WESSON'S GRAVITY AND MACH'S PRINCIPLE

Guang-wen MA
Department of Physics, Zhengzhou University, Zhengzhou, Henan 450052, PR China

Received 8 January 1990; revised manuscript received 22 March 1990; accepted for publication 10 April 1990
Communicated by J.P. Vigier

The general expressions of the variation of the rest mass of a test body with proper time and spatial position in a gravitational field are given in the framework of Wesson's 5D space–time–mass gravity. The position-dependent variation of the rest mass of a test body in a quasi-static and spherically symmetric gravitational field is discussed, and it is indicated that such variation embodies the spirit of Mach's principle in the sense that the inertial mass depends on the distribution of matter in the universe.

1. In 1880, Mach first started a constructive criticism of Newton's "absolute space". He pointed out that the inertia depends on the distribution of matter in the universe. This is called Mach's principle [1]. Many authorities on physics, such as Einstein and Dirac, greatly admired this viewpoint, and have made attempts to formulate it. Einstein's theory of general relativity embodies Mach's principle to a certain extent [2]. Dirac's large number hypothesis [3] is a generalization of Mach's principle. However, it is not explained satisfactorily even in more complete theories of gravity with variable gravitational constant, such as the Brans–Dicke scalar–tensor theory [4]. Recently, Wesson proposed a new variable gravity, that is 5D space–time–mass gravity [5,6]. In the framework of Wesson's gravity, we have proposed an explanation of the physical meaning of the fifth dimension subspace, in which we supposed that the rest mass of a body corresponds to the length of a "line segment" of the fifth subspace [7]. According to this explanation, we will give, in the present Letter, the expressions of the variable rest mass of a test body in a gravitational field, study the position-dependent variation of the rest mass of a test body in a quasi-static and spherically symmetric gravitational field and then discuss the relation of this result to Mach's principle.

2. It is mentioned above that the rest mass of a body corresponds to the length of a line segment of the fifth dimension subspace in 5D space–time–mass. It can be formulated as

$$m=\frac{c^2}{G}\int_{x^4}^{x^4+l}\sqrt{-g_{44}}\,\mathrm{d}x^4\,, \tag{1}$$

where l is constant for a given body. The differentiation of m with respect to the proper time can be found as

$$\frac{\mathrm{d}m}{\mathrm{d}\tau}=-\frac{c^2}{2G}\int_{x^4}^{x^4+l}\frac{1}{\sqrt{-g_{44}}}\frac{\partial g_{44}}{\partial x^i}\frac{\mathrm{d}x^i}{\mathrm{d}\tau}\mathrm{d}x^4 +\frac{c^2}{G}\sqrt{-g_{44}(x^\mu,x^4+l)} -\frac{c^2}{G}\sqrt{-g_{44}(x^\mu,x^4)}\,, \tag{2}$$

where the indices i and μ run from 0 to 4 and from 0 to 3, respectively. If g_{44} is independent of the fifth coordinate x^4, the expressions (1) and (2) reduce to

$$m=\frac{c^2}{G}\sqrt{-g_{44}}\,l\,, \tag{3}$$

$$\frac{\mathrm{d}m}{\mathrm{d}\tau}=\frac{1}{2}\frac{m}{g_{44}}\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}\frac{\partial g_{44}}{\partial x^\mu} \tag{4}$$

respectively. The partial differentiations of m with respect to spatial coordinates are

$$\frac{\partial m}{\partial x^a}=\frac{1}{2}\frac{m}{g_{44}}\frac{\partial g_{44}}{\partial x^a}, \tag{5}$$

here index a runs from 1 to 3.

These results show that even if the metric coefficients do not depend on x^4, the rest mass of a test body in a gravitational field may vary with time and/or position. When it is further assumed that the metric coefficients are independent of t, the rest mass of a test body varies with the spatial position only. This variation can give an explanation and a formulation for Mach's principle. The basic idea is as follows. The rest mass of a test body depends on the spatial position, thereby on the gravitational field, and the latter depends on the distribution of matter in the universe. Noting that the inertial mass is equivalent to the gravitational mass, we are led to conclude that the inertia of a body depends on the distribution of matter in the universe. In the next section we will give a concrete illustration of this idea by solving a quasi-static and spherically symmetric gravitational field.

Of course, there is, strictly, no static gravitational field in an evolving universe. However, we can sometimes neglect the effect of the evolution for many astrophysical problems and have a so-called quasi-static gravitational field.

Furthermore, considering that the fifth coordinate itself has not any longer direct physical meaning, we assume, for simplicity, that the metric coefficients are independent of the fifth coordinate in the following.

3. For a quasi-static and spherically symmetric gravitational field, the line element in an isotropic coordinate system can be written as

$$\mathrm{d}s^2=A^2(r)\,\mathrm{d}t^2-B^2(r)\,(\mathrm{d}r^2+r^2\,\mathrm{d}\theta^2+r^2\sin^2\theta\,\mathrm{d}\varphi^2)$$
$$+C^2(r)\,\mathrm{d}\psi^2\,. \tag{6}$$

The nonzero Christoffel symbols are

$$\Gamma^0_{01}=\Gamma^0_{10}=A'/A\,,\quad \Gamma^1_{00}=AA'/B^2\,,$$
$$\Gamma^1_{11}=B'/B\,,\quad \Gamma^1_{22}=-(B'/B+1/r)r^2\,,$$
$$\Gamma^1_{33}=-(B'/B+1/r)r^2\sin^2\theta\,,$$
$$\Gamma^1_{44}=CC'/B^2\,,\quad \Gamma^2_{12}=\Gamma^2_{21}=B'/B+1/r\,,$$
$$\Gamma^2_{33}=-\sin\theta\cos\theta\,,\quad \Gamma^3_{13}=\Gamma^3_{31}=B'/B+1/r\,,$$
$$\Gamma^3_{23}=\Gamma^3_{32}=\tan\theta\,,\quad \Gamma^4_{14}=\Gamma^4_{41}=C'/C\,,$$

where the prime denotes differentiation with respect to r. The 5D vacuum field equations are

$$\frac{A''}{A}+\frac{A'B'}{AB}+\frac{A'C'}{AC}+2\frac{A'}{Ar}=0\,,$$
$$\frac{A''}{A}+\frac{C''}{C}+2\frac{B''}{B}-\frac{A'B'}{AB}-\frac{B'C'}{BC}$$
$$-2\frac{B'^2}{B^2}-2\frac{B'}{Br}=0\,,$$
$$\frac{B''}{B}+\frac{A'B'}{AB}+\frac{B'C'}{BC}+\frac{A'}{Ar}+3\frac{B'}{Br}+\frac{C'}{Cr}=0\,,$$
$$\frac{C''}{C}+\frac{B'C'}{BC}+\frac{A'C'}{AC}+2\frac{C'}{Cr}=0\,. \tag{7}$$

The solutions of eqs. (7), which have been found by Davidson and Owen in another 5D theory [8], are of the forms

$$A^2=\left(\frac{1-1/ar}{1+1/ar}\right)^{2\epsilon_0\kappa/\sqrt{\kappa^2-\kappa+1}}\,,$$
$$B^2=\frac{(1+1/ar)^{2[\epsilon_0(\kappa-1)/\sqrt{\kappa^2-\kappa+1}+1]}}{(1-1/ar)^{2[\epsilon_0(\kappa-1)/\sqrt{\kappa^2-\kappa+1}-1]}}\,,$$
$$C^2=C_0^2\left(\frac{1+1/ar}{1-1/ar}\right)^{2\epsilon_0/\sqrt{\kappa^2-\kappa+1}}\,, \tag{8}$$

where $\epsilon_0^2=1$, and a, κ and C_0 are constants. One of the geodesic equations gives $C^2\mathrm{d}\psi/\mathrm{d}\tau=\text{const}$. If we set this constant to zero, the rest are identical to those of the 4D theory. Thus, we can discuss the physical meaning of the constant a and estimate the value of κ by using the results of the post-Newtonian test in the 4D theory. For this purpose, we carry out the Robertson expansions of A^2 and B^2, which are

$$A^2=1-\frac{4\epsilon_0\kappa}{\sqrt{\kappa^2-\kappa+1}}\frac{1}{ar}+\frac{8\kappa^2}{\kappa^2-\kappa+1}\frac{1}{a^2r^2}+\dots$$
$$=1-2\alpha\frac{GM}{r}+2\beta\frac{G^2M^2}{r^2}+\dots\,,$$

$$B^2=1+\frac{4\epsilon_0(\kappa-1)}{\sqrt{\kappa^2-\kappa+1}}\frac{1}{ar}+\dots$$
$$=1+2\gamma\frac{GM}{r}+\dots\,, \tag{9}$$

where

$$\alpha=\frac{2\epsilon_0\kappa/\sqrt{\kappa^2-\kappa+1}}{aGM}\,,\quad \beta=\frac{4\kappa^2/(\kappa^2-\kappa+1)}{a^2G^2M^2}\,,$$

$$\gamma=\frac{2\epsilon_0(\kappa-1)/\sqrt{\kappa^2-\kappa+1}}{aGM}$$

are three post-Newtonian parameters, and M is the mass of the source of the gravitational field. According to the definition of M, we should set $\alpha=1$. Then we have $\beta=1$ and $\gamma=(\kappa-1)/\kappa$. The 4D Einstein theory gives $\alpha=\beta=\gamma=1$, which corresponds to the case $\kappa\to\infty$. In fact, the solutions (8) reduce to the Schwarzschild exterior solutions and $C^2=C_0^2$ if κ is infinite. Brans–Dicke theory gives $\alpha=\beta=1$, and $\gamma=(\omega+1)/(\omega-1)$, which corresponds to $\kappa=\omega+2$. The experimental result of the "time delay" of radar signals gives $\gamma=1.000\pm0.002$ [9]. This means that $\omega\geqslant500$. If we accept this result and take the upper limit of the value of γ, we have $a=2.002\epsilon_0/GM$. Denoting $m(\infty)=m_0$, we have

$$m=m_0\left(\frac{1+2.002GM/r}{1=2.002GM/r}\right)^{0.004} \qquad (10)$$

and

$$\frac{m'}{m}=-0.008\times\frac{2.002GM/r^2}{1-(2.002GM/r)^2}\,. \qquad (11)$$

Of course, expression (11) gives only an upper limit of the variation of mass.

4. From the point of view of Mach's principle, the inertia depends on the distribution of matter in the universe, while the inertial mass is the measure of the inertia. In this sense, the result obtained in this Letter shows that the 5D space–time–mass theory embodies Mach's principle.

From the solutions (8) we can see that C^2, thereby m, becomes infinite if $r=\epsilon_0/a$. This is, of course, unacceptable. Our explanation for this is that the treatment as a one-body problem is no longer correct when m becomes large enough along with the decreasing of r and the appropriate approach should be a treatment as a two-body problem.

The author would like to thank Professor Hong-ya Liu and Professor Li-shi Chang for useful discussions.

References

[1] E. Mach, The science of mechanics, transl. T.J. McCormach, 2nd. Ed. (Open Court Publishing Co., 1893).
[2] J.M. Cohen, in: Relativity theory and astrophysics, Vol. 1. Relativity and cosmology, ed. J. Ehlers (Am. Math. Soc., Providence, 1967).
[3] P.A.M. Dirac, Nature 139 (1937) 323.
[4] C.H. Brans and R.H. Dicke, Phys. Rev. 124 (1961) 925.
[5] P.S. Wesson, Astron. Astrophys. 119 (1983) 145.
[6] P.S. Wesson, Gen. Rel. Grav. 16 (1984) 193.
[7] G. Ma, Phys. Lett. A 143 (1990) 183.
[8] A. Davidson and D.A. Owen, Phys. Lett. B 155 (1985) 247.
[9] R.D. Reasenberg et al., Astrophys. J. 234 (1979) L219.