Virtual Reality
and the Unfolding of Higher Dimensions

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ABSTRACT

As virtual/augmented reality evolves, the need for spaces that are responsive to structures independent from three dimensional spatial constraints, become apparent. The visual medium of computer graphics may also challenge these self imposed constraints. If one can get used to how projections affect 3D objects in two dimensions, it may also be possible to compose a situation in which to get used to the variations that occur while moving through higher dimensions. The presented application is an enveloping landscape of concave and convex forms, which are determined by the orientation and displacement of the user in relation to a grid made of tesseracts (cubes in four dimensions). The interface accepts input from tridimensional and four-dimensional transformations, and smoothly displays such interactions in real-time. The motion of the user becomes the graphic element whereas the higher dimensional grid references to his/her position relative to it. The user learns how motion inputs affect the grid, recognizing a correlation between the input and the transformations. Mapping information to complex grids in virtual reality is valuable for engineers, artists and users in general because navigation can be internalized like a dance pattern, and further engage us to maneuver space in order to know and experience.

Keywords: Virtual reality, higher dimensions, tesseract , design, grid.

1. INTRODUCTION

1.1 Why higher dimensions?

Different projections of space can be mapped into an electronic image. Not only tridimensional spaces can be projected to it, but it is possible to use higher dimensions as well to navigate time varying datasets, such as geological data (8) or molecule configurations of sorts. Higher-dimensional worlds may allow more data to be placed around a certain topic in a non-sequential but spatial arrangement which contains more area than 3D space. Instead of navigating the familiar landscape of volumes, space may be attached to the shadows of higher dimensions, which can then be turned around, to stretch or compress their sides to face the user. Such shadows projected to three dimensions can use the distortion of 3D perspective to organize visual hierarchy, making them navigable and more understandable. Such a capability may eventually help cluster information in flexible tree-structures according to particular needs. The use of stereo visualization and real-time computer graphics for accessing higher dimensions may also allow us to integrate the movement abilities of the human body into assimilating complex configurations. The ability to coordinate the different parts of our body may be taken full advantage of, when interacting with higher dimensions.

1.2 Can we navigate four dimensions?

Tridimensional projections can be flattened to the screen of a computer, and so can higher dimensions. But higher dimensions may cause concern because we are used to three dimensions for navigating space since this is how we move around the world with our bodies. If we were two-dimensional beings, and were to be shown a tridimensional object, all we could see from it would be its two dimensional shadow crossing our plane of existence over time. Since we are not two-dimensional beings, but tridimensional ones, it is relevant to wonder if increasing the number of dimensions instead of decreasing them would make sense, and allow us to navigate the tridimensional shadow of a four dimensional virtual

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space. For doing so, we would need to see the warping distortion of four dimensions folding into 3D. The design tool to make space visible in such a way is called grid. A flat space can be shown through a grid of squares; tridimensional space can be revealed through a grid of cubes; four dimensional space, through a grid of tesseracts, and n-dimensional spaces in general can be shown through a grid of hypercubes. If building the shadow of such a grid is possible, then moving through it, or even according to it, is possible as well.

1.3 A malleable structure

If navigating higher dimensions is perfectly possible, then what we may end up seeing is a flexible space that expands and collapses, intersecting itself depending on our position in relation to it. The fourth dimension can cast a shadow in either two dimensional or tridimensional space, but in order to create a higher dimensional world, we may want to base its parameters on parts of our daily experience of the natural world because navigating the tridimensional shadow of a four dimensional image is closer to the natural experience of moving our bodies through space, than the two dimensional shadow of it.

In this presentation, four dimensions is introduced as a form of augmented reality, and compared to natural experience. Then the function and principles behind the use of the grid in designing space will be described. Finally, a grid made from tesseracts (cubes in four-dimensional space) will be used as example for higher dimensional navigation.

2. BACKGROUND: EVERYDAY HIGHER DIMENSIONS

2.1 Augmented reality and natural experience

When we talk about augmented reality, we may think of being able to see what is not visible to the naked eye, because it is not in human scale, or within human vision capabilities. It also comes to mind the fact that some phenomena does not occur simultaneously in nature, so we augment reality where two or more events are fit into a common space that they would not otherwise share. However, higher dimensions can be considered augmented reality, as they expand space beyond the three dimensions we inhabit with our bodies. Yet, human vision goes beyond the human body when perceiving images through natural experience.

2.2 Shadows, imprints and reflections

Images persist beyond their three dimensions in our daily experience when they overlap: through reflection, through shadows, through imprints. They join the dimensions of separate objects through intersection. We are used to them. They allow us to understand the world of dynamic spatial relationships we live in. The graphic artist Escher, who is famous for his work on spatial projections, grew up in the Netherlands a trade area for diamonds –highly reflective objects-. This is said to have affected his work on higher dimensional compositions (5).

Shadows are a two dimensional image of a three dimensional object. When they fall on three dimensional surfaces , they add a dimension to it, melding into a new pattern or form. The intricate foliage pattern of a forest, for example, takes a new dimension when the shadows of the leaves is cast on the leaves themselves.

A reflective tridimensional surface can associate with the tridimensional objects it reflects. The resulting intersection of tridimensional forms is perceived simultaneously by a viewer, and can be clear enough to discern what to look at: whether it is the reflective object, or the reflected object, or even the perfect balance of the overlapping of both images.

An imprint can, in some cases, point to a fourth dimension when an object gets an overlay of the printed image on its surface. The situation may be that of a permanent static reflection on the surface of the printed object.

2.3 From higher dimensions to projections

The previous examples show that we deal with simultaneous imagery in the natural world, that already represent in a way, the presence of other dimensions. As humans, we can make sense of very sophisticated visuals, which reveal complex spatial relationships. To give structure to relationships, designers use the grid (6). A grid can tie in the relative position of visuals that would otherwise appear random. A grid can be visible or transparent, as the alignment of the objects contained in it reveal its existence. Two dimensional grids of squares are used commonly in printed matter, and tridimensional grids of cubes are used in architectural installations. For the purpose of the project presented here, the grid being used is made from tesseracts. The grid of tesseracts, that is, the tridimensional shadows of 4D cubes, can be navigated with tridimensional or four dimensional motion. The intention behind working with a four dimensional grid is to experiment with the projection of a structure that is complex to begin with, and see if it makes sense, while allowing a method for the organization of data.

3. DESIGN: A FOUR DIMENSIONAL STRUCTURE

3.1 Responsive space

In a perspective projection of an electronic image, the way the edges of an object change as the viewer moves around, is directed by the relative position of the user to the three dimensions in which the object exists (4). The image is responsive in the sense that it follows the rules of perspective from the point of view of the user, to evoke a world that is not flat. We could say that the world itself has no motion, but the motion is attached to the point of view of the user.

In the other hand, if we are looking at the tridimensional shadow of a four dimensional object, and we manage to either rotate the object in 4D or move the viewer according to the tridimensional shadow of a four dimensional path, the object will appear to move on its own, or rather, move according to rules other than solely perspective. The reason for that is because four-dimensional rotation goes further than in fewer dimensions, causing displacement on every point of the area being moved. Rotation in two dimensions revolve around a point, and rotation in three dimensions revolve around a line which do not move. In the other hand, four dimensional rotation occurs around a plane, thus for example a tesseract will endlessly turn inside out as it rotates (2).

3.2 Navigating pattern

Being able to animate a four dimensional rotation on a grid made from tesseracts means we can visualize four-dimensional navigation. The distortion that happens to one hypercube, gets extended into pattern when replicated to form a grid (1). The sides of the grid that have the same orientation draw attention to the direction those sides are facing. The resulting image resembles a building carrying its own ghost as the walls intersect each other.

3.3 Body motion

The human body, with its limbs moving in relation to the whole, is in a way a multidimensional structure. When walking, its biped motion led by the attention directed by the head is quite complex. Our bodies allow us to see with motion, and being in a tridimensional virtual world of static blocks captures but a fragment of what we are capable of seeing. It makes sense then to try and connect the motion of the body to a moving pattern of space.

3.4 Grid environments

A grid's main purpose is to relate elements to each other on a common space through alignment and a common unit of measure. The grid reveals the projection of the space that is visible, and defines the thresholds of relevance. By thresholds of relevance I mean, the smallest and the largest visually perceivable unit that defines the resolution of what you can distinguish as a form. The grid from four-dimensional space can be visualized as the tridimensional shadow of the grid of that space. We live a world of scientific visualizations these days, which help us understand how the micro and the macrocosms works in the shape of galaxies, solar systems, ocean streams, bacteria and so on and in, which use grids according to the mathematical relationship being explained. This information tends to be placed in either flat diagrams or tridimensional diagrams that change over time. In design, a composition is constructed based on how the smallest significant part is related to the overall form of a construction. This smallest significant part is then used as a measure to construct a grid, which then becomes a base pattern for defining the overall form of the navigation.

Graphic Design is based on the creation of visual paths that walk you through a given concept. Professor Francisco Méndez (7) would say that steps in this visual path were like the flags you find in a ski trail. This metaphor is very useful for designers, because it helps envision the transfer the 3D paths to the 2D surface of a page or screen. Ultimately this is what we do today in creating programs, which allow us to navigate 2D or 3D worlds.

3.5 Grid and representation

The question of how to represent 3D into 2D was well studied in painting. A whole parade of characters could be placed in front of us following the laws of perspective to dynamically engage us in the sequence of events being represented. Duher and Da Vinci drawings are always used in explaining perspective nowadays. Cubism came about breaking that space over time. Projections of space were heavily used in constructivist and futurist designs of the early 20th century.

Karl Schwitters worked on abstractly inhabitted 3D spaces. The distorted realism of Escher represented higher dimensions mapped to 2D.

3.5 The dynamic grid

A possible example of dynamic grid, or a grid that contains higher dimensions, can be found in the stick charts of the Marshall Islanders who travelled open sea by canoe by means of palm sticks tied with coconut fiber, which map prevailing wave fronts. Shells stand for the relative location of islands while threads indicate when they come into view (9). The map looks like overlapping grids which fullfill the function of relating the dimension of the sea currents and the dimension of island location. The construction of the artifact suggests that since the sea currents have motion and strength associated with them, the distance of the islands would vary depending on the direction of the path being followed. We could say them that the dynamic grid captures an extra dimension of data, in this case, the sea currents.

4. IMPLEMENTATION

4.1 Software

The software is primarily designed to run in the CAVE, a multi-person, room-sized virtual reality system developed at EVL, or similar systems such as the Configurable Wall (C-Wall), a single wall version. The CAVE Library is capable of supporting a number of different VR platforms, including simpler graphics workstations. The CAVE is a 10 x 10 x 10 ft. room constructed of three translucent walls (3). High resolution stereoscopic images are rear-projected onto the walls and the floor and viewed with light-weight LCD stereo glasses to mediate the stereoscopic imagery, or in the case of the C-Wall, viewed with polarized glasses. Attached to the glasses is a location sensor. As the viewer moves within the confines of the CAVE or C-Wall, the correct perspective and stereo projection of the environment are updated and the user may walk around or through virtual objects. The CAVE's or C-Wall's room-sized structure allows for multiple users to move around freely, both physically and virtually. The user interacts with the environment by maneuvering ``the wand", a simple tracked input device containing a joystick and 3 buttons. It is used to navigate around the virtual world, and to manipulate virtual objects within that world.

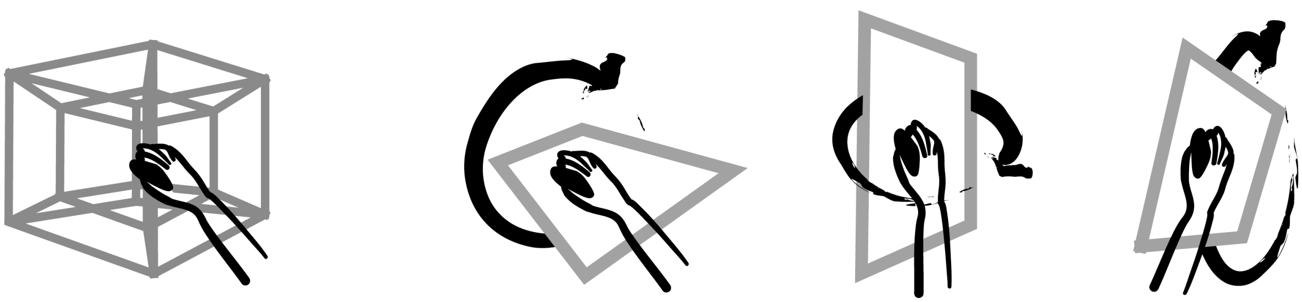

Figure 1: The tracked wand maneuvering to rotate the tesseract in 4D axis (planes).

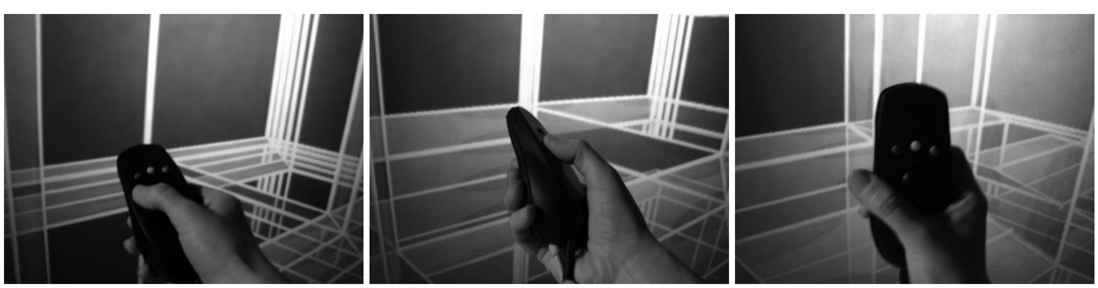

Figure 2: The tracked wand rotates the four dimensional plane, which results in the stretching the cells of the tesseract grid. This pictures depict the application controlled by the wand in front of the C-Wall (Configurable Wall). Even though the application is usually run in stereo with the use of two projectors and polarized glasses, one of the projectors has been turned off for the photograph to be more clear.

In computer graphics, the tridimensional world is flattened into a projection of space of two dimensions. To see a four-dimensional shadow projected onto three dimensions, the tridimensional space requires a means to manipulate the fourth dimension. In order to see this shadow from different four dimensional points of view, we have attached it to the orientation of the hand. When the hand rotates the wand, it does so over the wand's normal. As shown in Fig.1, the hand's orientation represents the planes over which the four dimensions can rotate. This creates a kaleidoscopic effect (Fig.2) where the alignment of the faces of the tesseracts result in expanding and collapsing corridors facing the user (Fig.3 and Fig.4). Let's imagine we are inside a tree, and we can tunnel through all the branches the tree could ever make. As we move, we see the different forks in the road that face us, leaving an opening to a new route.

The projection of the hypercube takes the method used by Pérez-Aguila (8) to draw the shadow of a tesseract in three dimensions. The W value is then set to 0 when the wand is pointing forward with the palm of the hand facing the ground. To form the grid, the center shape is drawn, then the shape is copied and transformed to the locations around the center that share faces or vertices with the central image. This process is repeated outwardly for as many layers as the operating system will allow. To avoid duplicating faces, only half of each tesseract is drawn, with the repeating cells of the grid forming the full shape of it. To create the illusion of an endless grid, the user is moved back to the center when he/she has reached a given distance from the center of the tesseract. To make the grid more apparent, faces are set to 50% transparency. Also, opposing faces can be turned on and off, creating corridors that allow the user to focus on the open directions.

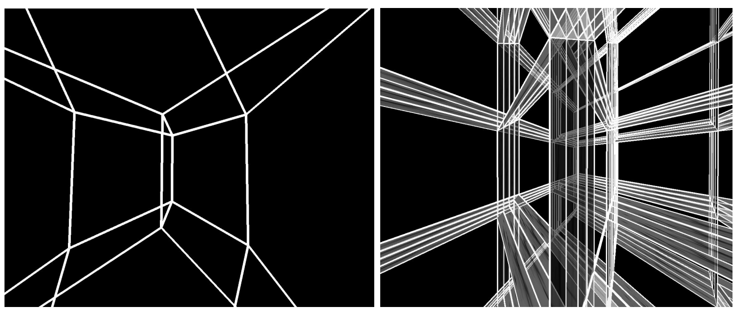

Figure 3: The mage to the left shows a wireframe view from inside a rotating tesseract. The image to the right shows a grid of tesseracts.

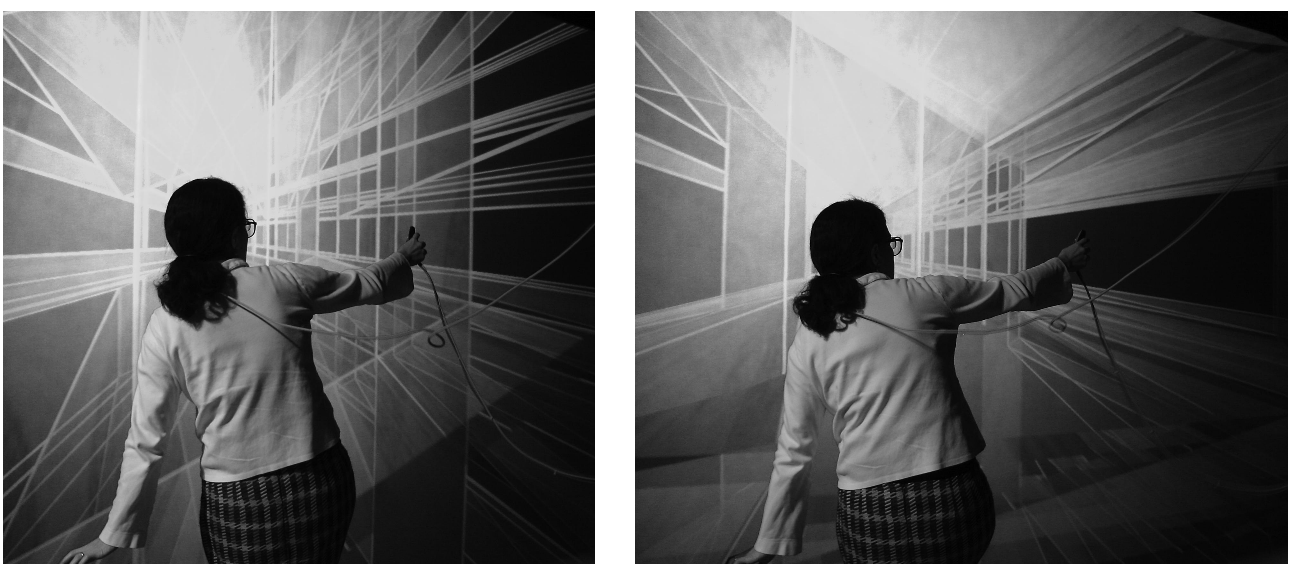

Figure 4: User manipulating the tracked wand in front of a C-Wall, causing the grid of tesseracts to rotate in four dimensions.

4.2 Hardware

The program is run on a Configurable Wall (C-Wall) with includes tracking and stereo projection. It is important to be able to handle a device that can be moved in three dimensions, so the motion can be synchronized with the body. The tracked wand fullfills this purpose by assigning it the control of 4D rotation, besides the 3D navigation.

4.3 Results

The grid made from tesseracts has allowed experimentation with a number of variations to see if one can get used to the motion attached to a four dimensional space when translated to three dimensions. The motion of the grid as a form results on the (repeated) pattern on the grid cells. This pattern works as the stepping stones or highlights of navigation.

The form of the grid can be appreciated in the corridors created by turning off pairs of the opposing faces of the tesseracts (Fig.5). This form is altered in real time in Virtual Reality, when the walls of the corridors self intersect or collapse once they are aligned exactly 90 degrees from the direction faced by the viewer.

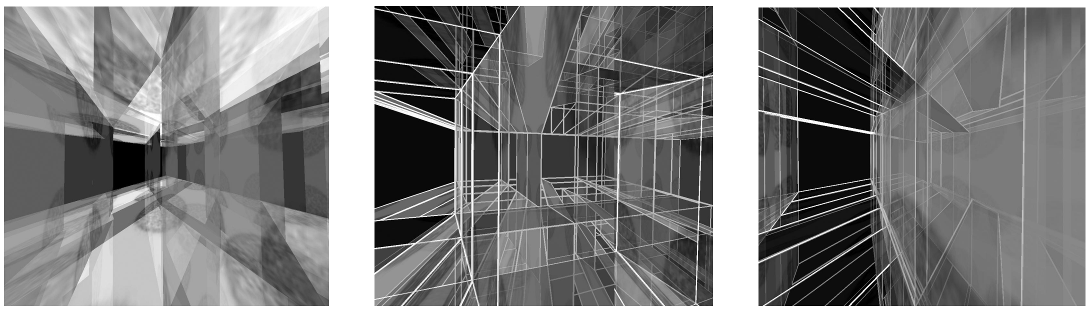

Figure 5: The self-intersecting sides of the tesseract still align with the same sides of the other grid cells, forming corridors when facing the user. This picture depicts the application run by the Simulator on a computer screen. The Simulator shows a wider field of view of applications which use the CAVE library.

The pattern produced by the cells of the grid was enhanced by attaching a 2D texture to the faces of the tesseract. The perspective projection of the virtual world causes the texture to shrink according to distance, as well as more clearly show the direction the face is pointed to. A repeated texture of 1/4 circle in each corner of the square face serves as pointer to the vertices of the four dimensional shadow. An alternate set of three textures of a human face –a left eye, a right eye, and the nose and mouth– was also used (Fig.6).

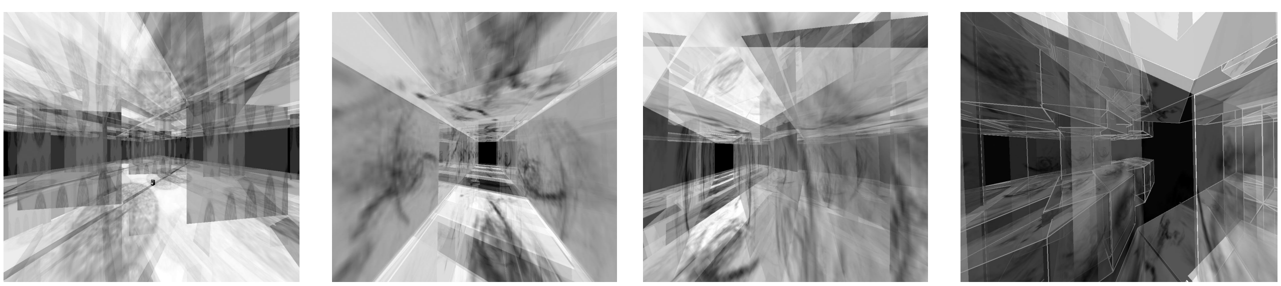

Figure 6: The First image in the row shows a grid of tesseracts with a uniform pattern of a quarter of a circle cornered to the four vertices of a square texture. The second and third image show the grid of tesseracts with the associative pattern of a face, where eyes and mouth are located in faces next to each other. The fourth image shows the same pattern enhanced by the wireframe image of the grid.

In regards to navigation, the simultaneous capability of 3D and 4D motion is supported visually by the tesseract grid of both form of the space and surface pattern.

5. CONCLUSION

A four dimensional grid can be used to augment reality in terms of structure and the relationship among its parts. To make four dimensions visible, a grid of tesseracts was put together in virtual reality (stereo projection with real-time computer graphics). The result is a tridimensional structure of surfaces that self intersect as the grid is rotated in four dimensions. In order to see the grid and enhance thei relative position of its cells, a 2D texture has been assigned to all the surfaces of the tesseracts, and has been set to be semi transparent. This produces a situation of colors and images being superimposed. The resulting image is an overlap of these faces. The same situation of image intersection and superimposition is familiar to us because we deal with similar phenomena in the natural world, specifically when we deal with shadows, reflections and imprints for understanding and navigating space. In the natural world, we can use the pattern and form of what we see for measure and continuity. The same design principles we use for designing 2D and 3D worlds can be used for higher dimensional navigation.

Interaction with a four dimensional structure can be facilitated by using body motion. The limbs of our bodies move as a whole but are capable of moving independently. The degrees of freedom of an arm and hand can be used to represent the projection of a 4D rotation, and relate the resulting form to the natural experience and intention of the user. The shadow of a tesseract grid can be then maneuvered to hide and reveal the surfaces of the structure while maintaining the coherence of a labyrinth structure, of knowing how much change to expect depending on how much the position of the body and the arm has changed. Ultimately, the space becomes an adaptive structure tied to the structure of the body which allows us to inhabit higher dimensions in computer graphics.

The use of higher dimensional grids will be explored further by attaching several three dimensional objects to the tesseract grid, in order to observe the dynamic forms that can be structured with it. Grids constructed from other polytopes in 4D, or hypercubes of n dimensions may be explored as well.

Can higher dimensional spaces be coherent in holding more information than three dimensions? If possible, such an idea would challenge our assumptions about design and architecture altogether, opening up a world related both to language and space, which only exists with body actions. The creation of higher dimensional worlds is a design challenge for people working on electronic imaging as the rendering of three dimensions into two dimensions was a challenge in the past. We may not know the use of ideas like this yet, but computer graphics has made the connection between math and space more apparent. The medium of electronic imaging should be developed, not just as a tridimensional collage, or a metaphor for technology, but as an entirely different space that rely on as many dimensions as are needed to represent a complex situation through a complex structure.

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