# Fourth component of relativistic forces

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The Lorentz scalar interaction gives rise to a fourth component of the relativistic four-force which has a different form than that of the vector interaction, the difference arising from the variable mass for particles interacting through the scalar potential as compared to the constant mass for particles interacting through the vector potential. The Lorentz scalar potential, with its pedagogically interesting dynamics, is rarely mentioned in intermediate-level or first-year, graduate-level textbooks.

## I. INTRODUCTION

Although interactions involving particles with a constant mass are the ones most frequently encountered in physical situations, those for which the mass varies with the potential (Lorentz scalar potential) or with the potential and the particle velocity (tensor potential) are of interest to theorists (meson interactions, gravitation), and would be very instructive to the student. Pleas to include the Lorentz scalar interaction in intermediate-level and first-year, graduate-level textbooks have been largely ignored.[1-3] This paper is in part another such plea, as well as an example of when the fourth component of a relativistic force is not of the form usually given in those texts which include relativistic dynamics.

## II. CONSTANT MASS

Let the mass $m'$ of a particle be defined by its four-momentum

$$\mathsf{P} = m'\mathsf{U}, \tag{1}$$

where U is the four-velocity[4]

$$\mathsf{U} = (1 - v^2/c^2)^{-1/2}(\mathbf{v}, ic) = \gamma(\mathbf{v}, ic). \tag{2}$$

The mass $m'$ might or might not be constant. If the particle is acted on by a four-force

$$\mathsf{F} = (\mathbf{F}, iF_4), \tag{3}$$

then its equation of motion is

$$\frac{d\mathsf{P}}{d\tau} = \mathsf{F}, \tag{4}$$

where $\tau$ is the proper time for the particle.

The scalar product of this equation of motion and its four-velocity, combined with the property

$$\mathsf{U}\cdot\mathsf{U} \equiv (1 - v^2/c^2)^{-1}(\mathbf{v}, ic)\cdot(\mathbf{v}, ic) = -c^2, \tag{5}$$

yields a relation between the mass $m'$ and the force[5]

$$-d(m'c^2)/d\tau = \mathsf{F}\cdot\mathsf{U}. \tag{6}$$

The mass $m'$ will be constant (proper mass) if and only if

$$\mathsf{F}\cdot\mathsf{U} = 0. \tag{7}$$

This holds for the vector potential (electromagnetic force), but not for a Lorentz scalar potential or for a tensor potential.[2,6]

For any particle with constant mass, the spatial and the fourth components of the four-force Eq. (3) are related through Eq. (7):

$$F_4 = \mathbf{F}\cdot\mathbf{v}/c. \tag{8}$$

This is the only case considered by most intermediate-level or first-year, graduate-level textbooks on electrodynamics, mechanics, or modern physics. Sometimes Eq. (8) is presented without an explicit statement that it only holds for interactions involving a constant mass.

## III. LORENTZ SCALAR POTENTIAL

The Lorentz scalar potential $\Phi(\mathbf{r}, t)$ gives rise to the four-force

$$\mathsf{F} = -g\Box\Phi, \tag{9}$$

where $g$ plays the same role as does charge in electromagnetism, and the quad is the four-dimensional differential operator

$$\Box = \left(\nabla, \frac{\partial}{ic\,\partial t}\right). \tag{10}$$

However, the Lorentz scalar potential must not be confused with the fourth component of the electromagnetic vector potential, which is often called the scalar potential in three-dimensional calculations; the latter is not Lorentz invariant. Using the fact that

$$\frac{d}{d\tau} = \gamma\left(\mathbf{v}\cdot\nabla + \frac{\partial}{\partial t}\right) \equiv \mathsf{U}\cdot\Box, \tag{11}$$

Eq. (6) can be written as

$$\frac{d(m'c^2)}{d\tau} = g\mathsf{U}\cdot\Box\Phi \equiv \frac{gd\Phi}{d\tau}; \tag{12}$$

therefore the mass $m'$ is not constant but rather

$$m' = m + g\Phi/c^2, \tag{13}$$

where $m$ is the constant of integration of Eq. (12) which is identified as the field-free particle mass; hence the momentum defined by Eq. (1) is

$$\mathsf{P} = (m + g\Phi/c^2)\mathsf{U}. \tag{14}$$

These results can also be obtained using Lagrangian methods[1,3,6,7] which might not be appropriate for use at the intermediate level.

The relation of the fourth component to the spatial components of the four-force is obtained by combining Eqs. (6) and (13):

$$\frac{-d(mc^2 + g\Phi)}{d\tau} = \gamma(\mathbf{F}\cdot\mathbf{v} - F_4c) \tag{15}$$

so that

$$F_4 = \mathbf{F}\cdot\mathbf{v}/c + gd\Phi/cdt, \tag{16}$$

  

which by virtue of Eq. (11) is in agreement with Eq. (9),

$$F_4 = \frac{g}{c}\frac{\partial \Phi}{\partial t}, \tag{17}$$

but is not the same as that for a vector interaction Eq. (8). The first term on the right-hand side of Eq. (16) can as usual be interpreted as the work done on the particle by the three-force per unit time, while the second term can be interpreted as power expended in changing the potential "environment" of the particle. Leibovitz points out[2] that the potential energy contributes to the mass $m'$ and that the potential energy is localized in the frame of the moving particle.

## IV. DISCUSSION

There are pedagogic advantages in discussing the Lorentz scalar potential in intermediate-level and first-year, graduate-level textbooks:

(a) The significance of and conditions for Eq. (8) are impressed on the student by including a counter-example.

(b) Using the standard techniques for elliptical orbital motion of a point particle around a fixed force-center (the field approximation), the relativistic equations of motion can be used to calculate the perihelion advance when the particle is in a vector potential, and the perihelion regression when it is in a Lorentz scalar potential.[8] This would be most effectively done with one case being worked out as an example and the other case left as an exercise for the student. At present very few intermediate mechanics textbooks include these illustrative examples of dynamics.[3]

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[1] O. Bergmann, Am. J. Phys. **24**, 38 (1956).
[2] C. Leibovitz, Am. J. Phys. **37**, 834 (1969).
[3] I. Bloch and H. Crater, Am. J. Phys. **49**, 67 (1981).
[4] Most intermediate-level books use the imaginary number $i = (-1)^{1/2}$ notation for the time-like component of four-vectors, rather than the use of a metric.
[5] J. L. Synge, *Relativity: The Special Theory* (North Holland, Amsterdam, 1965), pp. 165–167.
[6] G. Kalman, Phys. Rev. **123**, 384 (1961).
[7] J. L. Anderson, *Principles of Relativity Physics* (Academic, New York, 1967), pp 288–290.
[8] C. M. Andersen and H. C. von Baeyer, Ann. Phys. (NY) **62**, 120 (1971); H. P. Robertson and T. W. Noonan, *Relativity and Cosmology* (Saunders, Philadelphia, 1968), Chap. 6. This latter treatment of the orbital equation is rather abstract for the intermediate level.

# Simple laboratory demonstration of the Doppler shift of laser light

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The edge of a smoothly rotating turntable was used as a reflecting surface in a Michelson interferometer configuration to demonstrate the Doppler effect on laser light. Attached to the edge of the turntable was reflective tape which greatly enhanced the backscattered light. Spectrum analysis of the detected signal indicated that all of the signal and noise components were contained in a frequency range of about 15% of the central Doppler frequency. As a quantitative test of the method, the wavelength of the laser source, known to be 633 nm, was measured to be $(612 \pm 43)$ nm using an oscilloscope as the output device and $(634 \pm 12)$ nm using a spectrum analyzer for the output. All components that were used with the exception of the spectrum analyzer are commonly available items. The techniques described in this paper provide a quantitative demonstration of the Doppler effect of light and overcome the usual problems caused by motional instabilities associated with the moving mirror.

## I. INTRODUCTION

Although the Doppler shift is a well-understood effect that is usually introduced to undergraduate physics students in elementary courses, its demonstration using light is not a simple matter. Demonstration experiments are usually performed with sound[1–5] or with microwaves.[6–10] The main problem involved in using light is that the moving mirror from which light is reflected cannot usually be moved in a sufficiently smooth and uniform manner to keep the level of noise well below that of the signal. An uncontrollable "wobble" with excursions smaller than an optical wavelength can cause serious noise problems. We have attempted to observe the Doppler effect on light using a mirror mounted on a motor driven screw and also a mirror placed on the car of an air track. Although both these

  