Partially Localized Intersecting BPS Branes

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Abstract

We present the explicit forms of supergravity solutions for the various intersecting two BPS branes in eleven and ten dimensions, where one brane is localized at the delocalized other brane. Our partially localized supergravity solutions describe brane configurations in the near horizon region of the delocalized branes, where the two constituent branes meet in the overall transverse space and the delocalized branes coincide. We also give the brane worldvolume interpretations for some of such supergravity solutions.

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1 Introduction

Over the past years, there has been active development in constructing classical supergravity solutions of p-branes and other solitons in string theories. Such classical solutions in string theories made it possible to study quantum aspects of black holes such as statistical interpretation of black hole entropy and absorption and decay rates of black holes within the framework of string theories. In such studies, black holes are regarded as being obtained by wrapping (intersecting) higher-dimensional p-branes around compact manifolds. In the course of compactifying p-branes, the supergravity solutions become delocalized along the compactified directions [2], which include relative transverse directions (the transverse directions which are longitudinal to some of other constituent branes) and possibly some of overall transverse directions. So, the corresponding intersecting p-brane solutions in higher dimensions become localized only along the overall transverse directions. The most of intersecting brane solutions that have been constructed are such delocalized type. For the purpose of studying black hole physics in string theories, mostly it has been therefore sufficient to consider delocalized intersecting brane solutions.

In the recent AdS/CFT correspondence conjecture [1] and its generalizations (for example [2, 3, 4]), supergravity solutions in string theories also play important roles. In this conjecture, the decoupling limits of the worldvolume theories of brane configurations are dual to the supergravity or superstring theories on the manifolds of the near horizon geometry of the corresponding supergravity brane solutions. Thereby, one can understand gauge theories, which are the decoupling limits (where massive string modes, the Kaluza-Klein (KK) modes and gravity modes decouple from the massless modes of open strings) of the worldvolume theories of D-branes and other solitons, in terms of the supergravity or superstring theories on the near horizon manifolds, and vice versa. In the D-brane interpretation of gauge theories, the transverse locations of D-branes are interpreted as moduli of the gauge theories. For example, transverse locations of N numbers of D-branes, which are interpreted as scalar fields in the $U(N)$ gauge theories, parameterize the Coulomb branch. When more than one types of branes intersect, whereas locations of the "light" D-branes are dynamical moduli, locations of the "heavy" branes become couplings, i.e. mass of quarks, since the kinetic energy of their excitations is infinite, thereby being frozen at their classical values.

Therefore, if the AdS/CFT correspondence (or generally bulk/boundary holographic

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[2]When compactifying the supergravity solutions, one has to take an infinite uniform array of branes along the directions to be compactified in order to have an isometry necessary for the compactification. Also, by definition, compactification means the identification of the corresponding points in each cell of the compactification lattice, thereby the branes are periodically distributed along the compactified directions. Therefore, in the small size limit of the compactification manifold, the supergravity solutions become delocalized along the compactified directions.

correspondence) is correct, then all the parameters of the gauge theories, which are boundary theories at infinity of the AdS/CFT correspondence, have to be mapped one-to-one to the corresponding parameters of the corresponding supergravity solutions in the bulk, namely the locations of the constituent branes along the transverse directions. However, in delocalized intersecting p-brane solutions, locations of constituent branes along the worldvolume directions of the other branes are not specified, thereby not suitable for studying the bulk/boundary correspondence.

Some attempts have been made to construct localized intersecting brane solutions [5, 6, 7] with the restricted metric Ansatz which has the same form as the corresponding delocalized intersecting BPS brane solutions. Consistency of equations of motion along with such simplified metric Ansatz requires that one of the branes has to be delocalized on the relative transverse directions. Even with such simplified metric Ansatz, however, obtaining the "explicit" analytical form of localized solutions is almost an impossible task, since harmonic functions [3] that specify constituent branes now satisfy coupled partial differential equations instead of the Laplace's equations in the flat (overall) transverse space. Solutions to such differential equations in general do not have simple explicit form in terms of elementary functions. In the case of localized intersecting non-extreme brane solutions, even the metric Ansatz (in terms of harmonic functions) which would generalize such restricted metric Ansatz for the BPS case is not known, not to mention the explicit expressions for harmonic functions. "Partially" localized intersecting brane solutions, where constituent branes are localized along the relative transverse directions but delocalized along the overall transverse directions, have been constructed [8, 9, 10, 11]. However, these solutions satisfy different intersecting rules from the ordinary intersecting branes and therefore they are only useful for studying brane configurations with exotic intersecting rules.

However, in some special cases in the near-core limit of one of constituent branes, one can construct explicit solutions. The first attempt was made in Ref. [12], where the use was made of the fact that the near-core limit or the large charge limit of the KK monopole in $D = 11$ is $M^{(6,1)}$ times an A_{N-1} singularity, which can be obtained from the $D = 11$ flat spacetime by the Z_N identification. After the dimensional reduction, this A_{N-1} singularity metric becomes near-horizon limit of either $D6$-branes or the KK monopole in $D = 10$. Since an A_{N-1} singularity is Ricci flat (thereby satisfying the Einstein's equations), one can just replace the flat transverse space of M-branes by an A_{N-1} singularity. After the compactification down to $D = 10$, the resulting solutions are localized brane solutions in the core of either $D6$-branes or the KK monopoles. The other type of localized intersecting brane solutions that were explicitly constructed is

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[3]Harmonic function is defined as a solution $f(x_i)$ to the Laplace's equation $\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}g^{ij}\partial_j)f(x_i) = 0$, where $g \equiv \det(g_{ij})$. But in the following we will continue to call the solutions to such coupled differential equations as harmonic functions.

the $D(-1)$-brane solution in the core of $D3$-brane [13, 14, 15, 16]. In this case, it was possible to obtain the explicit form of the harmonic function for $D(-1)$-brane, since the equations of motion reduce to the Laplace's equation in the background of the near-horizon geometry of $D3$-branes, i.e. AdS_5 space, which is conformaly flat.

One may argue that other localized intersecting BPS brane solutions (in the core of one of the constituents) can be obtained by just applying duality transformations on the above localized solutions. However, in order to apply T-duality transformations in the transverse directions [4], one has to first compactify this direction, which becomes delocalized through smearing or uniform array of branes along this direction. Thereby, the power of radial coordinate in the harmonic function (of the above $D6$-brane, KK monopole and the $D3$-brane) changes. This implies that after the required delocalization along the T-duality direction the near-horizon geometries of $D6$-brane and $D = 10$ KK monopole do not get uplifted to an A_{N-1} singularity and the near-horizon geometry of $D3$-branes is no longer conformaly flat. So, the above tricks for constructing localized intersecting branes cannot be applied. Therefore, one has to construct such localized intersecting BPS brane solutions case by case. It is the purpose of this paper to construct various explicit partially localized intersecting BPS brane solutions in the core of the delocalized constituent in various dimensions. We will apply simple coordinate transformations to the differential equations satisfied by the harmonic functions in order to bring them to the forms of partial differential equations which have known explicit solutions.

The paper is organized as follows. In section 2, we setup the general formalism for constructing partially localized BPS intersecting brane solutions where one of the constituents is delocalized. We will apply this formalism to construct various explicit partially localized intersecting M-brane solutions in section 3, and various partially localized intersecting brane solutions in ten dimensions in section 4. Also, in sections 3 and 4, we discuss some worldvolume interpretations of such partially localized intersecting brane solutions for the purpose of illustrating possible usefulness of our solutions in studying the bulk/boundary holographic correspondence. Namely, although partially delocalized, our solutions still contain the corresponding parameters of the gauge theories on the "boundary", which delocalized intersecting brane solutions lack. In these sections, we identify the parameters of our supergravity solutions with the

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[4]The T-duality transformations on supergravity brane solutions along the longitudinal directions are not allowed, since these transformations introduce additional transverse directions, which the resulting transformed supergravity solutions have to depend on. One cannot arbitrarily let the solutions depend on this new coordinates through naive generalization of the form of the harmonic functions before the T-duality transformations, as the satisfaction of the field equations is not always guaranteed. In fact, the true "localized" harmonic functions, which satisfy the equations of motion, take different forms from the harmonic functions before the T-duality transformation on the longitudinal directions. This can be seen from the various intersecting brane solutions involving D-branes, which are presented in the following.

corresponding parameters in the gauge theories on the boundary.

2 General Setup

In this section, we setup general formalism, which can be applied to any types of intersecting branes in any dimensions, for obtaining the explicit expressions for harmonic functions.

In the BPS case, supergravity solutions for partially localized intersecting branes can be obtained with the same metric Ansätze and the same harmonic superposition rules as the delocalized cases. So, in the following, we shall assume the same forms of the metric Ansätze (which differ for different types of constituent branes and therefore will be given in sections 3 and 4 case by case) as the delocalized intersecting BPS branes. Schematically, in general the intersecting brane configuration is given by the following table. (This table is given also for the purpose of fixing the notations for the spacetime coordinates, which we shall follow in the following sections.)

	t	$\vec{w}$	$\vec{x}$	$\vec{y}$	$\vec{z}$
brane 1	●	●		●	
brane 2	●	●	●		

Here, t is the time coordinate, $\vec{w}$ is the possible overall longitudinal coordinate, $\vec{x} = (x_1, \ldots, x_p)$ [$\vec{y} = (y_1, \ldots, y_q)$] is the relative transverse coordinate for the brane 1 [the brane 2], and $\vec{z} = (z_1, \ldots, z_r)$ is the overall transverse coordinate. Generally, for any type of intersecting brane 1 (with the harmonic function $H_1 = H_1(\vec{x}, \vec{z})$) and brane 2 (with the harmonic function $H_2 = H_2(\vec{y}, \vec{z})$) in any dimensions with the above configuration, the harmonic functions satisfy the following coupled partial differential equations [5] [5, 6, 17, 12, 7]:

$$\partial_{\vec{z}}^2 H_1 + H_2 \partial_{\vec{x}}^2 H_1 = 0,$$
$$\partial_{\vec{z}}^2 H_2 + H_1 \partial_{\vec{y}}^2 H_2 = 0, \tag{1}$$

along with the constraint

$$\partial_{\vec{x}} H_1 \partial_{\vec{y}} H_2 = 0. \tag{2}$$

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[5]More precisely, the coordinates $\vec{z}$, $\vec{x}$ and $\vec{y}$ are the coordinates in which constituent branes are localized. Namely, when some of the coordinates are delocalized due to, for example, dimensional reduction, the harmonic functions H_1 and H_2 still satisfy the same coupled differential equations (1) but just do not depend on the delocalized coordinates. As will be seen in the following, in some cases it is necessary to delocalize the configuration along some of the overall transverse directions for the purpose of localizing one brane to the other. In such cases, $\vec{z}$ is the coordinate in the part of the overall transverse space where the intersecting branes are localized.

The constraint (2), i.e. $\partial_{\vec{x}} H_1 = 0$ or $\partial_{\vec{y}} H_2 = 0$, implies that either (i) the two branes are delocalized (localized only along the overall transverse directions) or (ii) while one brane is completely localized the other brane has to be localized along the overall transverse directions, only. So, the supergravity solutions with the above mentioned simplified metric Ansätze have such limited description of the microscope brane configurations on the boundary. However, this is not a disadvantageous situation for studying bulk/boundary correspondence, since the decoupling limit (where only the massless modes of open strings, which describe gauge theories, survive) of the brane worldvolume theories requires delocalization of some types of brane configurations. For example, for the configurations where one type of branes suspends between other type of branes, the distance between the latter type of branes has to approach zero so that the associated massive KK modes on the worldvolume theory of the former branes decouple from the massless open string modes. So, these directions, which are the relative transverse directions of the former branes, become delocalized [6].

Without loss of generality, for the sake of obtaining harmonic functions, we assume that the brane 2 is delocalized:

$$\partial_{\vec{z}}^2 H_1 + H_2 \partial_{\vec{x}}^2 H_1 = 0, \quad \partial_{\vec{z}}^2 H_2 = 0. \tag{3}$$

In general, the harmonic function H_2 that satisfies the second differential equation in (3) has the form $H_2 = 1 + \sum_i \frac{Q_i}{|\vec{z} - \vec{z}_{0\,i}|^{r-2}}$, where $\vec{z}_{0\,i}$ are locations of the i-th brane 2 with charge Q_i. However, in this paper we will consider the case in which the brane 2's coincide at the same location in the overall transverse $\vec{z}$-directions. This is also required as the decoupling limit of some types of brane configurations, as pointed out in the previous paragraph. Therefore, we choose the following form of the harmonic function H_2 in the near core region of the brane 2:

$$H_2 = \frac{Q}{z^{r-2}}, \tag{4}$$

where $z \equiv |\vec{z} - \vec{z}_0|$ and $\vec{z}_0$ is the location of the stack of N_2 brane 2's. There might exist solutions for the harmonic function H_1 in the case where each brane 2's are located at different points along the $\vec{z}$-directions. However, it may not be possible to find expression for H_1 in closed form in terms of elementary functions. Furthermore, naive inspection of the structure of the differential equation satisfied by H_1 seems to indicate that in general there does not exist a closed form of "localized" solution for H_1 in terms of elementary functions where brane 1's are located at arbitrary locations along the $\vec{z}$-directions away from the brane 2.

Then, the first differential equation in (3) reduces to the following form:

$$Q^{-1} z^{-1} \partial_z (z^{r-1} \partial_z H_1) + \partial_{\vec{x}}^2 H_1 = 0. \tag{5}$$

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[6]We assume here that the directions along which the latter type of branes stack are compactified with the distance between the stack of the branes being the size of the compactification manifold.

In the case where the harmonic function H_1 depends on its relative transverse coordinates $\vec{x}$ only through the radial coordinate $x \equiv |\vec{x}|$, this differential equation becomes of the following form:

$$Q^{-1}z^{-1}\partial_z(z^{r-1}\partial_z H_1) + x^{-p+1}\partial_x(x^{p-1}\partial_x H_1) = 0. \tag{6}$$

We will first obtain solution $H_1 = H_1(x, z)$ to the latter differential equation (6). This solution can be easily generalized as a general solution $H_1 = H_1(\vec{x}, z)$ to the former differential equation (5).

The differential equation (6) can be solved by transforming it to either of the following forms:

$$[X^{-a}(\partial_X X^a \partial_X) + Y^{-b}(\partial_Y Y^b \partial_Y)]F(X, Y) = 0, \tag{7}$$

$$[W^c \partial_W^2 + Z^d \partial_Z^2]G(W, Z) = 0, \tag{8}$$

where a, b, c and d are real numbers. These two partial differential equations are related through the following coordinate transformations:

$$\begin{cases} W = \left(\frac{2-c}{2}X\right)^{\frac{2}{2-c}} \\ Z = \left(\frac{2-d}{2}Y\right)^{\frac{2}{2-d}} \end{cases}, \qquad \begin{cases} X = (1-a)W^{\frac{1}{1-a}} \\ Y = (1-b)Z^{\frac{1}{1-b}} \end{cases}, \tag{9}$$

where the constants (a, b) and (c, d) are related as

$$a = \frac{c}{c-2}, \qquad b = \frac{d}{d-2}. \tag{10}$$

For a localized brane, the corresponding harmonic function has to be of the non-trivial form which is neither sum nor product of a function of X (or W) and a function of Y (or Z) [7]. Such non-trivial solutions of the partial differential equations (7) and (8) are respectively given by

$$F(X, Y) = 1 + \frac{P}{(X^2 + Y^2)^{\frac{a+b}{2}}}, \tag{11}$$

$$G(W, Z) = 1 + \frac{P}{[\frac{4}{(c-2)^2}W^{2-c} + \frac{4}{(d-2)^2}Z^{2-d}]^{\frac{cd-c-d}{(c-2)(d-2)}}}, \quad (c \neq 2 \neq d), \tag{12}$$

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[7] Obtaining the solutions to the partial differential equations (7) and (8) by applying the method of the additive or multiplicative separation of variables is very straightforward. However, when the harmonic functions are of such forms, either the supergravity solution does not match onto a delta-function brane source [6] or the point singularity of each term in the harmonic function represents the brane that is delocalized in the other directions. The possibility of getting solutions with the additive separation of variables were pointed out in Ref. [18]. Also, the expressions for harmonic functions in terms of the infinite series of special functions by applying the method of multiplicative separation of variables were obtained in Refs. [17, 19].

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where the integration constants in the constant terms have been set to 1 so that the harmonic functions take ordinary forms. One can solve the differential equation (6) by transforming it to the either of the forms (7) and (8).

The differential equation (6) can be put into the form (7) through the following change of variables:

$$x \to X = x, \quad z \to Y = \frac{2\sqrt{Q}}{|4 - r|} z^{\frac{4-r}{2}}, \tag{13}$$

resulting in the form (7) with $a = p - 1$ and $b = r/(4 - r)$. Or the equation (6) can be put into the form (8) by applying the following change of variables:

$$x \to W = \left(\frac{x}{2 - p}\right)^{2-p}, \quad z \to Z = \left[\frac{Q}{(2 - r)^2}\right]^{\frac{2-r}{4-r}} z^{2-r}, \tag{14}$$

resulting in the form (8) with $c = (2p - 2)/(p - 2)$ and $d = r/(r - 2)$. Therefore, the harmonic function $H_1 = H_1(x, z)$ that satisfies the differential equation (6) has the form:

$$H_1(x, z) = 1 + \frac{P}{\left[x^2 + \frac{4Q}{(4-r)^2} z^{4-r}\right]^{\frac{1}{2}(p-1+\frac{r}{4-r})}}. \tag{15}$$

The first transformation (13) already indicates that the harmonic function (15) for the brane 1 localized at the brane 2 is not valid when the dimensionality of the the overall transverse space is 4 ($r = 4$). Also, the second transformation (14) indicates that our method cannot be applied when the overall transverse space is two-dimensional ($r = 2$), in which case H_2 in Eq. (4) is logarithmic, and the solution (15) is not valid.

Note, as pointed out in the previous paragraph, when the dimensionality r of the overall transverse space with the coordinates $\vec{z}$ is 4, the coordinate transformations (13) and the expression (15) for the harmonic function H_1 are not valid. The only non-trivial solution, which is not a product of a function of x and a function of z, to the differential equation (6) with $r = 4$ that we have found so far has the form:

$$H_1(x, z) = 1 + P(x^2 - pQ \ln z), \tag{16}$$

although this looks quite extraordinary as a harmonic function associated with (localized) branes and this implies that the associated brane is delocalized. However, when the solution is smeared in one of the overall transverse directions $\vec{z}$, one can find more acceptable expression for the harmonic function H_1 from Eq. (15) (with $r = 3$), just as in Ref. [12], which represents brane 1's that are completely localized except for one delocalized overall transverse direction.

Also, for some other intersecting brane configurations to be discussed in the following sections, we note that the corresponding supergravity solutions become delocalized since the power in the harmonic function H_1 is positive instead of negative. This

happens when the overall transverse space has large enough dimensionality, i.e. when $r > 4$ as can be seen from Eq. (15). As can be in seen in the general expression for the harmonic function $H_1(x, z)$ in Eq. (15), which is valid for any type of brane (except for the case $r = 4$) in any spacetime dimensions, such delocalization depends on the dimensionality of the overall transverse space and possibly that of the relative transverse space of the brane 1, independently of the dimensionality of the overall longitudinal space. If one delocalizes some of the overall transverse directions, the power in the harmonic function H_1 becomes negative, thereby describing the brane 1 that are completely localized except for the delocalized overall transverse directions.

This seems to imply that, if there do not exist other class of localized solutions (where the brane 1 and the brane 2 meet in the overall transverse space), then the corresponding microscopic brane configurations have to be delocalized in the relative transverse directions [8] unless some of the overall transverse directions are delocalized. So, our delocalized intersecting brane solutions (with $r \geq 4$) correctly describe the corresponding delocalized microscope brane configurations on the boundary. On the other hand, this might be due to our choice of the simplified form of the metric Ansatz, which is the same form as the delocalized solutions. With more general form of metric Ansatz, there might exist completely localized intersecting brane solutions without any delocalized overall transverse directions.

In the case when some of the overall transverse directions are delocalized for the purpose of localizing the brane 1 at the brane 2, it is understood in the following sections that $\vec{z}$ in the harmonic functions is the coordinates for the part of the overall transverse directions where the brane configuration is localized.

It can be proven that the original differential equation (5) for the harmonic function $H_1 = H_1(\vec{x}, \vec{z})$ is solved by

$$H_1(\vec{x}, \vec{z}) = 1 + \sum_i \frac{P_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + \frac{4Q}{(4-r)^2}|\vec{z} - \vec{z}_0|^{4-r}]^{\frac{1}{2}(p-1+\frac{r}{4-r})}}, \tag{17}$$

where the i-th brane 1 with charge P_i is located at $(\vec{x}, \vec{z}) = (\vec{x}_{0\,i}, \vec{z}_0)$. Note, this general form of the "modified" harmonic function for the brane 1 is for the near horizon region ($|\vec{z} - \vec{z}_0| \approx 0$) of the brane 2 when the brane 1 and the brane 2 meet in the overall transverse space and when all the brane 2's coincide at $\vec{z}_0$.

Note, in the above expressions for harmonic functions H_1 and H_2, Q and P_i are just integration constants that result from solving the differential equations. When one wants to study the partially localized intersecting brane solutions in this paper within the frameworks of string or M theory, one has to express the constants Q and P_i in terms of the numbers N_1 and N_2 of the brane 1 and brane 2, and the charge quantization

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[8]This possibility was later studied in Ref. [20] by noticing such properties of the solutions presented in this paper.

constants (expressed in terms of the fundamental string scale l_s, the string coupling constant g_s, the eleven dimensional Planck scale l_p, etc.) It turns out that although the constant Q of H_2 is proportional to the number N_2 of brane 2's, the constant P of H_1 is related to the product N_1N_2 of the numbers N_1 and N_2 of brane 1's and brane 2's [9].

When more than two branes intersect, one can apply the above procedure with coupled differential equations with constraints that generalize (1) and (2). (For example, see Ref. [17].) For such general cases, one may have to transform the coupled partial differential equations to one of the following forms:

$$[\sum_i X_i^{-a_i}(\partial_{X_i}X_i^{a_i}\partial_{X_i})]F(X_i) = 0, \tag{18}$$

$$[\sum_i W_i^{c_i}\partial_{W_i}^2]G(W_i) = 0, \tag{19}$$

where a_i and c_i are real numbers. The solutions to these partial differential equations are respectively given by

$$F(X_i) = 1 + P/(\sum_i X_i^2)^{\frac{\sum_i a_i}{2}}, \tag{20}$$

$$G(W_i) = 1 + P/[\sum_i \frac{4}{(c_i - 2)^2}W_i^{2-c_i}]^{\sum_i \frac{c_i}{2(c_i-2)}}, \quad (c_i \neq 2). \tag{21}$$

In the following sections, we will present the explicit forms of every possible partially localized intersecting two brane solutions in eleven and ten dimensions in the case when the explicit form of the "modified" harmonic function can be obtained by applying the method discussed in this section. Localized intersecting brane solutions in other dimensions can be similarly constructed just by applying the procedure discussed in this section. All the possible delocalized intersecting brane configurations are studied in Refs. [21, 22], which we generalize to the localized case. In the following, we will just write down expressions for spacetime metric and the explicit forms of harmonic functions, since the expressions for other fields (dilaton and form fields) can be straightforwardly constructed by applying the same harmonic function superposition rules as the delocalized intersecting brane cases but now with new "localized" harmonic functions replaced.

3   Localized Intersecting M-branes

In eleven dimensions, the basic constituents of intersecting branes are $M2$- and $M5$-branes, which respectively carry electric and magnetic charges of the three-form field,

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[9]I would like to thank Y. Oz for pointing out this point.

the KK monopole and the pp-wave. In the following, we write down the spacetime metrics for all the possible combinations of intersecting pairs of these branes along with the explicit forms of the harmonic functions.

3.1 Intersecting $M2$- and $M5$-branes

There are 4 overall transverse directions ($r = 4$). The $M2$-branes have 4 relative transverse directions ($p = 4$), and the $M5$-branes have 1 relative transverse direction ($q = 1$). Since this configuration is interpreted as $M2$-branes ending on $M5$-branes, it is natural to let the solution be delocalized along the relative transverse direction of the $M5$-branes. Formally, one can however construct solution for the other case, as well. The spacetime metric has the following form:

$$
\begin{aligned}
ds^2_{11} &= H_2^{1/3} H_5^{2/3} [(H_2 H_5)^{-1}(-dt^2 + dw^2) + H_5^{-1}(dx_1^2 + \ldots + dx_4^2) \\
&\quad + H_2^{-1} dy^2 + dz_1^2 + \ldots + dz_4^2],
\end{aligned}
\tag{22}
$$

where the harmonic functions H_2 and H_5 are respectively associated with the $M2$- and $M5$-branes. For the purpose of obtaining more physically acceptable form of solution, we delocalize the solution along one of the overall transverse directions $\vec{z}$ [10]. Then, the harmonic functions are given by

$$
H_2 = 1 + \sum_i \frac{Q_i}{(|\vec{x} - \vec{x}_{0\,i}|^2 + 4P|\vec{z} - \vec{z}_0|)^3}, \qquad H_5 = \frac{P}{|\vec{z} - \vec{z}_0|}.
\tag{23}
$$

The effective worldvolume theory of the $M5$-brane is the $5 + 1$ dimensional $(2, 0)$ tensor multiplet containing 5 scalars, an anti-symmetric tensor with self-dual 3-form field strength, and 4 chiral fermions. The ends of $M2$-branes on the $M5$-branes are regarded as (self-dual) strings (in the $M5$-brane worldvolume theory) charged under this tensor. The charges Q_i and locations $\vec{x}_{0\,i}$ in the above supergravity solution are related to charges and locations of these strings in the worldvolume theory.

3.2 Intersecting two $M5$-branes

There are 3 overall transverse directions ($r = 3$) and 2 relative transverse directions for each $M5$-brane ($p = 2 = q$). So, in the core region of one of the $M5$-branes, the metric has the following form:

$$
\begin{aligned}
ds^2_{11} &= (H_1 H_2)^{2/3} [(H_1 H_2)^{-1}(-dt^2 + dw_1^2 + dw_2^2 + dw_3^2) \\
&\quad + H_2^{-1}(dx_1^2 + dx_2^2) + H_1^{-1}(dy_1^2 + dy_2^2) + dz_1^2 + dz_2^2 + dz_3^2],
\end{aligned}
\tag{24}
$$

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[10]When the solution is completely localized along all the overall transverse directions, the harmonic function takes an unacceptable form (16) with a logarithm

with the harmonic functions given by:

$$H_1 = 1 + \sum_i \frac{P_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + 4P|\vec{z} - \vec{z}_0|]^2}, \quad H_2 = \frac{P}{|\vec{z} - \vec{z}_0|}. \tag{25}$$

From the perspective of $M5$-brane worldvolume theory, the 3-dimensional intersection is $1/2$ supersymmetric 3-branes that carry the self-dual 3-form central charges of the worldvolume superalgebra of the $M5$-brane [23]. The parameters P_i and $\vec{x}_{0\,i}$ are related to the charges and the locations of these 3-branes in the $M5$-brane worldvolume theory.

3.3   Intersecting two $M2$-branes

In this case, there are 6 overall transverse directions ($r = 6$) and the dimensions of the relative transverse spaces of both of $M2$-branes are 2 ($p = 2 = q$). In the core region of one of the $M2$-branes, the solution has the following form:

$$\begin{aligned} ds_{11}^2 &= H_1^{1/3} H_2^{1/3} [-H_1^{-1} H_2^{-1} dt^2 + H_2^{-1}(dx_1^2 + dx_2^2) + H_1^{-1}(dy_1^2 + dy_2^2) \\ &\quad + dz_1^2 + \ldots + dz_6^2], \end{aligned} \tag{26}$$

where the harmonic functions are given by

$$H_1 = 1 + \sum_i Q_i [|\vec{x} - \vec{x}_{0\,i}|^2 + Q|\vec{z} - \vec{z}_0|^{-2}], \quad H_2 = \frac{Q}{|\vec{z} - \vec{z}_0|^4}. \tag{27}$$

When n of the overall transverse directions are delocalized, the harmonic functions take the following form:

$$H_1 = 1 + \sum_i \frac{Q_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + \frac{4Q}{(n-2)^2}|\vec{z} - \vec{z}_0|^{n-2}]^{\frac{2}{n-2}}}, \quad H_2 = \frac{Q}{|\vec{z} - \vec{z}_0|^{4-n}}. \tag{28}$$

So, when 3 overall transverse directions are delocalized, the harmonic function H_1, as well as H_2, takes a standard form where branes are localized except for the 3 delocalized overall transverse directions.

From the perspective of the $M2$-brane worldvolume theory, the $0 + 1$ dimensional intersection is interpreted as a 0-brane coupled to the zero form central charge in the worldvolume superalgebra. Such 0-brane is charged with respect to the Hodge-dual of a transverse scalar. The parameters $\vec{x}_{0\,i}$ and Q_i are related to the locations and charges of these worldvolume 0-branes.

3.4 $M2$-brane with the KK monopole

When the flat transverse space of the $M2$-brane is replaced by the Taub-NUT terms of the KK monopole, the spacetime metric takes the following form:

$$ds_{11}^2 = H_2^{-2/3} [-dt^2 + dw_1^2 + dw_2^2] + H_2^{1/3} [dx_1^2 + \ldots + dx_4^2]$$

$$+H_K(dz_1^2 + dz_2^2 + dz_3^2) + H_K^{-1}(dy + A_i dz_i)^2], \tag{29}$$

where the harmonic functions H_2 and H_K for the $M2$-brane and the KK monopole and a 1-form potential $\mathbf{A} = (A_i)$ satisfy the equations:

$$\partial_{\vec{z}}^2 H_K = 0, \quad \partial_{z_i} H_K = \epsilon_{ijk} \partial_{z_j} A_k, \quad \partial_{\vec{z}}^2 H_2 + H_K \partial_{\vec{x}}^2 H_2 = 0. \tag{30}$$

In the core region of the KK monopole or in the limit of large KK monopole charge, the harmonic functions are therefore given by:

$$H_K = \frac{Q_{KK}}{|\vec{z} - \vec{z}_0|}, \quad \mathbf{A} = Q_{KK} \cos\theta d\phi,$$

$$H_2 = 1 + \sum_i \frac{Q_i}{(|\vec{x} - \vec{x}_{0\,i}|^2 + 4Q_{KK}|\vec{z} - \vec{z}_0|)^3}. \tag{31}$$

This solution reproduces the one in Ref. [12].

The 0-form central charges of the $M2$-brane worldvolume superalgebra are equivalent to the space components of the dual 2-forms, which are carried by a worldvolume 2-branes. This 2-brane is interpreted as the intersection of the $M2$-brane with the KK monopole [23]. The locations and the charges of these worldvolume 2-branes are related to $\vec{x}_{0\,i}$ and Q_i of the above supergravity solutions.

3.5 $M5$-brane with the KK monopole

When the flat transverse space of the $M5$-brane is replaced by the Taub-NUT terms of the KK monopole, the spacetime metric takes the following form:

$$\begin{aligned} ds_{11}^2 &= H_5^{-1/3}[-dt^2 + dw_1^2 + \ldots + dw_5^2] + H_5^{2/3}[dx^2 \\ &\quad + H_K(dz_1^2 + dz_2^2 + dz_3^2) + H_K^{-1}(dy + A_i dz_i)^2], \end{aligned} \tag{32}$$

where the harmonic functions H_5 and H_K for the $M5$-branes and the KK monopole and a 1-form potential $\mathbf{A} = (A_i)$ satisfy the equations:

$$\partial_{\vec{z}}^2 H_K = 0, \quad \partial_{z_i} H_K = \epsilon_{ijk} \partial_{z_j} A_k, \quad \partial_{\vec{z}}^2 H_5 + H_K \partial_x^2 H_5 = 0. \tag{33}$$

In the core region of the KK monopole or in the limit of large KK monopole charge, the harmonic functions are therefore given by:

$$H_K = \frac{Q_{KK}}{|\vec{z} - \vec{z}_0|}, \quad \mathbf{A} = Q_{KK} \cos\theta d\phi,$$

$$H_5 = 1 + \sum_i \frac{P_i}{(|\vec{x} - \vec{x}_{0\,i}|^2 + 4Q_{KK}|\vec{z} - \vec{z}_0|)^{3/2}}. \tag{34}$$

This solution is also constructed in Ref. [12].

The time component of the 1-form central charge in the $M5$-brane worldvolume superalgebra can be viewed as the space components of a worldvolume dual 5-form. This charge is carried by the KK monopole [23]. Therefore, this configuration is interpreted as the $M5$-brane inside of the KK monopole [24, 25].

3.6 The pp wave in the background of $M2$-branes

For the pp wave which travels along one of the worldvolume directions, which we choose to be w, of the $M2$-brane, the metric takes the following form:

$$
\begin{aligned}
ds_{11}^2 &= H_2^{-2/3}[-dt^2 + dw^2 + dx^2 + (H_W - 1)(dt - dw)^2] \\
&\quad + H_2^{1/3}(dz_1^2 + \ldots + dz_8^2),
\end{aligned}
\tag{35}
$$

with the harmonic functions H_2 and H_W for the $M2$-brane and the pp wave satisfying the following equations:

$$
\partial_{\vec{z}}^2 H_2 = 0, \qquad \partial_{\vec{z}}^2 H_W + H_2 \partial_x^2 H_W = 0.
\tag{36}
$$

In the core region of the $M2$-brane, the harmonic functions are therefore given by:

$$
H_2 = \frac{Q}{|\vec{z} - \vec{z}_0|^6}, \qquad H_W = 1 + Q_W\left(x^2 + \frac{Q}{4} \frac{1}{|\vec{z} - \vec{z}_0|^4}\right).
\tag{37}
$$

When n of the overall transverse directions are delocalized, the harmonic functions take the following forms:

$$
H_2 = \frac{Q}{|\vec{z} - \vec{z}_0|^{6-n}}, \qquad H_W = 1 + \frac{Q_W}{\left[x^2 + \frac{4Q}{(n-4)^2}|\vec{z} - \vec{z}_0|^{n-4}\right]^{\frac{8-n}{2(n-4)}}}.
\tag{38}
$$

So, with 5 of the overall transverse directions delocalized the solutions become localized except for these delocalized overall transverse directions.

Upon dimensional reduction to $D = 10$, the $D = 11$ pp-wave becomes $D0$-brane in the type-IIA theory. The 0-form central charges in the $D = 11$ pp-wave superalgebra are decomposed into two sets of the 0-form central charges in the $D = 10$ $D0$-brane superalgebra, which can be respectively interpreted in the transverse 9-space as the central charges carried by a $D4$-brane or the KK monopole and a fundamental string or a $D8$-branes [23]. When uplifted to $D = 11$, the resulting configurations are $M2$-brane and $M5$-brane and the KK monopole involving the pp-wave, whose supergravity solutions are presented in this section and the following sections. On the other hand, the 3-momentum in the $M2$-brane superalgebra, interpreted as a 0-form in the transverse 8-space, is the null 3-momentum of the pp-wave inside the $M2$-brane [23]. Similarly, the 5-momentum in the $M5$-brane superalgebra, interpreted as a 0-form in the transverse 5-space, is the null 5-momentum of the $D = 11$ pp-wave inside of the $M5$-brane, whose supergravity solution is given in the next subsection.

3.7 The pp-wave in the background of $M5$-branes

For the pp wave which travels along one of the worldvolume directions, which we choose to be w, of the $M5$-brane, the metric takes the following form:

$$
\begin{aligned}
ds_{11}^2 &= H_5^{-1/3}[-dt^2 + dw^2 + (H_W - 1)(dt - dw)^2 + dx_1^2 + \ldots + dx_4^2] \\
&\quad + H_5^{2/3}(dz_1^2 + \ldots + dz_5^2),
\end{aligned} \tag{39}
$$

with the harmonic functions H_5 and H_W for the $M5$-brane and the pp wave satisfying the following equations:

$$
\partial_{\vec{z}}^2 H_5 = 0, \quad \partial_{\vec{z}}^2 H_W + H_5 \partial_{\vec{x}}^2 H_W = 0. \tag{40}
$$

In the core region of the $M5$-brane, the harmonic functions are therefore given by:

$$
H_5 = \frac{P}{|\vec{z} - \vec{z}_0|^3}, \quad H_W = 1 + Q_W\left(x^2 + \frac{4P}{|\vec{z} - \vec{z}_0|}\right). \tag{41}
$$

When n of the overall transverse directions are delocalized, the harmonic functions take the following forms:

$$
H_5 = \frac{P}{|\vec{z} - \vec{z}_0|^{3-n}}, \quad H_W = 1 + \frac{Q_W}{[x^2 + \frac{4P}{(n-1)^2}|\vec{z} - \vec{z}_0|^{n-1}]^{\frac{n+1}{n-1}}}. \tag{42}
$$

So, with 2 of the overall transverse directions delocalized, the solutions become localized except for these two delocalized overall transverse directions.

The pp-wave localized at the $M5$-brane is interpreted from the perspective of the worldvolume theory at the boundary as the neutral (with respect to the self-dual 3-form field strength in the $M5$-brane worldvolume theory) string in the $M5$-brane worldvolume.

3.8 The pp-wave in the background of the KK monopole

The spacetime metric for the pp wave which propagates in the background of the KK monopole has the following form:

$$
\begin{aligned}
ds_{11}^2 &= -dt^2 + dw^2 + (H_W - 1)(dt - dw)^2 + dx_1^2 + \ldots + dx_5^2 \\
&\quad + H_K(dz_1^2 + dz_2^2 + dz_3^2) + H_K^{-1}(dy + A_i dz_i)^2,
\end{aligned} \tag{43}
$$

where the harmonic functions H_K and H_W for the KK monopole and the pp wave and a 1-form potential $\mathbf{A} = (A_i)$ satisfy the equations:

$$
\partial_{\vec{z}}^2 H_K = 0, \quad \partial_{z_i} H_K = \epsilon_{ijk} \partial_{z_j} A_k, \quad \partial_{\vec{z}}^2 H_W + H_K \partial_{\vec{x}}^2 H_W = 0. \tag{44}
$$

In the core region of the KK monopole or in the limit of large KK monopole charge, the harmonic functions are therefore given by:

$$
\begin{aligned}
H_K &= \frac{Q_{KK}}{|\vec{z} - \vec{z}_0|}, \quad \mathbf{A} = Q_{KK} \cos \theta d\phi, \\
H_W &= 1 + \frac{Q_W}{(x^2 + 4Q_{KK}|\vec{z} - \vec{z}_0|)^{7/2}}.
\end{aligned}
\tag{45}
$$

4 Intersecting Branes in Ten Dimensions

In ten dimensions, the basic constituents of intersecting branes are D-branes, fundamental string, solitonic $NS5$-brane, the KK monopole and the pp wave. In the following, we present intersecting brane configurations of all the possible combinations.

4.1 Two Dp-branes self-intersecting over $(p-2)$ dimensions

In this case, the overall transverse space has $7 - p$ dimensions and there are 2 relative transverse directions for both of Dp-branes. The spacetime metric has the following form:

$$
\begin{aligned}
ds_{10}^2 &= (H_1 H_2)^{-1/2}(-dt^2 + dw_1^2 + \ldots + dw_{p-2}^2) + H_1^{1/2} H_2^{-1/2}(dx_1^2 + dx_2^2) \\
&\quad + H_1^{-1/2} H_2^{1/2}(dy_1^2 + dy_2^2) + (H_1 H_2)^{1/2}(dz_1^2 + \ldots + dz_{7-p}^2),
\end{aligned}
\tag{46}
$$

where the harmonic functions H_1 and H_2 for each Dp-branes satisfy the following equations:

$$
\partial_{\vec{z}}^2 H_1 + H_2 \partial_{\vec{x}}^2 H_1 = 0, \quad \partial_{\vec{z}}^2 H_2 = 0.
\tag{47}
$$

The harmonic functions are therefore given by

$$
H_1 = 1 + \sum_i \frac{Q_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + \frac{4Q}{(p-3)^2}|\vec{z} - \vec{z}_0|^{p-3}]^{\frac{2}{p-3}}}, \quad H_2 = \frac{Q}{|\vec{z} - \vec{z}_0|^{5-p}}.
\tag{48}
$$

Note, for $p = 2$, this solution becomes delocalized. So, one has to delocalize some of the overall transverse directions to obtain supergravity solution representing the intersecting brane localized except for the delocalized overall transverse directions. When the overall transverse space is 4-dimensional, i.e. self-intersecting $D3$-branes, the above expression for the harmonic function H_1 is singular. In this case, one has to delocalize one of the overall transverse directions. With two [one] overall transverse directions are delocalized for $p = 2$ [for $p = 3$], the harmonic functions are given by:

$$
H_1 = 1 + \sum_i \frac{Q_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + 4Q|\vec{z} - \vec{z}_0|]^2}, \quad H_2 = \frac{Q}{|\vec{z} - \vec{z}_0|}.
\tag{49}
$$

The Dp-brane worldvolume theory contains a scalar (interpreted as a Goldstone mode of spontaneously broken translational invariance by the Dp-brane), which is Hodge-dualized to a worldvolume $(p-1)$-form potential that the $(p-2)$-brane (common intersection of the two Dp-branes) couples to.

4.2 Dp-branes ending on $D(p+2)$-branes

There are $6-p$ overall transverse directions. The dimensions of the relative transverse space are 3 for Dp-branes and 1 for $D(p+2)$-branes. Since Dp-branes stretch between $D(p+2)$-branes, it is natural to let $D(p+2)$-branes to be delocalized along their relative transverse direction. The spacetime metric is given by

$$
\begin{aligned}
ds_{10}^2 &= (H_p H_{p+2})^{-1/2}(-dt^2 + dw_1^2 + \ldots + dw_{p-1}^2) + H_p^{1/2} H_{p+2}^{-1/2}(dx_1^2 + dx_2^2 + dx_3^2) \\
&\quad + H_p^{-1/2} H_{p+2}^{1/2} dy^2 + (H_p H_{p+2})^{1/2}(dz_1^2 + \ldots + dz_{6-p}^2),
\end{aligned} \tag{50}
$$

where the harmonic functions H_p and H_{p+2} for Dp- and $D(p+2)$-branes satisfy

$$
\partial_{\vec{z}}^2 H_p + H_{p+2}\partial_{\vec{x}}^2 H_p = 0, \quad \partial_{\vec{z}}^2 H_{p+2} = 0. \tag{51}
$$

The harmonic functions are given by

$$
H_p = 1 + \sum_i \frac{Q_i}{[|\vec{x}-\vec{x}_{0\,i}|^2 + \frac{4Q}{(p-2)^2}|\vec{z}-\vec{z}_0|^{p-2}]^{\frac{(p+2)}{2(p-2)}}}, \quad H_{p+2} = \frac{Q}{|\vec{z}-\vec{z}_0|^{4-p}}. \tag{52}
$$

The fluctuations of locations $\vec{x}_{0\,i}$ of the Dp-branes along the $D(p+2)$-brane directions, together with the y-component of the Dp-worldvolume gauge field, forms a massless hypermultiplet with free boundary conditions at $y=0$. The scalars describing the fluctuations of location $\vec{z}_0$ of the Dp-branes along the directions perpendicular to the $D(p+4)$-branes, together with the t- and $\vec{w}$-components of the Dp-worldvolume gauge field, form a vectormultiplet with each component field satisfying the Dirichlet boundary conditions.

From the point of view of the $D(p+2)$-branes, the ends of the Dp-branes are charged objects in the $D(p+2)$-brane worldvolume. For $D1$-branes ending on $D3$-branes ($p=1$ case), the ends of the $D1$-branes are magnetic sources for the $D3$-brane worldvolume gauge fields. Therefore, the worldvolume theory is the magnetic monopoles in the 4-dimensional $U(N_{p+2})$ Yang-Mills theory. The locations $\vec{x}=\vec{x}_{0\,i}$ of the $D1$-branes along the directions of the $D3$-branes, together with the Wilson lines of the $\sum_i N_{p\,i}$ $U(1)$ worldvolume gauge fields of $D1$-branes along the y-direction, parameterize the monopole moduli space. The charges Q_i of the $D1$-branes are interpreted as magnetic charges of the monopoles. Under the S-duality of the type-IIB theory, this supergravity brane solution transforms to the supergravity solution for fundamental strings ending

on $D3$-branes, which is presented in the following subsection. From the $D3$-brane worldvolume point of view, this S-duality is the Montonen-Olive's strong-weak coupling electric-magnetic duality of the $N = 4$ super-Yang-Mills theory in 4 dimensions. The ends of the fundamental strings on the $D3$-branes are electric charge sources, interpreted as charged gauge bosons. Just as the $D1$-brane and the fundamental string form a multiplet under the type-IIB S-duality, the charged gauge boson and the magnetic monopole transform as a multiplet under the Montonen-Olive duality.

When $p = 1$, the above harmonic function H_p represents delocalized Dp-branes, thereby requiring delocalization of some of overall transverse directions. When $p = 2$, the overall transverse space is 4-dimensional. So, one of the overall transverse directions has to be delocalized. With two [one] of the overall transverse directions delocalized for the $p = 1$ $[p = 2]$ case, harmonic functions are given by:

$$H_p = 1 + \sum_i \frac{Q_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + 4Q|\vec{z} - \vec{z}_0|]^{\frac{5}{2}}}, \quad H_{p+2} = \frac{Q}{|\vec{z} - \vec{z}_0|}. \tag{53}$$

4.3 Dp-branes inside of the worldvolume of $D(p+4)$-branes

There are $5 - p$ overall transverse directions. The dimensionalities of the relative transverse spaces are 4 for Dp-branes and 0 for $D(p+4)$-branes. The spacetime metric has the following form:

$$\begin{aligned}ds_{10}^2 &= (H_p H_{p+4})^{-1/2}(-dt^2 + dw_1^2 + \ldots + dw_p^2) + H_p^{1/2} H_{p+4}^{-1/2}(dx_1^2 + \ldots + dx_4^2) \\ &\quad + (H_p H_{p+4})^{1/2}(dz_1^2 + \ldots + dz_{5-p}^2),\end{aligned} \tag{54}$$

where the harmonic functions H_p and H_{p+4} for Dp- and $D(p+4)$-branes satisfy the differential equations:

$$\partial_{\vec{z}}^2 H_p + H_{p+4}\partial_{\vec{x}}^2 H_p = 0, \quad \partial_{\vec{z}}^2 H_{p+4} = 0. \tag{55}$$

The harmonic functions are therefore given by

$$H_p = 1 + \sum_i \frac{Q_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + \frac{4Q}{(p-1)^2}|\vec{z} - \vec{z}_0|^{p-1}]^{\frac{p+1}{p-1}}}, \quad H_{p+4} = \frac{Q}{|\vec{z} - \vec{z}_0|^{3-p}}. \tag{56}$$

The decoupling limit of the worldvolume theory of the corresponding microscopic D-brane configuration is (i) the $(p + 1)$-dimensional $U(\sum_i N_{p\,i})$ gauge theory with N_{p+4} flavors (in the fundamental representation of $U(\sum_i N_{p\,i})$) with the $U(N_{p+4})$ gauge symmetry of the $D(p + 4)$-branes being a global symmetry from the point of view of Dp-branes, or (ii) small $U(N_{p+4})$ instantons from the point of view of $D(p + 4)$-branes with $\sum_i N_{p\,i}$ Dp-branes being pointlike defects in the fundamental of $U(N_{p+4})$.

From the point of view of the worldvolume theory of the Dp-branes, the locations of the branes are interpreted as follows. The locations of the $D(p+4)$-branes relative to the locations of Dp-branes in the $\vec{z}$-direction are masses $\vec{m}_j$ $(j = 1, \ldots, N_{p+4})$ for the N_{p+4}-fundamentals. Since all the $D(p+4)$-branes coincide in the above supergravity solution, the full $U(N_{p+4})$ symmetry is left intact, while the quarks in the fundamental of $U(N_{p+4})$ remain massless. The locations $\vec{x}_{0\,i}$ of Dp-branes parallel to the $D(p+4)$-branes correspond to expectation values of an adjoint hypermultiplet of $U(\sum_i N_{p\,i})$. Since all the Dp-branes coincide in the direction $\vec{z}$ transverse to the $D(p+4)$-branes at $\vec{z} = \vec{z}_0$ (parameterizing the Coulomb branch of the $U(\sum_i N_{p\,i})$ gauge theory) for the above supergravity solution, the full $U(\sum_i N_{p\,i})$ gauge symmetry is left unbroken.

In the case of $p = 3$, i.e. intersecting $D3$- and $D7$-branes, one has to include the orientifold 7 plane with 8 units of $D7$-brane charge in order to cancel the brane charges. Then, the $D3$- and $D7$-branes have to located in pairs as mirror images of the orientifold plane along the $\vec{z}$-directions. Note, the ground state of an open string that stretches between the same $D3$-brane is the neutral gauge boson W_μ^3, whereas those that stretch between a pair of $D3$-brane and its mirror image are the charged gauge bosons $W_\mu^\pm$. Since all the $D3$-branes in the above supergravity solution [11] coincide in the $\vec{z}$-direction, these charged gauge bosons are massless and therefore the full $SU(2)$ gauge symmetry on the worldvolume of $D3$-branes is left unbroken. Since the relative locations of $D7$-branes and their mirror images with respect to $D3$-brane locations along the $\vec{z}$-direction is zero, the quarks are massless and the $SO(8)$ symmetry on the worldvolume of $D7$-branes remains unbroken.

The $p = 2$ case is the supergravity solutions for $D2$-branes within $D6$-branes constructed in Ref. [12]. The above solution becomes singular for $p = 1$ and delocalized for $p = 0$. After one [two] of the overall transverse directions are delocalized for $p = 1$ [for $p = 0$], the harmonic functions take the following forms:

$$H_p = 1 + \sum_i \frac{Q_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + 4Q|\vec{z} - \vec{z}_0|]^3}, \quad H_{p+4} = \frac{Q}{|\vec{z} - \vec{z}_0|}. \tag{57}$$

One can also obtain the localized BPS solution for $D(-1)$-branes in the background of $D3$-branes (the $p = -1$ case) applying the the general formalism in section 2, although the harmonic function for the $D(-1)$-brane is shown to satisfy the Laplace's equation in the AdS$_5$ space. After the Wick rotation to the Euclidean time coordinate

---

[11]When $p = 3$, the harmonic function H_7 for the $D7$-branes has to be logarithmic. Therefore, the solution (56) is not valid and one cannot obtain the explicit expression for the harmonic functions applying the method discussed in section 2. However, there might exist the explicit expressions for harmonic functions where all the D-branes coincide in the $\vec{z}$-direction. On the other hand, such supergravity solution cannot be trusted since the conservation of brane charges requires that there can be only 4 $D3$-branes and 4 $D7$-branes.

$x_0 \equiv \tau$, the spacetime metric takes the following form:

$$ds_{10}^2 = H_{-1}^{1/2} H_3^{-1/2} (dx_0^2 + \ldots + dx_3^2) + (H_{-1} H_3)^{1/2} (dz_1^2 + \ldots + dz_6^2), \qquad (58)$$

and the dilaton and the 0-form field in the RR sector are respectively given by $e^\phi = H_{-1}$ and $(\chi - \chi_\infty) = \pm(e^{-\phi} - e^{-\phi_\infty})$. Here, the subscript ∞ denotes the value of a field at the AdS$_5$ boundary. The harmonic functions H_{-1} and H_3 for the $D(-1)$-branes and $D3$-branes satisfy the following equations:

$$\partial_{\vec{z}}^2 H_{-1} + H_3 \partial_{\vec{x}}^2 H_{-1} = 0, \quad \partial_{\vec{z}}^2 H_3 = 0, \qquad (59)$$

where $\vec{x} = (x_0, \ldots, x_3)$ now the time coordinate $x_0 = \tau$ included. This system of partial differential equations cannot be solved by applying the method discussed in section 2. But it can be easily proved that the following harmonic functions satisfy (59):

$$H_{-1} = 1 + \sum_i \frac{Q_i |\vec{z} - \vec{z}_0|^4}{[|\vec{x} - \vec{x}_{0\,i}|^2 |\vec{z} - \vec{z}_0|^2 + Q]^4}, \quad H_3 = \frac{Q}{|\vec{z} - \vec{z}_0|^4}. \qquad (60)$$

This is the solution constructed in Ref. [15], where the radial coordinate $z = |\vec{z} - \vec{z}_0|$ in (60) is the inverse of that in their solution. The worldvolume theory of the corresponding microscopic D-brane configuration is the multi-Yang-Mills instantons in the 4-dimensional $U(N_3)$ gauge theory with the instanton with the instanton number $N_{-1\,i}$ located at $\vec{x} = \vec{x}_{0\,i}$. Since the $D(-1)$-brane is located at the position where the $D3$-branes coincide, the size of the Yang-Mills instantons is infinite [26].

4.4 Fundamental strings ending on Dp-branes

There are $8 - p$ overall transverse directions. The fundamental strings have p relative transverse directions and the Dp-branes have 1 relative transverse direction. It is natural to let the solution to be delocalized along the relative transverse direction of the Dp-branes. The spacetime metric has the following form:

$$\begin{aligned}ds_{10}^2 &= -H_F^{-1} H_p^{-1/2} dt^2 + H_p^{-1/2}(dx_1^2 + \ldots + dx_p^2) \\ &\quad + H_F^{-1} H_p^{1/2} dy^2 + H_p^{1/2}(dz_1^2 + \ldots + dz_{8-p}^2),\end{aligned} \qquad (61)$$

where the harmonic functions H_F and H_p for the fundamental string and Dp-branes satisfy the equations:

$$\partial_{\vec{z}}^2 H_F + H_p \partial_{\vec{x}}^2 H_F = 0, \quad \partial_{\vec{z}}^2 H_p = 0. \qquad (62)$$

The harmonic functions are therefore given by

$$H_F = 1 + \sum_i \frac{Q_i}{\left[|\vec{x} - \vec{x}_{0\,i}|^2 + \frac{4Q}{(p-4)^2}|\vec{z} - \vec{z}_0|^{p-4}\right]^{\frac{p^2-6p+12}{2(p-4)}}}, \quad H_p = \frac{Q}{|\vec{z} - \vec{z}_0|^{6-p}}. \qquad (63)$$

Note that for $p < 4$ the above solution becomes delocalized and for $p = 4$ the solution becomes singular. After $5-p$ overall transverse directions are delocalized, the harmonic functions take the following forms:

$$H_F = 1 + \sum_i \frac{Q_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + 4Q|\vec{z} - \vec{z}_0|]^3}, \quad H_p = \frac{Q}{|\vec{z} - \vec{z}_0|}, \tag{64}$$

representing fundamental strings localized except for the delocalized $5-p$ overall transverse directions.

For $p = 3$, this supergravity solution is S-dual to the D-strings ending on $D3$-branes discussed in the previous section. Once again, the ends of the fundamental strings are interpreted as (electrically) charged gauge bosons in the worldvolume gauge theory of the $D3$-branes.

4.5 Dp-branes ending on $NS5$-branes

There are 3 overall transverse directions. The dimensions of the relative transverse spaces are $6 - p$ for Dp-branes and 1 for $NS5$-branes. It is natural to delocalize in the direction relatively transverse to the $NS5$-branes. The spacetime metric for the Dp-branes ($p \le 6$) ending on $NS5$-branes is given by

$$
\begin{aligned}
ds_{10}^2 &= H_p^{-1/2}(-dt^2 + dw_1^2 + \ldots + dw_{p-1}^2) + H_p^{1/2}(dx_1^2 + \ldots + dx_{6-p}^2) \\
&\quad + H_p^{-1/2}H_{NS}dy^2 + H_p^{1/2}H_{NS}(dz_1^2 + dz_2^2 + dz_3^2),
\end{aligned} \tag{65}
$$

where the harmonic functions H_p and H_{NS} for the Dp-branes and $NS5$-branes satisfy the equations:

$$\partial_{\vec{z}}^2 H_p + H_{NS}\partial_{\vec{x}}^2 H_p = 0, \quad \partial_{\vec{z}}^2 H_{NS} = 0. \tag{66}$$

The harmonic functions are therefore given by

$$H_p = 1 + \sum_i \frac{Q_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + 4Q|\vec{z} - \vec{z}_0|]^{\frac{8-p}{2}}}, \quad H_{NS} = \frac{Q}{|\vec{z} - \vec{z}_0|}. \tag{67}$$

The $p = 6$ case is the supergravity solution for $NS5$-branes within $D6$-branes constructed in Ref. [12].

Since the y-direction, in which the $NS5$-branes stack, is delocalized, one can think of the $NS5$-branes as being periodically arrayed in the y-direction with the periodicity given by the circumference of the compactification circle. The scalars describing the fluctuations of the location $\vec{z} = \vec{z}_0$ of the Dp-branes in the perpendicular directions to the $NS5$-branes, along with the y-component of the Dp-worldvolume gauge field, form the hypermultiplet. The scalars corresponding to the locations $\vec{x} = \vec{x}_{0\,i}$ of the Dp-branes in the parallel directions of the $NS5$-branes, together with the t- and

$\vec{x}$-components of the Dp-worldvolume gauge field, form the vector multiplet. Since the Dp-branes are finite in y, the worldvolume gauge theory is effectively $1 + (p - 1)$ dimensional with the Kaluza-Klein excitations on the Dp-branes invisible at large distances. For $p = 3, 4$, the corresponding worldvolume theory is $1 + (p - 1)$ dimensional $\prod_{i=1}^{N_5} U(N_4)$ supersymmetric gauge theory with N_5 (bifundamental) hypermultiplets transforming in the $(N_4, \bar{N}_4)$ of $U(N_4) \times U(N_4)$. Here, each $U(N_4)$ gauge group is associated with N_4 Dp-branes that stretch between two adjacent $NS5$-branes.

4.6   Fundamental strings with $NS5$-branes

For fundamental strings parallel to $NS5$-branes, there are 4 overall transverse directions. So, the delocalization along one of the overall transverse directions is required. The fundamental strings have 4 relative transverse directions and the $NS5$-branes have no relative transverse direction. The metric has the following form:

$$ds_{10}^2 = H_F^{-1}(-dt^2 + dw^2) + dx_1^2 + \cdots + dx_4^2 + H_{NS}(dz_1^2 + \cdots + dz_4^2), \qquad (68)$$

where the harmonic functions H_F and H_{NS} for the fundamental strings and the $NS5$-branes satisfy

$$\partial_{\vec{z}}^2 H_F + H_{NS}\partial_{\vec{x}}^2 H_F = 0, \quad \partial_{\vec{z}}^2 H_{NS} = 0. \qquad (69)$$

The harmonic functions are therefore given by

$$H_F = 1 + \sum_i \frac{Q_i}{[|\vec{x} - \vec{x}_{0\,i}|^2 + 4Q|\vec{z} - \vec{z}_0|]^3}, \quad H_{NS} = \frac{Q}{|\vec{z} - \vec{z}_0|}. \qquad (70)$$

This solution is also constructed in Ref. [12].

4.7   Two $NS5$-branes intersecting over 3 dimensions

The spacetime metric has the following form:

$$\begin{aligned} ds_{10}^2 &= -dt^2 + dw_1^2 + dw_2^2 + dw_3^2 + H_1(dx_1^2 + dx_2^2) \\ &\quad + H_2(dy_1^2 + dy_2^2) + H_1 H_2(dz_1^2 + dz_2^2), \end{aligned} \qquad (71)$$

where the harmonic functions H_1 and H_2 for each $NS5$-branes satisfy:

$$\partial_{\vec{z}}^2 H_1 + H_2\partial_{\vec{x}}^2 H_1 = 0, \quad \partial_{\vec{z}}^2 H_2 = 0. \qquad (72)$$

The overall transverse space is 2-dimensional. This means that the harmonic function H_2 in Eq. (4) has to be logarithmic. So, one cannot construct the localized solution by applying the method developed in this paper.

4.8 The KK monopole in the transverse space of Dp-brane with $p \le 4$.

The spacetime metric has the following form:

$$\begin{aligned}
ds_{10}^2 &= H_p^{-1/2}(-dt^2 + dw_1^2 + \ldots + dw_p^2) + H_p^{1/2}[dx_1^2 + \ldots + dx_{5-p}^2 \\
&\quad + H_K(dz_1^2 + dz_2^2 + dz_3^2) + H_K^{-1}(dy + A_i dz_i)^2],
\end{aligned} \tag{73}$$

where the harmonic functions H_p and H_{KK} for the Dp-branes and the KK monopoles and a 1-form potential $\mathbf{A} = (A_i)$ satisfy

$$\partial_{\vec{z}}^2 H_p + H_{KK}\partial_{\vec{x}}^2 H_p = 0, \quad \partial_{\vec{z}}^2 H_{KK} = 0, \quad \partial_{z_i} H_K = \epsilon_{ijk}\partial_{z_j} A_k. \tag{74}$$

In the core region of the KK monopole or in the limit of large KK monopole charge, the harmonic functions are given by:

$$\begin{aligned}
H_K &= \frac{Q_{KK}}{|\vec{z} - \vec{z}_0|}, \quad \mathbf{A} = Q_{KK}\cos\theta d\phi, \\
H_p &= 1 + \sum_i \frac{Q_i}{(|\vec{x} - \vec{x}_{0i}|^2 + 4Q_{KK}|\vec{z} - \vec{z}_0|)^{\frac{7-p}{2}}}.
\end{aligned} \tag{75}$$

The corresponding worldvolume theory is the one with the flat transverse space replaced by an ALE space with $A_{N_{KK}-1}$ singularity.

4.9 The KK monopole in the transverse space of the fundamental string

The spacetime metric is given by

$$\begin{aligned}
ds_{10}^2 &= H_F^{-1}(-dt^2 + dw^2) + dx_1^2 + \ldots + dx_4^2 \\
&\quad + H_K(dz_1^2 + dz_2^2 + dz_3^2) + H_K^{-1}(dy + A_i dz_i)^2,
\end{aligned} \tag{76}$$

where the harmonic functions H_F and H_{KK} for the fundamental strings and the KK monopole and a 1-form potential $\mathbf{A} = (A_i)$ satisfy

$$\partial_{\vec{z}}^2 H_F + H_{KK}\partial_{\vec{x}}^2 H_F = 0, \quad \partial_{\vec{z}}^2 H_{KK} = 0, \quad \partial_{z_i} H_K = \epsilon_{ijk}\partial_{z_j} A_k. \tag{77}$$

In the core region of the KK monopole or in the limit of large KK monopole charge, the harmonic functions are given by:

$$\begin{aligned}
H_K &= \frac{Q_{KK}}{|\vec{z} - \vec{z}_0|}, \quad \mathbf{A} = Q_{KK}\cos\theta d\phi, \\
H_F &= 1 + \sum_i \frac{Q_i}{(|\vec{x} - \vec{x}_{0i}|^2 + 4Q_{KK}|\vec{z} - \vec{z}_0|)^3}.
\end{aligned} \tag{78}$$

The corresponding worldvolume theory is described by a conformal field theory in the target manifold including an ALE space with $A_{N_{KK}-1}$ singularity.

4.10 The KK monopole in the transverse space of the $NS5$-brane

The spacetime metric has the following form:

$$ds_{10}^2 = -dt^2 + dw_1^2 + \ldots + dw_5^2 + H_{NS}[H_K(dz_1^2 + dz_2^2 + dz_3^2) + H_K^{-1}(dy + A_i dz_i)^2], \quad (79)$$

where H_{NS} and H_K are the harmonic functions for the $NS5$-branes and the KK monopole and $\mathbf{A} = (A_i)$ is a 1-form potential. There are no relative transverse directions. So, there is no point in considering the localized intersecting configuration.

4.11 The pp wave propagating in the background of the KK monopole

The spacetime metric has the following form:

$$
\begin{aligned}
ds_{10}^2 &= -dt^2 + dw^2 + (H_W - 1)(dt - dw)^2 + dx_1^2 + \ldots + dx_4^2 \\
&\quad + H_K(dz_1^2 + dz_2^2 + dz_3^2) + H_K^{-1}(dy + A_i dz_i)^2,
\end{aligned}
\quad (80)
$$

where the harmonic functions H_K and H_W for the KK monopole and the pp wave and a 1-form potential $\mathbf{A} = (A_i)$ satisfy the equations:

$$\partial_{\vec{z}}^2 H_K = 0, \quad \partial_{z_i} H_K = \epsilon_{ijk}\partial_{z_j} A_k, \quad \partial_{\vec{z}}^2 H_W + H_K \partial_{\vec{x}}^2 H_W = 0. \quad (81)$$

In the core region of the KK monopole or in the limit of large KK monopole charge, the harmonic functions are therefore given by:

$$
\begin{aligned}
H_K &= \frac{Q_{KK}}{|\vec{z} - \vec{z}_0|}, \quad \mathbf{A} = Q_{KK}\cos\theta d\phi, \\
H_W &= 1 + \frac{Q_W}{(x^2 + 4Q_{KK}|\vec{z} - \vec{z}_0|)^3}.
\end{aligned}
\quad (82)
$$

4.12 Fundamental string with the pp wave propagating along its longitudinal direction

The spacetime metric has the following form:

$$ds_{10}^2 = H_F^{-1}[-dt^2 + dw^2 + (H_W - 1)(dt - dw)^2] + dz_1^2 + \ldots + dz_8^2, \quad (83)$$

where H_F and H_W are respectively the harmonic functions for the fundamental strings and the pp wave. Since there is no relative transverse directions, it is of no point to discuss the special localized solution.

4.13 $NS5$-brane with the pp wave propagating along one of its longitudinal directions

The spacetime metric has the following form:

$$\begin{aligned}
ds_{10}^2 &= -dt^2 + dw^2 + (H_W - 1)(dt - dw)^2 + dx_1^2 + \ldots + dx_4^2 \\
&\quad + H_{NS}(dz_1^2 + \ldots + dz_4^2),
\end{aligned} \tag{84}$$

with the harmonic functions H_{NS} and H_W for the $NS5$-brane and the pp wave satisfying the following equations:

$$\partial_{\vec{z}}^2 H_{NS} = 0, \quad \partial_{\vec{z}}^2 H_W + H_{NS}\partial_{\vec{x}}^2 H_W = 0. \tag{85}$$

In the core region of the $NS5$-brane with one of its overall transverse directions delocalized, the harmonic functions are therefore given by:

$$H_{NS} = \frac{P}{|\vec{z} - \vec{z}_0|}, \quad H_W = 1 + \frac{Q_W}{(x^2 + 4P|\vec{z} - \vec{z}_0|)^3}. \tag{86}$$

4.14 Dp-brane with the pp wave propagating along one of its longitudinal directions

The spacetime metric has the following form:

$$\begin{aligned}
ds_{10}^2 &= H_p^{-1/2}[-dt^2 + dw^2 + (H_W - 1)(dt - dw)^2 + dx_1^2 + \ldots + dx_{p-1}^2] \\
&\quad + H_p^{1/2}(dz_1^2 + \ldots + dz_{9-p}^2),
\end{aligned} \tag{87}$$

with the harmonic functions H_p and H_W for the Dp-brane and the pp wave satisfying the following equations:

$$\partial_{\vec{z}}^2 H_p = 0, \quad \partial_{\vec{z}}^2 H_W + H_p\partial_{\vec{x}}^2 H_W = 0. \tag{88}$$

In the core region of the Dp-branes, the harmonic functions are therefore given by:

$$H_p = \frac{Q}{|\vec{z} - \vec{z}_0|^{7-p}}, \quad H_W = 1 + \frac{Q_W}{\left(x^2 + \frac{4Q}{(p-5)^2}|\vec{z} - \vec{z}_0|^{p-5}\right)^{\frac{p^2-8p+19}{2(p-5)}}}. \tag{89}$$

For the $p < 5$ the pp wave becomes delocalized and for $p = 5$ the above solution becomes singular. After $6 - p$ of the overall transverse directions are delocalized, the harmonic functions take the following forms:

$$H_p = \frac{Q}{|\vec{z} - \vec{z}_0|}, \quad H_W = 1 + \frac{Q_W}{(x^2 + 4Q|\vec{z} - \vec{z}_0|)^3}. \tag{90}$$

The $p = 6$ case is the supergravity solution for the pp wave localized within $D6$-branes constructed in Ref. [12].

Acknowledgements

I would like to thank Y. Oz for discussions.

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