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Proton Stability in Six Dimensions

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We show that Lorentz and gauge invariance explain the long proton lifetime within the standard model in six dimensions. The baryon-number violating operators have mass dimension 15 or higher. Upon TeV-scale compactification of the two universal extra dimensions on a square T^2/Z_2 orbifold, a discrete subgroup of the 6-dimensional Lorentz group continues to forbid dangerous operators.

The stability of the proton is one of the most intriguing problems in physics. The current limit on the proton lifetime is 1.6×10^{33} years if the dominant decay mode is $p \rightarrow e^+\pi^0$, and 1.6×10^{25} years independent of the decay mode [1]. The conserved quantity associated with proton stability, the baryon number B, is the charge corresponding to a global $U(1)_B$ symmetry, or one of its discrete subgroups. It is important to explain why B is conserved to such a high degree of accuracy. In this letter we demonstrate that the spatial symmetry of two universal extra dimensions, accessible to all the standard model fields, naturally constrains the proton lifetime to be greater than the experimental limits.

The minimal standard model, including operators of dimension four and lower, automatically conserves B. This theory, however, is valid only up to some scale $\Lambda_{\rm SM}$ where new physics becomes manifest. The effect of physics at scales above $\Lambda_{\rm SM}$ is described most generally by higher dimension operators, suppressed by the appropriate power of $\Lambda_{\rm SM}$. Among these are B violating operators of dimension six: $(\overline{q}^c_L q_L)(\overline{q}^c_L l_L)$, $(\overline{d}^c_R u_R)(\overline{q}^c_L l_L)$, $(\overline{q}^c_L q_L)(\overline{u}^c_R e_R)$, $(\overline{u}^c_R d_R)(\overline{e}^c_R u_R)$, where only one Lorentz invariant form is displayed for each four-fermion term; the weak-eigenstate quarks, q_L, u_R, d_R, and leptons, l_L, e_R, may belong to any generation. The experimental limits on the proton lifetime impose a bound on the coefficients of these operators: $C_{\rm SM}/\Lambda^2_{\rm SM} \lesssim 10^{-24}{\rm TeV}^{-2}$. If $\Lambda_{\rm SM} \gtrsim 10^{16}$ GeV, then the coefficients $C_{\rm SM}$ can safely be as large as order one at that scale. However, the naturalness of the Higgs sector in the standard model restricts $\Lambda_{\rm SM}$ to lie in the TeV range. The $C_{\rm SM}$'s must therefore be extremely small — below 10^{-23} or so.

Various explanations for the smallness of B violation have been proposed. In the minimal supersymmetric standard model, a discrete symmetry is needed to banish B-violating terms of dimension three and four from the superpotential. The discrete symmetry may be a remnant of a $U(1)$ gauge group [2], or else one could hope that it is preserved by quantum gravity. In the case of large extra dimensions, the smallness of $C_{\rm SM}$ has been suggested to arise from the localization of the quarks and leptons at different points inside a thick brane [3].

Here we concentrate on the chiral 6-dimensional standard model, in which all standard model fields propagate in two extra dimensions. The current bound on the compactification scale of two universal extra dimensions is $1/R \gtrsim 500$ GeV [4], suggesting a rich phenomenology [5]. The standard model in two universal extra dimensions is especially appealing because of the properties of the Lorentz group in six dimensions, as well as the constraints imposed by anomaly cancellation. The 6-dimensional standard model is an effective theory valid up to a scale $M_s \approx 5/R$, above which the 6-dimensional $SU(3)_C \times SU(2)_W \times U(1)_Y$ gauge interactions become non-perturbative.

In six dimensions, the $SO(1,5)$ Lorentz symmetry has two irreducible spin-1/2 representations generated by $\Sigma^{\alpha\beta}/2$, where $\Sigma^{\alpha\beta} \equiv i[\Gamma^\alpha, \Gamma^\beta]/2$ in terms of the anticommuting 8×8 matrices Γ^α, $\alpha = 0,1,...,5$. The product $\Gamma^0\Gamma^1...\Gamma^5$ has eigenvalues ± 1 defining the two 6-dimensional chiralities. The irreducible gauge $[SU(3)_C \times SU(2)_W \times U(1)_Y]$ and gravitational anomalies cancel for only two chirality assignments, up to an overall sign [6,7]:

$$\mathcal{Q}_+ \, , \, \mathcal{U}_- \, , \, \mathcal{D}_- \, , \, \begin{cases} \mathcal{L}_+ \, , \; \mathcal{E}_- \, , \; \mathcal{N}_- \, , \\ \qquad \text{or} \\ \mathcal{L}_- \, , \; \mathcal{E}_+ \, , \; \mathcal{N}_+ \, , \end{cases} \tag{1}$$

where $\mathcal{Q}_+, \mathcal{U}_-, \mathcal{D}_-$ and $\mathcal{L}_\pm, \mathcal{E}_\mp, \mathcal{N}_\mp$ are the 6-dimensional quarks and leptons, respectively, and a generational index is implicit. We assume that only one of these two assignments applies to all fermion generations, since then the $SU(2)_W$ global anomaly demands 3 mod 3 generations [6]. Each fermion generation includes a gauge singlet field, $\mathcal{N}_\mp$, such that the gravitational anomaly cancels. The 6-dimensional chiral quark and lepton fields have four components, and if two dimensions are appropriately compactified on an orbifold, then their zero-modes may be identified with the left- and right-handed standard model fermions q_L, u_R, d_R, l_L, e_R, as well as three right-handed neutrinos ν_R. We first describe the constraints on B violation from 6-dimensional Lorentz invariance, and then show that the constraints are equally tight even with the lesser symmetry remaining after orbifold compactification.

We begin by observing that $SO(1,5)$ has an $SO(1,3) \times U(1)_{45}$ subgroup: the combination of the 4-dimensional Lorentz symmetry associated with the x^μ coordinates, $\mu = 0,1,2,3$, and the symmetry under rotations of the x^4, x^5 coordinates. Any chiral 6-dimensional fermion, $\Psi_\pm$, may be decomposed under this subgroup as

$$\Psi_\pm = \Psi_{\pm L} + \Psi_{\pm R}, \tag{2}$$

Fermion	$U(1)_{45}$ charge	zero-mode
$\mathcal{Q}_{+L}$	$-1/2$	$q_L = (u_L, d_L)$
$\mathcal{U}_{-R}$, $\mathcal{D}_{-R}$	$-1/2$	u_R, d_R
$\mathcal{Q}_{+R}, \mathcal{U}_{-L}, \mathcal{D}_{-L}$	$+1/2$	—
$\mathcal{L}_{\pm L}$	$\mp 1/2$	$l_L = (\nu_L, e_L)$
$\mathcal{E}_{\mp R}$, $\mathcal{N}_{\mp R}$	$\mp 1/2$	e_R, ν_R
$\mathcal{L}_{\pm R}$, $\mathcal{E}_{\mp L}$, $\mathcal{N}_{\mp L}$	$\pm 1/2$	—

TABLE I. Transformation properties of the quark and lepton fields under $SO(1,3) \times U(1)_{45}$, and the corresponding zero-modes under orbifold compactification.

where L and R are $SO(1,3)$ chiralities, defined by the projection operators

$$P_{\pm L} = P_{\mp R} = \frac{1}{2}\left(1 \mp \Sigma^{45}\right) \ . \quad (3)$$

Since $\Sigma^{\alpha\beta}/2$ are the generators of the spin-1/2 representations of $SO(1,5)$, the $SO(1,3)$-chiral fermions have $U(1)_{45}$ charges given by the eigenvalues of $\Sigma^{45}/2$: $\mp 1/2$ for $\Psi_{\pm L}$, and $\pm 1/2$ for $\Psi_{\pm R}$. In Table I we list the charges of the quarks and leptons. The covariant derivative, D_α, transforms as a vector under $SO(1,5)$, so that $D_4 + iD_5$ has $U(1)_{45}$ charge -1.

To construct the B-violating operators in six dimensions, we first note that since all quark fields carry $B = +1/3$, $SU(3)_C$ gauge invariance demands that for any such operator the number of quark fields (minus the number of charge-conjugated fields) is a multiple of three. Only operators constructed of fields that have zero modes can induce at tree-level the B-violating processes searched for in experiments so far. From Table I we see that all quark fields containing zero modes have $U(1)_{45}$ charge $-1/2$, so that an operator with $|\Delta B| \geq 1$ has a $U(1)_{45}$ charge given by $(-3/2)\Delta B$ plus the sum of the charges of all lepton fields included in that operator. Hence, *all B-violating nucleon-decay operators allowed by Lorentz invariance in six dimensions must involve at least three quarks and three leptons.*

Using this key fact, it is straightforward to find the lowest dimension operators, invariant under $SO(1,5)$ and standard-model gauge transformations, that (after compactification) can induce B-violating nucleon decays. We consider first the $\mathcal{L}_+$ chirality assignment. The operators then first appear at mass-dimension 16 in the 6-dimensional theory, each involving six fermions and one covariant derivative $\not{D} = D_\beta \Gamma^\beta$:

$$\left(\overline{\mathcal{L}}_+\mathcal{D}_-\right)^2\left(\overline{\mathcal{N}}_-\not{D}\mathcal{D}_-\right) \ , \ \left(\overline{\mathcal{E}}_-\not{D}\mathcal{D}_-\right)\left(\overline{\mathcal{N}}_-\Gamma^\alpha\mathcal{D}_-\right)^2 \ ,$$
$$\left(\overline{\mathcal{L}}_+\mathcal{D}_-\right)\left(\overline{\mathcal{N}}_-\mathcal{Q}_+\right)\left(\overline{\mathcal{N}}_-\not{D}\mathcal{D}_-\right) \ ,$$
$$\left(\overline{\mathcal{N}}_-\not{D}\mathcal{U}_-\right)\left(\overline{\mathcal{N}}_-\Gamma^\alpha\mathcal{D}_-\right)^2 \ , \ \left(\overline{\mathcal{N}}_-\mathcal{Q}_+\right)^2\left(\overline{\mathcal{N}}_-\not{D}\mathcal{D}_-\right) \ , \quad (4)$$

where we have exhibited only the Lorentz covariant bilinears with the smallest number of Γ matrices; permutations of $\not{D}$ and Γ^α are also allowed. The $SO(1,3)$ chiralities of the zero modes imply that only the terms in (4) which do not contain $\Gamma^{4,5}$ can induce nucleon decay. Each of the above dimension-16 operators enters with a coefficient proportional to M_s^{-10} leading to a strong (and adequate) suppression of B violation after compactification to four dimensions. We estimate their effects after first considering dimension-17 operators, whose contributions to proton decay turn out to dominate.

There is only one proton-decay operator of dimension 17 (modulo Fierz transformations, and the insertion of two or three $\Sigma^{\alpha\beta}$ matrices), invariant under $SO(1,5)$ and standard model gauge transformations (in particular $U(1)_Y$), that does not involve the gauge singlets $\mathcal{N}_-$:

$$\mathcal{O}_{17} = \frac{C_{17}}{M_s^{11}}\left(\overline{\mathcal{L}}_+\mathcal{D}_-\right)^3\tilde{\mathcal{H}} \ , \quad (5)$$

where $\tilde{\mathcal{H}}$ is the charge-conjugated Higgs doublet in six dimensions. Dimension-17 operators involving $\mathcal{N}_-$ have a similar form, and will be discussed below. All other B-violating nucleon-decay operators require more derivatives, $\tilde{\mathcal{H}}$ fields, or fermion bilinears.

Upon integrating over the compact dimensions, x^4, x^5, restricting the fields to their zero modes, and replacing the zero-mode Higgs doublet by its VEV ($v_h \approx 174$ GeV), $\mathcal{O}_{17}$ gives rise in the 4-dimensional theory to the operator

$$\frac{v_h C_{17}}{A_{45}^{5/2} M_s^{11}}\left(\overline{\nu}_L d_R\right)\left(\overline{l}_L d_R\right)^2 \ , \quad (6)$$

where A_{45} is the area spanned by x^4 and x^5. For the T^2/Z_2 square orbifold of radius R constructed in [4], $A_{45} = 2\pi^2R^2$. This operator is non-vanishing only if the generational indices of the three $\overline{\mathcal{L}}_+\mathcal{D}_-$ bilinears in $\mathcal{O}_{17}$ are not all identical. Hence, $\mathcal{O}_{17}$ induces proton decays into $e^-\pi^+\pi^+\nu\nu$ or $\mu^-\pi^+\pi^+\nu\nu$ with the two neutrinos belonging to different generations. Final states with more pions or a K^+ are kinematically suppressed. The proton width is given by:

$$\Gamma(p \to e^-\pi^+\pi^+\nu\nu) \approx \frac{v_h^2 C_{17}^2}{A_{45}^5 M_s^{22}}\left(\frac{m_p}{3}\right)^{11}\Phi_5 F(\pi\pi) \ , \quad (7)$$

where m_p is the proton mass. If the process occurs predominantly via the decay of the d valence quark [8], whose constituent mass $m_p/3$ is used to set the scale in Eq. (7), then the form factor $F(\pi\pi)$, which describes the probability for the two pions to be formed, is expected to be of order unity. The kinematical phase-space factor, Φ_5, defined here as the dimensionless quantity that relates the squared amplitude to the decay width, is tiny for a five-body decay. We estimate $\Phi_5 \lesssim (4\pi)^{-7} \times O(10^{-4})$, where the factors of 4π account for angular integration and the additional suppression is due to integration over the magnitudes of final state momenta. The process $p \to \mu^-\pi^+\pi^+\nu\nu$ has an even smaller Φ_5. Thus, $\mathcal{O}_{17}$ yields a finite proton lifetime:

$$\tau_p \approx \frac{10^{35}\mathrm{yr}}{C_{17}^2}\left[\frac{(4\pi)^{-7}10^{-4}}{\Phi_5 F(\pi\pi)}\right]\left[\frac{1/R}{0.5\ \mathrm{TeV}}\right]^{12}\left[\frac{RM_s}{5}\right]^{22}, \quad (8)$$

where the quantities in square brackets are of order one or larger. There are no published experimental limits for five-body proton decays. For comparison, the searches for $p \to e^-\pi^+\pi^+$ [9] set a limit of $\tau_p > 3\times 10^{31}$ yr. Thus, for $C_{17} \le O(1)$, the proton lifetime is orders of magnitude longer than the current experimental limits.

The $\mathcal{O}_{17}$ operator also induces B-violating neutron decays, including $n \to e^-\pi^+\nu\nu$, $\mu^-\pi^+\nu\nu$. The four-body decays proceed via the fusion of two d valence quarks, which is suppressed by their wave function overlap inside the neutron [8]. This is compensated though by the larger constituent mass, $\sim 2m_p/3$, of the dd pair. Compared with $\Gamma(p \to e^-\pi^+\pi^+\nu\nu)$ given in Eq. (7), the neutron partial width in these modes is enhanced by Φ_4/Φ_5, where $\Phi_4 \lesssim (4\pi)^{-5} \times O(10^{-3})$ is the four-body phase-space factor. Thus, the inverse width of the B-violating neutron decays induced by $\mathcal{O}_{17}$ is smaller than τ_p by three orders of magnitude. The experimental limit on the partial mean life of $n \to e^-\pi^+$ is 6.5×10^{31} yr [10]. If a dedicated search for $n \to e^-\pi^+\nu\nu$ were to yield a comparable limit, then only a weak bound of $C_{17} \lesssim O(1)$ would be imposed, with uncertainties especially due to the sensitive dependence on M_s and R.

These are striking results. B-violating nucleon decays are adequately suppressed even with the scale of B violation in the TeV range, providing only that the coefficient C_{17} is not larger than order unity. In fact, C_{17} is likely to be significantly smaller than unity because the operator $\mathcal{O}_{17}$ is composed of chirality flipping bilinears and therefore can be expected to arise with strength proportional to small Yukawa couplings.

We now return to the other operators. For the dimension-16 operators of Eq. (4), the presence of the covariant derivative leads to an additional factor of $m_p/3$ in the 4-dimensional amplitude in place of a $A_{45}^{-1/2}v_h/M_s$ factor. Thus, there is a supression factor of 10^3 in the widths relative to those arising from dimension-17 operators. As we mentioned above, all dimension-17 operators other than $\mathcal{O}_{17}$ involve at least one $\mathcal{N}_-$ field. The tiny neutrino masses suggest that the couplings of its zero mode, ν_R, are small. Thus, the coefficients of these operators may naturally be substantially smaller than C_{17}. Nonetheless, some could be relevant because they allow new decay modes: $p \to \pi^+\nu\nu\nu$, $n \to \pi^0\nu\nu\nu$, and other final states with mesons and three neutrinos. [$n \to \nu\nu\nu$ requires a three-quark fusion [8] whose small probability is not compensated by the gain in phase space.] The experimental limits on such modes are of order 10^{32} yr [1]. There exists an operator of this type with a single $\mathcal{N}_-$ field, $(\overline{\mathcal{L}}_+\mathcal{D}_-)^2(\overline{\mathcal{N}}_-\mathcal{Q}_+)\tilde{\mathcal{H}}$, and a few with more $\mathcal{N}_-$ fields. The bounds on the coefficients of these operators are as loose as those on C_{17}.

Consider next the $\mathcal{L}_-$ chirality assignment. It leads to a different set of proton-decay operators, invariant under $SO(1,5)$ and gauge transformations: the dominant ones are six-fermion operators of dimension 15,

$$\frac{1}{M_s^9}\left[C_{15}(\overline{\mathcal{E}}_+^c\mathcal{U}_-) + C_{15}'(\overline{\mathcal{L}}_-^c\mathcal{Q}_+)\right]\left(\overline{\mathcal{L}}_-^c\Gamma^\alpha\mathcal{U}_-\right)^2 . \quad (9)$$

Other six-fermion operators involve $\mathcal{N}_+$ fields, but again, their coefficients may be small. The operators (9) lead to proton decay into three anti-leptons (one or two are positively charged) and a number of mesons. For the dominant decay mode, $p \to e^+e^+\pi^-\bar{\nu}$, we estimate

$$\tau_p \approx \frac{10^{26}\mathrm{yr}}{C_{15}^2}\left[\frac{(4\pi)^{-5}10^{-3}}{\Phi_4 F(\pi)}\right]\left[\frac{1/R}{0.5\ \mathrm{TeV}}\right]^{10}\left[\frac{RM_s}{5}\right]^{18}, \quad (10)$$

where $F(\pi)$ is a form factor of order unity. Despite the reduced phase space, the process $p \to \mu^+e^+\pi^-\bar{\nu}$ is more constraining due to better data on decays into muons [1]: $\tau_p/\mathrm{Br}(p \to \mu^+X) > 10^{31}$ yr, where $\mathrm{Br}(p \to \mu^+X)$ is the branching fraction for inclusive decays. We derive a constraint, $C_{15} \lesssim 10^{-2}(R\times 0.5\ \mathrm{TeV})^{-5}$, which is rather loose given that C_{15} may naturally be as small as some Yukawa couplings. Nonetheless, the prospects for observing B violation look better compared with the $\mathcal{L}_+$ chirality assignment. Recall that no known theoretical argument determines which of the two chirality assignments of Eq. (1) is preferred.

We have so far considered proton decay arising from operators that respect the $SO(1,5)$ symmetry. But since the compactification of two dimensions breaks $SO(1,5)$, including its $U(1)_{45}$ subgroup, we must next study whether the reduced symmetry allows operators that could induce proton decay at an unacceptable level. A simple and symmetric choice for the compactification of the x^4 and x^5 dimensions is a T^2/Z_2 orbifold of equal radii [4]. The "torus" T^2 is taken to be a square of size $2\pi R$ in the x^4, x^5 plane, with periodic boundary conditions, $\varphi(x^4,x^5) = \varphi(x^4+2\pi R, x^5) = \varphi(x^4, x^5+2\pi R)$ for any field φ. The square has a Z_4 symmetry, being invariant under $\pi/2$ rotations in the x^4, x^5 plane. Now, since the generator of the $U(1)_{45}$ rotations acting on fermions is $\Sigma_{45}/2$ [the spin-1/2 representation is "double-valued" under 2π rotations], upon compactification on a square, the $U(1)_{45}$ symmetry is broken down to a Z_8 group whose elements are given by U^n with $U = \exp[i(\pi/2)\Sigma_{45}/2]$ and $n = 0, 1, ..., 7$.

Compactification on T^2 would allow only vectorlike zero-mode fermions in the 4-dimensional theory, so it is necessary to introduce the observed $SO(1,3)$ chirality of the standard model fermions by orbifolding the square. Each field is taken to be either even or odd under the Z_2 orbifold transformation, defined by a π rotation in the x^4, x^5 plane. The assigned Z_2 parity of the quarks and

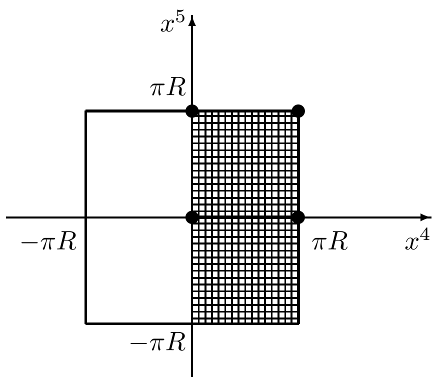

FIG. I. The square T^2/Z_2 orbifold of radius R: the fundamental region is shaded, and the four fixed points under Z_2 are marked by $\bullet$.

leptons can be read from Table I: a field has a zero mode if and only if it is even. The fundamental region in the x^4, x^5 plane of the square orbifold is shown in Fig. I.

The important point is that the square T^2/Z_2 orbifold does not break further the Z_8 symmetry of the 4-dimensional effective theory. This is because the Z_2 orbifold projection commutes with the Z_8 transformation. A fermion ψ of charge z under Z_8 transforms as follows:

$$U\psi(x^4, x^5) = \sigma^k e^{iz\pi/2} \psi(|x^5|, \sigma x^4) \ , \qquad (11)$$

where $k = 1$ ($k = 2$) if ψ is odd (even) under Z_2. The transformation of $\psi(x^4, x^5)$ depends on the sign of x^5, through $\sigma = +1$ ($\sigma = -1$) for $x^5 < 0$ ($x^5 \geq 0$). Note also that the presence of orbifold fixed points (see Fig. 1) does not break the Z_8 symmetry.

Having established that the residual symmetry of the effective theory is $SO(1,3) \times Z_8$, where Z_8 is the subgroup of $U(1)_{45}$ defined above, it follows from $SU(3)_C$ gauge invariance and from the charges given in Table I that all operators constructed only of zero-mode fields satisfy the selection rule $\frac{3}{2}\Delta B \pm \frac{1}{2}\Delta L = 0 \mod 4$. In particular, no proton-decay operators with less than 6 fermions are allowed and our previous operator analysis remains valid (with the dominant B-violating effects given by dimension-17 (-15) operators in the $\mathcal{L}_+$ ($\mathcal{L}_-$) chirality assignment). Furthermore, it is also clear that in this model there are no $n-\bar{n}$ oscillations (which require a $\Delta B = 2$ operator) and no Majorana masses ($\Delta L = 2$).

The remarkable suppression of B violation established here is due to the Z_8 symmetry. But if, for example, x^4 and x^5 were compactified on a rectangular orbifold (T^2/Z_2 with different radii), then the Z_8 would be reduced to a Z_4. It would follow that dimension-10 operators such as $(\overline{\mathcal{Q}}_+^c \Gamma^\alpha \mathcal{Q}_+)(\overline{\mathcal{Q}}_+^c \Gamma_\alpha \mathcal{L}_+)$, which do not include terms with only zero-modes, would induce proton decay via loops. [For the $\mathcal{L}_-$ chirality assignment, the leading B-violating operators have dimension 12, *e.g.* $(\overline{\mathcal{Q}}_+^c \Gamma^\alpha \mathcal{Q}_+)(\overline{\mathcal{L}}_-^c \Gamma_\alpha \mathcal{D}_-)\tilde{\mathcal{H}}$.] It is therefore important to explore whether the Z_8 symmetry is indeed left intact by the compactification. Here we offer only a few comments regarding the choice of vacuum. It has been noted that the square configuration is an extremum of the Casimir energy of bulk fields, including gravity, while the rectangular one is not [11]. Although the lowest order computation indicates that the square configuration is a saddle point of the effective potential (the minimum corresponds to a "rhombus" that has a Z_6 symmetry, still forbidding B-violating operators with less than three quarks and three leptons), it could be that a complete computation will reveal this configuration to be a minimum. Alternatively, it is possible to freeze the shape of the torus to be square by a Z_4 orbifold identification, in which case the Z_8 symmetry remains unbroken. Either of these possibilities would leave intact our conclusions based on the T_2/Z_2 square orbifold regarding B violation.

We finally mention that some of the fields describing gravitational fluctuations are also charged under the Z_8 (the components of the metric along x^4 and x^5). However, we need not worry about the effects of these fields for nucleon decay if the radius stabilization mechanism makes them heavier than the proton or if they are projected out by a Z_4 orbifold identification.

In conclusion, we have shown that the combination of standard-model gauge invariance, 6-dimensional Lorentz invariance, and compactification of the two extra dimensions on a T^2/Z_2 orbifold of equal radii naturally suppresses proton decay to acceptable levels, even with the scale of baryon number violation in the TeV range.

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