

THE EQUIVALENCE PRINCIPLE AS A SYMMETRY

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Abstract

It is shown that the extra coordinate of 5D induced-matter and membrane theory is related in certain gauges to the inertial rest mass of a test particle. This implies that the Weak Equivalence Principle is a geometric symmetry, valid only in the limit in which the test mass is negligible compared to the source mass. Exact solutions illustrate this, and show the way to possible resolutions of the cosmological-constant and hierarchy problems.

1 Introduction

The Weak Equivalence Principle is commonly taken to mean that in a gravitational field the acceleration of a test particle is independent of the properties of the latter, including its rest mass. Recently, however, the extension of 4D general relativity to 5D has led to the isolation of a fifth force, which exists for both induced-matter theory [1, 2] and membrane theory [3, 4]. These two versions of what used to be called Kaluza-Klein theory allow dependence on an extra coordinate l , and it is now known that their field

equations are essentially the same [5]. In both theories, the extra force per unit rest mass is an acceleration which is inertial in the Einstein sense, arising from the motion in the fifth dimension with respect to the 4D part of the manifold which we call spacetime [6]. This extra acceleration has already been related to the (inertial) rest mass m of a test particle [1, 3] in certain choices of coordinate frame (or gauge), and in general its presence represents a technical violation of the 4D WEP. Such violations of the 4D WEP in $N (> 4)$ D field theory have been mentioned before [7-11; for a short review see ref. 6, pp. 85-88]. However, the WEP is known from experiments conducted from the time of Galileo to now to be obeyed with an accuracy of at least 1 part in 10^{11} [12]. The purpose of the present work is to clarify the status of the 4D Weak Equivalence Principle in $N (> 4)$ D extensions of general relativity. We will do this for 5D; but the extension to higher N as in 10D superstrings, 11D supergravity and 26D string theory is straightforward, and in fact guaranteed by Campbell's theorem [13-15]. The plan is as follows: (a) Marshall extant mathematical results [16-22], showing that they have the consistent physical interpretation that the extra coordinate l measures the (inertial) rest mass of a test particle m ; (b) Illustrate the cogency of this inference by giving 3 exact l -dependent solutions of the 5D

field equations which generalize the 4D de Sitter solution of general relativity as widely used in particle physics [23-27], thereby generalizing the concept of “the vacuum” and opening a way to a resolution of the cosmological-constant problem; (c) Use the scalar potential as a classical analog of the Higgs field [6, 28], leading to an expression for the masses of real particles which avoids the hierarchy problem; (d) Conclude that the WEP is a geometric symmetry, valid only in the limit where the mass of a test particle is negligible compared to the mass of the source, thus supporting new endeavors [29, 30] to look for violations.

2 The Nature of the Fifth Coordinate

There are 5 degrees of coordinate freedom in an unrestricted 5D Riemannian manifold, of which 4 can be used to remove the potentials of electromagnetic type, giving the line element $dS^2 = g_{AB}dx^A dx^B = g_{\alpha\beta}(x^\gamma, l) dx^\alpha dx^\beta + \epsilon\Phi^2 dl^2$ ($A = O, 123, 4$; $\alpha = O, 123$). The signature is $+(---)\epsilon$ where $\epsilon = \pm 1$ is not restricted by Campbell’s theorem [13-15], the usual $\epsilon = -1$ admitting particle-like solutions and $\epsilon = +1$ admitting wave-like solutions [22]. The coordinates are $x^0 = t$ for time, $x^{123} = xyz$ (or $r\theta\phi$) for space and an ex-

tra one $x^4 = l$. All will be taken to have physical dimensions of length, and the constants c, G, h will usually be absorbed by a choice of units. It will turn out to be useful to defer usage of the fifth degree of coordinate freedom, though in principle it is available to suppress the scalar potential (Φ) or to restrict the velocity in the fifth dimension. With regard to velocities, we wish to make contact with 4D physics couched in terms of $ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ and 4-velocities $u^\alpha \equiv dx^\alpha/ds$. We will therefore parametrize motions in terms of the elements of 4D proper time ds , a choice which also allows us to handle null 5D paths with $dS = 0$ [2, 4]. With this setup, we can make several observations on the physical nature of the fifth coordinate.

(i) The extra force which appears when the manifold is extended from 4D to 5D has been derived in different ways for induced-matter theory [1, 2] and brane theory [3, 4]. But a generic and shorter way is as follows. The relation

$$g_{\alpha\beta}(x^\gamma, l) u^\alpha u^\beta = 1 \tag{1}$$

is a normalization condition on the 4-velocities. When multiplied by the inertial rest mass m of a test particle, it gives the usual relation $E^2 - p^2 = m^2$ where E is the energy and p is the 3-momentum. (Alternatively, $p^\alpha p_\alpha = m^2$ where $p^\alpha \equiv mu^\alpha$ are the 4-momenta.) There is actually no information in

(1) about the possibility that $m = m(s)$, which applies for example to the case of a rocket which loses mass as it burns fuel and so accelerates. The acceleration in such a case is given by appeal to the law of conservation of linear momentum (see below). However, we can consider the effect of a slight change in the 5D coordinates (including l) by differentiating (1) with respect to s . Doing this and using symmetries under the exchange of α and β to introduce the Christoffel symbols $\Gamma_{\alpha\beta}^{\mu}$, there comes

$$2g_{\alpha\mu}u^{\alpha} \left(\frac{du^{\mu}}{ds} + \Gamma_{\beta\gamma}^{\mu}u^{\beta}u^{\gamma} \right) + \frac{\partial g_{\alpha\beta}}{\partial l} \frac{dl}{ds} u^{\alpha}u^{\beta} = 0 \quad . \quad (2)$$

This reveals that in addition to its usual 4D geodesic motion (the part inside the parenthesis), a particle feels a new acceleration (or force per unit mass). It is due to the motion of the 4D frame with respect to the fifth dimension, and is parallel to the 4-velocity u^{μ} . Explicitly, the parallel acceleration is

$$P^{\mu} = -\frac{1}{2} \left(\frac{\partial g_{\alpha\beta}}{\partial l} u^{\alpha}u^{\beta} \right) \frac{dl}{ds} u^{\mu} \quad . \quad (3)$$

This has no analog in 4D field theory, including Einstein gravity and Maxwell electromagnetism, where forces are orthogonal to the velocities and obey $F^{\mu}u_{\mu} = 0$. (Another way of seeing that an extra force must appear in the extension from 4D to 5D is to note as in ref. 1 that if $F^A u_A = 0$ then $F^{\mu}u_{\mu} = -F^4 u_4 \neq 0$.) To investigate (3), we can evaluate it in the canonical

coordinate system, which is so called because it leads to great algebraic simplification of the geodesic equation and the field equations (see below) and has been extensively used [6, 17-19, 22]. Then $g_{\alpha\beta}(x^\gamma, l) = (l^2/L^2)\bar{g}_{\alpha\beta}(x^\gamma)$, where L is a length introduced for dimensional consistency, and for vacuum 4D spacetimes is given by $L^2 = 3/\Lambda$ where Λ is the cosmological constant [17]. The acceleration (3) can now be evaluated and simplified using (1). Its nature becomes clear in the Minkowski limit, when the motion of the particle is given by

$$\frac{du^\mu}{ds} = P^\mu = -\frac{1}{l} \frac{dl}{ds} u^\mu \quad , \quad (4)$$

$$\text{or} \quad \frac{d}{ds}(l u^\mu) = 0 \quad . \quad (5)$$

The last is just the expected law of conservation of linear momentum, provided $l = m$.

(ii) The action can be used to confirm this. Let us write the 5D interval in terms of its 4D and extra parts using a coordinate system which is perturbed from the pure canonical one noted above. Then with $d\bar{s}^2 \equiv \bar{g}_{\alpha\beta} dx^\alpha dx^\beta$ we

have

$$dS^2 = \frac{l^2}{L^2} \bar{g}_{\alpha\beta}(x^\gamma, l) dx^\alpha dx^\beta + \epsilon \Phi^2(x^\gamma, l) dl^2 \quad (6)$$

$$L^2 dS^2 = l^2 d\bar{s}^2 + \epsilon (\Phi L)^2 dl^2 \quad . \quad (7)$$

Clearly the first term on the right-hand side here involves the conventional element of action $m d\bar{s}$ if $l = m$. It should be noted that even in 4D the action should be written $\int m d\bar{s}$ to account for the possibility that the mass changes along the path, and that in 5D the expression (6) is still general. So the conventional action is the 4D part of a 5D one.

(iii) The 5D geodesic equation minimizes paths via $\delta(\int dS) = 0$, which generalizes the equations of motion in 4D and adds an extra component for the motion in the fifth dimension. The working requires the specification of a starting gauge, and is generally tedious. (See ref. 6, pp. 132-138 and pp. 161-167 for the cases where electromagnetism is and is not included respectively, as well as references to other work.) We therefore quote here two results which are relevant. First, for 5D metrics which are canonical in form, the fifth force noted above is proportional to $dl/d\bar{s}$, and disappears if the latter is zero, making the 4D part of the motion geodesic in the usual sense. It should be noted in passing that the conventional geodesic equation is a statement about accelerations (not forces) caused by the motion of reference

frames, so this result means that 4D geodesic motion is a special case of 5D motion, which latter is inertial in the Einstein sense. Second, for 5D metrics which are independent of $x^0 = t$, there is a constant of the motion which is the analog of the 4D particle energy. When the metric is l -factorized as in the canonical case, electromagnetic terms are absent and the 3-velocity v is projected out, this constant is

$$E = \frac{l}{(1 - v^2)^{\frac{1}{2}}} \quad . \quad (8)$$

One does not have to be Einstein to see that this gives back the conventional 4D energy provided l is identified with the particle rest mass m .

(iv) The field equations for 5D relativity are commonly taken in terms of the Ricci tensor to be $R_{AB} = 0$ ($A = 0, 123, 4$); and by Campbell's theorem [13-15] these contain those of general relativity, which in terms of the Einstein tensor and the energy-momentum tensor are $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ ($\alpha = 0, 123$). Here $G_{\alpha\beta}$ is constructed as usual from the 4D, l -independent parts of the 4D Ricci tensor and scalar. However, $T_{\alpha\beta}$ is an effective or induced source, constructed from the l -dependent parts of these quantities and the scalar field ($g_{44} = \epsilon\Phi^2$). As such, the latter includes parts which can be identified with conventional matter and parts which by default refer to the "vacuum". We will return to the latter concept below, but here we note that the general

expression for the source can be written down after some lengthy algebra.

With the metric in the general form $dS^2 = g_{\alpha\beta}(x^\gamma, l) dx^\alpha dx^\beta + \epsilon\Phi^2 dl^2$, it is

$$8\pi T_{\alpha\beta} = \frac{\Phi_{,\alpha;\beta}}{\Phi} - \frac{\epsilon}{2\Phi^2} \left\{ \frac{\Phi_{,4} g_{\alpha\beta,4}}{\Phi} - g_{\alpha\beta,44} + g^{\lambda\mu} g_{\alpha\lambda,4} g_{\beta\mu,4} - \frac{g^{\mu\nu} g_{\mu\nu,4} g_{\alpha\beta,4}}{2} + \frac{g_{\alpha\beta}}{4} [g^{\mu\nu}_{,4} g_{\mu\nu,4} + (g^{\mu\nu} g_{\mu\nu,4})^2] \right\} . \quad (9)$$

Here a comma denotes the partial derivative with respect to $x^4 = l$ and a semicolon denotes the usual 4D covariant derivative. The expression (9) is known to give back the conventional matter content of a wide variety of 4D solutions [6], but in order to bolster the physical identification of l we note a generic property of it. For $g_{\alpha\beta,4} = 0$, (9) gives $8\pi T \equiv 8\pi g^{\alpha\beta} T_{\alpha\beta} = g^{\alpha\beta} \Phi_{,\alpha;\beta} / \Phi \equiv \Phi^{-1} \square \Phi$; but the extra field equation $R_{44} = 0$, which we will examine below, gives $\square \Phi = 0$ for $g_{\alpha\beta,4} = 0$. Thus $T = 0$ for $g_{\alpha\beta,4} = 0$, meaning that the equation of state is that of radiation when the source consists of photons with zero rest mass. This is as expected.

(v) Algebraic arguments for $l = m$ can be understood from the physical perspective by simple dimensional analysis. The latter is actually an elementary group-theoretic technique based on the Pi theorem, and one could argue that a complete theory of mechanics ought to use a manifold in which spacetime is extended so as to properly take account of the three mechanical bases M, L, T. Obviously, this has to be done in a manner which does not

violate the known laws of mechanics and recognizes their use of the three dimensional constants c , G and h . The canonical metric of induced-matter theory, as employed in several instances above, clearly satisfies these criteria [1, 2]. But the warp metric of brane theory leads to similar results [3, 4]; and it has indeed been argued that the two theories are essentially the same one, expressed in different ways [5]. This leads to an important point: the physical identification of $x^4 = l$ requires a choice of 5D coordinates or gauge. To illustrate this, consider a 5D metric given by

$$dS^2 = \left(\frac{L}{l}\right)^{2a} \bar{g}_{\alpha\beta}(x^\gamma) dx^\alpha dx^\beta - \left(\frac{L}{l}\right)^{4b} dl^2 \quad . \quad (10)$$

Here a , b are constants which can be constrained by the full set of 5D field equations $R_{AB} = 0$ [22]. There are 3 choices: $a = b = 0$ gives general relativity embedded in a flat and physically innocuous extra dimension; $a = -1$, $b = 0$ gives the pure-canonical metric already discussed; while $a = b = 1$ gives a metric which looks different but is actually the canonical one after the coordinate transformation $l \rightarrow L^2/l$. We see that the last two cases describe the same physics but in terms of different choices of l . Temporarily introducing the relevant constants, these are

$$l_E = \frac{Gm}{c^2} \quad , \quad l_P = \frac{h}{mc} \quad (11)$$

in what may be termed the Einstein and Planck gauges. These represent convenient choices of $x^4 = l$, insofar as they represent parametrizations of the inertial rest mass m of a test particle which fit with known laws of 4D physics such as the conservation of momentum (see above: the fifth force conserves $l_E u^\mu$ or $l_P^{-1} u^\mu$). However, 5D relativity as based on the field equations $R_{AB} = 0$ is covariant under the 5D group of transformations $x^A \rightarrow \bar{x}^A(x^B)$, which is wider than the 4D group $x^\alpha \rightarrow \bar{x}^\alpha(x^\beta)$. Therefore 4D quantities $Q(x^\alpha, l)$ will in general change under a change of coordinates that includes l . This implies that we can only recognize m in certain gauges.

The import of the preceding comments (i)-(v) is major for the Weak Equivalence Principle. In gauges like those of Einstein or Planck, or ones close to them, the dependence of the ordinary 4D metric of spacetime on the extra coordinate $l = m$ will in general cause the acceleration of a test particle to depend to a degree (determined by the solution) on the rest mass of the latter. This is a clear violation of the WEP. Even in other gauges, l and its associated potential Φ must be connected with the concept of particle (as opposed to source) mass. We will formalize this using the field equations below, but here we point out that such a dependency can be expected on physical grounds: a test particle of mass m in the field of a source mass M

only has a negligible effect on the metric in the limit $m/M \rightarrow 0$. The effects that follow from $m/M \neq 0$ have traditionally been handled in areas such as gravitational radiation by considering the “back reaction” of the test particle on the field of the source [28]. This is clearly an approximation to the real physics, and must break down when m/M is significant. In other words, the WEP as viewed from 5D is a geometric symmetry which must break down at some level.

3 Vacua in 5D

To illustrate the argument that the 4D WEP is a symmetry of a 5D metric, it is natural to look at solutions of the field equations that represent a test particle in an otherwise empty space. Many l -dependent solutions of the field equations are known, including ones for cosmology and the solar system which are in agreement with observations [6]. However, the class of solutions which represents empty 3D space has not been much studied. There are technical and conceptual reasons for this. Technically, the field equations $R_{AB} = 0$ involve in general 15 nonzero components of the Ricci tensor. Even if we look for static non-electromagnetic solutions, it is still not easy to find

ones of the desired type, which should have 3D spherical symmetry and be (r, l) -dependent. Conceptually, the idea of a vacuum in 5D is blurry. Even in 4D, $R_{\alpha\beta} = 0$ admits solutions which are empty of ordinary matter but have 4D curvature, the prime example being the de Sitter solution in which spacetime is curved by the cosmological constant Λ , or alternatively by a vacuum fluid with density and pressure given by $\rho_v = -p_v = \Lambda/8\pi$. This solution has been extensively used in models of the origin of the classical universe based on quantum effects, such as tunneling [23, 24]. In 5D, the equations $R_{AB} = 0$ admit solutions which are apparently empty, but whose 4D subspaces may be curved and contain “ordinary” matter as determined by the embedded Einstein equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$ (see above). A clever but only partially successful way to sidestep these issues is to look for 5D solutions which are not only Ricci-flat with $R_{AB} = 0$ but also Riemann-flat with $R_{ABCD} = 0$ [25-27]. We will present 3 such solutions below, but wish to make a cautionary remark based on the contents of the preceding section: The physical application in 4D of any l -dependent solution in 5D depends on the choice of gauge. The solutions which follow are all equivalent to a flat 5D (Minkowski) manifold, but the 5D coordinate transformations which must exist between them are for technical reasons unknown, and their

different forms describe different 4D physical vacua.

The following solutions may be confirmed by hand or computer to satisfy

$R_{AB} = 0$ and $R_{ABCD} = 0$:

$$dS^2 = \frac{l^2}{L^2} \left\{ \left(1 - \frac{r^2}{L^2}\right) dt^2 - \frac{dr^2}{(1 - r^2/L^2)} - r^2 d\Omega^2 \right\} - dl^2 \quad (12)$$

$$dS^2 = \frac{l^2}{L^2} \left\{ \left[\left(1 - \frac{r^2}{L^2}\right)^{1/2} + \frac{\alpha L}{l} \right]^2 dt^2 - \frac{dr^2}{(1 - r^2/L^2)} - r^2 d\Omega^2 \right\} - dl^2 \quad (13)$$

$$dS^2 = \frac{l^2}{L^2} \left\{ \left[\left(1 - \frac{r^2}{L^2}\right)^{1/2} + \frac{\alpha L}{l} \right]^2 dt^2 - \frac{dr^2}{(1 - r^2/L^2)} - \left(1 + \frac{\beta L^2}{rl}\right)^2 r^2 d\Omega^2 \right\} - dl^2 \quad (14)$$

Here $d\Omega^2 \equiv (d\theta^2 + \sin^2 \theta d\phi^2)$, so all 3 solutions are spherically symmetric in 3D. The first is a 5D canonical embedding of the 4D de Sitter solution provided the identification $L^2 = 3/\Lambda$ is made (see above). However, in general L measures the size of the potential well associated with $x^4 = l$, as shown by the de Sitter form (12). Solutions like (12)-(14) depend in general on two dimensionless constants α, β . We have examined the properties of (12)-(14) extensively, but here note only their generic features. These can

be appreciated by combining (12)-(14) in the following form:

$$dS^2 = \frac{l^2}{L^2} \{A^2 dt^2 - B^2 dr^2 - C^2 r^2 d\Omega^2\} - dl^2 \quad (15)$$

$$A \equiv \left(1 - \frac{r^2}{L^2}\right)^{1/2} + \frac{\alpha L}{l}, \quad B \equiv \frac{1}{(1 - r^2/L^2)^{1/2}}, \quad C \equiv 1 + \frac{\beta L^2}{rl}. \quad (16)$$

The 4D subspaces defined by these solutions are curved, with a 4D Ricci scalar 4R which by Einstein's equations is related to the trace of the 4D energy-momentum tensor by ${}^4R = -8\pi T$. The general expression for 4R for any 5D metric of the form $dS^2 = g_{\alpha\beta} dx^\alpha dx^\beta + \epsilon \Phi^2 dl^2$ as used before is:

$${}^4R = \frac{\epsilon}{4\Phi^2} [g^{\mu\nu}{}_{,4} g_{\mu\nu,4} + (g^{\mu\nu} g_{\mu\nu,4})^2] \quad . \quad (17)$$

The special expression for (15), (16) is:

$${}^4R = -8\pi T = -\frac{2}{L^2} \left[\frac{1}{AB} + \frac{2}{ABC} + \frac{1}{C^2} + \frac{2}{C} \right] \quad . \quad (18)$$

This shows that stress-energy is concentrated around singular shells where one of A , B or C is zero. The equation of state is in general anisotropic ($T_1^1 \neq T_2^2$). If one replaces $1/L^2$ in (18) by its de Sitter limit $\Lambda/3$, it becomes obvious that the meaning of the cosmological “constant” requires a drastic rethink. The effective Λ is in general a function of r and l , opening a way to a resolution of the cosmological-constant problem. Indeed, there is

no such thing as “the vacuum” in 5D physics, but rather structured vacua.

4 Particle Masses in 5D

A common view, notably from inflationary quantum theory, is that particles are intrinsically massless, gaining masses from the Higgs field [28]. This view is in principle compatible with the recent demonstration that particles which move on null paths in 5D can move on timelike paths in 4D, both for induced-matter theory [2] and brane theory [4]. The scalar field $g_{44} = \epsilon\Phi^2$ of 5D relativity can be suppressed by use of one of the 5 degrees of coordinate freedom (see above); but solutions are known for both solitons and cosmologies where Φ contains significant physics, and it has been suggested that Φ is the classical analog of the Higgs field [6]. There are in fact several ways to define the mass of a particle in 5D. Here, we wish to give a short account of one which is mathematically straightforward [16, 22] and builds on the physical identification of the extra coordinate arrived at in section 2.

There we saw that $m = l$ for metrics of the canonical form with $|g_{44}| = 1$. For metrics which are of other forms, we can define an effective mass by

$$m \equiv \int |\Phi| dl = \int |\Phi (dl/ds)| ds \quad . \quad (19)$$

This is in line with how proper distance is defined in 3D. In practice, Φ would be given by a solution of the 5D field equations, and dl/ds would be given by a solution of the extra component of the 5D geodesic equation (or directly from the metric for a null 5D path and a particle at rest in 3D). We note that a potential problem with this approach is that Φ may show horizon-like behaviour. An example is the Gross/Perry/Davidson/Owen/Sorkin monopole, which in terms of a radial coordinate r which makes the 3D part of the metric isotropic has $g_{44} = -\Phi^2 = -[(1 - a/2r)/(1 + a/2r)]^{2\beta/\alpha}$ where a is the source strength and α, β are dimensionless constants constrained by the field equations to obey $\alpha^2 = \beta^2 + \beta + 1$ [ref. 6 p. 70]. This problem may be avoided by restricting the physically-relevant size of the manifold [6, 28]. Another potential problem is that real particles may have $\Phi = \Phi(x^\gamma, l)$ so complicated as to preclude finding an exact solution. This problem may be avoided by expanding Φ in a Fourier series:

$$\Phi(x^\gamma, l) = \sum_{n=-\infty}^{+\infty} \Phi^{(n)}(x^\gamma) \exp(inl/L) \quad . \quad (20)$$

Here L is the characteristic size of the extra dimension, which by (17) is related to the radius of curvature of the embedded 4-space which the particle inhabits. It should be noted that in both modern versions of 5D relativity, namely induced-matter theory and brane theory, the extra dimension is not

compactified [1-5]. Thus we do not expect a simple tower of states based on the Planck mass, but a more complicated spectrum of masses that offers a way out of the hierarchy problem.

Underlying the comments of the preceding paragraph is the field equation $R_{44} = 0$ which governs Φ . The full set of field equations $R_{AB} = 0$ contains 15 components. These can be reduced by tiresome algebra for the general metric noted before, namely $dS^2 = g_{\alpha\beta}(x^\gamma, l) dx^\alpha dx^\beta + \epsilon\Phi^2(x^\gamma, l) dl^2$, which only uses 4 of the 5 degrees of coordinate freedom to remove the potentials ($g_{4\alpha}$) of electromagnetic type. The result is sets of 10, 4 and 1 equations [6]. The first set comprises the Einstein equations $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$, with $T_{\alpha\beta}$ given by (9). The second set comprises the conservation equations

$$P_{\alpha;\beta}^\beta = 0 \quad (21)$$

$$P_\alpha^\beta \equiv \frac{1}{2\Phi} (g^{\beta\sigma} g_{\sigma\alpha,4} - \delta_\alpha^\beta g^{\mu\nu} g_{\mu\nu,4}) \quad . \quad (22)$$

These are usually easy to satisfy in the continuous fluid of induced-matter theory as developed by Wesson and others, and are related to the stress in the surface ($l = 0$) of membrane theory with the Z_2 symmetry as developed by Randall and Sundrum (see ref. 5 for a discussion of both). The remaining

field equation is the scalar relation

$$\square\Phi = -\frac{\epsilon}{2\Phi} \left[\frac{g^{\lambda\beta}{}_{,4}g_{\lambda\beta,4}}{2} + g^{\lambda\beta}g_{\lambda\beta,44} - \frac{\Phi_{,4}g^{\lambda\beta}g_{\lambda\beta,4}}{\Phi} \right] . \quad (23)$$

Here as before $\square\Phi \equiv g^{\alpha\beta}\Phi_{,\alpha;\beta}$ and some of the terms on the right-hand side are present in the energy-momentum tensor of (9). In fact, one can rewrite (23) for the static case as Poisson's equation with an effective source density for the Φ -field. In general, (23) is a wave equation with a source induced by the fifth dimension. This supports the series expansion (20), and implies that the inertial rest mass of a particle as defined by (19) arises from the scalar field.

5 Conclusion

Gravity in general relativity is a force which is encoded in the Christoffel symbols as coupled to the 4-velocities, and is inertial in the sense that it arises from the motion of a particle with respect to a 4D frame of reference or manifold which is not flat. The fifth force of induced-matter and membrane theory is similar [1, 3]. The normalization condition for the 4-velocities (1) shows that ordinary 4D geodesic motion is augmented by a fifth force (per unit mass) or acceleration (2), which while it depends on the velocity

in the fifth dimension has the unique property of acting parallel to the 4-velocity (3). This force depends in general on $x^4 = l$, the fifth coordinate of the particle, and therefore violates the Weak Equivalence Principle, at least technically. However, it is compatible with the principle of conservation of linear momentum (5), which leads to the identification of l with the (inertial rest) mass of the test particle m . Other aspects of 4D gravity support this. The presence of $x^4 = l$ in exact solutions of the 5D field equations (12)-(14), which would otherwise be called empty, lead to the realization that there are 5D vacua with structure. A definition for the rest mass m , analogous to that of proper distance and valid for any 5D metric (19), is compatible with the identification of the scalar field of classical 5D relativity with the Higgs field of particle physics, its field equation (23) describing a wave with a source. The above conclusions clearly open ways to resolving well-known problems that arise from mismatches of classical and quantum physics, notably the cosmological-constant and hierarchy problems.

The WEP, however, is rendered particularly transparent. It is a geometric symmetry, valid only in the limit in which the metric is independent of $x^4 = l$, that is the limit where the mass of a test particle is negligible compared to other terms such as the mass of the source. New techniques to

measure departures from the WEP are technically challenging [28-30]. But if the 4D world is part of one with 5 or more dimensions, violations of the WEP must exist.

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