep-th/0308070 (2003)
204. Yilmaz H., Toward a field theory of gravitation, Nuovo Cimento, 107 B, 941 (1992)
205. Zakharov V.I., Spin of virtual gravitons, Zh. Eksp.& Teor. Fiz., 48, 303 (1965)
206. Zeldovich Ya.B., Novikov I.D., *Relativistic astrophysics*, Vol.1,2, Univ. Chicago Press, (1984)