Appendix B
Comparison of the Configurations in the 4D-OPP's Defined by Hill and Aguilera & Pérez

We will refer to each configuration by according two ways:

- Through their binary representation: we will have a binary string with 16 bits that indicate each one of the 4D space's hyper-octants. If a bit has as value equal to one then the referred hyper-octant is occupied by a *hyper-box*; otherwise, the hyper-octant is empty. We will assume that a *hyper-box* is initially in the 4D space's hyper-octant defined by the positive parts of the X_1, X_2, X_3 and X_4 axes (or in other words, its coordinates have the values presented in **Table 2.3,** i.e. the vertices with decimal representation). Then, to build a configuration, we must translate a set of *hyper-boxes* to their respective hyper-octant according to their corresponding bit's position:

Bit's Position	**Translation to Apply**				
	T(	**t_1,**	**t_2,**	**t_3,**	**t_4)**
0	T(	-1,	-1,	-1,	-1)
1	T(	1,	-1,	-1,	-1)
2	T(	-1,	1,	-1,	-1)
3	T(	1,	1,	-1,	-1)
4	T(	-1,	-1,	1,	-1)
5	T(	1,	-1,	1,	-1)
6	T(	-1,	1,	1,	-1)
7	T(	1,	1,	1,	-1)
8	T(	-1,	-1,	-1,	1)
9	T(	1,	-1,	-1,	1)
10	T(	-1,	1,	-1,	1)
11	T(	1,	1,	-1,	1)
12	T(	-1,	-1,	1,	1)
13	T(	1,	-1,	1,	1)
14	T(	-1,	1,	1,	1)
15	T(	1,	1,	1,	1)

For example, the string 0110100010000000 defines a configuration with four *hyper-boxes*. The translation to apply to each *hyper-box* (initially collocated in the 4D space's main hyper-octant) is then:

  - Bit 1: T(1,-1,-1,-1).
  - Bit 2: T(-1, 1, -1, -1).

- Bit 4: T(-1, -1, 1, -1).
- Bit 8: T(-1, -1, -1, 1).

By this way we collocate the *hyper-boxes* in their respective hyper-octants. This representation will be used to refer each one of the Hill's 402 configurations.

- Through their adjacencies counting: each configuration is distinguished from the others by using each one of its adjacencies counting (this is the reference used in **Appendix A**). We will consider the number of volume, face, edge and vertex adjacencies. This representation will be used to refer each one of the Aguilera & Pérez's 253 configurations.

Keys:

H: Number of *Hyper-boxes*.

Dist: *Hyper-boxes* distribution in the 4D space.

Conf: Configuration.

C: Number of volume adjacencies.

F: Number of face adjacencies.

E: Number of edge adjacencies.

V: Number of vertex adjacencies.

Hill's Configurations (402)		**Equivalent Pérez & Aguilera's Configurations (253)**					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
0	0000000000000000	0	1	0	0	0	0
1	1000000000000000	1	2	0	0	0	0
2	1100000000000000	2	3	1	0	0	0
	0110000000000000		4	0	1	0	0
	0001100000000000		5	0	0	1	0
	0000000110000000		6	0	0	0	1
3	1110000000000000	3	7	2	1	0	0
	0110100000000000		8	0	3	0	0
	1001100000000000		9	1	1	1	0
	0001100010000000		10	0	1	2	0
	1000000110000000		11	1	0	1	1
	0100000110000000		12	0	2	0	1
4	1111000000000000	4	13	4	2	0	0
	1110100000000000		14	3	3	0	0
	1101100000000000		15	3	2	1	0
	0111100000000000		16	2	3	1	0

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
4	0011110000000000 1001100010000000	4	17	2	2	2	0
	1001011000000000 0110100010000000		18	0	6	0	0
	0101100010000000		19	1	3	2	0
	0011010010000000		20	1	2	3	0
	0001011010000000		21	0	3	3	0
	1100000110000000		22	2	2	1	1
	0110000110000000		23	0	5	0	1
	1001000110000000 0001010110000000		24	2	1	2	1
	0001100110000000		25	1	2	2	1
	0000001111000000		26	2	0	2	2
	0000011001100000		27	0	4	0	2
	0000100101100000		28	0	2	4	0
5	1111100000000000	5	29	5	4	1	0
	1011110000000000 1101100010000000		30	4	4	2	0
	1101011000000000		31	3	6	1	0
	1110100010000000		32	4	6	0	0
	0111100010000000		33	2	6	2	0
	1011010010000000		34	3	4	3	0
	0011110010000000 0101011010000000		35	2	4	4	0
	1001011010000000		36	1	6	3	0
	1110000110000000		37	3	5	1	1
	1101000110000000 0101010110000000		38	4	3	2	1
	0110100110000000		39	0	9	0	1
	1001100110000000 1001010110000000 0011010110000000 0001011110000000		40	3	3	3	1
	0101100110000000		41	2	5	2	1
	0001101011000000 0000110101100000		42	2	3	4	1
	0001011011000000		43	1	5	3	1
	1000001111000000		44	3	2	3	2
	1000011001100000		45	0	8	0	2
	0100011001100000		46	2	4	2	2
	1000100101100000		47	1	4	5	0
	1000000101101000		48	0	6	4	0

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
6	1111110000000000	6	49	7	6	2	0
	1111011000000000 1111100010000000		50	6	7	2	0
	0111111000000000 1111010010000000		51	6	6	3	0
	1011110010000000		52	5	6	4	0
	0111110010000000 1101011010000000		53	4	7	4	0
	0111011010000000		54	4	6	5	0
	1111000110000000		55	6	6	2	1
	1110100110000000		56	4	9	1	1
	1101100110000000 0111010110000000		57	5	6	3	1
	0111100110000000		58	3	9	2	1
	1101010110000000		59	6	5	3	1
	1011010110000000 0101011110000000		60	5	5	4	1
	0011110110000000 1001011110000000 0110101011000000 1001101011000000 0101011011000000 1000000111101000		61	4	6	4	1
	0011110011000000 1100100101100000		62	3	6	6	0
	0101101011000000 0100110101100000		63	4	5	5	1
	1001011011000000		64	2	9	3	1
	0011011011000000 1000110101100000		65	3	6	5	1
	1100001111000000 1010001111000000		66	5	4	4	2
	0110001111000000 0010101111000000 0110011001100000 0100111001100000 0000111101100000		67	4	5	4	2
	0001101111000000 0000110111100000		68	4	4	5	2
	1100011001100000		69	3	8	2	2
	1001011001100000		70	0	13	0	2
	0001111001100000		71	2	8	3	2

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
6	0110100101100000 1100000101101000	6	72	2	7	6	0
	0010110101100000		73	3	5	6	1
	0000011111100000		74	4	4	4	3
	0001011001101000		75	0	12	0	3
	0110000101101000		76	2	6	7	0
	1001000101101000		77	1	9	4	1
	0001100101101000		78	2	6	6	1
7	1111111000000000	7	79	9	9	3	0
	1111110010000000		80	8	9	4	0
	1111011010000000		81	7	9	5	0
	0111111010000000 1011110011000000		82	6	9	6	0
	1111100110000000		83	7	10	3	1
	1111010110000000 1110101011000000		84	8	8	4	1
	1011110110000000 1101011110000000 0111011110000000 1101101011000000		85	7	8	5	1
	0111110110000000 1101011011000000		86	6	10	4	1
	0111101011000000 0111011011000000 1100110101100000 1100000111101000		87	6	8	6	1
	1011011011000000		88	5	10	5	1
	0011111011000000 1010110101100000 0110110101100000 0111000101101000		89	5	8	7	1
	1110001111000000 1010101111000000		90	7	7	5	2
	0110101111000000 0101111001100000 1000111101100000 1001000111101000		91	5	9	5	2
	1001101111000000 0011101111000000 0110111001100000 0100111101100000 1000110111100000		92	6	7	6	2

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
7	1110011001100000 1100111001100000	7	93	6	9	4	2
	1101011001100000		94	4	13	2	2
	1001111001100000		95	3	13	3	2
	1110100101100000		96	5	9	7	0
	1001110101100000 1101000101101000		97	4	10	6	1
	0001111011100000 0011010101101000		98	4	9	6	2
	0010110111100000 0001100111101000		99	5	7	7	2
	1000011111100000		100	5	8	5	3
	0100011111100000 0000111111100000		101	6	6	6	3
	1001011001101000		102	0	18	0	3
	0101011001101000		103	3	12	3	3
	1110000101101000		104	4	9	8	0
	0110100101101000		105	3	9	9	0
	1001100101101000		106	3	10	7	1
	0101100101101000 1000011110011000		107	4	8	8	1
	1001010101101000		108	2	13	4	2
8	1111111100000000	8	109	12	12	4	0
	1111111010000000 1111110011000000		110	10	12	6	0
	1111110110000000		111	10	12	5	1
	1111011110000000 1111101011000000		112	10	11	6	1
	0111111110000000 1111011011000000		113	9	12	6	1
	1011111011000000 1101110101100000		114	8	12	7	1
	0111111011000000 1110110101100000 1110000111101000		115	8	11	8	1
	1111001111000000		116	10	10	6	2
	1110101111000000 1110111001100000		117	9	11	6	2
	1101101111000000 1011101111000000 1100110111100000		118	9	10	7	2

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
8	0111101111000000 0111111001100000 1100111101100000 1101000111101000	8	119	8	11	7	2
	0011111111000000 0110111101100000 0110111011100000 1010110111100000 0111000111101000 1000111110011000		120	8	10	8	2
	1111011001100000		121	8	14	4	2
	1101111001100000		122	7	14	5	2
	1111100101100000		123	8	12	8	0
	1011110101100000 1111000101101000		124	7	12	8	1
	1001111101100000 1001111011100000 1101010101101000		125	6	14	6	2
	0101111011100000 0110110111100000 0111010101101000 1001100111101000		126	7	11	8	2
	0011110111100000 0101100111101000 0101011110011000		127	7	10	9	2
	1100011111100000 1000111111100000		128	8	10	7	3
	0110011111100000 0100111111100000 0001111110011000		129	8	9	8	3
	1001011111100000 0111011001101000 0101011101101000 1001010111101000		130	6	13	6	3
	0001111111100000 0011010111101000		131	7	10	8	3
	0000111111110000 0001101111011000		132	8	8	8	4
	1101011001101000		133	4	18	3	3
	1110100101101000 1110011100011000		134	6	12	10	0
	1101100101101000 0110100111101000		135	6	12	9	1

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
8	0111100101101000 1100011110011000	8	136	6	11	10	1
	1011010101101000		137	5	14	7	2
	0011110101101000 1001011110011000		138	6	11	9	2
	1001011101101000		139	3	18	4	3
	0001011111101000		140	6	12	6	4
	0110011110011000		141	5	12	10	1
	0010011111011000		142	6	10	10	2
	1000011101111000		143	4	14	8	2
	1100001100111100		144	4	12	12	0
	0110100110010110		145	0	24	0	4
9	0110101010010111	9	146	4	24	4	4
	1010011010010111 0111100001100111		147	7	15	13	1
	0110011010010111		148	6	19	8	3
	1001011010010111		149	7	16	11	2
	0001111010010111		150	8	14	12	2
	1010100110010111		151	8	15	10	3
	0110100110010111		152	8	15	12	1
	1011100000011111 1111000000011111		153	9	14	9	4
	0111100000011111		154	7	18	7	4
	1101001000011111 1110011000010111		155	9	13	11	3
	1110000100011111 1100101010010111		156	10	15	10	1
	0110001010011111 0010111010010111		157	7	19	7	3
	0001011010011111		158	9	14	11	2
	0110000110011111		159	8	16	10	2
	0010100110011111		160	9	16	9	2
	0001100110011111 0011000110011111		161	9	15	11	1
	0110010000111111 1100010000111111 1001000110011111 1011000010011111 0111001000011111		162	8	19	6	3

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
9	1001010000111111 1010000110011111 0111000010011111 0110111000010111	9	163	10	12	10	4
	0001110000111111 0101010000111111		164	9	15	9	3
	1100000100111111 0101001010011111 1001001010011111 1000111010010111		165	10	13	10	3
	0100100100111111		166	10	15	8	3
	1000010100111111 1000100100111111 0011001010011111 0011111000010111		167	11	13	9	3
	1000001001111111 0010100100111111		168	10	14	10	2
	0100001001111111 0010100001111111 1000100001111111 0010010100111111		169	10	16	8	2
	0000101001111111 0001010100111111		170	11	15	9	1
	0000011001111111		171	11	16	7	2
	1000000101111111 0100001100111111		172	11	14	9	2
	0000100101111111		173	12	14	8	2
	0000001101111111		174	12	15	8	1
	0000000111111111		175	13	15	7	1
10	1110011010010111	10	176	8	24	8	5
	0110111010010111		177	8	25	8	4
	1001111010010111		178	10	18	14	3
	1110100110010111		179	9	21	12	3
	1111100000011111		180	10	18	15	2
	1101001010011111		181	12	16	12	5
	1001011010011111 0011111010010111		182	10	20	11	4
	1110000110011111		183	11	17	14	3
	0110100110011111		184	10	21	11	3
	0011100110011111		185	10	19	14	2
	1110010000111111 1111001000011111		186	12	16	13	4

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
10	1001110000111111 1101010000111111 1001100110011111 1011000110011111 1111000010011111	10	187	11	18	13	3
	0011110000111111 0101110000111111		188	11	18	14	2
	1100100100111111 0111001010011111		189	12	18	13	2
	0110100100111111		190	11	20	10	4
	1010010100111111 1011001010011111		191	12	17	13	3
	1100001100111111 0011011010011111		192	13	18	12	2
	1100001001111111 0110100001111111 1001010100111111 0110010100111111 1010100100111111 0111111000010111		193	12	18	12	3
	0100101001111111 1010100001111111		194	11	21	10	3
	0010101001111111		195	12	19	12	2
	1000011001111111		196	12	21	9	3
	0010011001111111 1000101001111111		197	12	17	12	4
	0001011001111111		198	13	16	12	4
	0000111001111111		199	13	17	12	3
	1000100101111111		200	13	18	11	3
	1000001101111111 0010100101111111		201	14	17	11	3
	0100001101111111		202	14	18	10	3
	1000000111111111 0000101101111111		203	14	18	11	2
	0000100111111111 0000011101111111		204	14	19	10	2
	0000001111111111		205	15	18	10	2
11	0111111010010111	11	206	12	26	12	5
	0111011010011111		207	12	24	16	3
	1011100110011111		208	12	27	12	4
	0111100110011111		209	13	23	15	4
	0111110000111111		210	13	22	17	3
	1110100100111111		211	13	24	15	3

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
11	1110010100111111 1111001010011111	11	212	15	20	15	5
	1010011001111111		213	14	22	14	5
	0110011001111111 0110101001111111 1100101001111111 1110100001111111		214	14	21	16	4
	1001011001111111		215	14	23	14	4
	0010111001111111 1010101001111111		216	14	22	16	3
	0001111001111111		217	14	24	14	3
	0110100101111111		218	15	22	15	3
	1100001101111111 1010100101111111		219	15	21	15	4
	0100101101111111		220	15	23	13	4
	1000011101111111		221	16	22	14	3
	0001011101111111		222	16	24	12	3
	0010100111111111		223	15	24	13	3
	0100001111111111 0010011101111111		224	16	21	14	4
	0000011111111111		225	17	22	13	3
12	1111011010011111	12	226	16	26	20	4
	1111100110011111		227	16	28	16	6
	1111110000111111		228	16	27	19	4
	1110011001111111		229	16	29	16	5
	0110111001111111 1110101001111111		230	16	30	16	4
	1001111001111111		231	18	24	18	6
	0011111001111111		232	17	26	18	5
	1110100101111111		233	17	26	19	4
	1100101101111111		234	17	27	18	4
	1010011101111111		235	18	25	18	5
	0110100111111111 1001011101111111		236	18	26	17	5
	1100001111111111 0110011101111111		237	18	26	18	4
	1000011111111111		238	18	27	17	4
	0010011111111111		239	19	26	17	4
	0001011111111111		240	19	27	16	4
	0000111111111111		241	20	26	16	4
13	1011111001111111	13	242	20	31	22	5
	0111111001111111		243	20	32	20	6

Hill's Configurations		Equivalent Pérez & Aguilera's Configurations					
H	**Dist**	**H**	**Conf**	**C**	**F**	**E**	**V**
13	1110011101111111	13	244	20	33	20	5
	0110011111111111		245	21	30	21	6
	1001011111111111		246	21	31	21	5
	0001111111111111		247	22	31	20	5
14	1111111001111111	14	248	24	36	24	7
	1110011111111111		249	24	36	25	6
	1001111111111111		250	24	37	24	6
	0011111111111111		251	25	36	24	6
15	0111111111111111	15	252	28	42	28	7
16	1111111111111111	16	253	32	48	32	8